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Experimental bounds for spacetime torsion

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OUTLINE

- **Theoretical foundations of spacetime torsion**
- **Dirac particle in gravity with torsion**
- **Foldy-Wouthuysen transformation for a Dirac particle with allowance for spacetime torsion**
- **Extension of the covariant Dirac equation on the anomalous magnetic and electric dipole moments**
- **Experimental bounds on spin-torsion coupling**
- **Summary**

Theoretical foundations of spacetime torsion

In local coordinates, the covariant derivative of a contravariant vector field A^i in an affine manifold is defined as

$$A^i_{;k} = A^i_{,k} + \Gamma^i_{jk} A^j,$$

where $i, j, k, \ldots$ are coordinate indices, comma denotes partial differentiation and $\Gamma^i{}_{jk}$ is the affine connection. The Riemann tensor is defined as

$$R^i_{jkl} = \Gamma^i_{jl,k} - \Gamma^i_{jk,l} + \Gamma^i_{mk}\Gamma^m_{jl} - \Gamma^i_{ml}\Gamma^m_{jk}.$$

We can split the connection $\Gamma^i{}_{jk}$ into its symmetric part $\underline{\Gamma}^i_{jk}$ and its antisymmetric part $T^i{}_{jk}$:

$$\Gamma^i_{jk} = \underline{\Gamma}^i_{jk} + T^i_{jk},$$

where

$$\underline{\Gamma}^i_{jk} = \frac{1}{2}\left(\Gamma^i_{jk} + \Gamma^i_{kj}\right), \qquad T^i_{jk} = \frac{1}{2}\left(\Gamma^i_{jk} - \Gamma^i_{kj}\right).$$

$T^i{}_{jk}$ is called Cartan's torsion tensor or, simply, torsion.

The covariant derivative of the metric is called the nonmetricity:

$$Q_k{}^{ij} = g^{ij}{}_{;k}.$$

$$\Gamma^i_{\ jk} = \left\{ {}^{\ i}_{jk} \right\} + T^i_{\ jk} + T_k{}^i{}_j + T_j{}^i{}_k + \frac{1}{2}\left(Q^i_{\ jk} + Q^i_{\ kj} - Q_k{}^i{}_j \right).$$

The Christoffel symbol is defined by

$$\left\{ {}^{\ i}_{jk} \right\} = \frac{1}{2} g^{im} \left(g_{jm,k} + g_{mk,j} - g_{jk,m} \right).$$

The contorsion is defined by

$$K^i_{\ jk} = T^i_{\ jk} + T_k{}^i{}_j + T_j{}^i{}_k.$$

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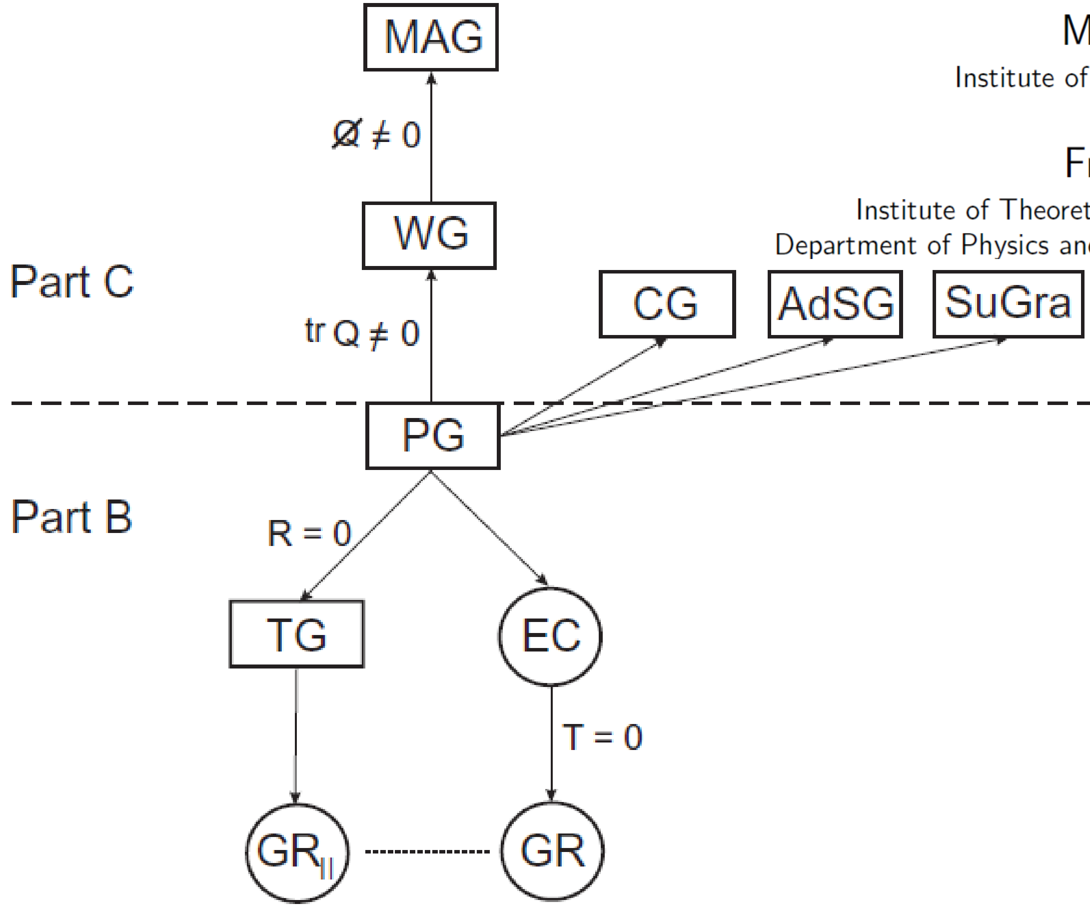

