

*FFK09, 1 – 4 декабря, 2009,
Дубна, Россия*

Ньютоновская гравитационная постоянная: современный статус экспериментального определения



В. К. Милюков,

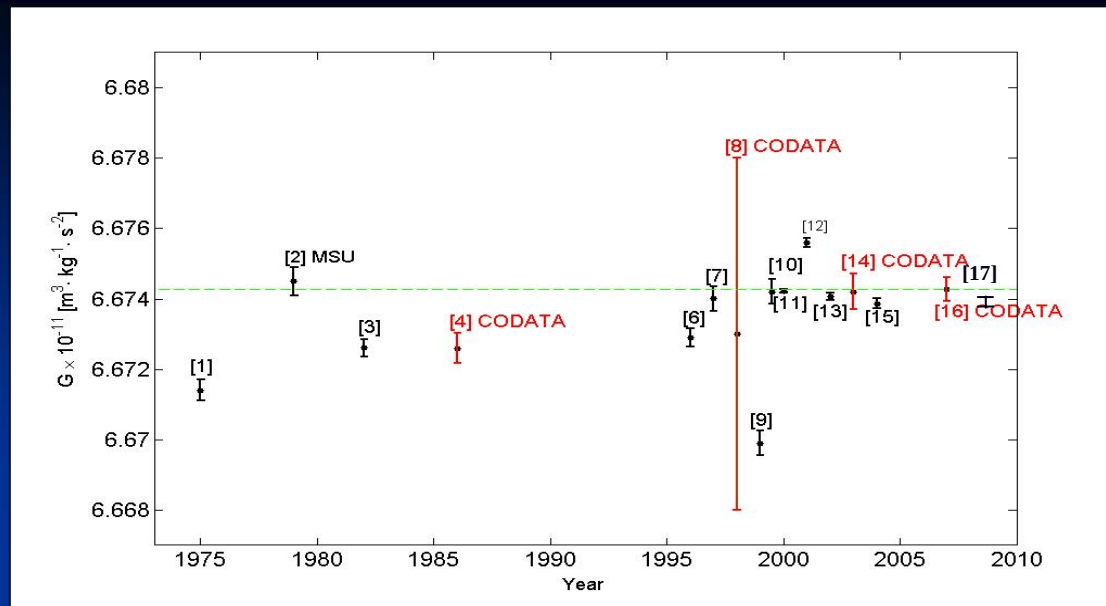
ГАИШ МГУ First G value:

1797-98, Henry Cavendish:

$$G = (6.67 \pm 0.07) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

CODATA 2006 value

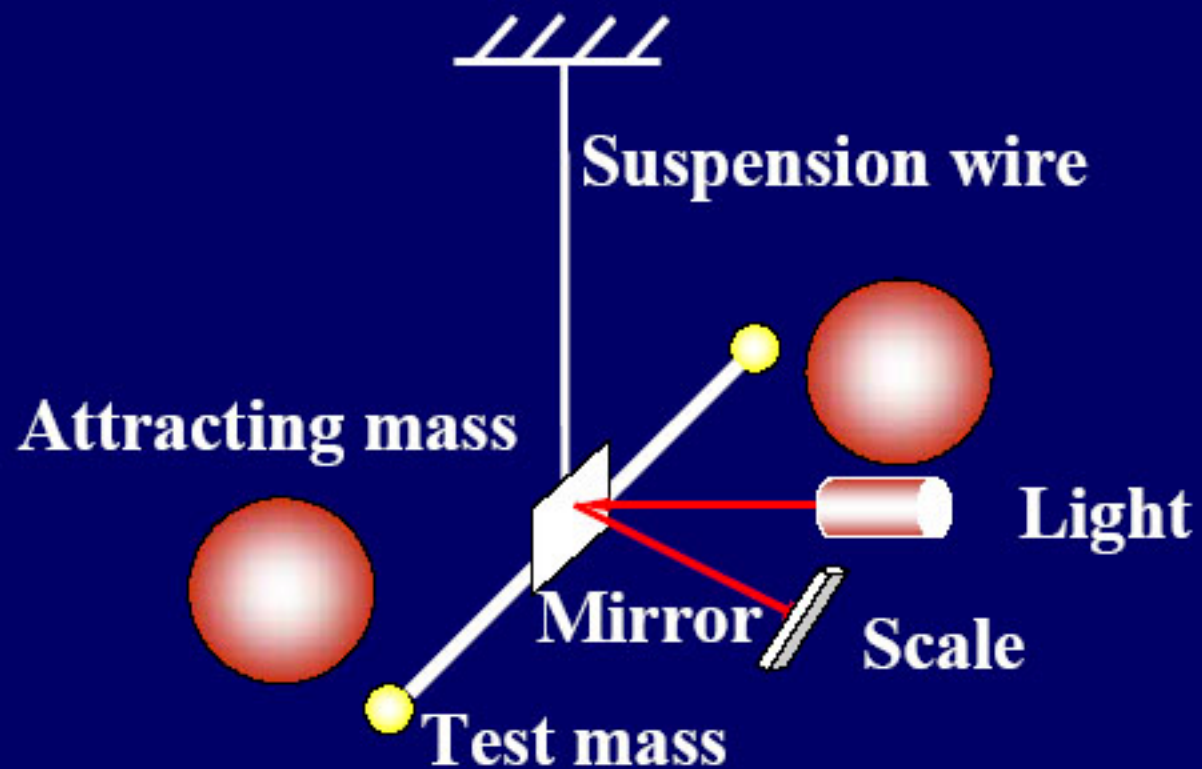
$$G = (6.67428 \pm 0.00067) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2},$$



The best world experiments on the measurement of G and CODATA values

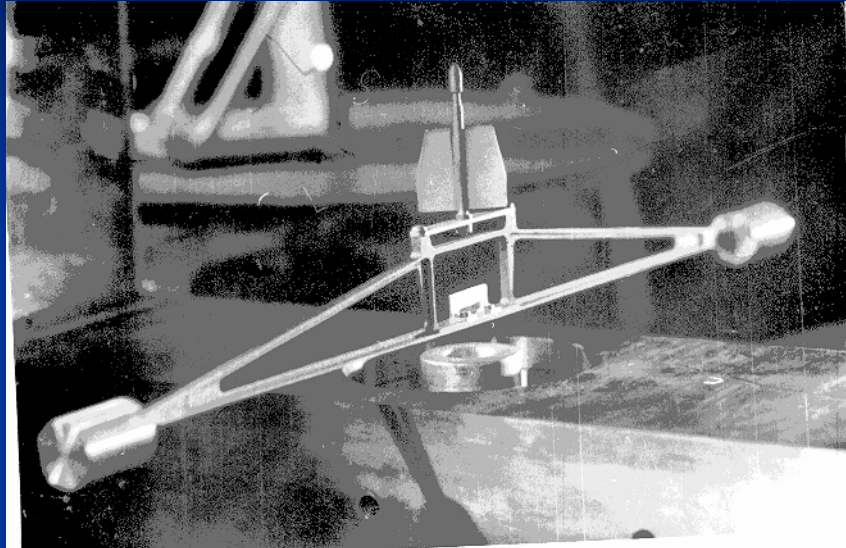
Authors, year of publication	Value of $G \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	STD $\times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	ppm
[1] Facy and Ponticis 1972	6.6714	0.0006	90
[2] Sagitov, Milyukov, et al. 1979	6.6745	0.0008	120
[3] Luther and Towler, 1982	6.6726	0.0005	75
[4] CODATA 1986	6.67259	0.00085	128
[5] Michaelis, et al. 1995	6.7154	0.0006	90
[6] Karagioz, Izmailov, 1996	6.6729	0.0005	75
[7] Bagley and Luther, 1997	6.6740	0.0007	105
[8] CODATA 1998	6.673	0.010	1500
[9] Jun Luo, et al., 1999	6.6699	0.0007	105
[10] Fitzgerald and Armstrong 1999	6.6742	0.0007	105
[11] Gundlach and Merkowich, 2000	6.674215	0.000092	14
[12] Quinn, Speake et al. 2001	6.67559	0.00027	41
[13] Schlamminger et al. 2002	6.67407	0.00022	33
[14] CODATA 2002	6.6742	0.0010	150
[15] Armstrong and Fitzgerald 2003	6.67387	0.00027	40
[16] CODATA 2006	6.67428	0.00067	100
[17] Jun Luo, et al. 2009	6.67349	0.00018	26

Cavendish' experiment



The torsion balances and time of swing method

No 1

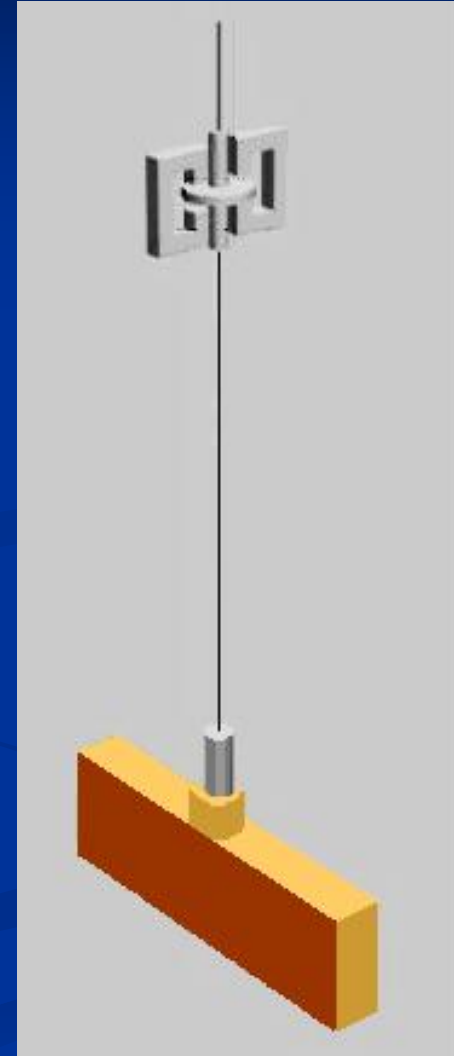


$$J\ddot{\varphi} + [D + G(\partial\Gamma/\partial\varphi)]\varphi = 0$$

$$\omega_1^2 = \frac{D + G(\partial\Gamma/\partial\varphi)}{J}$$

$$G = \frac{J[(\omega_1^2)_1 - (\omega_1^2)_2]}{(\partial\Gamma/\partial\varphi)_1 - (\partial\Gamma/\partial\varphi)_2}$$

