Lattice study of the sphaleron rate in gluodynamics

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Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics

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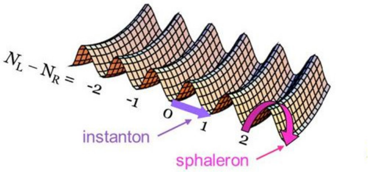
Topology changing transitions in YM



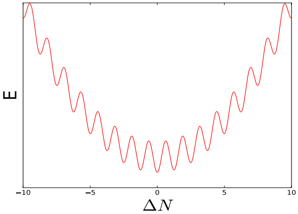
$$\Gamma_{CS} = \lim_{t\to\infty} \frac{\langle \Delta N_{CS}^2 \rangle}{Vt}$$

$$\Delta N_{CS} = Q_4 = \int d^4x q(x)$$

$$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

Sphaleron rate and chiral charge

- EW theory: baryon number n_B violation
- QCD: quark chirality non-conservation
- Fluctuation-dissipation theorem



$$\frac{dQ_5}{dt} = -CQ_5 \frac{\Gamma_{CS}}{2T}$$

- C depends on the fermionic part of the theory

Sphaleron rate

Weak coupling

$$\Gamma_{CS} \sim \alpha^5 \log 1/\alpha\, T^4$$

"Desperate extrapolation" to finite α (semiclassical field theory):

$$\Gamma_{CS} \sim 30\alpha_s^4 T^4$$

G.D. Moore and M.Tassler, JHEP 1102 (2011) 105, arXiv:1011.1167

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Holography

- $N = 4SYM$: $\Gamma = \frac{(g^2 N)^2}{256\pi^3} T^4$

  D.T. Son, A.O. Starinets, JHEP 0209 (2002) 042, arXiv: hep-th/0205051
- Improved holographic QCD (large-N_c): $\Gamma(T_c)/T_c^4 > 1.64$

  U. Gursoy et al., JHEP 1302 (2013) 119, arXiv:1212.3894

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Lattice

First attempt in lattice $SU(2)$ gluodynamics: $\Gamma/T^4 \sim 10^{-3}$

E.M. Ilgenfritz and H. Panagopoulos, Phys.Lett. B258 (1991) 415-420

Sphaleron rate and lattice QCD

Topological charge density correlator $G(t) = \langle \int d^3x q(0) q(\bar{x}, t) \rangle$

$$\Gamma_{CS} = -\lim_{\omega \to 0} \frac{2T}{\omega} \mathrm{Im}[G^R(\omega, \vec{k} = 0)] = -\pi T \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$$G_E(t) = \int_0^\infty \rho(\omega) \frac{\cosh \omega(\beta/2 - t)}{\sinh \omega\beta/2} d\omega$$

Similar to

- shear and bulk viscosity N. Astrakhantsev, V. Braguta, AK, 2017, 2018 H.B. Meyer, 2007
- electric conductivity G. Aarts et al., 2014
- heavy quark momentum diffusion O. Kaczmarek, 2014
- gluon spectral function E.M. Ilgenfritz et al., 2018

Topological charge density and Gradient flow

Gradient flow

$$\partial_\tau A_\mu(\tau, t, x) = -\frac{\partial S}{\partial A_\mu}$$

Topological charge density and Gradient flow

Gradient flow

$$\partial_\tau A_\mu(\tau, t, x) = -\frac{\partial S}{\partial A_\mu}$$

- $\partial_\tau q^\tau = \partial_\rho \omega^\tau_\rho$, $\partial_\tau Q^\tau = 0$
- $\langle q^\tau(x) q^\tau(0)\rangle = \langle q^{\tau=0}(x) q^{\tau=0}(0)\rangle + O_\tau(a^2) + O(\tau)$
- GF gives renormalized $q(x)$
- $C(x) = \lim_{\tau\to 0} \lim_{a\to 0}\langle q^\tau(x) q^\tau(0)\rangle$

M. Ce, G. P. Engel, L. Giusti, Phys.Rev. D92 (2015) no.7, 074502, arXiv:1506.06052

Stress-energy tensor and GF arXiv:1803.05656, R. Yanagihara et al.

