

Lattice study of QCD at nonzero chiral density

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The results are presented in papers:

- JHEP 1506 (2015) 094
- Phys.Rev. D93 (2016) 034509
- Phys.Rev. D93 (2016) 105025
- Preliminary results

Outline:

- Introduction
- SU(2) QCD with nonzero chiral chemical potential
- SU(3) QCD with nonzero chiral chemical potential
- Chiral catalysis
- Chiral density and confinement
- Conclusion

Introduction

Motivation:

- QCD action can be separated into right and left parts:
 $S_{QCD} = S_R + S_L$
- There is symmetry between right and left parts
- One can introduce asymmetry of the form $\mu_5 Q_5$, $Q_5 = Q_R - Q_L$
- $S_{QCD}(\mu_5) = S_R(\mu = +\mu_5) + S_L(\mu = -\mu_5)$
- $\mu_5 \neq 0$ can be created in
 - Heavy ion collisions
 - Neutron stars and supernovae
 - Early Universe
 - Dirac and Weyl semimetals ($\vec{E} \parallel \vec{H}$)
 - Elastic deformation

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How nonzero chiral chemical potential influences the properties of QCD

Studies of the phase diagram of chiral QCD

- "Chiral magnetic effect in the PNJL model", Kenji Fukushima, Marco Ruggieri, Raoul Gatto, Phys.Rev. D81 (2010) 114031
- "Phase diagram of chirally imbalanced QCD matter", M.N. Chernodub, A.S. Nedelin, Phys.Rev. D83 (2011) 105008
- "Hot Quark Matter with an Axial Chemical Potential", Raoul Gatto, Marco Ruggieri, Phys.Rev. D85 (2012) 054013
- "Inverse magnetic catalysis induced by sphalerons", Jingyi Chao, Pengcheng Chu, Mei Huang, Phys.Rev. D88 (2013) 054009
- "Spontaneous generation of local CP violation and inverse magnetic catalysis", Lang Yu, Hao Liu, Mei Huang, Phys.Rev. D90 (2014) 7, 074009
- "The effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes", Lang Yu, Hao Liu, Mei Huang, arXiv:1511.03073
- ...

Results:

- Decrease of the critical temperature with chiral chemical potential
- Decrease of the chiral condensate with chiral chemical potential

Studies of the phase diagram of chiral QCD

- "Universality of phase diagrams in QCD and QCD-like theories", M. Hanada, N. Yamamoto, PoS LATTICE2011, 221 (2011), arXiv:1111.3391
- "Chemical potentials and parity breaking: the Nambu-Jona-Lasinio model", Alexander A. Andrianov, Domenc Espriu, Xumeu Planells, Eur.Phys.J. C74 (2014) 2, 2776
- "Effect of the chiral chemical potential on the position of the critical endpoint", Bin Wang, Yong-Long Wang, Zhu-Fang Cui, Hong-Shi Zong, Phys.Rev. D91 (2015) 3, 034017
- "Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations", Shu-Sheng Xu, Zhu-Fang Cui, Bin Wang, Yuan-Mei Shi, You-Chang Yang, Hong-Shi Zong, Phys.Rev. D91 (2015) 5, 056003
- "Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential", M. Ruggieri, G.X. Peng, arXiv:1602.05250
- "Nonlocal Nambu-Jona-Lasinio model and chiral chemical potential", Marco Frasca, arXiv:1602.04654
- "Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential", M. Ruggieri, G.X. Peng, arXiv:1602.03651
- "Quark matter with a chiral imbalance in the Nambu-Jona-Lasinio model" R.L.S. Farias, Dyana C. Duarte, Gastão Krein, Rudnei O. Ramos, arXiv:1604.04518
- ...

