Temperature dependence of bulk and shear viscosities from lattice $SU(3)$–gluodynamics

N. Astrakhantsev, V. Braguta, A. Kotov

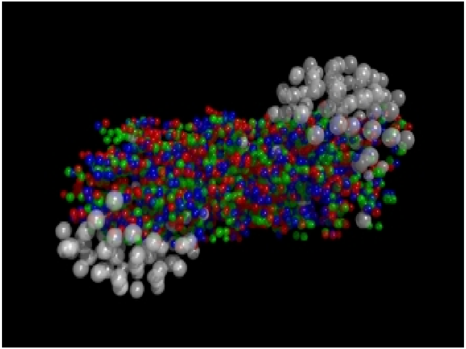
based on arXiv:1701.02266, JHEP 1704 (2017) 101
& new results

Dubna, 2018

Outline

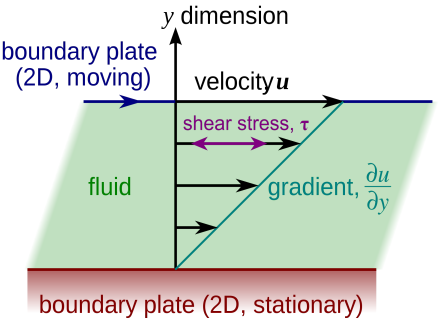
- Introduction
- Details of the calculation
- Shear viscosity
  - Fitting of the data
  - Backus-Gilbert method
- Bulk viscosity
  - Middle point method
  - Backus-Gilbert method
- Conclusion

Heavy ion collisions



In heavy ion collision experiments, the number of initial particles might be of order 10^5: hydrodynamic description is used.

Shear viscosity



Shear viscosity governs friction of layers moving with different velocities.

Relativistic Hydrodynamics

- $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + (\eta\nabla^{\langle\mu}u^{\nu\rangle} + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \ldots$
  $\nabla^\alpha = \Delta^{\alpha\nu}\partial_\nu, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
  $\nabla^{\langle\mu}u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$

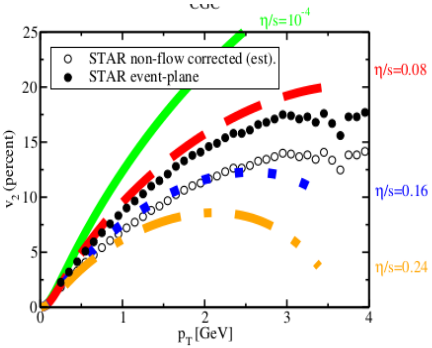
- EOM $\partial_\mu T^{\mu\nu} = 0$

- Non-relativistic limit $(u^\mu = (1, \vec{v}))$

  - *Continuity equation:* $\partial_t\rho + \rho(\vec{\partial}\vec{v}) + \vec{v}\vec{\partial}\rho = 0$
  - *Navier–Stokes equation:* $\frac{\partial v^i}{\partial t} + v^k\frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$
  - *Viscous stress tensor:* $\Pi^{ik} = -\eta\left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ik}\frac{\partial v^l}{\partial x^l}\right) - \zeta\delta^{ik}\frac{\partial v^l}{\partial x^l}$

- η — shear viscosity, ζ — bulk viscosity

Relativistic hydrodynamics & QGP



- Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\phi} \sim (1 + 2v_1 \cos(\phi) + 2v_2 \cos^2(\phi)), \phi\texttt{-scattering angle}$$

- Quark-gluon plasma is close to ideal liquid ($\frac{\eta}{s} = (1-3)\frac{1}{4\pi}$)

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)

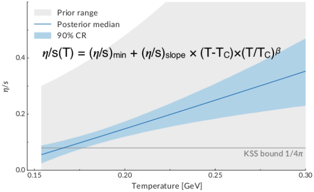
QM2017 (talk by S.Bass)

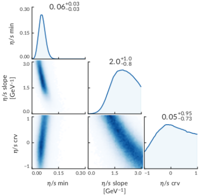
Temperature Dependence of Shear & Bulk Viscosities

temperature dependent shear viscosity:

- analysis favors small value and shallow rise
- results do not fully constrain temperature dependence:
  - inverse correlation between $(\eta/s)_{slope}$ slope and intercept $(\eta/s)_{min}$
  - insufficient data to obtain sharply peaked likelihood distributions for $(\eta/s)_{slope}$ and curvature β independently
- current analysis most sensitive to T< 0.23 GeV

▸ **RHIC data may disambiguate further**



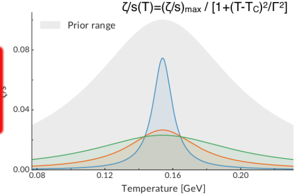


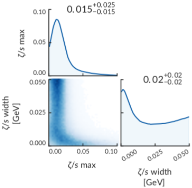
temperature dependent bulk viscosity:

- setup of analysis allows for vanishing value of bulk viscosity
- significant non-zero value at T_C favored, confirming the presence / need for bulk viscosity
- either high sharp peak or broad & shallow temperature dependence

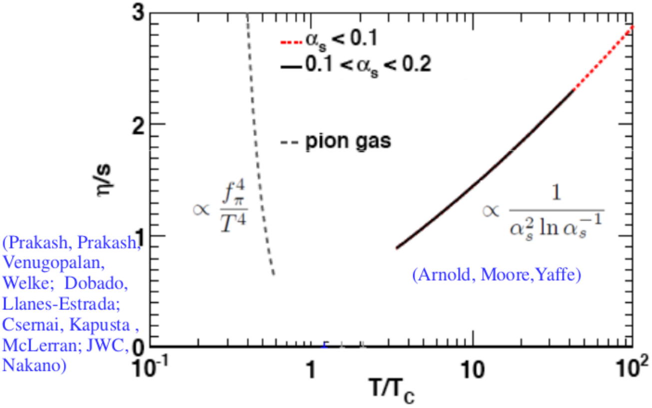
caveat of current analysis:

- bulk-viscous corrections are implemented using relaxation-time approximation & regulated to prevent negative particle densities

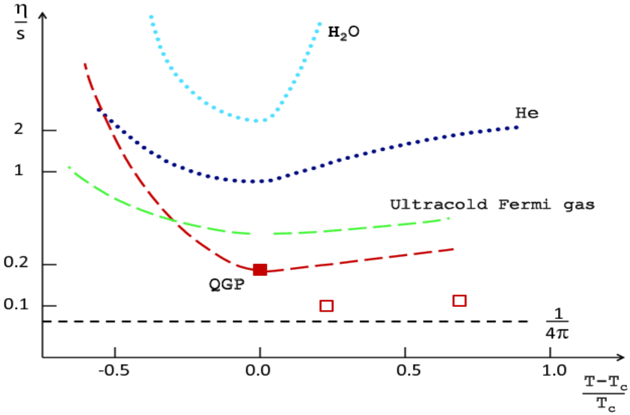




Shear viscosity in two limits



QGP is the most ideal fluid



The minimum of η/s is close to the prediction of $N = 4$ SYM at strong coupling.

Our goal

First-principle determination of shear and bulk viscosities!

Lattice calculation of shear & bulk viscocity

The first step:

Measurement of the correlation functions:

$$C_{sh}(t) = \int d^3\vec{x} \langle T_{12}(t,\vec{x}) T_{12}(0) \rangle$$

$$C_b(t) = \int d^3\vec{x} \langle T_{\mu\mu}(t,\vec{x}) T_{\nu\nu}(0) \rangle$$

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The second step (analytical continuation):

Calculation of the spectral function $\rho(\omega)$:

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho_{sh}(\omega)}{\omega}$$

$$\zeta = \frac{\pi}{9} \lim_{\omega\to 0} \frac{\rho_b(\omega)}{\omega}$$

Details of the calculation

- $SU(3)$–gluodynamics
- Two-level algorithm (only for gluodynamics)
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.2, 1.35, 1.5$
- Accuracy $\sim 2-3\%$ at $t = \frac{1}{2T}$
- For $\langle T_{12}(x)T_{12}(y)\rangle \sim (\langle T_{11}(x)T_{11}(y)\rangle - \langle T_{11}(x)T_{22}(y)\rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285

