Anisotropic flow fluctuations in heavy-ion collisions

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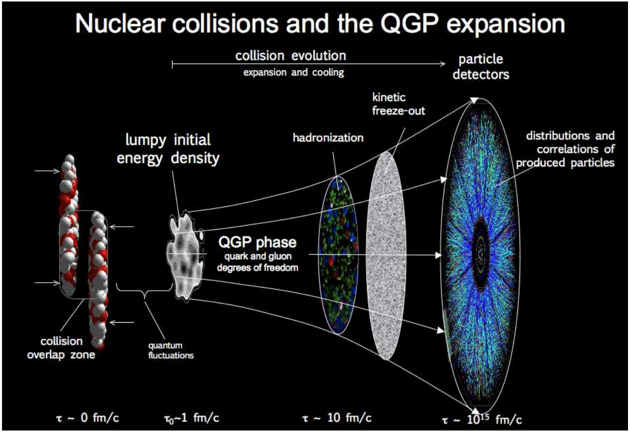
Outline

Structure

- Introduction
- Motivation
- Analysis
- Results
- Conclusion

Introduction

Introduction: Evolution of HI collision



P. Sorensen, http://arxiv.org/abs/arXiv:0905.0174

Fig.1. Stages of evolution in non-central nuclear collision.

Introduction: Flow

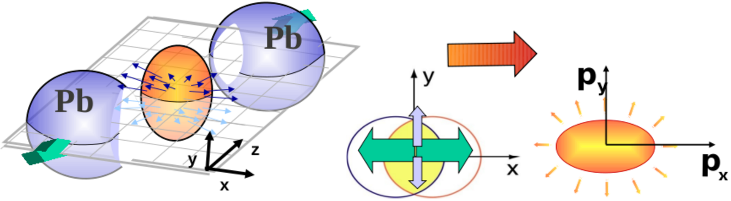


Fig.2. Formation of the overlap region:
asymmetry in the initial geometry $\longrightarrow$ anisotropy in particle momenta distributions

Azimuthal anisotropy can be described by Fourier expansion of particles' angular distribution around the beam direction:

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n cos(n(\varphi - \Psi_n))$$

where v_n and $\varphi - \Psi_n$ represent magnitude and phase of the n^{th} - order anisotropy of a given event in the momentum space.

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- Event-by-event (EbyE) analysis: studied **mean values** $\Longrightarrow$ can study **EbyE distributions** (like $p(v_n)$). Why?
    - Observation of non-zero v_3 at RHIC and LHC[1] $\Longrightarrow$ Participant eccentricity fluctuations ($v_n \rightarrow p(v_n)$)

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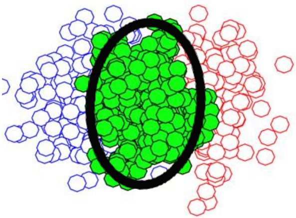

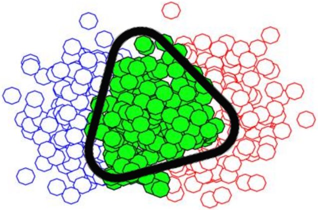

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  - Observation of non-zero v_3 at RHIC and LHC $\Longrightarrow$ Participant eccentricity fluctuations ($v_n \rightarrow p(v_n)$)
- Multiparticle cumulants $\Longrightarrow$ moments of $p(v_n)$[2]:
  - Observed $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$[3]:
  - Nature of fluctuations is Gaussian?

[2] S. A. Voloshin, A. M. Poskanzer, A. Tang and G. Wang, Phys.Lett. B659 (2008) 537-541

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- Measurement of $p(v_n)$ using EbyE unfolding[4]
  - Allows precise fluctuation studies
  - Cumulant extraction out of EbyE distributions
  - $p(\varepsilon_n)$ extraction

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- statistical fluctuations (due to finite number of particles)
- non-flow correlations (from resonance and jet decays)
- fluctuations of initial system distribution in the overlap area (dynamical fluctuations)

The method of EbyE unfolding analysis discussed here helps to get rid of the first two, $\longrightarrow$ leaves pure flow fluctuations connected with initial geometry of created system.

