

Поправки порядка $(Z\alpha)^n$ в процессах
образования лептонов и фотонов
на ускорителях
с тяжелыми ядрами RHIC и LHC

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ПЛАН:

1. Введение

2. Поправки порядка $(Z\alpha)^n$ в образовании e^+e^- пар при соударении тяжёлых релятивистских ядер

3. Поправки порядка $(Z\alpha)^n$ в образовании $\mu^+\mu^-$ пар при соударении тяжёлых релятивистских ядер

4. Поправки порядка $(Z\alpha)^n$ в обычном тормозном излучении ядер

5. Большой вклад виртуального дельбрюковского рассеяния в тормозное излучение ядер

Эффекты сильного поля рассмотрены во многих учебниках и обзорах:

Heitler “The quantum theory of radiation,” 1954;

Greiner, Müller, Rafelsky “QED of strong fields,” 1985

Baur, Hencken, Trautmann. Phys. Rep. 453, 1 (2007)

Baltz et al Phys. Rep. 458, 1 (2008)

Этот доклад составлен в основном по материалам следующих статей::

Ivanov, Schiller, Serbo. *Phys. Lett. B* **454**, 155 (1999);

Lee, Milstein, Serbo. *Phys. Rev. A* **65**, 022102 (2002);

Ginzburg, Jentschura, Serbo. *Phys. Lett. B* **658**, 125 (2008);

Ginzburg, Jentschura, Serbo. *Eur. Phys. J. C* **54**, 267 (2008).

Jentschura, Hencken, Serbo. *Eur. Phys. J. C* **58**, 281 (2008)

Jentschura, Serbo. *Eur. Phys. J. C* **64**, 309 (2009).

Поправки $\sim (Z\alpha)^n$ в электромагнитных процессах на встречных пучках тяжелых ультра-релятивистских ядер

Цитата из обзора:

Baur, Hencken, Trautmann, Phys. Rep. 453 (2007) 1:

“Higher order effects were studied in the last decades, mainly by the Russian schools”.

1. Введение

For the RHIC and LHC colliders, the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given as follows:

Collider	Z	γ
RHIC, Au-Au	79	108
LHC, Pb-Pb	82	3000

Only a few EM processes are related to Fundamental Physics, for example:

G.L. Kotkin, E.A. Kuraev, A. Schiller, V.G. Serbo. Production of parapositronium and orthopositronium at relativistic heavy ion colliders. *Physical Review C* v. C 59 (1999) p. 2734

I.F. Ginzburg, U.D. Jenschura, S.G. Karshenboim, F. Krauss, V.G. Serbo, G. Soff. Production of bound $\mu^+\mu^-$ system in relativistic heavy ion collisions. *Physical Review C* v. C 58 (1998) p. 3565

but

some of EM processes are of great importance mainly for two reasons:

they are **dangerous** or they are **useful**.

Two examples:

1) The e^+e^- pair production. The number of the produced electrons is so huge that some of them can be captured by nuclei, that immediately leads to loss of these nuclei from the beam. Thus, this very process is determined mainly **the life time of the beam and a possible luminosity of a machine.**

2) Coherent bremsstrahlung, not ordinary bremsstrahlung

$$Z_1 Z_2 \rightarrow Z_1 Z_2 \gamma$$

but coherent one! The number of the produced photons at the RHIC is so huge in the region of the infrared light, that this process can be used for **monitoring beam collisions**:

*R. Engel, A. Schiller, V.G. Serbo. A new possibility to monitor collisions of relativistic heavy ions at LHC and RHIC, Particle Accelerators **56**, 1 (1996)*

D. Trbojevic, D. Gasner, W. MacKay, G. McIntyre, S. Peggs, V. Serbo, G. Kotkin. Experimental set-up to measure coherent bremsstrahlung and beam profiles in RHIC. 8th European Particle Accelerator Conference (EPAC 2002, 3–7 June, 2002, Paris) p. 1986

Similar there are ideas to use multiple pair production as a possible trigger for ultra-peripheral collisions at ALICE. A good knowledge of the multiple pair production cross section is needed for this.

It means that various EM processes have to be estimated (their cross sections and distributions) **not to miss** something interesting or dangerous.

How **strong** is nuclear field?

The typical electric field of nucleus is of the order of

$$\mathcal{E} \sim \frac{Ze}{\rho^2} \gamma = \gamma Z\alpha \mathcal{E}_{\text{Schwinger}} \text{ at } \rho = \frac{\hbar}{m_e c},$$

$$\mathcal{E}_{\text{Schwinger}} = \frac{m_e^2 c^3}{e\hbar} = 1.3 \cdot 10^{16} \frac{\text{V}}{\text{cm}},$$

therefore,

$$\frac{\mathcal{E}}{\mathcal{E}_{\text{Schwinger}}} \sim \mathbf{60} \text{ for RHIC and } \sim \mathbf{1800} \text{ for LHC},$$

but interaction time is **very short**.

As a result, one can use **Perturbation Theory**, but the perturbation parameter $Z\alpha \approx \mathbf{0.6}$ for Au-Au and Pb-Pb collisions.

2. Поправки порядка $(Z\alpha)^n$ в образовании e^+e^- пар

The cross section of **one pair production in the Born approximation** (described by Feynman diagram of Fig. 1)

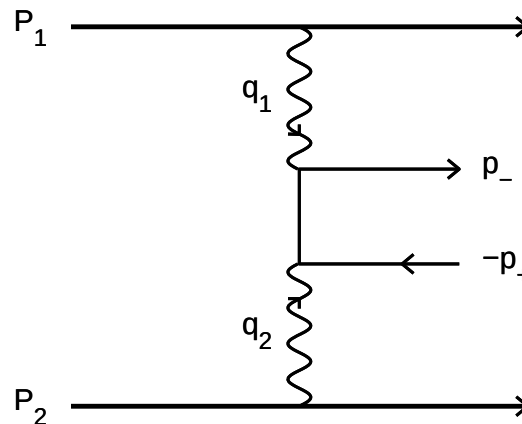


Fig. 1

with **two photon production** was obtained many years ago by

Landau, Lifshitz (1934) and *Racah (1937)*:

$$\sigma_{\text{Born}} = \sigma_0 \left[L^3 - 2.198 L^2 + 3.821 L - 1.632 \right],$$

where

$$\sigma_0 = \frac{28}{27\pi} \frac{(Z_1 \alpha Z_2 \alpha)^2}{m_e^2}, \quad \alpha = \frac{1}{137}, \quad L = \ln(\gamma_1 \gamma_2) \gtrsim 10,$$

m_e is the electron mass and $c = 1, \hbar = 1$.

