Vortex model of the QCD-vacuum

successes and problems

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Models of the QCD vacuum

Savvidy vacuum (1977): Infrared instability of the vacuum

dual superconductor picture: Nielsen and Olesen (1973), Nambu and Creutz (1974), 't Hooft, Parisi, Jevicki and Senjanovic (1975), Mandelstam (1976)
magnetic monopoles detected by Abelian Projection:
Kronfeld, Laursen, Schierholz, Wiese

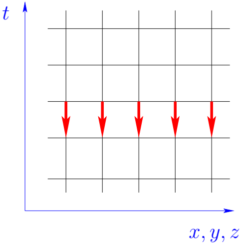
Belavin, Polyakov, Schwartz, Tyupkin (1975), 't Hooft (1976)
instanton liquid model
instanton-dyons (1998) invented by Kraan, van Baal, Lee, Lu
nonzero electric and magnetic charges, sources of Abelian gluons
instanton-dyon ensemble Diakonov,Petrov,Shuryak,Schäfer
V.G. Bornyakov, E.-M. Ilgenfritz, B.V. Martemyanov

center vortex condensation: 't Hooft, Vinciarelli, Yoneya (1978), Cornwall, Nielsen, Olesen, Mack, Petkova (1979)
vortices detected by Center Projection $\longrightarrow$ P-vortices

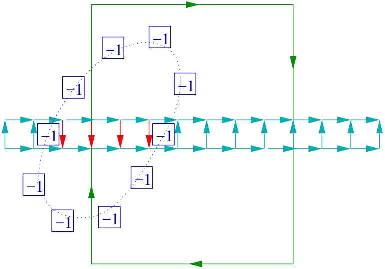
how far do models lead to non-vanishing gluon and quark condensate?

Preference by action or "entropy"

monopoles: by entropy
instantons: by local minima of the action: $S_{\rm inst} = \frac{8\pi^2}{g^2}$
vortices: center symmetry and entropy



multiply all links
in one time-slice
with a center element



Vortex as surface of Dirac volume

low action - high entropy

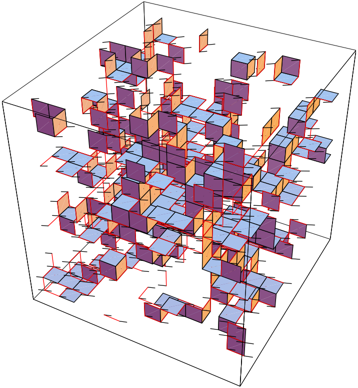
some Vortex properties

- form closed surfaces in dual space,
- vortices have a thick core,
- percolating in all directions
- deconfinement transition a de-percolation transition,
- in deconfinement: percolation in spatial directions only,
- scaling of the P-vortex density.

Shapes of projected vortices

3-dimensional cuts through dual lattices

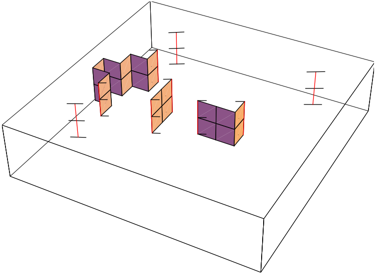
zero temperature



12^4-lattice

vortices percolate

finite temperature
above phase transition



2×12^3-lattice

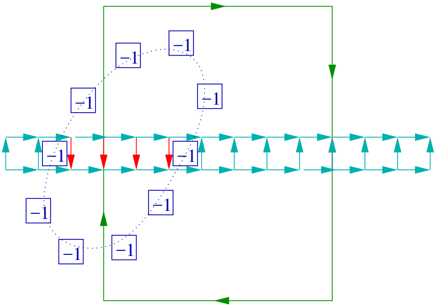
constant in time $\rightarrow$ cylinders

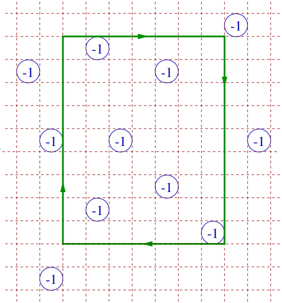
area law for spatial Wilson loops

Area law for center projected Wilson loops

Vortices are closed surfaces

only surface contribution to action



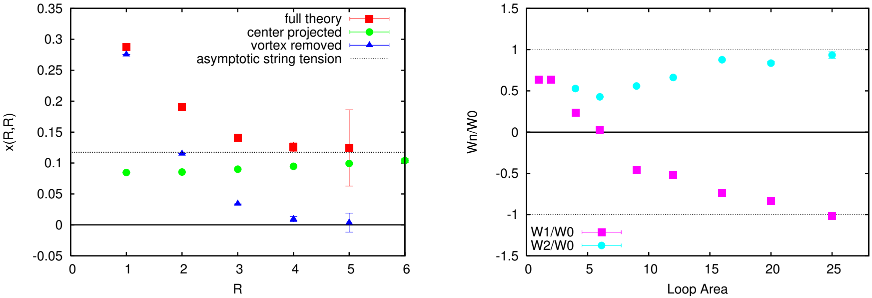


denote f the probability that a plaquette has the value -1

$$\langle W(A)\rangle = [(-1)f + (+1)(1-f)]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma} A] =$$

$$= \exp[-\sigma \overbrace{R \times T}^{A}], \qquad \sigma \equiv -\ln(1-2f) \approx 2f$$

Center vortex dominance



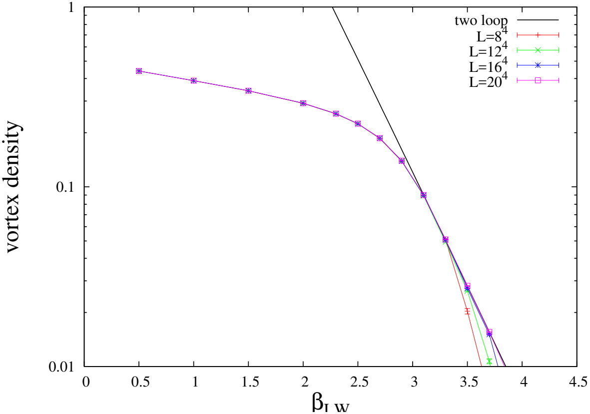
From: Höllwieser et al.:PhysRevD.78.054508.

