Petersburg Nuclear Physics Institute

Constraints from Oklo reactor analysis on parameters of BSBM model of varying α

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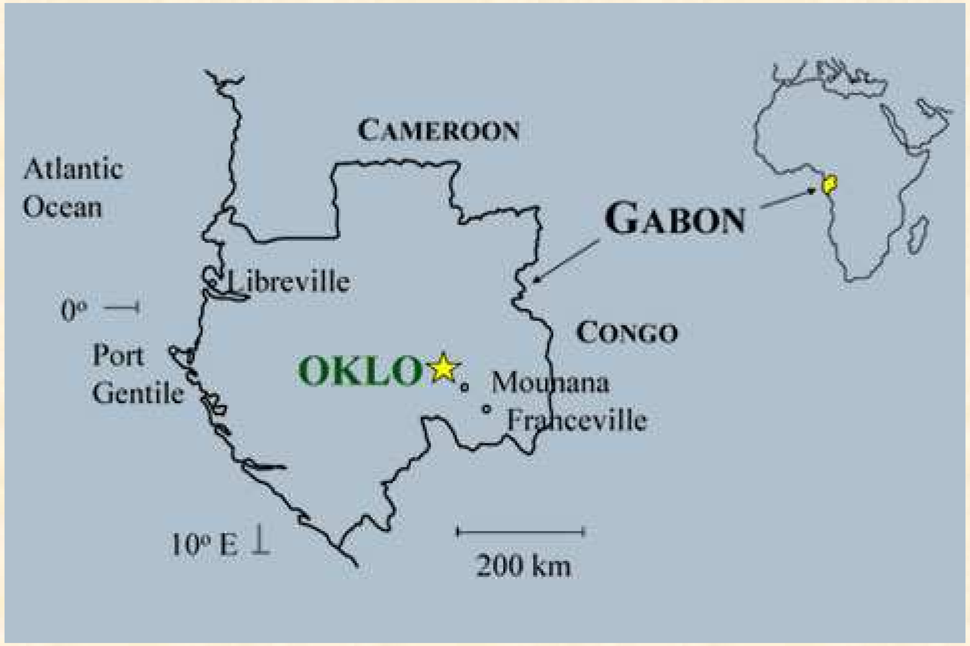
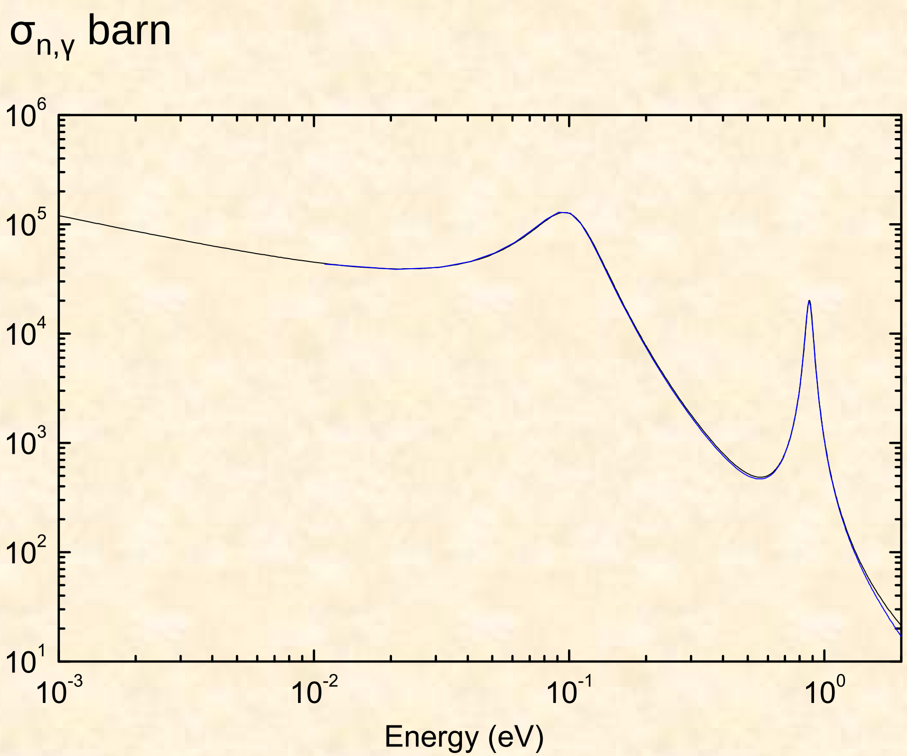
In 1972 during the mass spectrometry analysis of the samples in one of the laboratories of CEA (France) a discrepancy in the amount of the 235U in the samples from Oklo, Gabon was obtained. Normally the concentration is 0.720%, while these samples had 0.717 %. Further investigations into this uranium deposit discovered uranium ore with a 235U concentration as low as 0.440 %.