No 2



Determination of G in SAI MSU (1975-1978)

General view on experimental device

*Adjustment of torsion
pendulum support*

Vacuum lamps

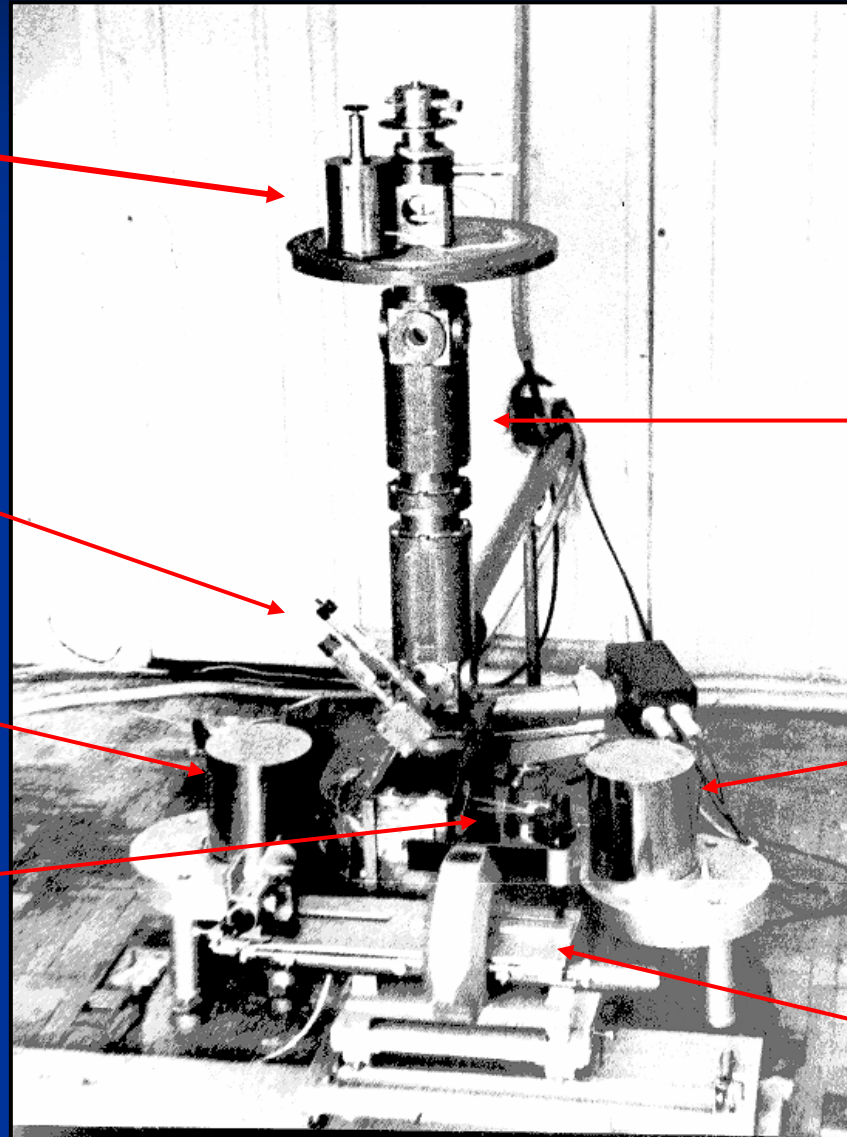
Attractive mass

Torsion balance

Vacuum chamber

Attractive mass

*Device for linear
measurements*



Results on measurement of Gravitational constant in SAI MSU in 1975-1978

Set No Exp. No	$G \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$			
	I	II	III	IV
1	6.6739	6.6708	6.6686	6.6721
2	6.6741	6.6746	6.6795	6.6749
3	6.6746	6.6730	6.6800	6.6826
4		6.7612	6.6732	6.6719
5		6.6747	6.6755	6.6688
6				6.6779
7				6.6757
8				6.6715
9				6.6799
10				6.6774
Average value	6.6742 ± 0.002	6.6729 ± 0.0009	6.6753 ± 0.0020	6.6752 ± 0.0013

According to STUDENT criterion (t-criterion) all these four sets on the 0.95 confidence level are belong to one statistical ensemble with the average value of

$$G = (6.6745 \pm 0.0008) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

The new experiment on determination of the gravitation constant in HUST (China)

Коллаборация HUST и
ГАИШ МГУ поддержана
международными грантами
РФФИ_ГФЕН

13 4:33PM



General view of the experimental setup



□ Measurements of test mass

➤ **Mass** 75.59 g

➤ **Dimension**

➤ **Temperature**



Rectangular glass block
coated with gold

91.52 x 12.01 x 27.58 mm

Параметры крутильных весов и пробных масс

а). Крутильные весы

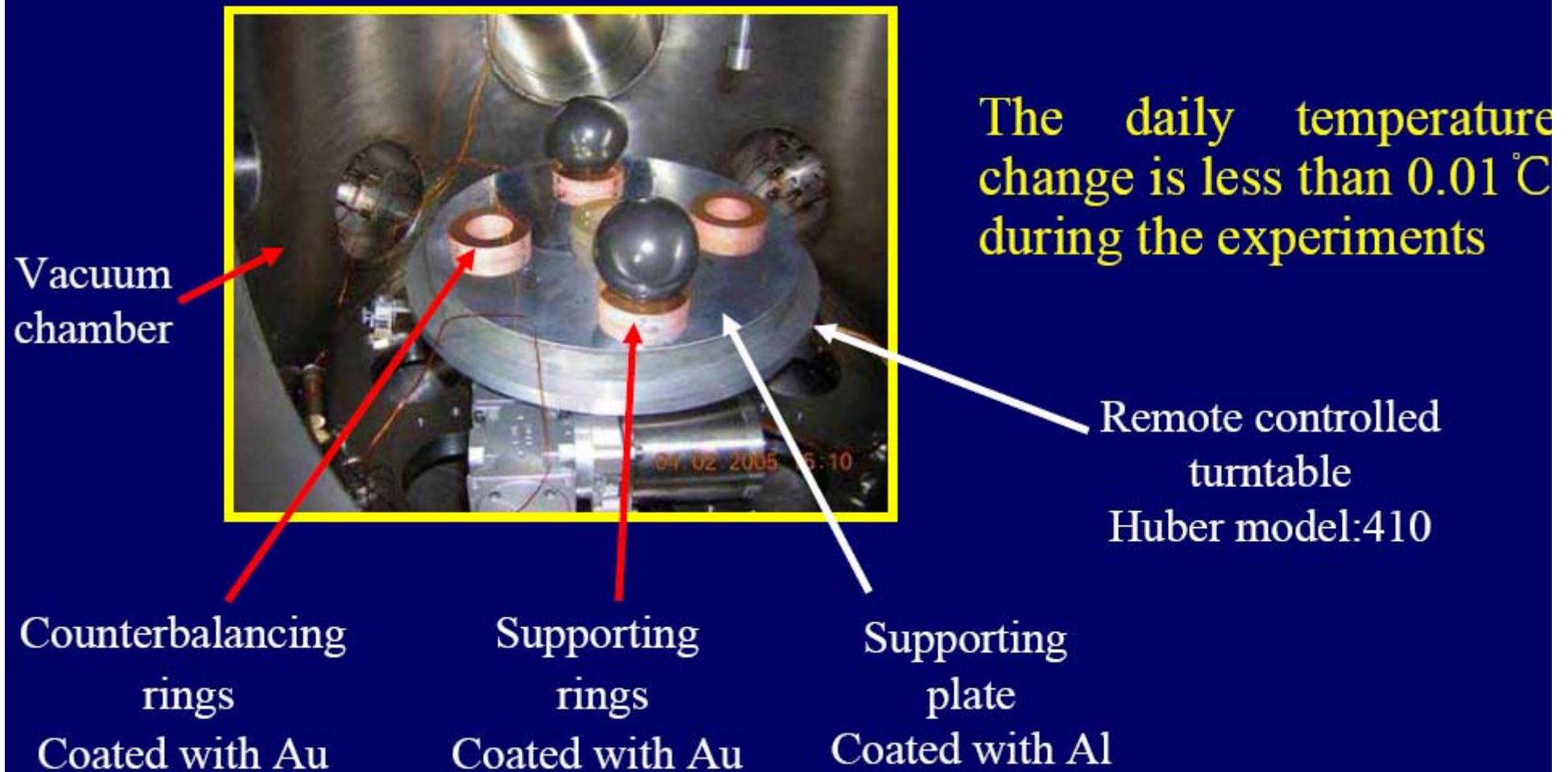
Масса бруска, m	75.591016 ± 0.00001 г
Длина бруска, b	9.15245 ± 0.00002 см
Высота бруска, h	2.75867 ± 0.00004 см
Толщина бруска, w	1.20194 ± 0.00001 см
Расстояние от центра бруска до точки подвеса, l_0	3.36 ± 0.00002 см
Нить подвеса (длина l / диаметр d)	89 см / 25 мкм
Крутильная жесткость нити подвеса, $K_{кр}$	6.17×10^{-2} дин·см ²
Момент инерции относительно оси X, J_x	(57.036 ± 0.00140) г·см ²
Момент инерции относительно оси Y, J_y	(575.608 ± 0.00269) г·см ²
Момент инерции относительно оси Z, J_z	(536.748 ± 0.00231) г·см ²