Motivation: Skewness

- Splitting of higher-order cumulants was observed[5]

[5] G. Aad et al., ATLAS Collaboration, Eur.Phys.J. C74 (2014), 3157

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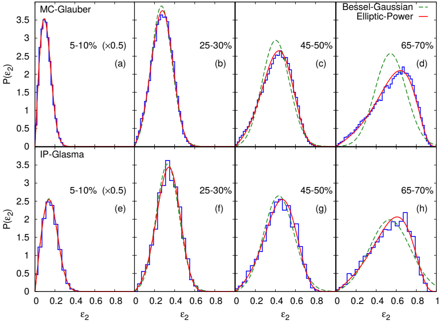
- Suggests that initial state fluctuations are non-Gaussian



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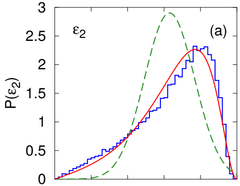
Motivation: Extraction of $p(\varepsilon_n)$

Two common eccentricity parametrizations:

- Bessel-Gaussian[7]



$$p(\varepsilon_n|\varepsilon_0, \delta) = \frac{\varepsilon_n}{\delta^2} Exp\left[-\frac{\varepsilon_n^2 + \varepsilon_0^2}{2\delta}\right] I_0\left(\frac{\varepsilon_n \varepsilon_0}{\delta^2}\right)$$

$$\varepsilon 2 > \varepsilon 4 = \varepsilon 6 = \varepsilon 8$$

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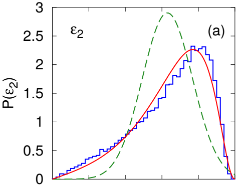

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- Elliptic Power Law[8]

$$p(\varepsilon_n|\varepsilon_0,\alpha) = \frac{2\alpha\varepsilon_n}{\pi}(1-\varepsilon_0^2)^{\alpha+1/2}\int_0^{\pi}\frac{(1-\varepsilon_n^2)^{\alpha-1}d\varphi}{(1-\varepsilon_0\varepsilon_n\cos\varphi)^{2\alpha+1}}$$

$|\varepsilon_n| \leq 1, \varepsilon 2 > \varepsilon 4 > \varepsilon 6 > \varepsilon 8$

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Analysis

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  - data:

$$\overrightarrow{V}_n^{obs} = \left(\frac{\Sigma \cos n\varphi_i/\epsilon_i}{\Sigma 1/\epsilon_i}, \frac{\Sigma \sin n\varphi_i/\epsilon_i}{\Sigma 1/\epsilon_i}\right) - \langle \overrightarrow{V}_n^{obs} \rangle$$

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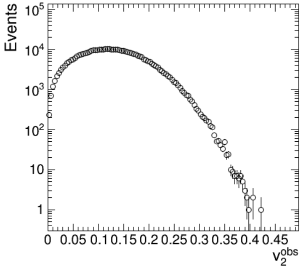

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[10] T. Adye, Proc. of PHYSTAT 2011 Workshop, CERN-2011-006, pp 313-318, 2011

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$$M_{ij}^{iter} = \frac{A_{ji} c_i^{iter}}{\sum_{m,k} A_{mi} A_{jk} c_k^{iter}},$$

$$\widehat{c}^{iter+1} = \widehat{M}^{iter} \widehat{e}, A_{ji} = p(e_j | c_i)$$

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- Perform D'Agostini Unfolding[9] (using RooUnfold package[10])

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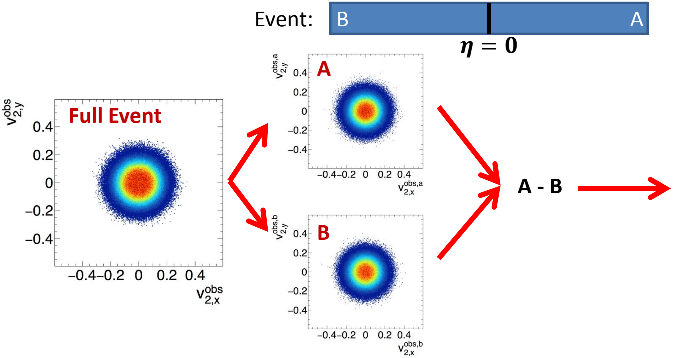
Analysis: Smearing Function