Strong neutron absorbers in reactor core



A.I. Shlyakhter, Nature ***264****, 340 (1976)*

$^{149}Sm\ (n,\gamma)\ ^{150}Sm$

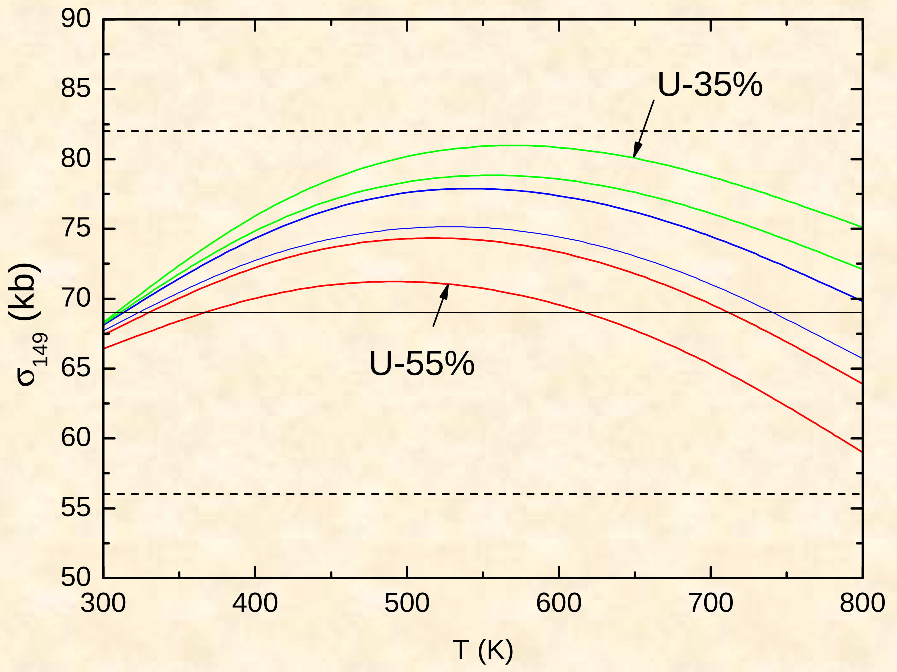
$$\sigma_{n,\gamma}(E) = g_0 \frac{\pi \hbar^2}{2mE} \frac{\Gamma_n(E)\cdot\Gamma_\gamma}{(E-E_r)^2 + \Gamma_{tot}^2/4}$$

$$\boxed{E_r = 97.3\ \text{meV}}$$

$$\Gamma_n(E) = \Gamma_n^0 \sqrt{\frac{E}{E_0}}$$

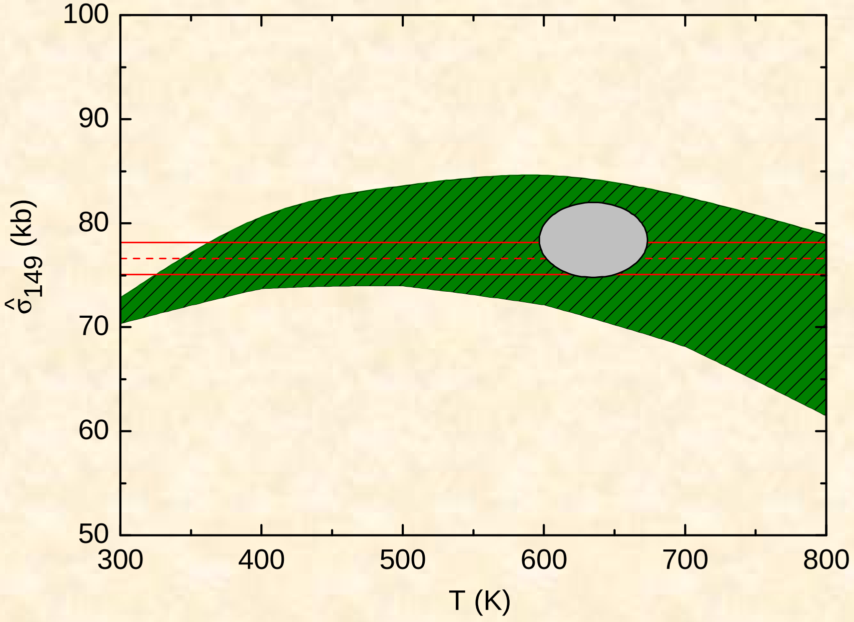
$$\hat{\sigma}_{149} = \frac{\int \sigma_{(n,\gamma)}(E)\Phi(E)dE}{\tau}$$

Sample	Fluence τ , кб$^{-1}$	σ, кб	Reference
KN50-3548	1,25	93	[9]
SC36-1408/4	0,81	73	[8]
SC36-1410/3	1,07	73	[8]
SC36-1413/3	1,43	83	[8]
SC36-1418	0,94	64	[8]
SC39-1383	0,68	66	[10, 19]
SC39-1385	0,80	69	[10, 19]
SC39-1387	1,01	36	[10, 19]
SC39-1389	1,02	64	[10, 19]
SC39-1390	0,87	82	[10, 19]
SC39-1391	1,02	82	[10, 19]
SC39-1393	0,77	68	[10, 19]
SC35bis-2126	0,92	57	[10, 19]
SC35bis-2130	1,44	81	[10, 19]
SC35bis-2134	1,21	71	[10, 19]
SC43-2421	0,85	48	[10]
SC63-1970	0,43	52	[10]
SC63-1972	0,83	58	[10]
SC63-1974	1,01	72	[10]
SC63-1976	0,88	87	[10]
SC63-1978	0,80	63	[10]
SC30-2035	0,49	70	[10]
SC30-2039	0,85	74	[10]
SC52-1472	0,23	75	[11]
		69 ± 13	



$$-73 \text{ meV} \leq \Delta E_r \leq 62 \text{ meV}$$

Yu.V.Petrov, A.I. Nazarov, M.S. Onegin, V.Yu. Petrov, and E.G. Sakhnovsky. PRC 74, 064610 (2006)



