Pole and running lepton masses in QED : results and by-products

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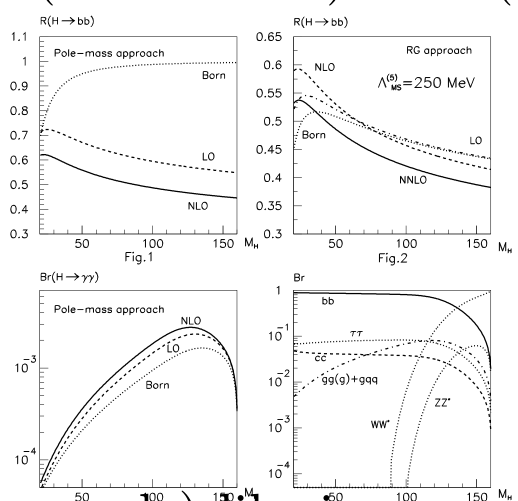
Abstract

Three discovered charged leptons have pole masses
m_e=0.510998910$\pm$13$\times$10^{-9} MeV (CODATA-06),
m_μ=105.6583668$\pm$38$\times$10^{-9} MeV (CODATA-06) and
$m_\tau = 1776.69^{+0.17}_{-0.19}\pm 0.15$ MeV from e^+e^- Novosibirsk data for threshold $e^+e^- \to \tau^+\tau^-$ (KEDR- collab (11)-still preliminary).
We study questions:
1. Definitions of running lepton masses in the MS-scheme- especially important for τ-lepton mass due to interest in $\Gamma(H \to \tau^+\tau^-)$- how many order $O(\alpha)$-corrections important? Indeed, the mass of τ-lepton is comparable with the mass of charm quark, where the concept of running is considered.

Why running lepton masses in QED ?

a) Not often considered. Interesting to use say for τ-lepton.

b) $H \to \tau^+\tau^-$- important decay mode: $H \to \tau^+\tau^- \to l + jets$ in W-boson fusion may be detectable at LHC if M_H = 115-135 GeV and L=60 fb^{-1} (CMS-collab note referred by **N.Krasnikov (07)**-at 13th Lomonosov Conf. talk). Up to $M_H \approx 150$GeV $Br(H \to \tau^+\tau^-) > Br(H \to c\overline{c})$ (see e.g. **Kataev,Kim (09)**:)



(m_τ-pole) like in some other codes

$\Gamma_{Hc\overline{c}} = 3G_F\sqrt{2}/8\pi \mathrm{M_H}\overline{\mathrm{m}}_\mathrm{c}^2(\mathrm{M_H})\mathrm{C_H}(\alpha_\mathrm{s})$ with running quark mass.

We used $\Gamma_{H\tau^-\tau^+} = G_F\sqrt{2}/8\pi \mathrm{M_H}\mathrm{m}_\tau^2$

At present $C_H(\alpha_s)$ is known up to $O(\alpha_s^4)$-terms (**Baikov, Chetyrkin, Kuhn (06)**. As was shown by **Kataev, Kim (07)**- 13 Lomonosov Conf, **Kataev, Kim (08)**- ACAT08- Erice and **Bakulev, Mikhaiklov, Stefanis (11)** Tevatron and LHC experimental precision do not need at present the inclusion of $O(\alpha_s^4)$- they may be important in case of finding Higgs-boson.

Therefore it is appropriate to study the similar approximation for $\Gamma_{H\tau^-\tau^+}$. In the $\overline{\rm MS}$-scheme its $O(\alpha^3)$-approximation takes the following form

$$\Gamma_{H\tau^-\tau^+} = \Gamma^0_\tau(\frac{\overline{m}_\tau(M_H)}{m_\tau})^2[1 + a(M_H)s_1 + a(M_H)^2 s_2 + a(M_H)^3 s_3 + a(M_H)^2 a_s(M_H)\delta^{QCD}] \quad ,$$

$a(M_H) = \frac{\alpha_{\overline{\rm MS}}(M_H)}{\pi}$, $\overline{m}_\tau(M_H)$ - are QED parameters (they are related to the on-shell α and m_τ- see below), $a_s(M_H) = \frac{\alpha_s^{\overline{\rm MS}}(M_H)}{\pi}$ is the QCD parameter.