б). Притягивающие массы

Масса пробной тела M_2	778.1754 ± 0.0010 г
Масса пробной тела M_3	778.1794 ± 0.0010 г
Диаметр пробного тела D_2	5.7151126 ± 0.0000107 см
Диаметр пробного тела D_3	5.7150765 ± 0.0000108 см

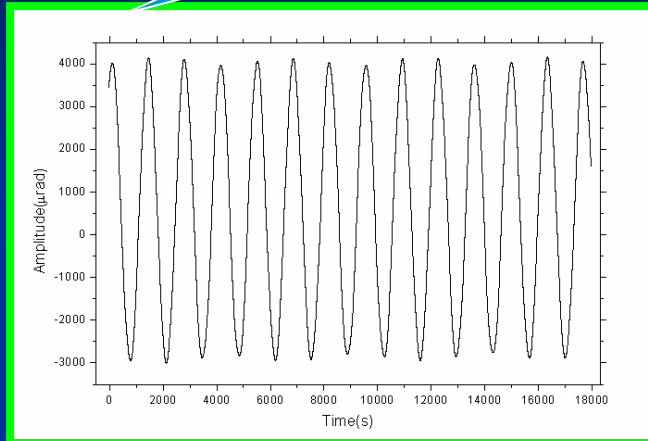
➤ Support rings and the spheres

Thermal expanding coefficient of the plate and the rings is
 $(0 \pm 1) \times 10^{-7} \text{ } ^\circ\text{C}^{-1}$

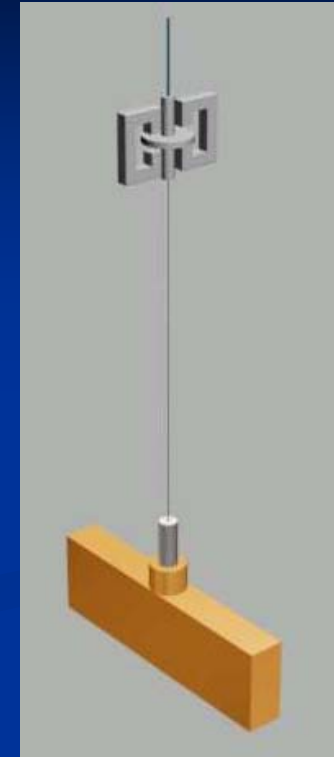
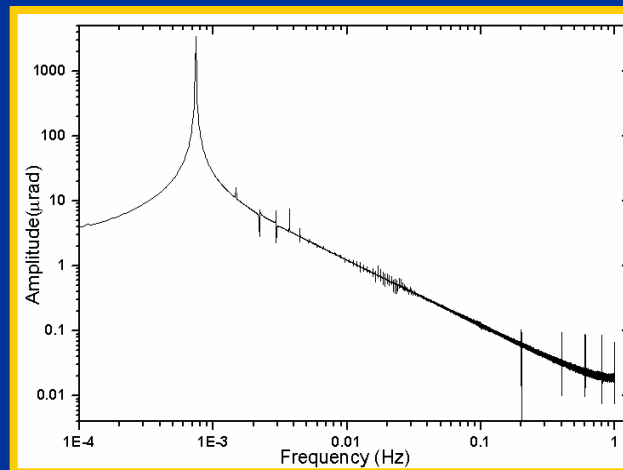
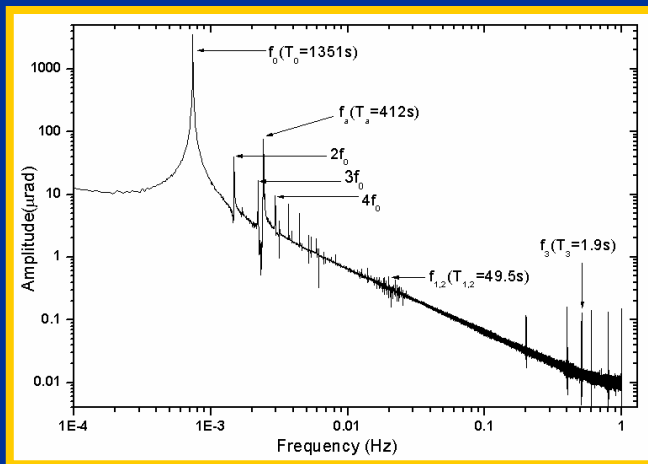
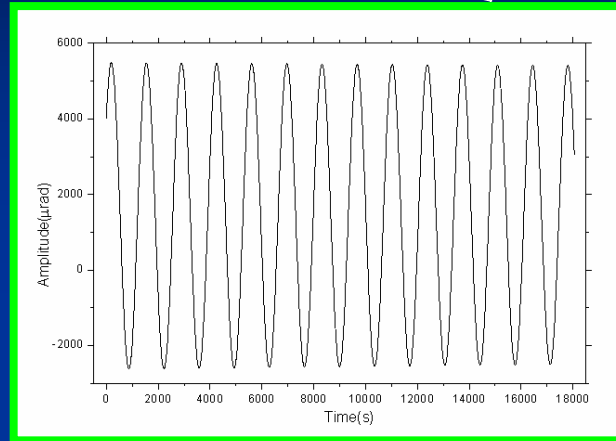


8. Suppression of vibration modes

Without damper



With damper



The damper can suppress the vibration modes effectively

5.Measurement of the separation of the two balls

Surface distance: S

Centre distance: $D=(S+r_1+r_2)$,

Length of gauge block: L

L is about $15\text{ }\mu\text{m}$ thinner than S.

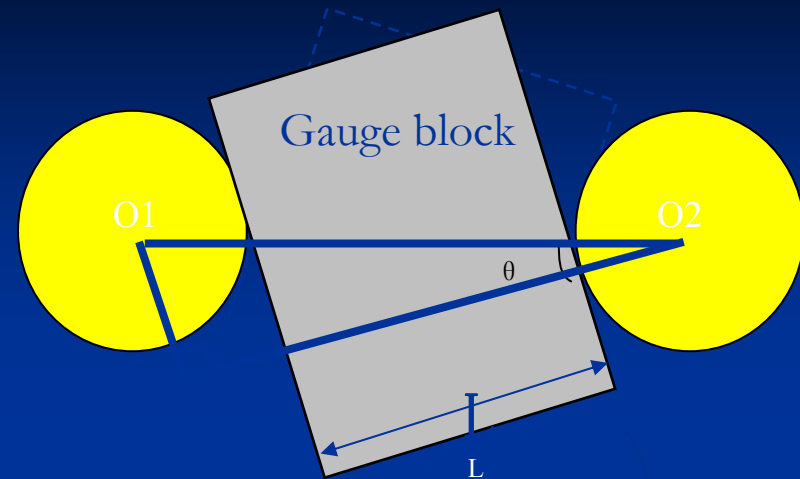
$$D = \overline{O_1O_2} = \frac{L + r_1 + r_2}{\cos\theta}$$

$$S = D - (r_1 + r_2) \approx L/\cos\theta + (r_1 + r_2)\theta^2/2$$

Result:

$$S=(100.015615 \pm 0.000034)\text{mm}$$

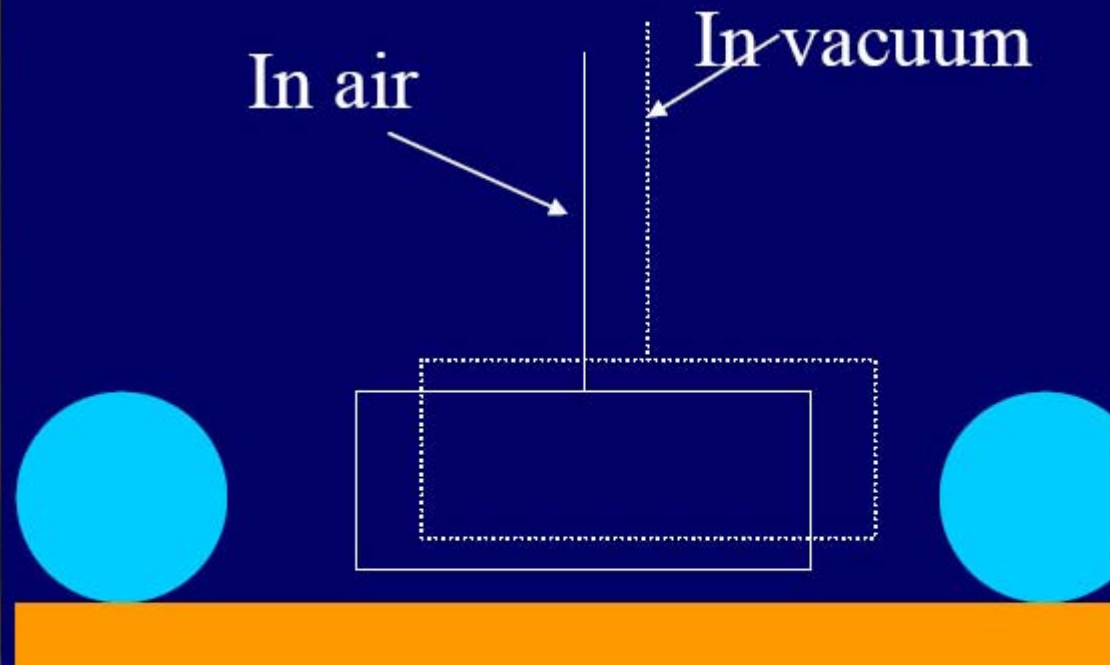
0.9ppm to G



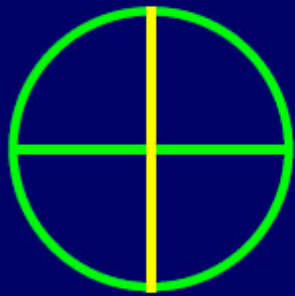
➤ Deformation of the vessel in vacuum condition