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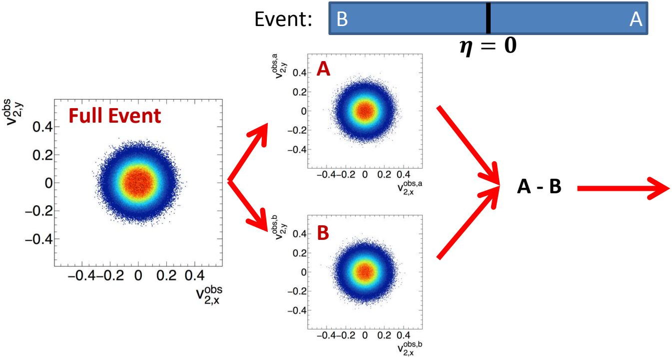

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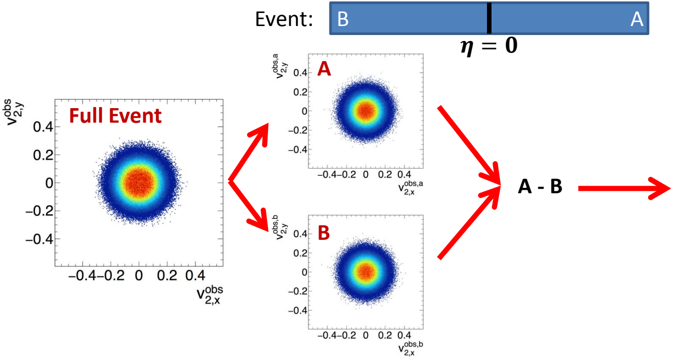

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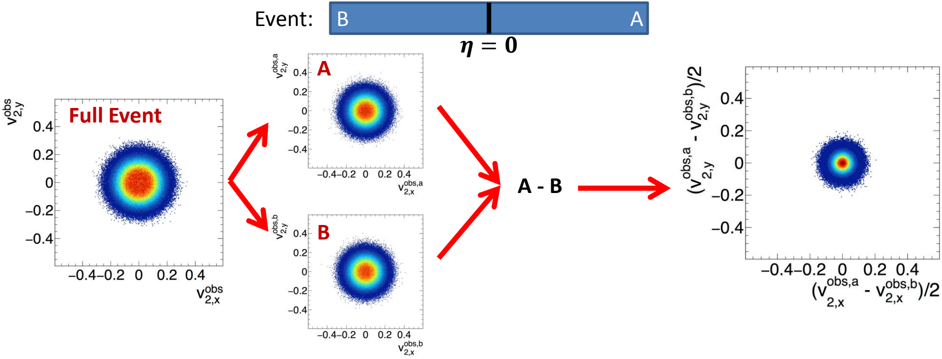

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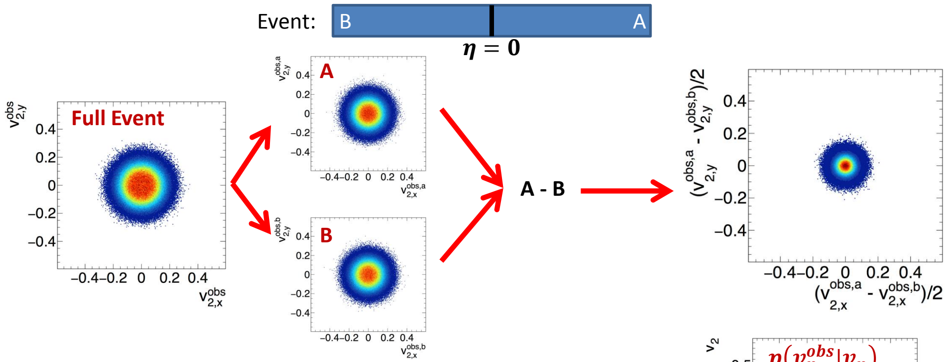