Evaluation of running τ-lepton mass: $\overline{m}_\tau(M_H) \to \overline{m}_\tau(m_\tau) \to m_\tau$:

$$\begin{aligned} \overline{m}_l(M_H) &= \overline{m}_l(m_l) \exp\left[- \int_{a(M_H)}^{a(m_l)} \frac{\gamma_m^{QED}(x)}{\beta^{QED}(x)} dx \right] \\ &= \overline{m}_l(m_l) \left(\frac{a(M_H)}{a(m_l)} \right)^{2\gamma_0/\beta_0} \left(\frac{AD(a(M_H))}{AD(a(m_l))} \right) \end{aligned} \quad (1)$$

where $AD(a)$ are defined up to $O(\alpha^3)$-corrections, which depend from first 4 terms of the RG-functions

$\frac{\partial \alpha}{\partial \ln \mu^2} = \beta_{\bar{M}S}^{QED}(a) = \beta_0 a^2 + \beta_1 a^3 + \beta_2 a^4 + \beta_3 a^5 + \beta_4 a^6$

$\frac{\partial \ln \bar{m}_l}{\partial \ln \mu^2} = \gamma_{\bar{m}}^{QED}(a) = -\gamma_0 a - \gamma_1 a^2 - \gamma_2 a^3 - \gamma_3 a^4 - \gamma_4 a^5$

β_3 known from **Gorishny et al (91)**; 1-fermion loop contribution to β_4 from **Baikov, Chetyrkin and Kuhn (07)**; QED γ_3 **Chetyrkin (97)** agrees with QED limit of **Vermaseren, Larin, van Ritbergen (97)** QCD γ_3; β_4, γ_4 are added for theoretical reasons. Next- relation $\overline{m}_\tau(m_\tau) \to m_\tau$

A.L.Kataev, G.Parente and V.T. Kim (11)-work in progress btained from analytical result of **Melnikov, van Ritbergen (00)**, which agrees with semi-analytical result of **Chetyrkin, Steinhauser(00)**.

$\overline{m}_\tau(m_\tau) = m_\tau[1 - (\frac{\alpha(\mu)}{\pi}) + (\frac{\alpha(\mu))}{\pi})^2 2.46 + 1.92(\frac{\alpha(m_\mu)}{\pi})^3]$; $\mu = m_\tau$

Hadronic vacuum polarization contributions may be important!

$\overline{m}_\mu(m_\mu) = m_\mu[1 - (\frac{\alpha(\nu)}{\pi}) + (\frac{\alpha(\nu)}{\pi})^2 0.9 + 3.17(\frac{\alpha(\nu)}{\pi})^3]$, (the scale $\nu = m_\mu$)

$\overline{m}_e(m_e) = m_e[1 - (\frac{\alpha(\nu)}{\pi}) - (\frac{\alpha(\nu)}{\pi})^2 0.7 + 4.36(\frac{\alpha(\nu)}{\pi})^3]$, (the scale $\nu = m_e$)

b) It is enough to take into account one term in PT (others are rather small). In the coefficient function- the same:

Indeed, in the $\overline{MS}$-scheme QED coefficients of the coefficient function read

$s_1 = d_1^E = \frac{17}{4} \approx 4.25 \qquad s_2 = d_2^E - \gamma_0(\beta_0 + 2\gamma_0)\pi^2/3$

$d_2^E = \frac{691}{64} - \frac{9}{4}\zeta_3 - [\frac{65}{16} - \zeta_3](N_L + 1 + 3\sum_F Q_F^2)$ (at $Q_F = 0$
$d_2^E = -.49$ (in case of τ) 2.37 (in case of μ)

$s_3 = d_3^E - [d_1(\beta_0 + \gamma_0)(\beta_0 + 2\gamma_0) + \beta_1\gamma_0 + 2\gamma_1(\beta_0 + 2\gamma_0)]\pi^2/3$

$\delta^{QCD} = \delta^E - \beta_1^{QED-QCD}\gamma_0\pi^2/3$

$d_3^E = \frac{23443}{768} - \frac{239}{16}\zeta_3 + \frac{45}{8}\zeta_5) - [\frac{88}{3} - \frac{65}{4}\zeta_3 - \frac{3}{4}\zeta_4 + 5\zeta_5](N_L + 1 + 3\sum_F Q_F^4)$
$+[\frac{15511}{3888} - \zeta_3](N_L + 1 + 3\sum_F Q_F^2)^2 \quad (d_3^E = -2.03 \;\; for \;\; \tau)$
$\delta^E = [\frac{15511}{2916} - \frac{4}{3}\zeta_3]4\sum_F Q_F^2 \approx 12.95$ (In Euclidean region $(\alpha/\pi)^2\alpha_s/\pi$-coefficient is larger- but overall contribution is smaller than 1-loop term.