Since **the parameter $Z\alpha$ is not small**

the **whole** series in $Z\alpha$ **has to be summed**

to obtain the cross section with sufficient accuracy.

Fortunately, there is an **important small parameter**

$$\frac{1}{L} < 0.11, \quad L = \ln(\gamma^2),$$

and therefore, in some (**but not in all!**) cases it is sufficient to calculate the corrections

in **the leading logarithmic approximation (LLA) only**.

In the literature, there were a lot of controversial and incorrect statements in papers devoted to this subject. Some corresponding references and **critical remarks** can be found in

Ivanov, Schiller, Serbo. Phys. Lett. B 454 (1999) 155;

Lee, Milstein. Phys. Rev. A 61 (2000) 032103;

Phys. Rev. A 64 (2001) 032106 (2001);

Lee, Milstein, Serbo. Phys. Rev. A 65, 022102 (2002);

Aste, Baur, Hencken, Trautmann, Scharf. EPJ C23 (2002) 545;

Jentschura, Hencken, Serbo. EPJ C58 (2008) 281.

The exact cross section for **one pair production** σ_1 can be written in the form

$$\sigma_1 = \sigma_{\text{Born}} + \sigma_{\text{Coul}} + \sigma_{\text{unit}},$$

where **two different types of SF-corrections** have been distinguished.

2.1. Results for the $(Z\alpha)^n$ -corrections

The **Coulomb** corrections σ_{Coul} correspond to **multi-photon exchanges** of the produced $e^\pm$ with the nuclei:

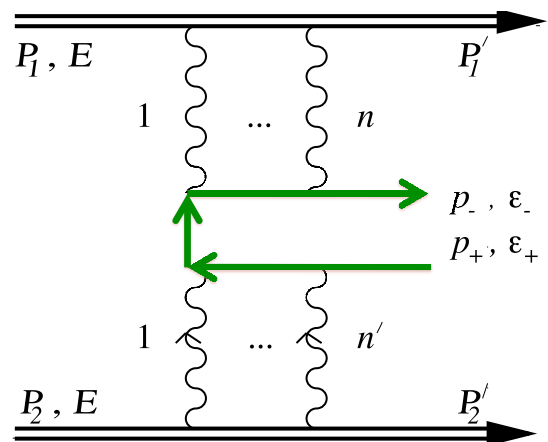


Fig. 2

$$\sigma_{\text{Coul}} = -A(Z\alpha) [L^2 - B(Z\alpha) L] \sigma_0 ,$$

where the leading coefficient

$$A(Z\alpha) = 6f(Z\alpha) = 6(Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)} \approx 1.9$$

was calculated ten years ago by *D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155* and next-to-leading coefficient

$$B(Z\alpha) \approx 5.5$$

was calculated recently by *R.N. Lee, A.I. Milstein. hep-ph 0903.0235*.

It was also shown by *ISS* that **the Coulomb corrections disappear** for **large transverse momenta of the produced leptons**, at $p_{\pm\perp} \gg m_e$.

The **unitarity** corrections σ_{unit} correspond to the exchange of the virtual light-by-light blocks between the nuclei (Fig. 3)

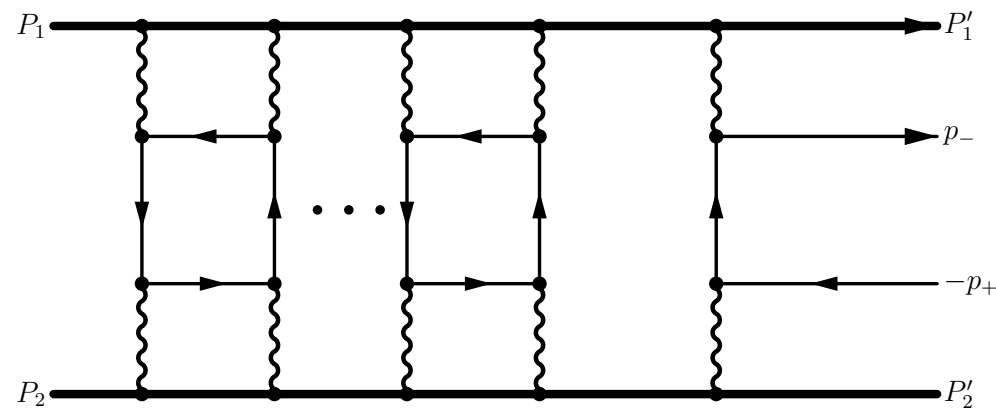


Fig. 3

They were calculated by *R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102*

and updated by

U.D. Jentschura, K. Hencken, V.G. Serbo. EPJ C58 (2008) 281.

It was found that **the Coulomb corrections are about 10 %** while **the unitarity corrections are about two times smaller:**

Coulomb and unitarity corrections to the e^+e^- pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$	
RHIC, Au-Au	−10%	−5.0%	
LHC, Pb-Pb	−9.4%	−4.0%	

Multiple production of e^+e^- pairs

$$Z + Z \rightarrow Z + Z + n(e^+e^-)$$

If $Z\alpha$ is small, the corresponding cross section grows as L^n :

$$\sigma_n = C_n \frac{(Z\alpha)^{4n}}{m_e^2} L^n, \quad n \geq 2,$$

$$C_2 = 2.21, \quad C_3 = 0.443, \quad C_4 = 0.119.$$

R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102

U.D. Jentschura, K. Hencken, V.G. Serbo (EPJ C58 (2008) 281)

For large values of $Z\alpha$ there are only numerical calculations of σ_n for a particular values of γ

A. Alscher, K. Henken, D. Trautman, G. Baur. Phys. Rev. C 59 (1999) 811

2.2. Unitarity corrections and σ_n

Due to $Z_1 Z_2 \alpha \gg 1$ for $\gamma \gg 1$ it is possible to treat the nuclei as **sources of the external field** and calculate the probability of n -pair production $P_n(\rho)$ in collision of two nuclei at a given impact parameter ρ .