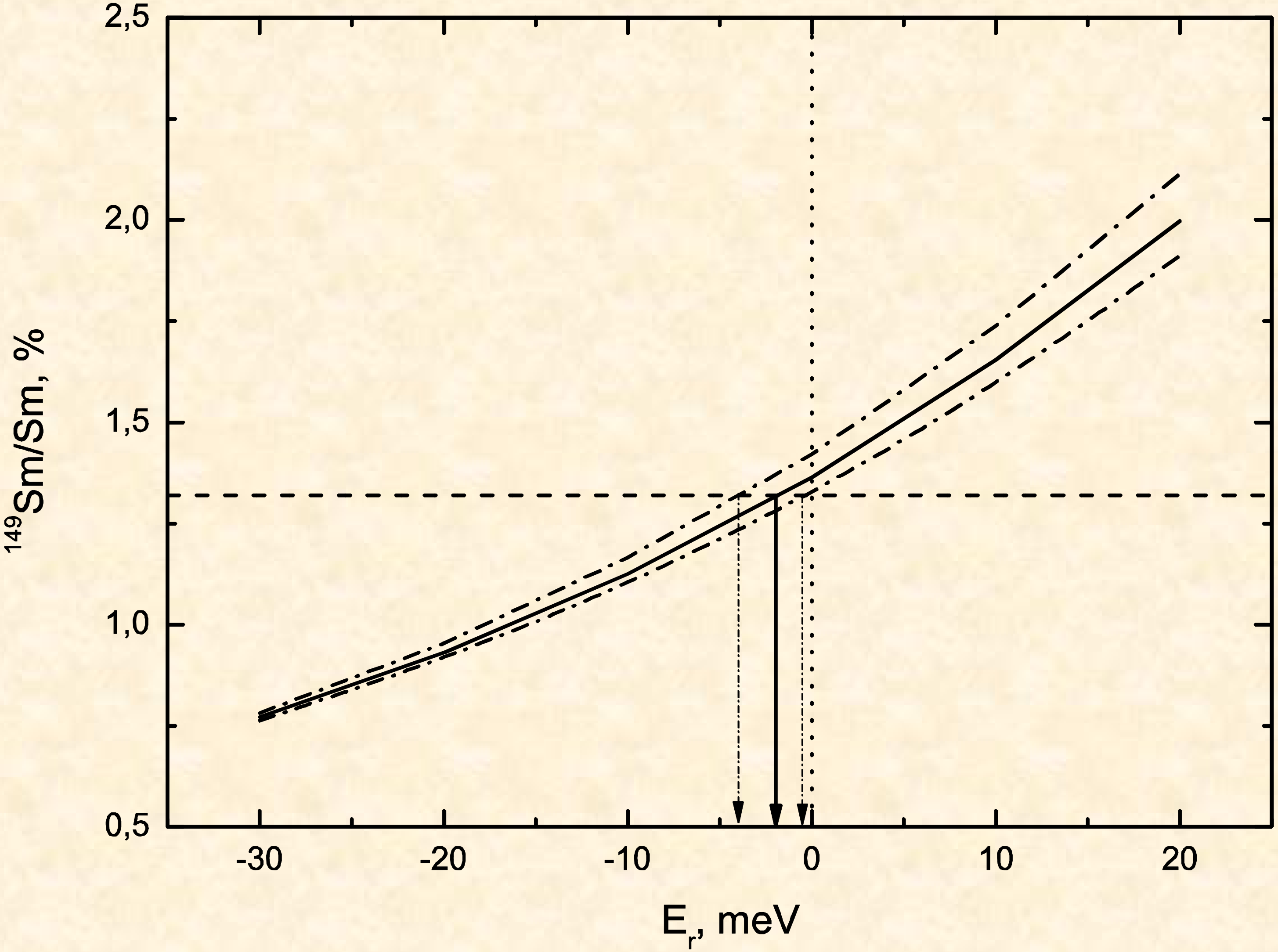
If we increase the accuracy of $\hat{\sigma}_{149}$ determination up to 2% and more strictly determine the conditions under which the chain nuclear reaction had been occurring, we can get better margin on the shift of the resonance.

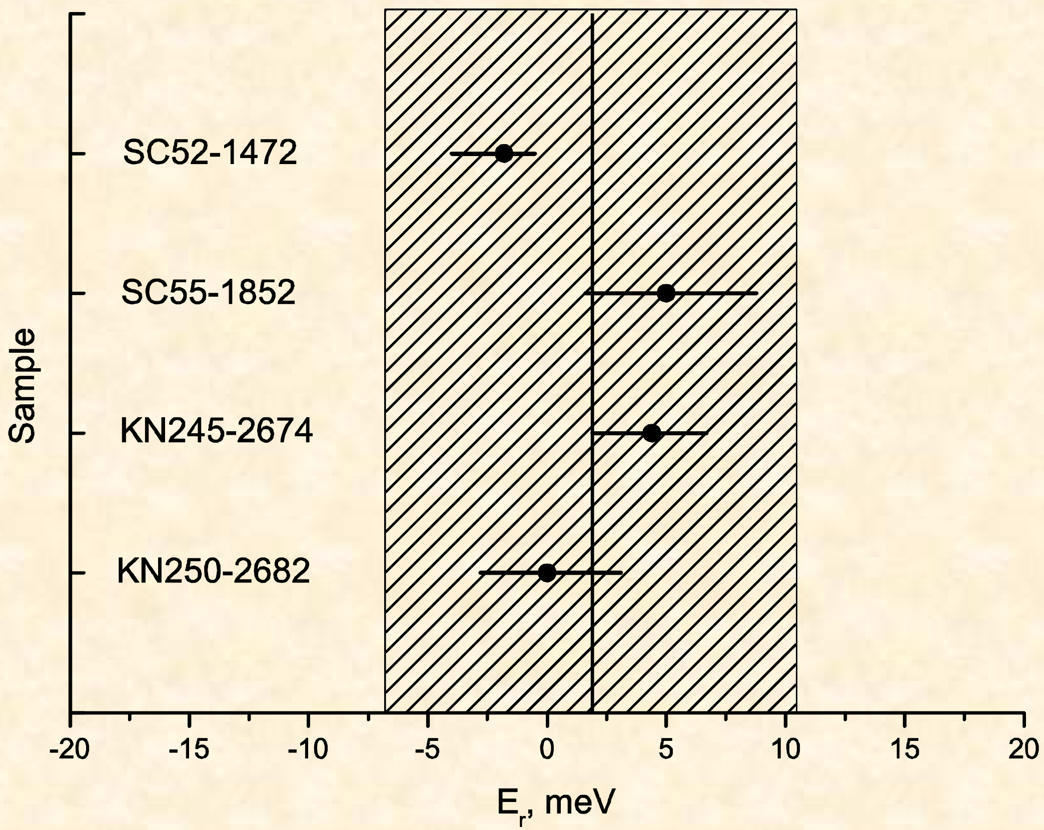
For example in the work of *G.R. Gould, E.I. Sharapov and S.K. Lamoreaux – Phys.Rev. C**74**, 024607 (2006) the analysis of exp. data was performed.* The authors used narrow interval of temperatures 473 – 573 K. They managed to narrow the allowed interval of the resonance shifting considerably:

$$-11.6\ \text{meV} \le \Delta E_r \le +26.0\ \text{meV}$$

Sample	ρ_{H_2O}, g/cm^3	Temperature, K
SC 52-1472	0.67	330 – 380
SC 55-1852	0.5	320 – 410
KN 245-2674	1.	380 – 470
KN 250-2682	1.	250 – 325