Left: Creutz ratios for full, center-projected, and vortex-removed gauge fields for $\beta_{LW} = 3.3$.

Right: Wilson loop pierced by n P-vortices W_n.
Expect $W_n \to (-1)^n W_0$ as area is increased.

Cancellations lead to area-law of confinement.

Center vortex dominance



P-vortex surface density vs. β_{LW}

From: Höllwieser et al.:PhysRevD.78.054508.

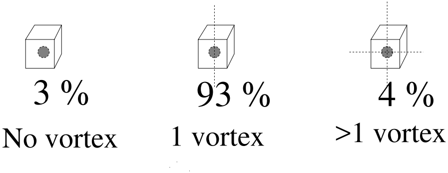
"Two-loop" line is scaling prediction with $\sqrt{\rho_v/6\Lambda^2} = 50$.

Scaling shows the vortex density is a physical quantity, with a well defined continuum limit.

Monopoles and Vortices

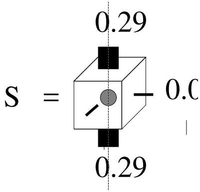
➜ *Greensite et al. (1997)*

Almost all monopole cubes are pierced by exactly one, P-vortex

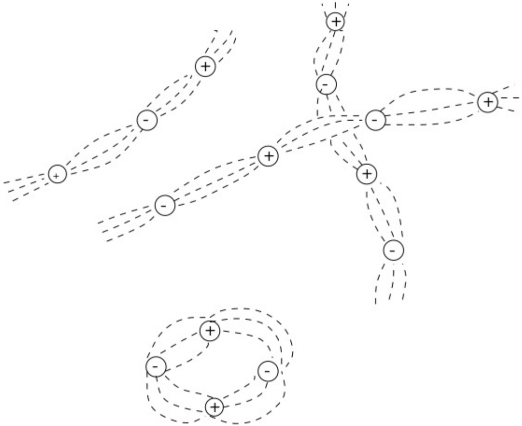


Monopole action is highly asymmetric:

Plaquette action
$S = (1 - \frac{1}{2}\mathrm{Tr}[U_\square]) - S_0$
mainly oriented in P-vortex direction



W-bosons change the field distribution



Monopoles arranged in monopole–antimonopole chains = Vortices

➜ *Ambjorm, Giedt, Greensite, 2000*

Vortices are colorful

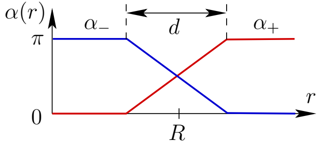
3D pictures

Colors are gauge dependent
In Abelian projection we use a color filter and find monopoles,
Monopoles are an indication of the color structure

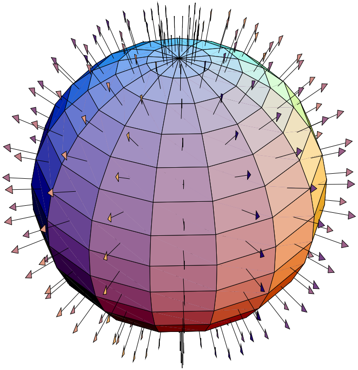
Monopoles as hint of color structure of vortices

Colorfull spherical vortex → *Höllwieser et al. 2012*

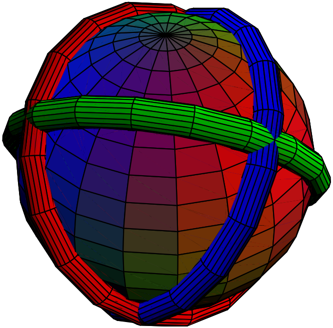
$$U_\mu(x) = \begin{cases} \exp\{\mathrm{i}\alpha(r)\,\vec{e}_r\cdot\vec{\sigma}\} & t=1,\mu=4 \\ \mathbb{1} & \text{elsewhere} \end{cases}$$



P-vortex

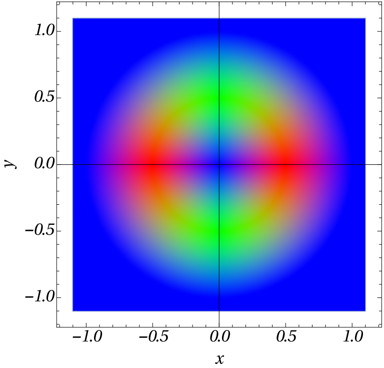


Abelian projection



Colorful plain vortex

plain xy-vortex: for $t = 1$ t-links vary in z-direction
bfrom $\mathbf{1}$ to $-\mathbf{1}$ in $|z - z_v| \le d$



$$U_i(x) = \mathbf{1}$$

$$U_4(x) \;=\; \begin{cases} U_4'(\vec{x}) & \text{for} \quad t = 1 \\ \mathbf{1} & \text{else} \end{cases}$$

where for $|z - z_v| \le d$

$$U_4'(\vec{x}) = \begin{cases} \mathrm{e}^{\mathrm{i}\alpha(z)\sigma_n}, & \rho \le R \\ \mathrm{e}^{\mathrm{i}\alpha(z)\sigma_3} & \text{else} \end{cases}$$

$$\sigma_n = \sigma_1 \sin\theta(\rho)\cos\phi \; + \; \sigma_2 \sin\theta(\rho)\sin\phi \; + \; \sigma_3 \cos\theta(\rho)$$

Vortices generate topological charge

Recall that the topological charge density is defined as

$$q(x) = \frac{1}{16\pi^2}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right) = \frac{1}{4\pi^2}\vec{E}\cdot\vec{B}\,, \qquad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}\,.$$

We need flux in all four directions.
A vortex has flux perpendicular to its world sheet.