The cross section is then found as:

$$\sigma_n = \int P_n(\rho) d^2\rho.$$

What we know about

$$P_n(\rho) ?$$

It was realized many years ago that in the Born approximation

$$P_1(\rho) \sim (Z\alpha)^4 L \quad \text{at } \rho \sim 1/m_e$$

and, therefore, this probability **can be greater than 1**
Baur. Phys. Rev. A 42 (1990) 5736.

It means:

- 1) that one **should take into account the unitarity corrections**,
which come from **the unitarity requirement for the S -matrix**;
- 2) that the cross section for multiple pair production should be large enough.

It was argued in papers

Baur. Phys. Rev. D 41, 3535 (1990); Roades-Brown and Wenes. Phys. Rev. A 44, 330 (1991); Best, Greiner, and Soff. Phys. Rev. A 46, 261 (1992); Henken, Trautman, and Baur. Phys. Rev. A 51, 998 (1995)

that the factorization of the multiple pair production probability is valid with a good accuracy given by **the Poisson distribution**:

$$P_n(\rho) = \frac{[\bar{n}(\rho)]^n}{n!} e^{-\bar{n}(\rho)},$$

where $\bar{n}(\rho)$ is **the average number of pairs**.

Recently, it was proved in paper

Bartoš, Gevorkyan, Kuraev, Nikolaev. Phys. Lett. B 538 (2002) 45

by a direct summation of the Feynman diagram in LLA.

The unitarity requirement is fulfilled by the Poisson distribution, whose sum over n gives one.

The probability for producing one pair, given in perturbation theory by $\bar{n}_e(\rho)$, should be modified to read $\bar{n}_e(\rho) \cdot \exp[-\bar{n}_e(\rho)]$.

For the one-pair production it corresponds to replacement:

$$\sigma_{e^+e^-} = \int \bar{n}_e(\rho) d^2\rho \quad \rightarrow \quad \sigma_{e^+e^-} + \sigma_{e^+e^-}^{\text{unit}} = \int \bar{n}_e(\rho) e^{-\bar{n}_e(\rho)} d^2\rho,$$

where

$$\sigma_{e^+e^-}^{\text{unit}} = - \int \bar{n}_e(\rho) [1 - e^{-\bar{n}_e(\rho)}] d^2\rho$$

is the **unitarity** correction.

The main contribution to $\sigma_{e^+e^-}$ comes from $\rho \gg 1/m_e$,
But, the main contribution to $\sigma_{e^+e^-}^{\text{unit}}$ comes from $\rho \sim 1/m_e$.

The function $\bar{n}_e(\rho)$ is a very important quantity for the evaluation of unitarity corrections.

It was found for $\gamma \gg 1$ in closed form
(taken into account $(Z\alpha)^n$ terms exactly) by

Baltz, McLerran. Phys. Rev. C 58 (1998) 1679;

Segev, Wells. Phys. Rev. A 57 (1998) 1849;

Baltz, Gelis, McLerran, Peshier. Nucl. Phys. A 695 (2001) 395 .

The problem of its **proper regularization** was solved by

Lee, Milstein. Phys. Rev. A 64 (2001) 032106.

But! The obtained close form for $\bar{n}_e(\rho)$ is, in fact, a **nine-fold integral**
and its calculation is **very laborious**.

A simpler approximate expression for $\bar{n}_e(\rho)$ is very desirable.

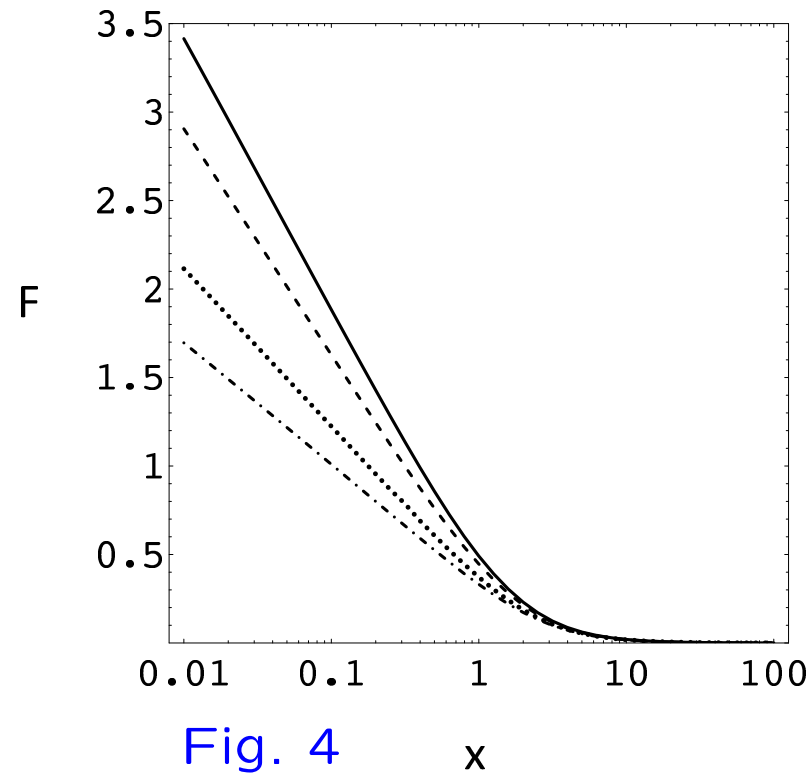
The functional form of this function in the region of interest reads

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4 F(x, Z) [L - G(x, Z)] , \quad L = \ln(\gamma^2), \quad x = m_e \rho .$$

The simple analytical expressions for functions $F(x, Z)$ and $G(x, Z)$ is obtained by [Lee, Milstein, Serbo \(2002\)](#) only at large values of the impact parameters, $\rho \gg 1/m_e$.

