Transverse momentum spectra of particles in pp and heavy-ion collisions with the Tsallis statistics

A.S. Parvan

BLTP, JINR, Dubna

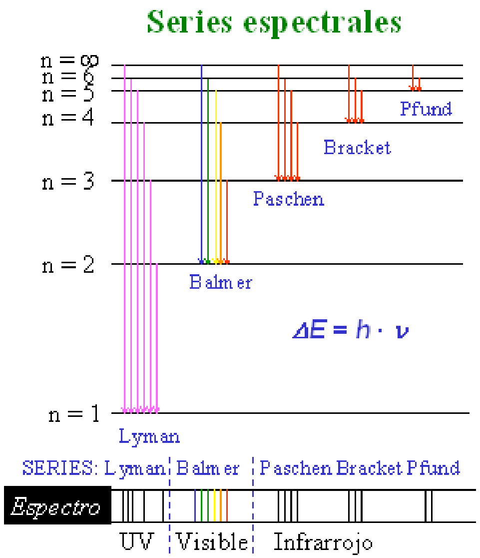
DFT, IFIN-HH, Bucharest

JINR

IFIN-HH

Heavy-Ion Collisions Thermometer

Electromagnetic Interactions
(Atomic Processes)



Black-body radiation

$$\langle n_{\vec{p}} \rangle = \frac{1}{e^{\frac{\varepsilon-\mu}{T}} - 1}$$

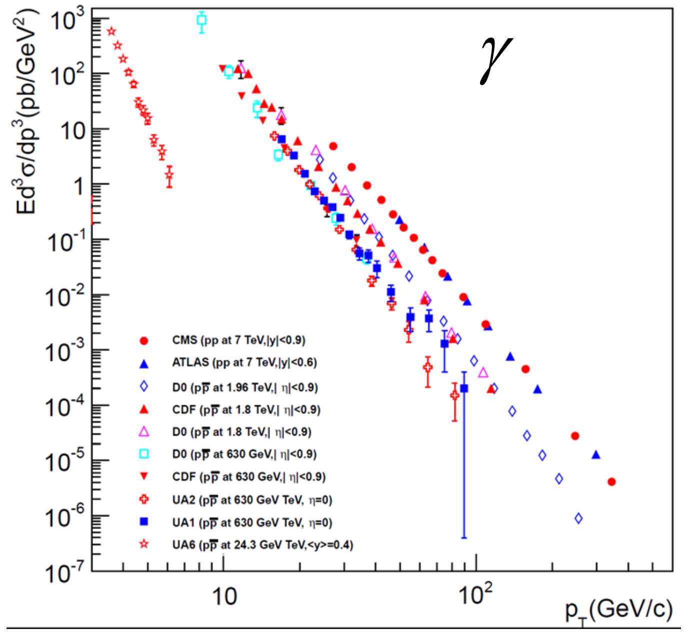
Photon Thermometer:

- Bose-Einstein distribution
- Boltzmann-Gibbs statistics
- Exponential functions

- It was discovered the quantum physics (radiation is transmitted in the form of quanta)

Strong Interactions
(Inelastic Nuclear Reactions)

C. Patrignani et al. (PDG), Chin. Phys. C, 40, 100001 (2016)



HIC and pp: Spectra of hadrons, leptons and gamma quanta

$$\langle n_{\vec{p}} \rangle = \frac{1}{e^{\frac{m_T\cosh y-\mu}{T}} + \eta}$$

- Boltzmann-Gibbs statistics failed to describe the transverse momentum spectra of particles in HIC and pp collisions
- These spectra follow power-law distributions

Is the power-law distribution a sign of new physics?

Phenomenological models for large transverse momentum particles

1) Independent emission model with total momentum conservation:

- A basic power-law pT dependence is a consequence of momentum conservation

$$f(p_T) = \frac{1}{\pi a^2}\left(1+\frac{p_T^2}{a^2}\right)^{-2}$$ -ansatz

C. Michael, L. Vanryckeghem, J. Phys. G: Nucl. Phys. 3 (1977) L151

2) Hydro-inspired models:

a. Blast-wave model of Siemens and Rasmussen: (the spherically symmetric flow)

$$\frac{d^3N}{d^3p} = \frac{V}{(2\pi)^3} e^{-\frac{1}{T}(E\gamma-\mu)}\left[\left(1+\frac{T}{\gamma E}\right)\frac{\sinh a}{a} - \frac{T}{\gamma E}\cosh a\right], \qquad \gamma = \left(1-v^2\right)^{-1/2}, \quad a = \frac{\gamma v p}{T}, \quad E = \sqrt{\vec{p}^{\,2}+m^2}$$

v- the radial collective velocity (radial flow)

P.J. Siemens, J.O. Rasmussen, Phys. Rev. Lett. 42 (1979) 880

b. Blast-wave model of Schnedermann, Sollfrank, and Heinz: (expansion with constant transverse flow)

$$\frac{d^3N}{d^3p} = Am_T K_1\left(\frac{m_T\cosh\rho}{T}\right) I_0\left(\frac{p_T\sinh\rho}{T}\right), \qquad \rho = \tanh^{-1}\beta_T$$

β_T - *the transverse flow velocity*

E. Schnedermann, J. Sollfrank, U.W. Heinz, Phys. Rev. C 48 (1993) 2462

3) Nonequilibrium statistical approach: Relativistic diffusion model (for rapidity distributions)

Fokker-Planck equation:
$$\frac{\partial}{\partial t}R_k(y,t) = \frac{1}{\tau_y}\frac{\partial}{\partial y}\left[(y-y_{eq})R_k(y,t)\right] + \frac{\partial^2}{\partial y^2}\left[D_y^k R_k(y,t)\right]$$

k=1,2,3 - three sources

τ_y - *the rapidity relaxation time*

G. Wolschin, J. Phys. G: Nucl. Part. Phys. 40 (2013) 045104

Phenomenological models for large transverse momentum particles

4) Statistical models:

a. Hagedorn's theory: (exponential decay)

$\sqrt{s} < 6$ GeV

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} = cp_T \int_0^\infty dp_L e^{-\frac{1}{T}\sqrt{p_L^2+m_T^2}}, \quad m_T = \sqrt{p_T^2 + m^2}$$

T - the Hagedorn temperature

R. Hagedorn, Suppl. Nuovo Cim. 3 (1965) 147

b. Tsallis-factorized distributions: (power-law distribution)

Definition of Bediaga, Curado, de Miranda, and Beck:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} = cp_T \int_0^\infty dp_L \left[1-(1-q)\frac{\sqrt{p_L^2+m_T^2}}{T}\right]^{\frac{q}{1-q}}$$

I. Bediaga, E.M.F. Curado and J.M. de Miranda, Phys. A 286 (2000) 156

C. Beck, Phys. A 286 (2000) 164

Definition of Cleymans et al.:

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\left[1-(1-q)\frac{m_T\cosh y - \mu}{T}\right]^{\frac{q}{1-q}}$$

- They are equivalent!

c. Erlang distribution: (m sources)

$$f(p_T) = \frac{p_T^{m-1}}{\Gamma(m)\langle p_T\rangle^m} e^{-\frac{p_T}{\langle p_T\rangle}}$$

F.-H. Liu et al., Eur. Phys. J. A 50, 94 (2014)

Tsallis-factorized statistics

Boltzmann-Gibbs Statistics

Ideal Gas: (Maxwell-Boltzmann particles)

- **Quantities obtained from the partition function:**

$$\langle n_{\vec{p}\sigma}\rangle = e^{-\frac{\varepsilon_{\vec{p}}-\mu}{T}}$$

$$S = -\sum_{\vec{p}\sigma}\left[\langle n_{\vec{p}\sigma}\rangle \ln\langle n_{\vec{p}\sigma}\rangle - \langle n_{\vec{p}\sigma}\rangle\right]$$

$$\langle N\rangle = \sum_{\vec{p}\sigma}\langle n_{\vec{p}\sigma}\rangle, \qquad E = \sum_{\vec{p}\sigma}\langle n_{\vec{p}\sigma}\rangle \varepsilon_{\vec{p}}$$

$$\Omega = E - TS - \mu\langle N\rangle = T\sum_{\vec{p}\sigma}\langle n_{\vec{p}\sigma}\rangle\left[\ln\langle n_{\vec{p}\sigma}\rangle - 1 + \frac{\varepsilon_{\vec{p}}-\mu}{T}\right]$$

- **Mean occupation numbers obtained from the maximization of the thermodynamic potential:**

$$\frac{\partial\Omega}{\partial\langle n_{\vec{p}\sigma}\rangle} = 0 \longrightarrow \langle n_{\vec{p}\sigma}\rangle = e^{-\frac{\varepsilon_{\vec{p}}-\mu}{T}}$$

The constrained maximization of the entropy of the ideal gas of the Boltzmann-Gibbs statistics with respect to the single-particle distribution function leads to the same results of the Boltzmann-Gibbs statistics obtained from the partition function

generalization

Tsallis-factorized Statistics

Ideal Gas: (Maxwell-Boltzmann particles)

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

- **Quantities given by definition:**

q_c − real parameter

$$S = -\sum_{\vec{p}\sigma}\left[f_{\vec{p}\sigma}^{q_c}\ln_{q_c} f_{\vec{p}\sigma} - f_{\vec{p}\sigma}\right], \qquad f_{\vec{p}\sigma}^{q_c} \equiv \langle n_{\vec{p}\sigma}\rangle$$

$$\langle N\rangle = \sum_{\vec{p}\sigma} f_{\vec{p}\sigma}^{q_c}, \qquad \ln_{q_c}(x) = \frac{x^{1-q_c}-1}{1-q_c}$$

$$E = \sum_{\vec{p}\sigma} f_{\vec{p}\sigma}^{q_c}\varepsilon_{\vec{p}}, \qquad 0 < q_c < \infty$$

$$\Omega = E - TS - \mu\langle N\rangle = T\sum_{\vec{p}\sigma} f_{\vec{p}\sigma}^{q_c}\left[q_c \ln_{q_c} f_{\vec{p}\sigma} - 1 + \frac{\varepsilon_{\vec{p}}-\mu}{T}\right]$$

- **Mean occupation numbers obtained from the maximization of the thermodynamic potential:**

$$\frac{\partial\Omega}{\partial f_{\vec{p}\sigma}} = 0 \longrightarrow \langle n_{\vec{p}\sigma}\rangle = \left[1 + (q_c - 1)\frac{\varepsilon_{\vec{p}}-\mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

The Tsallis-factorized statistics will be true only in the case when its mean occupation numbers obtained from the constrained maximization of the Tsallis-factorized entropy of the ideal gas (generalized from the Boltzmann-Gibbs entropy of the ideal gas) with respect to the single-particle distribution function will coincide with the mean occupation numbers of the full Tsallis statistics (Let us verify!)

