Захват двух электронов голым ядром

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Outline

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Motivation

- **REC** - Radiative Electron Capture - the main electron-capture channel;
- **RDEC** - Radiative Double Electron Capture: one-step process where the momentum and the energy of two correlated captured electrons are converted into one emitted photon:
  - probability is rather smaller then in REC,
  - interelectron interaction is taken into account;
- **DREC** - Double Radiative Electron Capture: two-step process (two uncorrelated electrons are captured and two photons are emitted)

These processes are under intensive investigation during last decades. There are some discrepancies between the experimental data and theoretical results.

Diagrams of the processes

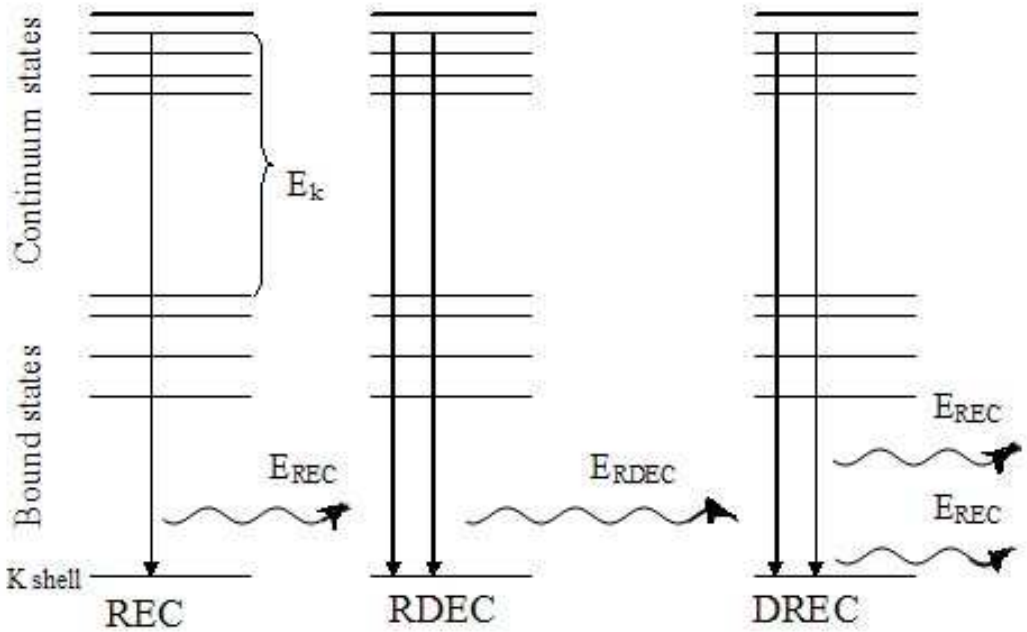


Figure: Diagrams for three processes REC, RDEC, DREC. In REC and RDEC processes capture occurs with the emission of one photon with the energy E_{REC}, E_{RDEC} respectively, E_{RDEC} is two times grater than E_{REC}. In DREC two photons are emitted each with the energy E_{REC}. Here E_k is the kinetic energy of the electron of the target.

Description of the experiments

- 1) 11.4 MeV/u Ar^{18+} ions go through a carbon foil, at GSI by **[1]**;
- 2) bare U^{92+} ions at 297 MeV/u in collisions with gas of Ar atoms, at GSI **[2]**;
- 3) 38 MeV O^{8+} ions go through thin carbon foil, at Western Michigan University in 2009 **[3]**;
- 4) 42 MeV $F^{9+} + C$, at Western Michigan University **[4]**;
- 5) 30 MeV/u $Cr^{24+} + He/N_2$ at GSI data analisis are in progress **[4]**;
- 6) $Xe^{54+} + C$ at 20 MeV/u is planned at GSI **[5]**.

[1] A. Warczak, et. al., Nucl. Instrum. Methods Phys. Res. Sect. B, **98**, 303, 1995

[2] G. Bednarz, et. al., Nucl. Instr. Meth. Phys. Res. Sec B, **235**, 573, 2003

[3] A. Simon, A. Warczak, T. ElKafrawy, and J. A. Tanis , Phys. Rev. Lett., **104**,123001, 2010

