Screening masses and quark free energies at small baryonic density (2+1 QCD)

(Andreoli et. al, Phys.Rev. D97 (2018) 5, 054515)

M. Andreoli[1,2], C. Bonati[1,2], M. D'Elia[1,2], M. Mesiti[4], F. Negro[2], **A. Rucci**[1,2] and F. Sanfilippo[3]

[1] Dipartimento di Fisica dell'Università di Pisa, Pisa, Italy
[2] INFN Sezione di Pisa, Pisa, Italy [3] INFN Sezione di Roma Tre, Roma, Italy
[4] Academy of advanced computing, Swansea University, United Kingdom

II International Workshop on *"Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics"* Dubna, September 4 - 6, 2018

Contents

- Introduction
- Screening masses at finite density
- Single quark free energy
- Conclusions

Introduction

$Q\bar{Q}$ interaction

Static color charges are useful probes of the properties of strongly interacting matter.

- **Low temperatures**: the Cornell potential

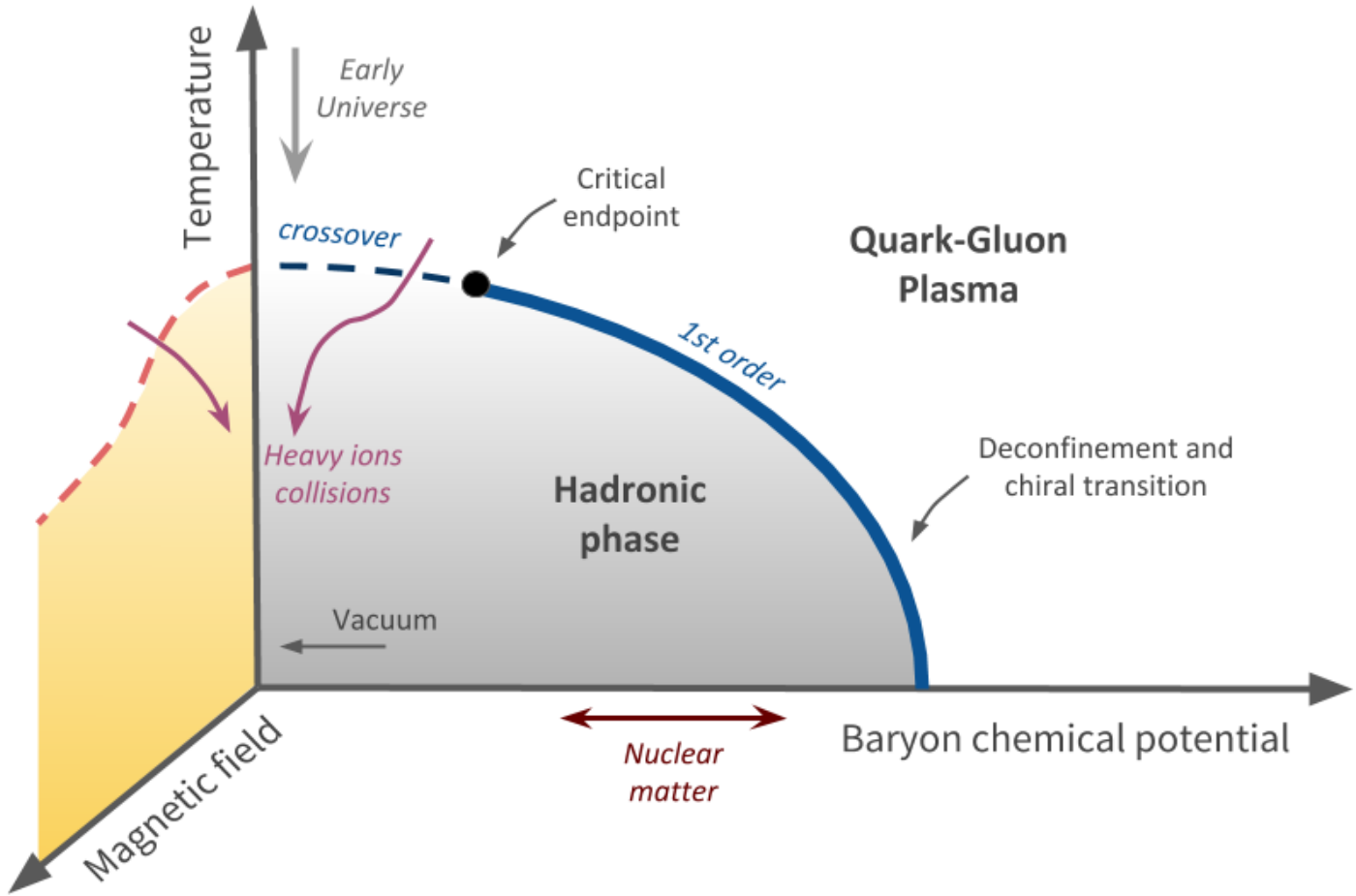
$$V_{Q\bar{Q}} = -\frac{\alpha}{r} + \sigma r + O(\frac{1}{m^2})$$

  describes the confining properties of the medium and the spectrum of heavy bound states

- **High temperatures**: interaction screened by the medium in the Quark-Gluon Plasma, dissociation of heavy quark bound states (Matsui and H. Satz, 1986)

QCD phase diagram

Screening effects can be influenced by external parameters such as **chemical potential** or **magnetic fields**

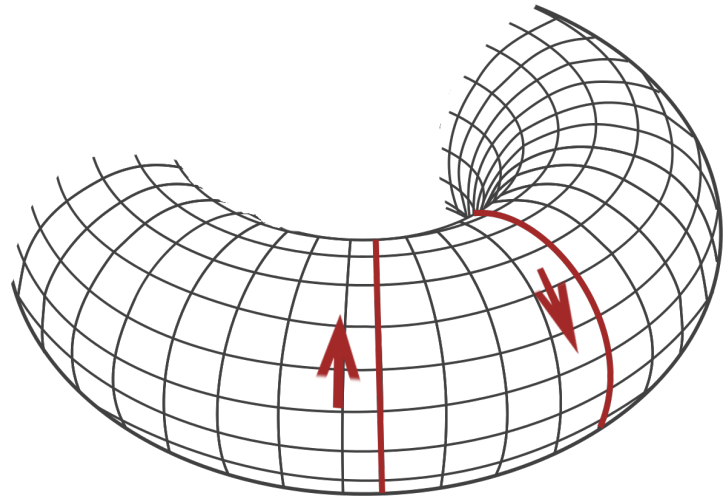


$Q\bar{Q}$ interaction on the lattice

At finite temperature T, correlator between Polyakov loops

$$L(\vec{x}) = \frac{1}{N_c}\mathcal{P}e^{-ig\int_0^{1/T} A_0(\vec{x},\tau)d\tau}$$