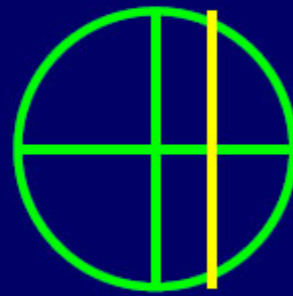
The air pressure would deform the vacuum vessel



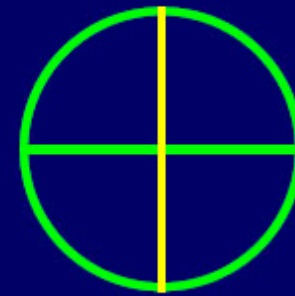
Two telescopes were used to monitor the position of the fiber at orthogonal directions



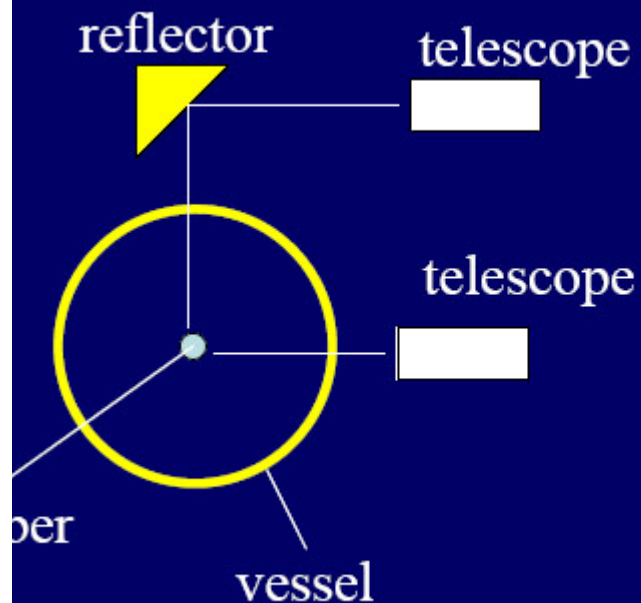
Before vacuum
acquired



After vacuum
acquired



Adjust the XY stages with
accuracy of 10 μm



$\Delta X: (0 \pm 12) \mu\text{m}$
2.59ppm

$\Delta Y: (0 \pm 13) \mu\text{m}$
4.61ppm

$\Delta Z: (0 \pm 10) \mu\text{m}$
0.12ppm

total: 5.29ppm

Error budget (1)

Error Sources	Corrections	$\Delta G/G$, ppm	
Pendulum			5.07
Dimensions		1.95	
Attitude		0.13	
Nonalignment with fiber		0.45	
Flatness		0.34	
Clamp		1.65	
Density inhomogeneity		≤ 0.21	
Coating layer	-24.28	4.33	
Edge flaw	-0.12	0.17	
Source masses			10.68
Masses		0.82	
Distance of GC		9.64	
Density inhomogeneity		4.50	
XYZ positions		0.48	

Error budget (2)

Error Sources	Corrections	$\Delta G/G$, ppm	
Fiber			18.76
Nonlinearity		<0.70	
Thermoelasticity	-39.83	1.52	
Anelasticity	-211.80	18.69	
Aging		<0.01	
Gravitation Nonlinearity			0.30
Magnetic damper			0.31
Magnetic field			0.40
Electrostatic field			0.10
Statistical $\Delta(\omega^2)$			14.18
Total			26.33

New value of Gravitational Constant

- $G = (6.67349 \pm 0.00018) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
with a standard uncertainty 26 ppm

Jun Luo, et al // Phys. Rev. Lett., **102**, 240801 (2009)

Accuracy of measurement of G



Precision of estimation of torsional frequency

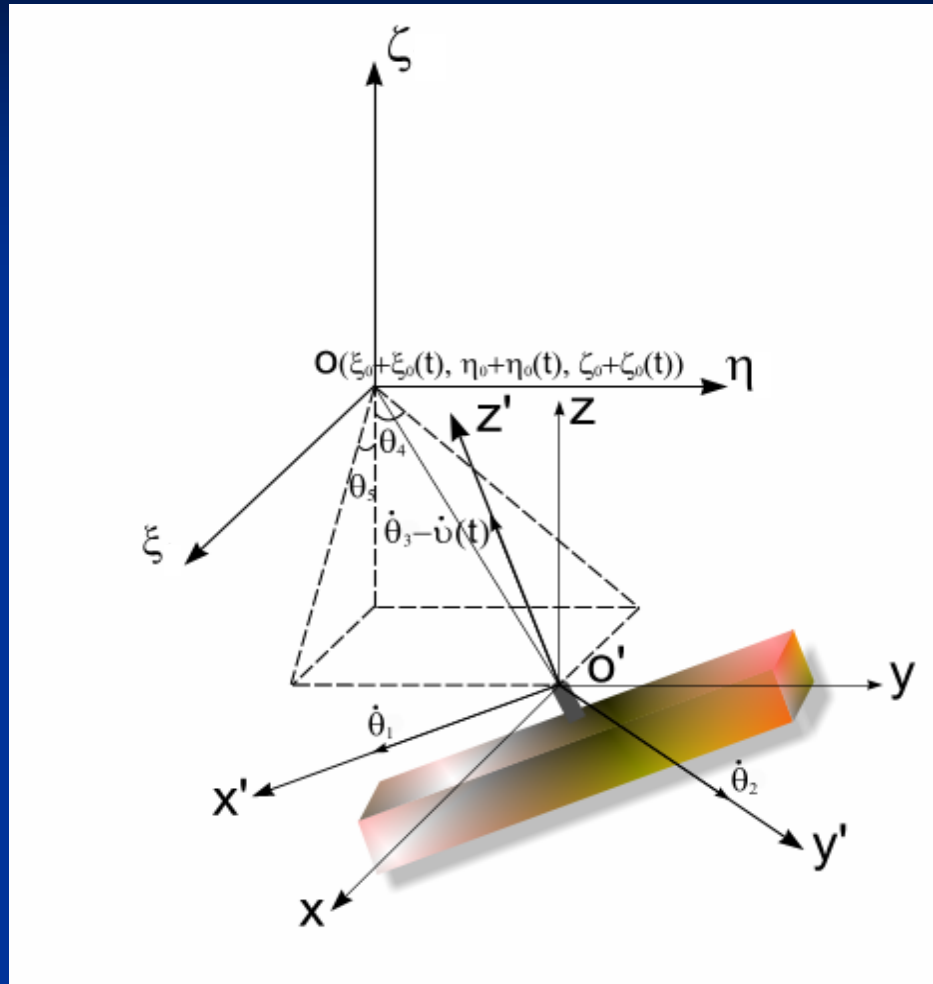


Stability of torsion balance oscillations

The problem:

**Torsion balance and Environmental
noise**

The torsion balance is the system with five degrees of freedom



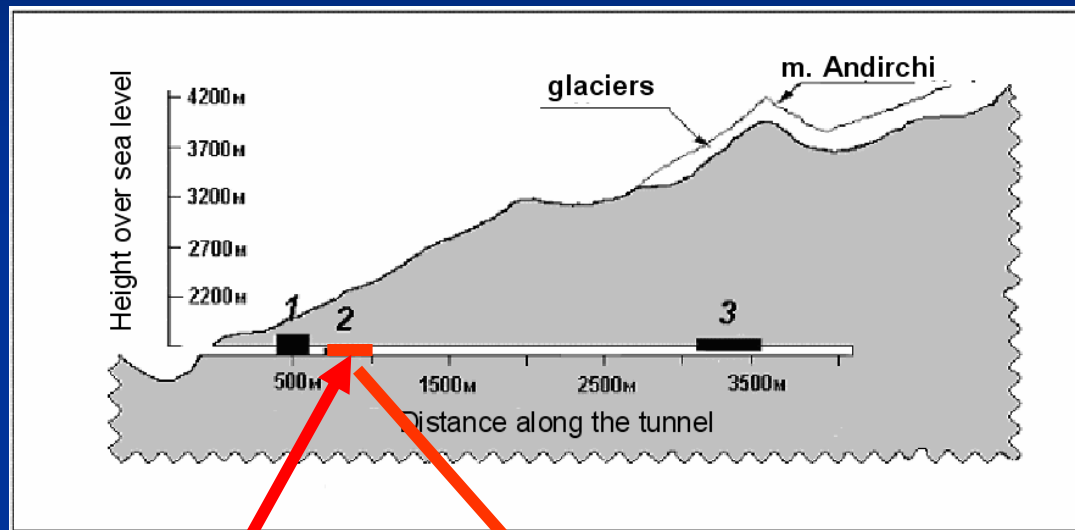
**Co-ordinates Θ_1 and Θ_4 is swing oscillations in the plane $\xi \zeta$,
Co-ordinates Θ_2 and Θ_5 is swing oscillations in the plane $\eta \zeta$.
 Θ_3 is the torsion oscillations**

