Fluctuation patterns of Polyakov loop clusters in SU(2) gluodynamics

Kyrill Bugaev, D. Savchenko, A. Ivanytskyi, V. Sagun, B. Grinyuk,

G. Zinovjev

Bogolyubov ITP, Kiev, Ukraine

E. Nikonov

Laboratory for Information Technologies, JINR, Dubna, Russia

E.-M. Ilgenfritz

Laboratory of Theoretical Physics, JINR, Dubna, Russia

L. Bravina

University of Oslo, Oslo, Norway

Dubna, September 6, 2018

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4. Size distributions of gaseous (anti)clusters and Liquid Droplet Formula

5. Surface tension of anticluster liquid droplet

6. Negative total surface tension of monomer clusters

7. Fluctuations

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A. I. Ivanytskyi, K. A. Bugaev et al., Nuclear Physics A 960 (2017) 90; arXiv:1606.047 [hep-lat]

Motivation 1

Traditionally, the deconfinement in SU(N) color gluodynamics is described as the break down of Z(N) symmetry

However, such a language is well suited for the phase transitions of solid-liquid and solid-solid types.

Furthermore, i) hadronic matter at low energy densities is a gas!

ii) at high energy densities the QGP is (probably) the most perfect fluid!

=> we need a language which is suited for GAS-to-LIQUID phase transition (PT)

Moreover, i) the language of symmetry breaking does not work for deconfinement PT in presence of quarks

ii) the same is true for the chiral symmetry restoration PT, if one uses non-vanishing quark masses

=> we need a language which can be used in presence of quarks with realistic masses

Motivation II

There are several exactly solvable cluster models for the LIQUID-GAS PT:

Fisher Droplet Model and its successors for ordinary liquid-gas PT

. M. E. Fisher, Physics 3, 255 (1967)

Statistical Multifragmentation Model for nuclear liquid-gas PT

. K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. 62 (2000)

Quark-Gluon Bags with Surface Tension Model of deconfinement PT

. K. A. Bugaev, Phys. Rev. C 76, 014903 (2007)

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. 76 (2013), 341

However, to use this framework we need to know

i) the T-dependence of surface tension of QGP bags

ii) the Fisher exponent of QGP bags

=> Lattice QCD allows us to determine all these quantities and to verify whether the known cluster models are suited to study deconfinement PT

Motivation III

These exactly solvable cluster models for the LIQUID-GAS PT
have different mechanisms of 1-st and 2-nd order PTs!

Model with tricritical endpoint (3CEP) K. A. Bugaev, Phys. Rev. C 76, 014903 (2007)

Model with critical endpoint (CEP)

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. 76 (2013), 341

The MOST IMPORTANT COMMON FEATURE
of Quark-Gluon Bags with Surface Tension Model (QGBSTM)
of deconfinement PT is that above T_cep the surface tension of bags
must be negative. This feature explains the cross-over existence!

=> Lattice QCD allows us to verify these models and to readjust them.

Furthermore, Lattice SU(2) gluodynamics allows us to study the properties of the
2-nd order PT which is expected to exist at (3)CEP and at which experimental
programs of heavy ion collisions planned at RHIC, NICA, FAIR and J-PARK
are aimed

Goals

Using the cluster approach to LQCD we hope

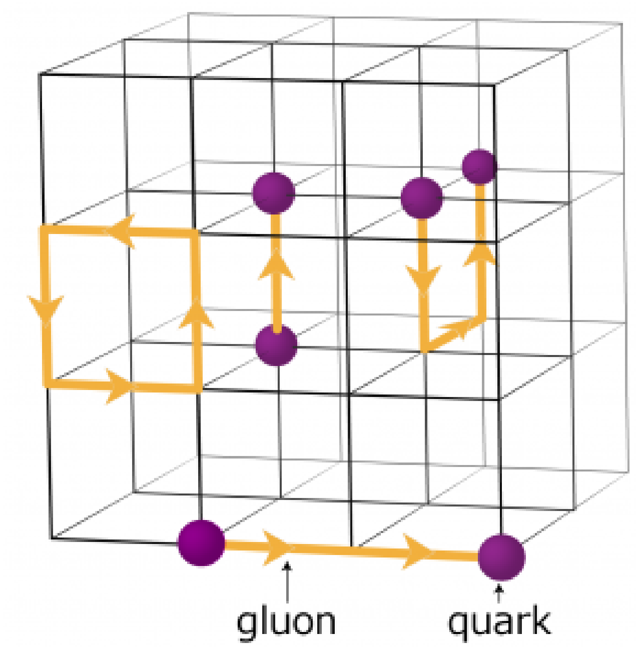
1. **to give a physical meaning to the concept of QGP bags**
2. **to formulate appropriate order parameters of this PT**
3. **to formulate the signals of 2-nd order LIQUID-GAS PT which maybe observed in the experiments and will help to locate (tri)CEP**

Definition of Polyakov loop

$$L(\tilde{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\tilde{x}, t)$$

$U_4(\tilde{x}, t)$ – temporal gauge link defined by gluon field

$SU(2) \Rightarrow L(\tilde{x}) \in [-1, 1]$, real



SU(2) Polyakov loop L (x) = Continuous Spin

L.G. Yaffe, B. Svetitsky, Phys. Rev. D 26 (1982) 963

existing at each spatial point of lattice

Similarly to Gattringer we define spins via cut-off L_cut

C. Gattringer, Phys. Lett. B 690, (2010) 179.

C. Gattringer and A. Schmidt, JHEP 1101 (2011) 051.

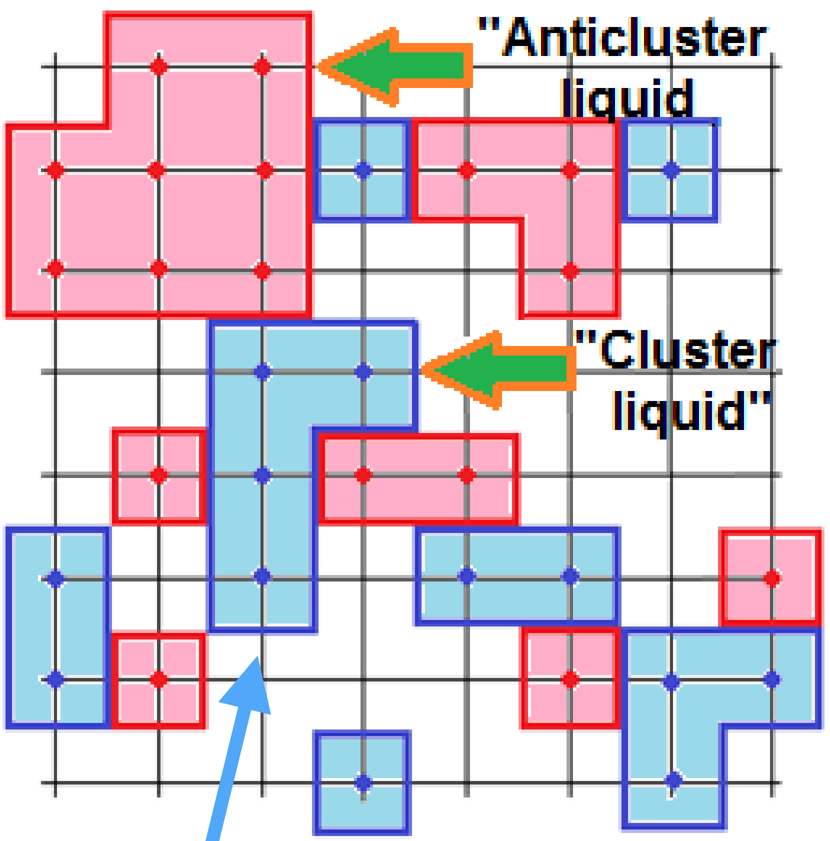
1. If $L > +|L_{cut}|$ it is spin Up,
2. If $L < -|L_{cut}|$ it is spin Down,
3. If $L: -|L_{cut}| < L < |L_{cut}|$ it is aux. Vacuum. **(will not be discussed)**

Definition of Polyakov loop Clusters

Monomer Up has all neighbors spin Down or aux Vac.

Dimer Up =Two neighboring monomers Up have all other neighbors spin Down or aux Vac.

Geometrical cluster of N same sign spin monomers is surrounded by opposite sign spins or aux Vac:



(Anti)clusters can be either "spin up"or "spin down"ones

- Largest fragment - "anticluster liquid droplet"
- Next to largest fragment of opposite sign - "cluster liquid droplet"
- Gas of (anti)clusters has the same Polykov loop sign as their "liquids"

Size Distributions of Clusters I

- Numerical simulations on $3+1$ dimensional lattice of size $N_\sigma = 24$, $N_\tau = 8$
- 13 values of inverse coupling $\beta \in [2.31, 3]$ $\Rightarrow$ 13 values of physical temperature
- vacuum cut-off parameter $L_{cut} = 0.1$ and 0.2
- Average over 1600 independent configurations for all β and L_{cut}

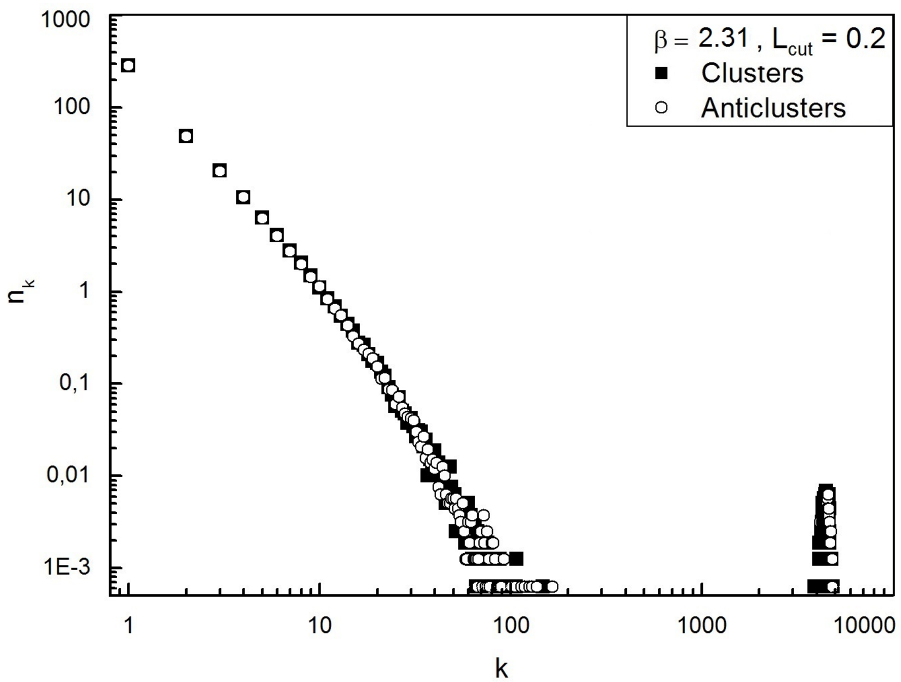
In thermodynamic limit the critical value is $\beta c = 2.5115$

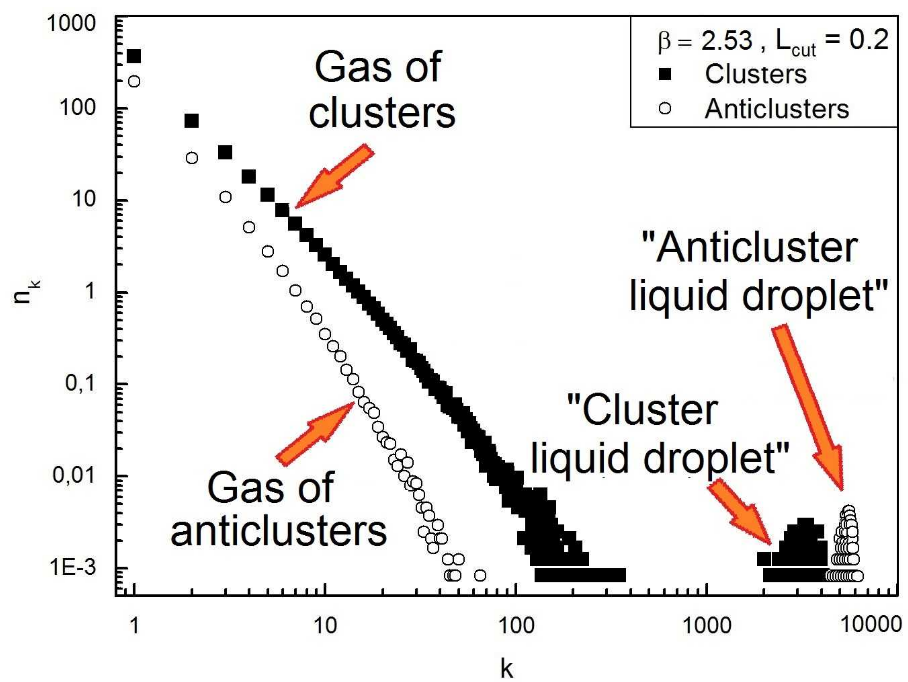
Distributions for $\beta \leq \beta_c \simeq 2.52$

- symmetry between (anti)cluster distributions
- gas and “liquid” domains are well separated

Distributions for $\beta > \beta_c \simeq 2.52$

- no symmetry between (anti)cluster distributions
- “cluster liquid” evaporates to cluster gas
- anticluster gas condensates to “anticluster liquid”