[4] A. Simon, ICPEAC XXVII, progress report, 2011

[5] A. Simon, PhD thesis, ArXiv 1008.5317v1, 2010

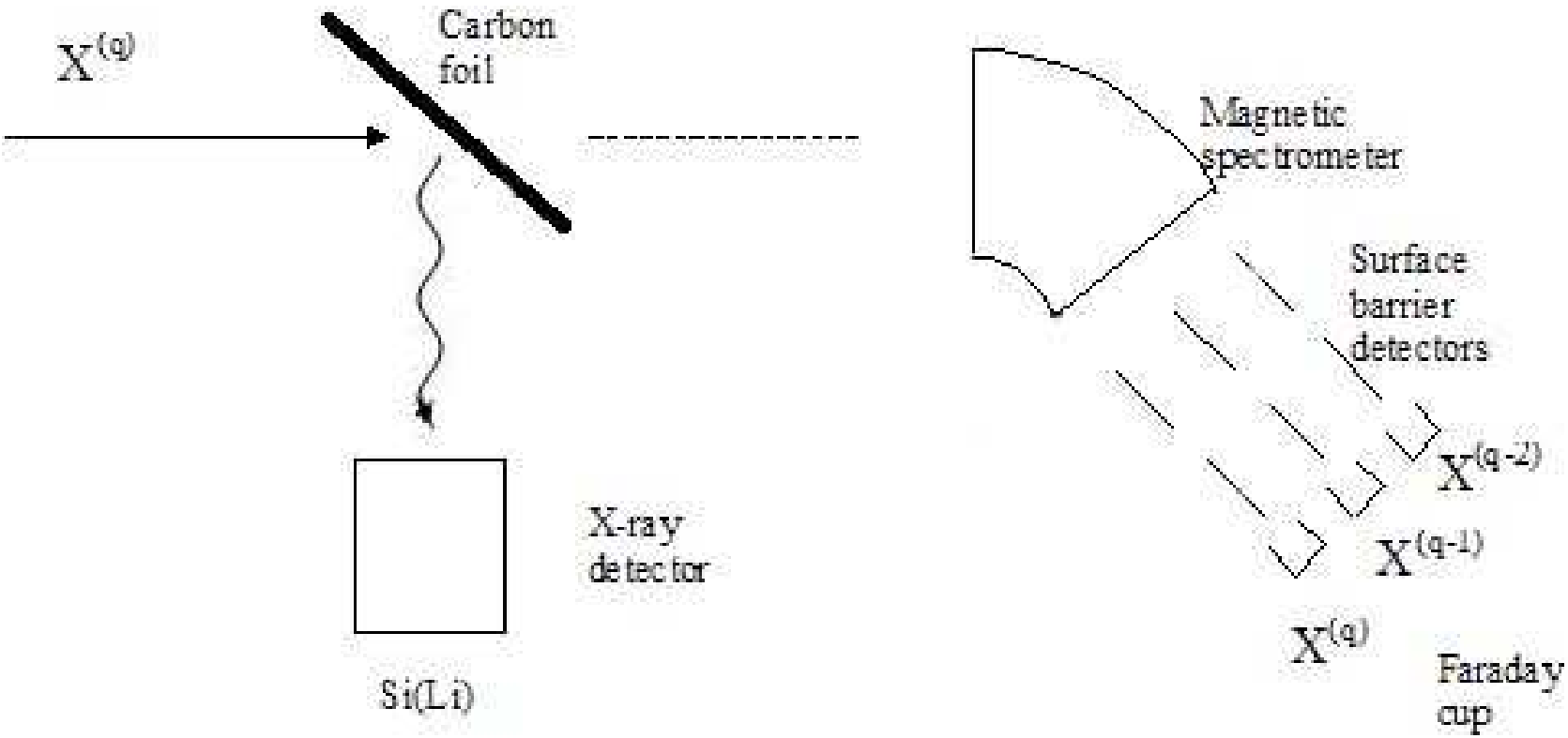


Figure: Experimental setup. Ions $X^{(q)}$ go though carbon foil, emitted photon is registered by X-ray detector, then beam go through magnetic spectrometer, then ions go to target chamber where surface barrier detectors count the ions with different charges q, q-1, q-2, where q is the charge of the incoming beam of the ions.

Theoretical model

- 1) all calculations are done within the framework of **Quantum Electrodynamics** (**QED**) and **Line-Profile Approach** (**LPA**) **[1]**;
- 2) **scheme of the RDEC:**

$$2e^-(\epsilon) + X^{(Z)+} \rightarrow X^{(Z-2)+}(1s1s) + \gamma(\omega); \qquad (1)$$

- 3) **initial state**: two incident electrons with the same energies (ϵ) and a bare atomic nucleus (the nuclear charge Z);
- 4) **final state**: two-electron ion in the ground state and a single photon ($\gamma(\omega)$);
- 5) the momenta of the electrons are equal and electrons go along one axis;
- 6) we integrate over the direction of the emitted photon;
- 7) in the experiments: electrons are quasifree $\Rightarrow$ take into account the properties of the target

[1] O. Yu. Andreev, L. N. Labzowsky, G. Plunien and D. A. Solovyev, Phys. Rep., **455**, 135, (2008)

Feynman graphs describing the process

Feynman graphs depicted in the Figure correspond to the first order of interelectron interaction.