The system of differential equations describing the motion of torsion balance with five degrees of freedom

- $(J_x + ml_0^2)\ddot{\theta}_1 + \tau_1^{-1}(J_x + ml_0^2)\dot{\theta}_1 + mgl_0\theta_1 = -mll_0\ddot{\theta}_4 - ml_0\eta_0(t) + (J_y - J_x)\ddot{\theta}_2\theta_3$
- $(J_y + ml_0^2)\ddot{\theta}_2 + \tau_2^{-1}(J_y + ml_0^2)\dot{\theta}_2 + mgl_0\theta_2 = mll_0\ddot{\theta}_5 + ml_0\xi_0(t) + (J_y - J_x)\ddot{\theta}_1\theta_3$
- $J_z\ddot{\theta}_3 + \tau_3^{-1}J_z\dot{\theta}_3 + K_e\theta_3 = (J_y - J_x)(\dot{\theta}_1^2 - \dot{\theta}_2^2)\theta_3 + 2(J_y - J_x)\dot{\theta}_1\dot{\theta}_2\theta_3^2 - \frac{2}{3}(J_y - J_x)(\dot{\theta}_1^2 - \dot{\theta}_2^2)\theta_3^3 - J_z\ddot{\theta}_1\dot{\theta}_2 + (J_x - J_y - J_z)\dot{\theta}_1\dot{\theta}_2 + K_{kp}v(t)$
- $ml^2\ddot{\theta}_4 + \tau_4^{-1}ml^2\dot{\theta}_4 + mgl\theta_4 = -mll_0\ddot{\theta}_1 + ml\eta_0(t)$
- $ml^2\ddot{\theta}_5 + \tau_5^{-1}ml^2\dot{\theta}_5 + mgl\theta_5 = mll_0\ddot{\theta}_2 - ml\xi_0(t)$

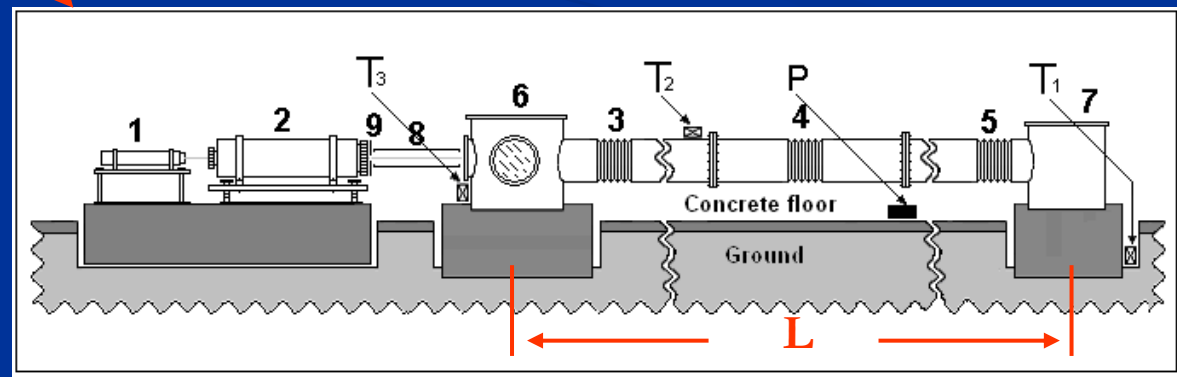
$\xi_0(t)$ and $\eta_0(t)$ random seismic accelerations, acting on suspension point in horizontal plane.
 $v(t)$ — fluctuations, describing random action on torsion degree of freedom.

Experimental estimation of wideband seismic noise in the underground laboratory condition (Baksan Laser Interferometer)

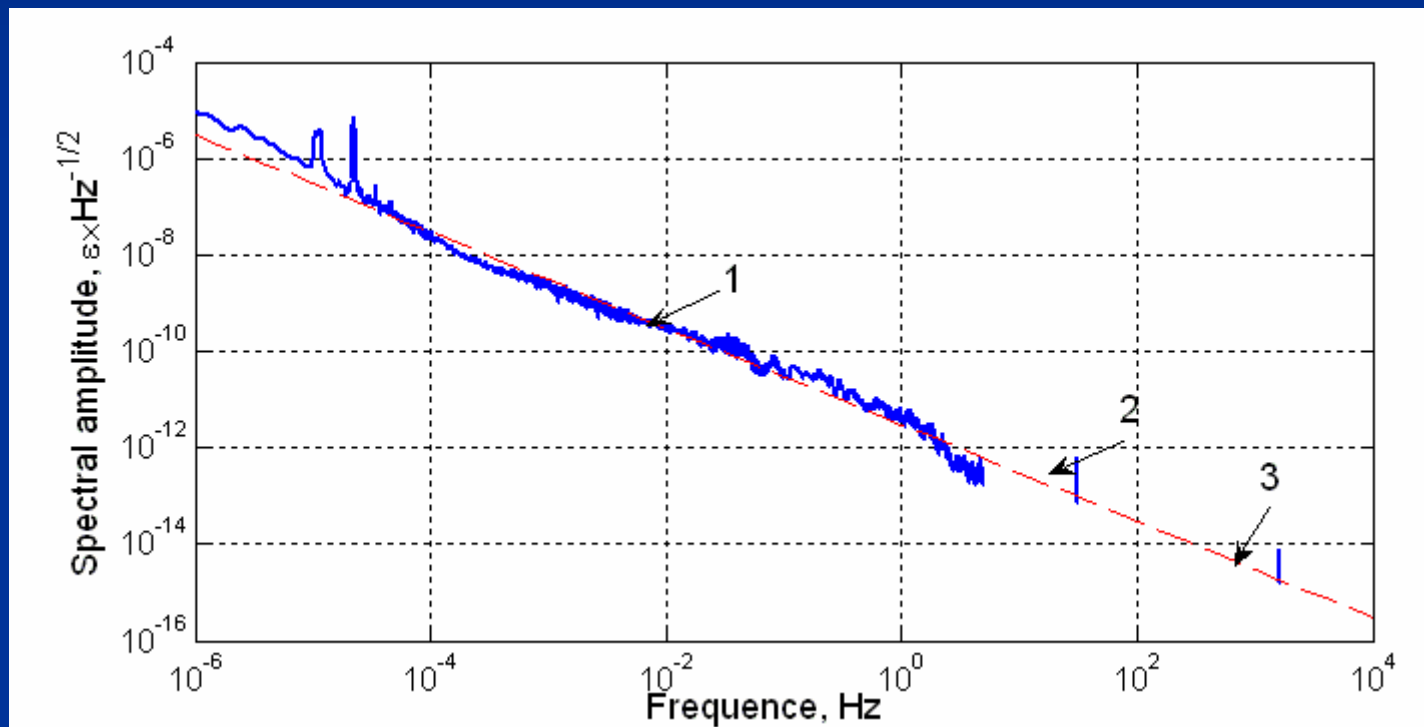


Baksan Laser Interferometer

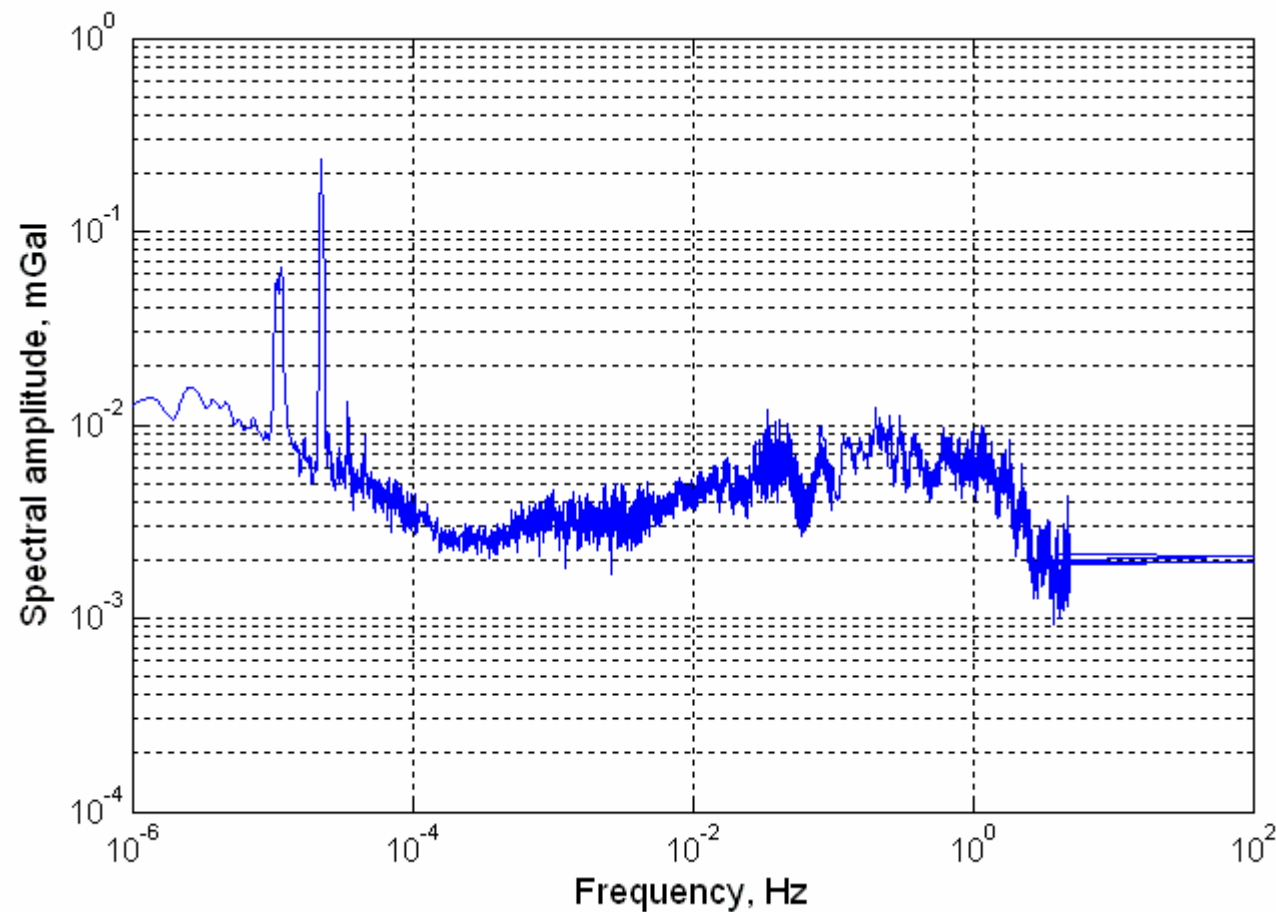
resolution: 2.3×10^{-13}
(1.6×10^{-11} m)