(Full) Tsallis statistics

1.) Definition:

C. Tsallis, J. Stat. Phys. 52, 479 (1988)

Boltzmann-Gibbs Statistics	Tsallis-1 Statistics	Tsallis-2 Statistics	
$S = -\sum_i p_i \ln p_i , \quad q = 1$	$S = -\sum_i \frac{p_i - p_i^q}{1-q} \quad 0 < q < \infty$	$S = -\sum_i \frac{p_i - p_i^{q_c}}{1-q_c} \quad 0 < q_c < \infty$	-entropy
$\sum_i p_i = 1$	$\sum_i p_i = 1$	$\sum_i p_i = 1$	-norm equation
$E = \sum_i p_i E_i$	$E = \sum_i p_i E_i$	$E = \sum_i p_i^{q_c} E_i$	-energy
$\langle N \rangle = \sum_i p_i N_i$	$\langle N \rangle = \sum_i p_i N_i$	$\langle N \rangle = \sum_i p_i^{q_c} N_i$	-number of particles
Standard expectation values	Standard expectation values	Generalized expectation values	

$p_i -$ probability of *i*-th microstate of the system

2.) Legendre Transform:

$$\Omega = E - TS - \mu \langle N \rangle$$

3.) Thermodynamic potential:

B-G

$$\Omega = T \sum_i p_i \left[\ln p_i + \frac{E_i - \mu N_i}{T} \right]$$

T-1

$$\Omega = T \sum_i p_i \left[\frac{1 - p_i^{q-1}}{1-q} + \frac{E_i - \mu N_i}{T} \right]$$

T-2

$$\Omega = T \sum_i p_i^{q_c} \left[\frac{p_i^{1-q_c} - 1}{1-q_c} + \frac{E_i - \mu N_i}{T} \right]$$

(Full) Tsallis statistics

4.) Constrained Local Extrema of the Thermodynamic Potential: (Method of Lagrange Multipliers)

$$\Phi = \Omega - \lambda \phi,$$ - *Lagrange function*

$$\phi = \sum_i p_i - 1 = 0,$$ - *constrained equation*

$$\frac{\partial \Phi}{\partial p_i} = 0$$ - *extremization*

5.) Many-body distribution function: (Probabilities of Microstates of the System)

Boltzmann-Gibbs Statistics	Tsallis-1 Statistics	Tsallis-2 Statistics
- many-body distribution function: $p_i = \frac{1}{Z}\exp\left(-\frac{E_i - \mu N_i}{T}\right)$	$p_i = \left[1 + \frac{q-1}{q}\frac{\Lambda - E_i + \mu N_i}{T}\right]^{\frac{1}{q-1}}$	$p_i = \frac{1}{Z}\left[1 - (1-q_c)\frac{E_i - \mu N_i}{T}\right]^{\frac{1}{1-q_c}}$
- norm function (partition function):	A.S. P., Eur. Phys. J. A 51 (2015) 108	A.S. P., Eur. Phys. J. A 53 (2017) 53
$Z = \sum_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)$	$\sum_i \left[1 + \frac{q-1}{q}\frac{\Lambda - E_i + \mu N_i}{T}\right]^{\frac{1}{q-1}} = 1$	$Z = \sum_i \left[1 - (1-q_c)\frac{E_i - \mu N_i}{T}\right]^{\frac{1}{1-q_c}}$
$\Omega = -T \ln Z = \lambda - T$	$\Lambda = \lambda - T$	$-Tq_c \frac{Z^{1-q_c} - 1}{1-q_c} = \lambda - T$

Ultrarelativistic Ideal Gas in the Tsallis-2 Statistics $q_c > 1$

1) Exact Tsallis-2 Statistics:

cut-off parameter

A.S.P., Eur. Phys. J. A 53 (2017) 53

$$Z = 1 + \sum_{N=1}^{N_0} \frac{\tilde{\omega}^N}{N!} \frac{\Gamma\left(\frac{1}{q_c-1} - 3N\right)}{(q_c-1)^{3N}\,\Gamma\left(\frac{1}{q_c-1}\right)} \left[1 - (q_c-1)\frac{\mu N}{T}\right]^{\frac{1}{1-q_c}+3N}$$

- The partition function is divergent
- The series should be truncated
- The physical terms are only preserved

$$\langle n_{\vec{p}\sigma}\rangle = \frac{1}{Z^{q_c}}\left[1 + (q_c-1)\frac{\varepsilon_{\vec{p}} - \mu}{T}\right]^{\frac{q_c}{1-q_c}} + \frac{1}{Z^{q_c}} \sum_{N=1}^{N_0} \frac{\tilde{\omega}^N}{N!} \frac{\Gamma\left(\frac{q_c}{q_c-1} - 3N\right)}{(q_c-1)^{3N}\,\Gamma\left(\frac{q_c}{q_c-1}\right)} \left[1 + (q_c-1)\frac{\varepsilon_{\vec{p}} - \mu(N+1)}{T}\right]^{\frac{q_c}{1-q_c}+3N}$$

$$\tilde{\omega} = \frac{gVT^3}{\pi^2}$$

2) Zeroth term approximation of the Tsallis-2 Statistics: (The terms with $N \geq 1$ in the series given above are deleted by hand)

$N_0 = 0, \quad Z = 1$

$$\langle n_{\vec{p}\sigma}\rangle = \left[1 + (q_c-1)\frac{\varepsilon_{\vec{p}} - \mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

- The zeroth term approximation is valid only for large deviations of q from unity

3) Tsallis-factorized Statistics:

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

$$\langle n_{\vec{p}\sigma}\rangle = \left[1 + (q_c-1)\frac{\varepsilon_{\vec{p}} - \mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

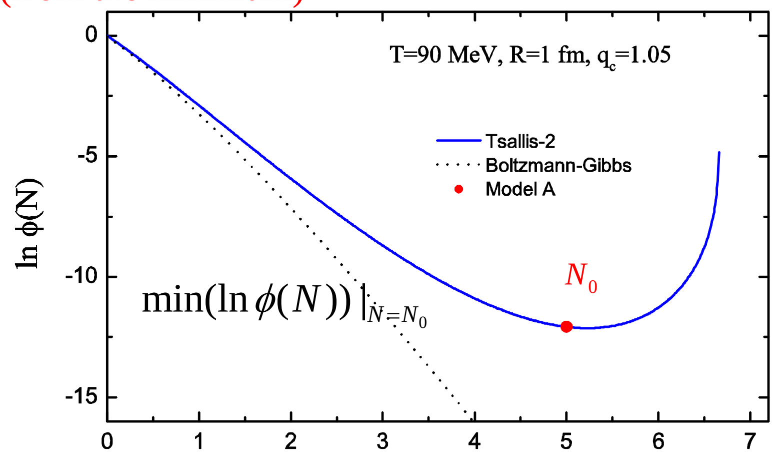
- The Tsallis-factorized statistics is not equivalent to the Tsallis-2 statistics

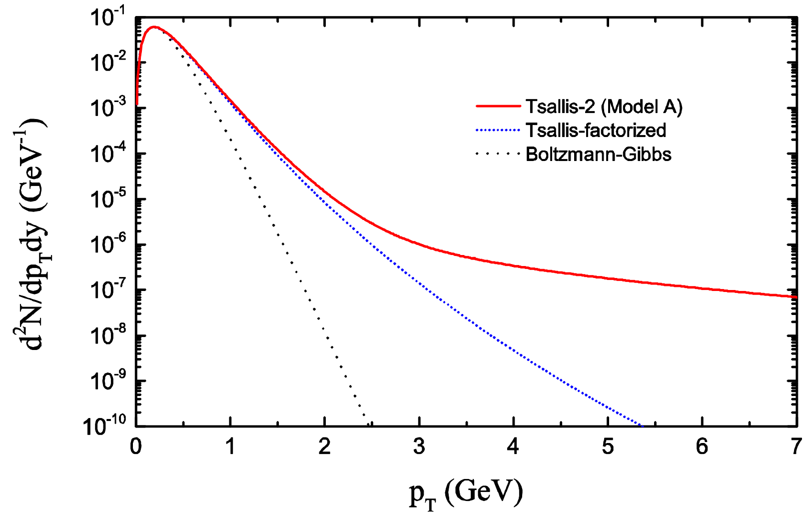
- The constrained maximization of the Tsallis-factorized entropy of the ideal gas (generalized from the Boltzmann-Gibbs entropy of the ideal gas) with respect to the single-particle distribution function does not lead to the true results for the Tsallis-2 statistics

- The Tsallis-factorized distribution is equivalent to the distribution of the Tsallis-2 statistics in the zeroth term approximation
- The Tsallis-factorized distribution does not recover the exact distribution of the Tsallis-2 statistics

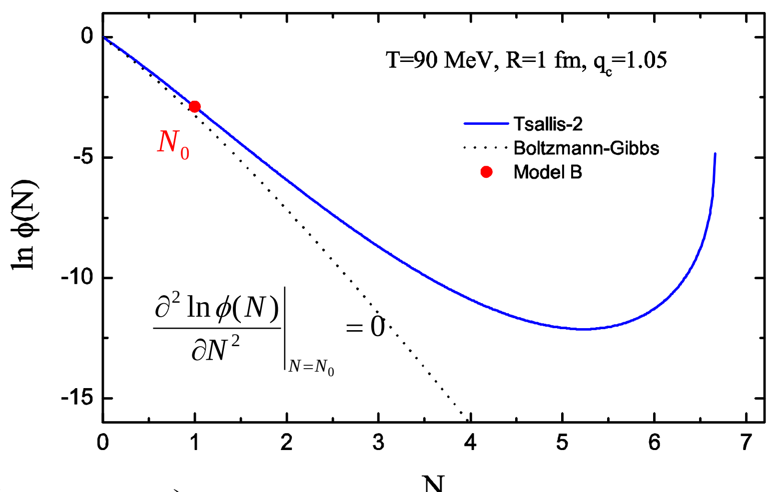
The cut-off parameter of the Tsallis-2 statistics