PG = Poincaré gauge theory (of gravity), **EC** = Einstein–Cartan(–Sciama–Kibble) theory (of gravity), **GR** = general relativity (Einstein's theory of gravity), **TG** = translation gauge theory (of gravity) also known as teleparallel theory (of gravity), $\mathbf{GR}_{||}$ = a specific TG known as teleparallel equivalent of GR (spoken "GR teleparallel"), **WG** = Weyl(–Cartan) gauge theory (of gravity), **MAG** = metric-affine gauge theory (of gravity), **CG** = conformal gauge theory (of gravity), **AdSG** = (anti-)de Sitter gauge theory (of gravity), **SuGra** = supergravity (super-Poincaré gauge theory of gravity).

The symbols in the figure have the following meaning: rectangle □ → class of theories; circle ○ → definite viable theories; nonmetricity $Q = \not{Q} + \frac{1}{4}(\mathrm{tr}\,Q)1$, torsion T, curvature R.

1979/80, Stoeger and Yasskin [14,15] asked the question: "Can a macroscopic gyroscope feel a torsion?". They used the general theory of spin motion of Mathisson and Papapetrou. Their verdict is unequivocal: "Our results show that the torsion couples to spin but not to rotation. Thus a rotating test body with no net spin will ignore the torsion and move according to the usual Papapetrou equations. Hence *the standard tests of gravity are insensitive to a torsion field*" (emphasis by us). Is there anything more to be said? This should have been the (definite) end of the story.

Eventually, we have also an optimistic message: Already in 1983, Ni [44] suggested to build gyroscopes with spin-polarized balls as active elements consisting of solid helium-three (^3He) and to put them into orbit around the Earth. Ni has also used for experiments in the gravitational field dysprosium–iron compounds Dy_6Fe_{23}, see [45], with a relatively high net spin of about 0.4 electron spins per atom, but with no disturbing magnetic moment. With such tools one could hope to find torsion, if it exists in nature.

Dirac particle in gravity with torsion

The quantum theory is based on the covariant Dirac equation:

$$\left(i\hbar\gamma^a D_a - mc\right)\psi = 0, \quad D_a = e_a^{\mu}\partial_\mu + \frac{i}{4}\sigma^{bc}\Gamma_{bca},$$

$$\sigma^{bc} = \frac{i}{2}(\gamma^b\gamma^c - \gamma^c\gamma^b), \quad \Gamma_{abc} = -\Gamma_{bac}, \quad a,b,c = 0,1,2,3.$$

General metric:

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_c W^{\hat{b}}{}_d\left(dx^c - K^c cdt\right)\left(dx^d - K^d cdt\right)$$

Schwinger gauge:

$$e_\mu^{0} = V\delta_\mu^0, \quad e_\mu^{i} = W\left(\delta_\mu^i - K^i\delta_\mu^0\right)$$

Tetrad indexes are blue

Nonunitary transformation of the initial equation

$$\psi = \left(\frac{1}{c}\sqrt{-g}e_{\widehat{0}}^{0}\right)^{\frac{1}{2}}\Psi$$

brings the Dirac Hamiltonian:

$$\mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2}\left(\pi_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \pi_b\right)$$
$$+ \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right),$$

Here $V = e_0^{\widehat{0}}$, $\mathcal{F}^b{}_a = \sqrt{-g}e_{\widehat{a}}^b = VW^b{}_{\widehat{a}}$, and

$$\Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\left(\Gamma_{\widehat{0}\widehat{b}\widehat{c}} + \Gamma_{\widehat{b}\widehat{c}\widehat{0}} + \Gamma_{\widehat{c}\widehat{0}\widehat{b}}\right).$$

Yu. N. Obukhov, A.J. Silenko and O.V. Teryaev, Phys. Rev. D **84**, 024025 (2011); **88**, 084014 (2013); arXiv:1410.6197.