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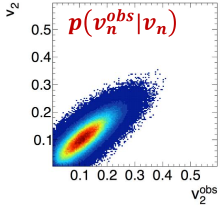
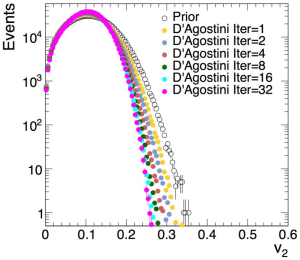


Figure by James Castle, QM2017

Analysis: Unfolding regularization

- Unfolding iteratively removes smearing
- Need to choose "final distribution"



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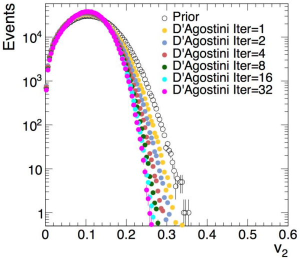

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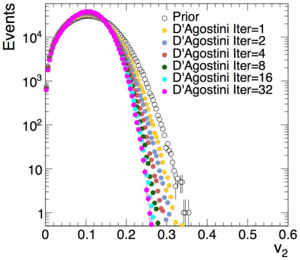
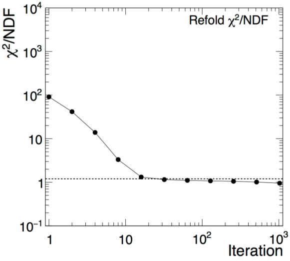


- Regularization:
  - Refolding
  - Cut-off criteria: χ^2/NDF



Results

HYDJET++ event generator

- Together with my colleagues from MSU and UiO performed the EbyE Unfolding analysis on HYDJET++[11]

- HYDJET++[12] is the event generator to simulate relativistic HI collisions as a superposition of two independent components:

  - The soft component is hydro-type state with preset freeze-out conditions

  - The hard state results from the in-medium multi-parton fragmentation with taking into account jet quenching effect

[11] L.V. Bravina et al., Eur.Phys.J. C75 (2015) 588

[12] I.P. Lokhtin et al., Comp. Phys. Commun., 779 (2009)

Elliptic flow in original version of HYDJET++

The direction and strength of the elliptic flow (v_2):

- $\epsilon(b)$ - the spatial anisotropy - represents the elliptic modulation of the final freeze-out hyper-surface at a given impact parameter b
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- $\epsilon(b)$ - the spatial anisotropy - represents the elliptic modulation of the final freeze-out hyper-surface at a given impact parameter b
- $\delta(b)$ - the momentum anisotropy - deals with the modulation of flow velocity profile
- the relation between $\delta(b)$ and $\epsilon(b)$ has the form:

$$\delta = \frac{\sqrt{1 + 4B(\epsilon + B)} - 1}{2B}, \; B = C(1 - \epsilon^2)\epsilon, \; \epsilon = k\epsilon_0$$