$$C_{LL^\dagger}(\vec{x},T) = \langle \mathrm{Tr}L(\vec{0})\mathrm{Tr}L^\dagger(\vec{x})\rangle$$



can be used to extract the **free energy** of a static $Q\bar{Q}$ pair

$$F_{Q\bar{Q}}(\vec{r},T) = -T\log C_{LL^\dagger}(\vec{x},T)$$

for a single quark

$$2F_Q(T) = -T\log|\langle \mathrm{L}\rangle|^2$$

LQCD at finite μ_B

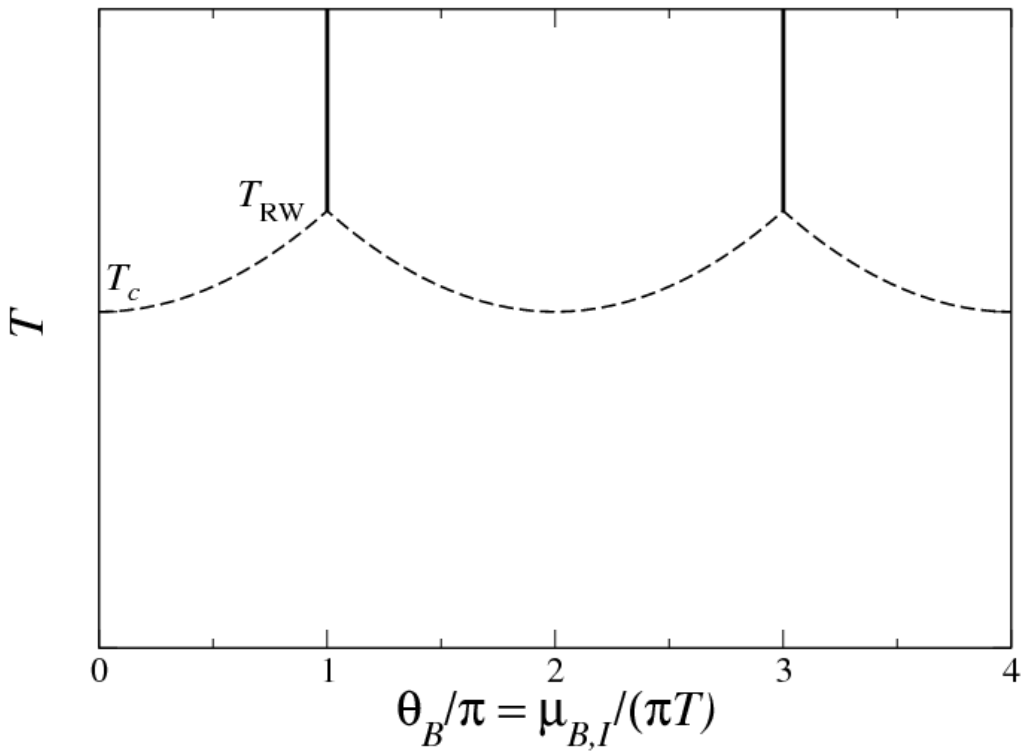
Introducing naively a finite baryon chemical potential μ_B on the lattice causes the so-called **sign problem**

Our approach: **imaginary chemical potential**

- perform MC simulations with $\mu_B = i\mu_{B,i}$
- degenerate quark chemical potentials $\mu_{f,I} = \mu_I = \mu_{B,I}/3$
- introduced in the Dirac operator attaching suitable phases to temporal gauge links
- analytic continuation to real values

Imaginary μ_B phase diagram

At imaginary $\mu_{B,I} = 3\mu_I$ the partition function of the system with fermions is $2\pi/3$ periodic in μ_I/T



Charge-conjugation recovered for $\mu_I = (2k+1)/(3\pi T)$ at low temperatures but spontaneously broken above $T_{RW} \simeq 200$MeV.

Screening masses at finite density

Definition of the masses

Naive approach: Debye masses (inverse screening length) from the pole in the gluon self-energy propagator at leading order

$$m_D^E = \left(\frac{N_C}{3} + \frac{N_f}{6}\right)^{1/2} gT + \mathcal{O}(g^2 T)$$

- m_D^E is the "electric" mass
- unscreened "magnetic" field (at this order) $m_B = 0$

But at higher orders perturbation theory breaks down due to non-perturbative contributions of magnetic gluons

(Nadkarni '86)

- at small $r \ll (g^2 T)^{-1}$ electric gluons dominates
- at larger $r \gtrsim (g^2 T)^{-1}$ the magnetic contribution is no longer negligible

Non-perturbative masses

Another approach needed: study the large distance behaviour of a suitable gauge-invariant correlator in high T regime

(Nadkarni '86, Arnold and Yaffe '95, Braaten and Nieto '94)

Consider the correlator between Polyakov loops

$$C_{LL^\dagger}(\vec{r}, T) = \langle \mathrm{Tr} L(\vec{0}) \mathrm{Tr} L^\dagger(\vec{r}) \rangle$$

and look at its decay

- with correlation length $1/m_E$ dominant at small r

$$C_{LL^\dagger}(\vec{r}, T) \sim e^{-m_E(T) r}/r$$

- with length $1/m_M$ at larger distances

$$C_{LL^\dagger}(\vec{r}, T) \sim e^{-m_M(T) r}/r$$

Magnetic and electric correlators

How we can discern the two contributions? Using symmetries to separate the electric and magnetic terms

(Arnold and Yaffe '95, Maezawa et al. '10, Borsányi et al. '15)

Idea:

- Using Euclidean time-reversal $A_i(\vec{r},\tau)$ and $A_0(\vec{r},\tau)$ are, respectively, even and odd, hence

$$L_M = (L + L^\dagger)/2 \qquad L_E = (L - L^\dagger)/2$$

  receive contributions only from magnetic or electric sectors

- Further decompose using charge conjugation $L \to L^*$

$$L_{M\pm} = (L_M \pm L_M^*)/2 \qquad L_{E\pm} = (L_E \pm L_E^*)/2$$

Magnetic and electric correlators

...and define magnetic and electric correlators as

$$C_M(\vec{r},T) = \langle \mathrm{Tr}L_{M^+}(\vec{0})\mathrm{Tr}L_{M^+}(\vec{r})\rangle - |\langle \mathrm{Tr}L\rangle|^2$$

$$C_E(\vec{r},T) = \langle \mathrm{Tr}L_{E^-}(\vec{0})\mathrm{Tr}L_{E^-}(\vec{r})\rangle$$