SC 52-1472





$-7.1 \text{ meV} < \Delta E_r < 10.9 \text{ meV}$

Obtained from Oklo analisys α variation constraints

$$\frac{\delta\alpha}{\alpha} = \frac{\Delta E_r}{M} \qquad M = -1.1\ \mathrm{MeV}$$

As a result:

*M.S. Onegin, M.S. Yudkevich, E.A. Gomin. Mod. Phys. Lett. A **27** (2012) 1250232*

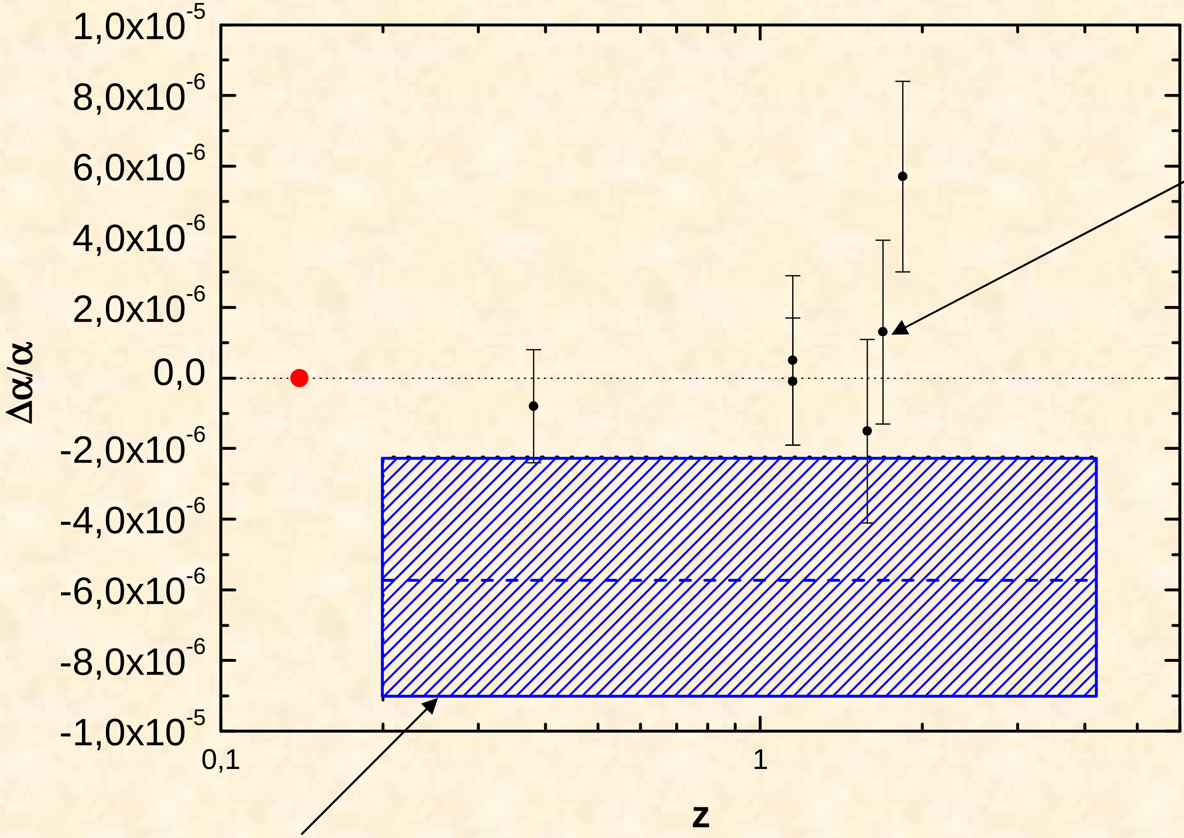
$$\boxed{-0.7\times10^{-8} < \delta\alpha/\alpha < 1.0\times10^{-8}}$$

Petrov Yu.V. et al Phys.Rev. C74 (2006) 064610

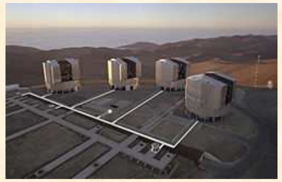
$$-5.8\times10^{-8} < \delta\alpha/\alpha < 6.6\times10^{-8}$$

Gould C.R. et al Phys.Rev. C74 (2006) 024607

$$-1.1\times10^{-8} < \delta\alpha/\alpha < 2.4\times10^{-8}$$



VLT/UVES



See: arXiv:1407.4099

Keck/HIRES

M.T. Murphy, J.K. Webb & V.V. Flambaum

Lec. Not. Phys. **648** (2004) 131

Quintessence model of varying α

If we adopt the assumption that *α* depends on *t,* we immediately get:

$$\frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g(t)^2}F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g(x)^2}F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g_0^2}B_F(\phi(x))F_{\mu\nu}F^{\mu\nu}$$

Usually function B_F can be taken in the linear form: $B_F(\phi(x)) = 1 - \zeta\kappa\phi,$
where $\kappa^2 = 8\pi G.$

The scalar field ϕ needs to be a dynamical field with the Lagrangian density:

$$L = \frac{1}{2}\partial^\mu\phi\, \partial_\mu\phi - V(\phi).$$

As we have $\alpha = \alpha_0 / B_F(\phi)$, then $\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = \zeta\kappa(\phi - \phi_0)$

