Two-loop electroweak corrections to the matching of α_s in the Standard Model

A.V. Bednyakov

FPC14, JINR,Dubna

02.12.2014

Outline

Motivation

- Three-loop RGE for all the SM Lagrangian parameters were calculated recently in the $\overline{MS}$ scheme [MSS12, BPV13, CZ13].
- Boundary values at the electroweak (EW) scale are required for a RGE analysis of the model
  - Matching predictions in terms of parameters with "observables" or "pseudo"-observables - in perturbation theory at two loops.
- In a vacuum stability analysis of the SM the uncertainty of the instability scale (or critical values of the SM parameters at the EW scale) is dominated by those of y_t, λ and α_s [BKKS12, DDVEM$^+$12]
  - When one determines $\alpha_s(\mu)$ in the SM (from that of $n_f = 5$ flavour QCD) usually only strong interactions are taken into account.
  - However, the electroweak corrections can be potentially enhanced by top Yukawa coupling.

Motivation

- Three-loop RGE for all the SM Lagrangian parameters were calculated recently in the $\overline{MS}$ scheme [MSS12, BPV13, CZ13].
- Boundary values at the electroweak (EW) scale are required for a RGE analysis of the model
  - Matching predictions in terms of parameters with "observables" or "pseudo"-observables - in perturbation theory at two loops.
- In a vacuum stability analysis of the SM the uncertainty of the instability scale (or critical values of the SM parameters at the EW scale) is dominated by those of y_t, λ and α_s [BKKS12, DDVEM$^+$12]
  - When one determines $\alpha_s(\mu)$ in the SM (from that of $n_f = 5$ flavour QCD) usually only strong interactions are taken into account.
  - However, the electroweak corrections can be potentially enhanced by top Yukawa coupling.

The SM RGEs and Vacuum instability

- RGEs allow one to predict the behavior of the higgs effective potential at large values of Higgs field $\phi \gg v$.
- The crucial parameters for the SM stability RGE analysis are the Higgs self-coupling λ,

$$V_{eff}(\phi \gg v) \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

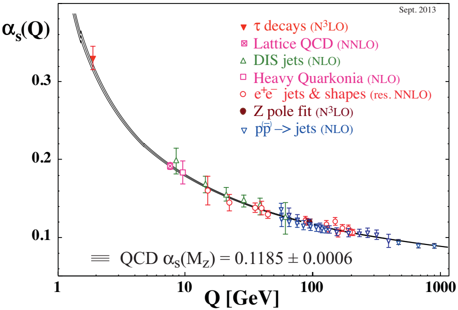
top Yukawa coupling y_t and the strong coupling $\alpha_s = g_s^2/(4\pi)$

$$(4\pi)^2 \frac{d\lambda}{dt} = 12\lambda^2 + 6y_t^2\lambda - 3y_t^4 + \ldots$$

$$(4\pi)^2 \frac{dy_t}{dt} = \frac{9}{4}y_t^3 - 4g_s^2 y_t + \ldots$$

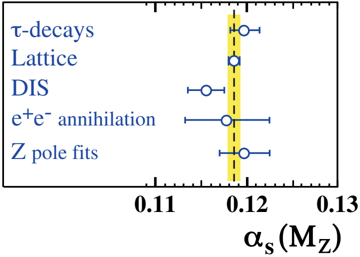
with $t = \ln \mu^2/\mu_0^2$

Observed running of α_s



- Observed running of the strong coupling (PDG'14 [O⁺14])
- World average at $\mu = M_Z$ scale in $\overline{MS}$ scheme
  - Dimensional regularization $D = 4 \to D = 4 - 2\epsilon$
  - (Modified) Minimal subtractions - only poles in ϵ (and a universal constant) go to the renormalization factors.

Experimental determination of α_s



Summary of values of $\alpha_s(M_Z)$ in $n_f = 5$ QCD obtained with "pre-averaging" in certain sub-classes.

We need $\alpha_s(\mu)$ in the SM!