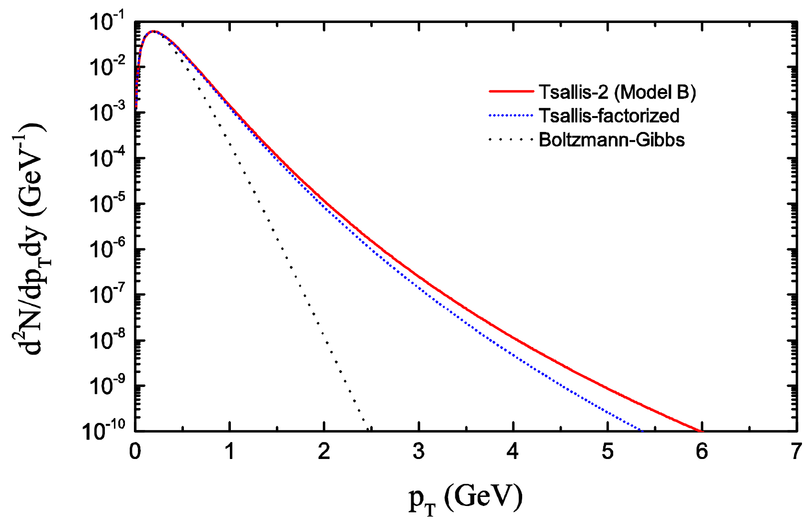
1. Model A: (from the minimum) $\ln\phi(N)$





2. Model B: (from the inflection point)





$$Z=\sum_{N=0}^{N_0}\phi(N),$$

$$\phi(N)=\frac{\tilde{\omega}^N}{N!}\frac{\Gamma\left(\frac{1}{q_c-1}-3N\right)}{(q_c-1)^{3N}\,\Gamma\left(\frac{1}{q_c-1}\right)}\left[1-(q_c-1)\frac{\mu N}{T}\right]^{\frac{1}{1-q_c}+3N}$$ *-Tsallis-2 statistics*

$$\phi(N)=\frac{\tilde{\omega}^N}{N!}e^{\frac{\mu N}{T}}$$ *-Boltzmann-Gibbs statistics*

Ultrarelativistic Ideal Gas in the Tsallis-1 Statistics $q<1$

1) Exact Tsallis-1 Statistics:

A.S.P., Eur. Phys. J. A 53 (2017) 53

$$\left[1+\frac{q-1}{q}\frac{\Lambda}{T}\right]^{\frac{1}{q-1}}+\sum_{N=1}^{N_0}\frac{\tilde{\omega}^N}{N!}\frac{\left(\frac{q}{1-q}\right)^{3N}\Gamma\left(\frac{1}{1-q}-3N\right)}{\Gamma\left(\frac{1}{1-q}\right)}\left[1+\frac{q-1}{q}\frac{\Lambda+\mu N}{T}\right]^{\frac{1}{q-1}+3N}=1$$ - *The norm equation* $\quad \tilde{\omega}=\frac{gVT^3}{\pi^2}$

$$\langle n_{\vec{p}\sigma}\rangle=\left[1+\frac{q-1}{q}\frac{\Lambda-\varepsilon_{\vec{p}}+\mu}{T}\right]^{\frac{1}{q-1}}+\sum_{N=1}^{N_0}\frac{\tilde{\omega}^N}{N!}\frac{\left(\frac{q}{1-q}\right)^{3N}\Gamma\left(\frac{1}{1-q}-3N\right)}{\Gamma\left(\frac{1}{1-q}\right)}\left[1+\frac{q-1}{q}\frac{\Lambda-\varepsilon_{\vec{p}}+\mu(N+1)}{T}\right]^{\frac{1}{q-1}+3N}$$

2) Zeroth term approximation of the Tsallis-1 Statistics: (The terms with $N\geq 1$ in the series given above are deleted by hand)

$N_0=0, \quad \Lambda=0$

$$\langle n_{\vec{p}\sigma}\rangle=\left[1-\frac{q-1}{q}\frac{\varepsilon_{\vec{p}}-\mu}{T}\right]^{\frac{1}{q-1}}$$

- The zeroth term approximation is valid only for $N_0=0$ at large deviations of q from the unity

$q\to 1/q_c$

3) Tsallis-factorized Statistics:

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

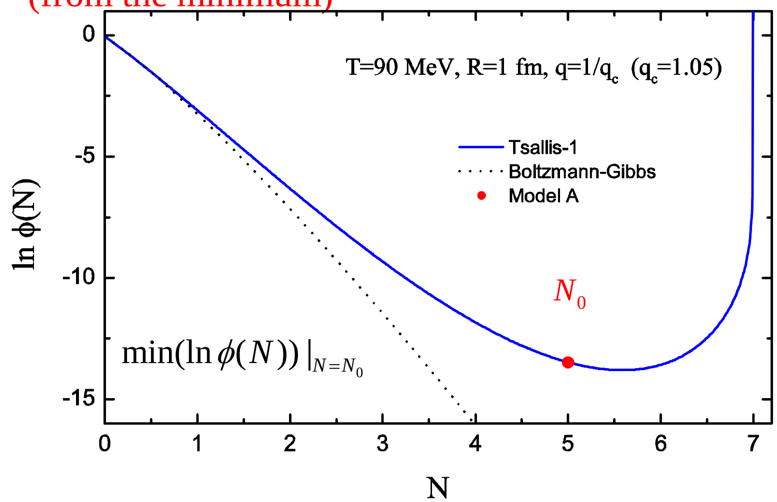
- The Tsallis-factorized distribution recovers the distribution of the Tsallis-1 statistics in the zeroth term approximation if $q_c\to 1/q$

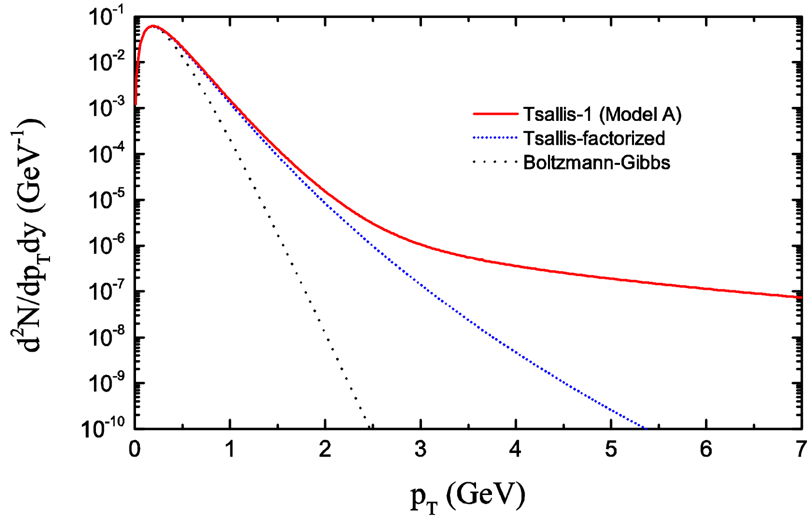
- The Tsallis-factorized distribution is not equivalent to the exact distribution of the Tsallis-1 statistics

$$\langle n_{\vec{p}\sigma}\rangle=\left[1+(q_c-1)\frac{\varepsilon_{\vec{p}}-\mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

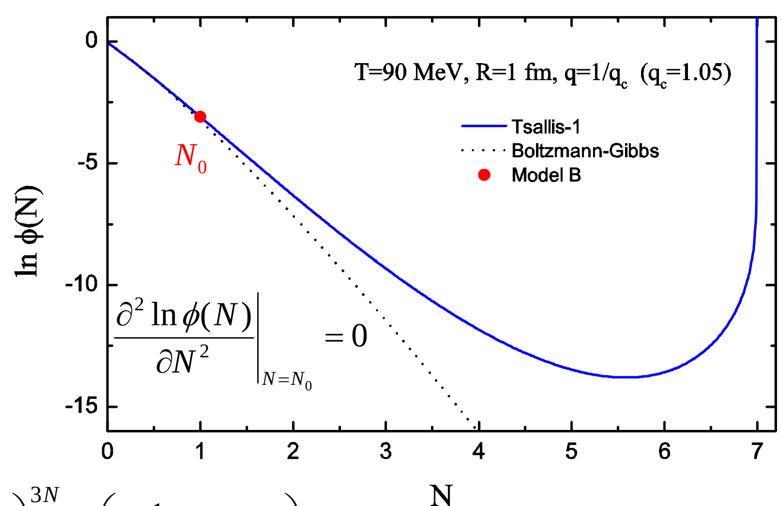
The cut-off parameter of the Tsallis-1 statistics

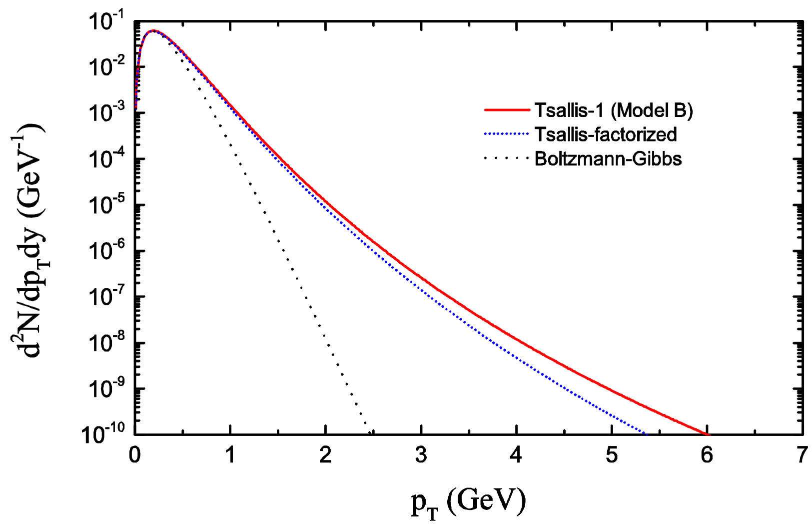
1. Model A: (from the minimum)