Spectral amplitude of deformation



Spectral amplitudes of horizontal seismic accelerations in the frequency band 10^{-6} -10 Hz for the Baksan underground tunnel

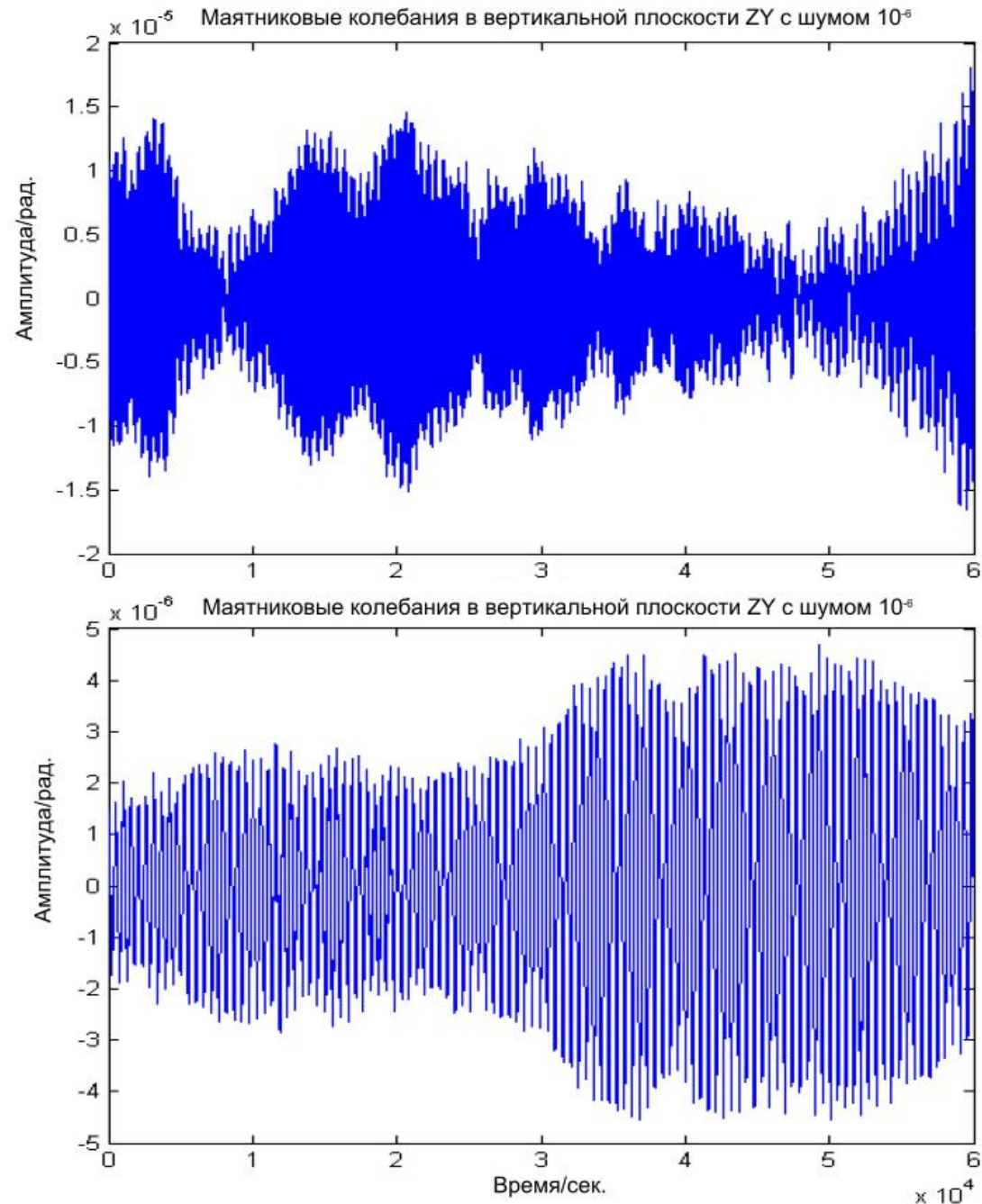


Excitation of swing oscillations by the seismic noise

Swing oscillations
on plane $\eta\xi$,
excited by seismic
noise with
amplitude about
0.1mGal.

a). For rotation
angle θ_1 ;

б). For rotation
angle θ_4 .

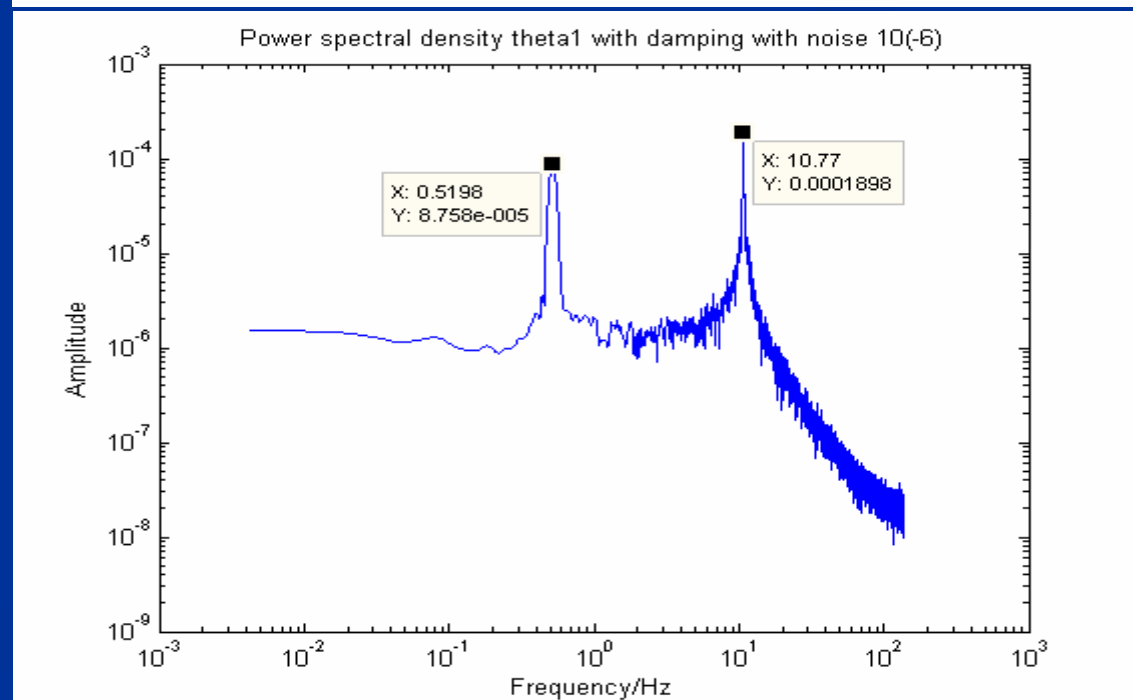
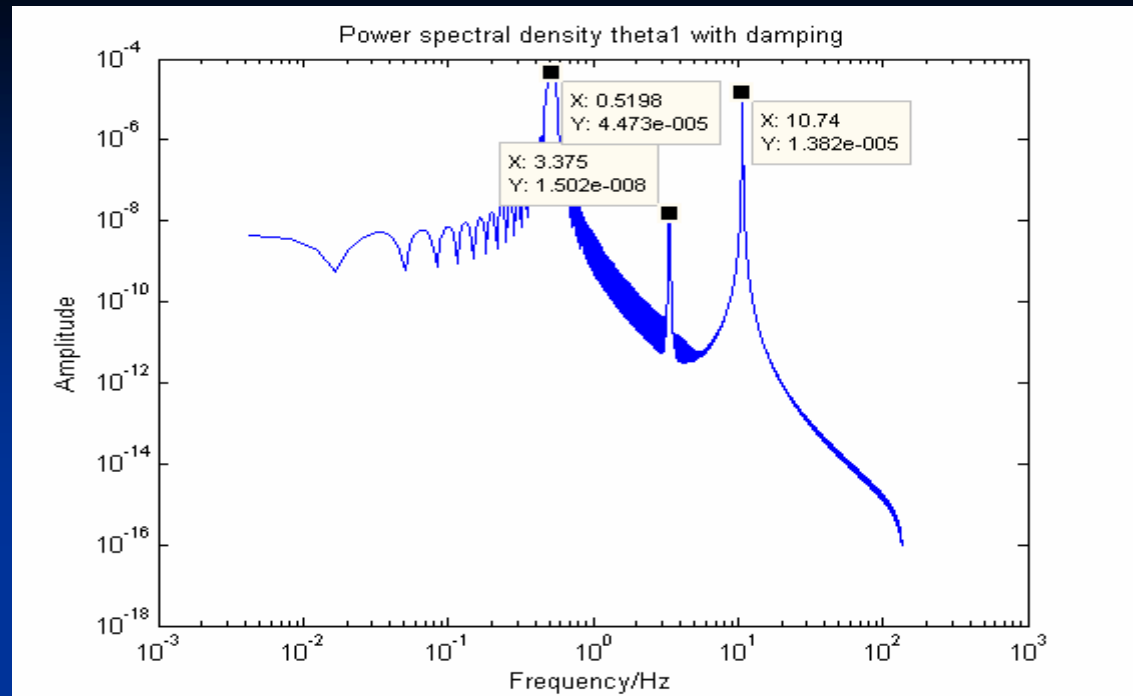