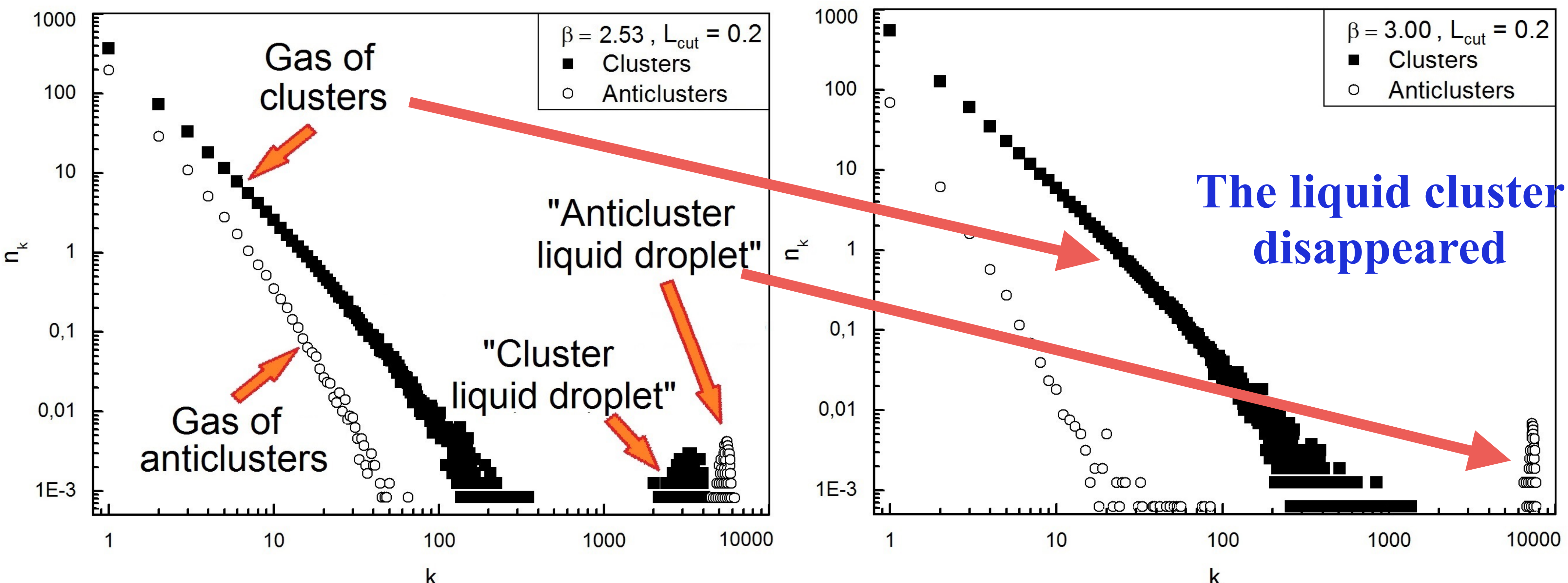


size (volume) of cluster in lattice units

Distributions at low $\beta \leq \beta_c \simeq 2.52$ (phase of restored global Z(2) symmetry)

- symmetry between (anti)cluster distributions
- gas and “liquid"domains are well separated

Size Distributions of Clusters II



size (volume) of cluster in lattice units

Distributions at high $\beta > \beta_c \simeq 2.52$ (phase of broken global Z(2) symmetry)

- no symmetry between (anti)cluster distributions
- “cluster liquid"evaporates to cluster gas
- anticluster gas condensates to “anticluster liquid"

=> **Distributions are rather sensitive to value of β!**

Can we describe the gas distributions by the liquid droplet formula?

Liquid Droplet Formula

Since a priori k-min is unknown we perform a free fit of

(anti)cluster size distributions for all k-min

according to Liquid Drop Model

0.667

$$n_k^{th} = C \exp\left(\mu k - \sigma k^{\kappa} - \tau \ln k\right), \qquad k > k\text{-}min - 1$$

normalization bulk surface Fisher index

M. E. Fisher, Physics **3**, 255 (1967).

K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. **C 62**, 044320 (2000)

Free fitting parameters: C, μ, σ, τ

μ and σ are reduced chemical potential and surface tension coeff.

For SU(2) (anti)clusters with volume $k \geq 2$
Fisher Exponent τ=1.806

Very important finding,

since in exactly solvable models τ defines the universality class:

Fisher droplet model: for d=2 => τ=2.07; for d=3 => τ=2.209

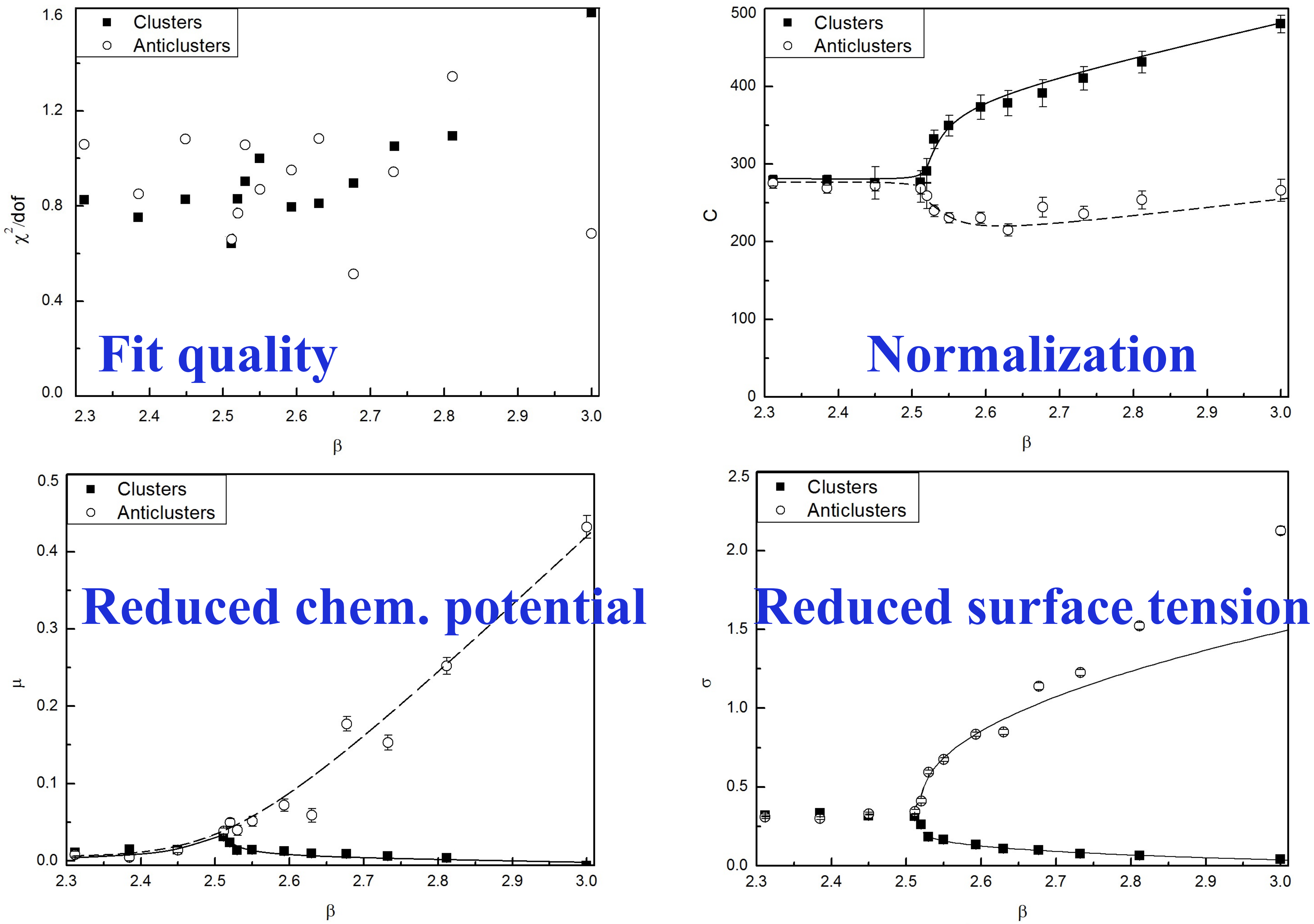
SMM and QGBags with surface tension with 3CEP: τ=1.825±0.025

QGBags with surface tension with CEP: τ >2

However, at the moment we cannot say that QCD has 3CEP!

For fixed T=1.806 Fit Results For Cut-off 0.2

Fitting parameters: C, μ, σ



β	$a_\sigma(\beta)/a_\sigma(\beta_c^\infty)$	T/T_c^∞
2.3115	1.7132	0.5837
2.3850	1.4057	0.7114
2.4500	1.1783	0.8487
2.5115	1.0000	1.0000
2.5200	0.9774	1.0231
2.5300	0.9514	1.0510
2.5500	0.9016	1.1092
2.5930	0.8030	1.2453
2.6300	0.7269	1.3757
2.6770	0.6405	1.5612
2.7325	0.5516	1.8128
2.8115	0.4459	2.2423
3.0000	0.2685	3.7244

At $\beta = 2.52$ global Z(2) symmetry breaks down $\Rightarrow$
chemical nonequilibrium between (anti)clusters ($\mu_{\mathrm{Cl}} \neq \mu_{\mathrm{aCl}}$)

=> Break down of symmetry leads to bifurcations in gas quantities!

=>Any quantity with bifurcation can be used as an order parameter!

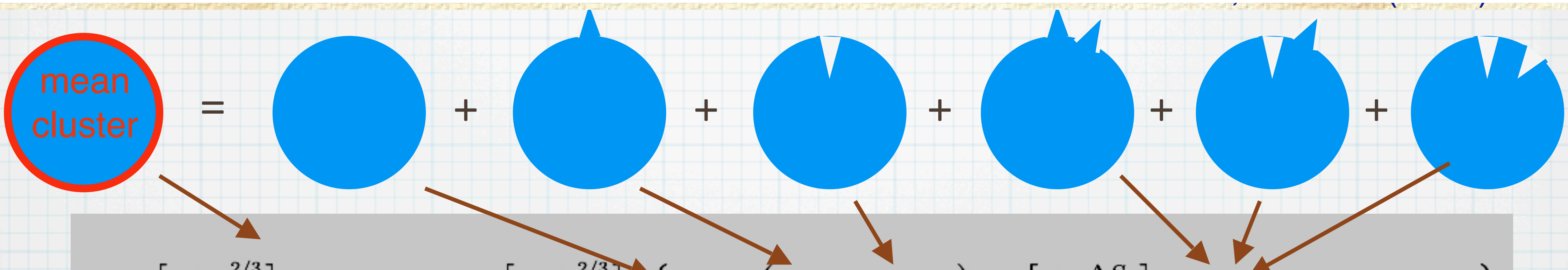
Negative Surface Tension Should Exist since

1. Exactly solvable models of surface deformations of physical clusters show that EIGEN surface tension coefficient must be negative at high T, since

F = E -TS

and since at high T the surface entropy is huge. Entropy is produced by the hedgehog shapes of clusters.

Hills and Dales Model of mean cluster: K. A. Bugaev et al., Phys. Rev. E 72 (2005); UJP 52 (2007)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy}\times\left\{1 + \left(\underbrace{w_H N_H}_{1\ Hill}+\underbrace{w_D N_D}_{1\ Dale}\right)\exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \geq F \geq \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

Negative Surface Tension Should Exist since

2. Cylindrical bag model for confining color tube also shows that at the moment of tube break off its surface tension coefficient MUST BE NEGATIVE.

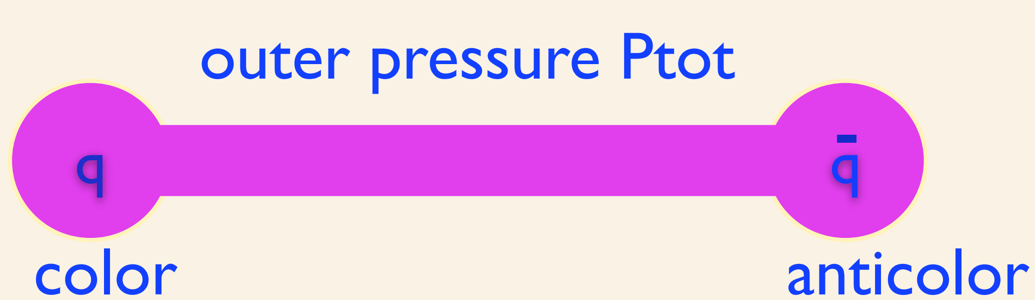
K. A. Bugaev and G. M. Zinovjev, Nucl. Phys. A 848 (2010) 443

In Edward Shuryak lectures Prog. Part. Nucl. Phys. 62:48-101 (2009)

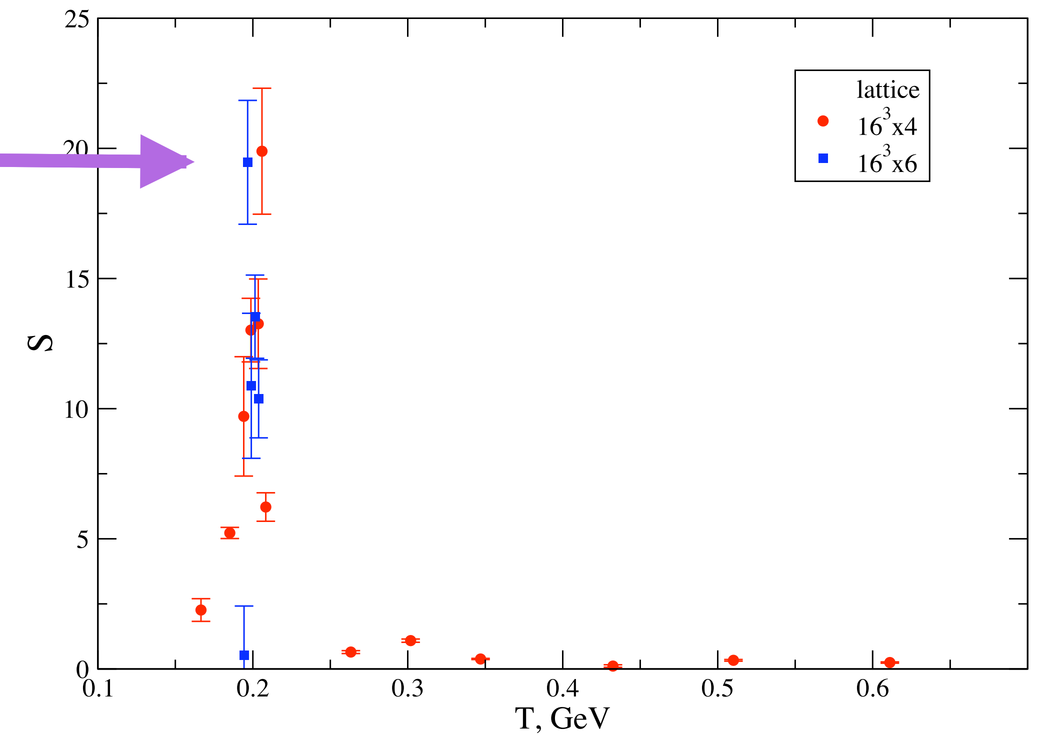
A HUGE maximum in color tube entropy S was called Mysterious

because it was unclear what are the dof with #dof = exp (S=20) = 485 000 000

Consider confining string between static q & anti q of length L and radius R<<L



Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$



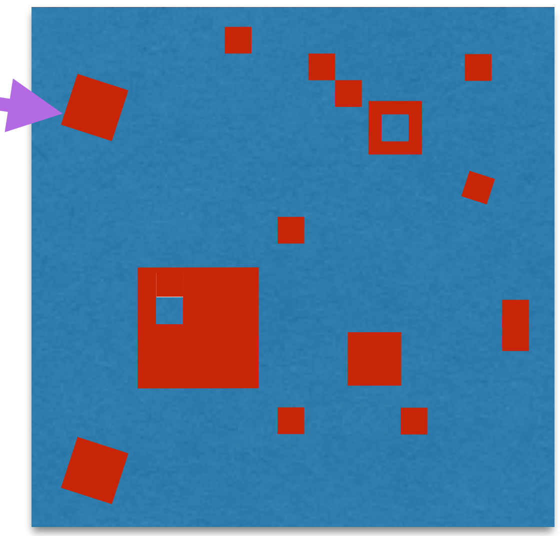
Due to negative value of surface tension coefficient there must appear the FRACTAL ripples on the surface of color tube. This explains a huge # of dof

However, the ripples must disappear, when the tube occupies the whole volume and, hence, it leaves no free surface!