Results:

- Increase of the critical temperature with chiral chemical potential
- Increase of the chiral condensate with chiral chemical potential

SU(2) QCD
with
nonzero chiral chemical potential

Details of the calculation:

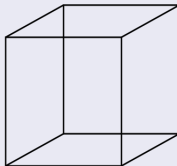
- Dynamical staggered fermions ($N_f = 4$) + Wilson action (SU(2))

- Link modification:

$$U \rightarrow Ue^{\mu_5\gamma_5}, \quad U^+ \rightarrow U^+e^{-\mu_5\gamma_5} \Rightarrow \text{nonlocal action}$$

- Chiral chemical potential:

$$\delta S_{\mu_5} = \frac{1}{2}\mu_5 a \sum_x (-1)^{x_2} (\bar{\psi}_{x+\delta} \bar{U}_{x+\delta,x} \psi_x - \bar{\psi}_x \bar{U}_{x+\delta,x}^\dagger \psi_{x+\delta})$$



- Correct continuum limit: $\delta S_{\mu_5}|_{a \rightarrow 0} \rightarrow \mu_5 \int d^4x \bar{Q}(\gamma_4\gamma_5 \times 1) Q$
- $6 \times 20^3 (m_\pi \sim 300\text{MeV}), 10 \times 28^3 (m_\pi \sim 500\text{MeV})$

Observables:

- The Polyakov loop (confinement/deconfinement transition)

$$L = \frac{1}{N_\sigma^3} \sum_{n_1, n_2, n_3} \langle \text{Tr} \prod_{n_4=1}^{N_\tau} U_4(n_1, n_2, n_3, n_4) \rangle$$

- The chiral condensate (chiral symmetry breaking/restoration transition)

$$a^3 \langle \bar{\psi} \psi \rangle = -\frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \frac{\partial}{\partial (ma)} \log Z = \frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \langle \text{Tr} \frac{1}{D+ma} \rangle$$

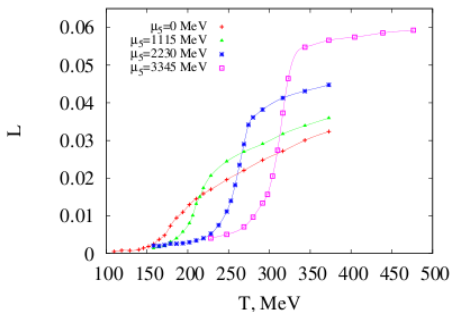
- The Polyakov loop susceptibility
(position of the transition)

$$\chi_L = N_\sigma^3 (\langle L^2 \rangle - \langle L \rangle^2)$$

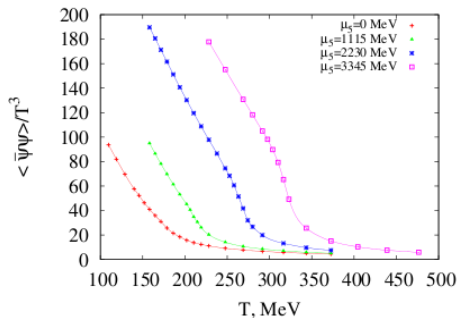
- The disconnected part of the chiral susceptibility
(position of the transition)

$$\chi_{disc} = \frac{1}{N_\tau N_\sigma^3} \frac{1}{16} (\langle (\text{Tr} \frac{1}{D+ma})^2 \rangle - \langle \text{Tr} \frac{1}{D+ma} \rangle^2)$$