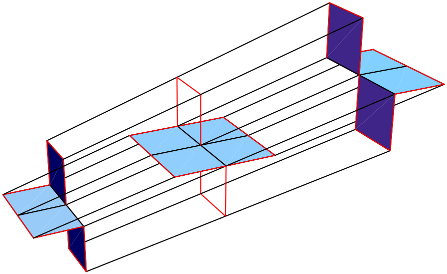
Generate topological charge by:

- intersecting vortices,
- vortex "writhing," *i.e.*, twisting around itself
- Color structure

P-vortices need an orientation
regions of different orientation are separated by monopole lines

➜ *Engelhardt, Reinhardt (2000)*

Topological charge from intersections and writhing points



➜ *Bruckmann, Engelhardt (2003)*

Intersections and writhing points contribute to the topological charge of a P-vortex surface

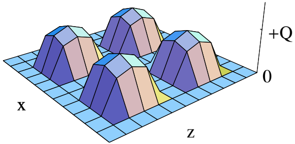
- intersections $Q = \pm\frac{1}{2}$
- writhing points $Q = \pm\frac{1}{8}$

H. Reinhardt, NPB628 (2002) 133 [hep-th/0112215], hep-th/0204194

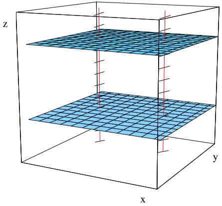
Intersecting plane vortices

Intersecting two orthogonal pairs of plane vortices we can generate topology. A *xy* vortex generates a chromo-electric field, E_z, and a *zt* vortex a chromo-magnetic field, B_z. Each intersection point contributes $Q = \pm 1/2$ to the total topological charge.

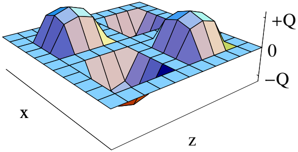
Parallel Vortices



Geometry



Antiparallel Vortices



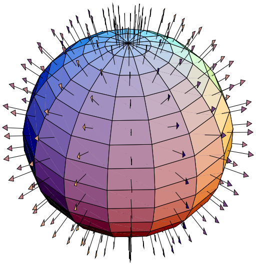
So we can get $Q = 2$ with parallel intersecting vortices and $Q = 0$ with antiparallel intersecting vortices.

Continuum Form of colorful spherical vortex

after time-dependent gauge transformation $\Omega(\vec{r}, t)$

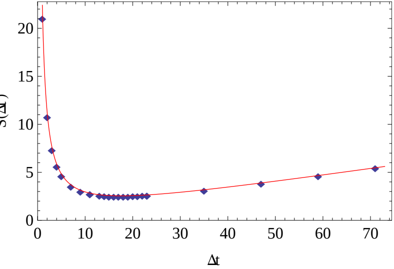
vortex $\equiv$ vacuum - vacuum transition

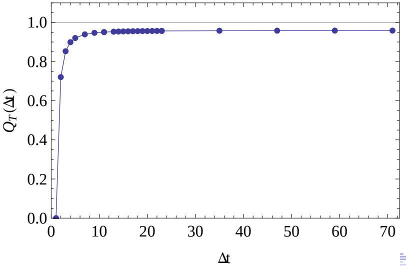
$$\left.\begin{array}{l} t=1 \\ t=2 \end{array}\right\} \begin{array}{c} \text{vacuum} \\ \text{pure gauge} \end{array} \left\{\begin{array}{ll} R^3 \mapsto 1 & \text{no winding} \\ R^3 \mapsto S^3 & \text{winding} \end{array}\right.$$



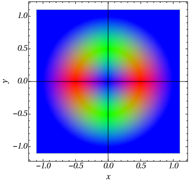
smoothing possible ➜ Schweigler, 2013

distribute to several time-slices Δt $\Rightarrow \mathcal{A}_\mu = \mathrm{i} f(t) \partial_\mu g^\dagger g$





Colorful plain vortices



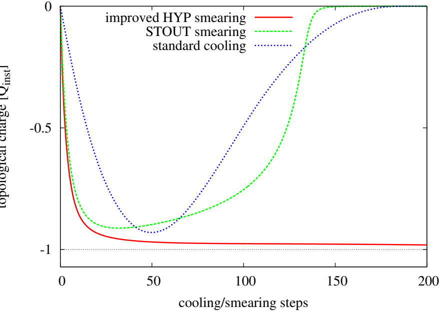
gauge transformation:

rotate time-links to $U_4(x) = \mathbb{1}$

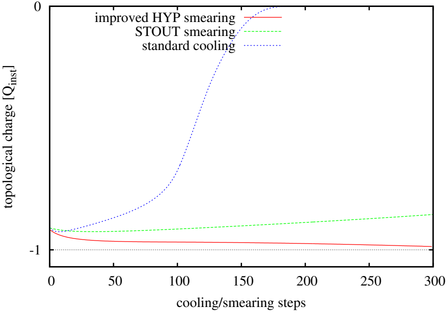
distribute transition over Δt time slices

topological charge during cooling for $R = d = 7$ on $28^3 \times 40$

$\Delta t = 1$



$\Delta t = 11$



Vortices and chiral symmetry breaking
Atiyah-Singer index theorem

- zero-modes of fermionic matrix: $D[A]\psi(x) = 0$
- ψ has definite chirality:

$$\psi_{\substack{R\\L}} = \frac{1}{2}(1 \pm \gamma_5)\psi, \quad \Rightarrow \quad \gamma_5 \psi_{\substack{R\\L}} = \pm \psi_{\substack{R\\L}}$$