Surface Free Energy of Anticluster Liquid Droplet (shown in blue)

Can be deduced from locations of gaseous clusters (shown in red)

T >> Tc



total geometrical volume of liquid anticluster

$$F_A^{surf} = \Sigma_A^{outer} \left[\max K_A + \widetilde{\sum_k} k\, n_{\bar{A}}(k) \right]^{\frac{D_A - 1}{D_A}} - T\sigma_{\bar{A}} \widetilde{\sum_k} k^{\frac{2}{3}}\, n_{\bar{A}}(k)$$

surface tension of outer surface

fractal dimension

total free energy of inner gaseous clusters of Polyakov loop of other sign

Due to periodic boundary conditions at T >> Tc there is no outer surface for anticluster LIQUID DROPLET!

=> at T >> Tc the largest droplet (liquid) may have NEGATIVE SURFACE TENSION!

A. I. Ivanytskyi, K. A. Bugaev et al., Nuclear Physics A 960 (2017) 90; arXiv:1606.047 [hep-lat]

This is true for T >> T_c, but what about T ≥ T_c, as it was expected?

Properties of Monomers

Idea: introduce effective volume and surface of monomers and fit their multiplicities!

A priori we do not know, if the Liquid Gas Model formula can work…

Include the monomers into fit

$$\chi_A^2 = \sum_{i=1}^{N_\beta} \left(\underbrace{\frac{\left[{}_A n_{k=1}^{th} - {}_A n_{k=1}\right]^2}{[\delta_A n_{k=1}]^2}}_{monomers} + \sum_{k=2}^{k_{max}(\beta)} \frac{\left[{}_A n^{th} - {}_A n\right]^2}{[\delta_A n]^2} \right),$$

$$dof = N_\beta - 3 + \sum_{i=1}^{N_\beta} (k_{max} - 2 - 3) = \sum_{i=1}^{N_\beta} k_{max} - 4N_\beta - 3$$

i counts for all β values: N_β is their number

Properties of Monomers

Parameterization of monomer multiplicity with effective volume and effective surface

Assume $$ln\left({}_A n_{k=1}^{th}\right) = ln\ C_A + \underbrace{ln\ g_A}_{correction} + \underbrace{\mu_A \cdot V_A}_{effective\ V} - \underbrace{\sigma_A \cdot S_A}_{effective\ S}$$

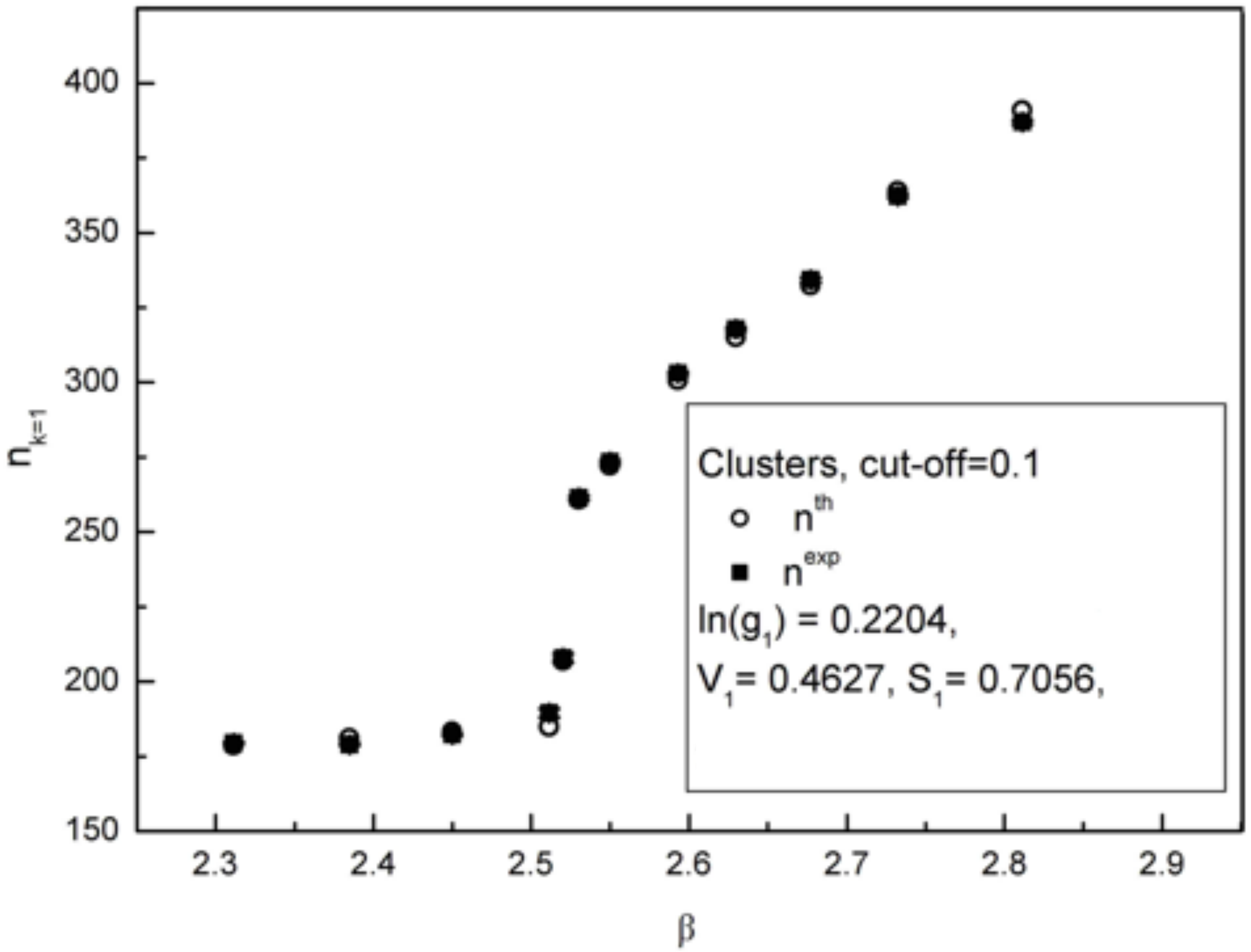
parameters C_A, μ_A and σ_A are taken from k-mers ($k > 1$)

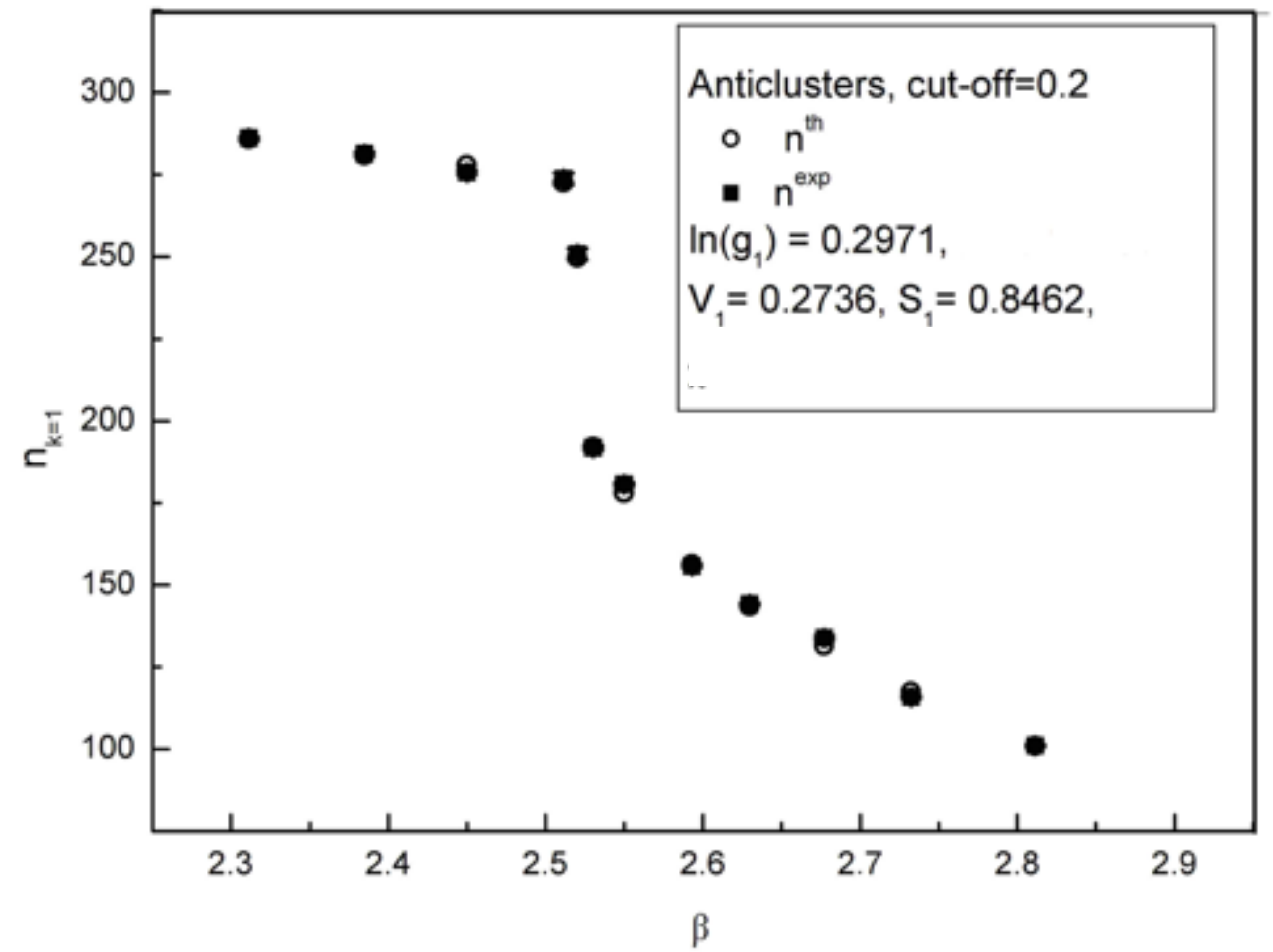
in order to assure full thermal and chemical equilibrium!

g_A, V_A and S_A can be found by the maximum-likelihood method as

$$\frac{\partial \chi^2_A}{\partial\left(ln\ g_A\right)} = 0\,, \qquad \frac{\partial \chi^2_A}{\partial V_A} = 0\,, \qquad \frac{\partial \chi^2_A}{\partial S_A} = 0\,.$$

Properties of Monomers (examples)





$data$	$cut-off$	$\ln g_A$	V_A	S_A	$\frac{S_A}{\ln g_A}$	$\frac{\chi^2_A}{dof}$
Anticlusters	0.1	0.30578±0.00103	0.42274±0.00927	0.85643±0.00099	2.80083	1.4685
Clusters	0.1	0.22039±0.00068	0.46265±0.04176	0.70556±0.00179	3.20143	0.8952
Anticlusters	0.2	0.29712±0.00081	0.27362±0.00952	0.84625±0.00115	2.84820	0.9567
Clusters	0.2	0.17568±0.00056	0.99028±0.05024	0.52508±0.00288	2.98878	0.9173

Please mark small volume and small surface of monomers!

Total fit quality is good! This ratio is puzzling since it is about 3!!!