Shear viscosity

Multilevel algorithms

One needs to calculate

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}(0) \rangle$$

A theory is called *local* if for any disjoint X and Y one can write:

$$p\left(\mathcal{X}, \mathcal{Y}\right) = \sum_{\mathcal{A}} p(\mathcal{A}) p_A(\mathcal{X}) \tilde{p}_A(\mathcal{Y}),$$

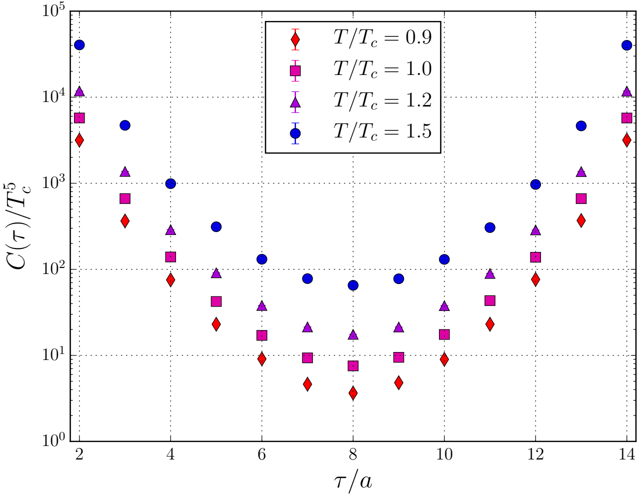
here p is some configuration probability functional. Thus, one can expand

$$\langle \mathcal{O}_x \mathcal{O}_y \rangle = \sum_{\mathcal{C}} \mathcal{O}_x(\mathcal{C}) \mathcal{O}_y(\mathcal{C}) p(\mathcal{C}) = \sum_{\mathcal{A}} p(\mathcal{A}) \langle \mathcal{O}_x \rangle_A \langle \mathcal{O}_y \rangle_A,$$

where $\langle \mathcal{O}_z \rangle_A = \sum_{\mathcal{Z}} p_A(\mathcal{Z}) \mathcal{O}_z(\mathcal{Z})$ — mean values of operator for fixed $\mathcal{A}$. So process factorizes: average at fixed BC and then varying BC.

Correlation functions (shear viscosity)

Configuration parts separated by boundary are considered independent: N updates give N^2 independent measurements.



Spectral function

$$C_{sh}(t) = \int_0^\infty d\omega \rho_{sh}(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0,\ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega\to\infty}^{NLO} = \frac{1}{10}\frac{d_A}{(4\pi)^2}\omega^4\left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  $\sim 90\%$ of the total contribution $t = 1/(2T)$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{\eta}{\pi}\omega$

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Methods of integral equation inversion

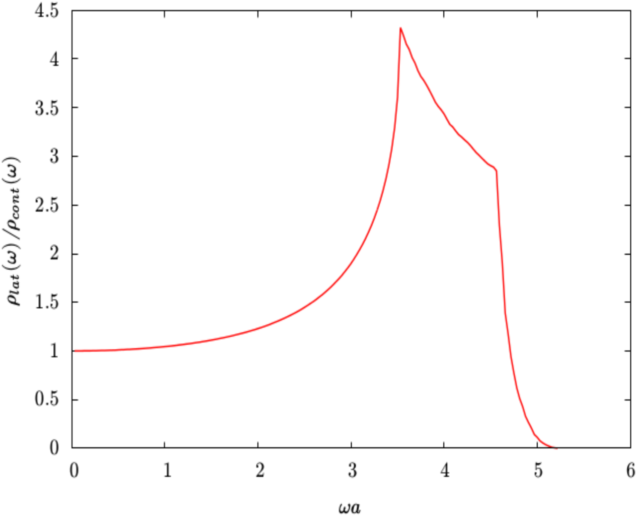
The problem is ill-posed, regularization should be applied.

- Physically-motivated fitting procedure
- Classical non-parametric extimation procdeures (Maximum Entropy method, Backus-Gilbert method)
- Regularization using Neural Networks (approach is only being developed now).

Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

Lattice spectral function



Properties of the spectral function

- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H.B. Meyer, arXiv:0809.5202)

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- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H.B. Meyer, arXiv:0809.5202)
- Asymptotic freedom works well from $\omega > 3$ GeV
- Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV
  $\Rightarrow$ Main source of uncertainty in the fitting procedure

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int\limits_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\cosh\left(\frac{\omega}{2T} - \omega x_i\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) \rho(\omega)$$

- Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

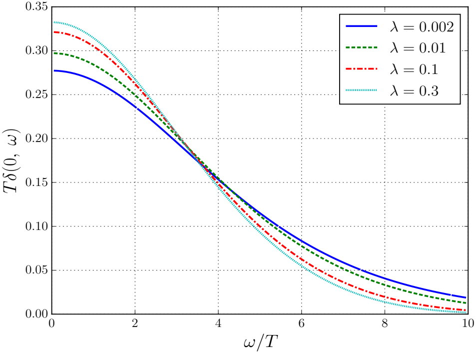
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution function $\delta(0,\omega)$ $(T/T_c = 1.35)$



- Width of the resolution function $\omega/T \sim 4$
- Hydrodynamical approximation works up to $\omega/T < \pi$
- Problem: large contribution from ultraviolet tail ($\sim 50\%$)
- Solution: UV contribution can be subtracted as we know UV part from the fitting procedure quite well

Subtraction of UV contribution

We assume

$$f_{\rm UV}(\omega) = \rho_{\rm UV}(\omega),$$

and rescale kernel

$$K(\omega, t) = \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

as

$$K \to K \times f_{\rm UV}(\omega).$$

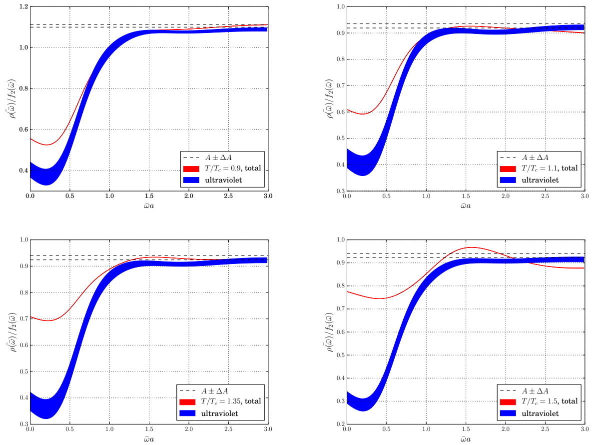
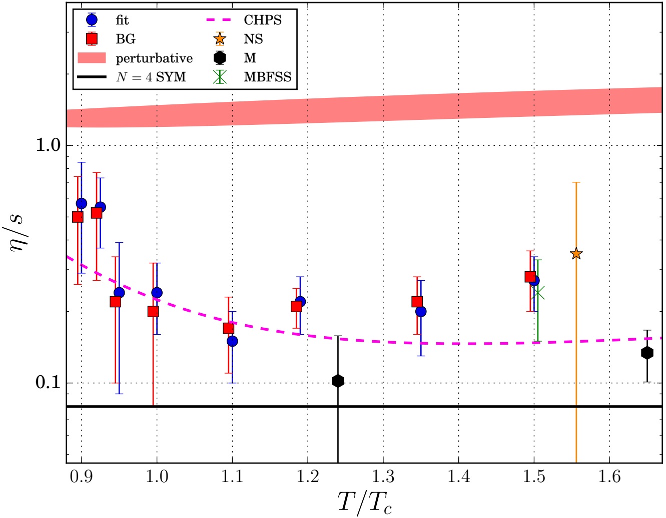


Рис.: The ratios $\bar{\rho}(\bar{\omega})/f_2(\bar{\omega})$ as a function of $\bar{\omega}a$ for the temperatures $T/T_c = 0.9, 1.1, 1.35, 1.5$. Red curves correspond to spectral functions restored by the BG method from the data. Blue curves correspond to the ultraviolet contribution convoluted with the resolution function. Dashed lines are values of the constants A with uncertainties obtained within fitting procedure.