Spectra of swing oscillations on plane $\eta\zeta$ (rotation angle θ_1).

Peaks 0.5198 Hz and 10.77 Hz are eigen swing modes in this plane. Peaks 3.375 Hz is forced oscillations due to nonlinear coupling with oscillations on plane $\xi\zeta$.

a). Oscillations without action of seismic noise

б). Oscillation with action of seismic noise

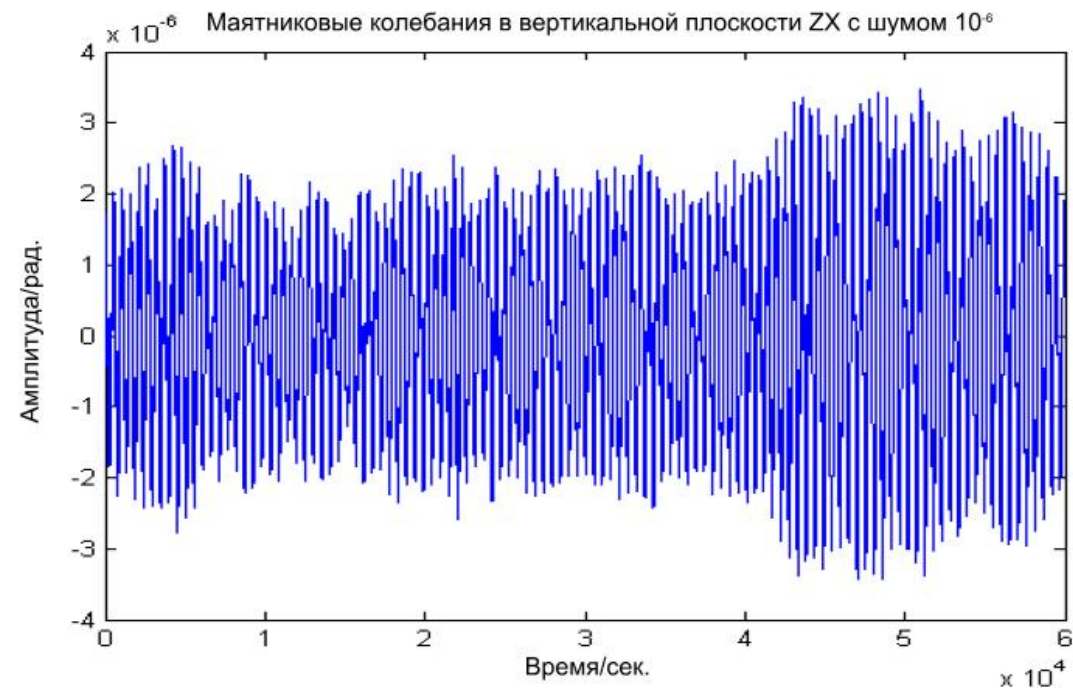


Swing oscillations on plane $\xi\zeta$, excited by seismic noise with amplitude about 0.1mGal.

a). For rotation angle θ_2 ;



б). For rotation angle θ_5 .



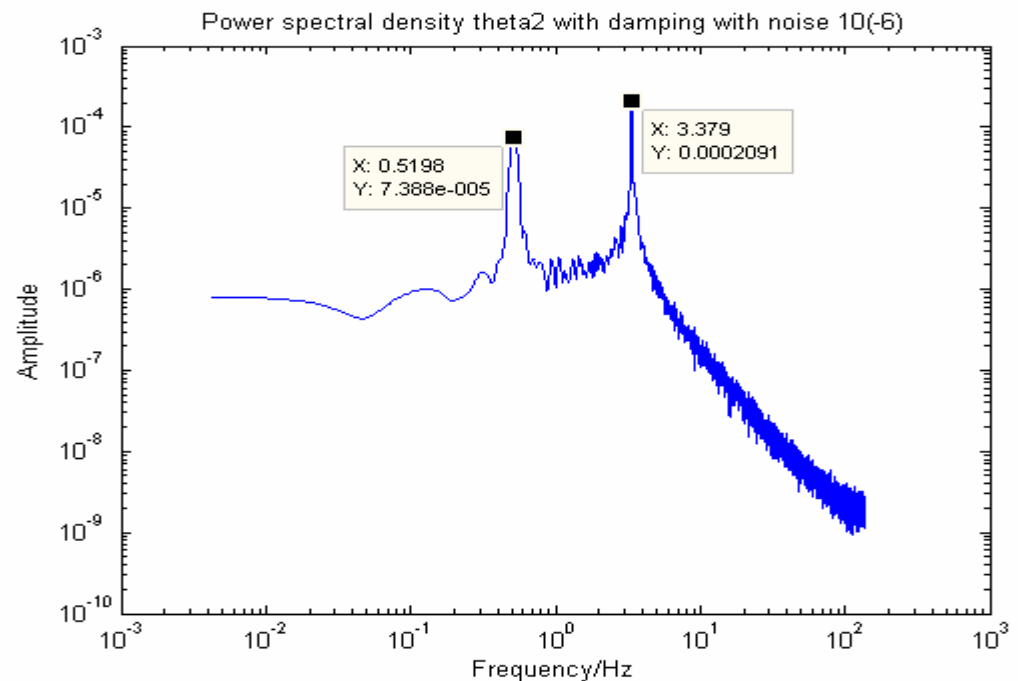
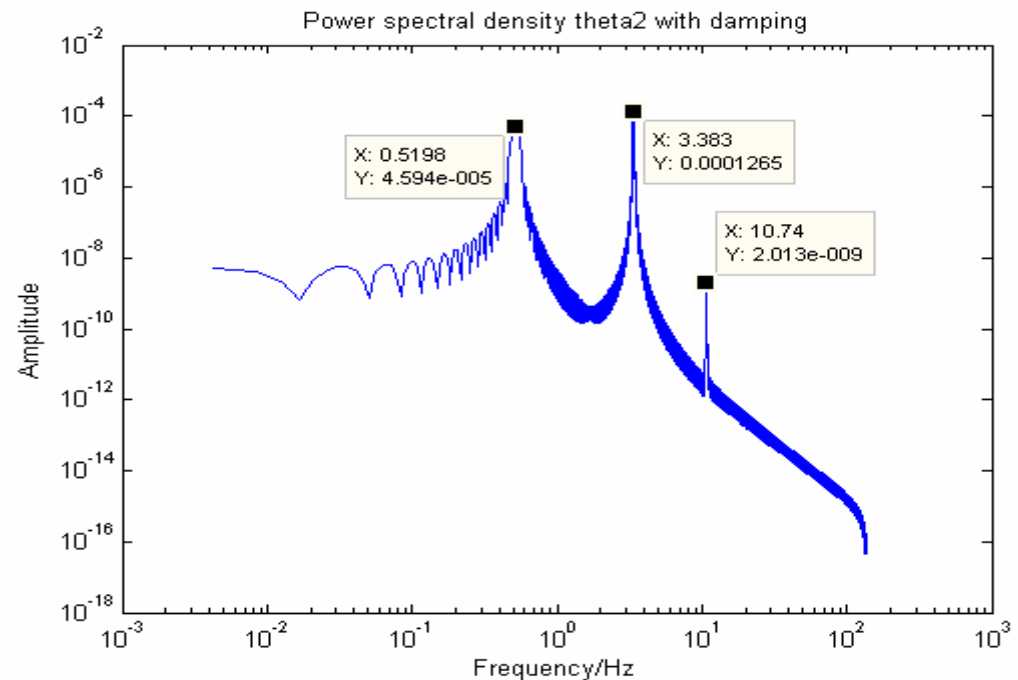
Spectra of swing oscillations on plane $\xi\zeta$ (rotation angle θ_2).

Peaks 0.5198 Hz and 3.383 Hz are eigen swing modes in this plane.

Peaks 10.74 Hz is forced oscillations due to nonlinear coupling with oscillations on plane $\eta\zeta$.