- Index theorem (wilson, overlap fermions):

$$n_-, n_+: \text{ number of left-/right-handed zeromodes}$$

$$\mathrm{ind} D[A] = n_- - n_+ = Q[A]$$

- (Asqtad) staggered fermions:

$$\mathrm{ind}\ D[A] = 2Q[A] \text{ (SU(2), double degeneracy)}$$

- Adjoint overlap fermions:

$$\mathrm{ind}\ D[A] = 2NQ[A] = 4Q[A] \text{ (real representation)}$$

➜ *Neuberger, Fukaya (1999)*

Banks-Casher relation

Chiral symmetry breaking $\Longrightarrow$

$\Longrightarrow$ Low-lying eigenmodes of Dirac operator

Dirac equation: $D[A]\,\psi_n = \mathrm{i}\lambda_n\psi_n$,
$\{\gamma_5, \gamma_\mu\} = 0$, $D[A]\,\gamma_5\psi_n = -\mathrm{i}\lambda_n\gamma_5\psi_n$
Non-zero eigenvalues appear in imaginary pairs $\pm\mathrm{i}\lambda_n$.

$$\langle\bar{\psi}\psi\rangle = -\lim_{m\to 0}\lim_{V\to\infty}\left\langle \frac{1}{V}\sum_n \frac{1}{m+\mathrm{i}\lambda_n}\right\rangle =$$

$$= -\lim_{m\to 0}\lim_{V\to\infty}\left\langle \frac{1}{V}\int \mathrm{d}\lambda\,\rho_V(\lambda)\frac{1}{2}\left(\frac{1}{m+\mathrm{i}\lambda}+\frac{1}{m-\mathrm{i}\lambda}\right)\right\rangle$$

$$-\lim_{m\to 0}\frac{m}{m^2+\lambda^2} = \lim_{m\to 0}\frac{\mathrm{d}}{\mathrm{d}\lambda}\arctan\frac{m}{\lambda} \quad\longrightarrow\quad \pi\delta(0)$$

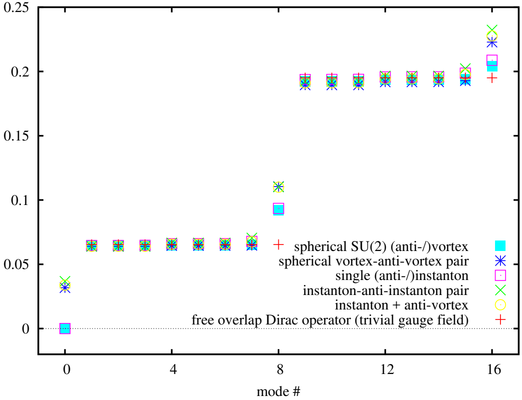
Chiral condensate $\Longrightarrow$ Density of Near-Zero-modes

$$\langle\bar{\psi}\psi\rangle = \frac{\pi\rho_V(0)}{V}$$

Banks, Casher (1980)

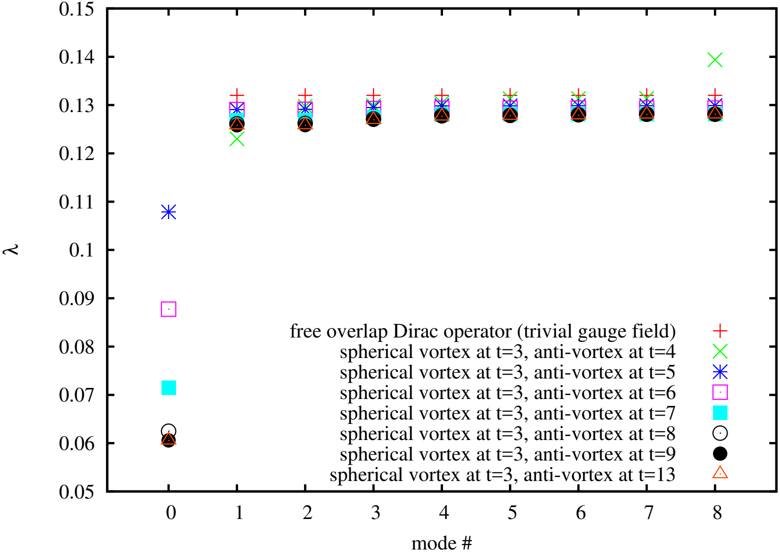
Dirac spectra, spherical vortices and instantons

The overlap Dirac eigenvalues, and even the eigenmodes, in the background of spherical vortices are very similar to those with instantons.



With objects of opposite topological charge, the would-be zero modes interact and become near-zero modes.