Lattice details

- $SU(3)$ Gluodynamics + Gradient flow
- No multilevel algorithm
- Clover gluonic discretization of $F\tilde{F}$
- Continuum extrapolation
  Lattice sizes 12×36^3, 16×48^3, 20×60^3, 24×70^3
- Scale setting using GF (reference scale ω_0):
  M.Kitazawa et al., Phys.Rev. D94 (2016) no.11, 114512, arXiv:1610.07810
- $T/Tc = 1.24, \quad 1.50, \quad 1.70$

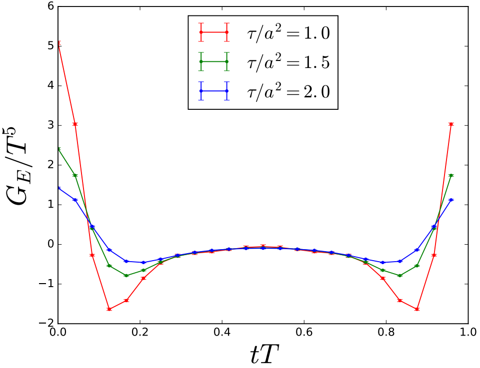
Outline of procedure

1. For given T/T_c generate configurations for various lattice steps a (and N_t).
2. Perform GF on each configuration
3. Measure average $G_q(t, \tau, a)$
4. Extrapolate $G_q(t, \tau) = \lim_{a \to 0} G_q(t, \tau, a)$
5. Extrapolate $G_q(t) = \lim_{\tau \to 0} G_q(t, \tau)$
6. Use spline interpolation, if needed
7. Use $G_q(t)$ to invert integral relation and extract Γ_{CS}

Detailed discussion for $T/T_c = 1.24$, for other T/T_c only final results

How does the correlator $G_q(t, \tau, a)$ look like?

$24 \times 70^3,\ T/T_c = 1.24$



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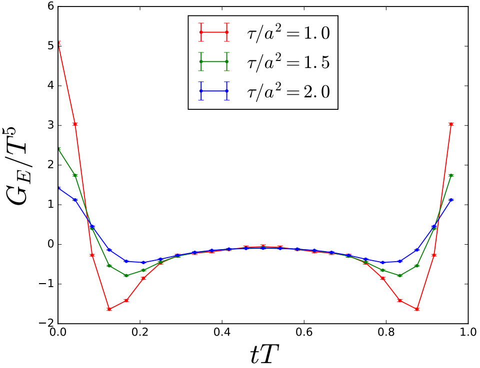
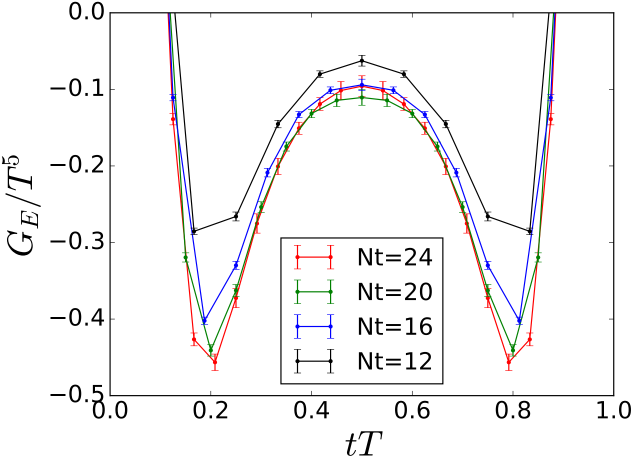


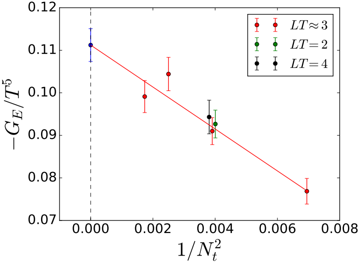
One should use points $a^2 \lessapprox \tau \lessapprox t^2/8$