Total Surface Tension of Monomers

g_A is nearly the same for both cut-offs and for both types of clusters ($\pm 7\%$)

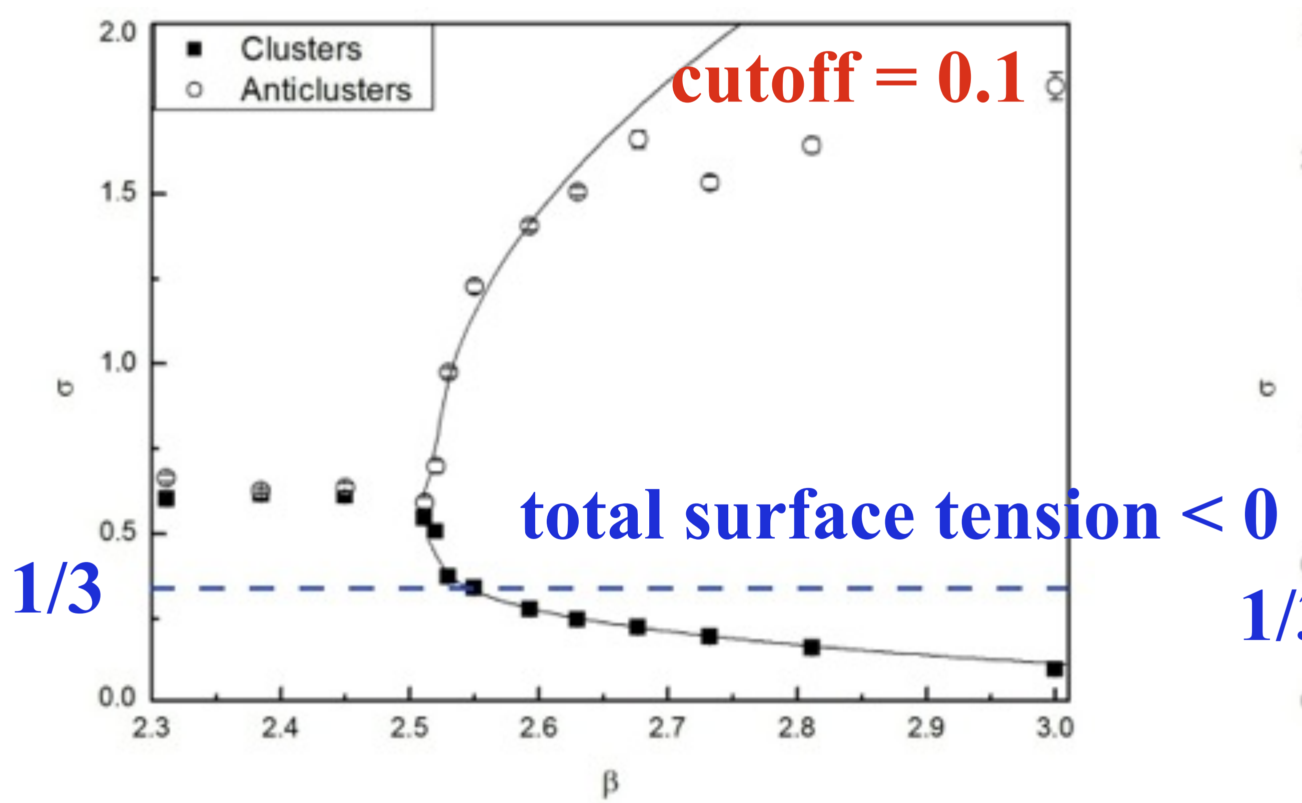
It seems that the correction of normalization g_A is redundant!

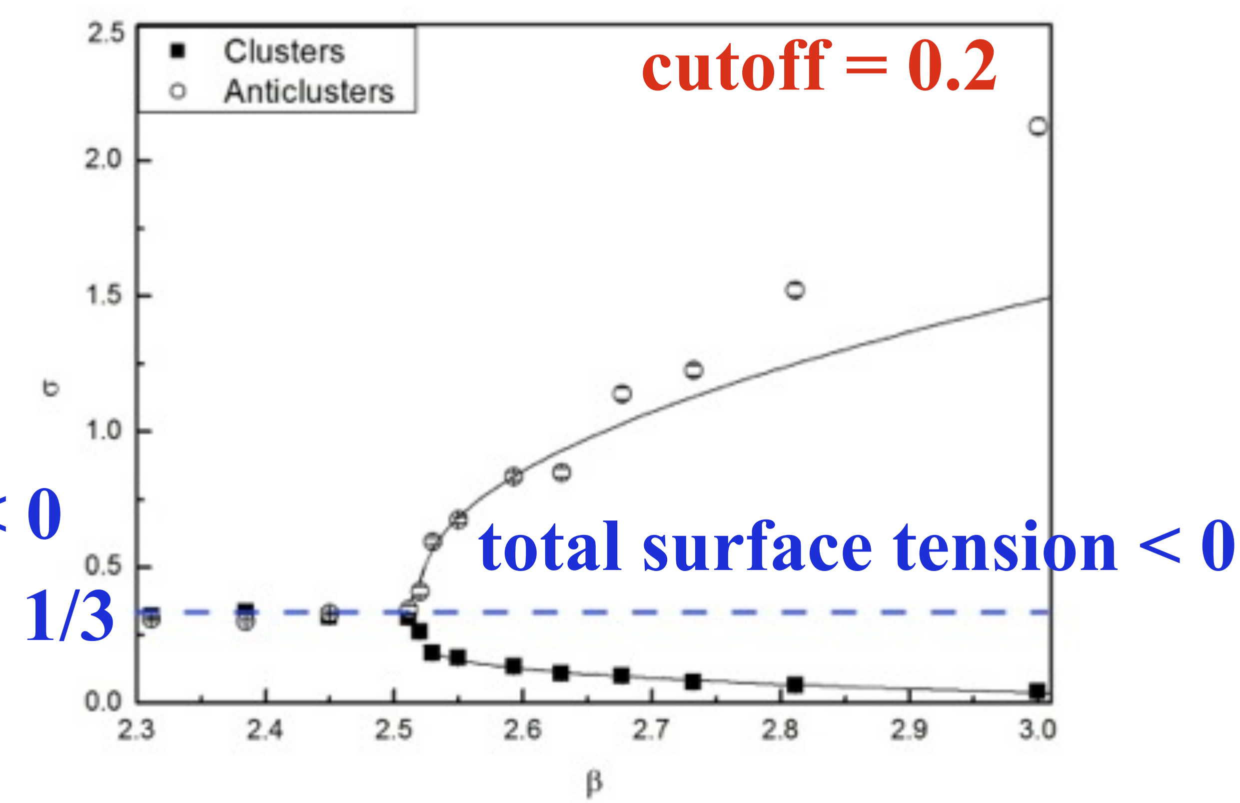
Then one can get rid of it

$$ln\left({}_{A}n_{k=1}^{th}\right) \simeq \ln\ C_A + \mu_A V_A - S_A\left(\sigma_A - \frac{1}{3}\right)$$

total surface tension

For monomer anticlusters the total surface tension > 0 always!





But for monomer clusters the total surface tension < 0 for β > βc!

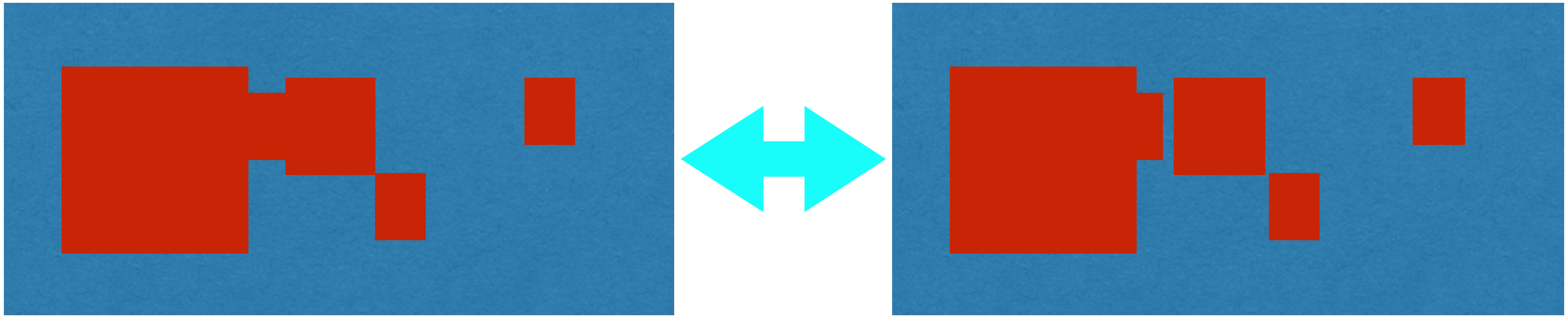
Why Only the Monomer Clusters Have Negative Surface Tension?

The Hills and Dales Model of mean cluster and Cylindrical Bag Model are dealing with Eigen Surface Tension of a separate mean cluster!

However, in a medium the clusters should have an additional Surface Tension Induced by the hard-core repulsion between them!

V.V. Sagun, A.I. Ivanytskyi, K.A.B., I.N. Mishustin, Nucl. Phys. A 848 (2010) 443

Consider vacuum-like reactions of clusters: L. G. Moretto, K.A.B. et al., PRL 94 (2005) 202701

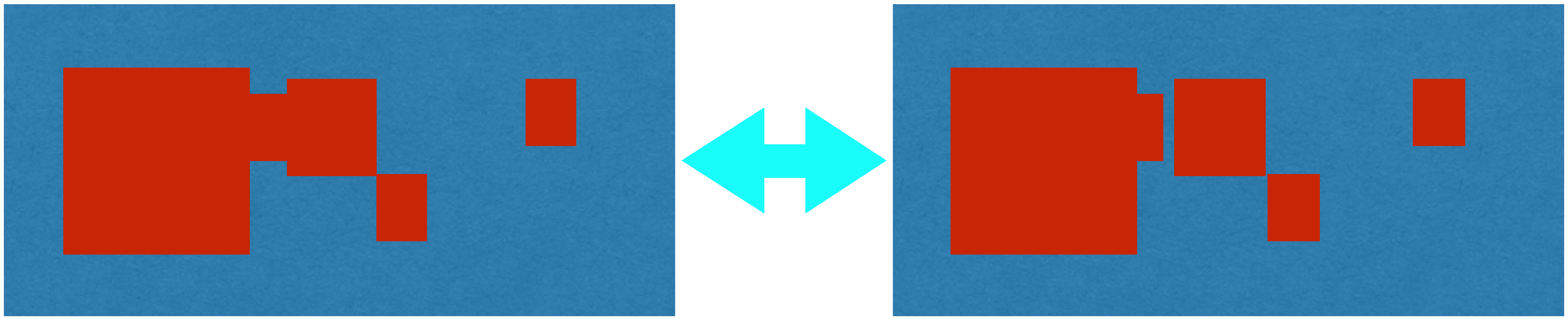


Repulsion: presence of third party affects the decay/fusion probability of a cluster and its liquid droplet!

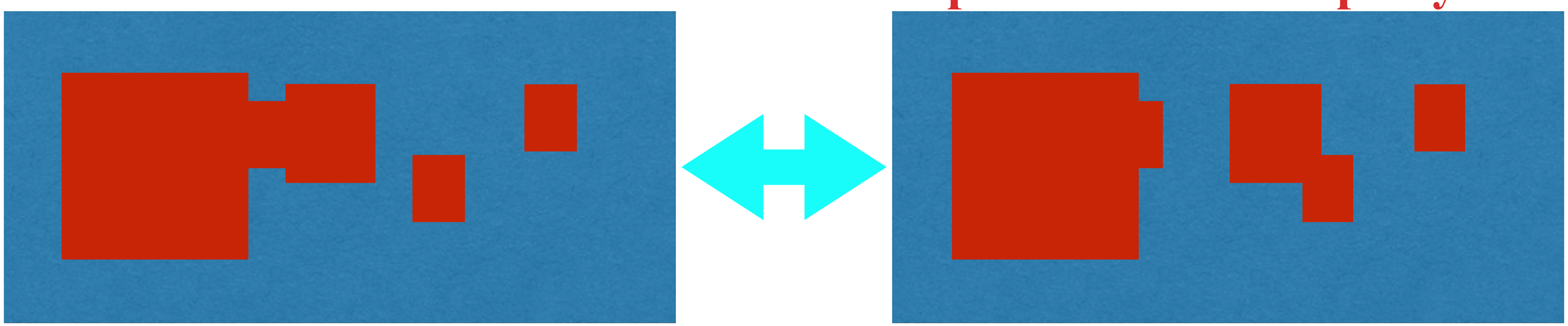
This is one finite size effect!

Why Only the Monomer Clusters Have Negative Surface Tension?

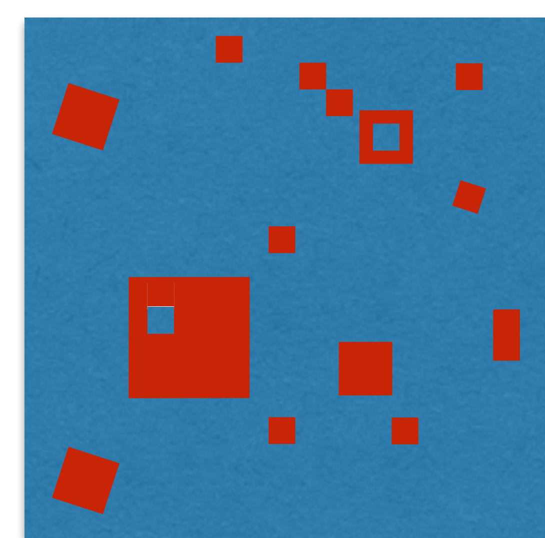
In addition to vacuum-like reactions: L. G. Moretto, K.A.B. et al., PRL 94 (2005) 202701



There are cluster reactions due to the presence of third party:



Which create the other finite size effect (kind of attraction)!



Moreover, one should remember that reactions change the free energy of anticluster liquid!

Why Only the Monomer Clusters Have Negative Surface Tension?

=> **At the moment there is NO COMPLETE UNDERSTANDING of this fact due to an interplay of short range interaction and finite size effects.**

Working guess: due to smaller volume (<1 lattice units) and smaller surface (<<6 lattice units) they are least affected by the presence of third party!?