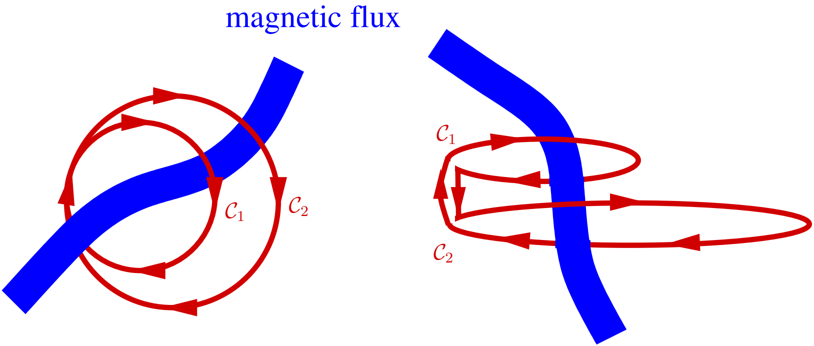
changing distance between Vortex and Anti-vortex



Abelian or Center degrees of freedom

Double-winding Wilson loops

➜ *Greensite, Höllwieser, 2015*



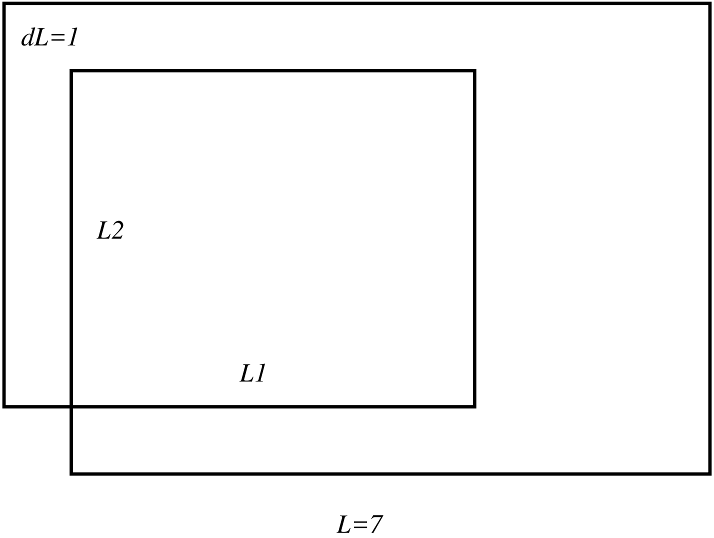
Spherical symmetric monopole flux is spreading with $1/A$
and may lead to small contributions to Wilson loops

$$W_{\mathcal{C}_1+\mathcal{C}_2} = \langle \exp\{\mathrm{i}\tfrac{\sigma_3}{2}(\alpha_{\mathcal{C}_1} + \alpha_{\mathcal{C}_2})\}\rangle \approx \alpha_a \exp[-\sigma(A_1 + A_2) - \mu P]$$

Center vortex flux doesn't spread

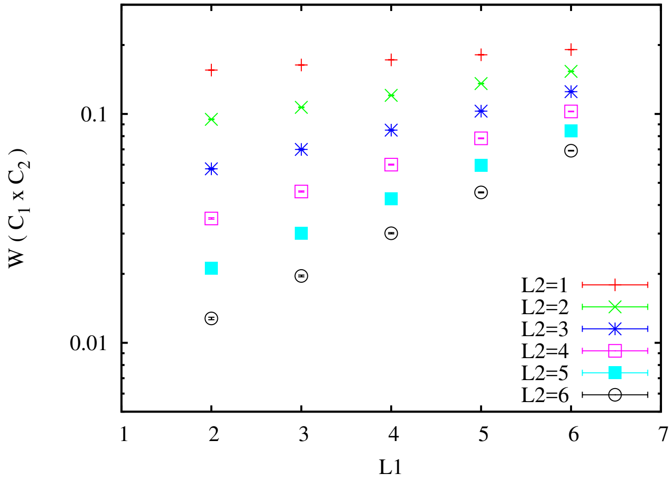
$$W_{\mathcal{C}_1+\mathcal{C}_2} = \langle (-1)^{n_{\mathcal{C}_1}+n_{\mathcal{C}_2}} \rangle = \langle (-1)^{|n_{\mathcal{C}_1}-n_{\mathcal{C}_2}|} \rangle \approx \alpha_c \exp[-\sigma|A_1 - A_2|]$$

Double-winding Wilson loops

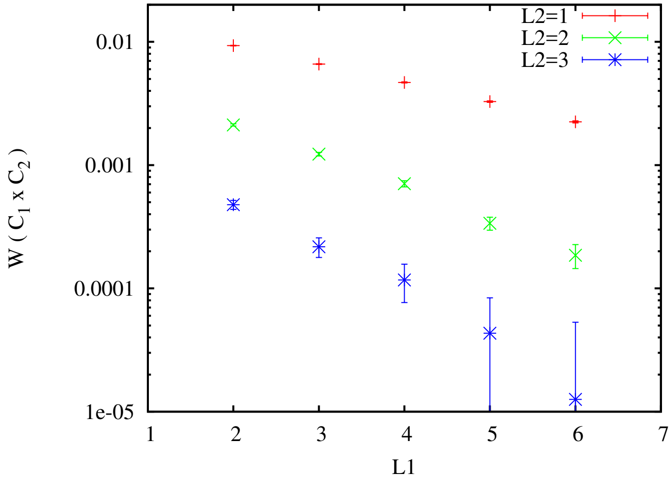


$$A_1 = 8(L_2 + 1) - 1, \quad A_2 = L_1 L_2$$

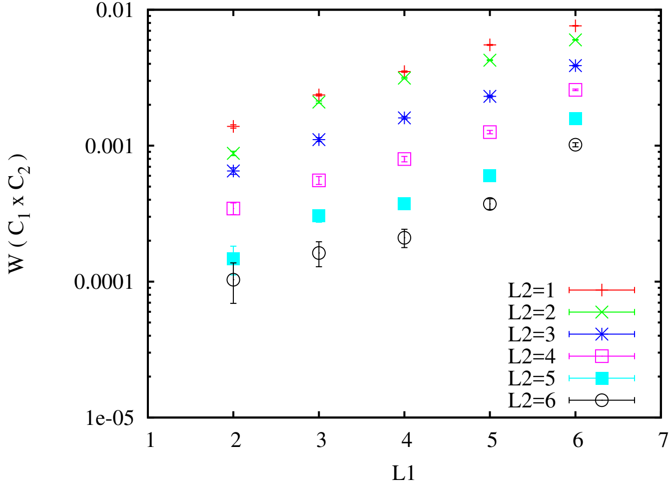
Double-winding Wilson loops: Z(2)