a-extrapolation, $T/T_c = 1.24$

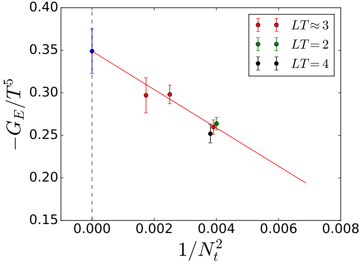


$\tau/a_{24}^2 = 2.0$

a-extrapolation, $T/T_c = 1.24$



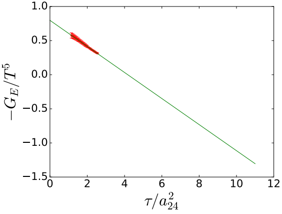
$\tau/a_{24}^2 = 4$
$t/a_{24} = 12$



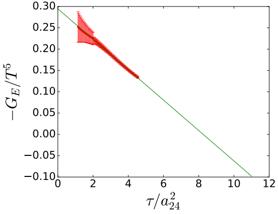
$\tau/a_{24}^2 = 1.5$
$t/a_{24} = 7$

Linear (in a^2) extrapolation works
FV effects are small

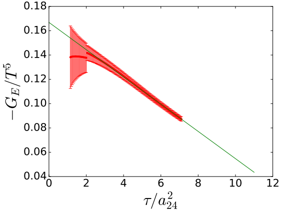
τ-extrapolation, $T/T_c = 1.24$



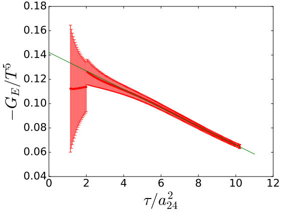
$t/a_{24} = 6$



$t/a_{24} = 8$



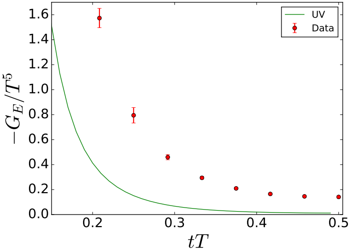
$t/a_{24} = 10$



$t/a_{24} = 12$

Linear (in τ) extrapolation works

Extrapolated correlator



$T/T_c = 1.24$

Asymptotic freedom

$$\rho_q(\omega) = -\frac{d_A \alpha_s^2}{256\pi^4}\omega^4 \qquad G_E(t) = \int_0^\infty \rho(\omega)\frac{\cosh \omega(\beta/2 - t)}{\sinh \omega\beta/2}d\omega$$

N. Iqbal, H. B. Meyer, 0911 (2009) 029, arXiv:0909.0582

UV part seems to be small!

Inversion of the integral relation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\cosh \omega\left(\frac{\beta}{2} - x_i\right)}{\sinh\left(\frac{\omega\beta}{2}\right)}$$

$\rho \to \rho/\omega$
$K(x,\omega) \to \omega K(x,\omega)$

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \omega \frac{\cosh \omega\left(\frac{\beta}{2} - x_i\right)}{\sinh\left(\frac{\omega\beta}{2}\right)}$$

We are interested in $\lim_{\omega \to 0} \rho(\omega) \sim const$

Two methods for the inversion:

- Backus-Gilbert method
- Maximum entropy method (preliminary results)

Backus-Gilbert method for the spectral function

Problem: find $\rho(\omega)$ from the integral equation

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Let $b_i(\omega)$ be arbitrary functions. Define $\tilde{\rho}(\bar{\omega})$:

$$\tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

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How should we choose $b_i(\omega)$?
Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

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Regularization by the parameter λ: numerically more stable, but wider resolution function

$$W_{ij} \to \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method. $T/T_c = 1.24$

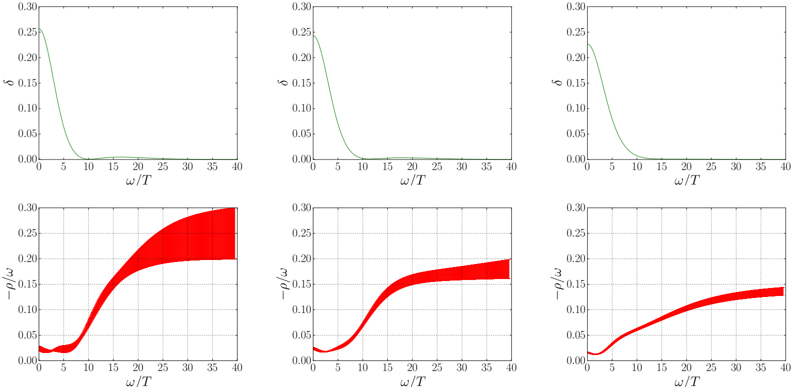
We obtain an estimator

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega)\rho(\omega)$$

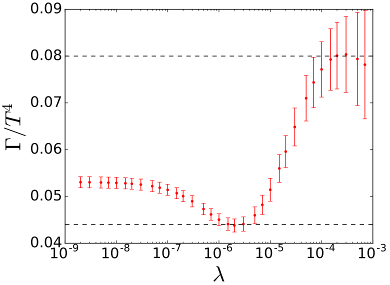
Regulator λ:

Statistical error $\leftrightarrow$ Width of the resolution function $\hat{\delta}(\bar{\omega}, \omega)$

$\lambda = 10^{-3}$ $\lambda = 10^{-4}$ $\lambda = 10^{-5}$



Backus-Gilbert. λ-dependence.



$$T/T_c = 1.24$$

$$\Gamma/T^4 = 0.062 \pm 0.018$$

The main source of uncertainty comes from analytical continuation.
Confirms the smallness of UV.