On the other hand, for the calculation of the unitarity corrections we need $F(x, Z)$ and $G(x, Z)$ in the range $\rho \sim 1/m_e$.

In the recent paper by [Lee, Milstein nucl-th/0610008](#): detailed consideration of the function $F(x, Z)$ including **tables and compact integral form — (“only” a five-fold integral).**



As an example, in Fig. 4 it is shown the function $F(x, Z)$ from [Lee, Milstein](#) paper, for $Z = 92$ (dash-dotted line) , $Z = 79$ (dotted line), $Z = 47$ (dashed line), and the Born approximation (solid line).

Using some numerical calculations for the function $\bar{n}_e(\rho, \gamma, Z)$, **we find a simple approximation** (Jentschura, Hencken, Serbo (*EPJ C*58 (2008) 281))

$$G(x, Z) \approx 1.5 \ln(x + 1.4) + 1.9.$$

As a result, the approximate expression

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4 F(x, Z) [L - 1.5 \ln(x + 1.4) - 1.9],$$

$$L = \ln(\gamma^2), \quad x = m_e \rho$$

with the function $F(x, Z)$ from the paper of Lee, Milstein (2006) can be used for calculation of unitarity corrections with an accuracy on the order of few percents.

3. Поправки порядка $(Z\alpha)^n$ в образовании $\mu^+\mu^-$ пар

Motivation: **muon pair production may be easier for an experimental observation.**

This process was recently considered in detail by
Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903;
Jetschura, Hencken, Serbo. EPJ C58 (2008) 281;
Jentschura, Serbo. EPJ C64 (2009) 309.

It was found out that:

1. **The Coulomb** corrections are **small**, while **unitarity** corrections are **large**;
2. **The exclusive** cross section **differs** considerable from its Born value, but an experimental observation is difficult;
3. **The inclusive** cross section **coincides** with the Born cross section.

3.1. Born cross section for one $\mu^+\mu^-$ pair production

Let us consider the production of **one $\mu^+\mu^-$ pair**

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^-,$$

using EPA, but **taking into account** nuclear electromagnetic **form factors** (Fig. 5):

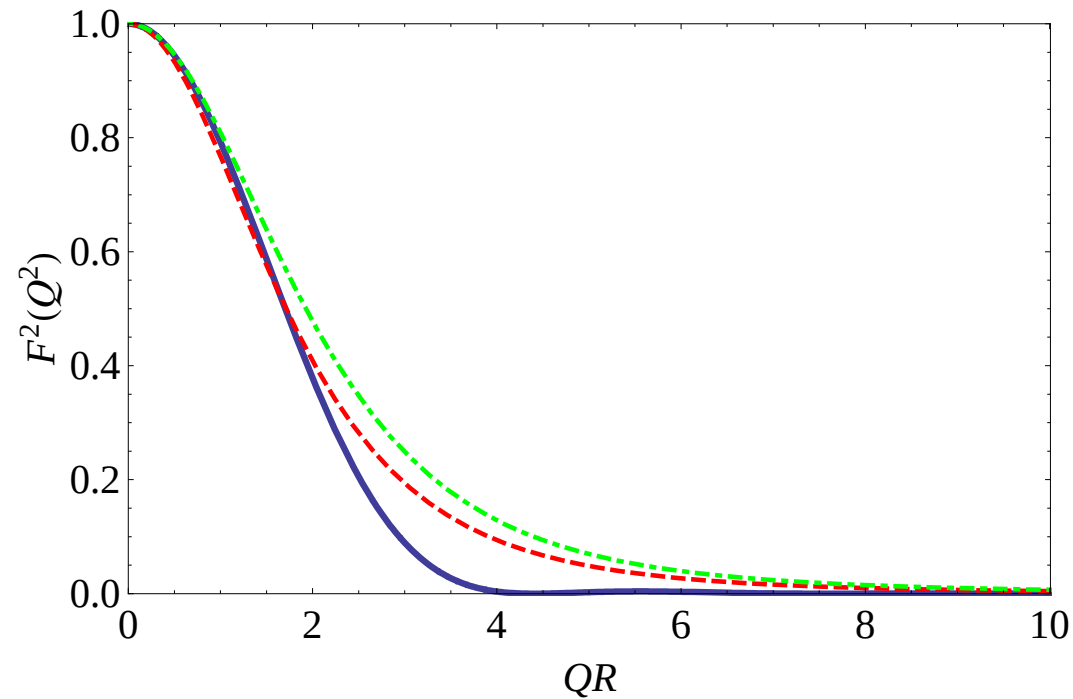


Fig. 5. Realistic (**solid blue line**) and simplified (**dashed red** and **dot-dashed green** lines) form factors vs. QR for Au

The Born differential cross section $d\sigma_B$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for **the real $\gamma\gamma \rightarrow \mu^+\mu^-$ process** by the equation

$$d\sigma_B = dn_1 dn_2 d\sigma_{\gamma\gamma},$$

where dn_i is the number of equivalent photons.

As a result, the cross section for the case of **the realistic nuclear form factor** reads:

$$\sigma_B = 0.21 \text{ barn for RHIC and } 2.5 \text{ barn for LHC.}$$

The accuracy of this calculation is of the order of few percents.