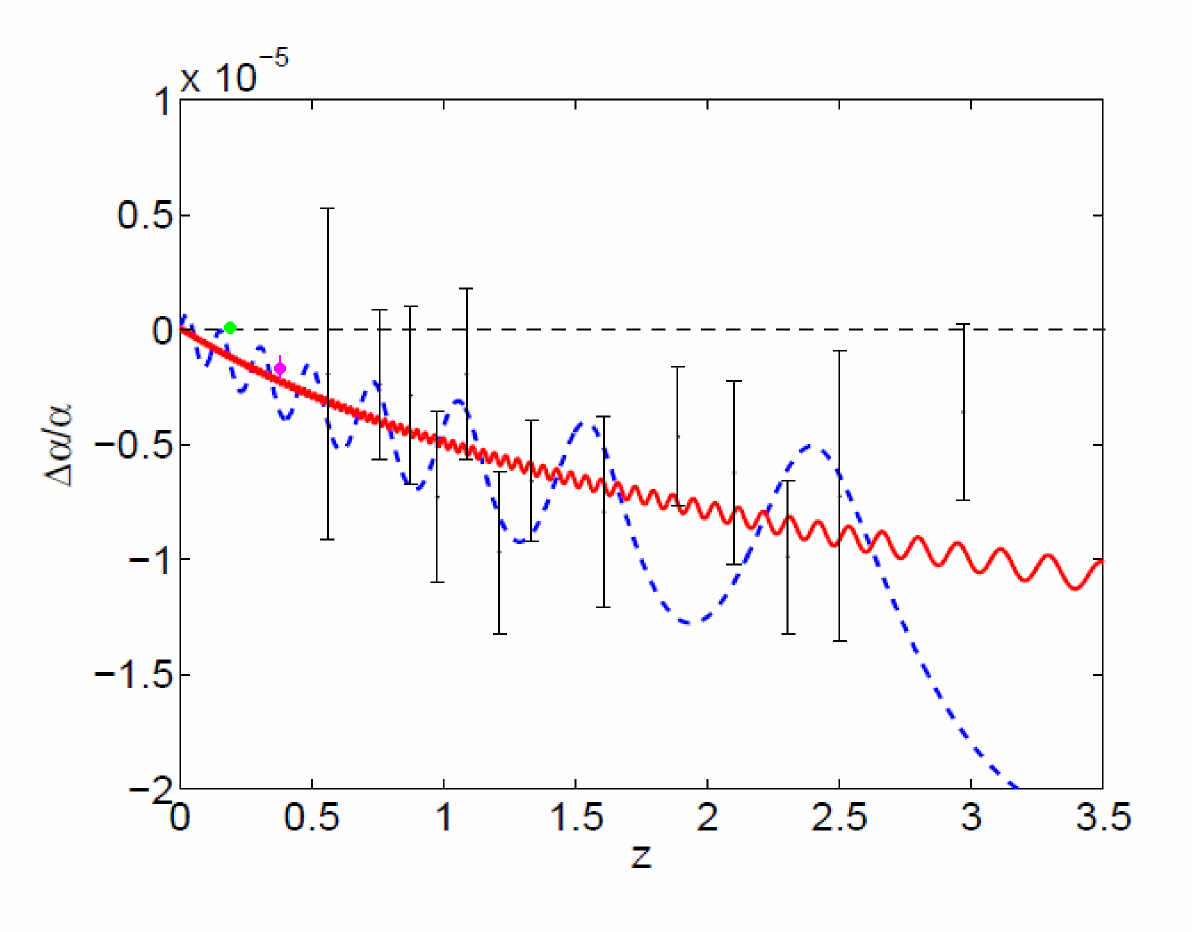
If affirmation of Murphy et al is right, we have

$$\zeta = -\frac{10^{-5}}{\kappa\phi(z=3) - \kappa\phi(z=0)}$$

We have a great arbitrariness in choosing the form of potential V. For example,

$$V(\phi) = V_0 \exp(-\lambda\kappa\phi); \quad V(\phi) = M^{4+n}/\phi^n; \quad V(\phi) = V_0 e^{\lambda(\kappa\phi)^\beta}(\kappa\phi)^n$$

Copeland E.J. et al hep-th/0603057



$$V(\phi) = V_0 e^{-\lambda\kappa\phi}$$

Solid line - λ = 100;
Dash line - λ = 10

Jacob D. Bekenstein theory of varying alpha

Phys. Rev. D 25 (1982) 1527

$$S_{BH} = C\ k_B \frac{A}{l_{pl}^2}$$ - 1972

C=1/4 - Hawking, 1974

The theory was based on several postulates:

1. For constant α electromagnetism is Maxwellian and the coupling of the vector potential to matter is minimal.
2. Variations of α result from dynamics.
3. Dynamics of electromagnetism and α are derivable from an invariant action.
4. The action is locally gauge invariant.
5. Electromagnetism is causal.
6. The electromagnetic action is time reversal invariant.
7. The shortest scale of length which can enter into physical theory is the Plank length.
8. Gravitation is described by the metric of spacetime which satisfies Einstein's equations.