In order to make the coupling of spin and torsion explicit, we now use the decomposition of the connection into the Riemannian and post-Riemannian parts

$$\Upsilon = \widetilde{\Upsilon} + V c \check{T}^{\hat{0}}, \qquad \Xi^{\hat{a}} = \widetilde{\Xi}^{\hat{a}} - V \check{T}^{\hat{a}}.$$

The tilde, as usual, denotes the Riemannian quantities

$$\widetilde{\Upsilon} = V \epsilon^{\hat{a}\hat{b}\hat{c}} \widetilde{\Gamma}_{\hat{a}\hat{b}\hat{c}} = -V \epsilon^{\hat{a}\hat{b}\hat{c}} C_{\hat{a}\hat{b}\hat{c}},$$

$$\widetilde{\Xi}_{\hat{a}} = \frac{V}{c} \epsilon_{\hat{a}\hat{b}\hat{c}} \widetilde{\Gamma}_{\hat{0}}{}^{\hat{b}\hat{c}} = \epsilon_{\hat{a}\hat{b}\hat{c}} Q^{\hat{b}\hat{c}},$$

$$C_{\hat{a}\hat{b}}{}^{\hat{c}} = W^{d}{}_{\hat{a}} W^{e}{}_{\hat{b}} \partial_{[d} W^{\hat{c}}{}_{e]}, \qquad C_{\hat{a}\hat{b}\hat{c}} = g_{\hat{c}\hat{d}} C_{\hat{a}\hat{b}}{}^{\hat{d}},$$

$$Q_{\hat{a}\hat{b}} = g_{\hat{a}\hat{c}} W^{d}{}_{\hat{b}} \left(\frac{1}{c} \dot{W}^{\hat{c}}{}_{d} + K^{e} \partial_{e} W^{\hat{c}}{}_{d} + W^{\hat{c}}{}_{e} \partial_{d} K^{e} \right).$$

$$C_{\hat{a}\hat{b}}{}^{\hat{c}} = -C_{\hat{b}\hat{a}}{}^{\hat{c}}$$

The non-Riemannian parts are constructed from the components of the axial torsion vector

$$\check{T}^{\alpha} = -\frac{1}{2}\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}, \qquad \eta_{\hat{0}\hat{1}\hat{2}\hat{3}} = 1,$$

where $\eta^{\alpha\mu\nu\lambda}$ is the totally antisymmetric Levi-Civita tensor.

As a result, we can explicitly identify the spin-torsion coupling

$$-\frac{\hbar c V}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\hat{0}}\right)$$

Foldy-Wouthuysen transformation for a Dirac particle with allowance for spacetime torsion

Foldy-Wouthuysen Hamiltonian:

$$\mathcal{H}_{FW} = \mathcal{H}^{(1)}_{FW} + \mathcal{H}^{(2)}_{FW} + \mathcal{H}^{(3)}_{FW}$$

$$\mathcal{H}^{(1)}_{FW} = \beta\epsilon' + \frac{\hbar c^2}{16}\left\{\frac{1}{\epsilon'}, \left(2\epsilon^{cae}\Pi_e\{p_b, \mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} + \Pi^a\{p_b, \mathcal{F}^b{}_a\widetilde{\Upsilon}\}\right)\right\} + \frac{\hbar mc^4}{4}\epsilon^{cae}\Pi_e\left\{\frac{1}{\mathcal{T}}, \{p_d, \mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_b V\}\right\}$$

$$\mathcal{H}^{(2)}_{FW} = \frac{c}{2}(K^a p_a + p_a K^a) + \frac{\hbar c}{4}\Sigma_a\widetilde{\Xi}^a$$
$$+\frac{\hbar c^2}{16}\left\{\frac{1}{\mathcal{T}}, \left\{\Sigma_a\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d\left(\delta^{db}\widetilde{\Xi}^a - \delta^{da}\widetilde{\Xi}^b\right)\right]\right\}\right\}\right\}$$

$$\mathcal{H}^{(3)}_{FW} = \frac{\hbar}{2}\Sigma^a\Omega^{(T)}_a$$

$$\Omega^{(T)}_a = -\frac{c}{2}V\delta_{ab}\check{T}^{\hat{b}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \{p_b, \mathcal{F}^b{}_a V\check{T}^{\hat{0}}\}\right\} + \frac{c^2}{16}\left\{\frac{1}{\mathcal{T}}, \left\{\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \mathcal{F}^f{}_d V(\delta^{db}\check{T}^{\hat{a}} - \delta^{da}\check{T}^{\hat{b}})\right\}\right\}\right\},$$

$$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b, \mathcal{F}^b{}_a\}\{p_d, \mathcal{F}^d{}_c\}}, \qquad \mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2V\}.$$

