Off-mass-shell muon anomalous magnetic moment

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Plan

- Some notes on higher-order corrections to elastic electron-proton scattering
- Off-mass-shell muon $(g-2)_\mu$

Elastic $e - p$ scattering (I)

Motivation:

J. C. Bernauer *et al.* [A1 Collaboration],
The electric and magnetic form factors of the proton,
Phys. Rev. C **90** (2014)

The proton charge radius is extracted from elastic *ep* scattering with precision of about 1%

Difference from $\langle r^2 \rangle$ defined from muonic hydrogen is about 7 sigma, see details in the talk by S. Karshenboim

Elastic $e - p$ scattering (II)

Experimental statistics and systematics at MAMI are really high.

That leads to 0.37% average point-to-point errors in cross section.

So, radiative corrections should be taken into account accurately.

$\mathcal{O}(\alpha)$ corrections are well known:

$$\left(\frac{d\sigma}{d\Omega}\right)_1 = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + \delta_{\mathrm{vac.pol}} + \delta_{1e} + \delta_{1p} + \delta_{1ep}\right]$$

N.B. Formulae for δ_{1e} contain one obvious misprint. I can not believe that it affected the data analysis.

Elastic $e-p$ scattering (III)

$\alpha/(2\pi) \simeq 2 \cdot 10^{-3}$, $\alpha^2/(2\pi)^2 \simeq 5 \cdot 10^{-6}$, but there are enhancement factors:

1) Large log $L \equiv \ln(Q^2/m_e^2)$ in $\delta_{\rm vac.pol}$ and δ_{1e}

2) The log $\ln((E-E')_{\rm max}/E) \equiv \ln\Delta$, where $(E-E')_{\rm max}$ is the maximal lost energy (experimental cut-off)

This makes higher order effects important

Elastic $e - p$ scattering (IV)

J. C. Bernauer *et al.* applied exponentiation:

$$1 + \delta \rightarrow \mathrm{e}^{1+\delta}$$

Well motivated by Yennie, Frautschi, and Suura.

But there are some drawbacks:

1) vacuum polarization by muons (etc.) already at $\mathcal{O}(\alpha)$ was not taken into account, while $\delta_{\mu-\mathrm{vac.pol}} \approx 1.5 \cdot 10^{-3}$ for $Q^2 = 0.1$ GeV^2 (measurements were for $0.003 < Q^2 < 1$ GeV^2)

2) higher order vacuum polarization

$$\frac{1}{1 - \delta_{\mathrm{vac.pol}}} \neq \mathrm{e}^{1+\delta_{\mathrm{vac.pol}}}$$

the difference makes about $1 \cdot 10^{-4}$

Elastic $e - p$ scattering (V)

3) Exponentiation of soft photon emission assumes that each photon is emitted independently and $E_{\gamma_i} \leq (E - E')_{\mathrm{max}}$, while the experiment makes the cut on the total lost energy. For two photons the difference makes

$$-\left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3}(L-1)^2 \approx -2.5 \cdot 10^{-3}$$

4) Soft and virtual pair corrections appear starting from $\mathcal{O}(\alpha^2)$

$$\delta_{\mathrm{pair}} \sim 2\frac{1}{3}\left(\frac{\alpha}{2\pi}(L-1)\right)^2 \left[2\ln\Delta + \frac{3}{2}\right] \sim 1 \cdot 10^{-3}$$

5) The leading and next-to-leading logarithmic QED corrections can be computed systematically in $\mathcal{O}(\alpha^2 L^{2,1})$ as shown in [A.A.& E.Scherbakova JETP Lett. 2006] for Bhabha scattering in a similar set-up

Elastic $e - p$ scattering (VI)