Gauge invariance

$$e = e_0 \varepsilon(x^\mu)$$

Where $\varepsilon(x^\mu)$ is the universal scale invariant function for all charged particles.

$$L = -mc(-u^\alpha u_\alpha)^{1/2} + (e_0 \varepsilon / c) u^\alpha A_\alpha \qquad (1)$$

Gauge transformation law for A_μ:

$$\varepsilon A_\alpha \to \varepsilon A_\alpha + \chi_{,\alpha}$$

From (1) the equation of motion follows:

$$d(mu_\alpha)/d\tau = -m_{,\alpha} c^2 + (e_0 / c)\left[\left(\varepsilon A_\beta\right)_{,\alpha} - \left(\varepsilon A_\alpha\right)_{,\beta}\right] u^\beta$$

then

$$F_{\alpha\beta} = \varepsilon^{-1}\left[\left(\varepsilon A_\beta\right)_{,\alpha} - \left(\varepsilon A_\alpha\right)_{,\beta}\right]$$

$$S_{EM} = -\frac{1}{16\pi}\int F^{\mu\nu} F_{\mu\nu} \left(-g\right)^{1/2} d^4 x$$

Lagrangian density ${}^{*}F^{\mu\nu} F_{\mu\nu}$ is forbidden by the time reversal invariance

Dynamics of α

Lagrangian density has to be constructed from the metric and the logarithmic gradient:

$$\varepsilon^{-1}\varepsilon_{,\mu}$$

For dimensional reason Bekenstein proposed to use fundamental length ℓ :

$$S_\varepsilon = -\frac{1}{2}\hbar c \ell^{-2} \int \varepsilon^{-2}\varepsilon_{,\mu}\varepsilon^{\mu} \left(-g\right)^{1/2} d^4x \qquad (2)$$

More general Lagrangian density $f\left[\ell^2\varepsilon^{-2}\varepsilon_{,\mu}\varepsilon^{,\mu}\right]$ is forbidden because it leads to causality violation.

The natural restriction on ℓ : it has to be larger than Plank length:

$$\ell \geq l_{pl} = \left(\hbar G / c^3\right)^{1/2} \approx 1.6 \cdot 10^{-33} \text{ cm.}$$

As well as, ℓ has to be smaller than 10^{-15} cm to pass tests of quantum electrodynamics.

$$S_m = \sum \int \left[-mc^2 + (e_0\varepsilon / c)u^{\mu} A_{\mu}\right]\gamma^{-1}\delta^3\left[x^i - x^i(\tau)\right] d^4x$$

Equations of motion

$$S = S_{EM} + S_{\varepsilon} + S_m$$

Variation with respect to A_μ gives

$$\left(\varepsilon^{-1} F^{\mu\nu}\right)_{;\nu} = 4\pi j^{\mu} \qquad (3)$$

with

$$j^{\mu} = \sum \left(e_0 / c\gamma\right) u^{\mu} \left(-g\right)^{-1/2} \delta^3\left[x^i - x^i(\tau)\right]$$

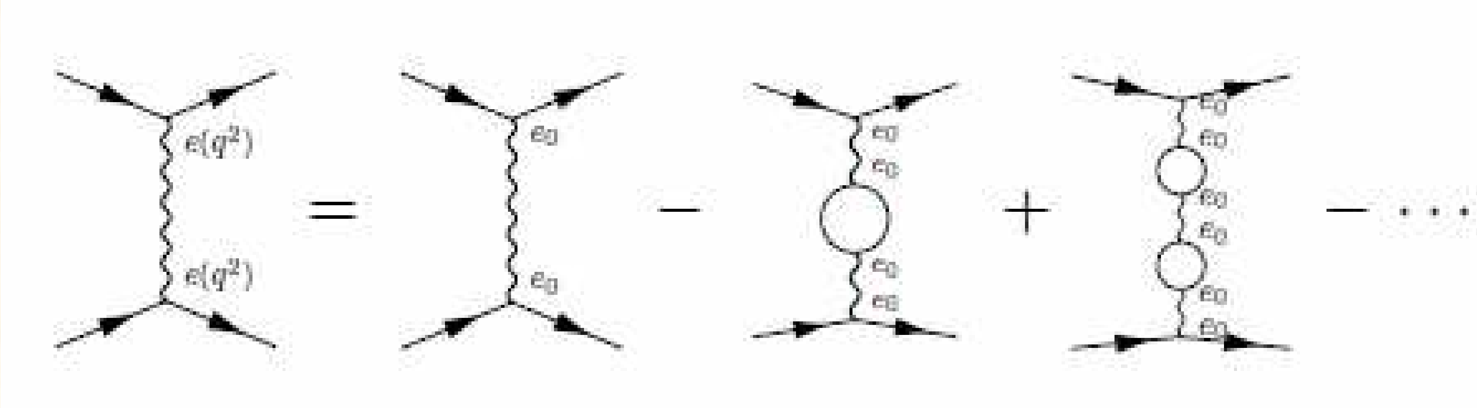
Because $\varepsilon^{-1} F_{\mu\nu}$ is antisymmetric, we have the identity

$$j^{\mu}_{;\mu} = 0$$

So, the conserved charge is $\sum e_0$ which is distinct from the charge $e_0\varepsilon$ which couple to A_μ in the action.

The dynamics of ε is scale invariant, so we are free to make ε=1 for our present epoch. With this choice, e_0 coincide with the usual charges of particles.

Running α



$$\alpha(-q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln\left(\frac{-q^2}{\mu^2}\right)}$$

$$\alpha(4m_e^2) = \frac{1}{137}$$

$$\alpha(Q^2) = \frac{\alpha(4m_e^2)}{1 - f\,\frac{\alpha(4m_e^2)}{3\pi} \ln\left(\frac{Q^2}{4m_e^2}\right)}$$

$$\alpha(m_Z^2) = \frac{1}{130}$$ about 5% greater than at m_e^2

Due to scale invariance of ε, scalar-field dynamic for lnε makes sense only:

$$\Box \ln \varepsilon = \frac{\ell^2}{\hbar c}\left[\varepsilon \frac{d\sigma}{d\varepsilon} - \frac{1}{8\pi} F^{\mu\nu} F_{\mu\nu}\right] \qquad (4)$$

where

$$\sigma = \sum mc^2 \gamma^{-1} \left(-g\right)^{-1/2} \delta^3\left[x^i - x^i(\tau)\right]$$

If we identify F^{0i}, i=1,2,3 with the electric field ***E***, equation (3) in the case the point located far from the charges leads to:

$$\vec{\nabla} \cdot (\varepsilon^{-1} \vec{E}) = 0$$

Which is solved by:

$$\vec{E} = \hat{r} \varepsilon Q / r^2, \qquad (5)$$

Where Q could be identified with $\sum e_0$.