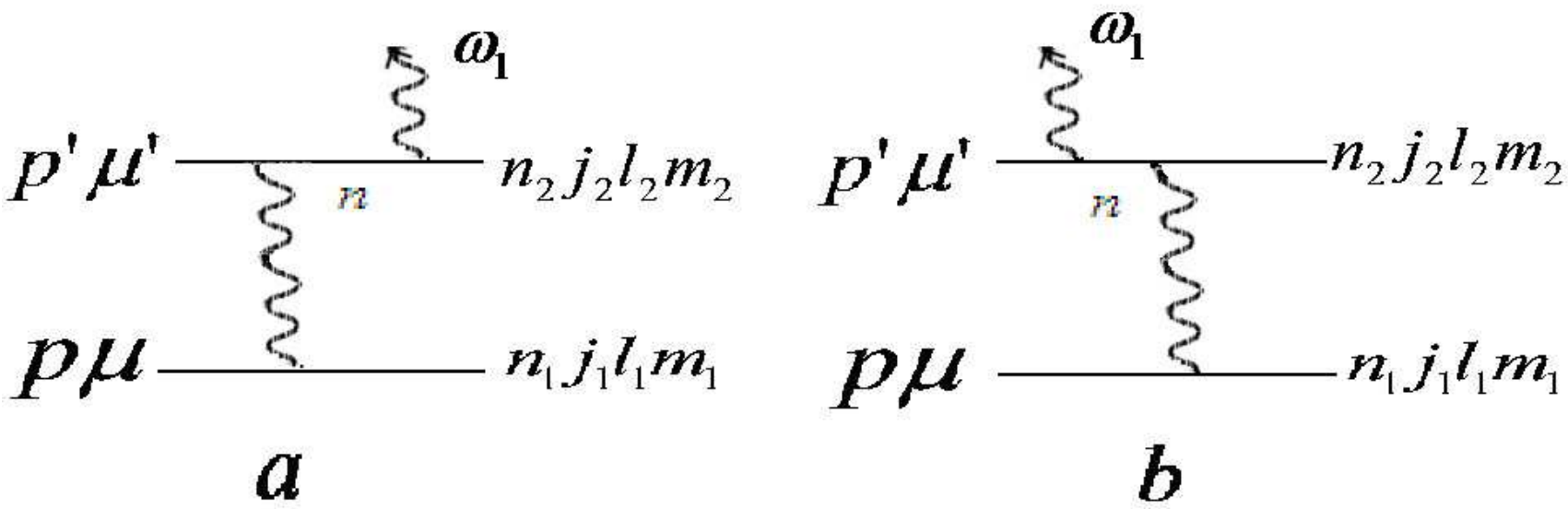


Figure: The inner wavy lines describe the interelectron interaction, the wavy lines with the arrows describe the emission of the photon. Index n corresponds to the intermediate one-electron state, indices $\mathbf{p}$, μ correspond to the initial electron with momentum $\mathbf{p}$ and polarization μ. Indices n_1, j_1, l_1, m_1, n_2, j_2, l_2, m_2 are the quantum numbers of electrons in the final state.

The wave function of the electron confined within the large sphere $\psi_{e_R}(\mathbf{r})$ is represented like

$$\psi_{e_R jlm}(\mathbf{r}) = \frac{1}{N_R}\psi_{ejlm}(\mathbf{r})\,\theta(R - |\mathbf{r}|)\,, \tag{1}$$

where $\theta(R - |\mathbf{r}|)$ is the Heavyside step function, N_R normalazing constant

$$(N_R)^2 = \frac{\epsilon R}{p\pi} \tag{2}$$

If we have one continuum electron in the intial or final states, the amplitude U_{e_R} calculated with functions $\psi_{e_R jlm}$ and U_e with ψ_{ejlm} are related like

$$U_e = \lim_{R\to\infty} N_{e_R} U_{e_R}\,. \tag{3}$$

If there are two continuum electron in the initial or final states, the amplitudes are related like

$$U_{e_1 e_2} = \lim_{R\to\infty} N_{e_1 R} N_{e_2 R} U_{e_1 R e_2 R}\,, \tag{4}$$

Here we take $N_{e_R} \equiv N_{e_1 R} = N_{e_2 R}$.

Two-electron functions for double electron capture process

Wave function of the incident electron is $\psi_{\mathbf{p}\mu}(\mathbf{r})$, where $(\mathbf{p})$ is the momentum , $(\epsilon = \sqrt{p^2 + m_e^2})$ - energy and (μ) - polarization. The wave function of incident electron is normalized like

$$\int d^3\mathbf{r}\, \psi^+_{\mathbf{p}'\mu'}(\mathbf{r})\psi_{\mathbf{p}\mu}(\mathbf{r}) = (2\pi)^3\delta^3(\mathbf{p}' - \mathbf{p})\delta_{\mu'\mu}\,. \tag{1}$$

Function $\psi_{\mathbf{p}\mu}(\mathbf{r})$ can be expanded like [1]

$$\psi_{\mathbf{p}\mu}(\mathbf{r}) = \int d\varepsilon \sum_{jlm} a_{\mathbf{p}\mu,\varepsilon jlm}\psi_{\varepsilon jlm}(\mathbf{r})\,. \tag{2}$$

This function $\psi_{\varepsilon jlm}(\mathbf{r})$ is normalized like

$$\int d^3\mathbf{r}\, \psi^+_{\varepsilon' j' l' m'}(\mathbf{r})\psi_{\varepsilon jlm}(\mathbf{r}) = \delta(\varepsilon' - \varepsilon)\delta_{j'j}\delta_{l'l}\delta_{m'm}\,. \tag{3}$$

[1] A. I. Akhiezer and V. B. Berestetskii, Quantum Electrodynamics, Wiley Interscience, New York, 1965

The wave function describing two incident electrons with the certain momenta and polarizations can be written as

$$\Psi_{\mathbf{p}_1\mu_1,\mathbf{p}_2\mu_2}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\det\{\psi_{\mathbf{p}_1\mu_1}(\mathbf{r}_1),\psi_{\mathbf{p}_2\mu_1}(\mathbf{r}_2)\}\,. \tag{1}$$

We suppose that the initial state of the system is given by two incident electrons with the equal momenta ($\mathbf{p}$) and the opposite polarizations ($\mu_1 = -\mu_2$). Accordingly, the wave function of the initial state is

$$\Psi^{\mathrm{ini}}(\mathbf{r}_1,\mathbf{r}_2)\frac{1}{\sqrt{2}}\det\{\psi_{\mathbf{p}\mu_1=1/2}(\mathbf{r}_1),\psi_{\mathbf{p}\mu_2=-1/2}(\mathbf{r}_2)\}\,. \tag{2}$$

Cross section

The amplitude of the process of electron capture (U_{if}) is defined via S-matrix

$$S_{if} = (-2\pi i)\delta(E_f - E_i)U_{if}\,. \tag{1}$$

Then, the transition probability is given by

$$dw_{if} = 2\pi \frac{1}{V^2}|U_{if}|^2 \delta(E_f - E_i)\frac{d^3\mathbf{k}}{(2\pi)^3}\,, \tag{2}$$

where E_i, E_f are the initial and final energies of the whole system. Cross section is recorded as

$$d\sigma_{if} = \frac{dw_{if}}{j}\,, \tag{3}$$

where j is the current of the incident electrons. We consider the current to be

$$j = n^{\mathrm{e}}\upsilon, \tag{4}$$

where $n^{\mathrm{e}} = \frac{1}{V}$ and $\upsilon = p/\epsilon$ are the density and velocity of the incident electrons.

