

FUNDAMENTAL CONSTANTS OF MODERN COSMOLOGY

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Present matter content of the Universe

(in terms of the critical density)

$$\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G}, \quad \Omega_i = \frac{\rho_i}{\rho_{\text{crit}}})$$

$$\approx 10^{-29} \text{ g/cm}^3$$

1. Baryons (p, n)
and leptons (e)

$(4.5 \pm 0.5) \%$

No antimatter

2. Photons (γ)

$5 \cdot 10^{-5}$

3. Neutrinos
(ν_e, ν_μ, ν_τ)

$\leq 1\%$

($\sum_i m_{\nu_i} < 0.6 \text{ eV}$)

$\sum_i m_{\nu_i} (\text{eV}) = 94 \Omega_\nu h^2$

4. Non-relativistic
non-baryonic
dark matter

$\approx 23\%$

5. Dark energy

$\approx 72\%$

} Not
known
from
laboratory
experiments

$$T_\gamma = 2.72548 \pm 0.00057 \text{ K}$$

(arXiv: 0911.1955)

SCM: Λ CDM + ($\mathcal{H}=0$) + ($n_s=1$
adiabatic)

$\sim 10\%$ accuracy

4 main fundamental constants
and related theories:

$$1. \frac{k^3 \epsilon_\Lambda}{c^5 M_{Pl}^4} = \frac{G^2 k \epsilon_\Lambda}{c^7} = 1.25 \cdot 10^{-123} \cdot \frac{\Omega_\Lambda}{0.7} \cdot \left(\frac{H_0}{70}\right)^2$$

Theory of a cosmological constant
(dark energy)

$$2. \frac{\epsilon_b}{\epsilon_m} = 0.156 \cdot \frac{\Omega_b R^2}{0.023} \cdot \left(\frac{70}{H_0}\right)^2 \cdot \frac{0.3}{\Omega_m} \quad R = \frac{H_0}{100}$$

$$\epsilon_m = \epsilon_b + \epsilon_{CDM}$$

Theory of dark non-baryonic matter

$$3. \frac{n_b}{n_\gamma} = 6.25 \cdot 10^{-10} \cdot \frac{\Omega_b R^2}{0.023} \cdot \left(\frac{2.73}{T_\gamma}\right)^3$$

Theory of baryogenesis

$$4. \Phi_0 = 2.94 \cdot 10^{-5} \cdot \left(\frac{\Delta_B^2}{2.4 \cdot 10^{-9}}\right)^{1/2}$$

$$\langle \Phi_{in}^2 \rangle = \Phi_0^2 \int \frac{d^3 k}{k} \text{ at the MD stage}$$