Results

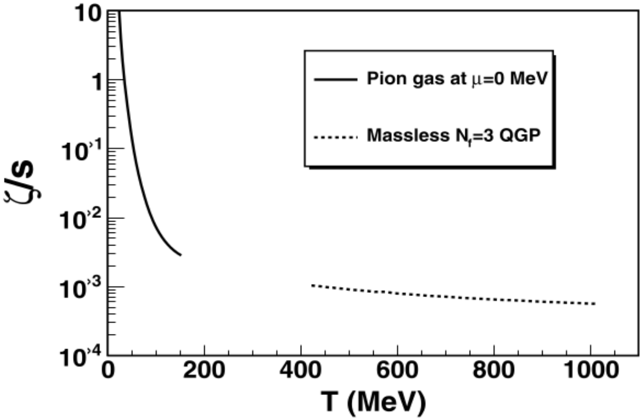

fit
BG
perturbative
$N = 4$ SYM
CHPS
NS
M
MBFSS
1.0
0.1
η/s
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
T/T_c

Results

- Shear viscosity of $SU(3)$–gluodynamics was measured on the lattice for $T/T_c \in [0.9,\, 1.5]$.
- We see closeness of $\eta(T)/s(T)$ to $1/4\pi$ prediction of $N = 4$ SYM at strong coupling,
- We also observe disagreement with the naive perturbative calculations for all temperatures: QGP remains strongly correlated at $1.5\,T_c$ (and even further).
- Shear viscosity of $SU(2)$ and $SU(3)$ theories agrees at $T = 1.2\,T_c$, which is surprising.

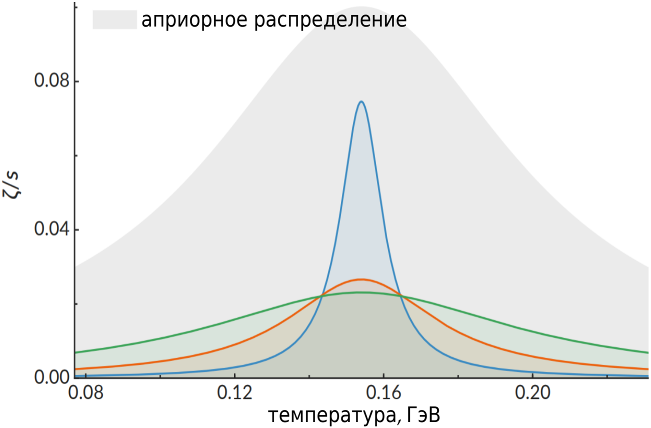
Bulk viscosity

Bulk viscosity in two limits



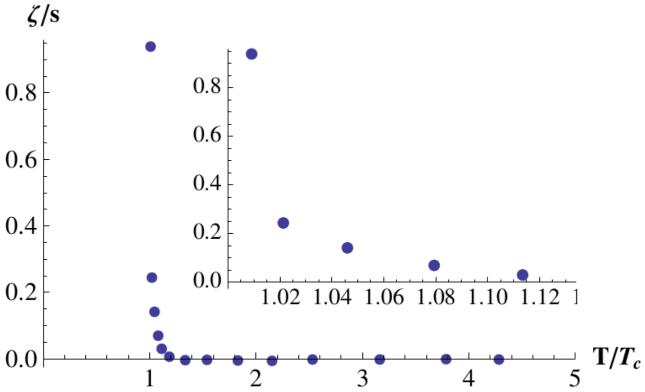
- **CHPT:** A. Dobado, F.J. Llanes-Estrada, J.M. Torres-Rincon, Physics Letters B 702 (2011) 43
- **Perturbative QCD:** P. Arnold, C. Dogan, G. Moore , Physical Review D 74, 085021 (2006)

Bayesian analysis of the experimental data (S. Bass)



Statistical analysis of many-variable hydrodinamic models predict a peak of smaller magnitude.

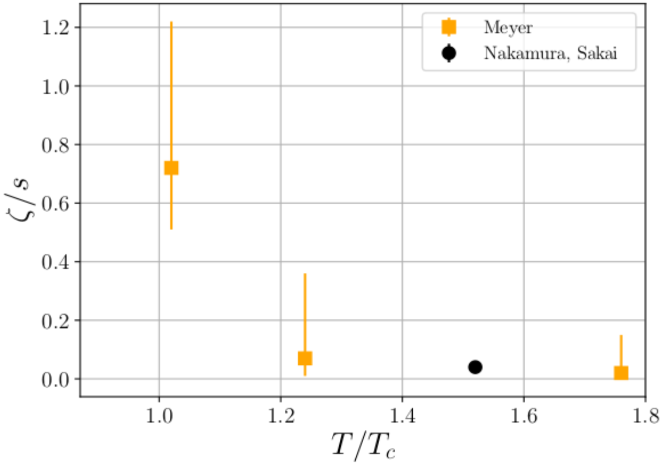
Low energy theorems of QCD



- $\zeta = \frac{1}{9\omega_0}\left(T^5 \frac{\partial}{\partial T}\frac{e-3p}{T^4} + 16\epsilon_v\right)$