The equation of spin motion is obtained from the commutator of the FW Hamiltonian with the polarization operator $\mathbf{\Pi} = \beta\mathbf{\Sigma}$:

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \mathbf{\Pi}] = \mathbf{\Omega}\times\mathbf{\Pi}.$$

Angular velocity of spin rotation conditioned by the spacetime torsion:

$$\Omega_a^{(T)} = -\frac{c}{2}V\delta_{ab}\check{T}^{\hat{b}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \{p_b, \mathcal{F}^b{}_a V\check{T}^{\hat{0}}\}\right\}$$

$$+\ \frac{c^2}{16}\left\{\frac{1}{\mathcal{T}}, \left\{\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \mathcal{F}^f{}_d V\left(\delta^{db}\check{T}^{\hat{a}} - \delta^{da}\check{T}^{\hat{b}}\right)\right\}\right\}\right\},$$

$$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b, \mathcal{F}^b{}_a\}\{p_d, \mathcal{F}^d{}_c\}},$$

$$\mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2V\}.$$

As a special case, let us consider the flat Minkowski metric with $V = 1, K^a = 0, W^{\widehat{a}}{}_b = \delta^a_b$. The spin precesses under the action of the torsion with the angular velocity

$$\Omega^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \left\{\boldsymbol{p}, \check{T}^{\hat{0}}\right\}\right\}$$

$$+\frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon' + mc^2)}, \left(\{\boldsymbol{p}^2, \check{\boldsymbol{T}}\} - \{\boldsymbol{p}, (\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}.$$

Extension of the covariant Dirac equation on the anomalous magnetic and electric dipole moments

nonminimal coupling terms

$$\frac{\mu'}{2c} F_{\alpha\beta} \overline{\Psi} \sigma^{\alpha\beta} \Psi + \frac{\delta'}{2} G_{\alpha\beta} \overline{\Psi} \sigma^{\alpha\beta} \Psi$$

modified Hamiltonian

$$\mathcal{H} = \beta m c^2 V + q\Phi + \frac{c}{2} \left(\pi_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \pi_b \right) + \frac{c}{2} \left(\boldsymbol{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \boldsymbol{K} \right) + \frac{\hbar c}{4} \left(\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - \Upsilon \gamma_5 \right) - \beta \left(\boldsymbol{\Sigma} \cdot \boldsymbol{\mathcal{M}} + i \boldsymbol{\alpha} \cdot \boldsymbol{\mathcal{P}} \right).$$

Here we defined

$$\mathcal{M}^a = V \left(\mu' \mathrm{B}^a + \delta' \mathrm{E}^a \right),$$

$$\mathcal{P}_a = V \left(c\delta' \mathrm{B}_a - \mu' \mathrm{E}_a / c \right),$$

Dirac particle in magnetic field and rotating frame

$$V = 1, \qquad W^{\hat{a}}{}_b = \delta^a_b, \qquad K^a = -\frac{(\boldsymbol{\omega} \times \boldsymbol{r})^a}{c}$$

Dirac Hamiltonian

$$\mathcal{H} = \beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{\pi} - \boldsymbol{\omega}\cdot\boldsymbol{\lambda} - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma} - \frac{\hbar c}{4}\left(\check{T}^{\hat{0}} c\gamma_5 + \check{\boldsymbol{T}}\cdot\boldsymbol{\Sigma}\right)$$

Here $\boldsymbol{\lambda} = \boldsymbol{r} \times \boldsymbol{\pi} = -\boldsymbol{\pi} \times \boldsymbol{r}$ denotes the orbital angular moment operator, and the torsion effects are encoded in the last term.