Electron density and description of the current

$$j = n^{\mathrm{e}} \upsilon, \quad n^{\mathrm{e}} = \frac{1}{V}, \quad \upsilon = p/\epsilon \tag{1}$$

We get the expression of the volume V as the volume taken by one incident electron is

$$V = \frac{V_T}{Z_T}, V = 2RS, S = S_T/Z_T \tag{2}$$

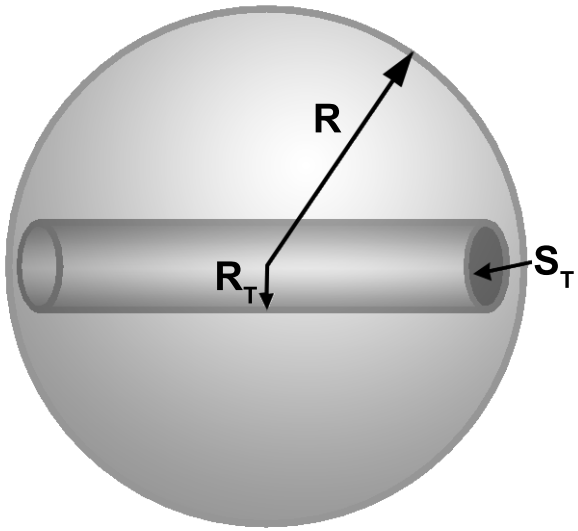


Figure: V_T is the volume of cylinder, Z_T is the number of the electrons in the target atom, R is the radius of the large sphere, $S_T = \pi R_T^2$ is the cross section area of the cylinder, R_T is the radius of the target atom

$$V = \frac{V_T}{Z_T} = 2RS, \tag{1}$$

Using

$$N_{e_R} = \frac{\varepsilon R}{p\pi}, \tag{2}$$

we obtain connection R with the normalization constant N_R:

$$R = \frac{\pi p N_{e_R}^2}{\epsilon} \tag{3}$$

We can write down the equation for the volume via constant N_R.

$$V = 2RS = \frac{2\pi p N_{e_R}^2 S}{\epsilon}, \tag{4}$$

Expression of the cross section for the double electron capture

$$\sigma_{if} = \frac{\omega^2}{(2\pi)^2}\left[\frac{\epsilon}{p}\frac{1}{4\pi}N_{\varepsilon_R}^4\frac{\varepsilon}{2\pi p N_{\varepsilon_R}^2 S}\right]\int d\boldsymbol{\nu}_k\, d\boldsymbol{\nu}_p\, |U_{i,\mathbf{k}\lambda s}|^2\,, \qquad (1)$$

where the photon frequency (ω) is defined from the energy conservation law, term ε/p comes fron the current of the incident electrons, the factor $1/4\pi$ represents the average over the direction of the momentum of the incident electrons ($\boldsymbol{\nu}_p$), N_{ε_R} is the normalization constant, the last term in square brackets is the contribution of the volume. The first index (i) of the amplitude represents the initial state, sub-indeces $\mathbf{k}\lambda$ describe the emitted photon, the subindex $s = (J_s M_s n_{s_1} j_{s_1} l_{s_1} n_{s_2} j_{s_2} l_{s_2})$ corrsponds to two-electron configuration.
We also integrate over the direction of the emitted photon ($\boldsymbol{\nu}_k$) and sum over the polarization (λ).

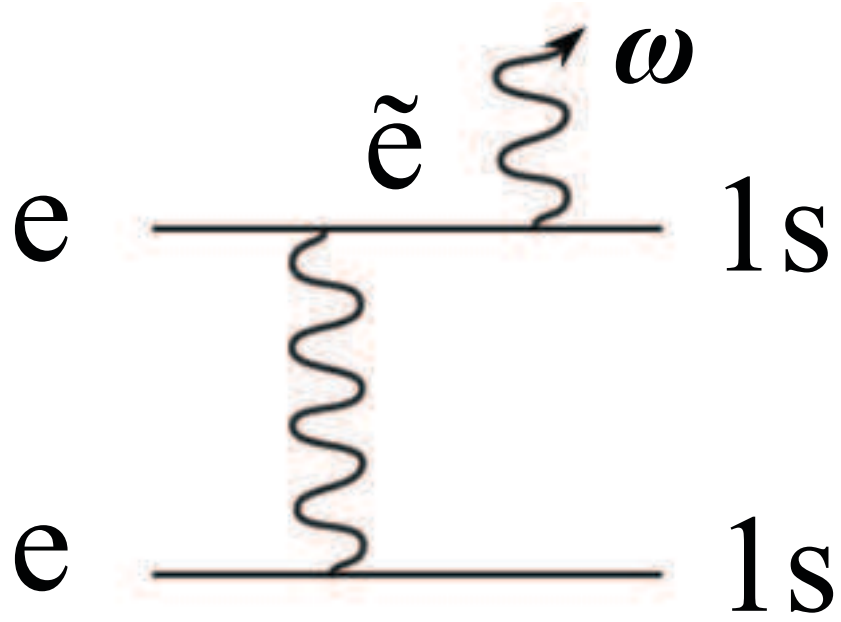


Figure: When in the spectrum the electron $\widetilde{e}$ is the artificial electron state according to the condition: $\epsilon_{\tilde{e}} + \epsilon_{1s} = \epsilon_e + \epsilon_e$, the contribution with this electron is the most significant. electrons e, e, $1s$, $\widetilde{e}$ are concerned as quasidegenerate, which we use applying the Line-Profile Approach (LPA) to calculate the amplitude

Link between amplitudes of one-electron capture and two-electron capture

We select an artificial electron state ($\tilde{e}$) by the condition:
$\epsilon_{\tilde{e}} + \epsilon_{1s} = \epsilon_e + \epsilon_e$.
The configurations $(1s, \tilde{e})$ have the same energy as the initial state. We compare two process:

$$e + e \to (1s1s) + \gamma(\omega) \quad \text{and} \quad 1s + \tilde{e} \to (1s1s) + \gamma(\omega)$$

the amplitudes of these processes are connected as

$$U[e + e] = \eta U[1s + \tilde{e}] \tag{1}$$

In calculations we really operate with $U[1s + \tilde{e}]$ and then with the help of the equation (1) obtain the amplitude of the double-electron capture.