Inserting (5) in (4) we can get the solution to ε(r):

$$\varepsilon(r) = \frac{1}{\cos(\ell Q (4\pi\hbar c)^{-1/2} r^{-1})}$$

So, we can see deviation from Coulomb law only for $r \ll \ell$

H. Sandvic, J. Barrow, J. Magueijo extension of Bekenstein theory
Phys.Rev.Lett. **88** (2002) 031302

$$\psi = \ln \varepsilon; \quad f_{\mu\nu} = \varepsilon F_{\mu\nu}$$

Total action becomes

$$S = \int d^4x \sqrt{-g}\,(L_g + L_{mat} + L_\psi + e^{-2\psi} L_{em}),$$

$$\text{where } L_\psi = -\frac{\omega}{2}\partial_\mu \psi\, \partial^\mu \psi \ \text{ and } \ L_{em} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$\text{and } \ \omega = \frac{\hbar c}{\ell^2}.$$

Varying *ψ,* we get the following equation:

$$\Box \psi = \frac{2}{\omega} e^{-2\psi} L_{em}$$

For pure radiation $L_{em} = (E^2 - B^2)/2 = 0$. So, during radiation domination epoch the variation of α was negligible. Only in matter epoch changes in α occur. The only contribution to variation of ψ come mainly from pure electrostatic or magnetostatic energy.

It's convenient to work in the following parameter: $\zeta_N = m_N^{-1} \left\langle N \left| \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right| N \right\rangle$

According to J. Gasser and H. Leutwyler (Phys.Rep. **87** (1982) 77)

$$\zeta_p = -0.0007, \quad \zeta_n = 0.00015$$

Varying the metric tensor and using Friedmann metric we get the following Friedmann equation:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_m(1+\zeta_m e^{-2\psi}) + \rho_r e^{-2\psi} + \frac{\omega}{2}\dot\psi^2 + \Lambda/8\pi\right]$$

And the equation for ψ takes form:

$$\ddot\psi + 3H\dot\psi = -\frac{2}{\omega}e^{-2\psi}\zeta_m\rho_m; \qquad \alpha = e^{2\psi}e_0^2/\hbar c$$

We have also energy conservation equations:

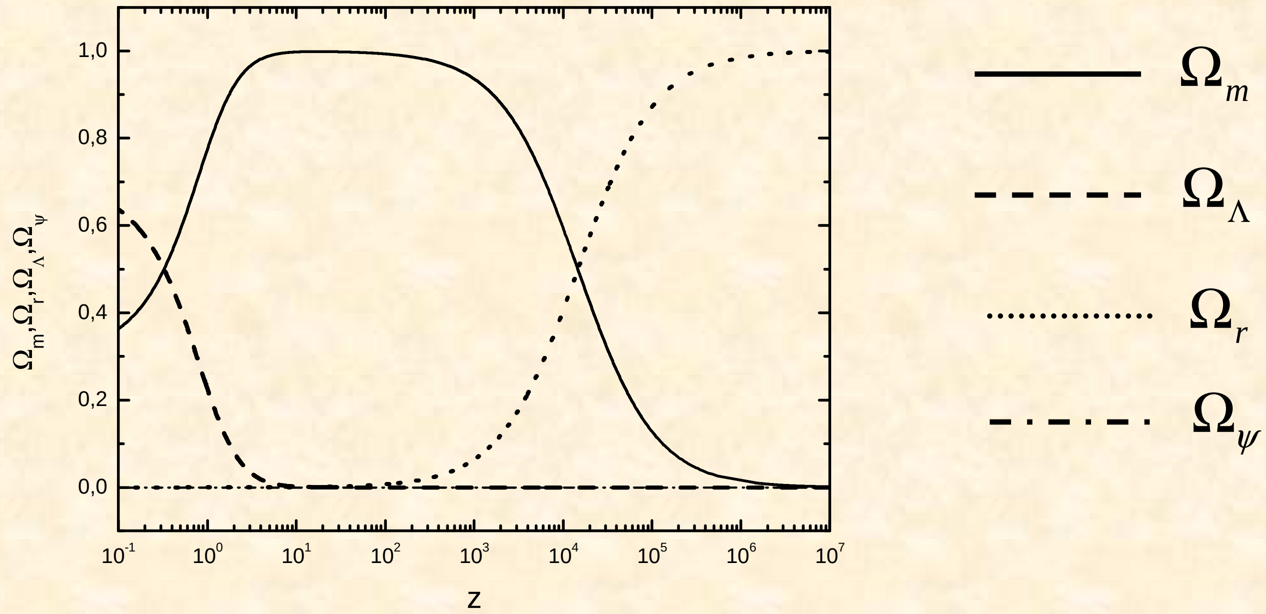
$$\dot\rho_m + 3H\rho_m = 0, \qquad \rho_m \sim a^{-3}$$

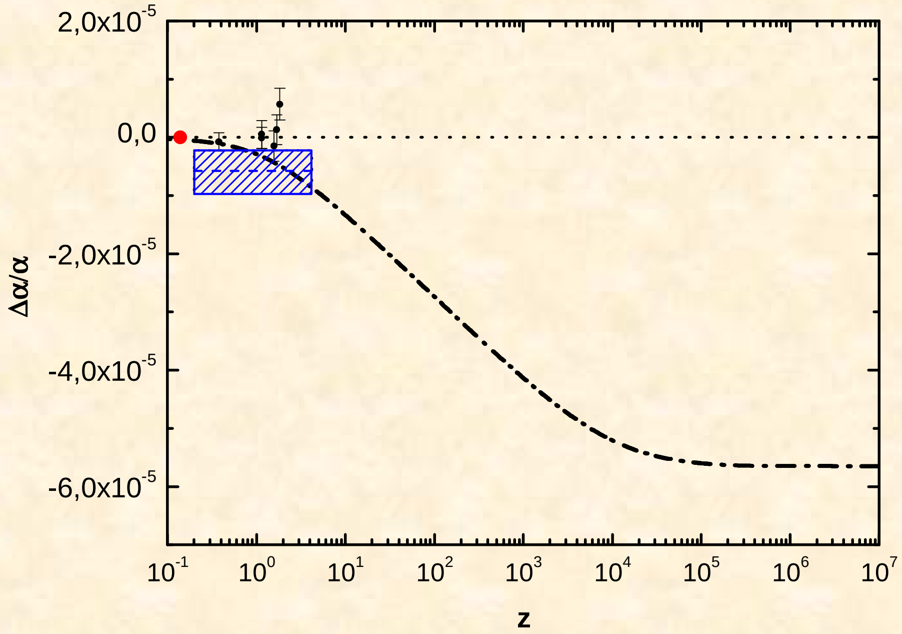
$$\dot\rho_r + 4\rho_r = 2\dot\psi\rho_r, \qquad e^{-2\psi}\rho_r \sim a^{-4}$$