=> **This direction is open for exploration!**

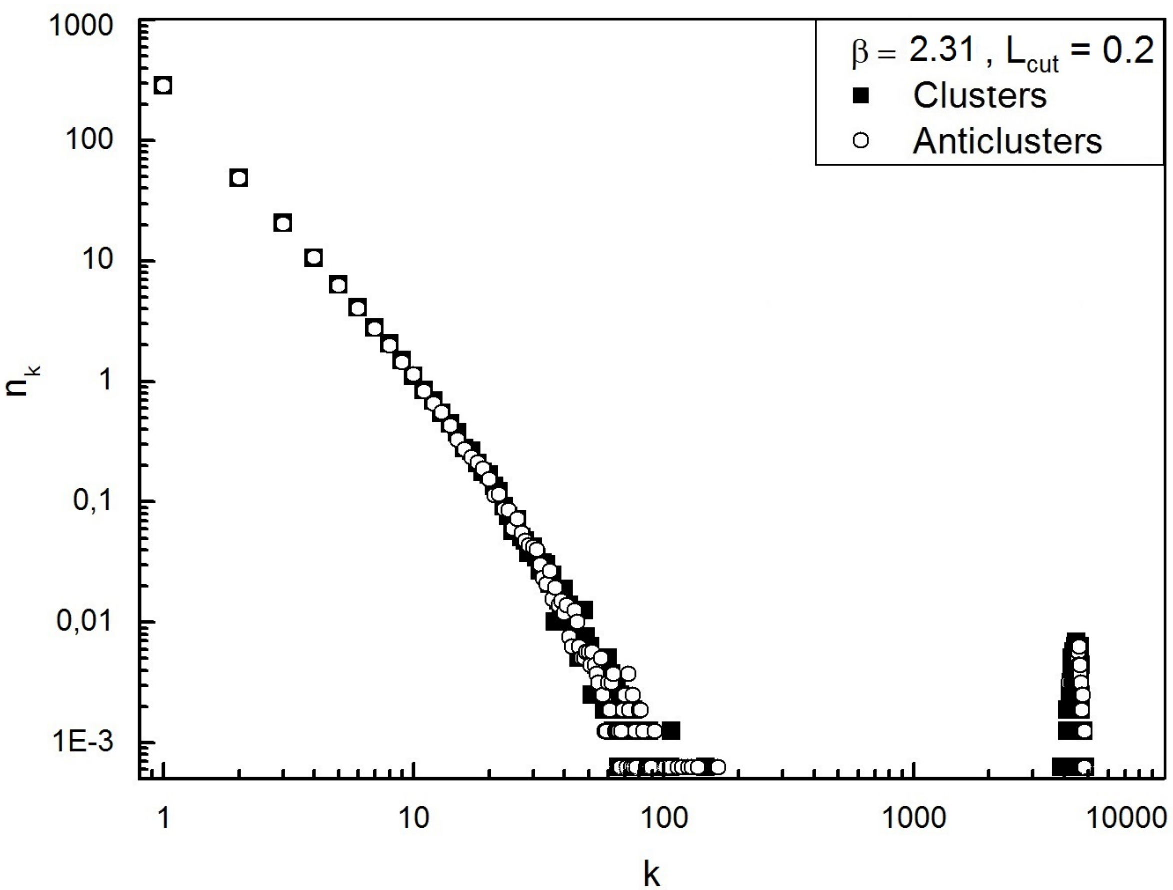
Fluctuations of (Anti)Cluster Multiplicities near 2-nd order Phase Transition (PT)

Motivation: Experimental searchers for (Tri)critical Endpoint ((3)CEP) in Heavy Ion Collisions, which is expected to have a 2-nd order PT

Multiplicities of small clusters (were shown)

Distributions for $\beta \leq \beta_c \simeq 2.52$

- symmetry between (anti)cluster distributions
- gas and "liquid" domains are well separated



Distributions for $\beta > \beta_c \simeq 2.52$

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- "cluster liquid" evaporates to cluster gas
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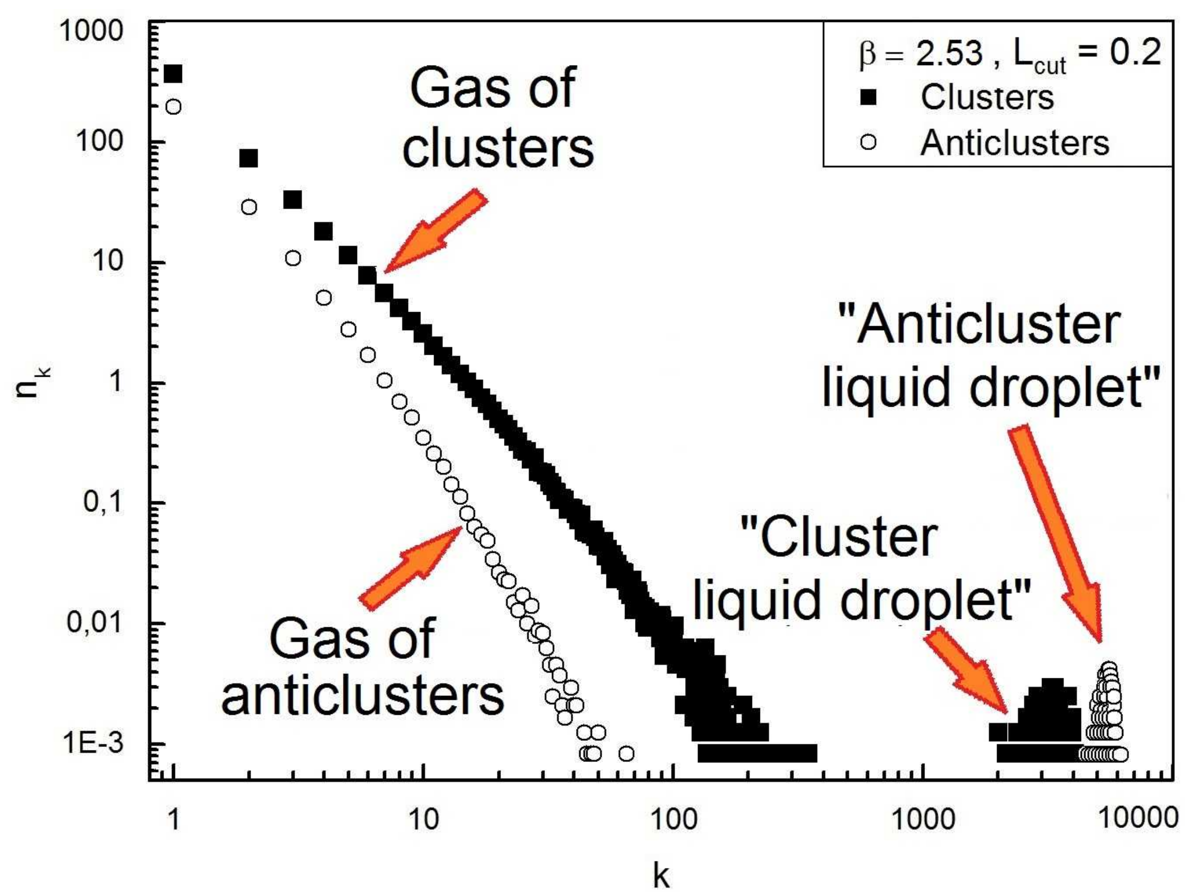
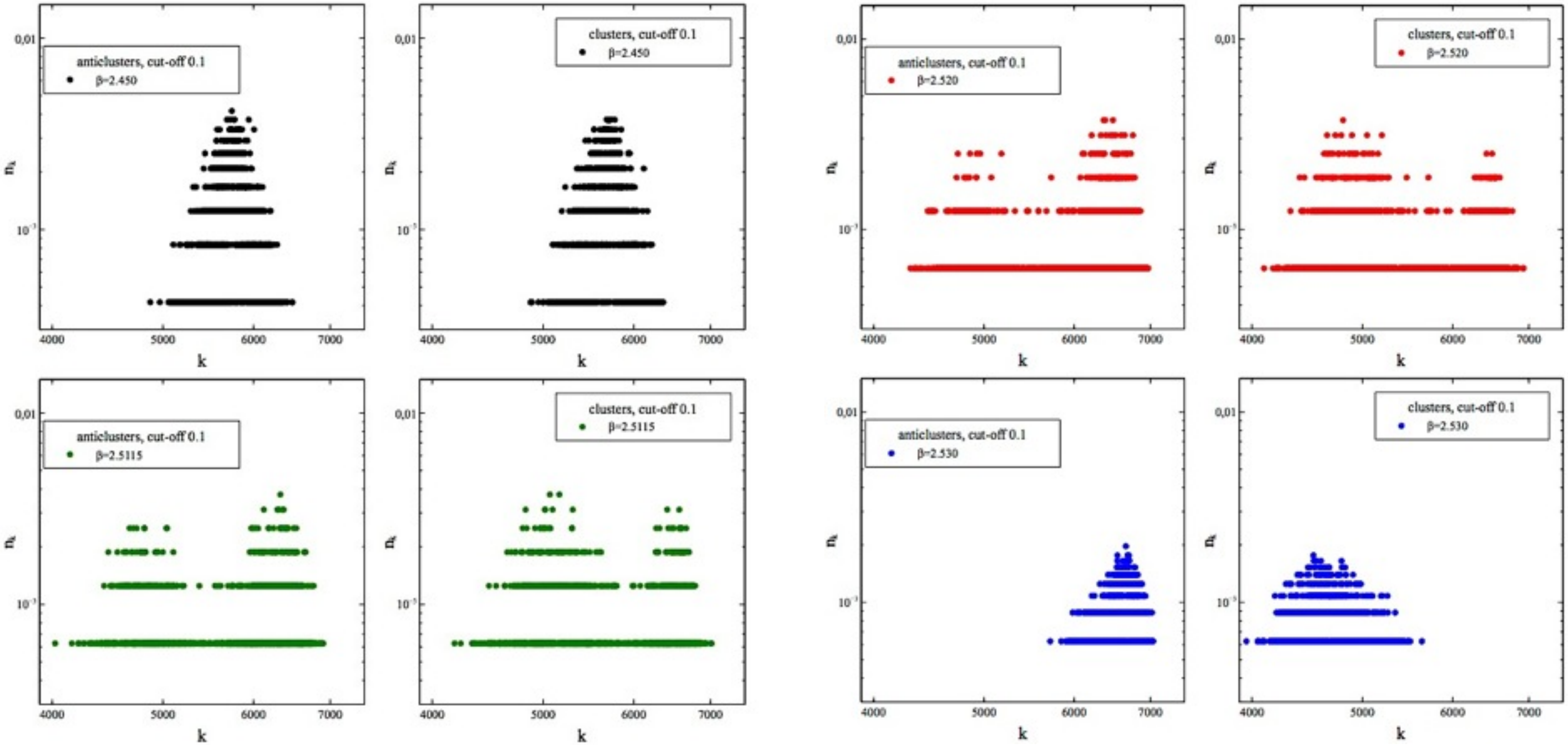


size (volume) of cluster in lattice units

Fluctuations of (Anti)Cluster Liquid Droplet Multiplicity near 2-nd order PT

In thermodynamic limit the critical value is $\beta c = 2.5115$

For finite volumes the critical value is $\beta c \sim 2.52$



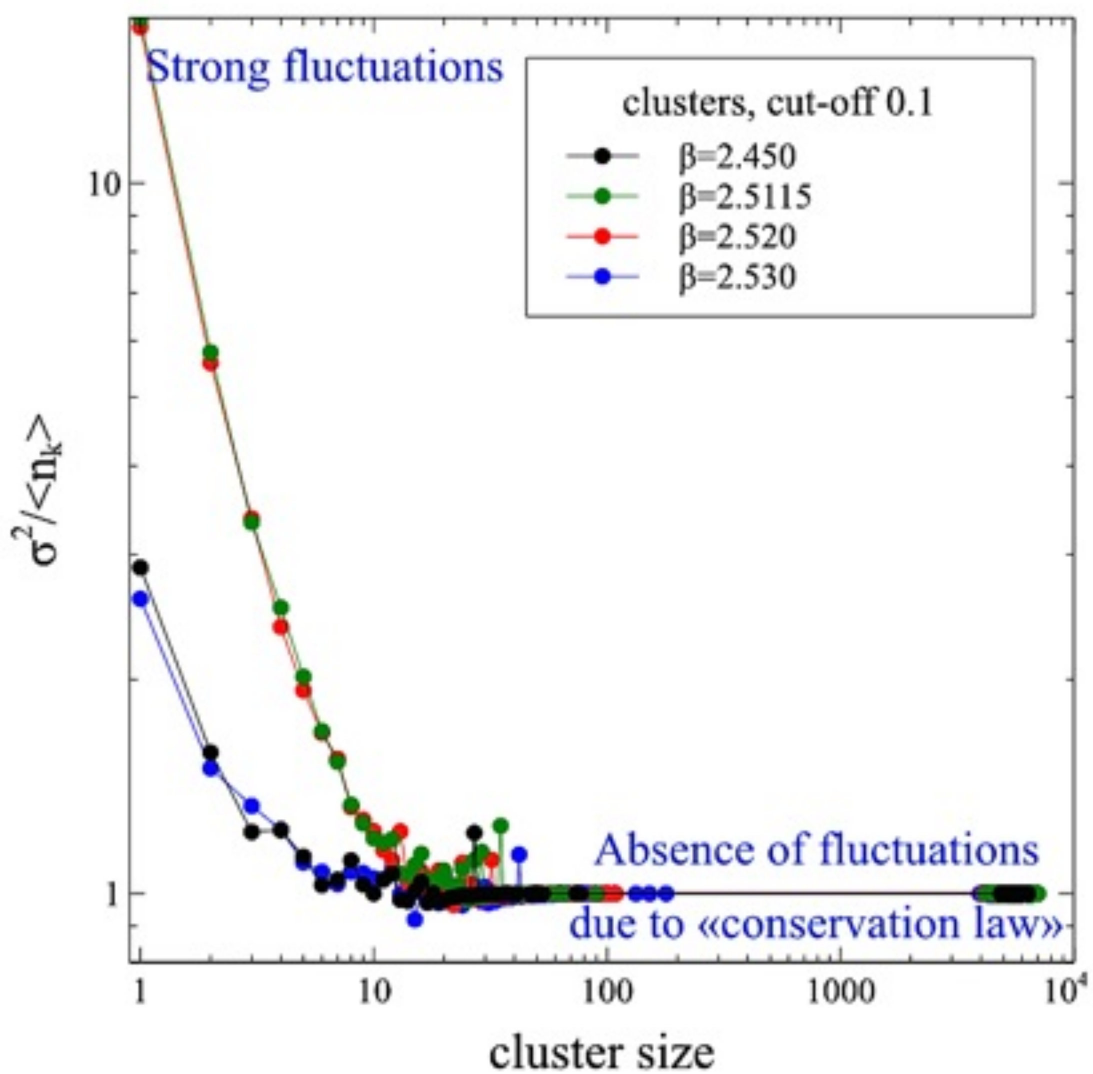
For $\beta c \sim 2.51$-2.52 there are strong fluctuations of Polyakov loop sign of both liquid droplets! Looks like a **mitosis**!?