Some facts **at zero chemical potential**:

- Sectors related to the fluctuations in the imaginary plane: $\mathrm{Tr}L_{M^+} = \mathrm{Re}\mathrm{Tr}L$ and $\mathrm{Tr}L_{E^-} = i\mathrm{Im}\mathrm{Tr}L$
- $\mathrm{Tr}L_{M^-} = \mathrm{Tr}L_{E^+} = 0$ then no overlap with these sectors
- Expected behaviour at large distances is

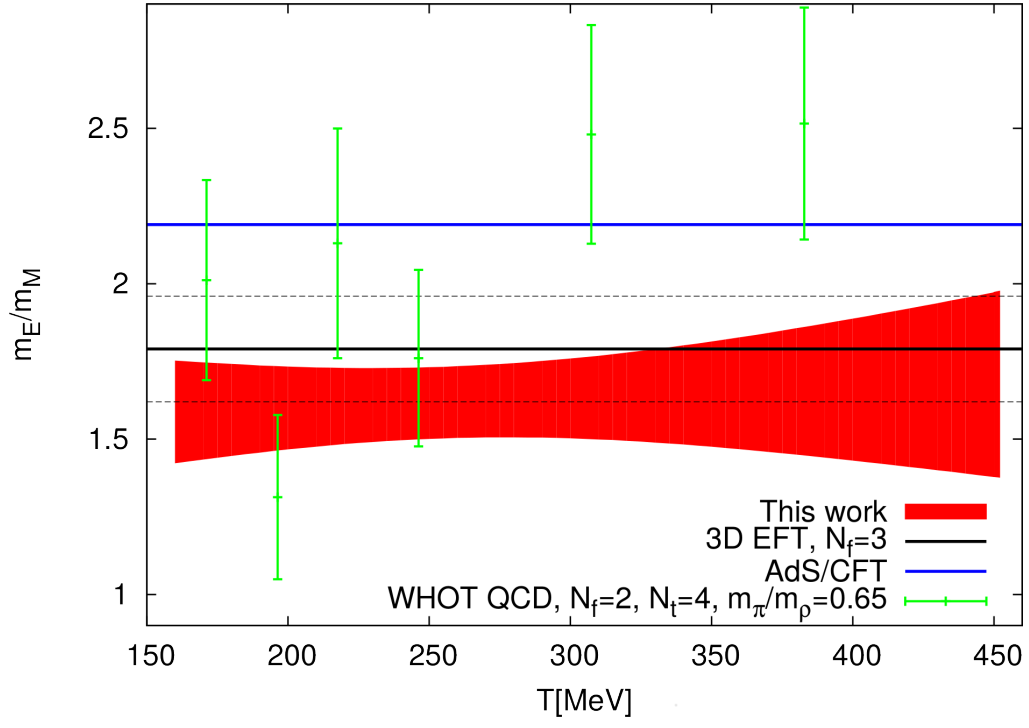
$$C_M(\vec{r},T) \sim \frac{1}{r}e^{-m_M(T)r} \qquad C_E(\vec{r},T) \sim \frac{1}{r}e^{-m_E(T)r}$$

Magnetic and electric correlators

Some results:

- $m_E > m_M$ with $m_E/m_M \simeq 1.5 - 2$
- masses grow linearly with T
- $m_E/T \simeq 7 - 8$ and $m_M/T \simeq 4 - 4.5$

(Maezawa et al. '10, Borsányi et al. '15 (lat), Hart et al. '00 (EFT))



Picture from (Borsányi et al. '15)

Mixed correlators with chemical potential

when $\mu_B \neq 0$ charge conjugation symmetry is explicitely broken
–> magnetic and electric sectors are not separate anymore

The correlators to be considered are

$$C_M(\vec{r},T) = \langle \mathrm{ReTr}L(\vec{0})\mathrm{ReTr}L(\vec{r})\rangle - \langle \mathrm{ReTr}L\rangle^2$$

$$C_E(\vec{r},T) = \langle \mathrm{ImTr}L(\vec{0})\mathrm{ImTr}L(\vec{r})\rangle - \langle \mathrm{ImTr}L\rangle^2$$

$$C_X(\vec{r},T) = \langle \mathrm{ReTr}L(\vec{0})\mathrm{ImTr}L(\vec{r})\rangle - \langle \mathrm{ReTr}L\rangle\langle \mathrm{ImTr}L\rangle$$

physical fluctuations diagonalize the matrix

$$\begin{pmatrix} C_M(\vec{r},T) & C_X(\vec{r},T) \\ C_X(\vec{r},T) & C_E(\vec{r},T) \end{pmatrix}$$

$$C_{1,2} = \frac{1}{2}(C_M + C_E) \pm \left[(C_M - C_E)^2 + 4C_X^2\right]^{1/2}$$

associated to two **new masses** m_1 and m_2. (Andreoli et al. '18)

Method and numerical setup

Idea: at large r the two new correlators go similarly to the magnetic and electric at $\mu = 0$

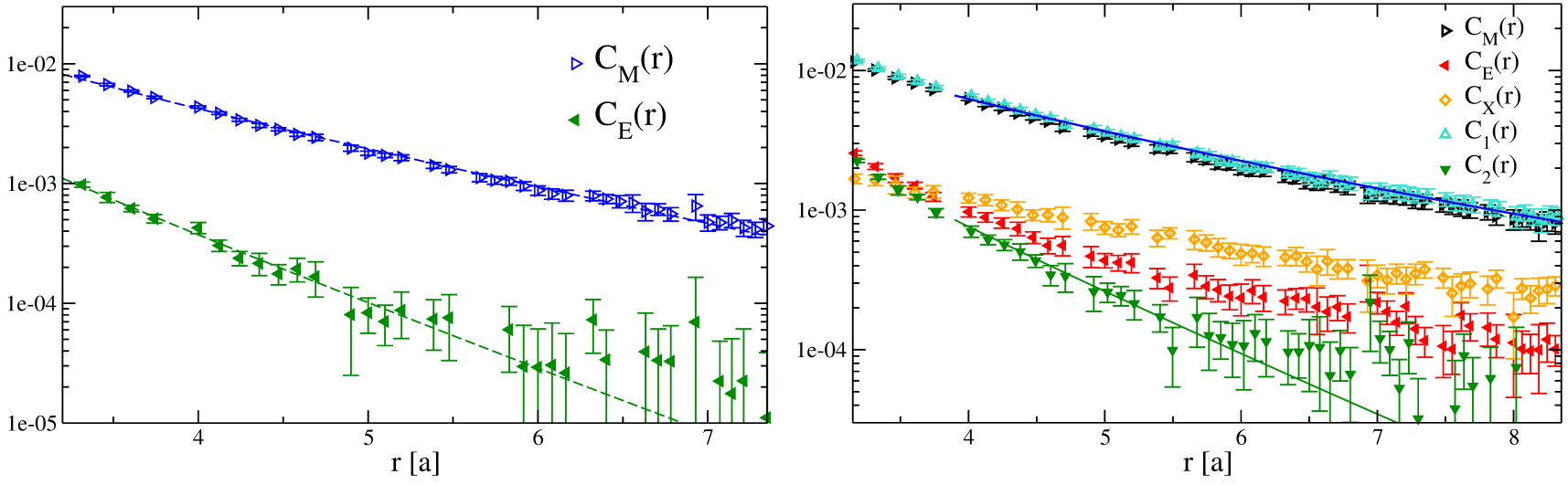
$$C_1(\vec{r}, T) \sim \frac{e^{-m_1(T,\mu)r}}{r} \qquad C_2(\vec{r}, T) \sim \frac{e^{-m_2(T,\mu)r}}{r}$$