Polyakov loop and chiral condensate

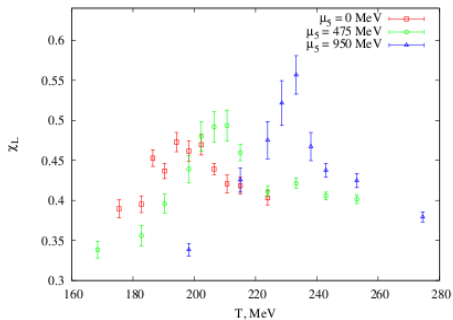


Polyakov loop

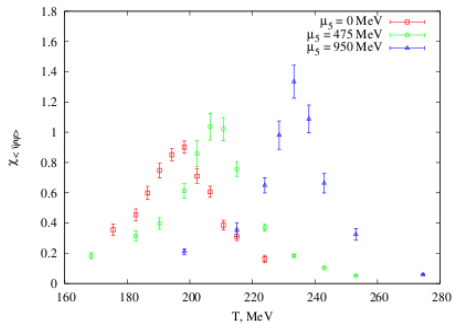


chiral condensate

Susceptibilities

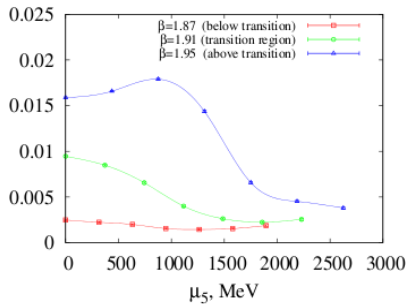


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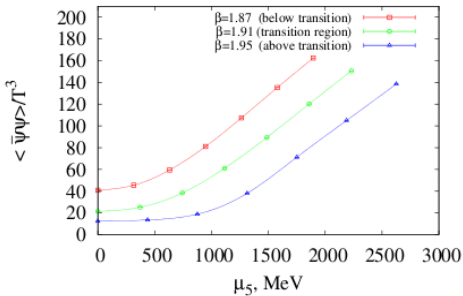


chiral susceptibility

Fixed temperature scan



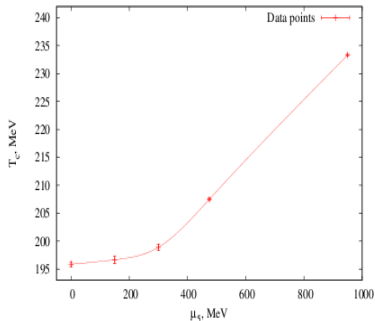
Polyakov loop



chiral condensate

Results of the calculation:

- The critical temperatures increase
- The critical temperatures of the confinement/deconfinement phase transition and of the chiral symmetry breaking/restoration coincide

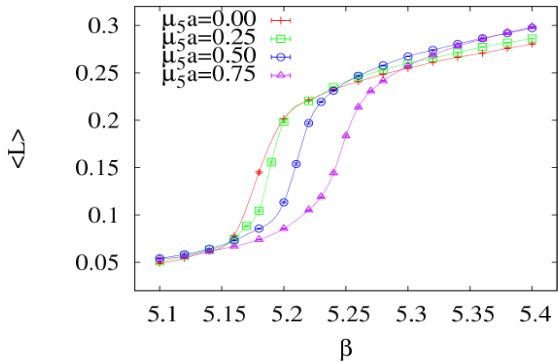


SU(3) QCD
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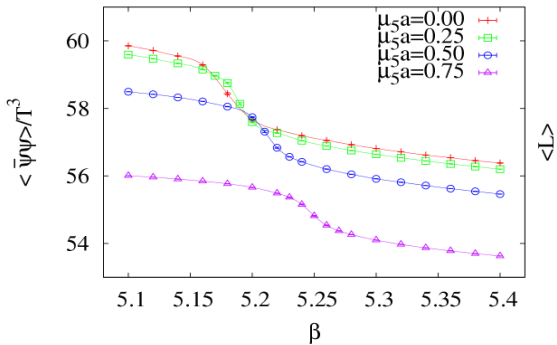
Details of the calculation:

- Dynamical Wilson fermions ($N_f = 2$) + Wilson action (SU(3))
- Link modification:
$$U \rightarrow Ue^{\mu_5\gamma_5}, \quad U^+ \rightarrow U^+e^{-\mu_5\gamma_5}$$
- Continuum limit: $\delta S_{\mu_5}|_{a \rightarrow 0} \rightarrow \mu_5 \int d^4x \bar{Q}(\gamma_4\gamma_5 \times 1)Q$
- $4 \times 16^3 (m_\pi \sim 400\text{MeV})$