3.2. Probability $P_B(\rho)$

Let us now consider **the probability of muon pair production** $P_B(\rho)$ in the Born approximation. In the LLA:

$$\begin{aligned} P_B(\rho) &= \int dn_1 dn_2 \delta(\rho_1 - \rho_2 - \rho) \sigma_{\gamma\gamma} = \\ &= \frac{28}{9\pi^2} \frac{(Z_1\alpha Z_2\alpha)^2}{(\mu\rho)^2} \Phi(\rho). \end{aligned}$$

There are **two scales** in dependence of function $\Phi(\rho)$ on ρ :

$$\Phi(\rho) = \left(4 \ln \frac{\gamma}{\mu \rho} + \ln \frac{\rho}{R} \right) \ln \frac{\rho}{R} \quad \text{at} \quad R \ll \rho \leq \gamma/\mu,$$

$$\Phi(\rho) = \left(\ln \frac{\gamma^2}{\mu^2 \rho R} \right)^2 \quad \text{at} \quad \gamma/\mu \leq \rho \ll \gamma^2/(\mu^2 R).$$

We compare Eqs. for $\Phi(\rho)$ with the numerical calculations based on **the exact matrix element**. **There is a good agreement** for the Pb-Pb collisions: **the discrepancy is less than 10 % at $\mu\rho > 10$ and it is less than 15 % at $\mu\rho > 2\mu R = 7.55$.**

3.3. Coulomb and unitarity corrections

The Coulomb correction corresponds to the Feynman diagram of Fig. 6 with **a multi-photon exchange**.

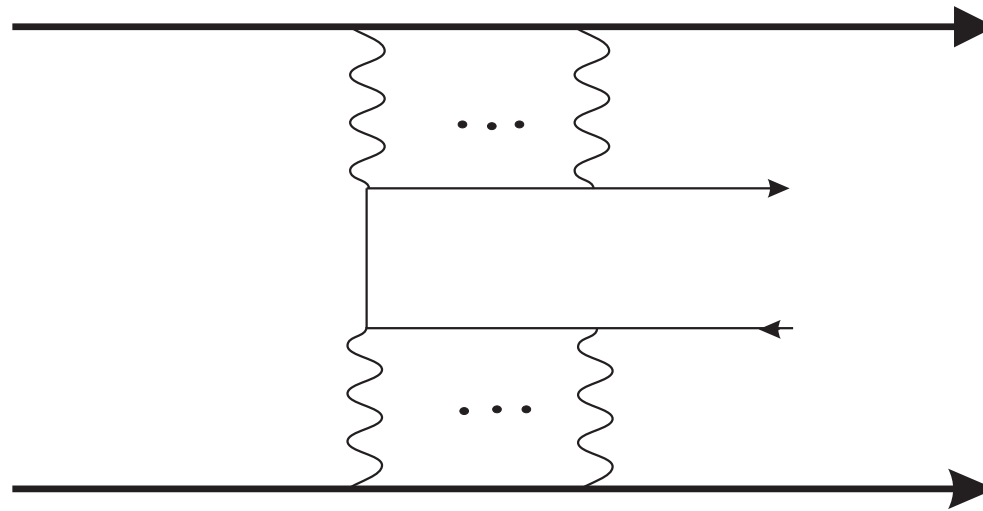


Fig. 6

Estimation: Due to the **restriction of transverse momenta** of additional exchange photons on the level of $1/R$ (**nuclear form factor!**), the effective parameter of the perturbation series is not $(Z\alpha)^2$, **the real suppression parameter** is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}, \quad L = \ln(\gamma^2),$$

which corresponds to **a Coulomb correction on the order of a percent**.

Our recent calculation shows that **Coulomb corrections** to the $\mu^+\mu^-$ pair production is **small**:

Coulomb corrections to the $\mu^+\mu^-$ pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}} [\text{Baltz}]$
RHIC, Au-Au	−3.7%	−22%
LHC, Pb-Pb	−1.3%	−14%

In the last column is shown the recent result of [A. Baltz. nucl-th/0901.0891](#). In fact, this calculations **do not** take into account the nuclear form factors **properly** and, therefore, may be **incorrect**. Their trend contradicts the physical requirement that Coulomb corrections should vanish for an infinite mass of the produced lepton pair, not grow with the lepton mass.

The unitarity correction σ_{unit} to one muon pair production is described by the exchange of blocks, corresponding to light-by-light scattering via a virtual electron loop, between the nuclei (Fig. 7).

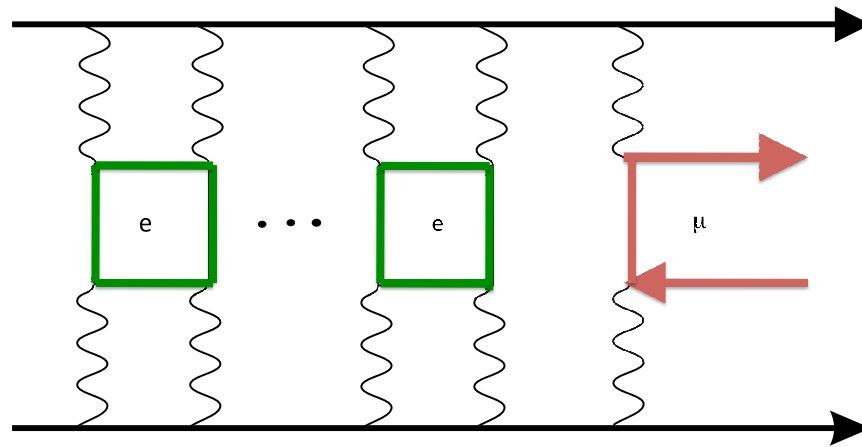


Fig. 7

As usual,

$$\sigma_B = \int_{2R}^{\infty} P_B(\rho) d^2\rho \rightarrow \sigma_B + \sigma_{\text{unit}} = \int_{2R}^{\infty} P_B(\rho) e^{-\bar{n}_e(\rho)} d^2\rho$$

and

$$\sigma_{\text{unit}} = - \int_{2R}^{\infty} [1 - e^{-\bar{n}_e(\rho)}] P_B(\rho) d^2\rho$$

is **the unitarity correction for the exclusive production of one muon pair**. In **LLA** we find

$$\delta_{\text{unit}} = \frac{\sigma_{\text{unit}}}{\sigma_B} = -49 \% \text{ for the Pb-Pb collisions at LHC.}$$

The correction is large because there is a logarithmic enhancement from the region of small impact parameters $2R < \rho < 1/m_e$.