- e^+e^- annihilation
  - $\alpha_s(M_Z) = 0.1177 \pm 0.0046$
- EW precision fits
  - $\alpha_s(M_Z) = 0.1197 \pm 0.0028$
- DIS
  - $\alpha_s(M_Z) = 0.1154 \pm 0.020$
- τ-lepton
  - $\alpha_s(M_\tau) = 0.330 \pm 0.014 \Rightarrow \alpha_s(M_Z) = 0.1197 \pm 0.016$
- Lattice
  - $\alpha_s(M_Z) = 0.1185 \pm 0.0005$

QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\text{gauge}}_{\text{QCD}} + \mathcal{L}^{\text{gauge}}_{\text{SU(2)}\times\text{U(1)}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{ghosts}}$$

- In the QCD embedded in the SM, quark mass terms are generated via Yukawa interactions with the Higgs vacuum expectation value v:

$$m_q = \frac{y_q v}{\sqrt{2}}$$

- Due to spontaneous symmetry breaking (SSB) all other SM masses are also proportional to v

$$M_W^2 = \frac{g_2^2 v^2}{4}, \qquad M_Z^2 = \frac{g_1^2 + g_2^2}{4} v^2, \qquad M_h^2 = 2\lambda v^2$$

QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\rm gauge}_{\rm QCD} + \mathcal{L}^{\rm gauge}_{\rm SU(2)\times U(1)} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm ghosts}$$

- Introducing fine-structure constant α and Weinberg angle θ_W

$$(4\pi)\alpha = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} = g_2^2 \sin^2\theta_W = g_1^2 \cos^2\theta_W$$

- Parametrization used in this work

$$y_q^2 = \frac{4\pi\alpha}{\sin^2\theta_W}\frac{m_q^2}{M_W^2}, \qquad \lambda = \frac{4\pi\alpha}{8\sin^2\theta_W}\frac{M_h^2}{M_W^2}$$

  - All the parameters here are bare (or $\overline{MS}$ renormalizied) ones.
  - NB: In the formal limit $v \to \infty$ the mass ratios are finite.

Parameter values and the choice of renormlization scheme

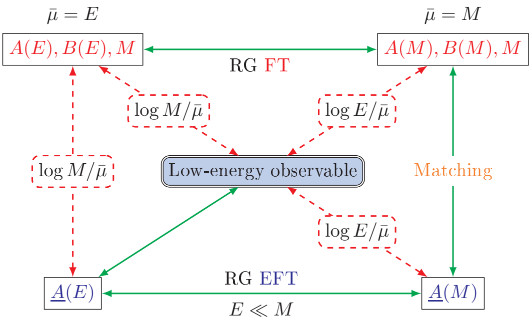
- The values of the SM parameter are not predicted by the theory but should be extracted from an experiment via matching procedure.

- In the QCD sector, due to confinement, one usually adopts $\overline{MS}$ scheme to define the running $\alpha_s(\mu)$.
- In order to determine the corresponding value, an observable $\mathcal{O}$ is matched to the corresponding theoretical prediction

$$\mathcal{O} = \alpha_s^k(\mu)\left[c_0(\mu) + c_1(\mu)\alpha_s(\mu) + c_2(\mu)\alpha_s^2(\mu) + \ldots\right],$$

so that $\alpha_s(\mu_0)$ at some matching μ_0 is extracted.
- To avoid large logarithms the scale μ_0 is usually chosen around the typical scale involved in the measurement of $\mathcal{O}$ (e.g. momentum transfer Q^2).
- However, in $\overline{MS}$ additional effort is required if a theory involves different mass scales (apparent violation of the Appelquist & Carazzone decoupling theorem[AC75])

Re-summation and effective theories



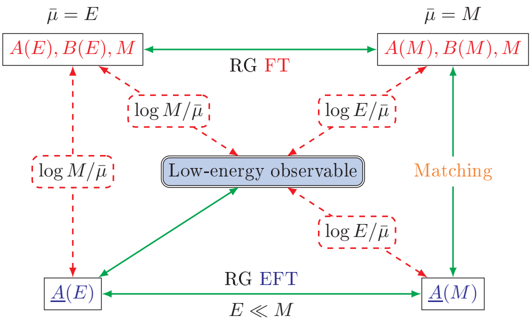
A well-known example:

- $A(\bar{\mu}) = \alpha_s^{(6)}(\bar{\mu})$,
- $M = M_t$,
- $\bar{A}(\bar{\mu}) = \alpha_s^{(5)}(\bar{\mu})$

Matching 6-flavor QCD with 5-flavor QCD without top quark.