Foldy-Wouthuysen Hamiltonian

$$\mathcal{H}_{FW} = \mathcal{H}_0 + \mathcal{H}_1,$$

$$\mathcal{H}_0 = \beta\epsilon - \boldsymbol{\omega}\cdot\boldsymbol{\lambda} - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma} + \frac{\hbar c^3}{16}\left\{\frac{1}{\epsilon},\left\{\boldsymbol{\pi}\cdot\boldsymbol{\Pi},\check{T}^{\hat{0}}\right\}\right\} - \frac{\hbar c}{4}\check{\boldsymbol{T}}\cdot\boldsymbol{\Sigma}$$
$$+\frac{\hbar c}{16}\left\{\frac{c^2}{\epsilon(\epsilon+mc^2)},\left[\left\{\boldsymbol{\pi}^2,\check{\boldsymbol{T}}\cdot\boldsymbol{\Sigma}\right\} - \frac{1}{2}\left\{\boldsymbol{\Sigma}\cdot\boldsymbol{\pi},(\boldsymbol{\pi}\cdot\check{\boldsymbol{T}}+\check{\boldsymbol{T}}\cdot\boldsymbol{\pi})\right\}\right]\right\},$$

$$\mathcal{H}_1 = \frac{e\hbar c}{8}\left\{\frac{1}{\epsilon(\epsilon+mc^2)},\boldsymbol{\Sigma}\cdot(\boldsymbol{\pi}\times\boldsymbol{G}-\boldsymbol{G}\times\boldsymbol{\pi})\right\}.$$

$$\epsilon = \sqrt{m^2c^4 + c^2\boldsymbol{\pi}^2 - e\hbar c^2\boldsymbol{\Sigma}\cdot\boldsymbol{B}}, \qquad \boldsymbol{G} = \boldsymbol{B}\times(\boldsymbol{\omega}\times\boldsymbol{r}).$$

Spin-1/2 particle with an anomalous magnetic moment in magnetic field and rotating frame

Dirac Hamiltonian

$$\mathcal{H} = \beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{\pi} - \boldsymbol{\omega}\cdot\boldsymbol{\lambda} - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma} - \mu'\boldsymbol{\Pi}\cdot\boldsymbol{B} - \frac{\hbar c}{4}\left(\check{T}^{\hat{0}}c\gamma_5 + \check{\boldsymbol{T}}\cdot\boldsymbol{\Sigma}\right)$$

Foldy-Wouthuysen Hamiltonian

$$\mathcal{H}_{FW} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2,$$

$$\mathcal{H}_2 = -\mu'\boldsymbol{\Pi}\cdot\boldsymbol{B} + \frac{\mu'}{4}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)},\left[(\boldsymbol{B}\cdot\boldsymbol{\pi})(\boldsymbol{\Pi}\cdot\boldsymbol{\pi}) + (\boldsymbol{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{B})\right]\right\}.$$

The angular velocity of spin rotation

$$
\begin{aligned}
\boldsymbol{\Omega} = -\boldsymbol{\omega} - \beta\left\{\frac{\mu_0 mc^2}{\hbar\epsilon'}, \boldsymbol{B}\right\} - 2\beta\frac{\mu'}{\hbar}\boldsymbol{B} \\
+\frac{\mu'}{2\hbar}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)}, [(\boldsymbol{B}\cdot\boldsymbol{\pi})\boldsymbol{\pi}+\boldsymbol{\pi}(\boldsymbol{\pi}\cdot\boldsymbol{B})]\right\} \\
-\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \left\{\boldsymbol{\pi}, \check{T}^{\hat{0}}\right\}\right\} \\
+\frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)}, [\{\boldsymbol{\pi}^2, \check{\boldsymbol{T}}\} - (\check{\boldsymbol{T}}\cdot\boldsymbol{\pi})\boldsymbol{\pi} - \boldsymbol{\pi}(\boldsymbol{\pi}\cdot\check{\boldsymbol{T}})]\right\}.
\end{aligned}
$$

Experimental bounds on spin-torsion coupling

Equivalence Principle for the spin and its experimental verification

- **Kobzarev – Okun relations define form factors at zero momentum transfer**

I.Yu. Kobzarev, L.B. Okun, Gravitational Interaction of Fermions. Zh. Eksp. Teor. Fiz. **43**, 1904 (1962) [Sov. Phys. JETP **16**, 1343 (1963)].

Kobzarev – Okun relations result in the absence of the anomalous gravitomagnetic moment and the gravitoelectric dipole one

Motion of gyroscopes and particle spins in Riemannian spacetimes is the same. However, the spacetime torsion couples to atomic spins but not to gyroscopes!

Yu. N. Obukhov, A.J. Silenko and O.V. Teryaev, Spin-torsion coupling and gravitational moments of Dirac fermions: theory and experimental bound; arXiv:1410.6197.