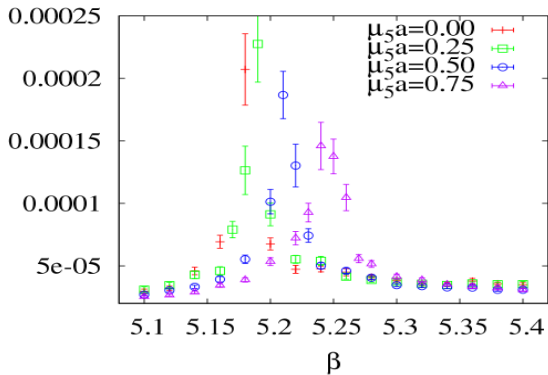
Polyakov loop



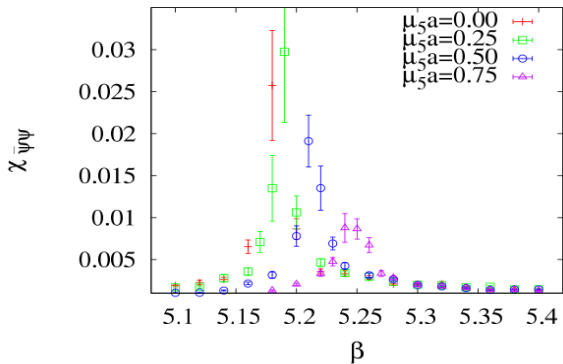
Chiral condensate



Polyakov loop susceptibility

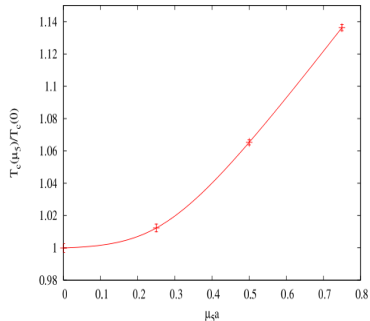


Chiral condensate susceptibility



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- QCD-like theories:
 - $SU(2)$ $N_f = 4$
 - $SU(3)$ $N_f = 2$
- Different ways of introduction of chiral chemical potential
- Similar results (enhancement of chiral symmetry breaking)

Chiral catalysis

NJL model ($U_L(1) \times U_R(1)$, N_c colors)

- $S_E = \int d^4x \left(\bar{\psi} (\partial + m - \mu_5 \gamma_4 \gamma_5) \psi - G [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2] \right)$

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- $S_E = \int d^4x \left(\bar{\psi} (\hat{\partial} + m - \mu_5 \gamma_4 \gamma_5 + \sigma + i \gamma_5 \pi) \psi + \frac{1}{4G} [\sigma^2 + \pi^2] \right)$

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Gap equation ($N_c \rightarrow \infty$, $m \rightarrow 0$)

- $\frac{\delta S_{\text{eff}}}{\delta \sigma} = \frac{\sigma}{2G} - N_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[\frac{1}{i \hat{k} - \mu_5 \gamma_4 \gamma_5 + \sigma + i \gamma_5 \pi} \right] = 0$

Dynamical mass $M = \sigma$

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- $\frac{\pi^2}{GN_c} = \int_0^\Lambda k^2 dk \left[\frac{1}{\sqrt{(|\vec{k}| - \mu_5)^2 + M^2}} + \frac{1}{\sqrt{(|\vec{k}| + \mu_5)^2 + M^2}} \right]$

Gap equation

$$\frac{1}{\alpha_{NJL}} - 1 = \left(y^2 - \frac{x^2}{2} \right) \log \frac{1}{x^2}$$

$$\alpha_{NJL} = \frac{GN_c \Lambda^2}{\pi^2}, \quad x = \frac{M}{\Lambda}, \quad y = \frac{\mu_5}{\Lambda}$$

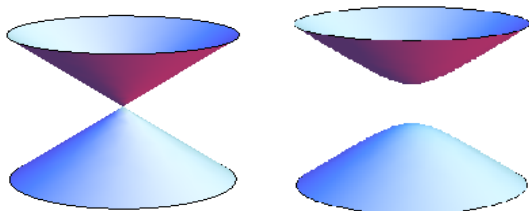
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Properties ($\mu_5 = 0$):