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\overline{A}(\bar{\mu})$ coupling .
- The latter can be expessed via matching in terms of (running) parameters of the "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and heavy masses M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

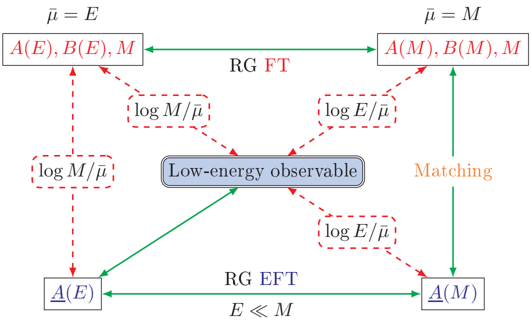
Re-summation and effective theories



Matching can be used to find $A(\bar\mu)$ given $\bar{A}(\bar\mu)$, $B(\bar\mu)$ and M.

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\overline{A}(\bar\mu)$ coupling .
- The latter can be expessed via matching in terms of (running) parameters of the "full" theory (FT) - $A(\bar\mu)$,$B(\bar\mu)$ and heavy masses M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar\mu = M$.

Re-summation and effective theories



Matching can be used to find $A(\bar{\mu})$ given $\bar{A}(\bar{\mu})$, $B(\bar{\mu})$ and M.

This is how $\alpha_s^{(6)}(\bar{\mu})$ is found from the quoted value of $\alpha_s^{(5)}(M_Z)$!

- Effective theory (ET) describes the interactions of light fields at low energies $E \ll M$ and parametrized by running $\overline{A}(\bar{\mu})$ coupling .
- The latter can be expessed via matching in terms of (running) parameters of the "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and heavy masses M.
- Large $\log E/M$ are re-summed by solving renormalization group (RG) equations in the effective theory with initial conditions at $\bar{\mu} = M$.

An example: QCD with n_f flavours

- Consider n_f flavour QCD with one heavy flavour having large mass M.
- At energies $E < M$, one can "integrate out" heavy quarks, leading to an effective Lagrangian for $n_f - 1$ flavors involving a tower of operators $\mathcal{O}_i$ with dimensions $d_i > 4$ (see [Pic98] for review)

$$\mathcal{L}_{QCD}^{(n_f)} \Leftrightarrow \mathcal{L}_{QCD}^{(n_f-1)} + \sum_{d_i>4} \frac{c_i}{M^{d_i-4}} O_i$$

An example: QCD with n_f flavours

- Consider n_f flavour QCD with one heavy flavour having large mass M.
- At energies $E < M$, one can "integrate out" heavy quarks, leading to an effective Lagrangian for $n_f - 1$ flavors involving a tower of operators $\mathcal{O}_i$ with dimensions $d_i > 4$ (see [Pic98] for review)

$$\mathcal{L}_{QCD}^{(n_f)}\left(\alpha_s^{(n_f)}\right) \Rightarrow \mathcal{L}_{QCD}^{(n_f-1)}\left(\alpha_s^{(n_f-1)}\right)$$

- At low scales $E \ll M$ one can neglect O_i and consider renormalizable version of ET.
- The two couplings are related through matching condition:

$$\underbrace{\alpha_s^{(n_f-1)}(\mu)}_{\overline{A}(\mu)} = \underbrace{\alpha_s^{(n_f)}(\mu)}_{A(\mu)} \underbrace{\left[1 + \sum_i \frac{\alpha_s^i(\mu)}{(4\pi)^i} C_i(L)\right]}_{\zeta_{\alpha_s}-\text{decoupling constant}}, \qquad L = \ln\frac{M^2}{\mu^2}$$

- Coefficients C_i are known up to the four-loop level, $i = 1, ..., 4$ (see, e.g., [CKS00, SS06, KKOV06, CKS06]).

QED x QCD as an effective low-energy theory

- As a “low-energy” effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five massless quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}\right)$$

- Similar to the QCD case we “integrate out” top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.

- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = \textit{fixed}$ usually implied in the discussions of “non-decoupling” feature of the models with SSB (see [Pic98]).

QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five massless quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}^{(n_f=5)}_{QCD\times QED}\left(\alpha_s^{(5)}, \alpha_{EM}\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar\psi\psi\bar\psi\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.

- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = \textit{fixed}$ usually implied in the discussions of "non-decoupling" feature of the models with SSB (see [Pic98]).
- From the phenomelogical point of view we miss a lot of electroweak physics, goverened at low energies by the Fermi constant G_F!

QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five massless quarks (u, d, c, s, b) and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{g_s^2}{M_W^2}$.
- Nevertheless, our task is to study the running of $\alpha_s^{SM}(\mu)$ in $\overline{MS}$ extracted from $\alpha_s^{(5)}(\mu)$ at some matching scale $\mu_0 \simeq 100 - 200$ GeV
- Due to the *chosen $\overline{MS}$ scheme*, the result is also valid in the effective QED$\times$QCD$\times$Fermi theory!

How to find matching relation?

- In order to match the SM and our effective theory, one, in principle, needs to consider some 1 PI Green functions predicted in both models.
- Asymptotic expansion in large mass M (LME) of the SM result should reproduce the dependence on **"soft"** scales given by effective theory prediction in each order of $\frac{1}{M^2}$.
- The dependence on **"hard"** scales is absorbed in the (re)definition of the effective theory couplings.
- The rules of LME tells us that the expansion (in terms of Feynman diagrams) consists of
  - the "hard part" [all internal momenta $q_i \sim M$
  - the "soft part" [all internal momenta $q_i \ll M$]
  - a mixture of hard and soft lines, some internal lines have $q_i \simeq M$ and some have $q_k \ll M$

It turns out that only the "hard part" contributes to the matching relation between the couplings of the theories at the given loop level.

How to find matching relation?

It turns out that only the "hard part" contributes to the matching relation between the couplings of the theories at the given loop level.

- Due to this, it is tempting to calculate the "hard" part via Taylor expansion of the integrand in small external momentum and masses.
- An obvious subtlety: such an expansion can generate (spurious) infra-red (IR) divergencies upon integration, which should be properly "subtracted".
- A convenient way to deal with this problem is to use dimensional regularization and perform matching at the bare level, e.g.,

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0} \times \alpha_{s,0}, \qquad \alpha_{s,0} \equiv \alpha_{s,0}^{SM}$$

Matching bare parameters

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_{s,0}}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^{2}\zeta_{c,0}^{-2}\zeta_{G,0}^{-1} = \zeta_{qGq,0}^{2}\zeta_{q,0}^{-2}\zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM so that

- $\zeta_{cGc,0}$and $\zeta_{qGq,0}$ correspond to the leading terms in Taylor expansion of the integrand of the ghost-gluon and (light)-quark-gluon vertices, respectively.
- $\zeta_{c,0}, \zeta_{G,0}, \zeta_{q,0}$ involve only $\ln M/\mu$ terms coming from ghost, gluon and quark propagators.

Matching bare parameters

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_{s,0}}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta_{cGc,0}^{2}\zeta_{c,0}^{-2}\zeta_{G,0}^{-1} = \zeta_{qGq,0}^{2}\zeta_{q,0}^{-2}\zeta_{G,0}^{-1} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM

Taylor expansion can produce spurious IR-divergent $\frac{1}{(q^2)^2}$ terms, which, upon integration, lead to additional IR poles in $\epsilon = (4-d)/2$ in bare ζs.

Matching bare parameters

$$\alpha_s^{(5)}(\mu) = \frac{Z_{\alpha_s}[\alpha_s, \alpha, M]}{Z_{\alpha_{s(5)}}\left[\alpha_s^{(5)}\right]} \zeta_{\alpha_s,0}[Z_{\alpha_s}\alpha_s, Z_\alpha \alpha, Z_M M] \times \alpha_s(\mu)$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_s,0}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta^2_{cGc,0}\zeta^{-2}_{c,0}\zeta^{-1}_{G,0} = \zeta^2_{qGq,0}\zeta^{-2}_{q,0}\zeta^{-1}_{G,0} = \ldots$$

in which different ζs are found by considering three- and two-point 1PI Green functions in the SM

But the spurious IR poles are canceled in the matching relation for the running couplings *after* renormalization.