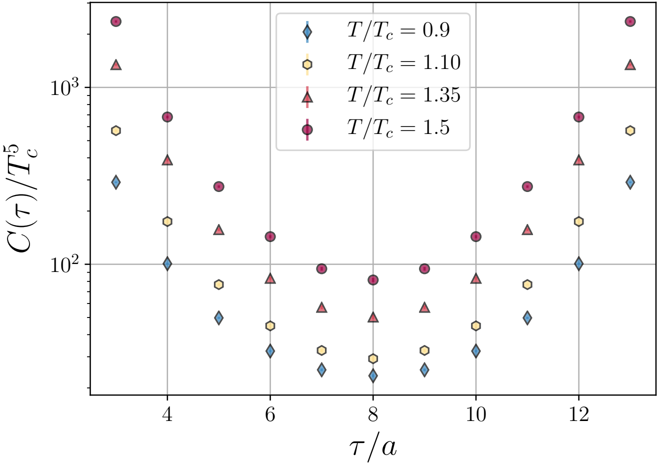
  D. Kharzeev, K. Tuchin, JHEP 0809 (2008) 093,
  D. Kharzeev, F. Karsch, K. Tuchin, Phys.Lett. B663 (2008) 217

Previous lattice works ($SU(3)$–gluodynamics)

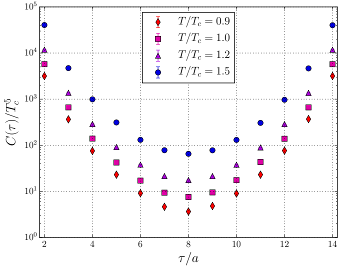


- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev.Lett. 100 (2008) 162001

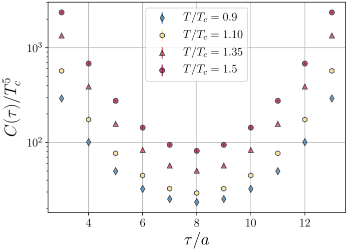
Correlation functions (bulk viscosity)



Correlation functions (shear & bulk viscosity)



shear



bulk

Spectral function

$$C_b(t) = \int_0^\infty d\omega \rho_b(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

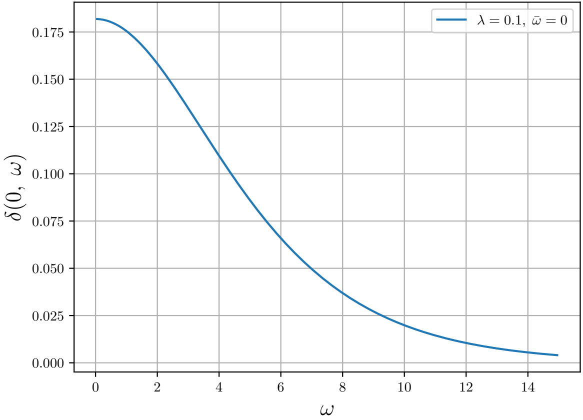
Properties of the spectral function:

- $\rho(\omega) \geq 0,\ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega\to\infty}^{NLO} = d_A \left(\frac{11\alpha_s}{(4\pi)^2}\right)^2 \omega^4$

  compare with shear channel $\sim d_A \frac{1}{10(4\pi)^2}\omega^4$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{9}{\pi}\zeta\omega$

Backus-Gilbert method

Resolution function $\delta(0,\omega)$ $(T/T_c = 1.5,\ \lambda = 0.1)$



- Width of the resolution function $\omega/T \sim 5$, ultraviolet contribution subtraction required.