We thus conclude that the anomalous gravitomagnetic moment is not allowed in the covariant Dirac-Pauli theory with a nonminimal coupling of a fermion to the Poincaré gauge gravitational field. This demonstrates a limited nature of analogies between gravitational and electromagnetic interactions observed in the weak-field approximation. The same conclusion is valid for the anomalous gravitoelectric moment. It is worthwhile to recall that the analysis of the gravitational form-factors of Dirac fermions by Kobzarev and Okun [81] (see also [82, 83]) have shown that the anomalous gravitomagnetic and gravitoelectric moments should be strictly zero.

- The restriction on the anomalous gravitomagnetic moment may be obtained from experimental data:

- B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson, Phys. Rev. Lett. **68**, 135 (1992).

- by the analysis in **A.J. Silenko and O.V. Teryaev**, Phys. Rev. D **76**, 061101(R) (2007).

$$\left|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})\right| < 0.042$$

(95% C.L.)

PHYSICAL REVIEW D 76, 061101(R) (2007)

Let us reconsider the earlier results [15] as restrictions on the AGM rather than on the dipole spin-gravity coupling. Recall that the latter violates not only the Kobzarev-Okun relation for the gravitoelectric dipole moment but also *CP* invariance, and may be neglected. The spin-dependent Hamiltonian for atoms in *S* states may be obtained by the modification of the coefficient of the term defining the spin-rotation coupling and has the form

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \qquad \zeta = 1 + \chi, \qquad (7)$$

where g is the nuclear g factor, μ_N is the nuclear magneton, and χ is the AGM. The measured ratio of energy differences in neighboring Zeeman levels, $R = |\nu_2|/|\nu_1|$, depends on the AGMs. The difference of these ratios for two opposite directions of the magnetic field is given by

$$R_+ - R_- = \pm\frac{2f\cos\theta}{|\nu_1|}(\zeta_2 - \mathcal{G}\zeta_1), \qquad \mathcal{G} = \frac{g_2}{g_1}, \qquad (8)$$

where θ is the angle between the directions of the magnetic field and the Earth's rotation axis, $f = \omega/(2\pi) = 11.6\ \mu\mathrm{Hz}$ is the Earth's rotation frequency, and $|\nu_1|$ is the Zeeman frequency for atoms of the first kind. The experimental conditions of [15] for ^{199}Hg and ^{201}Hg atoms correspond to $\theta \approx 0$, $\mathcal{G} = -0.369\,139$. Reconsidering the bound for $R_+ - R_-$ obtained in that reference, we drop the contribution of the *CP*-violating gravitoelectric dipole moment, but account for the possibility for nonzero AGM, which makes a difference between (8) and their Eq. (4). As a result, their data lead to the following restriction:

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \qquad (95\%\ \mathrm{C.L.}).$$

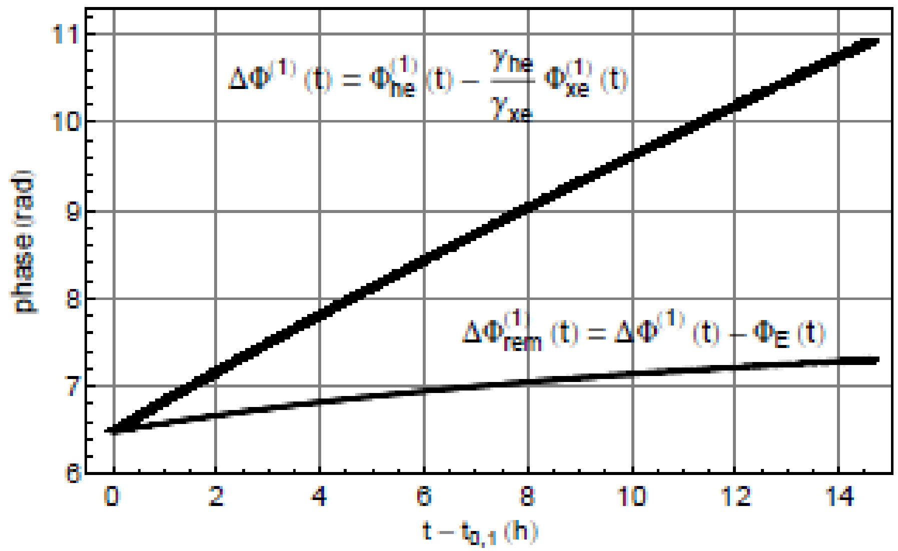

Figure 4.14: Weighted phase difference for run (1) before ($\Delta\Phi^{(1)}$) and after ($\Delta\Phi^{(1)}_{rem}$) the subtraction of the Earth's rotation term Φ_E (Eq. (4.21)). One data point comprises 20 sub-data sets.