Numerical methods

- 1) Spherical box with the radius $R^{\rm box} = 70/(\alpha Z)$, where α is the fine-structure constant, Z is the nuclear charge;
- 2) The Dirac spectrum is constructed in terms of B-splines (of the order 8 and a grid with 60 nonzero knots).
- 3) The electron states of the generated spectrum, which are close to the substituted electron state (e_n), are designated as e_{n-1} and e_{n+1};
- 4) The final state is characterized by $(1s1s)$ and $(1s2s)$ configurations. We consider the emission of the electric photons with $J = 1$;
- 5) $1s$, $2s$, e_{n-1}, e_{n+1} compose the set of all possible configuration in $j - j$ scheme;
- 6) We take into account the interelectron interaction including Breit contribution.

Results and discussion

We consider two approximation for the experiments:

1) we suppose that the electrons are distributed homogeneously in the atom ($\sigma^{\mathrm{RDEC,A}}$);

2) we neglect all the electrons of the atom except the K-shell electrons and suppose that the electrons are distributed homogeneously within the sphere of the K-shell radius of the atom ($\sigma^{\mathrm{RDEC,K}}$).

Table: 1 Radii (in pm) and areas of the cross-sections of the reaction volumes for one incident electron (in kilobarn) for the C target atom

$R_{\mathrm{T}}^{\mathrm{A}}$	S^{A}	$R_{\mathrm{T}}^{\mathrm{K}}$	S^{K}
133	919	14	31

$R_{\mathrm{T}}^{\mathrm{A}}$ is the radius of the target atom. The area of the cross-section of the reaction volume for one electron (S^{A}): $S^{\mathrm{A}} = \pi(R_{\mathrm{T}}^{\mathrm{A}})^2/Z_{\mathrm{T}}$, where Z_{T} is the charge of the nucleus of the target atom. The radius $R_{\mathrm{T}}^{\mathrm{K}}$ is the radius of the K-shell of the target atom. In the case of the second approximation for one electron (S^{K}) is set to $S^{\mathrm{K}} = \pi(R_{\mathrm{T}}^{\mathrm{K}})^2/2$.

Table: 1 Cross-section (in barn) for RDEC process O^{8+}+C, $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ contribution.

Experiment	Theory		
$\sigma^{\mathrm{RDEC}}_{(1s1s)}$ [1]	$\sigma^{\mathrm{RDEC}}_{(1s1s)}$ [3]	$\sigma^{\mathrm{RDEC,A}}_{(1s1s)}$ [4]	$\sigma^{\mathrm{RDEC,K}}_{(1s1s)}$ [4]
3.2(1.9)	0.15	0.55	0.019

Table: 2 Cross-section (in barn) for RDEC process O^{8+}+C, $\sigma^{\mathrm{RDEC}}_{(1s2s)}$ contribution

Experiment	Theory		
$\sigma^{\mathrm{RDEC}}_{(1s2s)}$ [1]	$\sigma^{\mathrm{RDEC}}_{(1s2s)}$ [2]	$\sigma^{\mathrm{RDEC,A}}_{(1s2s)}$ [4]	$\sigma^{\mathrm{RDEC,K}}_{(1s2s)}$ [4]
2.2(1.3)	0.105	0.05	0.002

Table: 3 Total cross-section (in barn) for RDEC process O^{8+}+C, σ^{RDEC} contribution.

Experiment	Theory		
σ^{RDEC} [1]	σ^{RDEC} [3]	$\sigma^{\mathrm{RDEC,A}}$ [4]	$\sigma^{\mathrm{RDEC,K}}$ [4]
5.5(3.2)	0.26	0.61	0.021

[1] A. Simon, PhD thesis ArXiv:1008.5317v1, 2010; Progress report, XXVII ICPEAC, Belfast, UK 2011
[2]A. V. Nefiodov, A.I. Mikhailov, and G. Plunien, Phyics Letters A, **346**, 158, 2005
[3] A.I. Mikhailov, I.A. Mikhailov, A.V. Nefiodov, G. Plunien, and G. Soff, Phys. Lett A **328**, 350, 2004

[4] E.A. Chernovskaya, O. Yu. Andreev, and L. N. Labzowsky ArXiv 1110.4791v1, 2011

Table: 1 Cross-section (in barn) for RDEC process F^{9+}+C, $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ contribution.

Experiment	Theory	
$\sigma^{\mathrm{RDEC}}_{(1s1s)}$ [1]	$\sigma^{\mathrm{RDEC,A}}_{(1s1s)}$ [this work]	$\sigma^{\mathrm{RDEC,K}}_{(1s1s)}$ [this work]
0.5(0.3)	0.94	0.035

Table: 2 Cross-section (in millibarn) for RDEC process Ar^{18+}+C, $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ contribution.