Double-winding Wilson loops: MAG

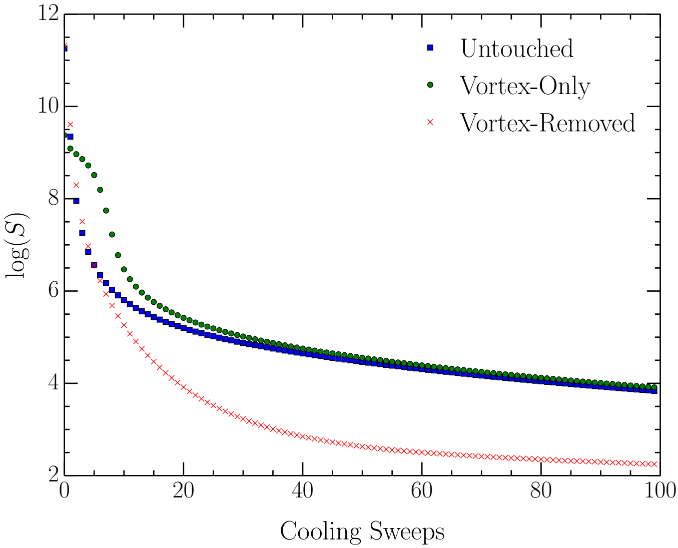


Double-winding Wilson loops: SU(2)



Examine instanton content in SU(3) by cooling

recent results of Adelaide group: Trewartha, Kamleh, Leinweber

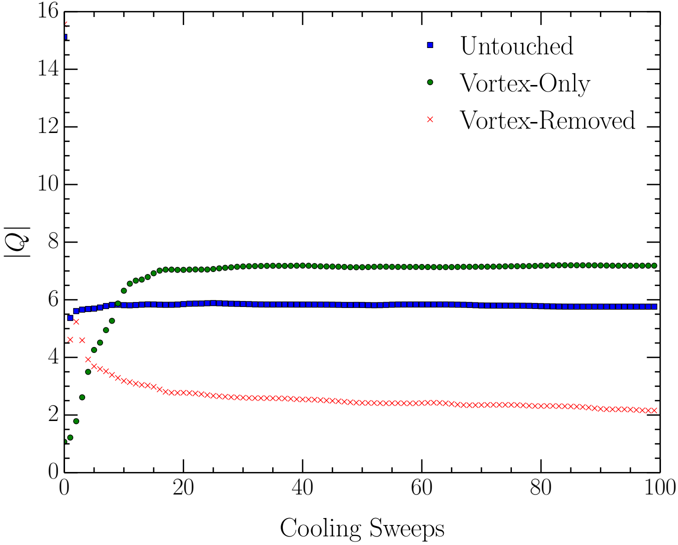


average action

Examine instanton content in SU(3) by cooling

Trewartha et al. (2015)

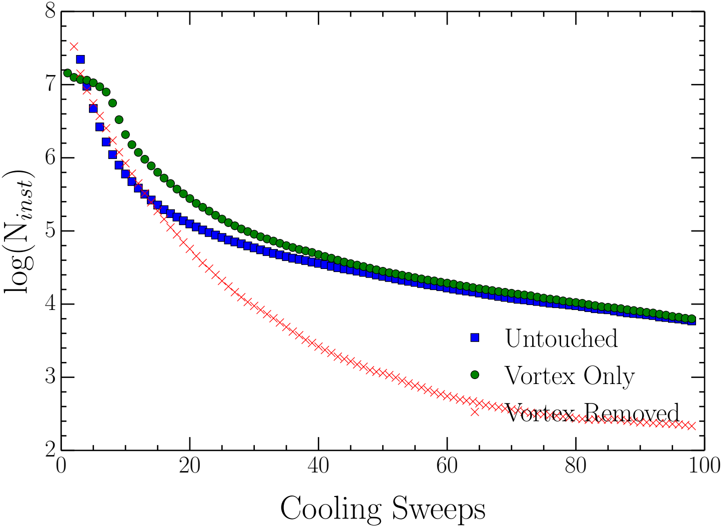
Average absolut value of topological charge



Examine instanton content in SU(3) by cooling

Trewartha et al. (2015)