Scaled Variance of (Anti)Cluster Multiplicities near 2-nd order PT

the scaled variance $\omega(k) = \frac{\sigma(k)^2}{<n(k)>}$

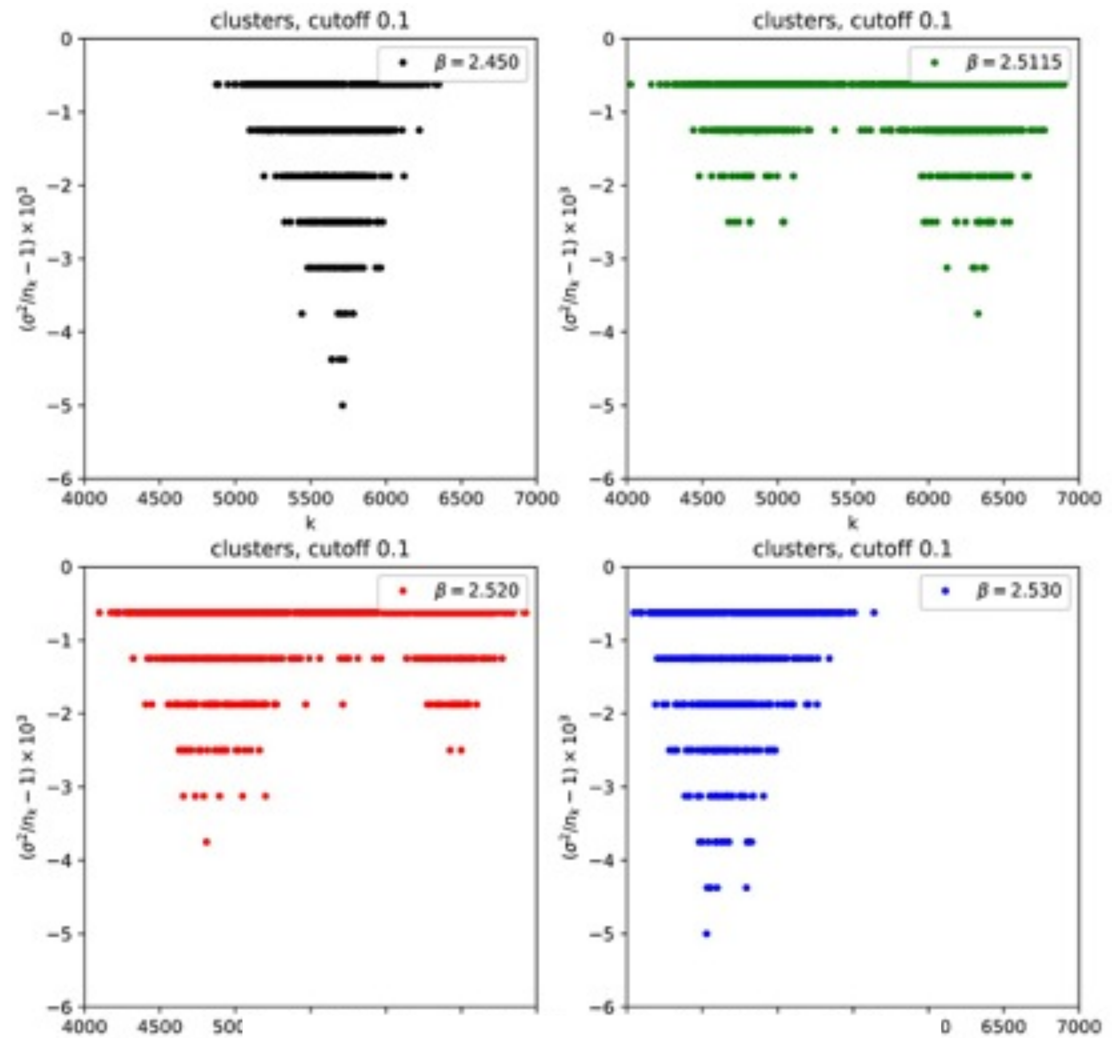
the standard deviation for (anti)cluster of size k is $\sigma(k) = \sqrt{< n(k)^2 > - < n(k) >^2}$

Small clusters



In grand canonical ensemble ω=1 means conservation of the number of droplet.

Largest cluster (droplet)



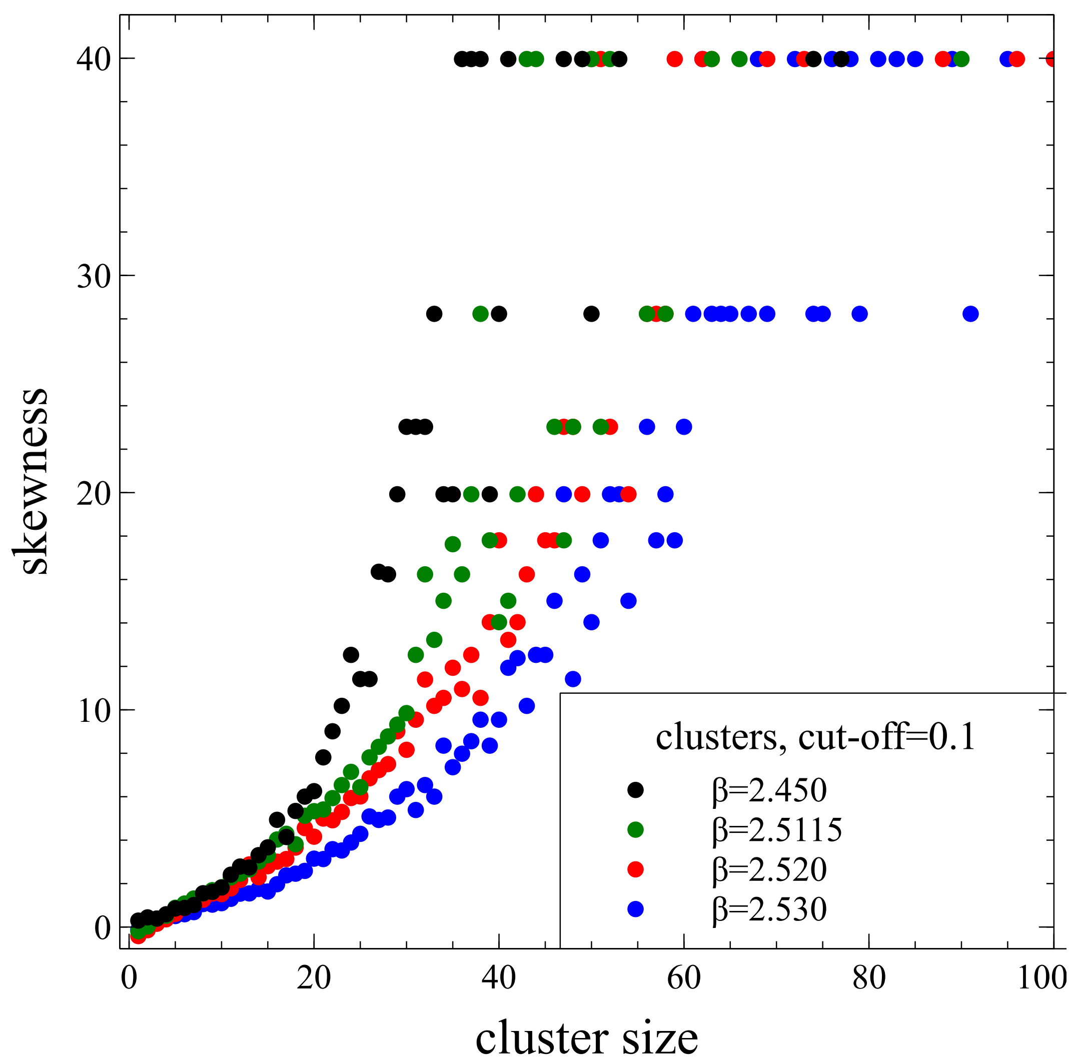
$$\Delta\omega(k) \equiv (\omega(k) - 1) \cdot 10^3$$

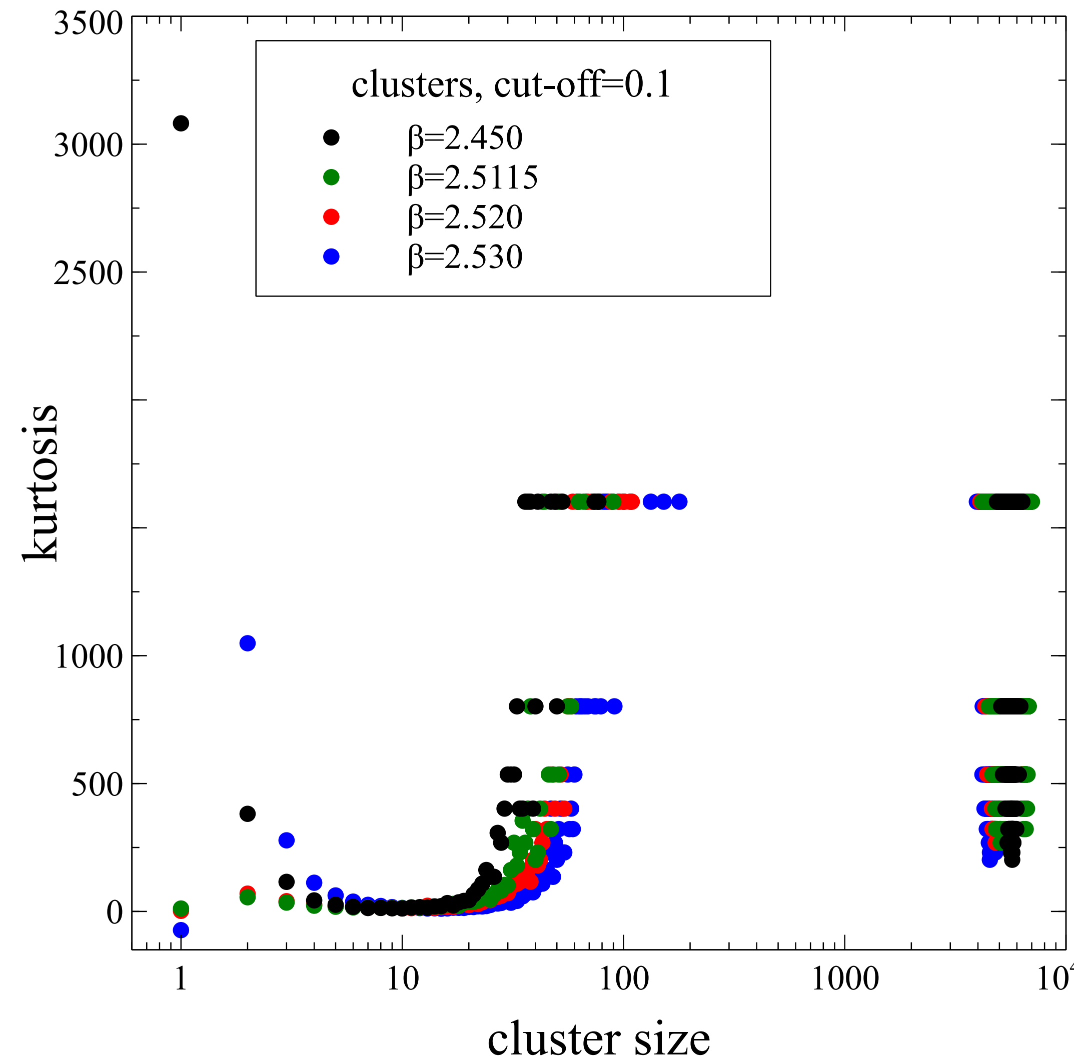
is very similar to n(k) of droplets!

Skewness and Kurtosis of Small Cluster Multiplicities near 2-nd order PT

the skewness $\gamma(k) = \frac{<n(k)^3> - 3\, n(k)\, \sigma(k)^2 - n(k)^3}{\sigma(k)^3}$

the kurtosis $Kurt = \frac{<n(k)^4>}{\sigma(k)^4}$





For β > 2.51 the skewness is NEGATIVE for k=1, 2 clusters only!

For anticlusters skewness is always positive.