then extract the masses fitting the correlators

Numerical setup:

- Nf=2+1 QCD with rooted staggered fermions + stout
- tree level Symanzic improved gauge action
- 32^3x8 lattices with four spacings / temperatures above Tc
- four $\mu_I/\pi T$ between 0 and 1/3
- ~5000 configurations for each run
- fit correlators with the model above + systematics on range

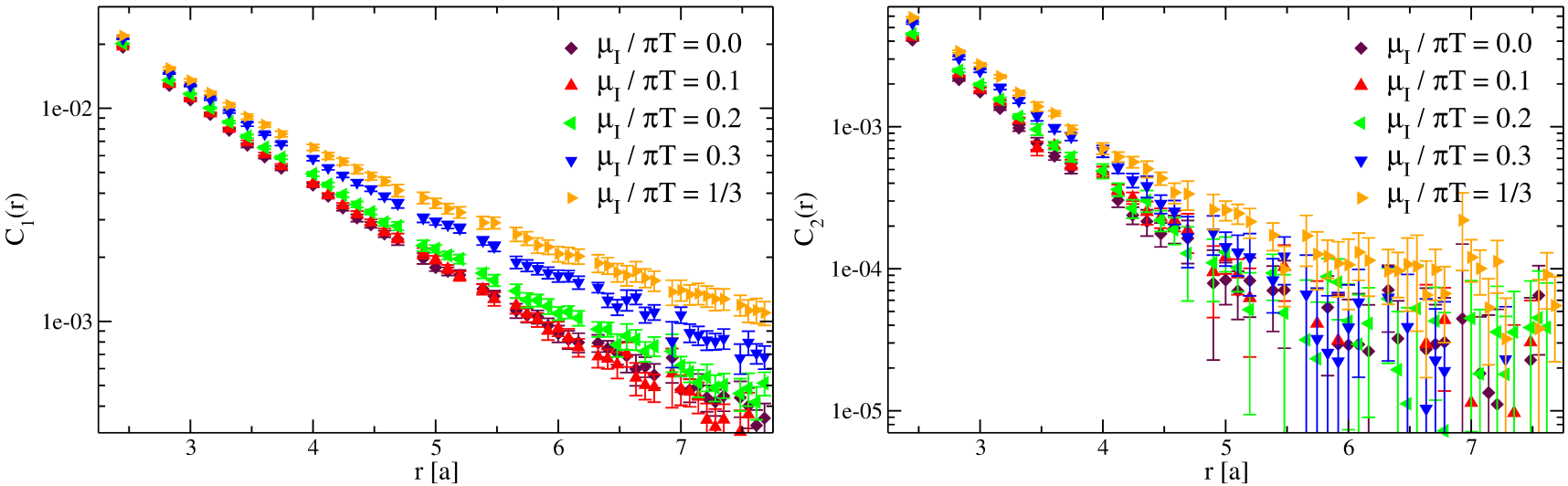
Results: correlators



$T = 217$MeV at $\mu/\pi T = 0, 0.32$; Picture from (Andreoli et al. '18)

- At $\mu = 0$ standard hierachy $m_M < m_E$ and $C_X \sim 0$
- At $\mu > 0$ there is a mixing $C_X \neq 0$
- Diagonalized correlators C_1 and C_2 verify the fit ansatz

Results: correlators



$T = 217$MeV; Picture from (Andreoli et al. '18)

- Both the correlators grow with μ_I
- Less steep decrease <–> smaller correlation lengths