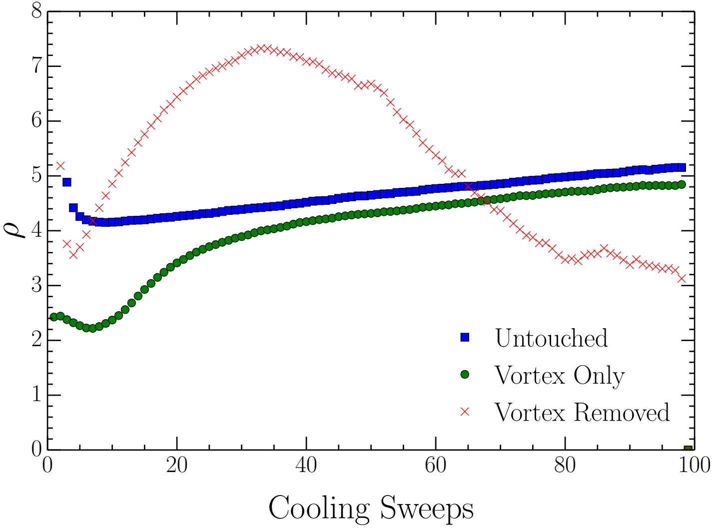
Average number of local action maxima



Examine instanton content in SU(3) by cooling

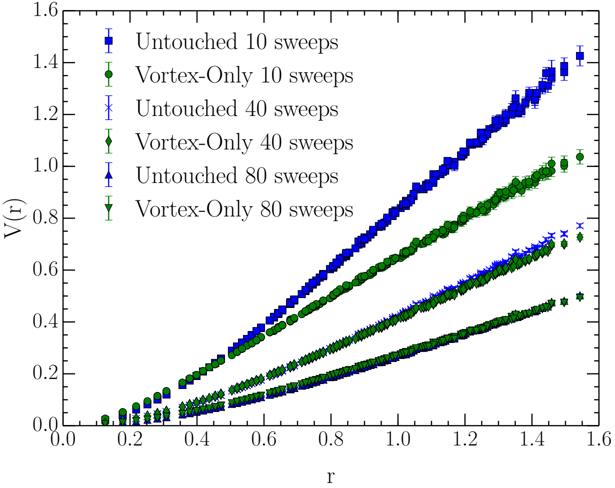
Trewartha et al. (2015)

Average radius ρ of instanton candidates



String tension in SU(3) by cooling

Trewartha et al. (2015)



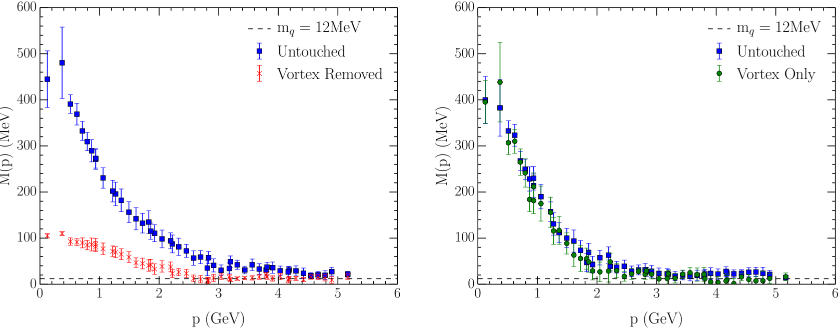
N	$\frac{\sigma_{\mathrm{VO}}}{\sigma_{\mathrm{UT}}}$
20	0, 67
40	0, 64
80	0, 97

Landau gauge quark propagator in SU(3) by cooling

Trewartha et al. (2015)

Lattice quark propagator $S(p) = \frac{Z(p)}{i\not{q} + M(p)}$

nonp-perturbative mass function $M(p)$



after 10 cooling sweeps

Examine instanton content in SU(3) by cooling

recent results of Adelaide group: Trewartha, Kamleh, Leinweber

same smoothing of
- original configurations
- vortex only configurations
- vortex removed configurations

- Vortex removal spoils and destabilizes instantons
- Spoiled instantons are removed via cooling
- Under cooling vortex only configurations produce background of instanton-like objects
- gauge field smoothing can restore agreement between untouched and vortex only configurations
- consistency with instanton model of dynamical mass generation

Support of hypothesis

Center vortices are the fundamental long-range structures underpinning chiral symmetry breaking

Vortex model explains

- non-trivial vacuum $\rightarrow$ gluon condensate
- area law of Wilson loops
- Casimir scaling of heavy-quark potential
- double winding Wilson loops
- finite temperature phase transition $\rightarrow$ Polyakov loops
- orders of phase transitions in SU(2) and SU(3)
- area law for spatial Wilson loops
- topological charge
- chiral symmetry breaking $\rightarrow$ quark condensate
- monopole picture of confinement
  $\rightarrow$ dual superconductor model
- color structure of vortices $\rightarrow$ instantons

Methods of vortex detection, problems

Laplacian center gauge: absence of scaling of P-vortex density

de Forcrand, D'Elia, Alexandrou and Langfeld, Reinhardt, Schäfke

Maximal center gauge = adjoint Landau gauge

$$R = \sum_x \sum_\mu |\mathrm{Tr}[U_\mu(x)]|^2 \quad \rightarrow \quad \mathrm{Maximum}$$

+ center projection

$$U_\mu(x) \quad \rightarrow \quad Z_\mu(x) \equiv \mathrm{sign}\ \mathrm{Tr}[U_\mu(x)]$$

Problems:

- cooled or RG-smoothed configurations, Kovacs-Tomboulis: string tension is drastically reduced after only a few cooling steps, why: vortex cores expand considerably,
  every region of the lattice is part of a vortex core,
  fits fail badly near the middle of the vortex.
- Gribov ambiguity: local maxima versus global maxima, extensive simulated annealing: Bornyakov, Komarov, Polikarpov, Veselov → loss of vortex finding property

The Gribov-copy problem

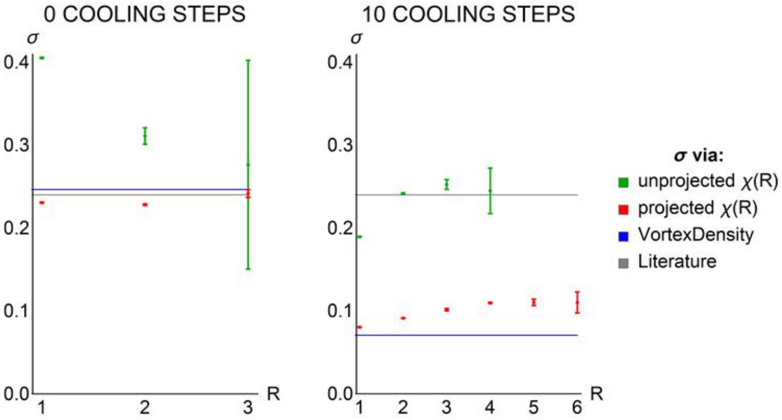
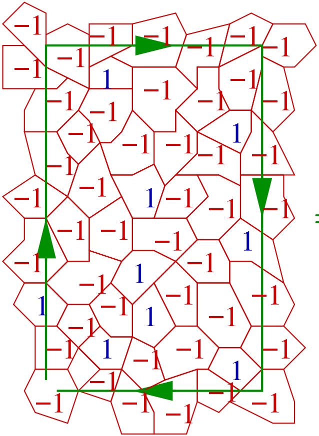