Theory of initial conditions - inflation

Table 6. Cosmological Parameter Summary

Description	Symbol	WMAP-only	WMAP+BAO+SN
Parameters for Standard Λ CDM Model ^a			
Age of universe	t_0	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr
Hubble constant	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
Baryon density	Ω_b	0.0441 ± 0.0030	0.0462 ± 0.0015
Physical baryon density	$\Omega_b h^2$	0.02273 ± 0.00062	0.02265 ± 0.00059
Dark matter density	Ω_c	0.214 ± 0.027	0.233 ± 0.013
Physical dark matter density	$\Omega_c h^2$	0.1099 ± 0.0062	0.1143 ± 0.0034
Dark energy density	Ω_Λ	0.742 ± 0.030	0.721 ± 0.015
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc ⁻¹ ^b	$\Delta_{\mathcal{R}}^2$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.796 ± 0.036	0.817 ± 0.026
$l(l+1)C_{220}^{TT}/2\pi$	C_{220}	5756 ± 42 μK^2	5748 ± 41 μK^2
Scalar spectral index	n_s	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
Redshift of matter-radiation equality	z_{eq}	3176^{+151}_{-150}	3280^{+88}_{-89}
Angular diameter distance to matter-radiation eq. ^c	$d_A(z_{eq})$	14279^{+186}_{-189} Mpc	14172^{+141}_{-139} Mpc
Redshift of decoupling	z_*	1090.51 ± 0.95	$1091.00^{+0.72}_{-0.73}$
Age at decoupling	t_*	380081^{+5843}_{-5841} yr	375938^{+3148}_{-3115} yr
Angular diameter distance to decoupling ^{c,d}	$d_A(z_*)$	14115^{+188}_{-191} Mpc	14006^{+142}_{-141} Mpc
Sound horizon at decoupling ^d	$r_s(z_*)$	146.8 ± 1.8 Mpc	145.6 ± 1.2 Mpc
Acoustic scale at decoupling ^d	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	$302.11^{+0.84}_{-0.82}$
Reionization optical depth	τ	0.087 ± 0.017	0.084 ± 0.016
Redshift of reionization	z_{reion}	11.0 ± 1.4	10.8 ± 1.4
Age at reionization	t_{reion}	427^{+88}_{-65} Myr	432^{+90}_{-67} Myr
Parameters for Extended Models ^e			
Total density ^f	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064
Equation of state ^g	w	$-1.06^{+0.41}_{-0.42}$	$-0.972^{+0.061}_{-0.060}$
Tensor to scalar ratio, $k_0 = 0.002$ Mpc ⁻¹ ^{b,h}	r	< 0.43 (95% CL)	< 0.20 (95% CL)
Running of spectral index, $k_0 = 0.002$ Mpc ⁻¹ ^{b,i}	$dn_s/d \ln k$	-0.037 ± 0.028	$-0.032^{+0.021}_{-0.020}$
Neutrino density ^j	$\Omega_\nu h^2$	< 0.014 (95% CL)	< 0.0065 (95% CL)
Neutrino mass ^j	$\sum m_\nu$	< 1.3 eV (95% CL)	< 0.61 eV (95% CL)
Number of light neutrino families ^k	N_{eff}	> 2.3 (95% CL)	4.4 ± 1.5

^aThe parameters reported in the first section assume the 6 parameter Λ CDM model, first using WMAP data only (Dunkley et al. 2008), then using WMAP+BAO+SN data (Komatsu et al. 2008).

^b $k = 0.002$ Mpc⁻¹ $\longleftrightarrow l_{eff} \approx 30$.

^cComoving angular diameter distance.

Comments about "cosmic coincidences"
for Λ

1. Why small? Not known why so small, but all known dimensionless densities are very small

$$\rho_E \sim \rho_{\text{water}}^{4/3} \sim m_\nu^4$$

2. Why now? Not an independent problem. Reduces to the first problem (plus relations between other fundamental constants) once "now" is defined in an objective way.

It is natural to use (weak) anthropic principle to define "now". However, in practice, very remote arguments are used.

Example. Let us, following Dicke, define $t_0 \sim t_{\text{active life main sequence star}} \sim t_{ge} \cdot \left(\frac{M_{ge}}{m_p}\right)^3$

Then the "second coincidence problem" is reduced to the first one because of the empirical relation

$$\rho_E \sim \left(\frac{m_p}{M_{ge}}\right)^6$$

One constant — one problem ("coincidence")

DARK ENERGY

Dark matter and dark energy -
two entities in the Universe seen
by their gravitational interaction
only

DM - non-relativistic
gravitationally clustered

DE - relativistic
unclustered

Definition through equations

I. DM - through the (generalized)
Poisson equation

$$\frac{\Delta \Phi}{a^2} = 4\pi G (\rho - \rho_0(t))$$

$\Phi(\vec{r})$ is measured using the motion
of 'test particles' in it

- a) Stars in galaxies → rotation curves
- b) Galaxies → peculiar velocities
- c) Hot gas in clusters → X-ray profiles
- d) Photons → gravitational
lensing
(strong and weak)

Result: DM is non-relativistic ($p \ll E$), collisionless, and has the same spatial distribution as visible matter for $L \gtrsim 1 \text{ Mpc}$

$$b \approx 1, \quad \Omega_{m, \text{tot}} = 0.28 \pm 0.03 \quad (26)$$

$$\Omega_{\text{bar}} = 0.046 \pm 0.003$$

II. DE - through relativistic gravitational field equations in the Einsteinian form

$$\frac{1}{8\pi G} \left(R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R \right) = \underbrace{T_{\mu}^{\nu}(\text{vis}) + T_{\mu}^{\nu}(\text{DM})}_{\text{approximately dust-like}} + T_{\mu}^{\nu}(\text{DE})$$