We have extracted the new restriction of about 0.9% from the experimental data presented in C. Gemmel et al, Eur. Phys. J. D **57**, 303 (2010).

$$|\chi(\mathrm{He}) - G\chi(\mathrm{Xe})| < 0.009, \qquad G = \frac{g_{\mathrm{He}}}{g_{\mathrm{Xe}}}$$

- **The absence of the gravitoelectric dipole moment results in the absence of direct spin-gravity coupling**

$$W \sim \mathbf{g}\cdot\mathbf{S}$$

see the discussion in B. Mashhoon, Lect. Notes Phys. **702,** 112 (2006).

Spin-dependent part of the Foldy-Wouthuysen Hamiltonian for atoms at rest:

$$\mathcal{H}_{FW} = -(\mu_0 + \mu')B \cdot \Pi - \frac{\hbar}{2}\omega \cdot \Sigma - \frac{\hbar c}{4}\check{T} \cdot \Sigma.$$

Classical limit of this Hamiltonian is given by

$$H = -g_N \frac{\mu_N}{\hbar} B \cdot s - \omega \cdot s - \frac{c}{2}\check{T} \cdot s,$$

where g_N is the nuclear g-factor and μ_N is the nuclear magneton.

For *Venema et al* experiment

$$\frac{\hbar c}{4}|\check{T}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV},$$

$$|\check{T}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}.$$

For *Gemmel et al* experiment

$$\frac{\hbar c}{2}|\check{T}| \cdot |(1-\mathcal{G})\cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV},$$

$$|\check{T}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}.$$

New experiment of the same collaboration:

F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Yu. Sobolev, and K. Tullney, *New limit on Lorentz-invariance- and CPT-violating neutron spin interactions using a free-spin-precession ^{3}He-^{129}Xe comagnetometer*, Phys. Rev. Lett. **112** (2014) 110801 [5 pages].

In this experiment, the Ramsey-Bloch-Siegert shift has been taken into account

The latter one gives the shift in Larmor frequency ω_L due to a rotating field with amplitude B_1 and frequency ω_D. Related to our case, this is generated by the precessing magnetization of the polarized gas,

$$\delta\omega_{\mathrm{RBS}}(t) = \pm\left(\sqrt{\Delta\omega^2 + \gamma^2 B_1^2(t)} - \Delta\omega\right),$$

with $\Delta\omega = |\omega_L - \omega_D|$. The plus sign applies to $(\omega_D/\omega_L) < 1$, the minus sign to $(\omega_D/\omega_L) > 1$, respectively. Two effects contribute to the RBS shift and have to be taken into account, i.e., cross-talk (ct) and self-shift (ss).