It is seen that unitarity corrections are **large**, in other words, **the exclusive production of one muon pair differs considerable from its Born value**.

However, the experimental study of the **exclusive** muon pair production seems to be **a very difficult task**.

Indeed, this process requires that the muon pair should be registered **without any electron–positron pair production**, including $e^{\pm}$ emitted at very small angles.

Otherwise, the corresponding inclusive cross section will be close to the Born cross section (for detail see

Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903).

4. Поправки порядка $(Z\alpha)^n$ в обычном тормозном излучении ядер

The ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by Feynman diagrams of Fig. 5

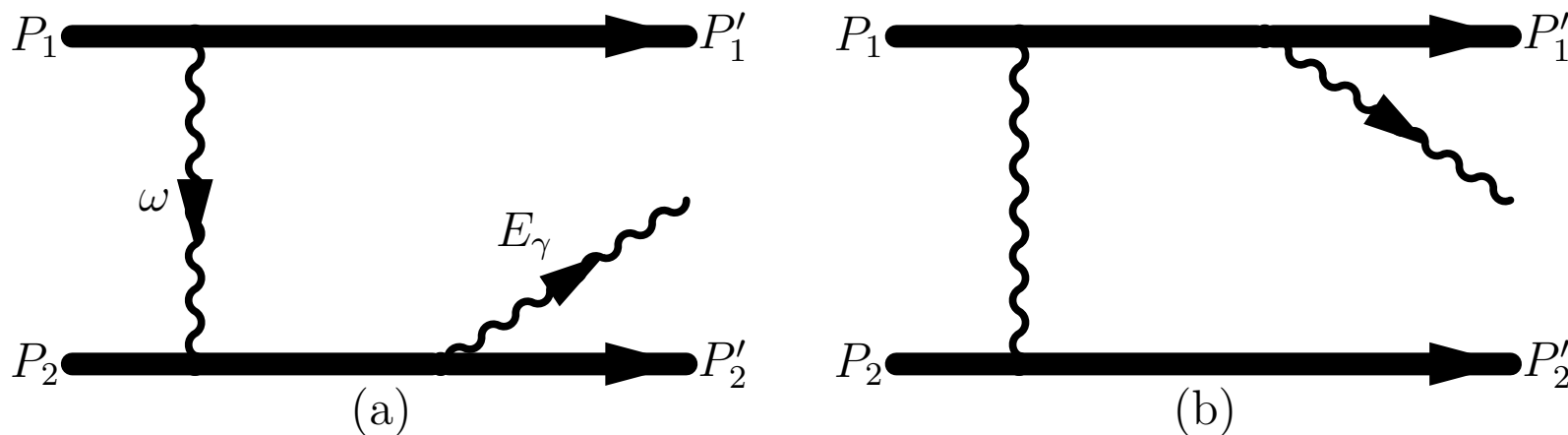


Fig. 5

and was known in detail many years ago

Bertulany, Baur Phys. Rep. 163, 299 (1988)

It can be described as **the Compton scattering** of the equivalent photon off opposite nucleus:

$$d\sigma_{\text{br}} = d\sigma_{\text{br}}^a + d\sigma_{\text{br}}^b ,$$

and

$$d\sigma_{\text{br}}^a = dn_1(\omega) d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2) .$$

Here, dn_1 is the number of equivalent photons emitted by nucleus Z_1 and $d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)$ is the differential cross section for the Compton scattering off nucleus Z_2 .

We can rewrite these Eqs. as

$$d\sigma_{\text{br}}^a = dP_a(\rho) d^2\rho,$$

where the differential probability $dP_a(\rho)$ assumes the form

$$dP_a(\rho) = \frac{Z_1^2 \alpha}{\pi^2} \frac{\sigma_{\text{T}}(Z_2)}{\rho^2} \left(1 - x_\gamma + \frac{3}{4} x_\gamma^2\right) \frac{dE_\gamma}{E_\gamma}, \quad x_\gamma = \frac{E_\gamma}{E_2}$$

with the **Thomson cross section** (M is the mass of nucleus)

$$\sigma_{\text{T}}(Z_2) = \frac{8\pi}{3} \frac{Z_2^4 \alpha^2}{M^2}.$$

The unitarity correction

$$\delta_{\text{unit}} = \frac{d\sigma_{\text{unit}}^a}{d\sigma_{\text{br}}^a},$$

reads (R is the nuclear radius)

$$\delta_{\text{unit}} = -\frac{1}{L_\gamma} \int_{2R}^{\infty} \frac{d\rho}{\rho} \left[1 - e^{-\bar{n}_e(\rho)} \right], \quad L_\gamma = \ln \left(\frac{2\gamma_1 \gamma_2^2 (1 - x_\gamma)}{R E_\gamma} \right).$$

An evaluation of this integral gives the following result

for a photon energy $E_\gamma = 1$ GeV,

$$\delta_{\text{unit}} = -\mathbf{19\%} \quad \text{for the RHIC,}$$

$$\delta_{\text{unit}} = -\mathbf{15\%} \quad \text{for the LHC.}$$

**5. Большой вклад
виртуального дельбрюковского
рассеяния
в тормозное излучение ядер**

Recently we consider emission of photons not via the virtual Compton subprocess, but via another one –

the virtual Delbrück scattering subprocess (Fig. 6)