NLO corrections to elastic $e - p$ scattering with $E_{\gamma_i} \leq \Delta E \ll E$

$$d\sigma_{\rm NLO} = D_{ee}^{\rm str}(\Delta, L)\left[d\sigma_{\rm Born} + d\bar{\sigma}_1(\Delta)\right] D_{ee}^{\rm frg}(\Delta, L),$$

$$d\bar{\sigma}_1(Delta) = \sigma_1 - 2\frac{\alpha}{2\pi} L P_{ee}^{(0)}(\Delta) - 2\frac{\alpha}{2\pi} d_1(\Delta),$$

$$P_{ee}^{(0)}(\Delta) = 2\ln\Delta + \frac{3}{2}, \qquad d_1(\Delta) = 2\ln^2\Delta + 2\ln\Delta - 2,$$

$$D_{ee}^{\rm str,frg}(\Delta, L) = 1 + \frac{\alpha}{2\pi} d_1(\Delta) + \frac{\alpha}{2\pi} L P_{ee}^{(0)}(\Delta)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left[\frac{1}{2} L^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)}(\Delta) + \frac{1}{3} L^2 P_{ee}^{(0)}(\Delta)\right.$$
$$\left. + L P_{ee}^{(0)} \otimes d_1(\Delta) + L P_{ee}^{(1)\rm str,frg}(\Delta)\right] + \mathcal{O}(\alpha^2 L^0, \alpha^3)$$

Elastic $e - p$ scattering (VII)

Analytic formulae for higher order corrections to elastic $e - p$ scattering

and the corresponding computer code

will be given to the experimental group. Only after insertion them into data analysis we will see the effect on the proton charge radius value.

But for the cross section itself the leading effects are of the order of the experimental uncertainty and thus they will lead to a sizable shift.

Outline for $(g-2)_\mu$

- Motivation
- Status of $(g-2)_\mu$ puzzle
- Off-mass-shell muon form factor in $\mathcal{O}(\alpha)$
- Future experiments
- Conclusions

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- Hadronic contributions are being scrutinized
- Two new experiments on $(g - 2)_\mu$ are forthcoming

$g_e - 2$ status

Harvard experiment (2008):

$$a_e^{\rm exp} = 1\ 159\ 652\ 180.73\ (0.28) \cdot 10^{-12}$$

The experimental error is only 0.24 ppb **!**

SM theory (T. Kinoshita's group):

$$a_e^{\rm SM} = 1\ 159\ 652\ 181.78\ (0.06)(0.04)(0.03)(0.77) \cdot 10^{-12}$$

The theoretical error [0.67ppb] is dominated by the hadronic contribution

$g_\mu - 2$ status (I)

E821 experiment at BNL (2006):

$$a_\mu^{\mathrm{exp}} = 116\ 592\ 089\ (54)(33) \cdot 10^{-11}$$

SM theory (PDG 2012):

$$a_\mu^{\mathrm{SM}} = 116\ 591\ 803\ (1)(42)(26) \cdot 10^{-11}$$

The difference:

$$\Delta a_\mu^{\mathrm{exp-SM}} = 288\ (63)(49) \cdot 10^{-11}$$

i.e. 3.6σ

$g_\mu - 2$ status (II)

SM contributions:

$$a_\mu^{\rm QED} = 116\ 584\ 718.95\ (0.08) \cdot 10^{-11}$$
$$a_\mu^{\rm EW} = 153.6\ (1.0) \cdot 10^{-11}$$

Hadronic vacuum polarization in LO and NLO

$$a_\mu^{\rm Had}[LO] = 6\ 923\ (42)(3) \cdot 10^{-11}$$
$$a_\mu^{\rm Had}[NLO] = 7\ (26) \cdot 10^{-11}$$

Light-by-Light contribution:

$$a_\mu^{\rm Had,LbL(a)}[NLO] = 105\ (26) \cdot 10^{-11}$$
$$a_\mu^{\rm Had,LbL(b)}[NLO] = 110\ (40) \cdot 10^{-11}$$
$$a_\mu^{\rm Had,LbL(c)}[NLO] = 136\ (25) \cdot 10^{-11}$$

Hadronic vacuum polarization

It is extracted from $e^+e^- \to$ *hadrons* and τ decays

See e.g. review

S. Actis, A. Arbuzov, G. Balossini *et al.* (Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies Collaboration)
Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data
Eur. Phys. J. C **66** (2010) 585

SUSY and "new physics"

SUSY predicts

$$a_\mu^{\mathrm{SUSY}} = \pm 130 \cdot 10^{-11} \times \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}} \right)^2 \tan\beta$$

But restrictions on m_{SUSY} go up rapidly because of LHC ...