A comment on Gauge independence and tadpole diagrams

The calculation was carried out in a general R_ξ gauge, parametrized by four gauge-fixing parameters $(\xi_G, \xi_W, \xi_Z, \xi_\gamma)$

$$\begin{aligned}\mathcal{L}_{\text{g.f.}} &= -\frac{1}{2\xi_G}(\partial_\mu G_\mu)^2 - \frac{1}{2}(\partial_\mu A_\mu)^2 \\ &\quad -\frac{1}{\xi_W}\left|\partial_\mu W_\mu^+ - i\xi_W M_W \phi^+\right|^2 - \frac{1}{2\xi_Z}(\partial_\mu Z_\mu - \xi_Z M_Z \chi)^2\end{aligned}$$

- The result for ζ_{α_s} expressed in terms of the pole masses is free from gauge-fixing parameters.
- However, the bare expression $\zeta_{\alpha_s,0}$ looks gauge-dependent (e.g., due to the top quark self-energy) if tadpoles are not properly accounted for (see [FJ81]).
- We follow [ACOV03] here.

A comment on Gauge independence and tadpole diagrams

- We assume that the bare vev v_0 minimizes the effective potential so that loop-generated tadpoles T are canceled by a tree-level term t_0(already at the bare level)

$$i \cdot t_0 \qquad - \qquad i \cdot T \qquad = 0$$

- It is convenient to cast the bare vev into the following form with non-minimal Z_{v0}. The latter is determined in PT by canceling tadpoles order by order ([ACOV03])

$$v_0 = Z_{v_0}^{\frac{1}{2}} \cdot v_{tree,0}, \qquad v_{tree,0}^2 \equiv \frac{m_0^2}{\lambda_0} \Rightarrow \frac{M_{h,0}^2}{2\lambda_0}$$

$$t_0 = \left[\frac{M_h^2 M_W \sin\theta_w}{e}\right]_0 (Z_{v_0} - 1) Z_{v_0}^{\frac{1}{2}}$$

Matching running parameters

(One of) our final expression (s):

$$\alpha_s^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s\alpha} + \ldots \right),$$

Matching running parameters

(One of) our final expression (s):

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2} \ \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2} \ \delta\zeta^{(2)}_{\alpha_s\alpha} + \ldots \right),$$

In terms of the top pole mass M_t (all μ-dependence of Xs is explicit)

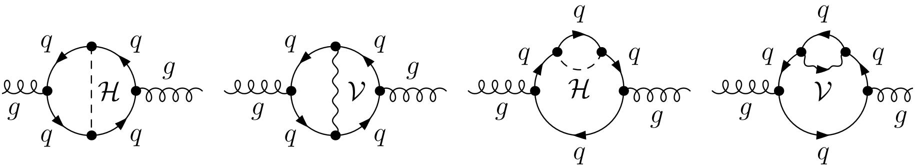
$$\begin{aligned}
\delta\zeta^{(1)}_{\alpha_s} &= X^{(1)}_{\alpha_s} \ln \frac{M_t^2}{\mu^2}, \qquad X^{(1)}_{\alpha_s} = \frac{4}{3} T_f = \frac{2}{3} \\
\delta\zeta^{(2)}_{\alpha_s} &= X^{(0)}_{\alpha_s^2} + X^{(1)}_{\alpha_s^2} \ln \frac{M_t^2}{\mu^2} + X^{(2)}_{\alpha_s^2} \ln^2 \frac{M_t^2}{\mu^2}, \\
X^{(0)}_{\alpha_s^2} &= \left(\frac{32}{9} C_A - 15 C_F \right) T_f = -\frac{14}{3} \\
X^{(2)}_{\alpha_s^2} &= \frac{16}{9} T_f^2 = \frac{4}{9}, \qquad X^{(1)}_{\alpha_s^2} = \left(\frac{20}{3} C_A + 4 C_F \right) T_f = \frac{38}{3}
\end{aligned}$$

Matching running parameters

(One of) our final expression (s):

$$\alpha_s^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\, \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2}\, \delta\zeta^{(2)}_{\alpha_s \alpha} + \dots \right),$$

Diagrams contributing to $\delta\zeta^{(2)}_{\alpha_s\alpha}$ ($\mathcal{H} = h_0, \phi^{\pm}, \chi$ - higgs and would be goldstone bosons, $\mathcal{V} = W^{\pm}, Z$, q - different quarks)



The corresponding integrands are expanded in external momentum Q and masses of light quarks (all but t). For consistency, Yukawa interactions of light quarks are also neglected.