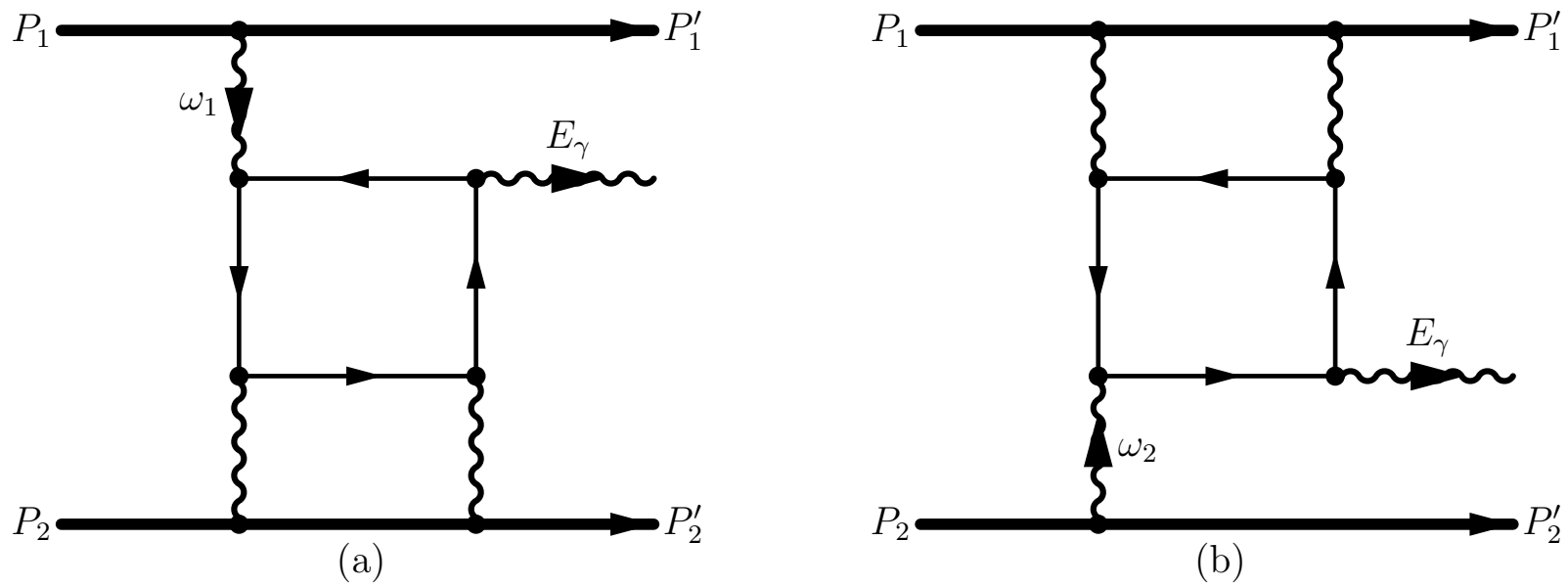


Fig. 6

First note: *Baur, Bertulany* *Z. f. Phys.* **A 330**, 77 (1988)

At first sight, this is a process of **a very small cross section** since

$$\sigma \propto \alpha^7.$$

But at second sight, we should **add a very large factor**

$$Z^6 \sim 10^{11}$$

and take into account that the cross section **scale** is

$$1/m_e^2.$$

And the last, but not the least, we found that this cross section has **an additional logarithmic enhancement** of the order of

$$L^2 \gtrsim 100, \quad L = \ln(\gamma^2).$$

Thus, the estimate is

$$\sigma \sim \frac{(Z\alpha)^6 \alpha}{m_e^2} L^2.$$

Our analytical result

Ginzburg, Jentschura, Serbo, *Phys. Lett. B* **658**, 125 (2008);

Ginzburg, Jentschura, Serbo, *Eur. Phys. J. C* **54**, 267 (2008)

$$\sigma = C \frac{(Z\alpha)^6 \alpha}{m_e^2} L^2$$

with

$$C \approx 0.4.$$

This cross sections is **considerably larger** than that for ordinary nuclear bremsstrahlung in the photon energy range:

$$m_e \ll E_\gamma \ll m_e \gamma.$$

Thus, the discussed cross section for Au-Au collisions at the RHIC collider is

$$\sigma = 14 \text{ barn}$$

and for Pb-Pb collisions at the LHC collider is

$$\sigma = 50 \text{ barn}.$$

That is quite a serious number!

Note for comparison, that the last cross section is **6 times larger than for the total hadronic/nuclear cross section in Pb–Pb collisions**, which is roughly **8 barn**.

Few words about calculations

First of all about Delbrück scattering (DS)

The DS is an elastic scattering of a photon in the Coulomb field of a nucleus via a **virtual electron-positron loop** (Fig. 7)

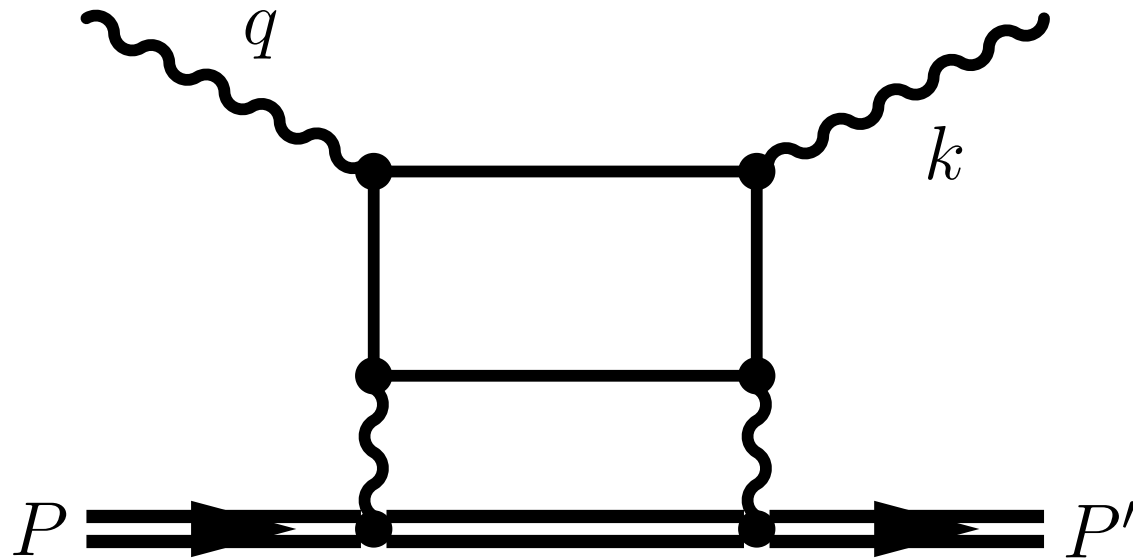


Fig. 7

Its properties are well known

see review *Milstein, Schumacher*, *Phys. Rep.* **243**, 183 (1994)

The total cross section of this process **vanishes** at small energies

$$\sigma_D(\omega_L, Z) \sim (Z\alpha)^4 \frac{\alpha^2}{m_e^2} \left(\frac{\omega_L}{m_e} \right)^4 \quad \text{at } \omega_L = qP/M \ll m_e,$$

but **tends to constant** at $\omega_L \gg m_e$.