Critical density: $\rho_c(t) = \frac{3H^2(t)}{8\pi G}$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left[\Omega_m^{(0)}\left(\frac{a_0}{a}\right)^3(1+\zeta_m e^{-2\psi}) + \Omega_r^{(0)}\left(\frac{a_0}{a}\right)^4 e^{-2\psi_0} + \Omega_\Lambda^{(0)} + \frac{\omega}{2}\frac{\dot\psi^2}{\rho_{c0}}\right]$$

$$\frac{d}{dt}\left(a^3\dot\psi\right) = Ne^{-2\psi}; \qquad N = -2\frac{\zeta_m}{\omega}\,\Omega_m^{(0)}\rho_{c0}. \qquad N \sim \zeta_m\left(\frac{\ell}{l_{pl}}\right)^2$$

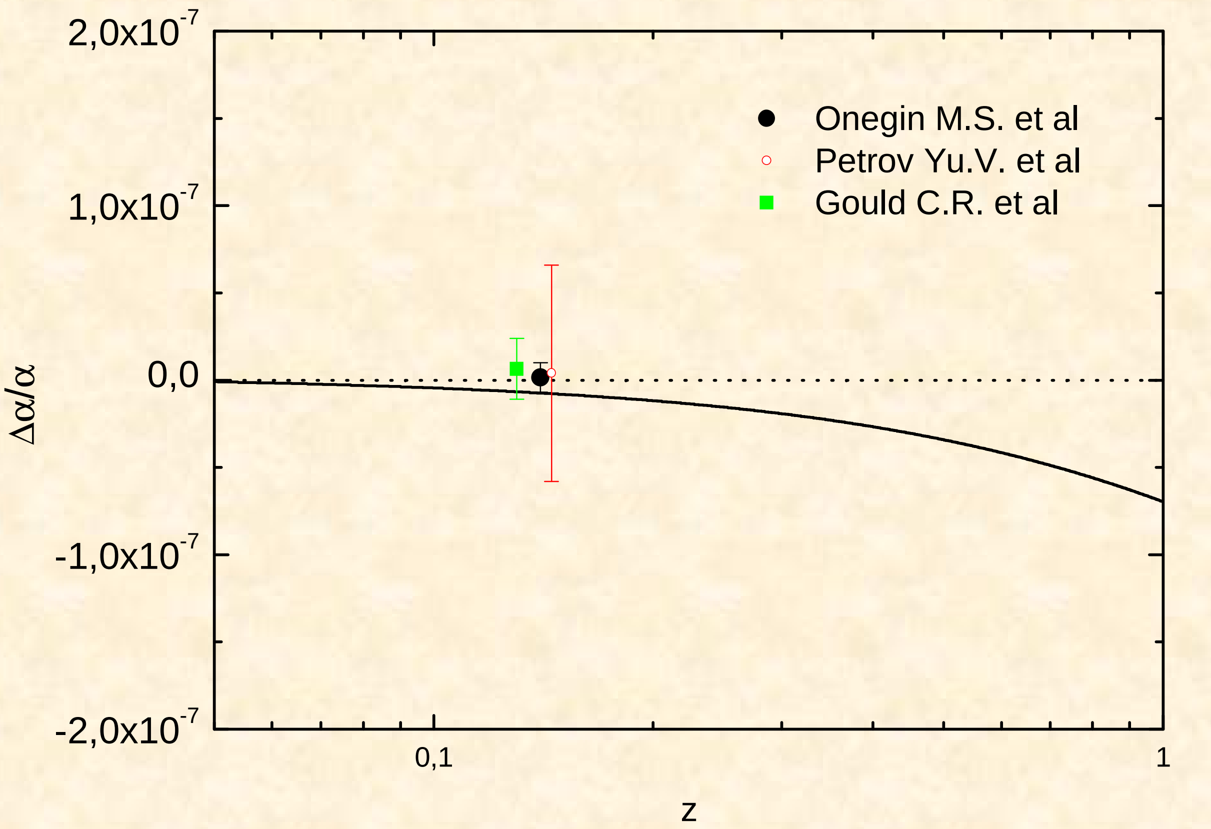




$$\ell = l_{pl}$$

$$\zeta_m = -0.01\%$$

$$\zeta_m \simeq \frac{\Omega_b}{\Omega_m}\zeta_p, \quad \Omega_b \simeq 0.03$$



$$\left|\zeta_m\left(\frac{\ell}{l_{pl}}\right)^2\right| < 6\times10^{-7}$$

If $\zeta_m \simeq 10^{-4} \Rightarrow \quad \dfrac{\ell}{l_{pl}} < 0.1$

Conclusions

A theoretical framework under very general assumptions was worked out by Bekenstein to admit the variation of the fine structure constant.
A characteristic length ℓ enters into it.
An experimental constraint rules out α variability of any kind if it is in clear conflict with predictions of the framework for ℓ no shorter than the fundamental length l_{pl}.
The Oklo geophysical constraints strongly rule out all α variability.

Thank you for

your attention!