$G = G_0 = \text{const}$ - the Newton gravitational constant measured in laboratory

$$T_{\mu(\text{DE}) ; \nu}^{\nu} = 0 \quad (\text{in the absence of decay of DM particles into DE})$$

Applications of the definition of $T_{\mu}^{\nu}(\partial E)$

1. To the FRW background

$$\epsilon_{\partial E} \equiv T_{00}^0(\partial E) = \epsilon(z)$$

$$z = z(t) = \frac{a_0}{a(t)} - 1$$

$$p_{\partial E} = -T_{\perp\perp}^{\perp}(\partial E) = p_{\partial E}(z)$$

(no summation on $\perp$)

$$\dot{\epsilon}_{\partial E} + 3H(\epsilon_{\partial E} + p_{\partial E}) = 0$$

$$H \equiv \frac{\dot{a}}{a}$$

1 function

2. To the evolution of density
perturbations in the matter component
(baryons + DM) at small scales

$\left(\frac{\delta p}{p}\right)_m(z)$ - second observational function

Main up-to-date result

In the zero approximation ($\sim 10\%$ accuracy)

$$\epsilon_{\partial E}(z) = \epsilon_0 = \text{const}$$

$$p_{\partial E} = -\epsilon_0, \quad T_{\mu}^{\nu}(\partial E) = \epsilon_0 \delta_{\mu}^{\nu}$$

$$\Omega_{\partial E} = 1 - \Omega_m$$

$$\rho_0 = \frac{\epsilon_0}{c^2} = 6.44 \cdot 10^{-30} \frac{\Omega_{\partial E}}{0.7} \cdot \left(\frac{H_0}{70}\right)^2 \text{ g cm}^{-3}$$

$$\frac{G^2 k \epsilon_0}{c^7} = 1.25 \cdot 10^{-123} \cdot \underbrace{\left(\frac{\Omega_{\partial E}}{0.7} \cdot \left(\frac{H_0}{70}\right)^2\right)}_{(\dots \downarrow \dots)}$$