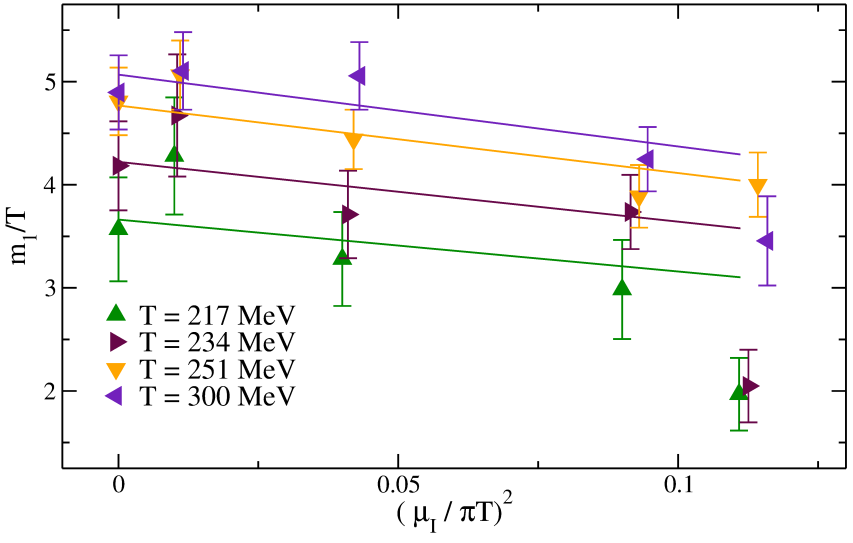
Results: masses

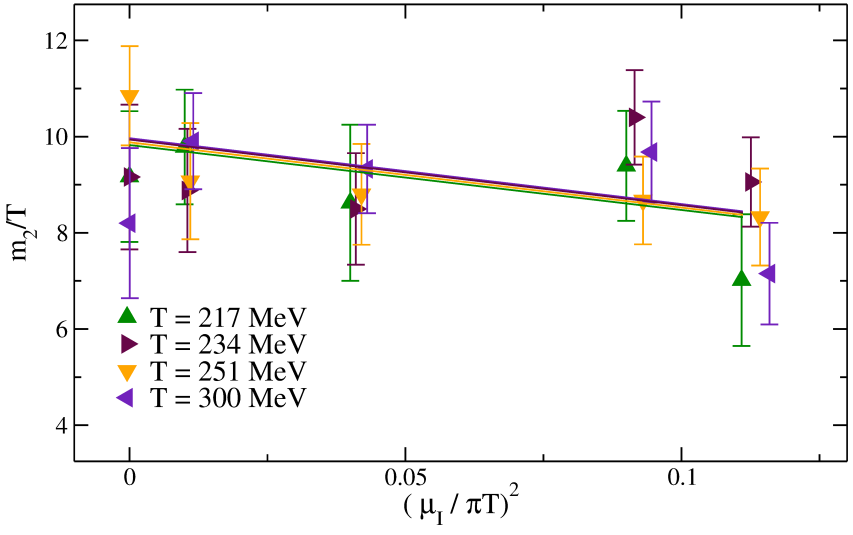
Diagonalized masses are well described by

$$\frac{m_{1,2}(\mu_B, T)}{T} = a_{1,2}(T)\left(1 + b_{1,2}(T)\left(\frac{\mu_B}{\pi T}\right)^2\right)$$

with $b_{1,2}(T) \sim b \sim$ const for both masses.

- masses increase with **real** chemical potential
- mass ratio constant with μ (within errors)

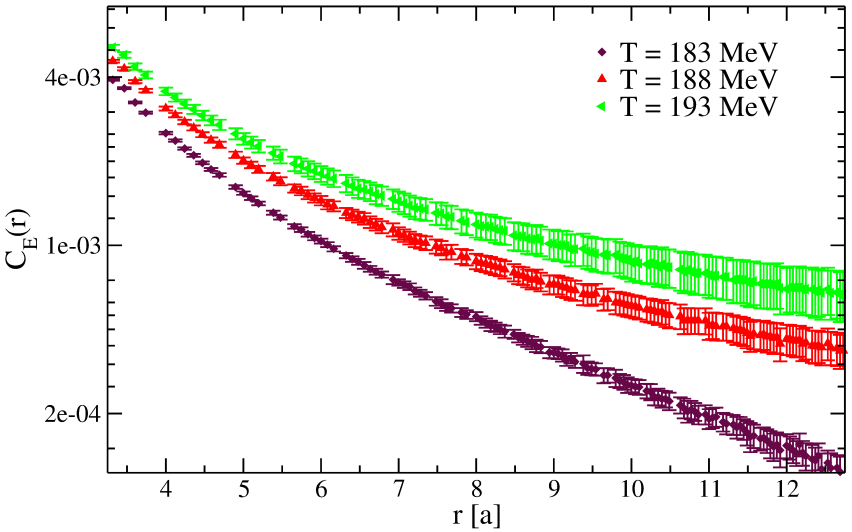


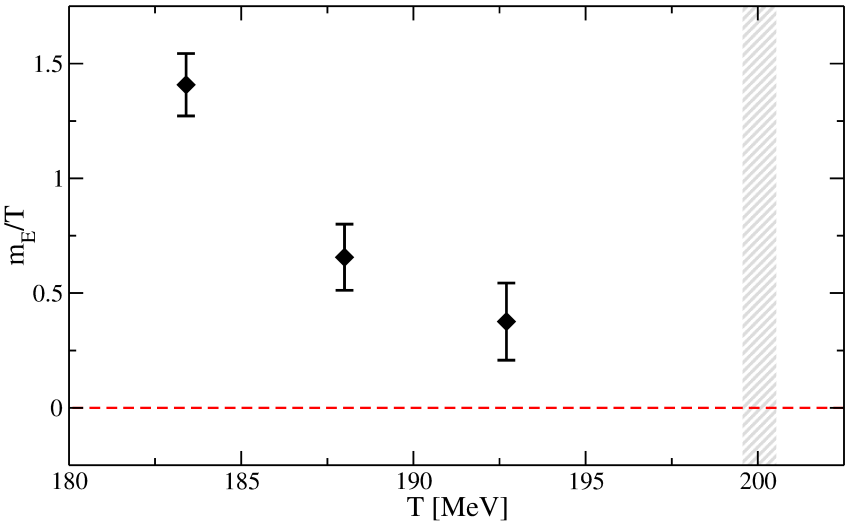


Picture from (Andreoli et al. '18)

Results: near the RW transition

At $\mu_B/\pi T = 1$ charge conjugation is **spontaneously** broken for $T > T_{RW}$, while **recovered** below