Experiment	Theory		
$\sigma^{\mathrm{RDEC}}_{(1s1s)}$ [2]	$\sigma^{\mathrm{RDEC}}_{(1s1s)}$ [3]	$\sigma^{\mathrm{RDEC,A}}_{(1s1s)}$ [this work]	$\sigma^{\mathrm{RDEC,K}}_{(1s1s)}$ [this work]
≤ 5.2	3.2	120	4.3

[1] A. Simon Progress report, XXVII ICPEAC, Belfast, UK 2011;
[2] Warczak et. al., Nucl. Instrum. Methods Phys. Res. Sect. B, **98**, 303, 1995
[3] Mikhailov, A.I., Mikhailov, I.A., Nefiodov, A.V. and Plunien, G. and Soff G., J Phys. B At. Mol. Opt. Phys., **39**, 3403, 2006

Table: 1 Radii (in pm) and areas of the cross-sections of the reaction volumes for one incident electron (in kilobarn) for the Ar target atom

$R_{\mathrm{T}}^{\mathrm{A}}$	S^{A}	$R_{\mathrm{T}}^{\mathrm{K}}$	S^{K}
188	617	3.0	1.4

Table: 2 Cross-section (in millibarn) for RDEC process U^{92+}+Ar, $\sigma_{(1s1s)}^{\mathrm{RDEC}}$ contribution.

Experiment	Theory		
$\sigma_{(1s1s)}^{\mathrm{RDEC}}$ [1]	$\sigma_{(1s1s)}^{\mathrm{RDEC}}$ [2]	$\sigma_{(1s1s)}^{\mathrm{RDEC,A}}$ [this work]	$\sigma_{(1s1s)}^{\mathrm{RDEC,K}}$ [this work]
< 10	2.5×10^{-2}	1.73	0.31×10^{-2}

[1] G. Bednarz, et. al., Nucl. Instr. Meth. Phys. Res. Sec B, **205**, 573, 2003

[2] A.I. Mikhailov, I.A. Mikhailov, A.V. Nefiodov, G. Plunien, and G. Soff, Phys. Lett A **328**, 350, 2004

Conclusion

- 1) We have developed the theory for the double electron capture within the Quantum Electrodynamics using the Line-Profile Approach;
- 2) We have calculated the cross section for different experiments;
- 3) The discrepancy between the experimental data and our theoretical results can be explained by the fact that we have not take into account the bound energy of the electron and use free electron model but in the experiments the electrons are quasifree;
- 4) We suggest to do experiments with free electrons.

Thank You for your attention!

Application of the Line-Profile Approach

The asymptotics ($r \to \infty$) of the Dirac wave function of the electron in continuum reads

$$\psi_{\epsilon jlm}(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} g_\epsilon(r)\, \Omega_{jlm}(\mathbf{n}) \\ i f_\epsilon(r)\, \Omega_{j,2j-l,m}(\mathbf{n}) \end{pmatrix} \tag{1}$$

$$g_\epsilon(r) = C_g \sqrt{\frac{\epsilon + m}{\pi p}} \cos(pr + \phi_g(r)) \tag{2}$$

$$f_\epsilon(r) = C_f \sqrt{\frac{\epsilon - m}{\pi p}} \sin(pr + \phi_f(r)) \tag{3},$$

where $|C_g| = |C_f| = 1$ and $\phi_g(r)$, $\phi_f(r)$ are the functions smoothly depending on r. The energy (ϵ) and momentum (p) of the electron are connected as $\epsilon^2 = m_e^2 c^4 + p^2$, where m_e is the electron mass, c is the speed of light. We confined the system into the sphere of radius R ($R \to \infty$), all spectrum becomes discrete.

Calculation of the amplitude

Amplitudes of our peocess are defined by Feigman graphs:

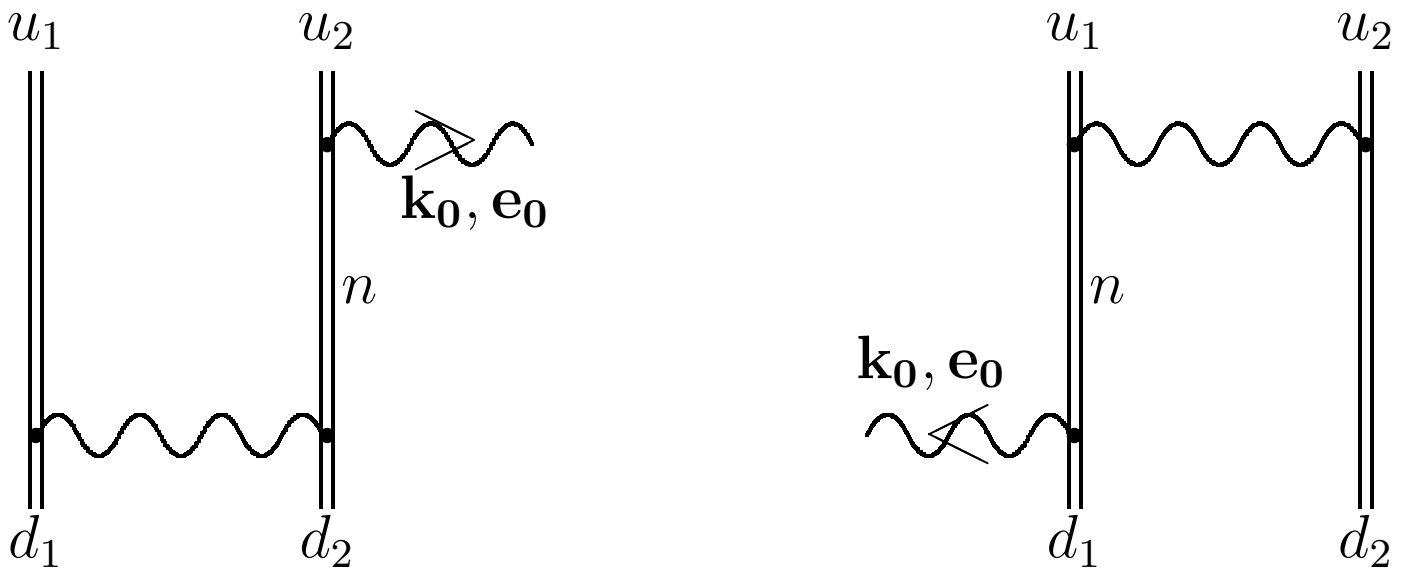


Figure: The Feynman graphs representing the first order interelectron interaction corrections to the process of electron recombination. The internal wavy line denotes the exchange by the photon between two electrons. The indices d_1, d_2 correspond to the initial one-electron states of a system; u_1, u_2 correspond to the final states. the index n corresponds to the intermediate one-electron states.