Matching running parameters

(One of) our final expression (s):

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi}\delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\,\delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s\alpha}{(4\pi)^2}\,\delta\zeta^{(2)}_{\alpha_s\alpha} + \dots\right),$$

In terms of PDG'14 particle pole masses (all μ-dependence of Xs is explicit) new result is given by ($x_{ij} \equiv M_i/M_j$)

$$\delta\zeta^{(2)}_{\alpha_s\alpha} = \frac{M_t^2}{M_W^2 s_W^2}\left(X^{(1)}_{\alpha_s\alpha}\ln\frac{M_t^2}{\mu^2} + X^{(0)}_{\alpha_s\alpha}\right), \qquad \frac{M_t^2}{M_W^2 s_W^2} = 20.8(2)$$

$$X^{(1)}_{\alpha_s\alpha} = -1 + x_{wt}^2\left(\frac{2}{9} + \frac{22}{9}x_{wz}^2\right) + \frac{11}{6}x_{zt}^2 = -0.034(15)$$

$$X^{(0)}_{\alpha_s\alpha} = -1.17(2) \quad \text{to be compared with } X^{(0)}_{\alpha_s^2} = -\frac{14}{3}$$

See arXiv:1410.7603 [Bed14] for analytic result in terms of x_{ij}

Enhancement factor due to the top Yukawa coupling y_t : $\alpha_s\alpha\frac{M_t^2}{M_W^2 s_W^2} \sim \alpha_s^2$

Extraction of α_s^{SM} from $\alpha_s^{(5)}$

- By construction, given the parameters of the SM one can find the value of the effective coupling $\alpha_s^{(5)}$.
- However, it is $\alpha_s^{(5)}(\mu)$ which is fitted to observables the QCD.
- Due to this, one is interested in the inverse relation (obtained in PT):

$$\alpha_s = {\alpha_s}^{(5)} \left(1 + \frac{{\alpha_s}^{(5)}}{4\pi}\delta\zeta^{(1)}_{\alpha_s'} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'} + \frac{{\alpha_s}^{(5)}\alpha}{(4\pi)^2}\delta\zeta^{(2)}_{\alpha_s'\alpha} \right)$$

$$\begin{aligned} \delta\zeta^{(1)}_{\alpha_s'} &= \delta\zeta^{(1)}_{{\alpha_s}^{(5)}} = -\delta\zeta^{(1)}_{\alpha_s} \\ \delta\zeta^{(2)}_{\alpha_s'} &= -\left(\delta\zeta^{(2)}_{\alpha_s} - 2(\delta\zeta^{(1)}_{\alpha_s})^2\right) \\ \delta\zeta^{(2)}_{\alpha_s'\alpha} &= -\delta\zeta^{(2)}_{\alpha_s\alpha} \end{aligned}$$

Numerical analysis of the $\mathcal{O}(\alpha_s \alpha)$ correction

- In order to analyze the calculated correction we take the matching scale is $\mu = M_Z$ and use PDG'14 values of the pole masses.
- The quoted world averages $\alpha_s^{(5)}(M_Z) = 0.1185$, $\alpha^{-1} = 127.04$ is assumed to be fitted within the effective theory.
- At Z - boson mass scale (three-loop contribution $\mathcal{O}(\alpha_s^3)$ is also shown):

$$\alpha_s(M_Z) = 0.1185 \cdot \left[1 - \underbrace{0.008067}_{\alpha_s} - \underbrace{0.000965}_{\alpha_s^2} + \underbrace{0.000143}_{\alpha_s \alpha} + \underbrace{0.000018}_{\alpha_s^3}\right],$$

- In principle, final result for the running $\alpha_s^{SM}(\mu \gg M_Z)$ should not depend on the matching scale. However, due to truncation of the series, there is a residual dependence on μ
- As a consequence, the matching scale is usually chosen of the order of electroweak scale so that no large logs appear in the relation (effectively re-sum logarithms $\ln M_Z/\mu$).