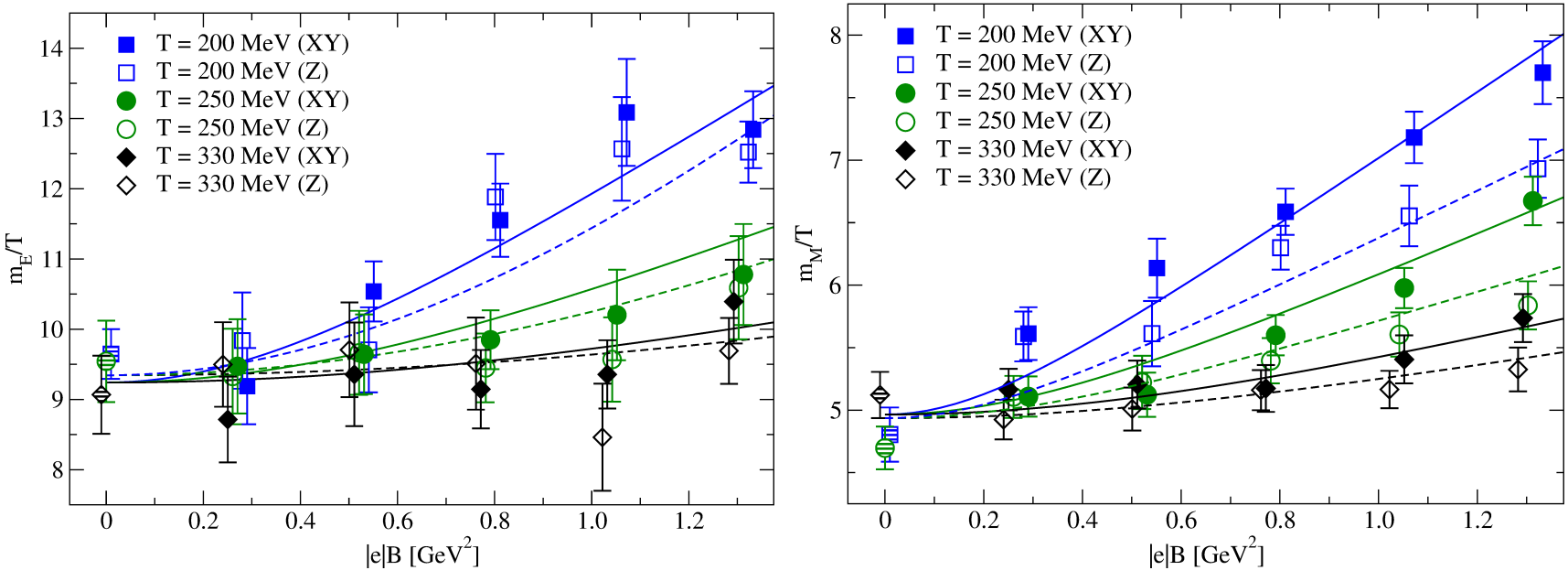


- Above T_C the correlators are reflected about $2\pi/3$
- Below T_C we can keep the magnetic C_M and electric C_E correlator definitions
- Hierarchy inverted, Im$\mathrm{Tr}L$ order parameter of RW

other extreme conditions: magnetic field

a comparison: screening masses with external constant $|e|B$

- external parameter increases the masses
- effect larger approaching T_C
- compatible with decreasing T_C due to B



Picture from (Bonati et al. '17)

Static quark free energy

Static quark free energy

The free energy F_Q of a single quark is given by

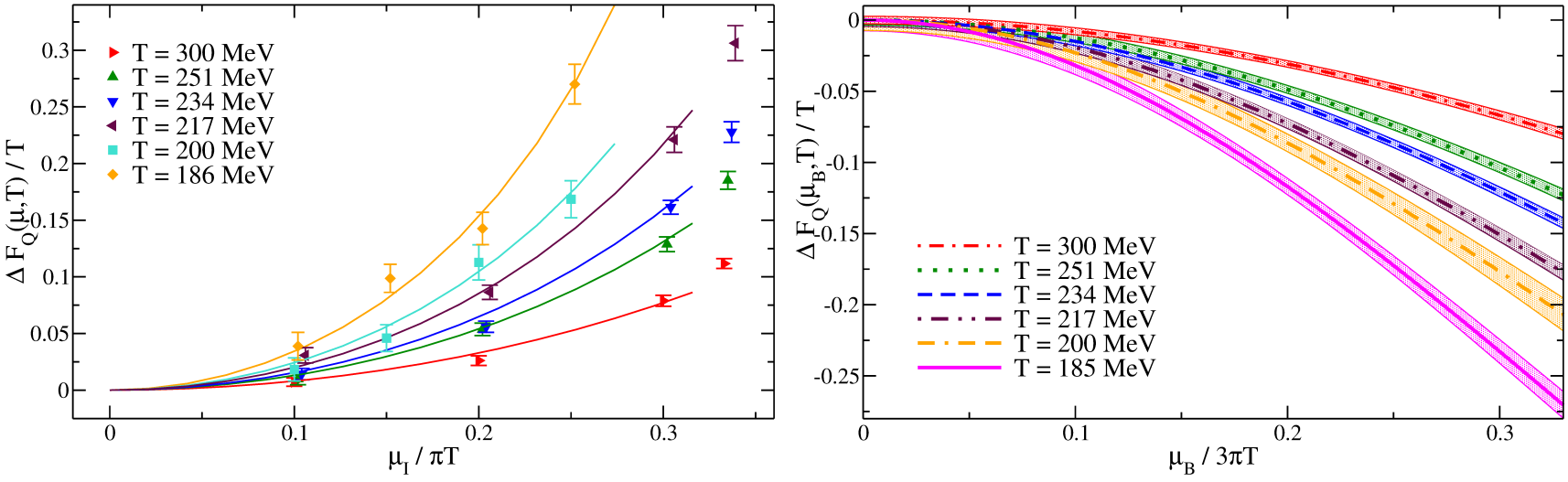
$$2F_Q(T) = -T \log |\langle \mathrm{L} \rangle|^2$$

i.e. the asymptotic value of the Polyakov loop correlator.

The quantity we measure is

$$\begin{aligned} \frac{\Delta F_Q(T, \mu_B, \beta)}{T} &= \frac{F_Q(T, \mu_B, \beta) - F_Q(T, 0, \beta)}{T} \\ &= -\log \left(\frac{|\langle \mathrm{Tr} L \rangle(T, \mu_B, \beta)|}{|\langle \mathrm{Tr} L \rangle(T, 0, \beta)|} \right) \end{aligned}$$

Results: free energy



- chemical potential enhances deconfinement
- F_Q decreases with μ_B, good description with

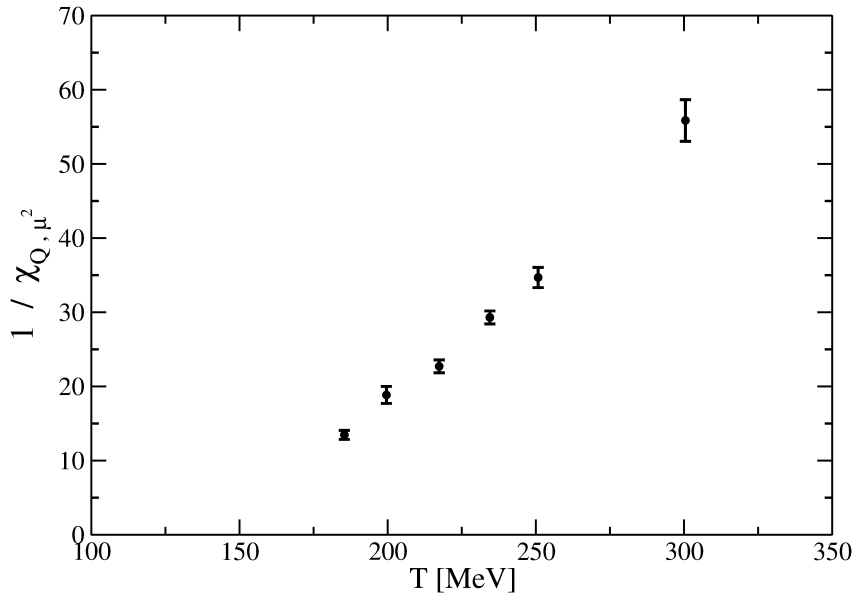
$$\frac{|\langle \mathrm{Tr}L\rangle(T,\mu_B)|}{|\langle \mathrm{Tr}L\rangle(T,0)|} = \exp\left(-2\frac{\Delta F_Q(T,\mu_B)}{T}\right) = 1 - \chi_{Q,\mu_B^2}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left((\mu_B/T)^4\right)$$

Computing the curvature of F_Q

What happens to

$$\chi_{Q,\mu_B^2} = \frac{\partial^2 F_Q/T}{\partial(\mu_B/T)^2}$$

near T_c?