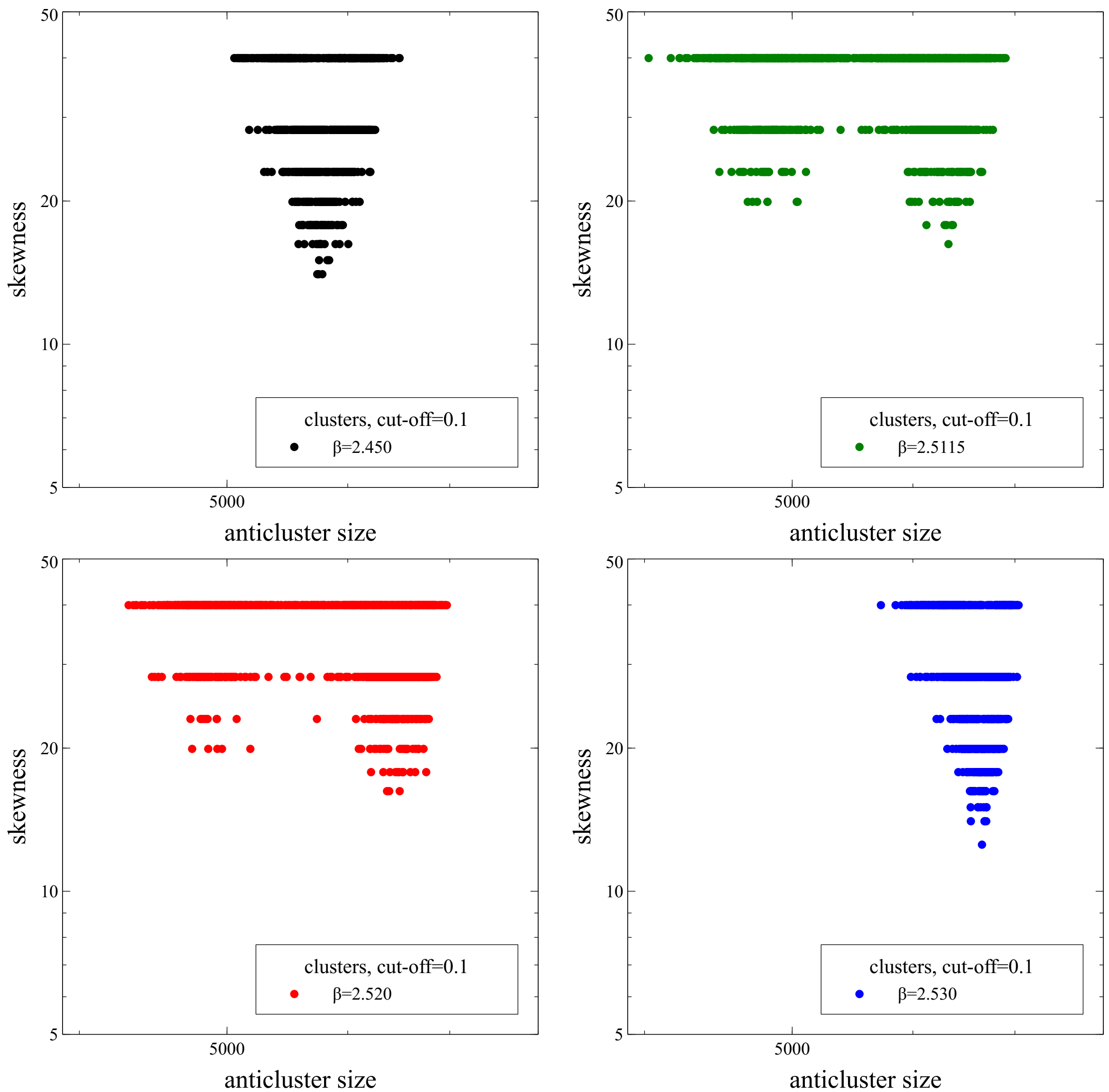
For β > 2.51 the kurtosis is NEGATIVE for k=1 clusters only!

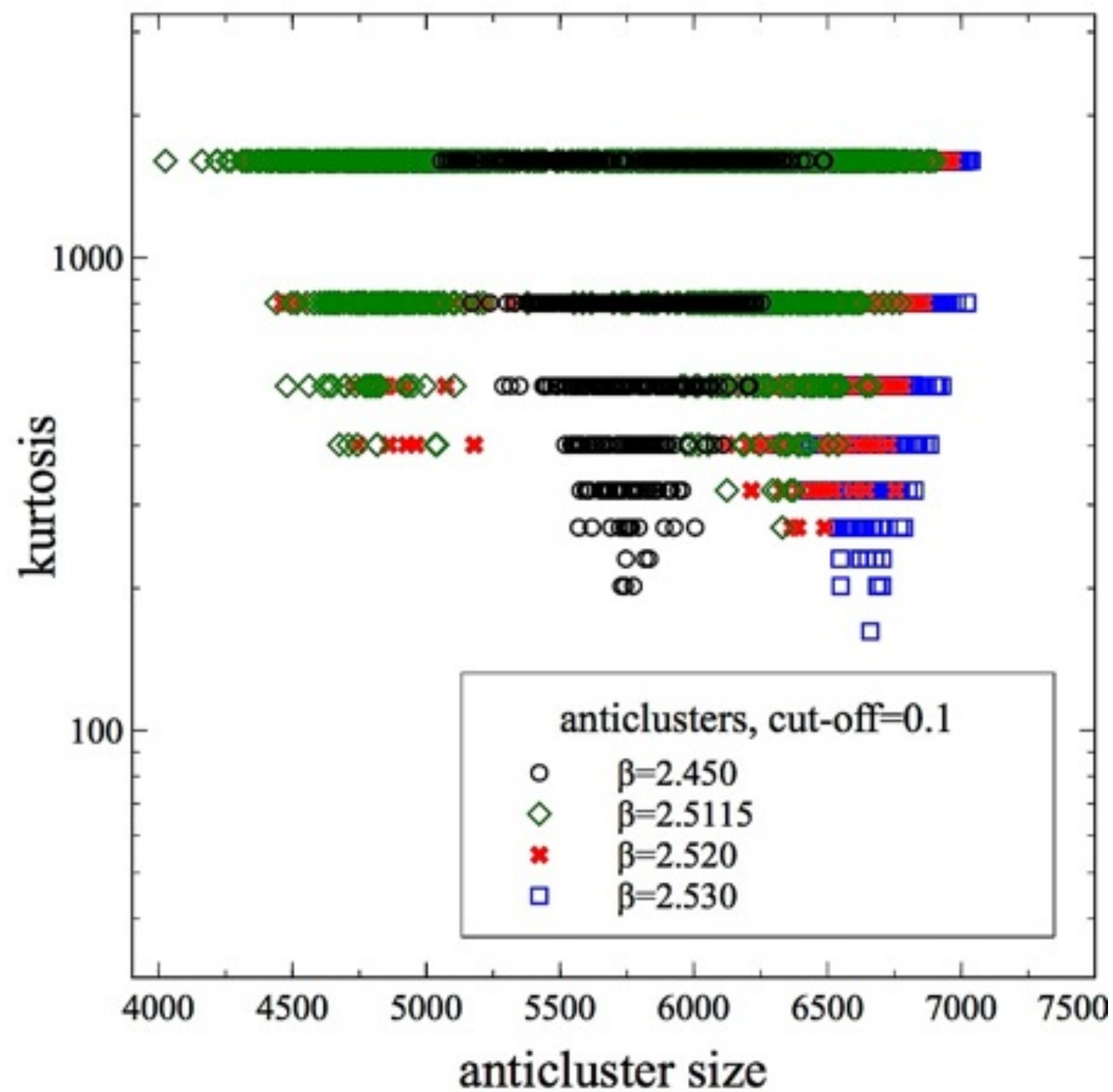
For anticlusters it is always positive.

Skewness and Kurtosis of (Anti)Cluster Droplet Multiplicities near 2-nd order PT

the skewness $\gamma(k) = \frac{<n(k)^3> - 3\, n(k)\, \sigma(k)^2 - n(k)^3}{\sigma(k)^3}$

the kurtosis $Kurt = \frac{<n(k)^4>}{\sigma(k)^4}$





Similarly to multiplicities the skewness and kurtosis of (anti)cluster liquid droplets demonstrate a mitosis!

Conclusions A

- The cluster approach based on Polyakov loop geometrical clusters is generalized to monomer (anti)clusters.
- In terms of liquid-gas cluster model the PT in SU(2) gluodynamics is an evaporation of smaller liquid droplet into corresponding gas and condensation of another gas onto the largest liquid droplet.
- The Fisher topological constant τ is found to be 1.806 ± 0.008 which disagrees with the Fisher Droplet Model value, but agrees with SMM and QGP bag with surface tension model with 3CEP.
- Any quantity which shows bifurcation can be used as the order parameter.
- In contrast to the existing cluster models of quark-gluon-hadron PT the lattice surface tension of dimers and larger clusters does not vanish above PT. Only the surface tension coefficient of monomer clusters is negative above βc. Still there is no understanding of this issue.

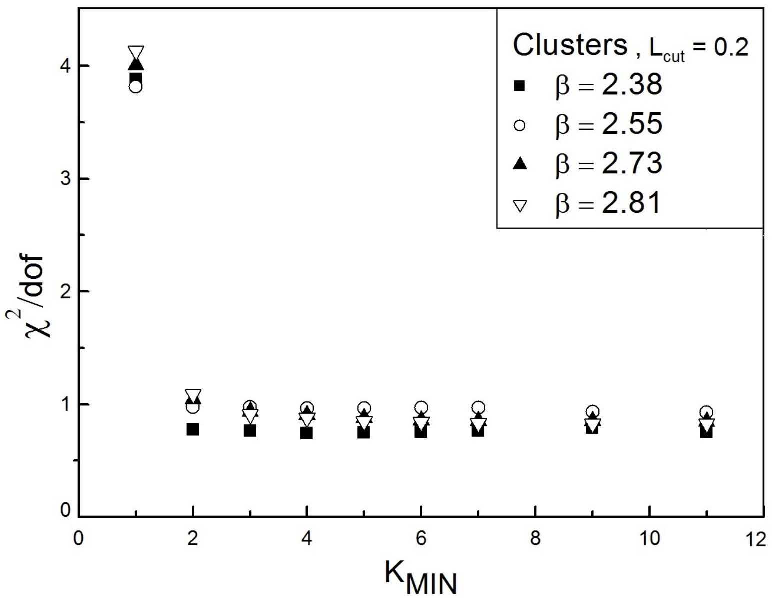
Conclusions B

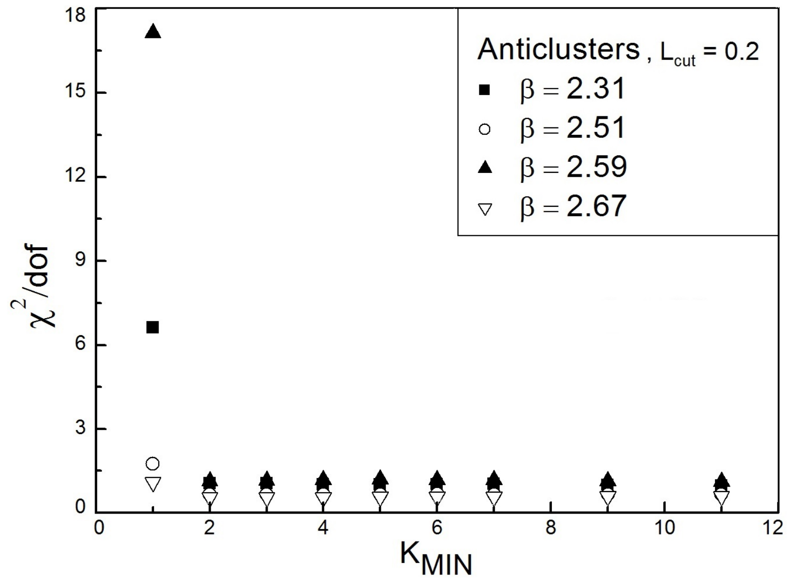
- Fluctuation patterns of (anti)cluster multiplicities demonstrate peculiar behavior.
- Bad news for experimentalists: strong fluctuations of small clusters and their droplets can be seen within a very narrow range of temperatures ~ 0.003 Tc.
- Good news for experimentalists: there is a similarity of n(k), σ^2(k), γ(k) and Kurt (k) of (anti)cluster liquid droplets. Hence, one can hope to measure (with good precision!) n(k) fluctuations only to recover γ(k) and Kurt (k). But at the moment it is unclear what and how to measure (deutrons? tritons? helium3?).

Thank you for your attention!

Defining the minimal N-mer and Fisher index τ

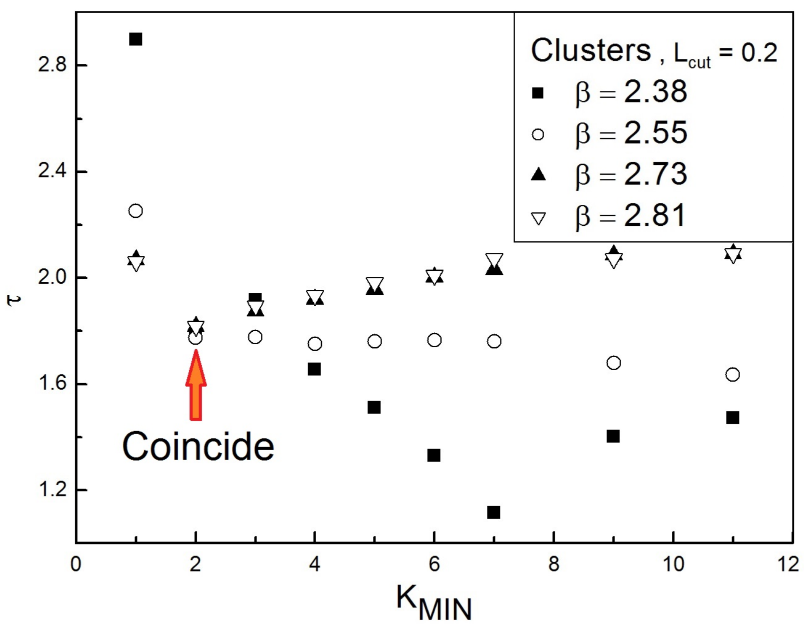
- LDF describes size distributions with almost the same quality for all $k_{min} \geq 2$

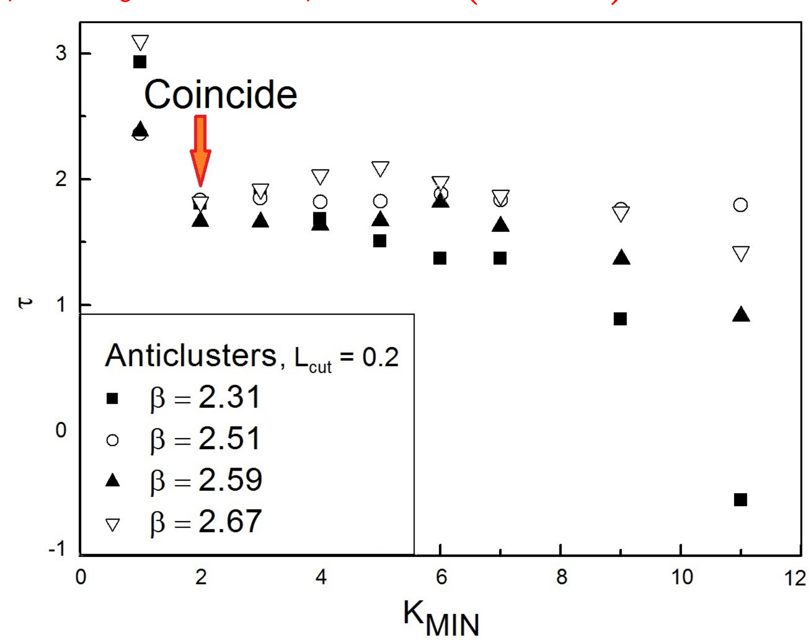




- Fisher topological exponent τ is temperature independent at $k_{min} = 2$

in agreement with Fisher droplet model M.E. Fisher, Physics 3, 255 (1967)





$$\mathbf{k_{min} = 2, \quad \tau = 1.806 \pm 0.008}$$

This value of τ = const of β as required by LDM!