Scale dependence of the decoupling corrections

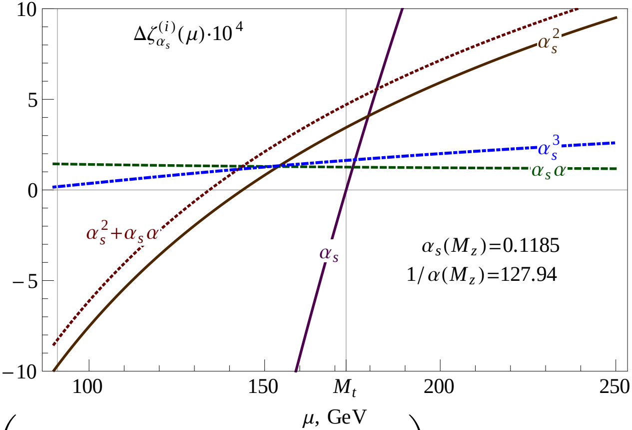
The scale dependence of different matching corrections:

α_s in terms of $\alpha_s^{(5)}$

$$\Delta\zeta_{\alpha_s}^{(\alpha_s)} \equiv \frac{\alpha_s^{(5)}}{(4\pi)}\delta\zeta^{(1)}_{\alpha_s^{(5)}},$$

etc

Four-loop running up to the matching scale via RunDec [CKS00] package.



$$\alpha_s(M_t) = 0.10800\cdot\left(1+\underbrace{0.00034}_{\alpha_s^2}+\underbrace{0.00013}_{\alpha_s\alpha}+\underbrace{0.00016}_{\alpha_s^3}\right)$$

Conclusions

- Electroweak corrections to the matching relation between α_s of the SM and effective ${\alpha_s}^{(5)}$ are found and expressed either in terms of particle pole masses or $\overline{MS}$ running masses in an explicit gauge-invariant way.
- The corrections,when evaluated at the electroweak scale, are found to be comparable with pure three-loop QCD contribution usually taken into account in three-loop RGE analysis of the SM.
- However, the relative value of $\mathcal{O}(\alpha_s\alpha)$ correction is typically around 10^{-4}, which currently below the uncertainty in determination of ${\alpha_s}^{(5)}$.
- Nevetherless, we hope that the result presented here is a necessary step towards future precise analysis of the SM.

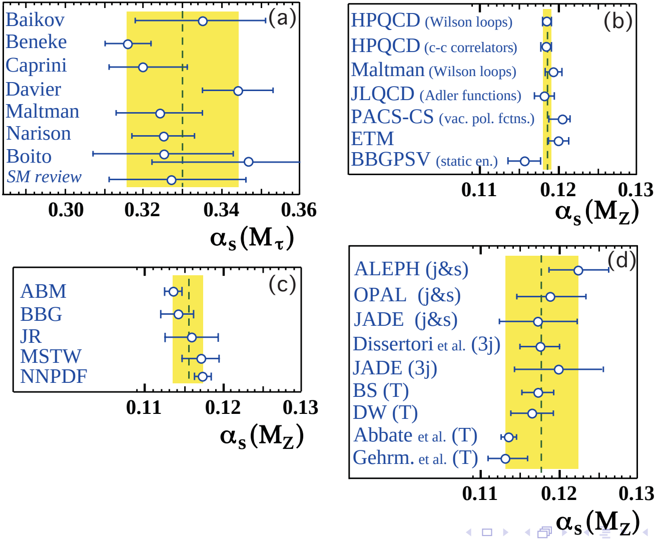
Thank you for your attention!