- $\alpha_{NJL} < 1$ no solutions, $M = 0$, $E^2 = \vec{p}^2$
- $\alpha_{NJL} > 1$ there is solution $M \neq 0$, $E^2 = \vec{p}^2 + M^2$



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$$\mu_5 \sim M: \quad M^2 \simeq 2\mu_5^2 \left(1 - \frac{1-\alpha_{NJL}}{\alpha_{NJL}} \frac{1}{2y^2 \log\left(\frac{1}{2y^2}\right)} \right).$$

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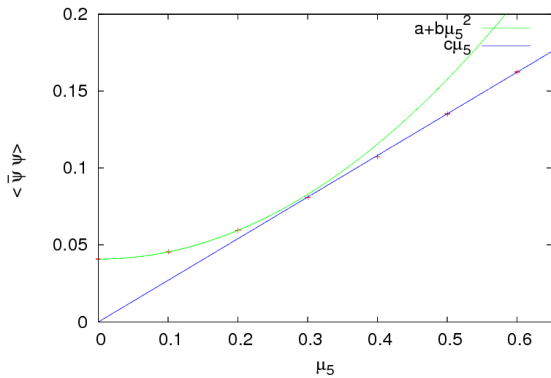
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μ_5 enhances dynamical chiral symmetry breaking

Prediction:

In strong coupling region NJL model (also CHPT) predicts that dynamical fermion mass is quadratically rising function at small μ_5 which switches to linear rising behaviour at large μ_5



Chiral symmetry breaking as condensation of Cooper pairs (BCS theory)

- Vacuum: $|vac\rangle = \hat{G}_1 \hat{G}_2 \hat{G}_3 |PF\rangle$,
$$\hat{G}_1 = \prod_{\mathbf{p}} \left(\cos(\theta_L) - \sin(\theta_L) \hat{a}_{L,\mathbf{p}}^+ \hat{b}_{L,-\mathbf{p}}^+ \right)$$
$$\hat{G}_2 = \prod_{\mathbf{p} > \mu_B} \left(\cos(\theta_R) + \sin(\theta_R) \hat{a}_{R,\mathbf{p}}^+ \hat{b}_{R,-\mathbf{p}}^+ \right)$$
$$\hat{G}_3 = \prod_{\mathbf{p} < \mu_B} \left(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R) \hat{b}_{R,-\mathbf{p}} \hat{a}_{R,\mathbf{p}} \right)$$

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- Energy:

$$E_{vac} = 2N_c \left[\int_{\mathbf{p} < \mu_5} \frac{d^3 p}{(2\pi)^3} (p-\mu) \cos^2 \tilde{\theta}_R + \int_{\mathbf{p} > \mu_5} \frac{d^3 p}{(2\pi)^3} (p-\mu) \sin^2 \theta_R + \int \frac{d^3 p}{(2\pi)^3} (p+\mu) \sin^2 \theta_L \right]$$
$$- GN_c^2 \left(\int_{\mathbf{p} < \mu_5} \frac{d^3 p}{(2\pi)^3} \sin 2\tilde{\theta}_R + \int_{\mathbf{p} > \mu_5} \frac{d^3 p}{(2\pi)^3} \sin 2\theta_R + \int \frac{d^3 p}{(2\pi)^3} \sin 2\theta_L \right)^2$$

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- Minimum energy is realized at the gap equation

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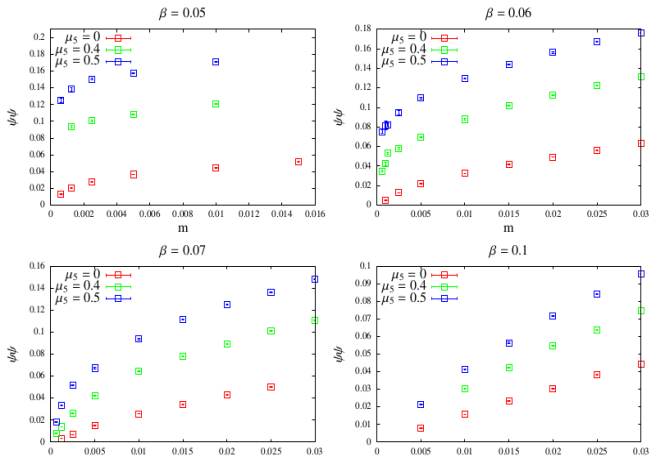
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We called this phenomena: **CHIRAL CATALYSIS**