2. Model B: (from the inflection point)





$$\sum_{N=0}^{N_0}\phi(N)=1,$$

$$\phi(N)=\frac{\tilde{\omega}^N}{N!}\frac{\left(\frac{q}{1-q}\right)^{3N}\Gamma\left(\frac{1}{1-q}-3N\right)}{\Gamma\left(\frac{1}{1-q}\right)}\left[1+\frac{q-1}{q}\frac{\Lambda+\mu N}{T}\right]^{\frac{1}{q-1}+3N}$$

-Tsallis-1 statistics

$$\phi(N)=\frac{\tilde{\omega}^N}{N!}e^{\frac{\Omega+\mu N}{T}}$$

-Boltzmann-Gibbs statistics

Ultrarelativistic p_T – distribution in the Tsallis–2 statistics $q_c > 1$

1) Exact Tsallis-2 Statistics:

A.S.P., Eur. Phys. J. A 53 (2017) 53

$$\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^2} p_T^2 \cosh y \frac{1}{Z^{q_c}} \sum_{N=0}^{N_0} \frac{\tilde{\omega}^N}{N!} \frac{\Gamma\left(\frac{q_c}{q_c-1} - 3N\right)}{(q_c-1)^{3N}\,\Gamma\left(\frac{q_c}{q_c-1}\right)} \left[1 + (q_c-1)\frac{p_T \cosh y - \mu(N+1)}{T}\right]^{\frac{q_c}{1-q_c}+3N}$$

$$\tilde{\omega} = \frac{gVT^3}{\pi^2}$$

N_0 is a function of (T,V,q,mu)

2) Zeroth term approximation of the Tsallis-2 Statistics:

A.S.P., Eur. Phys. J. A 53 (2017) 53

$$\frac{d^2N}{dp_T dy} = \frac{gVp_T^2 \cosh y}{(2\pi)^2} \left[1 + (q_c-1)\frac{p_T \cosh y - \mu}{T}\right]^{\frac{q_c}{1-q_c}}, \qquad N_0 = 0$$

3) Tsallis-factorized Statistics:

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

$$\frac{d^2N}{dp_T dy} = \frac{gVp_T^2 \cosh y}{(2\pi)^2} \left[1 + (q_c-1)\frac{p_T \cosh y - \mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

- The transverse momentum distribution of the Tsallis-factorized statistics is equivalent to the distribution of the Tsallis-2 statistics in the zeroth term approximation at $N_0 = 0$
- The transverse momentum distribution of the Tsallis-factorized statistics does not recover the exact distribution of the Tsallis-2 statistics. Thus, the Tsallis-factorized statistics is only an approximation of the Tsallis statistics.

Ultrarelativistic p_T – distribution in the Tsallis-1 statistics $q<1$

1) Exact Tsallis-1 Statistics:

A.S.P., Eur. Phys. J. A 53 (2017) 53; Eur. Phys. J. A 52 (2016) 355

$$\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^2} p_T^2 \cosh y \sum_{N=0}^{N_0} \frac{\tilde{\omega}^N}{N!} \frac{\Gamma\left(\frac{1}{1-q} - 3N\right)}{\left(\frac{1-q}{q}\right)^{3N} \Gamma\left(\frac{1}{1-q}\right)} \left[1 + \frac{q-1}{q} \frac{\Lambda - p_T \cosh y + \mu(N+1)}{T}\right]^{\frac{1}{q-1}+3N}$$

$$\tilde{\omega} = \frac{gVT^3}{\pi^2}$$

N_0 and Λ are the functions of (T,V,q,mu)

2) Zeroth term approximation of the Tsallis-1 Statistics:

A.S.P., Eur. Phys. J. A 53 (2017) 53; Eur. Phys. J. A 52 (2016) 355

$$\frac{d^2N}{dp_T dy} = \frac{gVp_T^2 \cosh y}{(2\pi)^2} \left[1 - \frac{q-1}{q} \frac{p_T \cosh y - \mu}{T}\right]^{\frac{1}{q-1}}, \qquad N_0 = 0$$

3) Tsallis-factorized Statistics:

J. Cleymans, D. Worku, Eur. Phys. J. A 48, 160 (2012)

$$\frac{d^2N}{dp_T dy} = \frac{gVp_T^2 \cosh y}{(2\pi)^2} \left[1 + (q_c - 1) \frac{p_T \cosh y - \mu}{T}\right]^{\frac{q_c}{1-q_c}}$$

- The transverse momentum distribution of the Tsallis-factorized statistics recovers the distribution of the Tsallis-1 statistics in the zeroth term approximation at $N_0 = 0$ if $q_c \to 1/q$
- The transverse momentum distribution of the Tsallis-factorized statistics does not resemble the exact distribution of the Tsallis-1 statistics. Thus, the Tsallis-factorized statistics is only an approximation of the Tsallis statistics when $q_c \to 1/q$

Equivalence of the ultrarelativistic p_T – distributions of the Tsallis-1 and Tsallis-2 statistics

A.S.P., EPJ Web Conf. 138 (2017) 03008

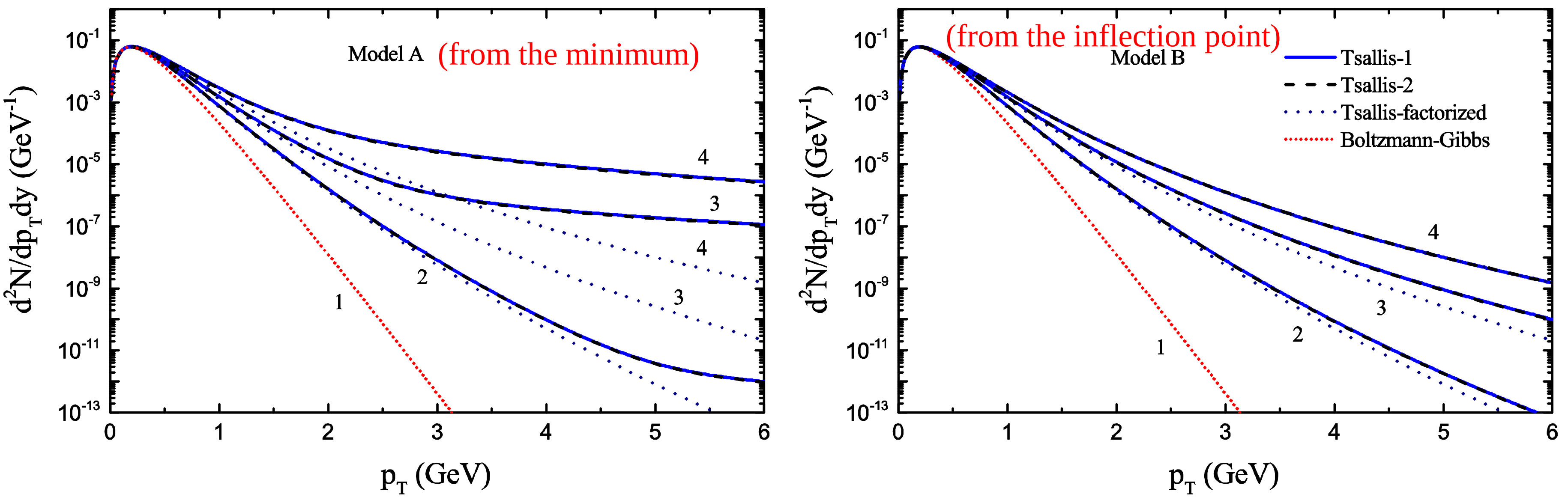


Figure 3. The transverse momentum distribution for the ultrarelativistic ideal gas of π^0 pions in the Model A (left panel) and the Model B (right panel) for the Tsallis-1 and Tsallis-2 statistics in the volume $V = 1$ fm at the temperature $T = 90$ MeV, chemical potential $\mu = 0$, rapidity $y = 0$ and different values of q_c ($q = 1/q_c$). The lines 1, 2, 3 and 4 correspond to $q_c = 1, 1.03, 1.05$ and 1.07, respectively.

The ultrarelativistic transverse momentum distributions of the Tsallis-1 and Tsallis-2 statistics are equivalent under the transformation of the parameter $q_c \to 1/q$ in both models A and B.

Heavy-ion collisions: SPS CERN

Transverse mass spectra of identified hadrons with Tsallis-factorized statistics

Ex.: NA49, Phys. Rev. C 66 (2002) 054902; Phys. Rev. C 77 (2008) 024903

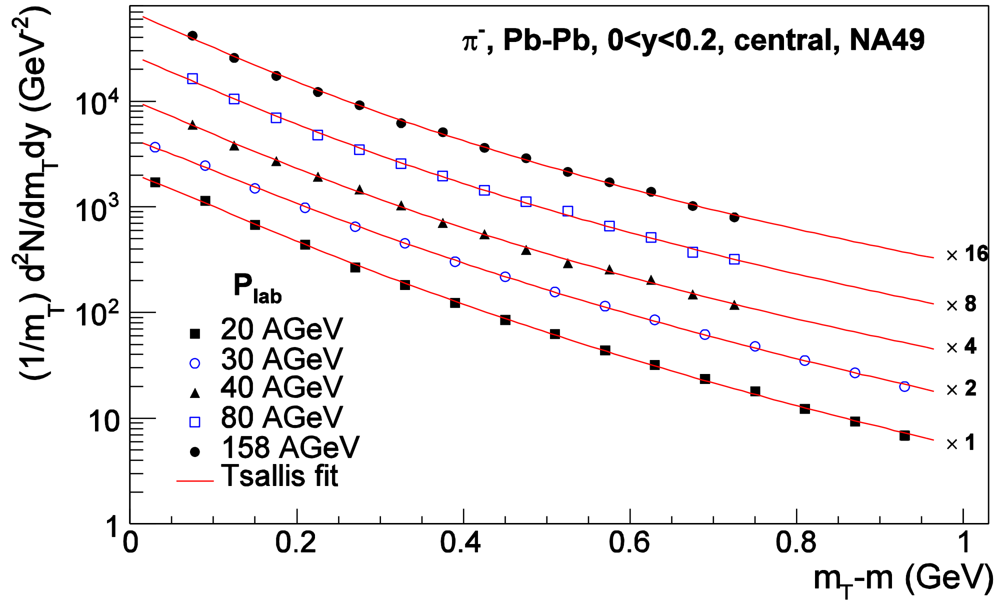
- mT – distribution in the Tsallis-factorized statistics:

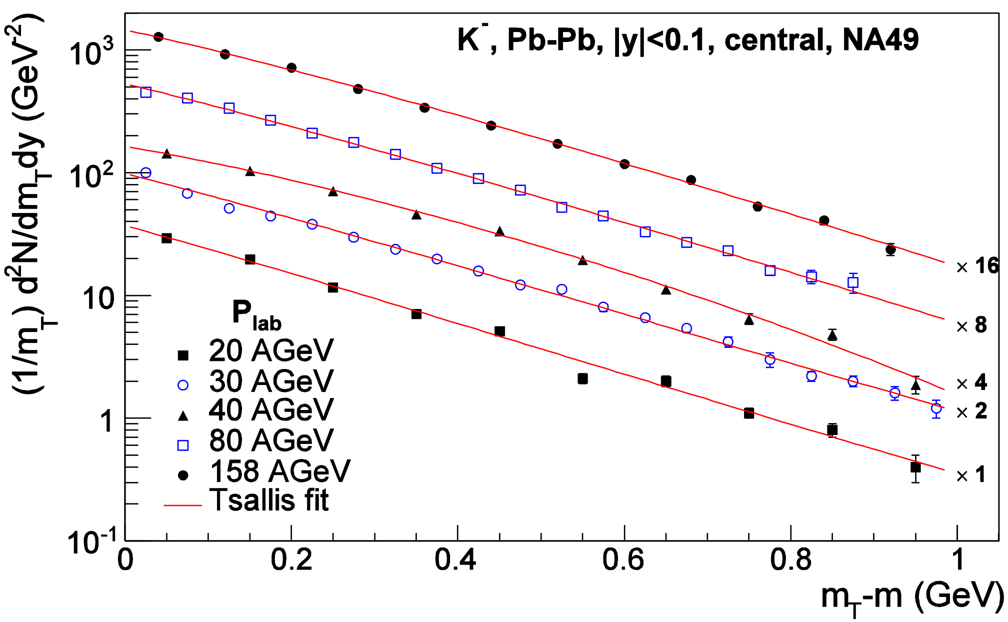
$$\left.\frac{1}{m_T}\frac{d^2N}{dp_T dy}\right|_{y_0}^{y_1} = gV\int_{y_0}^{y_1} dy\, \frac{p_T \cosh y}{(2\pi)^2} \times \left[1-(1-q)\frac{m_T \cosh y - \mu}{T}\right]^{\frac{q}{1-q}}$$

- π^-, K^- – mesons

- Central PbPb collisions in the energy range $\sqrt{s_{NN}} = 6.3 - 17.3$ GeV

- The data of π^- are very well described by the Tsallis-factorized statistics
- The data of K^- measured by NA49 Collaboration in the energy range 6.3-12.3 GeV contain irregularities and they should be corrected by NICA experiment
- The data of K^- at 17.3 GeV fits very well the Tsallis-factorized distribution





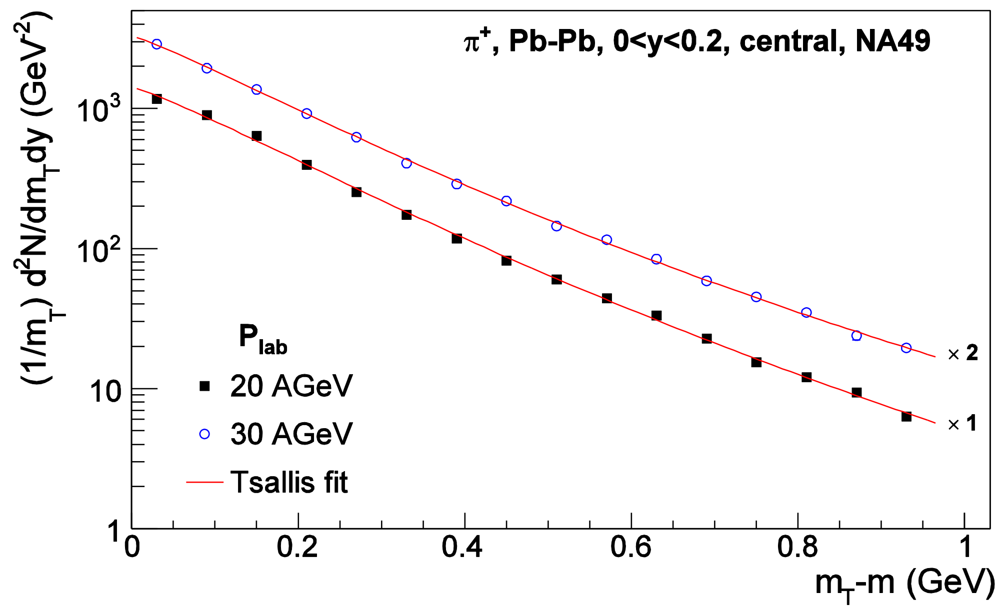
Transverse mass spectra of identified hadrons with Tsallis-factorized statistics

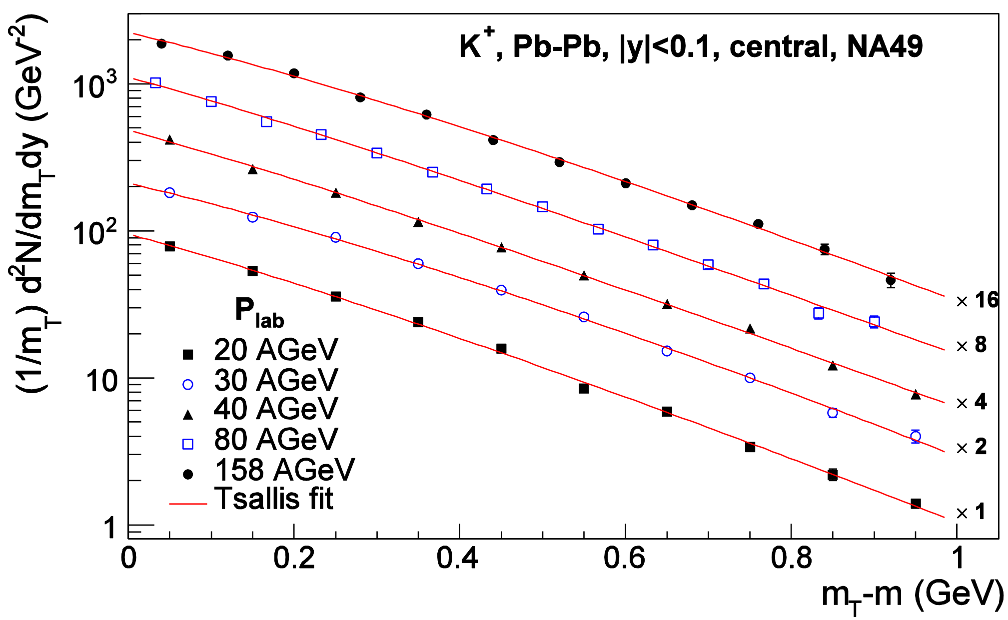
Ex.: NA49, Phys. Rev. C 66 (2002) 054902; Phys. Rev. C 77 (2008) 024903

- mT – distribution in the Tsallis-factorized statistics:

$$\frac{1}{m_T}\frac{d^2N}{dp_T dy}\bigg|_{y_0}^{y_1} = gV\int_{y_0}^{y_1} dy\, \frac{p_T \cosh y}{(2\pi)^2} \times\left[1-(1-q)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{q}{1-q}}$$

- π^+, K^+ – mesons
- Central PbPb collisions in the energy range $\sqrt{s_{NN}} = 6.3 - 17.3$ GeV
- The NA49 data for π^+, K^+ are very well described by the Tsallis-factorized statistics in the all its energy range





Heavy-ion collisions: RHIC BNL

Transverse momentum distribution with Tsallis-factorized statistics

- pT – distribution in the Tsallis-factorized statistics:

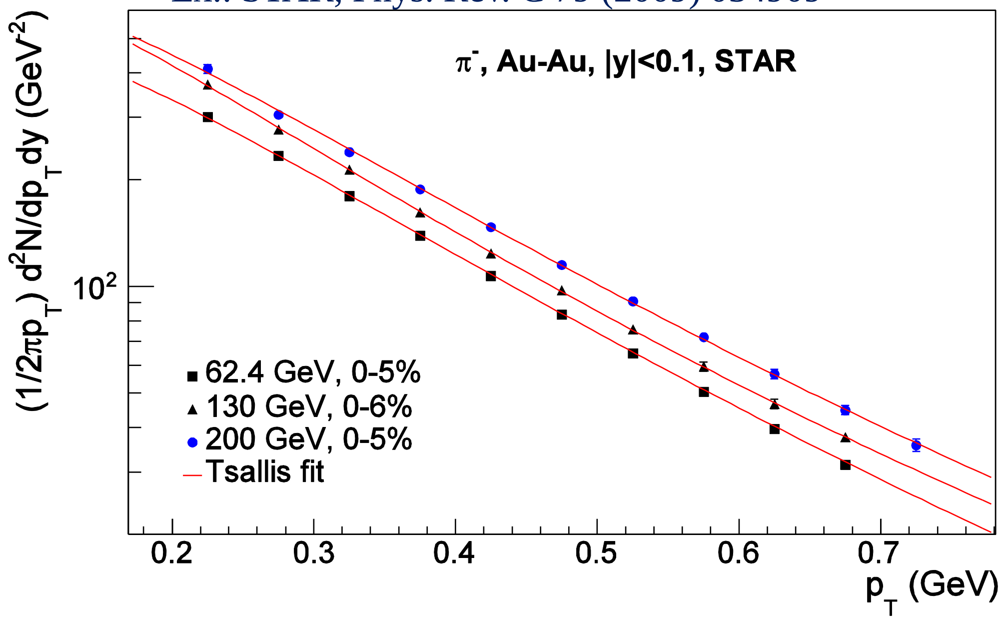
$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}\bigg|_{y_0}^{y_1} = gV\int_{y_0}^{y_1} dy \frac{m_T \cosh y}{(2\pi)^2} \times \left[1-(1-q)\frac{m_T \cosh y - \mu}{T}\right]^{\frac{q}{1-q}}$$