The amplitudes are proportional to the following expressions:

$$\xi_1 \sum_n \xi_{1,n} = \sum_n \frac{A_{u_2 n} I_{u_1 n d_1 d_2}}{\varepsilon_{u_1} + \varepsilon_n - \varepsilon_{d_1} - \varepsilon_{d_2}}\,, \tag{1}$$

$$\xi_2 \sum_n \xi_{2,n} = \sum_n \frac{I_{u_1 u_2 n d_2} A_{n d_1}}{\varepsilon_{u_1} + \varepsilon_{u_2} - \varepsilon_n - \varepsilon_{d_2}}\,. \tag{2}$$

where the photon emission matrix elements

$$A_{ud}^{(k,\lambda)} = \int d\mathbf{r}\, \bar{\psi}_u(\mathbf{r}) \gamma^\mu A_\mu^{(k,\lambda)}(\mathbf{k}) \psi_d(\mathbf{r}) \tag{3}$$

and the one-photon exchange matrix elements

$$I_{u_1 u_2 d_1 d_2}(\Omega) = \int d\mathbf{r}_1 d\mathbf{r}_2\, \bar{\psi}_{u_1}(\mathbf{r}_1) \bar{\psi}_{u_2}(\mathbf{r}_2) \gamma_1^{\mu_1} \gamma_2^{\mu_2} I_{\mu_1 \mu_2}(\Omega, r_{12}) \psi_{d_1}(\mathbf{r}_1) \psi_{d_2}(\mathbf{r}_2)\,. \tag{4}$$

The indices u_i, d_i designate one-electron Dirac states, Dirac matrices $\gamma_i^{\mu_i}$ act on the one-electron functions $\psi_{d_i}(\mathbf{r}_i)$, respectively.

Investigation of the behavior of the terms ξ with various values of intermediate electron states

For the states corresponding to continuum:

$$\psi_\epsilon(\mathbf{r}) \sim \frac{1}{R^{1/2}}\,, \qquad \Delta\epsilon \sim \frac{1}{R}\,, \tag{1}$$

where $\Delta\epsilon$ is the distance between two closest energy levels.
If n belongs to the descrete part of the spectrum: $\xi_{i,n}$ contain two wave functions of electrons from continuum spectrum, we can write

$$\xi_{i,n} \sim \frac{1}{R} \quad i = 1, 2\,. \tag{2}$$

If n belongs to continuous part, then in general case:

$$\xi_{i,n} \sim \frac{1}{R^2}\,, \quad i = 1, 2\,. \tag{3}$$

We define continuum electron $\tilde{e}$ with the energy

$$\epsilon_{\tilde{e}} = \epsilon_e + \epsilon_e - \epsilon_{1s}\,. \tag{1}$$

Three special cases when

$$\xi_{i,n} \sim \frac{1}{R^2}\,, \quad i = 1, 2 \tag{2}\,.$$

is violated:

1. $\varepsilon_n = \epsilon_e$

$$\xi_{1,n} \sim \frac{\log(R)}{R^2}\,. \tag{3}$$

2. $\varepsilon_n = \epsilon_{\tilde{e}}$, then $\xi_{1,n} \longrightarrow \infty$.
3. $\varepsilon_n \approx \epsilon_{\tilde{e}}$. If ε_n is the next state to $\epsilon_{\tilde{e}}$ (i.e., $\varepsilon_n = \epsilon_{\tilde{e}} \pm \Delta\epsilon$), then

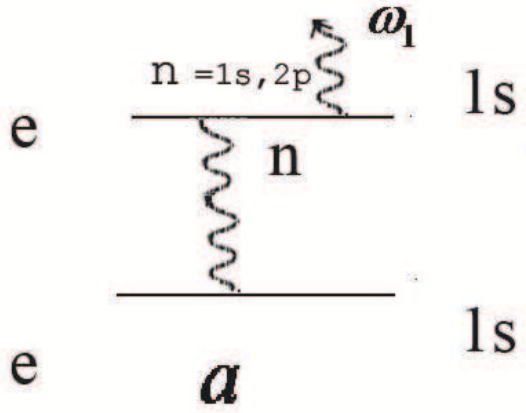
$$\xi_{1,n} \sim \frac{1}{R}\,. \tag{4}$$

Calculation of the amplitude

Ion is enclosed within a sphere of radius R, for the continuum states:

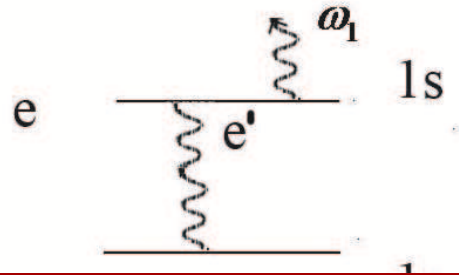
$$\psi_\epsilon(\mathbf{r}) \sim \frac{1}{R^{1/2}}, \qquad \Delta\epsilon \sim \frac{1}{R}, \qquad \Delta_k\epsilon \sim \frac{k}{R},$$

where $\Delta\epsilon$ is the distance between two closest energy levels;
$\Delta_k\epsilon$ is the distance between k closest energy levels.