Numerical simulation of Dirac semimetals

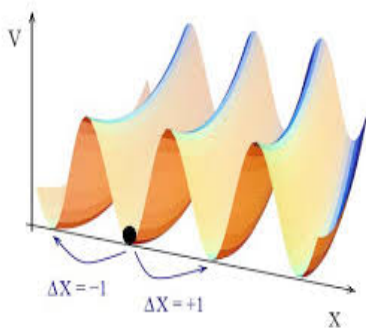
- Staggered fermions with rooting
- Instantaneous Coulomb interaction with α_{eff}
- Asymmetry in Fermi velocity
- The first study of the phase diagram in $(\alpha_{\text{eff}}, v_{\perp}/v_{\parallel})$ plane

V.V. Braguta, M.I. Katsnelson, A. Yu. Kotov, A.A. Nikolaev, Phys.Rev. B94 (2016) no.20, 205147

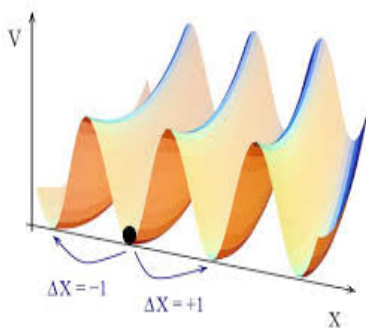


Chiral density and confinement

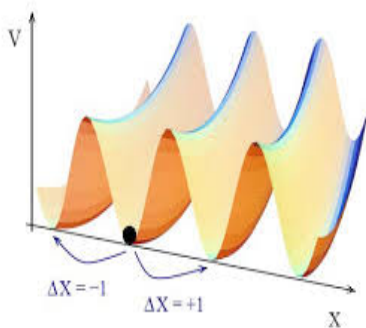
(Preliminary results)



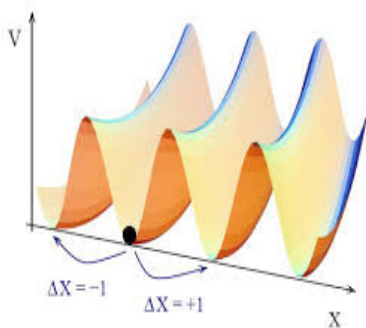
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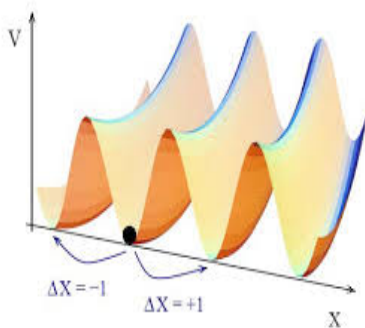
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- The larger μ_5 the larger the fluctuations of topological charge (μ_5 as a fluctuation catalyst)



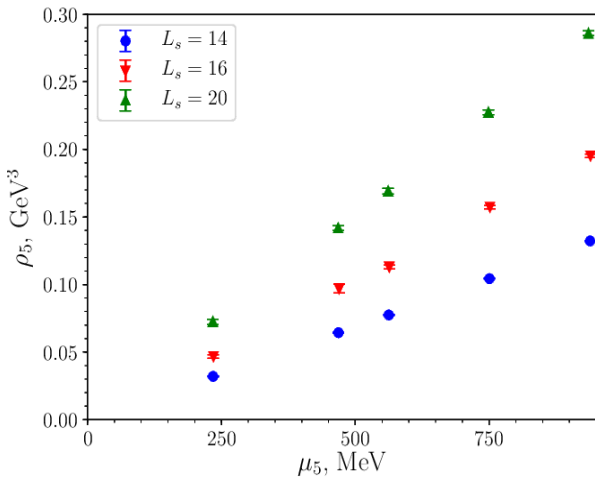
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Properties of QCD with enhanced topological charge fluctuations