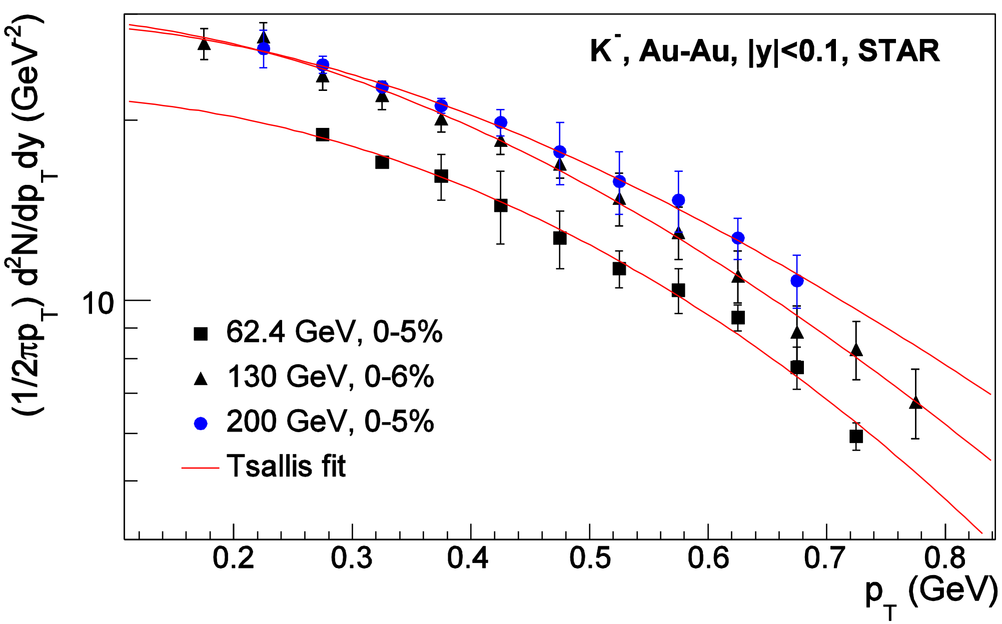
- π^-, K^- – mesons

- Central AuAu collisions in the energy range $\sqrt{s_{NN}} = 62.4 - 200$ GeV

- The data of π^- are very well described by the Tsallis-factorized statistics
- The data of K^- measured by STAR Collaboration 62.4 and 130 GeV contain irregularities which should be corrected by another experiment.
- The data of K^- at 200 GeV fits very well the Tsallis-factorized distribution

Ex.: STAR, Phys. Rev. C 79 (2009) 034909





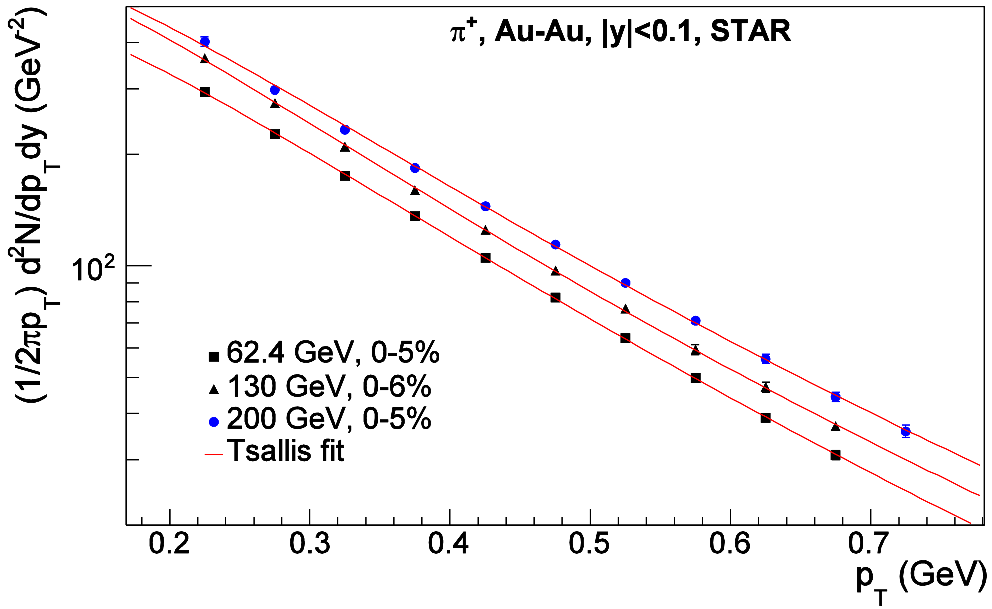
Transverse momentum distribution with Tsallis-factorized statistics

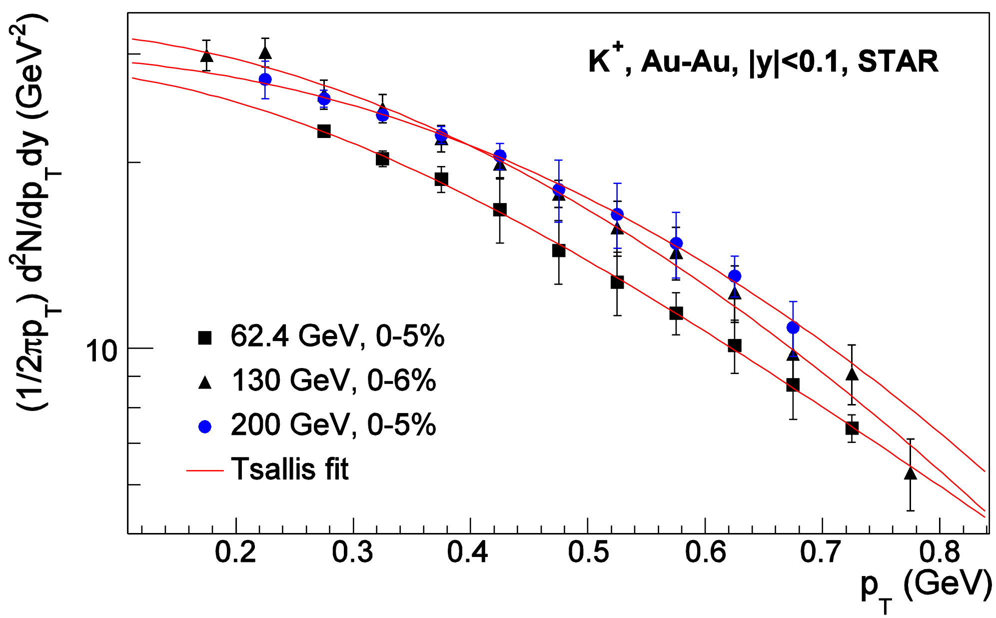
- pT – distribution in the Tsallis-factorized statistics:

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}\bigg|_{y_0}^{y_1} = gV\int_{y_0}^{y_1} dy \frac{m_T \cosh y}{(2\pi)^2} \times \left[1-(1-q)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{q}{1-q}}$$

- π^+, K^+ – mesons
- Central AuAu collisions in the energy range $\sqrt{s_{NN}} = 62.4 - 200$ GeV
- The data of π^+ are very well described by the Tsallis-factorized statistics
- The data of K^+ measured by STAR Collaboration at 130 GeV contain irregularities which should be corrected by another experiment.
- The data of K^+ at 62.4 and 200 GeV fits very well the Tsallis-factorized distribution

Ex.: STAR, Phys. Rev. C 79 (2009) 034909





Heavy-ion collisions: LHC CERN

Transverse momentum distribution with Tsallis-factorized statistics

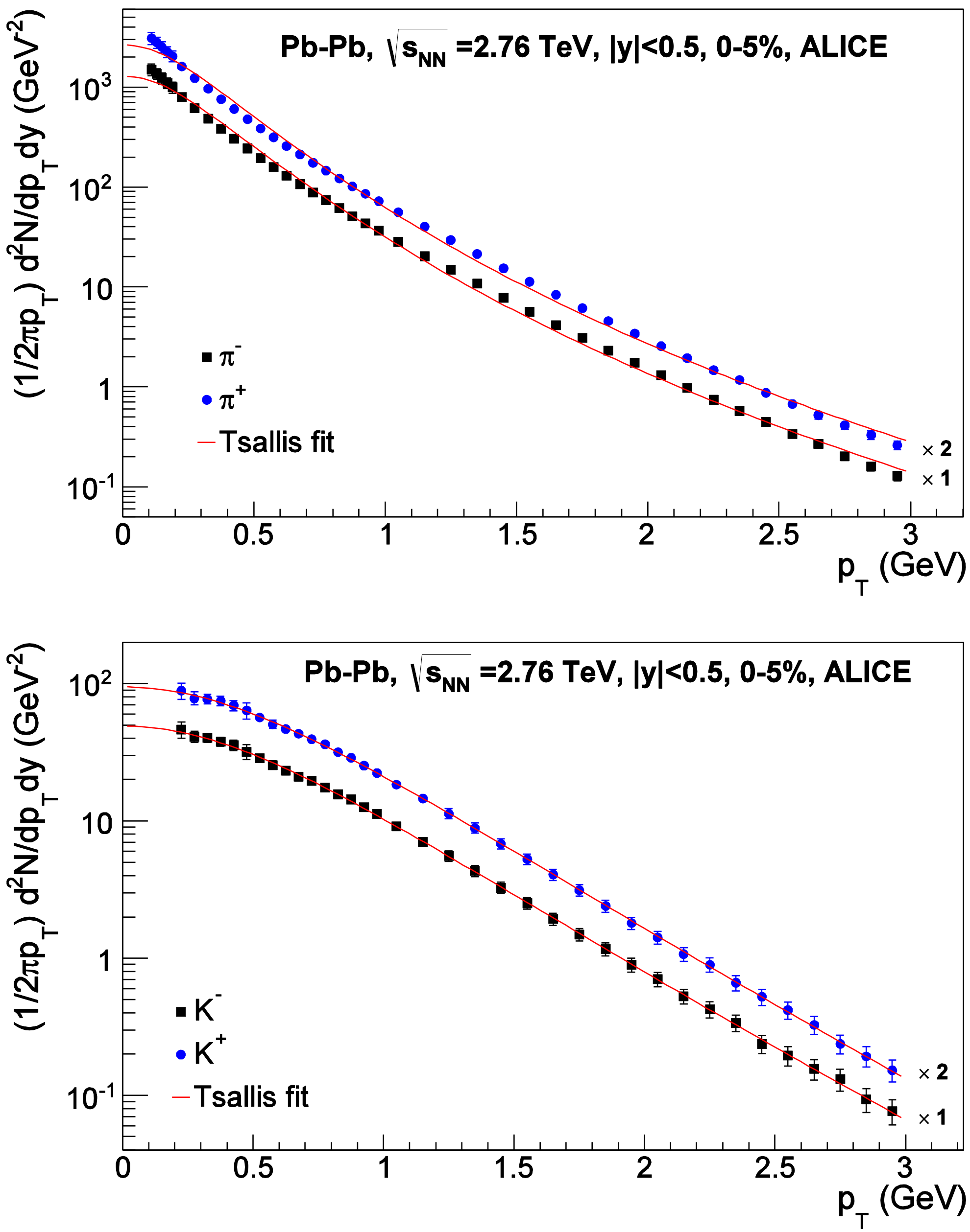
- pT – distribution in the Tsallis-factorized statistics:

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}\bigg|_{y_0}^{y_1} = gV\int_{y_0}^{y_1} dy\, \frac{m_T \cosh y}{(2\pi)^2}$$

$$\times\left[1-(1-q)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{q}{1-q}}$$

- $\pi^{\pm}, K^{\pm}$ – mesons
- Central PbPb collisions at 2.76 TeV
- The data of $K^{\pm}$ at 2.76 TeV are very well described by the Tsallis-factorized statistics
- The data of $\pi^{\pm}$ at 2.76 TeV can not be described by the Tsallis-factorized statistics for low pT momenta

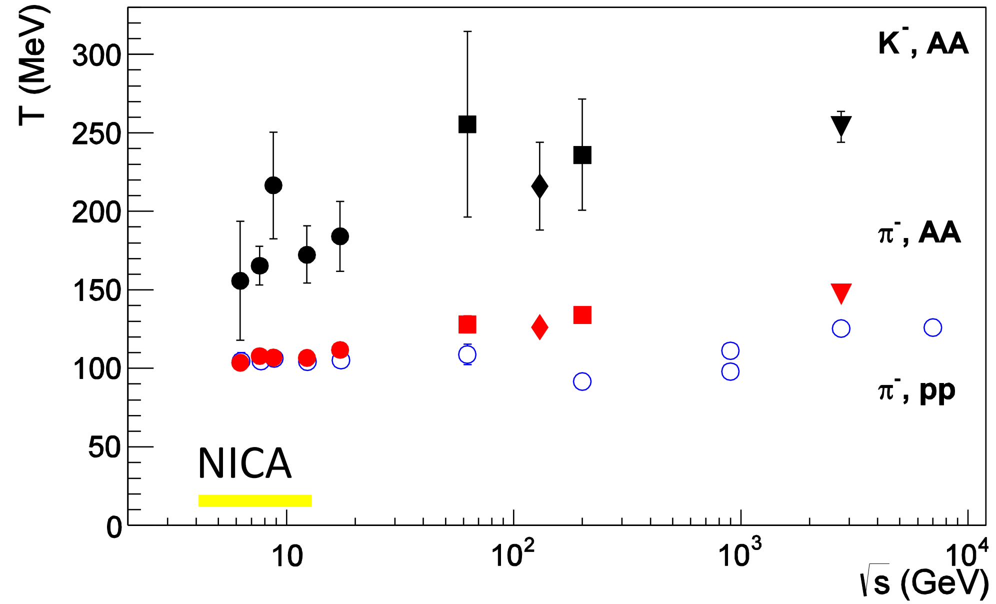
Ex.: ALICE, Phys. Rev. C 88 (2013) 044910

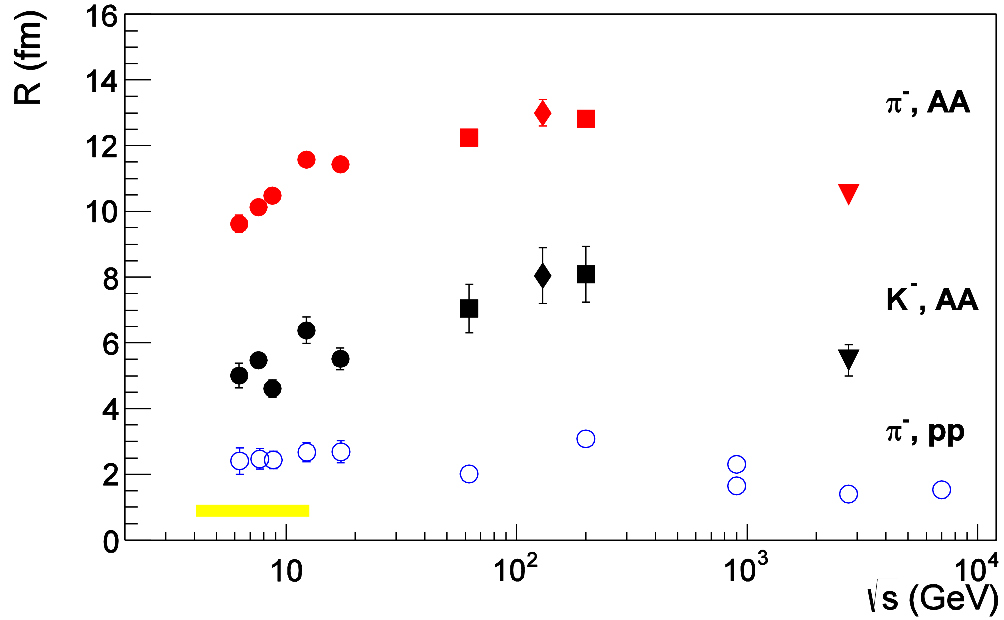


Parameters of the Tsallis−factorized statistics in AA and pp collisions

Temperature and volume for K- and π- mesons

- The experimental transverse momentum distributions from heavy-ion collisions clearly show that K- and pi- mesons have different temperatures T and are emitted from different volumes V.
- The temperature of K- kaons in AA collisions is higher than the temperature of pi- pions.
- However, K- kaons in AA collisions are emitted from the smaller volume than pi- pions.
- The volume for pi- pions in AA collision corresponds to the geometrical volume of two nuclei.
- And the volume for pi- pions in pp collision corresponds to the geometrical volume of two protons.
- The temperatures for pi- pions from AA and pp collisions are close to each other in comparison with the temperature of K-

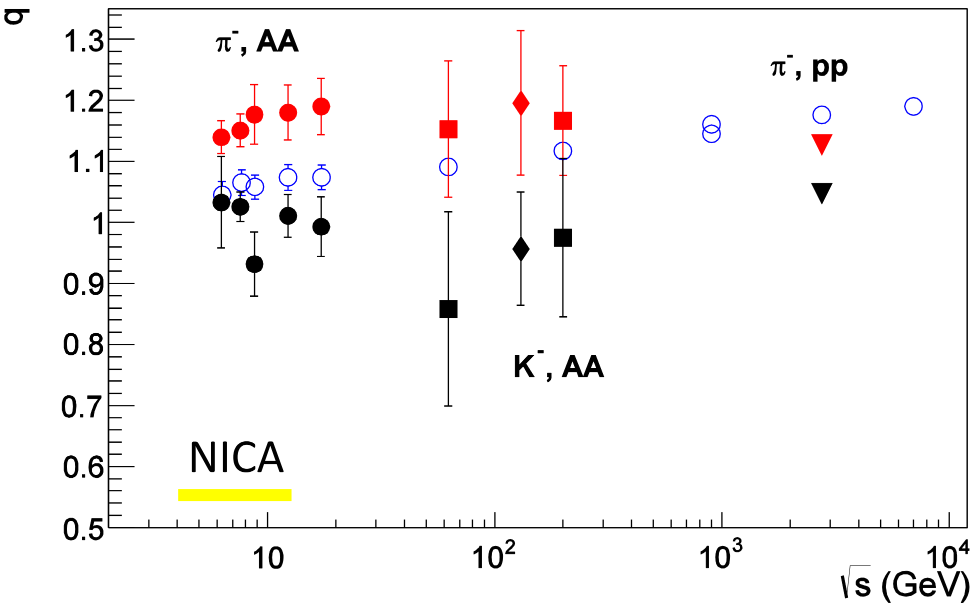


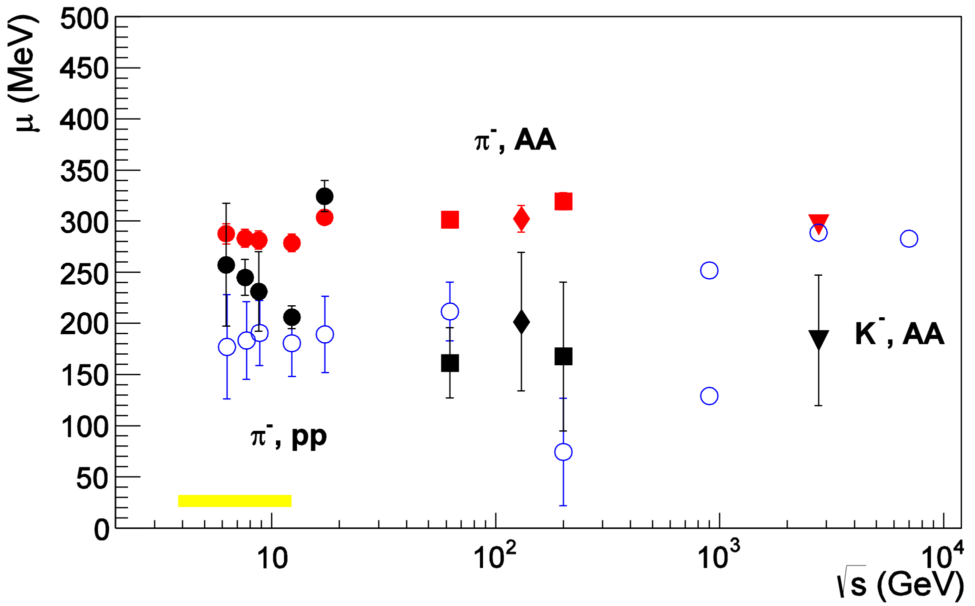


Parameters of the Tsallis-factorized statistics in AA and pp collisions

Parameter q and particle chemical potential μ for K- and π- mesons

- The value q=1 corresponds to the Boltzmann-Gibbs statistics (exponential function).
- The deviation of the value of the parameter q from unity indicates on the measure of deviation of the power-law distribution from the Gibbs exponential function.
- The deviations from Boltzmann-Gibbs statistics are monotonically growing with beam energy for pi- pions in pp collisions.
- The transverse momentum distribution of pi- pions in AA collisions deviates essentially from the Gibbs exponent.
- The distribution of K- kaons in AA collisions is close to the Gibbs exponent at low energies.
- The introduction of the non-vanishing particle chemical potential allows to correctly describe the values of the volume of the system.