Electron n belongs to discrete spectrum: asymptotic $\sim \frac{1}{R_e}$

n belongs to continuum part of the spectrum: two cases



$$\varepsilon_n + \varepsilon_{1s} = \varepsilon_e + \varepsilon_e :\sim \frac{\log(R)}{R^{\frac{3}{2}}}, \tag{1}$$

$$\varepsilon_n + \varepsilon_{1s} \approx \varepsilon_e + \varepsilon_e :\sim \frac{1}{R^{\frac{1}{2}}} \tag{2}$$

The description of the matrix V From QED perturbation theory we obtained the energy levels from the matrix V:

$$V(\omega) = V^{(0)} + \Delta V^{(1)}(\omega) + \Delta V^{(2)}(\omega) + \dots . \tag{1}$$

$V^{(0)}$ corresponds to zero order and consists of one-electron Dirac energies
$V^{(1)}$ includes the first order corrections related to α, such as self-energy (SE) and vacuum polarization (VP) corrections and one-photon exchange corrections.
The amplitude of the transition process from the initial state I to the final state F with emission of one photon with the frequency ω_0 can be written as

$$U_{I\to F} = (\Xi(\omega_0))_{\Phi_F \Phi_I} \ , \tag{2}$$

where $\Xi(\omega_0)$ is operator emission of the photon, Φ_I and Φ_F are the eigenvectors of the matrix $V(\omega)$ corresponding to the I and F states, respectively. The operator $\Xi(\omega_0)$ is evaluated with employment of the QED PT In zero order approximation this operator coincides with the photon emission operator ($A^{(k_0,\lambda_0)*}$). In this work we consider only the one-photon exchange corrections to the operator Ξ.

The description of the matrix V

The amplitude of the transition process from the initial state I to the final state F with emission of one photon with the frequency ω_0 can be written as

$$U_{I\to F} = \left(\Xi(\omega_0)\right)_{\Phi_F \Phi_I} \ , \tag{1}$$

where $\Xi(\omega_0)$ is operator emission of the photon, Φ_I and Φ_F are the eigenvectors of the matrix $V(\omega)$ corresponding to the I and F states, respectively. The operator $\Xi(\omega_0)$ is evaluated with employment of the QED PT In zero order approximation this operator coincides with the photon emission operator $(A^{(k_0,\lambda_0)*})$. In this work we consider only the one-photon exchange corrections to the operator Ξ.

set g

We choose the electron $\tilde{e}$ which is the state designed by the conclusion $\epsilon_{\tilde{e}} + \epsilon_{1s} = \epsilon_e + \epsilon_e$ where ϵ_e is the energy of the incident electron.

Then we determine the closest to the $\tilde{e}$ s,p-electrons: $(n-1)s_{\frac{1}{2}}$, $(n+1)s_{\frac{1}{2}}$ and $(n-1)p_{\frac{1}{2}}$, $(n+1)p_{\frac{1}{2}}$, $(n-1)p_{\frac{3}{2}}$, $(n+1)p_{\frac{3}{2}}$ respectively, where index n is not the principal quantum number it is the number of the state from B-spline spectrum and the value of the n corresponds to the $\tilde{e}$, indices $\frac{1}{2}$, $\frac{3}{2}$ corresponds to angular momentum $j = \frac{1}{2}$, $\frac{3}{2}$ respectively.

We use as $\tilde{e}$ p electron with different angular momentum $(j = \frac{1}{2}, j = \frac{3}{2})$

Finally we get the set g:

$1s_{\frac{1}{2}}$, $\tilde{e}p_{\frac{1}{2}}$, $(n-1)s_{\frac{1}{2}}$, $(n+1)s_{\frac{1}{2}}$, $(n-1)p_{\frac{1}{2}}$, $(n+1)p_{\frac{1}{2}}$, $(n-1)p_{\frac{3}{2}}$, $(n+1)p_{\frac{3}{2}}$

It is convenient to write the matrix V in a block form

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = \begin{bmatrix} V_{11}^{(0)} + \Delta V_{11} & \Delta V_{12} \\ \Delta V_{21} & V_{22}^{(0)} + \Delta V_{22} \end{bmatrix}, \qquad (1)$$

where the block V_{11} composes entirely the states from the set g and the block V_{22} does not contain states from the set g. The blocks V_{12} and V_{21} are made up with one configuration from the set g and another one not included in the set g. The matrix V_{11} can be diagonalized numerically (non-perturbatively)

$$V_{11}^{\mathrm{diag}} = B^t V_{11} B\,, \qquad (2)$$

where B is an orthogonal matrix, B^t is the transposed matrix.

The eigenvector of the matrix V can be written as

$$\Phi_{n_g} = \sum_{k_g \in g} B_{k_g n_g} \Psi^{(0)}_{k_g} + \sum_{\substack{k \notin g \\ l_g \in g}} (\Delta V_{21})_{k l_g} \frac{B_{l_g n_g}}{E_{n_g} - E^{(0)}_k} \Psi^{(0)}_k + \dots , \qquad (1)$$

where E_{n_g} are the eigenvalues of the matrix V_{11} and $E^{(0)}_k$ are the sums of the Dirac energies. The Dirac energies are assumed to include the rest energy of an electron $m_e c^2$. The functions $\Psi^{(0)}$ are the two-electron functions in the j–j coupling scheme. The indices k_g, l_g run over configurations from the set g, while the index k runs over configurations not included in the set g, *i.e.*, over all the other two-electron configurations. The first term in the right-hand side of the expression (1) can be considered as the zero order of the applied perturbation theory, the second term corresponds to the first order.

Cross-section (in millibarn) for RDEC process $Cr^{24+} + He/N_2$, $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ contribution.

For He target with $R^A_T = 105pm$ we get $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ =10.04 mb

For N_2 target with R^A_T =92 pm: $\sigma^{\mathrm{RDEC}}_{(1s1s)}$ =53.61 mb

Experimental analasis are still in progress.