Modification of HYDJET++

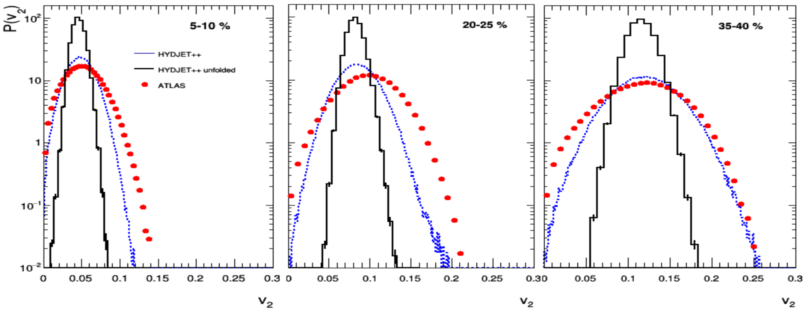

Fig. 3. EbyE $p(v_2)$ distributions in original HYDJET++

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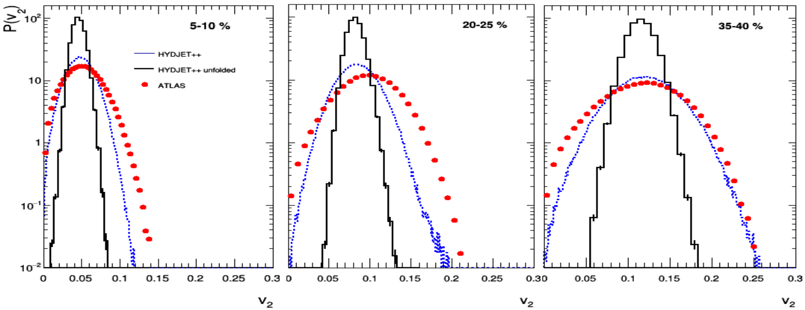

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Introducing additional "eccentricity" fluctuations in HYDJET++ model

The possible modification of HYDJET++:

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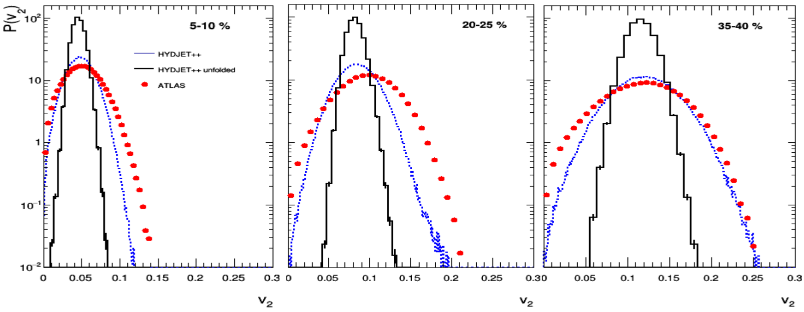

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The modification has been done by introducing EbyE Gaussian smearing of spatial anisotropy parameters $\epsilon(b)$ and $\epsilon_3(b)$.

Results: EbyE $p(v_2)$ in HYDJET++

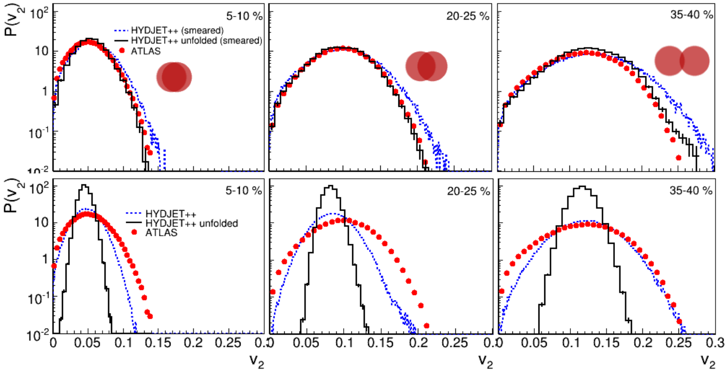

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Fig.4. Comparison of $p(v_2)$ in $5-10\%$ (left), $20-25\%$ (middle), $35-40\%$ (right) centrality intervals in HYDJET++ with ATLAS data

Top/bottom row shows model results with/without additional smearing of spatial anisotropy parameters. ATLAS data are shown by full circles.

Results: EbyE $P(v_3)$ in HYDJET++

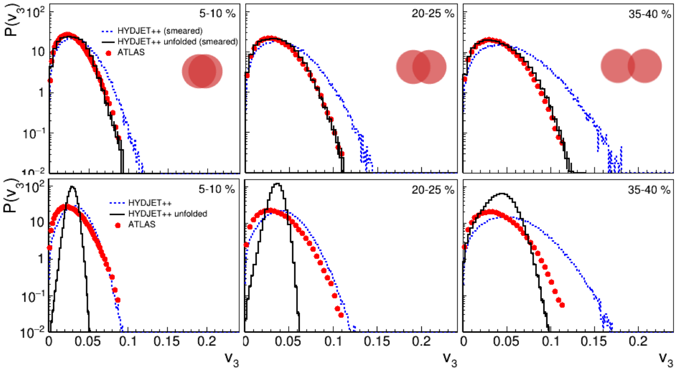

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Fig.5. Comparison of $p(v_3)$ in $5-10\%$ (left), $20-25\%$ (middle), $35-40\%$ (right) centrality intervals in HYDJET++ with ATLAS data.