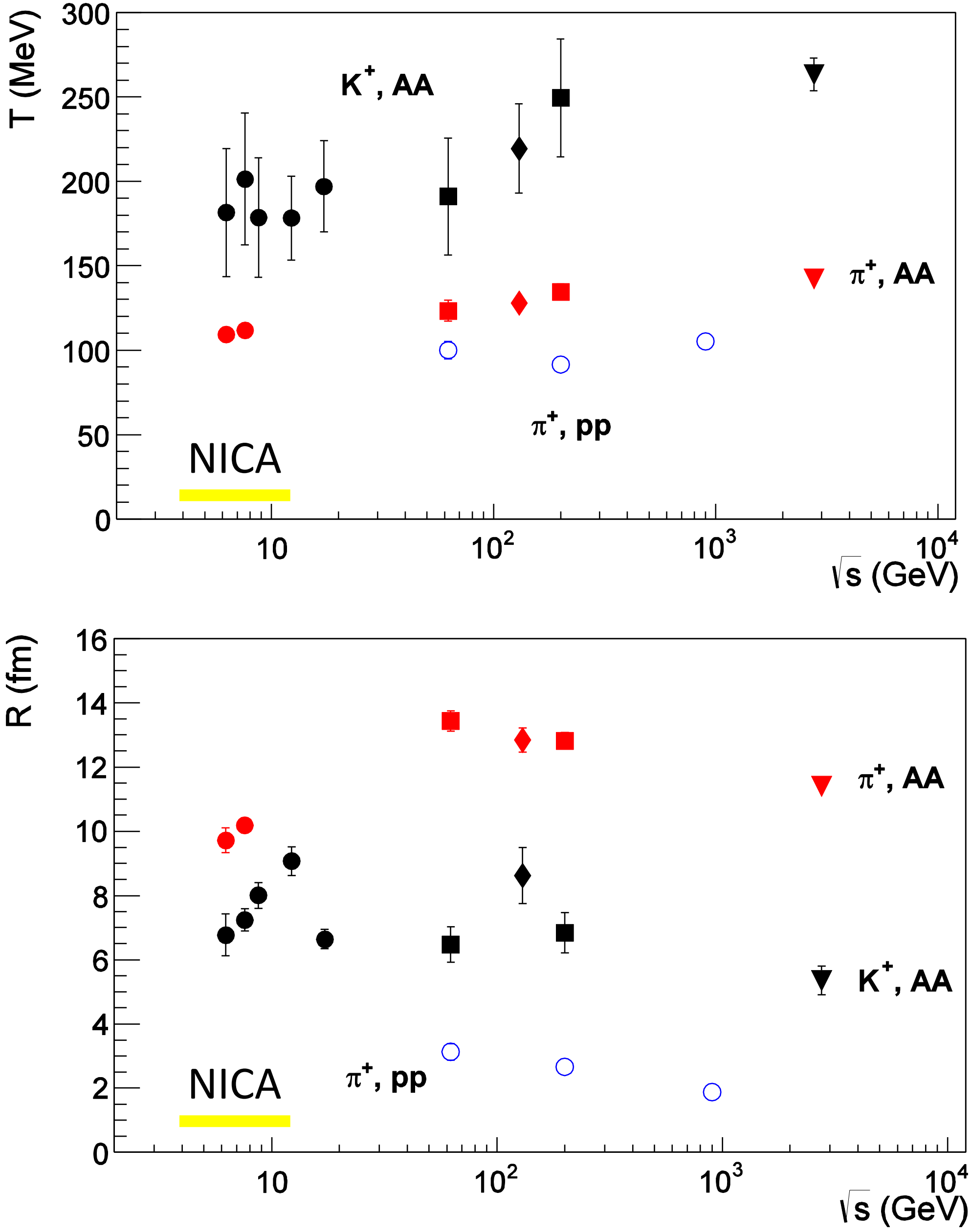




Parameters of the Tsallis-factorized statistics in AA and pp collisions

Temperature and volume for K+ and π+ mesons

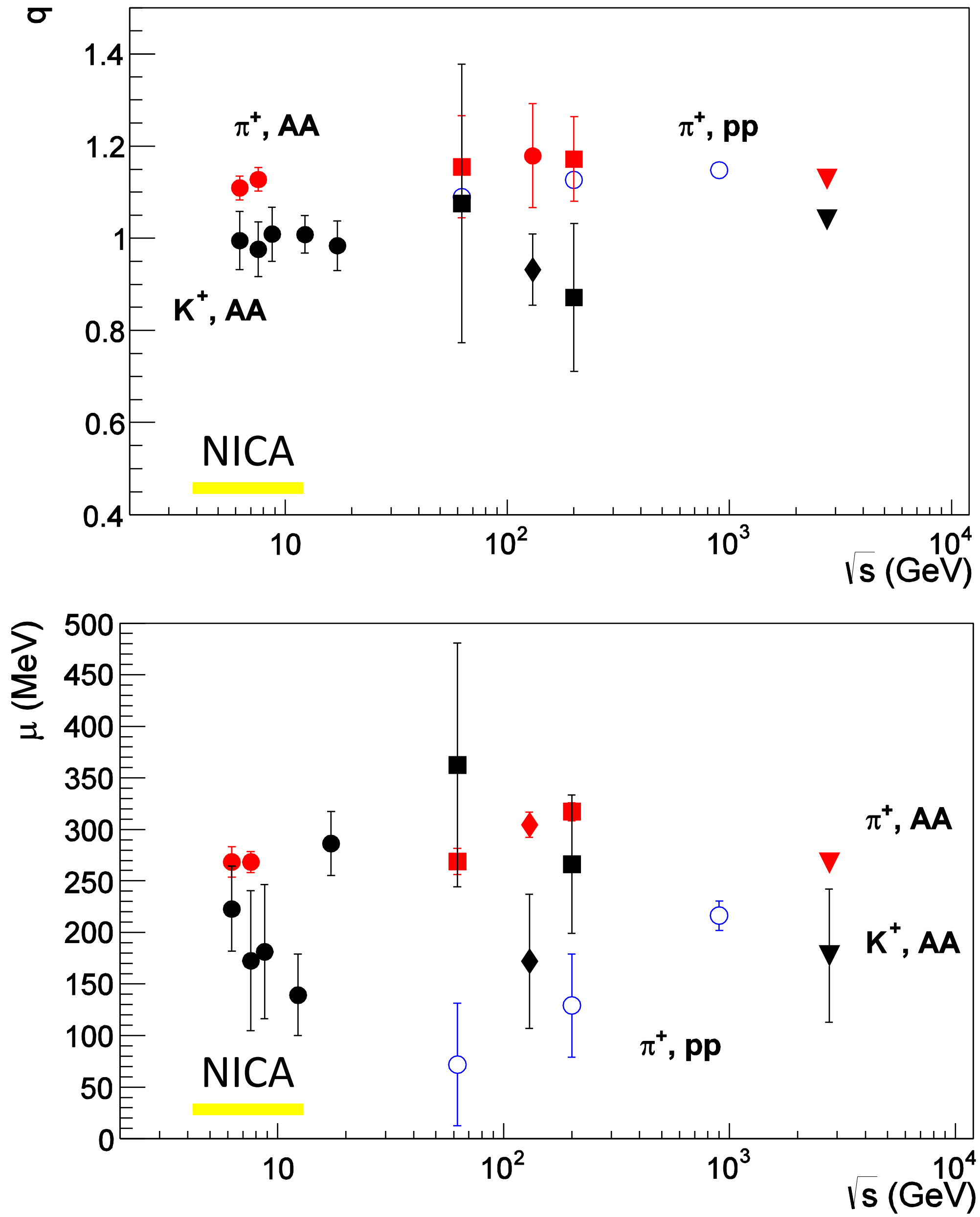
- At NICA energies the temperature and volume for K+ and pi+ have some structures as a function of energy.
- The temperature of K+ kaons in AA collisions is higher than the temperature of pi+ pions.
- However, K+ kaons in AA collisions are emitted from the smaller volume than pi+ pions.
- The volume for pi+ pions in AA collision corresponds to the geometrical volume of two nuclei.
- And the volume for pi+ pions in pp collision corresponds to the geometrical volume of two protons.
- The temperatures for pi+ pions from AA and pp collisions are close to each other in comparison with the temperature of K+ kaon



Parameters of the Tsallis-factorized statistics in AA and pp collisions

Parameter q and particle chemical potential μ for K+ and π+ mesons

- The deviations from Boltzmann-Gibbs statistics are monotonically growing with beam energy for pi+ pions in pp collisions.
- The transverse momentum distribution of pi+ pions in AA collisions deviates essentially from the Gibbs exponent.
- The distribution of K+ kaons in AA collisions is close to the Gibbs exponent at low energies.
- The introduction of the non-vanishing particle chemical potential allows to correctly describe the values of the volume of the system.
- The zero particle chemical potential leads to unphysical values of volume in AA and pp collisions



Conclusions

1. The analytical expressions for the ultrarelativistic transverse momentum distributions of the Tsallis-1 and Tsallis-2 statistics were obtained

2. We have demonstrated that the ultrarelativistic transverse momentum distribution of the Tsallis-factorized statistics is equivalent to the distribution of the Tsallis-2 statistics in the zeroth term approximation.

3. We also demonstrated that the ultrarelativistic transverse momentum distribution of the Tsallis-factorized statistics recovers the distribution of the Tsallis-1 statistics in the zeroth term approximation under the transformation of the parameter q to $1/q_c$

4. Applying the Tsallis-factorized statistics to the experimental data on the transverse momentum distributions of particles created in heavy-ion collisions we have revealed that the charged pions and kaons are emitted from the collision zone at different temperatures from different volumes.

Thank you for your attention!