Subtraction of UV contribution

We assume

$$f_{\text{UV}}(\omega) = \alpha_s^2(\omega)\rho_{\text{UV}}(\omega),$$

and rescale kernel

$$K(\omega, t) = \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

as

$$K \to K \times f_{\text{UV}}(\omega).$$

Subtraction of UV contribution

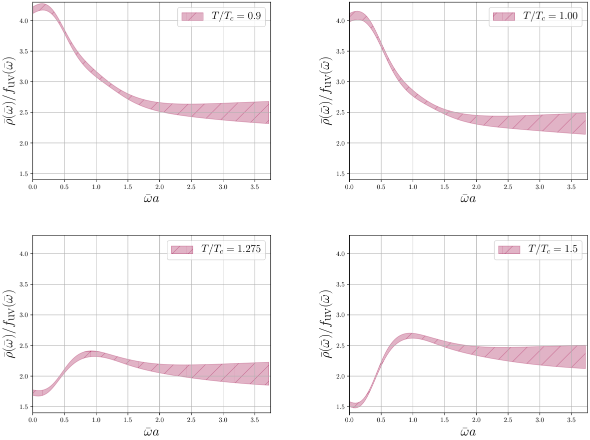
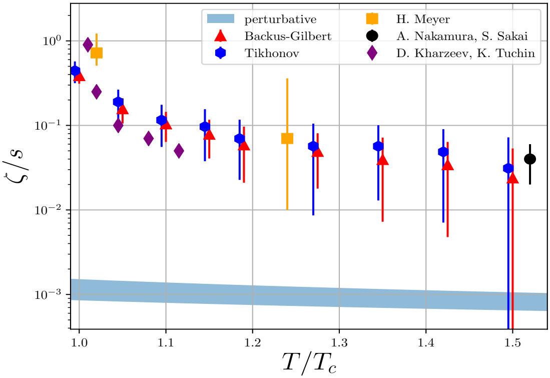


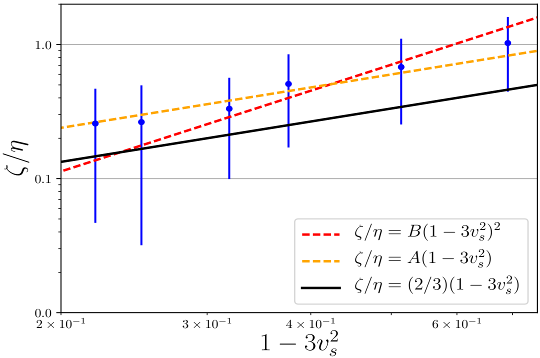
Рис.: The ratio $\bar{\rho}(\bar{\omega})/f_{\text{UV}}(\bar{\omega})$ reconstructed within the BG method as a function of $\bar{\omega}a$ for the temperatures: $T/T_c = 0.9$, 1.0, 1.275, 1.5.

Comparison with other approaches



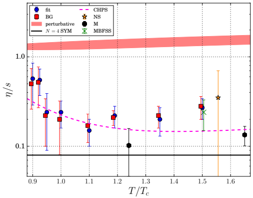
- Agreement with other lattice studies
- Large deviation from perturbative results

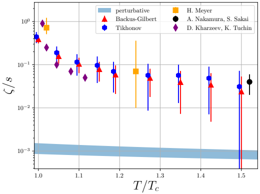
Is QGP weakly or strongly coupled?

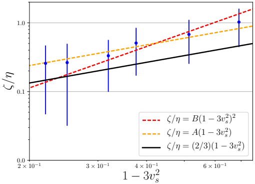


- Weakly coupled system $\zeta/\eta \sim (1-3v_s^2)^2$ ($\chi^2/\mathrm{ndof} \sim 1$)
- Strongly coupled system $\zeta/\eta \sim (1-3v_s^2)$ ($\chi^2/\mathrm{ndof} \sim 1$)
- $\zeta/\eta \geq \frac{2}{3}(1-3v_s^2)$ (A. Buchel, Physics Letters B663, 286 (2008))

Results and Conclusions







- We calculated η/s and ζ/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Agreement with previous lattice results and effective models
- Large deviation from perturbative calculation
- QGP reveals the properties of strongly coupled system