Picture from (Andreoli et al. '18)

- chemical potential drives the transition
- L is not an order parameter, but modifications arise near the critical temperature

Computing the curvature of F_Q

PRELIMINARY STUDY OF χ_{Q,μ_B^2}

Two methods:

- **analytic continuation**: compute $|L|^2$ for several imaginary μs, then fit and extract the (minus) quadratic term

  PROS: simple
  CONS: limited range between T_C and T_{RW}, systematic on the fits, simulations with many μs

- **Taylor exp.**: compute directly the second derivative of F_Q

  PROS: straightforward
  CONS: very noisy and costly observable on the lattice

Computing the curvature of F_Q

A direct computation of the curvature

$$\chi_{Q,\mu_B} = \frac{1}{2}\frac{\partial^2}{\partial\mu^2}\frac{|\langle \mathrm{Tr}L(\mu,T)\rangle|^2}{|\langle \mathrm{Tr}L(0,T)\rangle|^2}$$

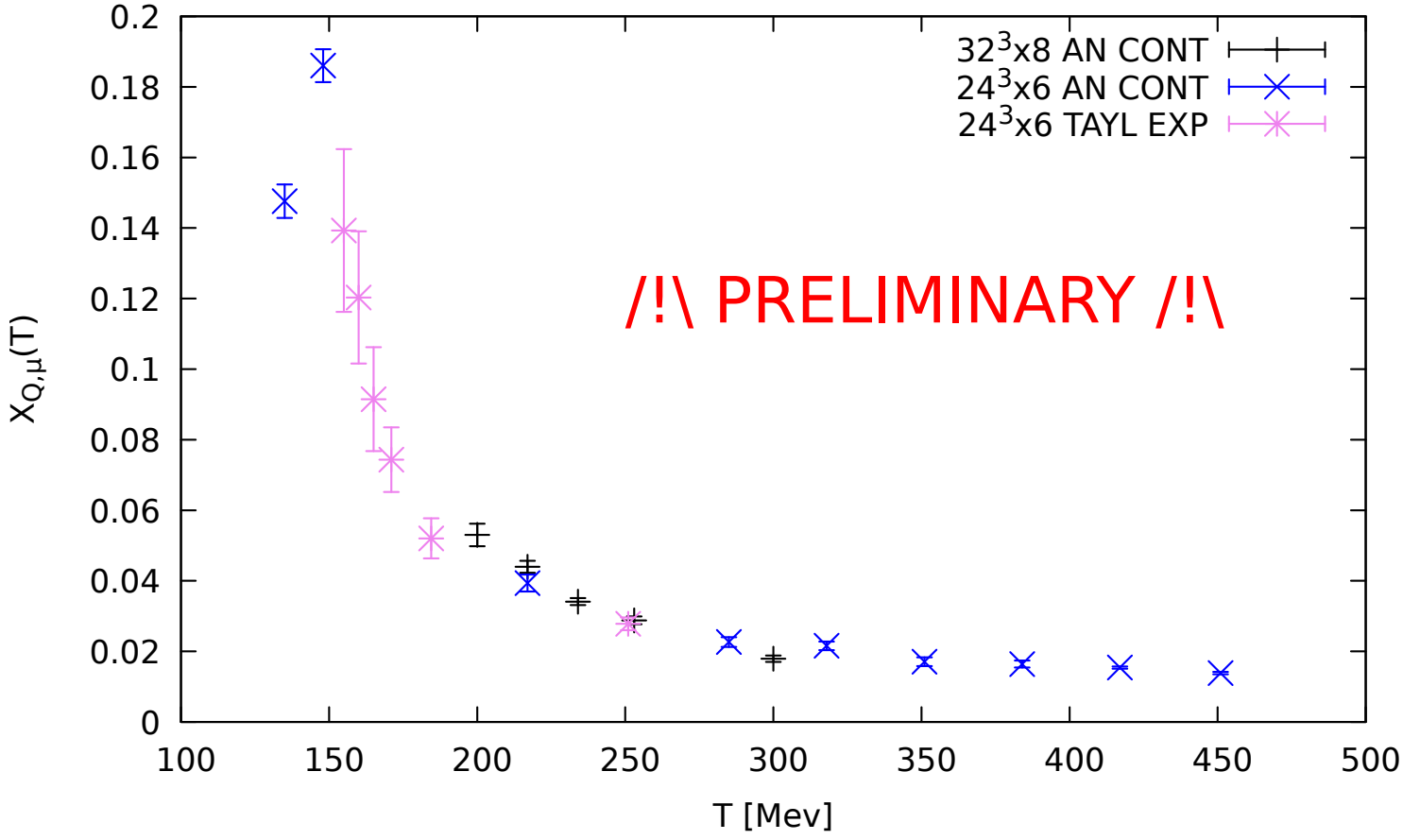
leads to

$$\begin{aligned}\frac{1}{2}\frac{\partial^2|\langle \mathrm{Tr}L\rangle|^2}{\partial\mu^2} &= \langle \mathrm{ReTrL}n\rangle^2 + 3\langle \mathrm{ReTrL}\rangle^2\langle n\rangle^2 - 4\langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}n\rangle\langle n\rangle^2 \\ &+ \langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}n^2\rangle - \langle \mathrm{ReTrL}n\rangle^2\langle n^2\rangle \\ &+ \langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}n'\rangle - \langle \mathrm{ReTrL}\rangle^2\langle n'^2\rangle + (\mathrm{ReTrL} \leftrightarrow \mathrm{ImTrL})\end{aligned}$$

with same μ for the flavours $f = u, d, s$

$$n = \sum_f n_f = \sum_f \frac{1}{4}\mathrm{Tr}\left(M_f^{-1}\frac{\partial M_f}{\partial\mu}\right) \qquad n' = \frac{\partial n}{\partial\mu}$$

Results



- Data seem to confirm the presence of a peak
- TODO: more data, several volumes and spacings