Backups

Some backup slides

Issues with α_s determination

Measurements within the sub-classes seems to be marginally compatible with each other within the quoted uncertainties



A comment on Gauge independence and tadpole diagrams

$$v_0 = Z_{v_0}^{\frac{1}{2}} \cdot v_{tree,0}, \qquad v_{tree,0}^2 \equiv \frac{m_0^2}{\lambda_0}$$

- The "tree-level" bare $v_{tree,0}$ is gauge-invariant by construction, since it is defined in terms of the Lagrangian parameters.
- This allows one to define gauge-invariant bare and $\overline{MS}$ renormalized particle masses, e.g., for the Higgs mass

$$\begin{aligned}
\left[3\lambda_0 v_0^2 - m_0^2\right] &\rightarrow M_{h,0}^2 + \frac{3}{2} M_{h,0}^2 (Z_{v_0} - 1) \\
M_{h,0}^2 &\equiv 2\lambda_0 v_{tree,0}^2 = 2m_0^2 \\
M_{h,0}^2 &= Z_{M_h^2}(\mu) m_h^2(\mu), \qquad Z_{M_h^2} = Z_\lambda Z_v = Z_{m_0^2}
\end{aligned}$$

with minimal renormalization constants $Z_{M_{h^2}}, Z_\lambda, Z_{m^2}$, and Z_v .

- The same is true for other masses (in particular, M_t)!

A comment on Gauge independence and tadpole diagrams

- This approach allows us to obtain bare $\zeta_{\alpha_s,0}$ free from gauge-fixing parameters and , as a consequence, an explicit gauge-independent expression for

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi}\delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s\alpha}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s\alpha} + \dots\right),$$

in which $\delta\zeta$s are given in terms of $\overline{MS}$ parameters and involve $\ln\frac{m_t^2(\mu)}{\mu^2}$ instead of $\ln\frac{M_t^2}{\mu^2}$.

Thomas Appelquist and J. Carazzone.
Infrared Singularities and Massive Fields.
Phys.Rev., D11:2856, 1975.

M. Awramik, M. Czakon, A. Onishchenko, and O. Veretin.
Bosonic corrections to Delta r at the two loop level.
Phys.Rev., D68:053004, 2003.

A.V. Bednyakov.
On the electroweak contribution to the matching of the strong coupling constant in the SM.
2014.

Fedor Bezrukov, Mikhail Yu. Kalmykov, Bernd A. Kniehl, and Mikhail Shaposhnikov.
Higgs Boson Mass and New Physics.
JHEP, 1210:140, 2012.

A.V. Bednyakov, A.F. Pikelner, and V.N. Velizhanin.
Yukawa coupling beta-functions in the Standard Model at three loops.
Phys.Lett., B722:336–340, 2013.

K.G. Chetyrkin, Johann H. Kuhn, and M. Steinhauser.
RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses.
Comput.Phys.Commun., 133:43–65, 2000.

K.G. Chetyrkin, Johann H. Kuhn, and Christian Sturm.
QCD decoupling at four loops.
Nucl.Phys., B744:121–135, 2006.

K.G. Chetyrkin and M.F. Zoller.
β-function for the Higgs self-interaction in the Standard Model at three-loop level.
JHEP, 1304:091, 2013.

Giuseppe Degrassi, Stefano Di Vita, Joan Elias-Miro, Jose R. Espinosa, Gian F. Giudice, et al.
Higgs mass and vacuum stability in the Standard Model at NNLO.
JHEP, 1208:098, 2012.

J. Fleischer and F. Jegerlehner.

Radiative Corrections to Higgs Decays in the Extended Weinberg-Salam Model.
Phys.Rev., D23:2001–2026, 1981.

B.A. Kniehl, A.V. Kotikov, A.I. Onishchenko, and O.L. Veretin.
Strong-coupling constant with flavor thresholds at five loops in the anti-MS scheme.
Phys.Rev.Lett., 97:042001, 2006.

Luminita N. Mihaila, Jens Salomon, and Matthias Steinhauser.
Gauge Coupling Beta Functions in the Standard Model to Three Loops.
Phys.Rev.Lett., 108:151602, 2012.

K.A. Olive et al.
Review of Particle Physics.
Chin.Phys., C38:090001, 2014.

Antonio Pich.
Effective field theory: Course.
pages 949–1049, 1998.

Y. Schroder and M. Steinhauser.
Four-loop decoupling relations for the strong coupling.
JHEP, 0601:051, 2006.