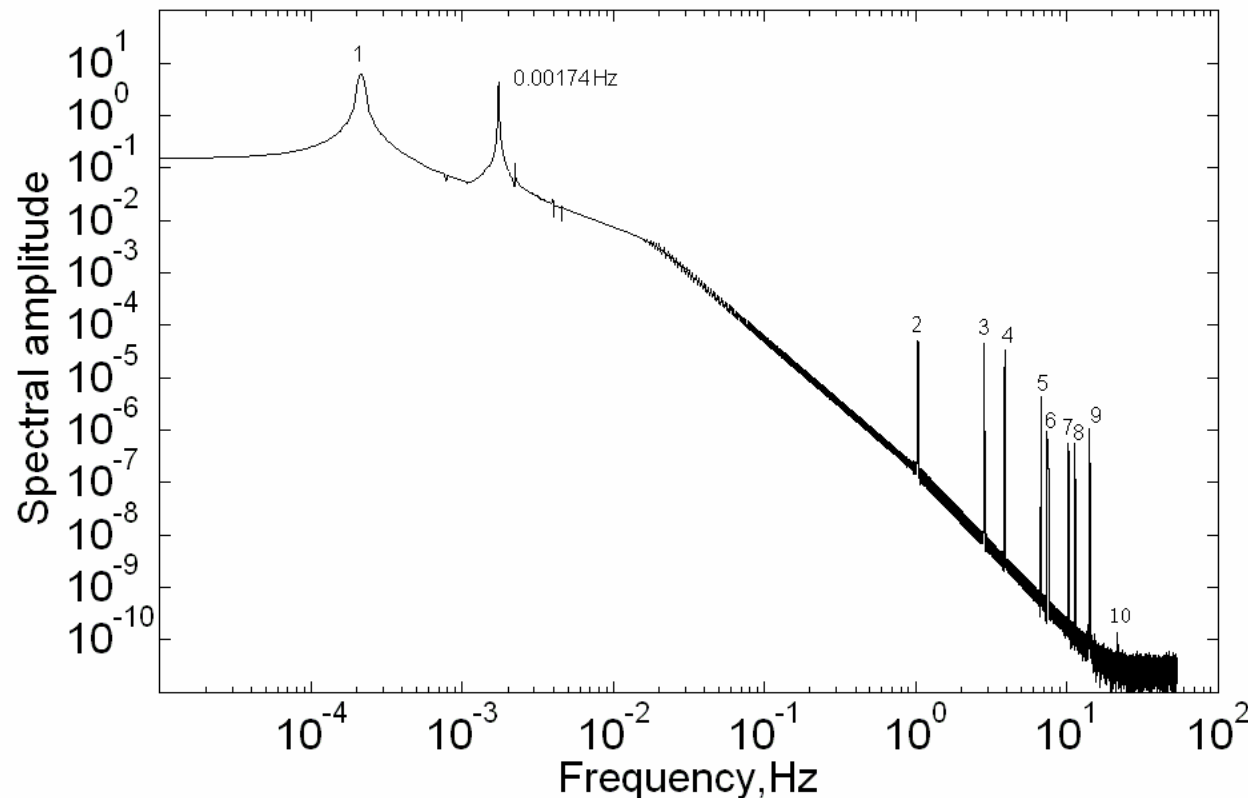
a). Oscillations without action of seismic noise

б). Oscillation with action of seismic noise



Torsion oscillations and mode couplings

The frequencies of coupling modes are linear combinations of pendulum frequencies



No.	Frequency, Hz
1	0.0002
2	1.0372
3	2.8562
4	3.8936
5	7.3401
6	10.20
7	11.2335
8	14.0899

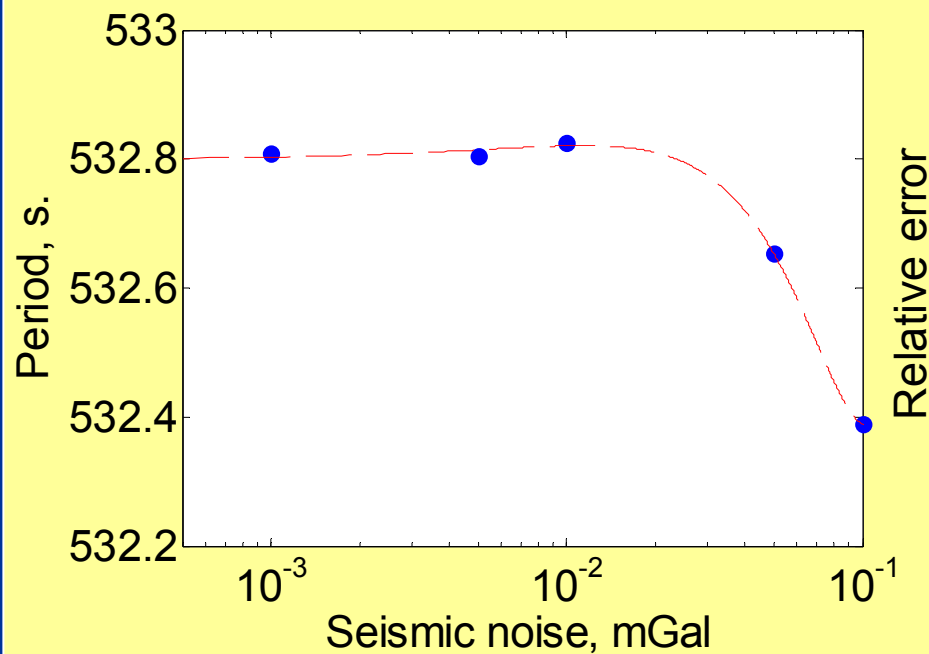
Dependence of period of torsion oscillation from the level of seismic noise

Torsion noise 10^{-4} rad.

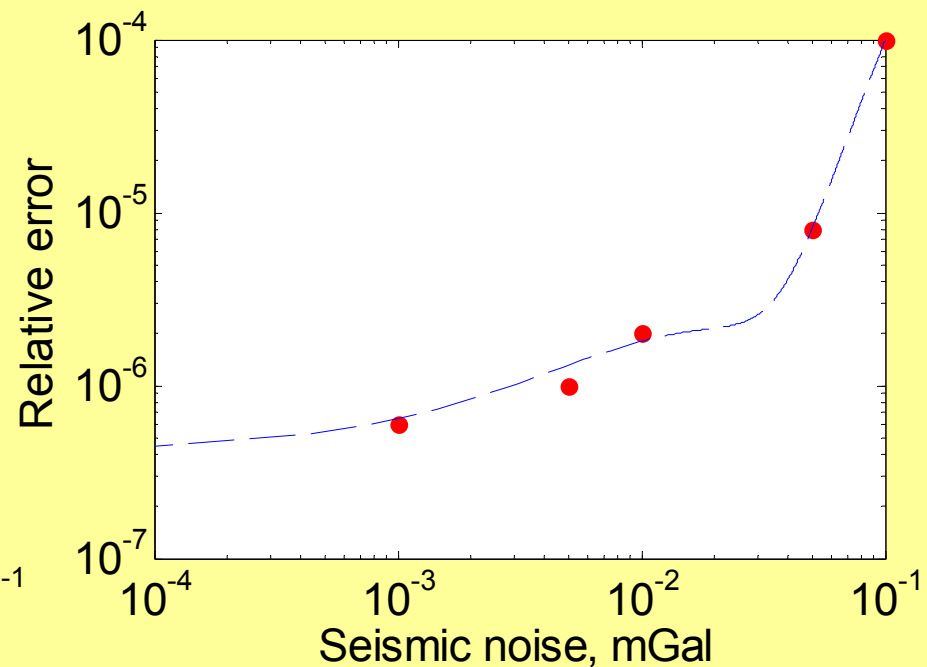
Seismic noise, mGal	Period of torsion oscillation (mean for 787 periods)	Period of torsion oscillation (mean for 20x40 periods)
0.0 (Tor. noise 0 rad)	532.80224 ± 0.00015 (0.3 ppm)	532.80162 ± 0.00035 (0.6 ppm)
0.001	532.80882 ± 0.00016 (0.3 ppm)	532.80788 ± 0.00036 (0.6 ppm)
0.005	532.80851 ± 0.00030 (0.6 ppm)	532.80463 ± 0.00068 (1·ppm)
0.01		532.8258 ± 0.0010 (2·ppm)
0.05		532.6541 ± 0.0046 (8·ppm)
0.1		532.390 ± 0.062 (100 ppm)

Dependence of period of torsion oscillation from the level of seismic noise

Mean for 787 periods



Mean for 20x40 periods



Dependence of period of torsion oscillation from the level of torsion noise

Seismic noise 0.01mGal

Level of torsion noise, rad.	Period of torsion oscillation (mean for 20x40 periods)
10^{-6}	532.88182 ± 0.0012 (2·ppm)
6×10^{-5}	532.79127 ± 0.00058 (1·ppm)
10^{-5}	532.76545 ± 0.00086 (2·ppm)
5×10^{-5}	532.84456 ± 0.0010 (2·ppm)
10^{-4}	532.82583 ± 0.0010 (2·ppm)
5×10^{-4}	532.80087 ± 0.0010 (2·ppm)
10^{-3}	532.73626 ± 0.0011 (2·ppm)

Conclusion

- Estimation of the torsion period on the level of few ppm is provided by the environmental seismic noise with amplitude, less than 10^{-2} mGal, and by torsion fluctuations less than 10^{-3} rad.
- Environmental noise changes the absolute value of the period of torsion oscillations
- To provide the absence of systematic errors in the period values caused by noise, the noise conditions have not to change during the experiment (Changing the level of seismic noise of one order leads to changing the value of torsion period on few decades ppm).