Conclusions

Conclusions

- Extension of the definition of non-perturbative screening masses in the presence of chemical potential
- Screening properties modified by μ_B:
  masses increase <–> system moves away from the transition
- Single quark free energy grows with μ_B
- TODO: study of the curvature χ_{Q,μ_B^2} of F_Q

Conclusions

- Extension of the definition of non-perturbative screening masses in the presence of chemical potential
- Screening properties modified by μ_B:
  masses increase <–> system moves away from the transition
- Single quark free energy grows with μ_B
- TODO: study of the curvature χ_{Q,μ_B^2} of F_Q

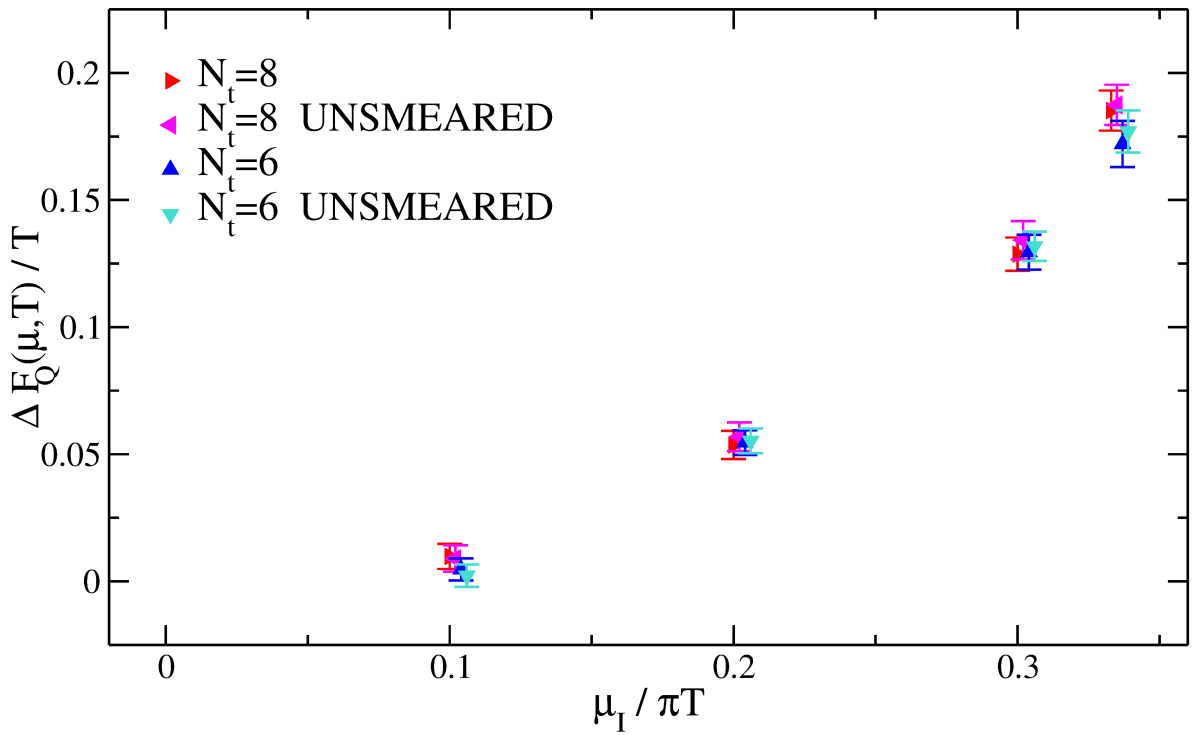
THANK YOU

Backup

check of renormalization of ΔF_Q

Chemical potential is not expected to introduce further divergencies. Checks:

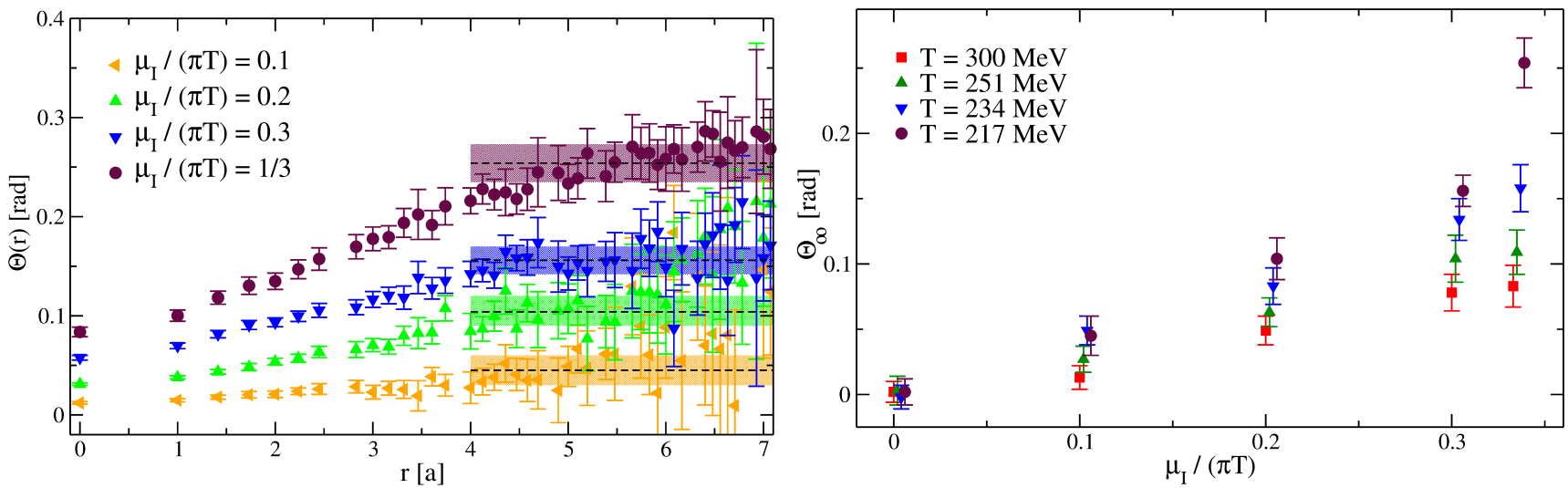
- With and without smearing on Polyakov loops (HYP)
- Different spacings



Correlators mixing angle

Mixing angle of the correlators is given by

$$\theta(r) = \frac{1}{2}\mathrm{atan}\left(\frac{2C_X(r)}{C_M(r) - C_E(r)}\right)$$



(left: $T = 217$MeV), Picture from (Andreoli et al. '18)