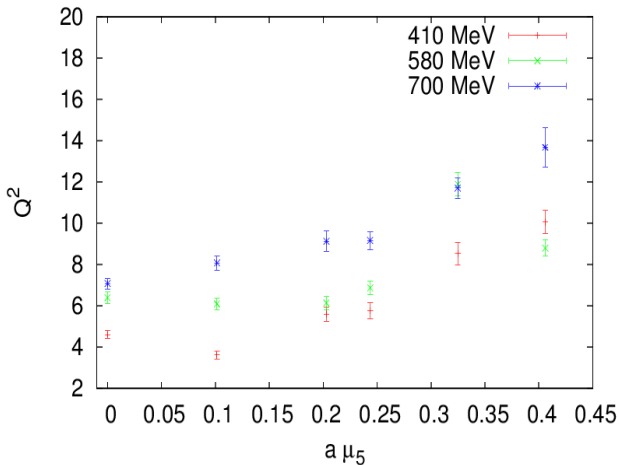
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- Idea: Confinement will be “enhanced”
“Color confinement from fluctuating topology”, D. Kharzeev, Int.J.Mod.Phys. A Vol. 31, (2016), 1645023

Details of lattice simulation:

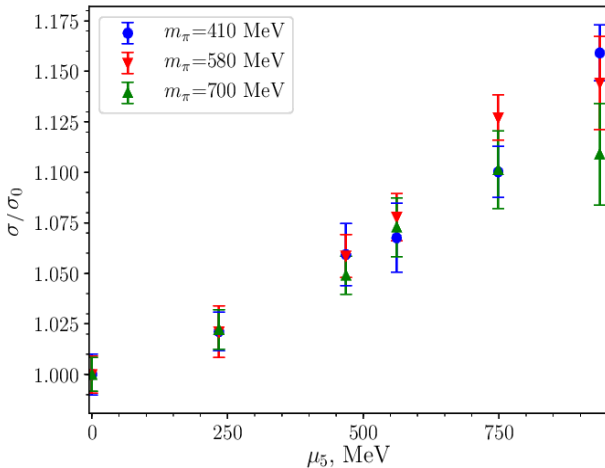
- SU(3)
- Dynamical staggered fermions with rooting
(In continuum $N_f = 2$)
- Lattices (The physical volume is fixed: $L \simeq 1.7$ fm):
 - $a = 0.128$ fm, $L_s = 14$
 - $a = 0.105$ fm, $L_s = 16$
 - $a = 0.086$ fm, $L_s = 20$
- Pion masses:
 - $m_\pi = 410$ MeV
 - $m_\pi = 580$ MeV
 - $m_\pi = 700$ MeV



- We observe only divergence $\rho_5 = \frac{0.057}{a^2} \mu_5$
- It is difficult to see physical result since $\rho_5 \sim \frac{0.057 + (af_\pi)^2}{a^2} \mu_5 = \frac{0.057 + 0.002}{a^2} \mu_5$
- We have predictions of CHPT: $\rho_5 \sim f_\pi^2 \mu_5$



Chiral chemical potential enhances the topological charge fluctuations



Chiral chemical potential enhances the string tension

Conclusion

Conclusion:

- We carried out numerical study of SU(2) and SU(3) QCD with nonzero chiral chemical potential
- Critical temperatures of confinement/deconfinement and breaking/restoration of chiral symmetry transitions rise with chiral chemical potential
- CHIRAL CATALYSIS: Chiral chemical potential creates/enhances chiral symmetry breaking
- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$
- Chiral chemical potential enhances the topological charge fluctuations and, as the result, the string tension is enhanced