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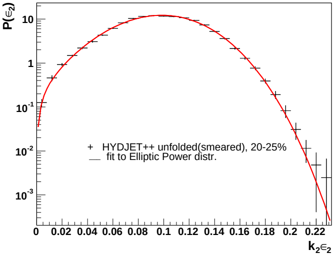
Results: EbyE $p(\varepsilon_2)$ in HYDJET++



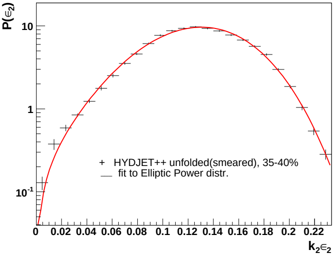


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Fig.6. The probability densities $p(\varepsilon_n)$ obtained in HYDJET++ (with smearing and unfolding) of PbPb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV.

Left/right panel shows model results for centrality 20-25 % and 35-40 %. Full lines represent the fits to the Elliptic Power distribution

Results: Cumulant ratios in CMS

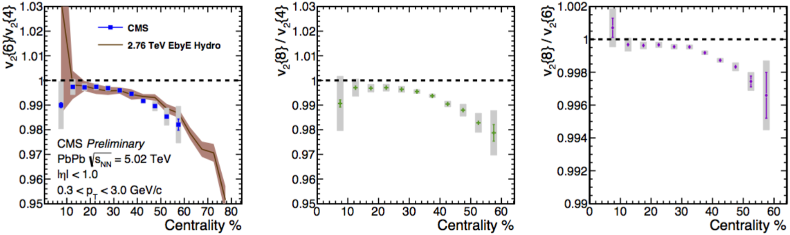

Fig. 7. Cumulant ratios

Figure by James Castle, QM2017

Results: Cumulant ratios and skewness in CMS

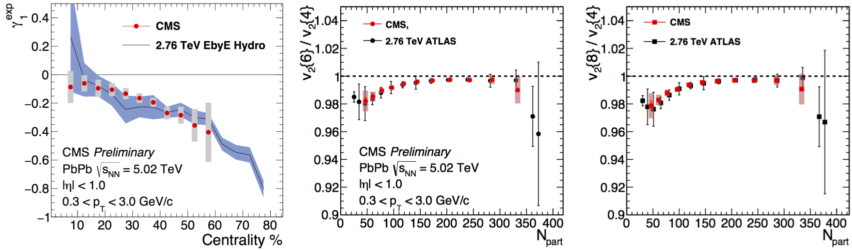

Fig. 8. Cumulant ratios and skewness

Figure by James Castle, QM2017

Results: Fitting EbyE distributions in CMS

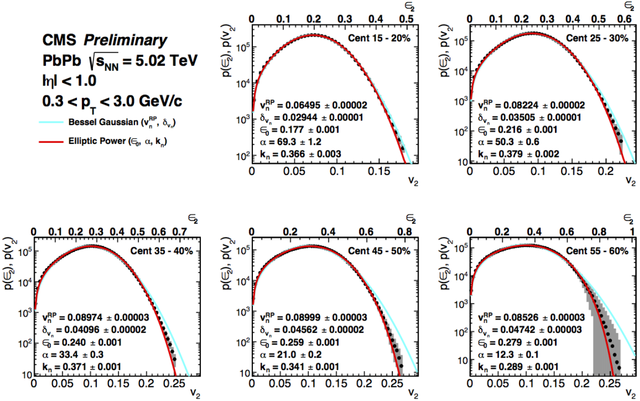

Fig. 9. Fitting with Elliptic Power and Bessel-Gaussian Parametrizations

Figure by James Castle, QM2017

Conclusion

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Plans & Conclusions:

- EbyE unfolding analysis makes it possible to study the anisotropic flow and its fluctuations on a new level:

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- EbyE unfolding analysis makes it possible to study the anisotropic flow and its fluctuations on a new level:
    - Extract the underlying EbyE $p(v_n)$ distribution

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- Possibility of EbyE Unfolding for NICA energies?