In the lowest order of the perturbative theory this constant is

$$\sigma_D^{(0)}(Z) = 1.07 (Z\alpha)^4 \frac{\alpha^2}{m_e^2} \quad \text{at } \omega_L \gg m_e.$$

The Coulomb corrections $\sim (Z\alpha)^{2n}$ **decrease** it significantly

$$\sigma_D(\omega_L, Z)_{\omega_L \gg m} \rightarrow \sigma_D(Z) = \frac{\sigma_D^{(0)}(Z)}{r_Z}.$$

For example, for DS off the Au ($Z = 79$) and Pb ($Z = 82$) nuclei

$$\sigma_D(Z = 79) = 5.5 \text{ mb}, \quad \sigma_D(Z = 82) = 6.2 \text{ mb},$$

this corresponds to $r_{79} = \mathbf{1.7}$ and $r_{82} = \mathbf{1.8}$.

Comparison:

Cross section for **the nuclear Thomson scattering** is

$$\sigma_{\text{T}}(Z) = \frac{8\pi}{3} \frac{Z^4 \alpha^2}{M^2},$$

where $M \approx Am_p$.

The ratio

$$\frac{\sigma_{\text{T}}(Z)}{\sigma_{\text{D}}(Z)} = 7.83 r_Z \left(\frac{m_e}{\alpha^2 Am_p} \right)^2 \approx \frac{1}{30} \quad \text{for } {}^{208}\text{Pb}$$

is small for heavy nuclei.

The cross section for the nuclear bremsstrahlung is given by (Fig. 8)

$$d\sigma = d\sigma_a + d\sigma_b$$

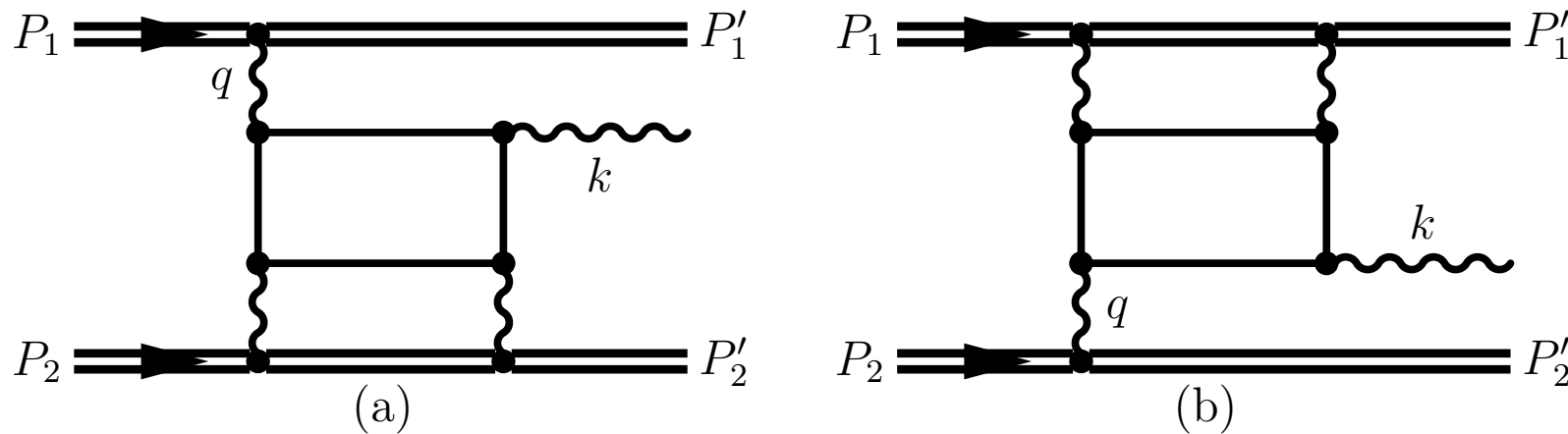


Fig. 8

The interference term is small and can be safely neglected.

In **the equivalent photon approximation**

$$d\sigma_a = dn_1(\omega) \sigma_D(\omega_L, Z),$$

where the number of equivalent photons is

$$dn_1(\omega) = 2 \frac{Z^2 \alpha}{\pi} \frac{d\omega}{\omega} \ln \frac{m_e \gamma}{\omega}.$$

Then integrating this cross section over ω in the region

$$\frac{m_e}{\gamma} \lesssim \omega \lesssim m_e \gamma,$$

we obtain **the total cross section** in the leading log approximation

$$\sigma = \sigma_a + \sigma_b = 2 \frac{Z^2 \alpha}{\pi} \sigma_D(Z) L^2, \quad L = \ln \frac{P_1 P_2}{2M_1 M_2} = \ln(\gamma^2). \quad (1)$$

We also calculated the energy and angular distribution of photons

Заключительные замечания:

Кулоновские и унитарные поправки оказываются весьма существенными для количественного описания процессов тормозного излучения и образования лептонных пар в столкновении ультрарелятивистских тяжелых ядер.

Экстремально сильные электромагнитные поля, не встречающиеся в других процессах, приводят к необычным эффектам типа доминантности вклада 7 порядка по константе α над борновским вкладом 3 порядка.

THANK YOU FOR YOUR ATTENTION!