In general

$$a_\mu^{\mathrm{new\ phys}} \sim a_\mu^{\mathrm{EW}} \times \left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{new\ phys}}} \right)^2$$

is unrealistic since

$$\Delta a_\mu^{\mathrm{exp-SM}} \approx 2 a_\mu^{\mathrm{EW}}$$

Off-mass-shell muon $g-2$ (I)

Let us consider the process

$$\mu(p) + \gamma(k) \longrightarrow \mu(p+k)$$
$$k^2 = 0, \qquad p^2 = -m^2, \qquad (p+k)^2 + m^2 = 2pk = \kappa m^2$$

where parameter κ defines the off-shellness degree

Notation of Akhiezer & Berestetskii book "*QED*" is used,
in particular metric $(-,+,+,+)$ is applied

$$\kappa \leq 0, \qquad \kappa \ll 1$$

Off-mass-shell muon $g-2$ (II)

In calculation of vertex $\Lambda_\mu^{(3)}(p, p+k; k)$ we have 3 integrals: J_0, J_σ, $J_{\sigma\tau}$. The scalar and the tensor integrals are infrared and ultraviolet divergent, respectively. But as known from the standard calculations of $g_f - 2$, only the vector integral contributes,

$$\frac{m^2}{i\pi^2} J_\sigma = \left(J_0 - \frac{\ln|\kappa|}{\kappa - 1} \right) p_\sigma + \left(2J_0 - \frac{\kappa - 2}{\kappa - 1} \ln|\kappa| - 2 \right) \frac{k_\sigma}{\kappa}$$

where J_0 is the scalar integral

$$\frac{m^2}{i\pi^2} J_0 = \frac{1}{\kappa} \left[\mathrm{Li}_2\left(1\right) - \mathrm{Li}_2\left(1-\kappa\right) \right], \qquad \mathrm{Li}_2\left(x\right) \equiv - \int\limits_0^x \frac{\ln(1-y)}{y} dy$$

Off-mass-shell muon $g-2$ (III)

Expanding in $\kappa \ll 1$ we get the first correction due to the off-shellness:

$$\Delta a_f^{(1,\kappa)} = \frac{\alpha}{2\pi}\left[1 + \delta a_f^{(\kappa)}\right]$$

$$\delta a_f^{(\kappa)} = \left(\frac{1}{4} + \frac{\ln|\kappa|}{2}\right)\kappa + \mathcal{O}(\kappa^2)$$

Off-mass-shell muon $g-2$ (IV)

Assuming that the difference between experimental result and theoretical predictions is due to the off-shellness effect, we get the equation to define the value of κ:

$$\Delta a_\mu^{\mathrm{exp-SM}} \approx 3 \cdot 10^{-9} = \frac{\alpha}{2\pi} \delta a_\mu^{(\kappa)}$$

Numerical solution of the latter equation gives

$$\kappa \approx -3.5 \cdot 10^{-7}$$

N.B. The shift has the proper sign and the solution exists

Such a value corresponds to off-shellness of a muon of the order

$$m|\kappa| \sim 35\ \mathrm{eV}$$

New experiments

New Muon $g - 2$ at Fermilab (upgrade of the BNL experiment)
start operation in 2016(?), new results after ~ 2 years of running

J-PARC $g - 2$ experiment: a completely different set-up with ultra-cold muon beam

New g-2 Fermilab

New Experimental Goal: 63 → 16 x 10^{-11}

- Statistics: 0.46 → 0.10 ppm
- Systematics on Precession: 0.21 → 0.07 ppm
- Systematics on Field: 0.17 → 0.07 ppm

21 x BNL

- **Need counts**
  - Note: E821 was already "rate limited"
    - Cleaner beam
    - Inject more often
    - Run longer
- **Reduce systematics**
  - Note: Many scale with counts; others were "good enough"
    - Modern detectors / electronics / DAQ critical
    - Improved field intrinsic uniformity
    - Better environment (building)
    - Improved injection

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- (Quantum) Single-body $\longleftrightarrow$ (Quantum) Collective
- Quantum Born-level $\longleftrightarrow$ Quantum Loop, e.g. $(g-2)_\mu$
- **A Collective Quantum Loop Effect?**

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- Potentially, this effect can describe the difference between the SM and experiment
- Experimental conditions of muon interactions inside the beam to be applied
- New experiments at Fermilab and J-PARC will have different conditions. They will "check" the effect in practice

THANK YOU FOR ATTENTION!