F. Bloch and A. Siegert, Phys. Rev. **57**, 522 (1940).
N. F. Ramsey, Phys. Rev. **100**, 1191 (1955).

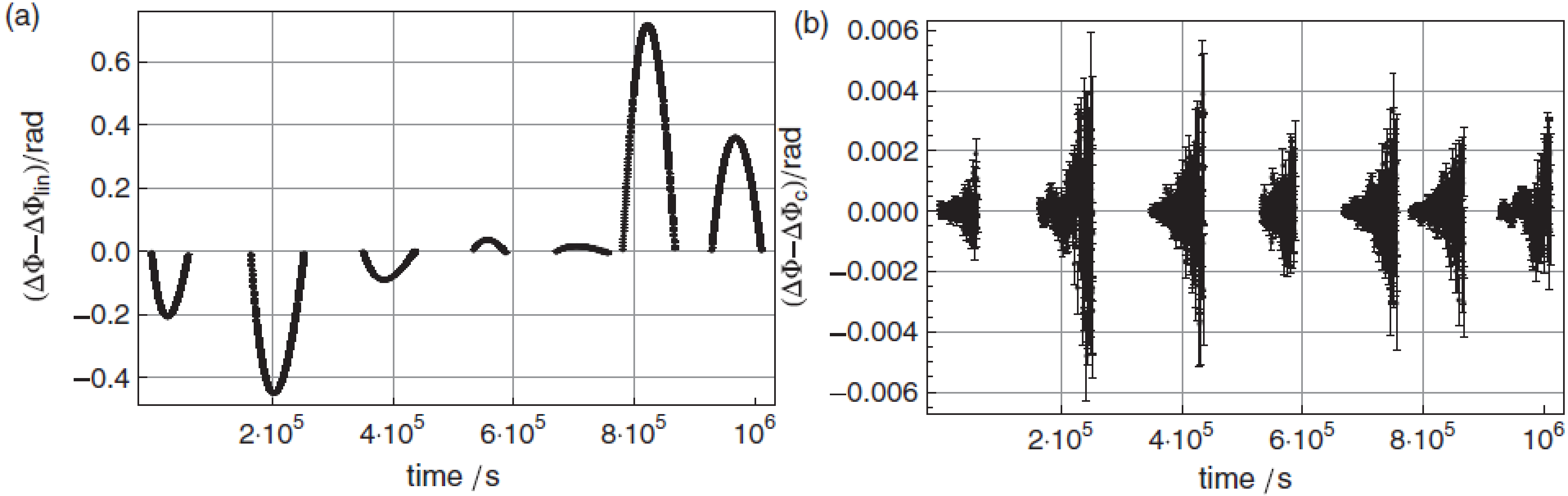

FIG. 1. (a) Weighted phase difference $\Delta\Phi$ (data bin: 320 s) after subtraction of the linear terms $\Delta\Phi_{\mathrm{lin}}$ in Eq. (9). The remaining parabolic shaped structure is the contribution of the RBS shift (in particular the self-shift). (b) Phase residuals after subtraction of the entire fit model $\Delta\Phi_c$ according to Eqs. (8) and (9). The time evolution of the phase noise is caused by the exponential decay of the signal amplitudes. Note that the phase noise is much less than the symbol size in (a).

An approximate estimate can be given:

$$\left|\mathbf{T}\right| \cdot \left|\cos\Theta\right| \leq 10^{-16}\ \mathrm{m}^{-1}.$$

Other experiments and estimates

C. Lämmerzahl, *Constraints on space-time torsion from Hughes-Drever experiments*, Phys. Lett. **A228** (1997) 223-231.

V.A. Kostelecký, N. Russell, and J.D. Tasson, *Constraints on torsion from bounds on Lorentz violation*, Phys. Rev. Lett. **100** (2008) 111102 [4 pages].

$$a_i = (\tfrac{1}{6})\varepsilon_{ijkl}T^{jlk}, \quad e^{ijkl} = \begin{cases} 1, & \text{if } (ijkl) \text{ is an even permutation of } (0123), \\ -1, & \text{if } (ijkl) \text{ is an odd permutation of } (0123), \\ 0, & \text{otherwise.} \end{cases}$$

By analyzing Hughes–Drever experiments in this context, Lämmerzahl (1997) obtained a constraint on the axial torsion $|a_\alpha| \leqslant 1.5 \times 10^{-15}\,\text{m}^{-1}$ where $|a_\alpha|$ $[= (a_1^2 + a_2^2 + a_3^2)^{1/2}]$ is the absolute value of the spatial part of the axial torsion.

Kostelecky et al estimate:

$$A^{\mu} \equiv \frac{1}{6}\epsilon^{\alpha\beta\gamma\mu}T_{\alpha\beta\gamma},$$

$$|A_T| < 2.9 \times 10^{-27}\ \mathrm{GeV} \simeq 1.5 \times 10^{-11}\ \mathrm{m}^{-1},$$

$$|A_X| < 2.1 \times 10^{-31}\ \mathrm{GeV} \simeq 1.1 \times 10^{-15}\ \mathrm{m}^{-1},$$

$$|A_Y| < 2.5 \times 10^{-31}\ \mathrm{GeV} \simeq 1.3 \times 10^{-15}\ \mathrm{m}^{-1},$$

$$|A_Z| < 1.0 \times 10^{-29}\ \mathrm{GeV} \simeq 5.3 \times 10^{-13}\ \mathrm{m}^{-1}.$$

R. Lehnert, W.M. Snow, H. Yan, A first experimental limit on in-matter torsion from neutron spin rotation in liquid 4He, Phys. Lett. B **730,** 353 (2014).

We report the first experimental upper bound to our knowledge on possible in-matter torsion interactions of the neutron from a recent search for parity violation in neutron spin rotation in liquid ^{4}He. Our experiment constrains a coefficient ζ consisting of a linear combination of parameters involving the time components of the torsion fields T^μ and A^μ from the nucleons and electrons in helium which violates parity. We report an upper bound of $|\zeta| < 5.4 \times 10^{-16}$ GeV at 68% confidence level and indicate other physical processes that could be analyzed to constrain in-matter torsion.

Summary

- Dirac particle in gravity with torsion is considered and the Dirac Hamiltonian is obtained
- Foldy-Wouthuysen transformation for the relativistic Dirac particle is performed with allowance for spacetime torsion
- The covariant Dirac equation is extended on the anomalous magnetic and electric dipole moments and the relativistic Foldy-Wouthuysen transformation is fulfilled
- New experimental bounds on spin-torsion coupling are found


Thank you for your attention