The End

Thank you for your attention!

Back Up: Response Matrix

Constructing the Response Matrix:

- Data-driven approach

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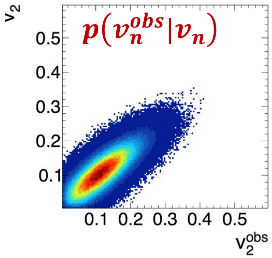
- Prior $p(\vec{v}_n)$ using $p(\vec{v}_n^{obs})$ and $p((\vec{v}_n^{obs,a} - \vec{v}_n^{obs,b})/2) = p(\vec{s}_n)$
- Sample a random $\vec{v}_n$ from the prior
- Sample a random $\vec{s}_n$ from the smearing function
- $\vec{v}_n^{obs} = \vec{v}_n + \vec{s}_n$
- Fill the response matrix with v_n and v_n^{obs}



Back Up: Response Matrix

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Constructing the Response Matrix:

- Analytical approach
  - Using Bessel-Gaussian function:
  $$p(v_n^{obs}|v_n^{true}) \approx \frac{v_n^{obs}}{\delta_{v_n}^2} Exp\left[-\frac{(v_n^{obs})^2 + (v_n^{true})^2}{2\delta_{v_n}}\right] I_0\left(\frac{v_n^{obs} v_n^{true}}{\delta_{v_n}^2}\right)$$
  - Using Student's T function: $p(v_n^{obs}|v_n^{true}, \delta_{v_n}, \nu) \approx$
  $$v_n^{obs} \oint \left[1 + \frac{(v_n^{obs})^2 + (v_n^{true})^2 - 2v_n^{obs} v_n^{true} \cos\phi}{\nu \delta_{v_n}^2}\right]^{-\frac{\nu - 1}{2}} d\phi$$

Back-up: Possibilities of EbyE

- Extract cumulants from $p(v_n)$ directly[1]:

$v_n\{2\}^2 \equiv \langle v_n^2 \rangle$
$v_n\{4\}^4 \equiv -\langle v_n^4 \rangle + 2\langle v_n^2 \rangle^2$
$v_n\{6\}^6 \equiv (\langle v_n^6 \rangle - 9\langle v_n^4 \rangle\langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3)/4$

Where

$$\langle v_n^{2k} \rangle \equiv \int v_n^{2k} p(v_n) dv_n$$

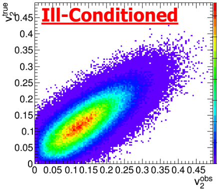
- Measure skewness of $p(v_n)$:

$$\gamma_1^{exp} = -6\sqrt{2} v_2\{4\}^4 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}$$

- Fit fluctuation models to data
  - Bessel Gaussian
  - Elliptic Power
- Couple to ESE
  - Extract detailed shape correlations

[1] **Phys.Lett. B659 (2008) 537-541**

Back-up: Smearing



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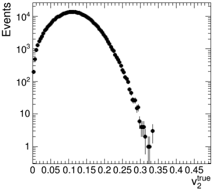

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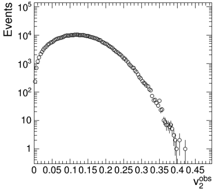

$$\left[\; P\left(v_2^{obs} \middle| v_2^{true}\right) \;\right]\left[v_2^{true} \right] = \left[v_2^{obs} \right]$$

$n_{true} \times n_{obs}$ $\qquad$ $n_{true} \times 1$ $\qquad$ $n_{obs} \times 1$

Back Up: Elliptic flow in HYDJET++

The altered radius of the freeze-out hyper-surface in azimuthal plane:

$$R(b, \phi) = R_{ell}(b, \phi)[1 + \epsilon_3(b)\cos[3(\phi - \Psi_3)]],$$

- $\epsilon_0 = b/2R_A$ - initial ellipticity,
- R_A - the nuclear radius,
- R_{ell} - former transverse radius of the fireball, which reproduces the elliptic deformation.