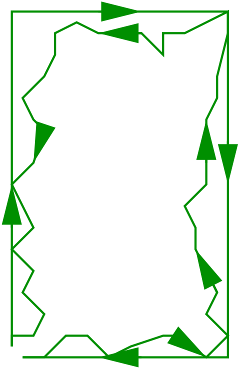
Figure: Wilson action on a 18^4-lattice, 458 configurations at $\beta = 2.2$,

Center projection underestimates the string tension in cooled configurations.

Non-abelian Stokes law



$$= (-1)^{22} \times$$

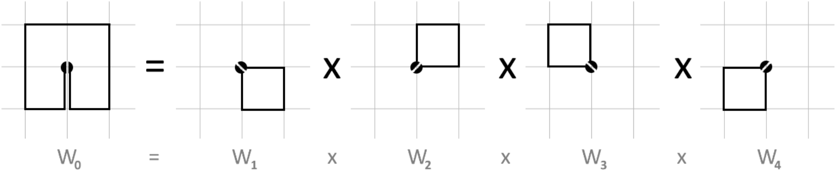


there are no holes between center regions

non-Abelian Stokes law $\rightarrow$ Abelian Stokes law

observables to identify these center regions?

Homogeneity of a 2×2-Wilson-Loop



$\boldsymbol{W}_j = \cos(\alpha_j)\, \boldsymbol{\sigma}_0 + \mathrm{i} \sum_{k=1}^{3} \sin(\alpha_j)\, (\boldsymbol{n}_j)_k\, \boldsymbol{\sigma}_k,$

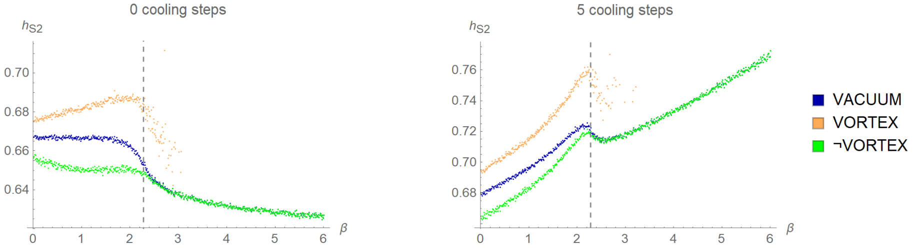
$\boldsymbol{n}_j \in \mathbb{S}^2, \ |\ \boldsymbol{n}_j\ | = 1,$

Definition: S2-homogeneity: $h_{S2} := \frac{1}{4} \mid \sum_{j=1}^{4} \boldsymbol{n}_j \mid \in [0, 1].$

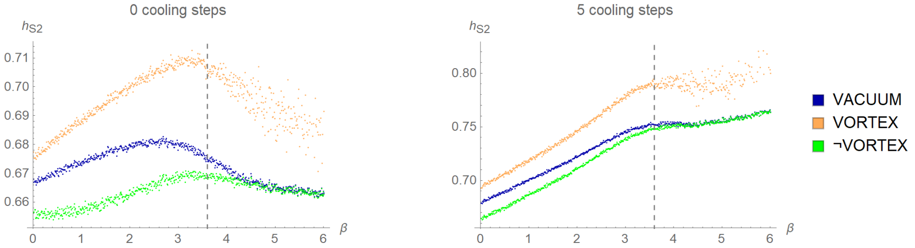
Homogeneity of 2 to 4 plaquettes

S2-homogeneity of plaquette pairs

Wilson action:

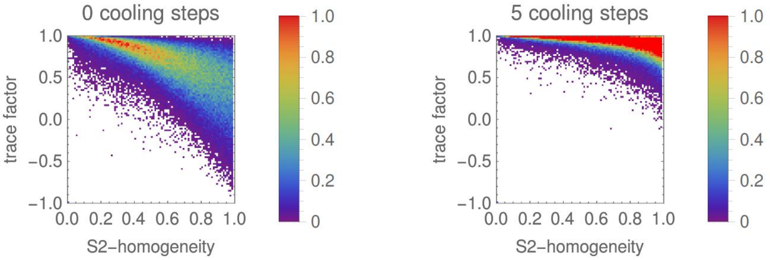


Lüscher-Weisz action:

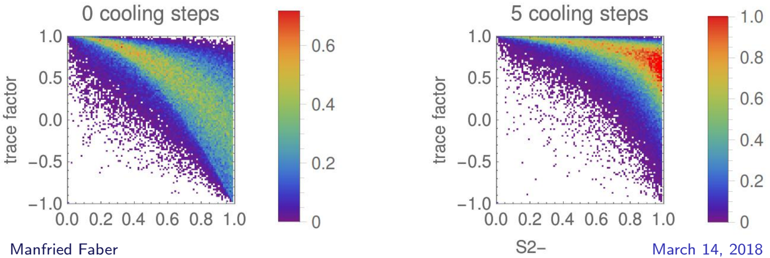


S2-homogeneity and trace of plaquette pairs

Wilson action with $\beta = 2.2$:



Lüscher-Weisz action with $\beta = 3.35$:



Conclusion

many successes of vortex model

- explains confinement
- explains phase transition
- explains topological charge
- explains chiral symmetry breaking
- explains success of abelian monopoles

still open problems to solve

- improve gauge fixing functional

Thank you for your attention!

Questions?