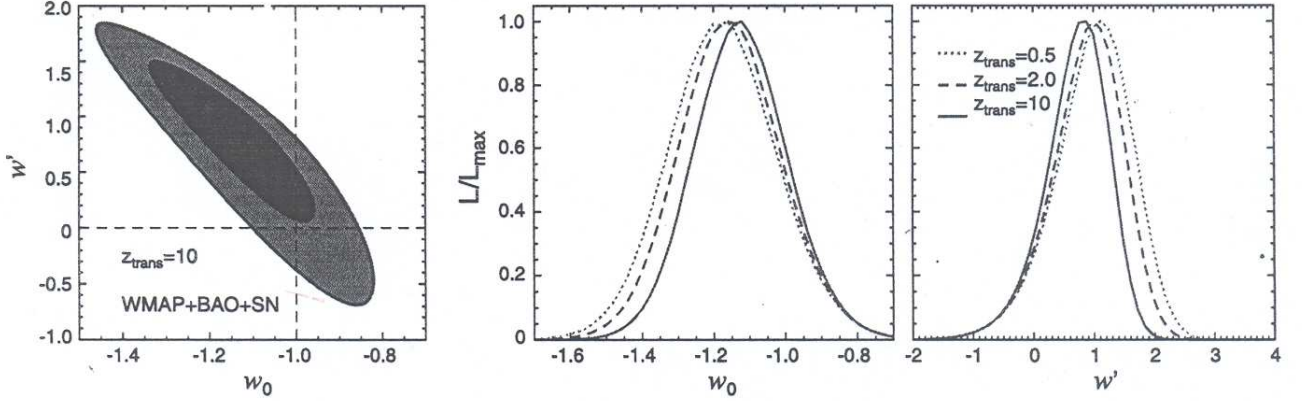


FIG. 14.— Constraint on models of time-dependent dark energy equation of state, $w(z)$ (Eq. [70]), derived from the WMAP distance priors (l_A , R , and z_*) combined with the BAO and SN distance data (§ 5.4.2). There are three parameters: w_0 is the value of w at the present epoch, $w_0 \equiv w(z=0)$, w' is the first derivative of w with respect to z at $z=0$, $w' \equiv dw/dz|_{z=0}$, and z_{trans} is the transition redshift above which $w(z)$ approaches to -1 . Here, we assume flatness of the universe, $\Omega_k = 0$. (Left) Joint two-dimensional marginalized distribution of w_0 and w' for $z_{\text{trans}} = 10$. The constraints are similar for the other values of z_{trans} . The contours show the $\Delta\chi^2_{\text{total}} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\text{total}} = 6.17$ (95.4% CL). (Middle) One-dimensional marginalized distribution of w' for $z_{\text{trans}} = 0.5$ (dotted), 2 (dashed), and 10 (solid). The constraints are similar for all z_{trans} . We do not find evidence for the evolution of dark energy.

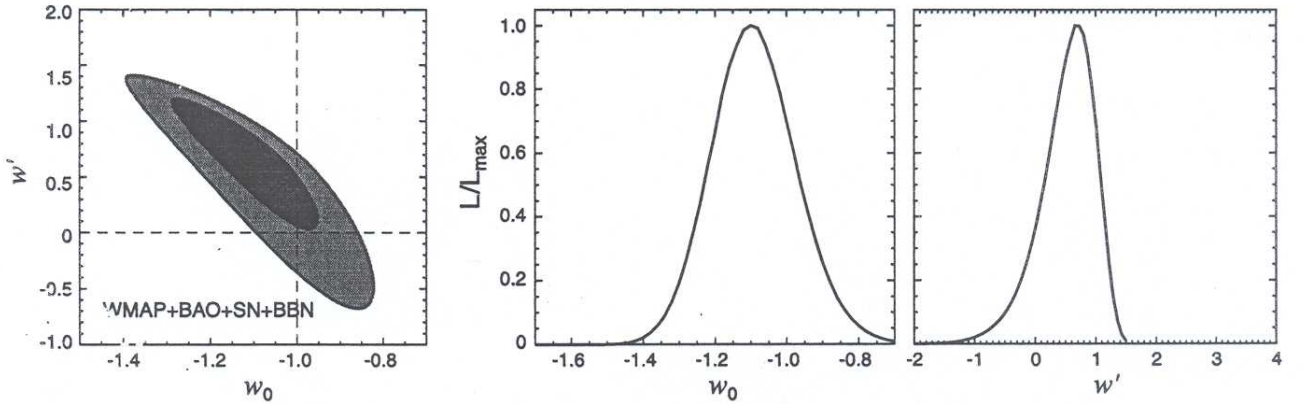


FIG. 15.— Constraint on the linear evolution model of dark energy equation of state, $w(z) = w_0 + w'z/(1+z)$, derived from the WMAP distance priors (l_A , R , and z_*) combined with the BAO and SN distance data as well as the Big Bang Nucleosynthesis (BBN) prior (Eq. [71]). Here, we assume flatness of the universe, $\Omega_k = 0$. (Left) Joint two-dimensional marginalized distribution of w_0 and w' . The contours show the $\Delta\chi^2_{\text{total}} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\text{total}} = 6.17$ (95.4% CL). (Middle) One-dimensional marginalized distribution of w_0 . (Middle) One-dimensional marginalized distribution of w' . We do not find evidence for the evolution of dark energy. Note that Linder (2003) defines w' as the derivative of w at $z=1$, whereas we define it as the derivative at $z=0$. They are related by $w'_{\text{Linder}} = 0.5w'_{\text{WMAP}}$.