Surface Tension in Physical Units for Fixed τ=1.806

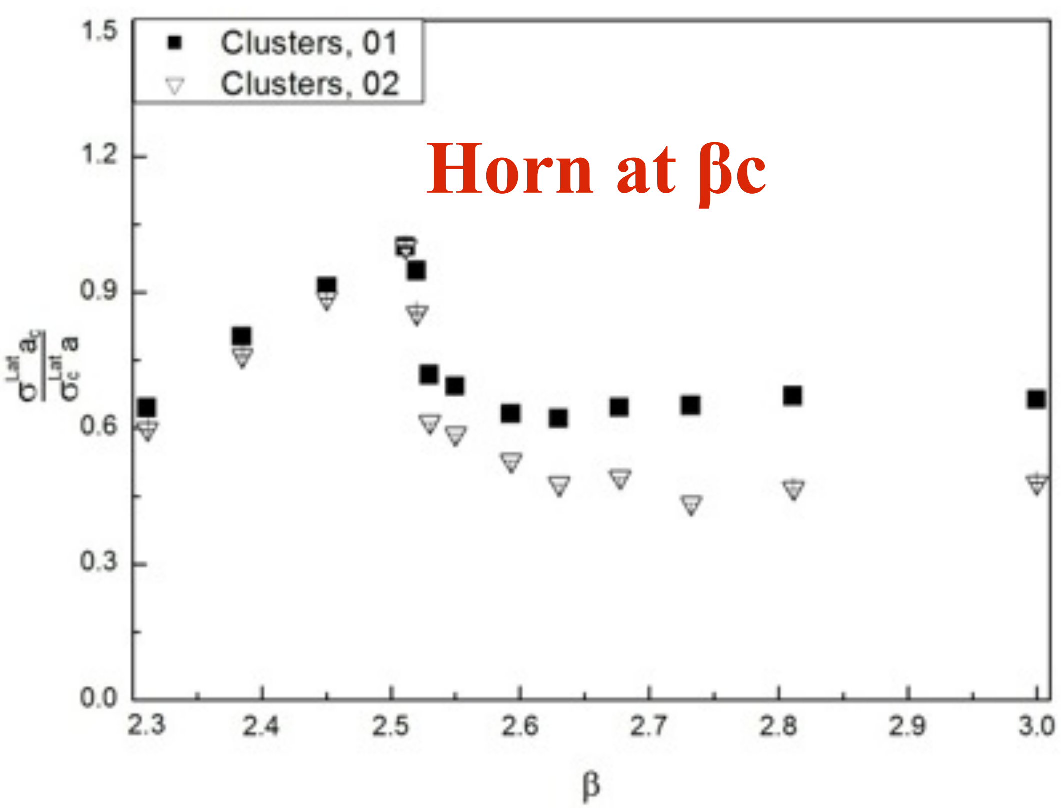
The β-dependence of physical surface tension defined as

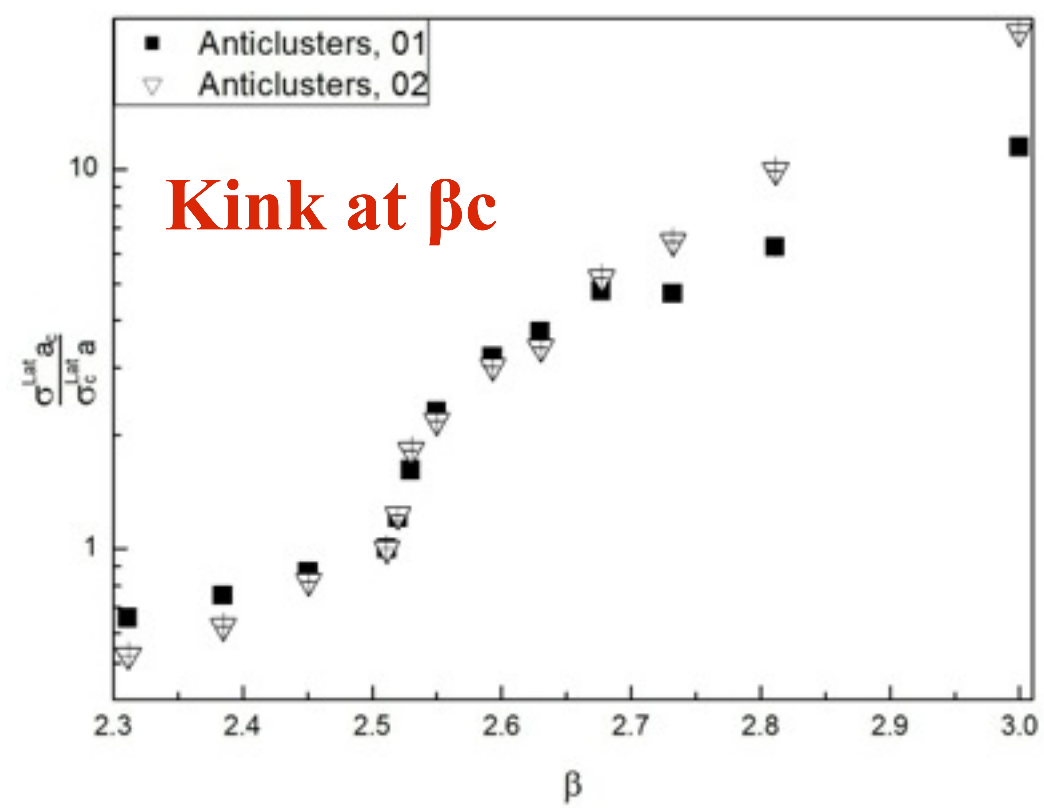
$$\sigma_A^{phys}(\beta) \equiv T \frac{\sigma_A(\beta)}{[\, a_\sigma(\beta)\,]^2} = T_c^\infty \, \frac{a_\sigma(\beta_c^\infty)}{a_\sigma(\beta)} \, \frac{\sigma_A(\beta)}{[\, a_\sigma(\beta)\,]^2} \, ,$$

But the ratio

$$\sigma_A(\beta)\, a_\sigma(\beta_c^\infty)/\sigma_A(\beta_c^\infty)/a_\sigma(\beta)$$

is more convenient





$$\sigma_{cl}^{phys}(T) = \frac{Const}{[\, a_\sigma(\beta)\,]^2} \sim T^2 \quad \text{for} \quad 1.25\, T_c^\infty < T \leq 3.7\, T_c^\infty$$

BUT $\sigma_{acl}^{phys}(T) \sim T^4$

Other Important Findings

In contrast to existing exactly solvable models of cluster type the physical surface tension of Polyakov loop (anti)clusters DOES NOT VANISH above PT!

=> Hence SU(2) gludynamics may have a different mechanism for 2-nd order PT!

However, the KINKs in physical surface tension are known from existing cluster models!

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. 76 (2013), 341

This is the surface tension induced PT which was invented to generate the CEP!

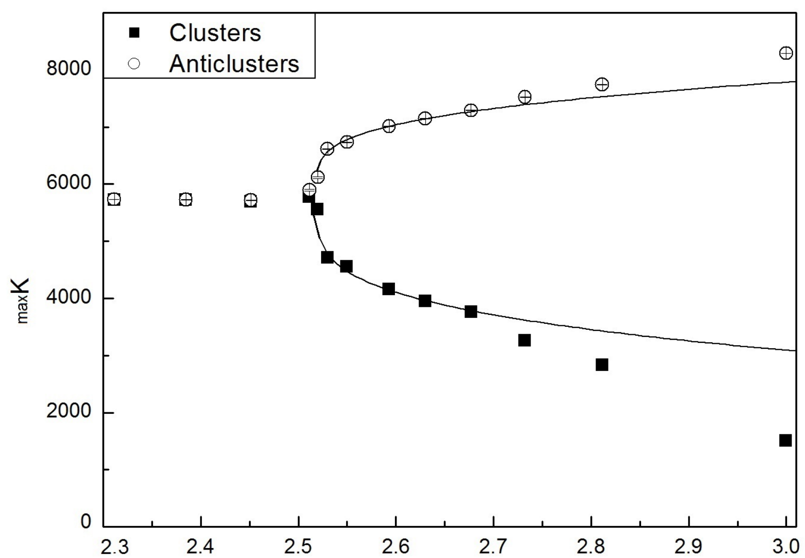
Properties of Liquid (Anti)Cluster

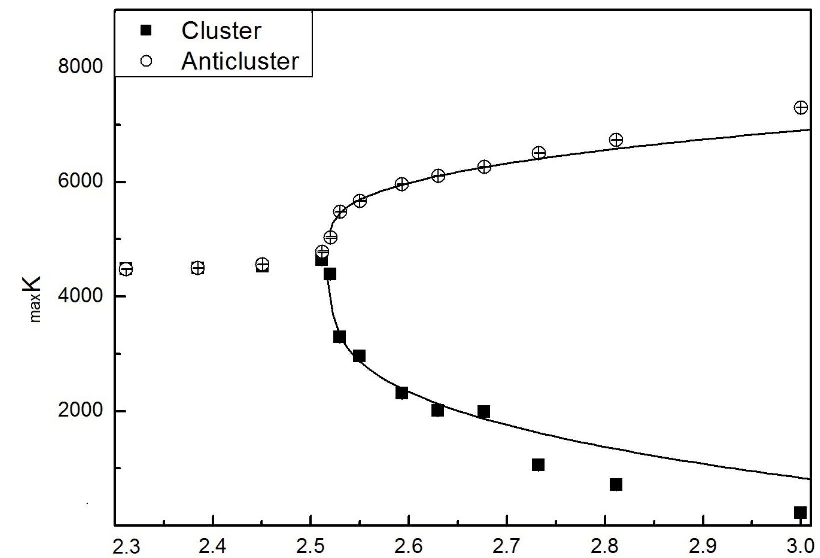
The mean value of Polyakov loop <L> is an order parameter in gluodynamics

One can show that $|L| \sim \max K_{aCl} - \max K_{Cl}$

$$\max K = \sum_{\tilde{x}} k^{1+\tau} n_k \Big/ \sum_{\tilde{x}} k^{\tau} n_k$$ **is the mean liquid (=largest) (anti)cluster**

$$\beta > \beta_c : \ \max K(\beta) - \max K(\beta_c) = a \cdot (\beta_c - \beta)^b$$





L_{cut}	type	a	b	χ^2/dof
0.1	Cl	-3056 ± 246	0.2964 ± 0.0284	$16.32/4 \simeq 4.08$
0.1	aCl	2129 ± 160	0.3315 ± 0.0269	$8.94/4 \simeq 2.235$
0.2	Cl	-4953 ± 443	0.3359 ± 0.0289	$12.3/3 \simeq 4.01$
0.2	aCl	2462 ± 87.7	0.3750 ± 0.0129	$2.068/4 \simeq 0.517$

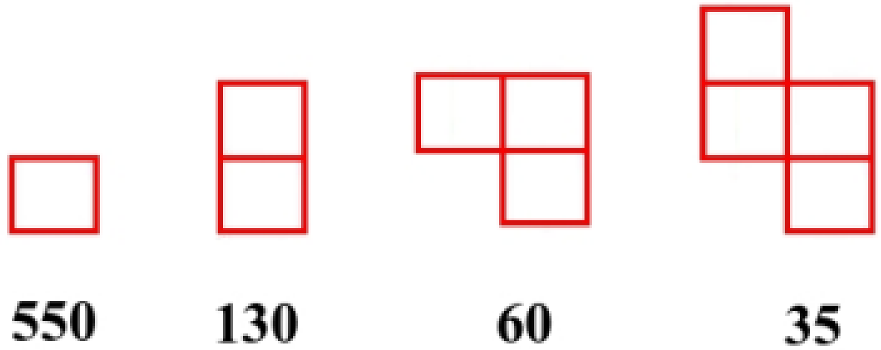
strong fluctuations

Except for aCL with cut-off 0.2 the exponent b corresponds to 3-d Ising model!

Space Inhomogeneity

Example: $\beta = 3$ and cut-off 0.2

$^{max}K_{acl}$ = 7300 - the volume of largest anticluster;
$^{max}K_{cl}$ = 223 - the volume of largest cluster;
V^{gas}_{acl} = 89 - the total volume of the gas of anticlusters;
V^{gas}_{cl} = 2848 - the total volume of the gas of clusters;
V_{vac} = 1707 - the volume of auxiliary vacuum;



Since at hight T the surface tension and chem. potential of clusters is about 0, then size distribution is a power like!

Assuming DENSE PACKING of all clusters one needs at least 3100 surrounding anticlusters or aux. Vacuum, but one can get 1796 only!

=> Gaseous clusters are located inside of anticluster LIQUID droplet!

=> High T is not an exception, hence, the clusters are located inside of anticluster LIQUID droplet and vice versa!