Maximum Entropy Method (preliminary!). $T/T_c = 1.24$

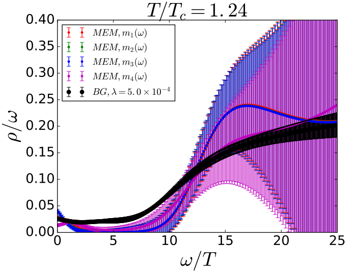
Instead of minimizing $\frac{1}{2}\chi^2$ minimize $\frac{1}{2}\chi^2 - \alpha S$

Entropy $S = \int_0^\infty d\omega[\rho(\omega) - m(\omega) - \rho(\omega)\log\frac{\rho(\omega)}{m(\omega)}]$

Prior function $m(\omega)$:

1. $m_1(\omega) = A\,\omega^3/\tanh^3(\omega/T)$
2. $m_2(\omega) = A\,\omega^3/\tanh^3(2\omega/T)$
3. $m_3(\omega) = A\,\omega^3/\tanh^3(4\omega/T)$
4. $m_4(\omega) = 2A\,\omega^3/\tanh^3(\omega/T)$

A is taken from estimation of UV part: $\rho_q(\omega) = -\frac{d_A\alpha_s^2}{256\pi^4}\omega^4$



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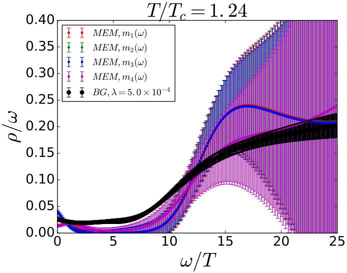
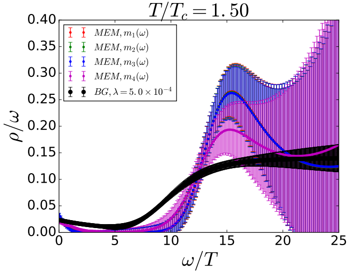
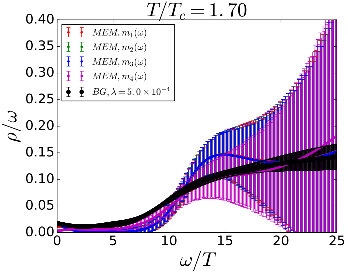
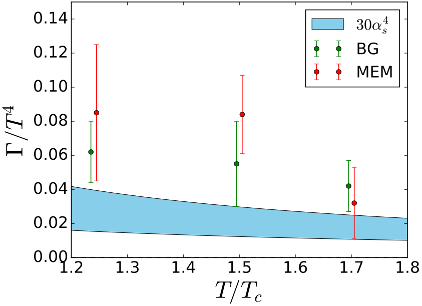






Results



"Desperate extrapolation" to finite α (semiclassical field theory)

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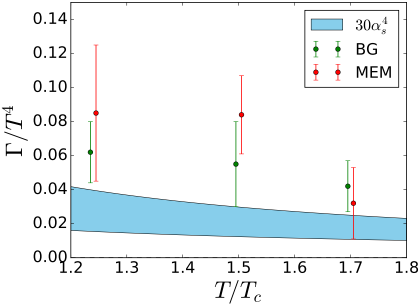
G.D. Moore and M.Tassler, *The Sphaleron Rate in SU(N) Gauge Theory*, JHEP 1102 (2011) 105, arXiv:1011.1167

Conclusions

- Continuum extrapolated correlator of $q(x)$
- Estimation of sphaleron rate in lattice gluodynamics



- The results are in agreement with

G.D. Moore and M.Tassler, *The Sphaleron Rate in SU(N) Gauge Theory*, JHEP 1102 (2011) 105, arXiv:1011.1167