The 95% limit on w_0 for $z_{\text{trans}} = 10$ is $-0.38 < 1 + w_0 < 0.14^{46}$, whose upper limit is surprisingly close to the limit for a constant w model in a flat universe, $-0.97 < 1 + w < 0.142$ (95% CL). Therefore, we have a fairly robust upper limit on the present-day value of the equation of state, $1 + w_0 < 0.14$. (This statement is, however, true only for a flat universe.) On the other hand, the lower limit has weakened significantly – by a factor of about four. Our results are consistent with the previous work using the WMAP 3-year data (see Wang & Mukherjee 2007; Wright 2007, for recent work and references therein). We find that our upper limit on w_0 and lower limit on w' are better than the previous work by a factor of ~ 2 .

Alternatively, one may take the linear form, $w(a) = w_0 + (1-a)w_a$, literally and extend it to an arbitrarily high redshift. This can result in an undesirable situation

⁴⁶ The 68% intervals are $w_0 = -1.12 \pm 0.13$ and $w' = 0.70 \pm 0.53$ (WMAP+BAO+SN; $\Omega_k = 0$).

in which the dark energy is as important as the radiation density at the epoch of the Big Bang Nucleosynthesis (BBN); however, one can constrain such a scenario severely using the limit on the expansion rate from BBN (Steigman 2007). We follow Wright (2007) to adopt a Gaussian prior on

$$\sqrt{1 + \frac{\Omega_\Lambda(1+z_{\text{BBN}})^{3[1+w_{\text{eff}}(z_{\text{BBN}})]}}{\Omega_m(1+z_{\text{BBN}})^3 + \Omega_r(1+z_{\text{BBN}})^4 + \Omega_k(1+z_{\text{BBN}})^2}} = 0.942 \pm 0.030, \quad (71)$$

where we have kept Ω_m and Ω_k for definiteness, but they are entirely negligible compared to the radiation density at the redshift of BBN, $z_{\text{BBN}} = 10^9$. Figure 15 shows the constraint on w_0 and w' for the linear evolution model derived from the WMAP distance priors, the BAO and SN data, and the BBN prior. The 95% limit on w_0 is

Is inflation falsifiable?

Any concrete model is falsifiable, and many of them have been already falsified.

Fortunately, there remain many viable models, and the first 3 are among them.

1. "Geometrical inflation" (A.S., 1980)

$R + \frac{R^2}{6M^2}$ model

$$\frac{M}{M_{\text{Pl}}} = 3.0 \cdot 10^{-6} \cdot \frac{50}{N} \quad t_{\text{reh}} \approx 10^{-26} \text{ s}$$

$$T_{\text{reh}} \sim 10^{-9} M_{\text{Pl}} = 10^{10} \text{ GeV}$$

$$n_s - 1 = -\frac{2}{N} = -0.04 \cdot \frac{50}{N}, \quad r = \frac{12}{N^2} = 5 \cdot 10^{-3} \left(\frac{50}{N}\right)^2$$

2. Inflation without phase transition

(Linde, 1983 - though the mathematics was known from A.S., 1978 where this model was used for the bouncing scenario)

$$V = \frac{m^2 \phi^2}{2}, \quad \frac{m}{M_{\text{Pl}}} = 1.4 \cdot 10^{-6}$$

$$n_s - 1 = \frac{2}{N} = -0.04 \cdot \frac{50}{N}, \quad r = \frac{8}{N} = 0.16 \cdot \frac{50}{N}$$

3. Inflation with phase transition
(Linde, Albrecht & Steinhardt - 1982)

$$V = V_0 - \frac{\lambda \phi^4}{4}, \quad \lambda = 2.9 \cdot 10^{-13} \cdot (50/N)^2$$

$$n_s - 1 = \frac{2}{N} = -0.06 \cdot \frac{50}{N}, \quad r \ll 1$$

CONCLUSIONS

1. Present cosmology requires 4 new fundamental dimensionless constants to describe observational data.
2. In the case of present dark energy:
 - a) no statistically significant deviation from an exact cosmological constant,
 - b) one constant is sufficient to describe its properties,
 - c) no more than one new "coincidence problem".
3. In the case of inflation driven by primordial dark energy:
a number of inflationary models having only one dimensionless adjustable parameter can explain all presently existing data.