These results are obtained from **Gorishny et al (90-91)** and **Chetyrkin (97)**

Relation between on-shell and running b-quark mass:

Kataev,Kim (11)

$$\frac{\mathrm{m_b}}{\overline{m_b}(m_b)} = 1 + \frac{4}{3}a_s(m_b)) + 12a_s(m_b)^2 + 131a_s(m_b)^3 + 1101a_s(m_b)^4$$

The estimate a_s^4 was ontained using **Chetyrkin,Kniehl and Sirlin (97)** application of the **Kataev, Starshenko (95)** PMS/ECH -inspired studies.

$$M_q = \overline{m}_q(\overline{m_q})\sum_{n=0}^{\infty} t_n a^n(\overline{m}_q(\overline{m}_q^2))$$

$$M_q = \frac{1}{2\pi i}\int_{-\overline{m}_q(\overline{m}_q)-i\varepsilon}^{-\overline{m}_q(\overline{m}_q)+i\varepsilon} ds' \int_0^{\infty} ds \frac{T(s)}{(s+s')^2}$$

$$f_0^E = t_0, f_1 = t_1^M \quad f_2^E = t_2^M + \frac{\pi^2}{6}t_0\gamma_0(\beta_0+\gamma_0)$$

$$f_3^E = t_3^M + \frac{\pi^2}{3}\left\{t_1(\beta_0+\gamma_0)\left(\beta_0+\frac{\gamma_0}{2}\right) + t_0\left[\frac{\beta_1\gamma_0}{2} + \gamma_1(\beta_0+\gamma_0)\right]\right\}$$

Effects of analytical continuation! Are they observed in the analytical α_s^3-result? Partial cancellation with typical to on-shell ζ_2-effects !

$$
\begin{aligned}
\frac{\overline{m}(M)}{M} = 1 - \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg(N_L \left(\frac{71}{144} + \frac{\pi^2}{18}\right) \\
-\frac{3019}{288} + \frac{1}{6}\zeta_3 - \frac{\pi^2}{9}\log 2 - \frac{\pi^2}{3}\Bigg) \\
+ \left(\frac{\alpha_s}{\pi}\right)^3 \Bigg(N_L^2 \left(-\frac{2353}{23328} - \frac{7}{54}\zeta_3 - \frac{13}{324}\pi^2\right) \\
+N_L \Bigg(\frac{246643}{23328} + \frac{241}{72}\zeta_3 + \frac{11}{81}\pi^2\log 2 - \frac{2}{81}\pi^2\log^2 2 \\
+\frac{967}{648}\pi^2 - \frac{61}{1944}\pi^4 - \frac{1}{81}\log^4 2 - \frac{8}{27}a_4\Bigg) \\
-\frac{9478333}{93312} + \frac{1439}{432}\zeta_3\pi^2 - \frac{61}{27}\zeta_3 - \frac{1975}{216}\zeta_5 \\
+\frac{587}{162}\pi^2\log 2 + \frac{22}{81}\pi^2\log^2 2 - \frac{644201}{38880}\pi^2 + \frac{695}{7776}\pi^4 \\
+\frac{55}{162}\log^4 2 + \frac{220}{27}a_4\Bigg)
\end{aligned}
$$

CONCLUSION: For

1) For careful study of $\Gamma_{H\tau^-\tau^+}$ is worth to take into account 2-loop running of mass and 1 -loop coefficient function to get the precision, available for $\Gamma(H \to b\bar{b})$ at α_s^3-level **Kataev, Kim (09), Bakulev et al- (08-09)**

2) QED structure of PT- no sign alternation- at least in the $\overline{MS}$-schemde !

3) In the relation bewteen QCD on-shell and running mass do esist effects of analytical continuation- are they summable?