

Шансы и вызовы для КХД из эксперимента BaBar по пион-фотонному переходному формфактору

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Plan of Presentation

$\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$, factorization, structure of $\mathbf{F}^{\gamma^*\gamma^*\pi}$

- ▶ Introduction to collinear factorization
- ▶ Meson Distribution Amplitudes (DA) φ
- ▶ Hard-scattering amplitudes in NLO, $\mathbf{T}_1$, and in NNLO, $\mathbf{T}_2$

Light Cone Sum Rules (LCSR) for $\gamma^*\gamma(q_2^2 \simeq 0)$

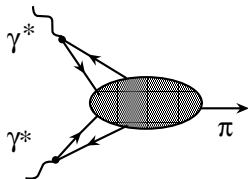
- ▶ Why Light Cone Sum Rules (LCSR)? Dispersion relations for $\mathbf{F}^{\gamma^*\gamma\pi}$
- ▶ Direct predictions of $\mathbf{F}_{\text{LCSR}}^{\gamma^*\gamma\pi}$ vs CELLO and CLEO data
- ▶ CLEO data on $\mathbf{F}_{\text{LCSR}}^{\gamma^*\gamma\pi} \Rightarrow$ endpoint-suppressed **Pion DA**
- ▶ Recent Lattice data confirm CLEO LCSR analysis and endpoint-suppressed **Pion DA** obtained from NLO QCD SR

Do the new BaBar data pose a puzzle?

Conclusions

$\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$,
factorization, and structure of $F^{\gamma^*\gamma^*\pi}$

$\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$ in pQCD



$$\int d^4x e^{-iq_1 \cdot z} \langle \pi^0(p) | T \{ j_\mu(z) j_\nu(0) \} | 0 \rangle =$$

$$i \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \cdot F^{\gamma^* \gamma^* \pi}(Q^2, q^2),$$

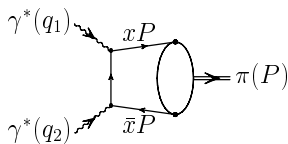
$$-q_1^2 = Q^2 > 0, \quad -q_2^2 = q^2 \geq 0$$

Collinear factorization at $Q^2, q^2 \gg (\text{hadron scale} \sim m_\rho)^2$

$$F^{\gamma^* \gamma^* \pi}(Q^2, q^2) = T(Q^2, q^2, \mu_F^2; x) \otimes \varphi_\pi^{(2)}(x; \mu_F^2) + O\left(\frac{1}{Q^4}\right)$$

μ_F^2 – boundary between large scale and hadronic one.

For leading twist 2 and at parton level



$$F^{\gamma^* \gamma^* \pi}(Q^2, q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \frac{1}{Q^2 x + q^2 \bar{x}} \varphi_\pi^{(2)}(x)$$

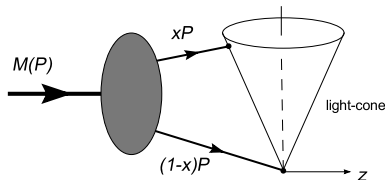
$$Q^2 F^{\gamma^* \gamma^* \pi}(Q^2, q^2 \rightarrow 0) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{dx}{x} \varphi_\pi^{(2)}(x)$$

$$\equiv \frac{\sqrt{2}}{3} f_\pi \langle x^{-1} \rangle_\pi$$

Distribution amplitudes in exclusive reactions

$$\langle 0 | \bar{q}(z) \gamma_\mu \gamma_5 E(z, 0) q(0) | \pi(P) \rangle \Big|_{z^2=0} = i P_\mu f_\pi \int dx e^{ix(zp)} \varphi_\pi^{(2)}(x, \mu_F^2)$$

$$E(z, 0) = P \exp\left(ig \int_0^z A_\mu(\tau) d\tau^\mu\right)$$



Distribution amplitudes are **nonperturbative** quantities to be derived from

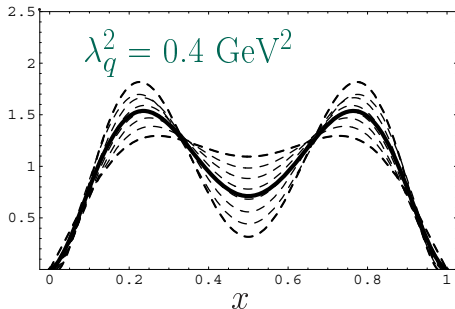
- ▶ QCD SR [CZ 1984],
NLC QCD SR [Bakulev&M&Stefanis 1998, 2001–04]
- ▶ instanton-vacuum approaches,
[Dorokhov *et al.* 2000; Polyakov *et al.* 1998, 2009]
- ▶ Lattice QCD, [Braun *et al.* 2006; Donnellan *et al.* 2007]
- ▶ from experimental data [Schmedding&Yakovlev 2000, BMS 2003–2006]

But DA evolves with μ_F^2 according to **ERBL equation in pQCD**

Pion distribution amplitudes in NLC QCD SR

NLC SRs for Pion DA produce “bunch” of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 = 1.35 \text{ GeV}^2$:

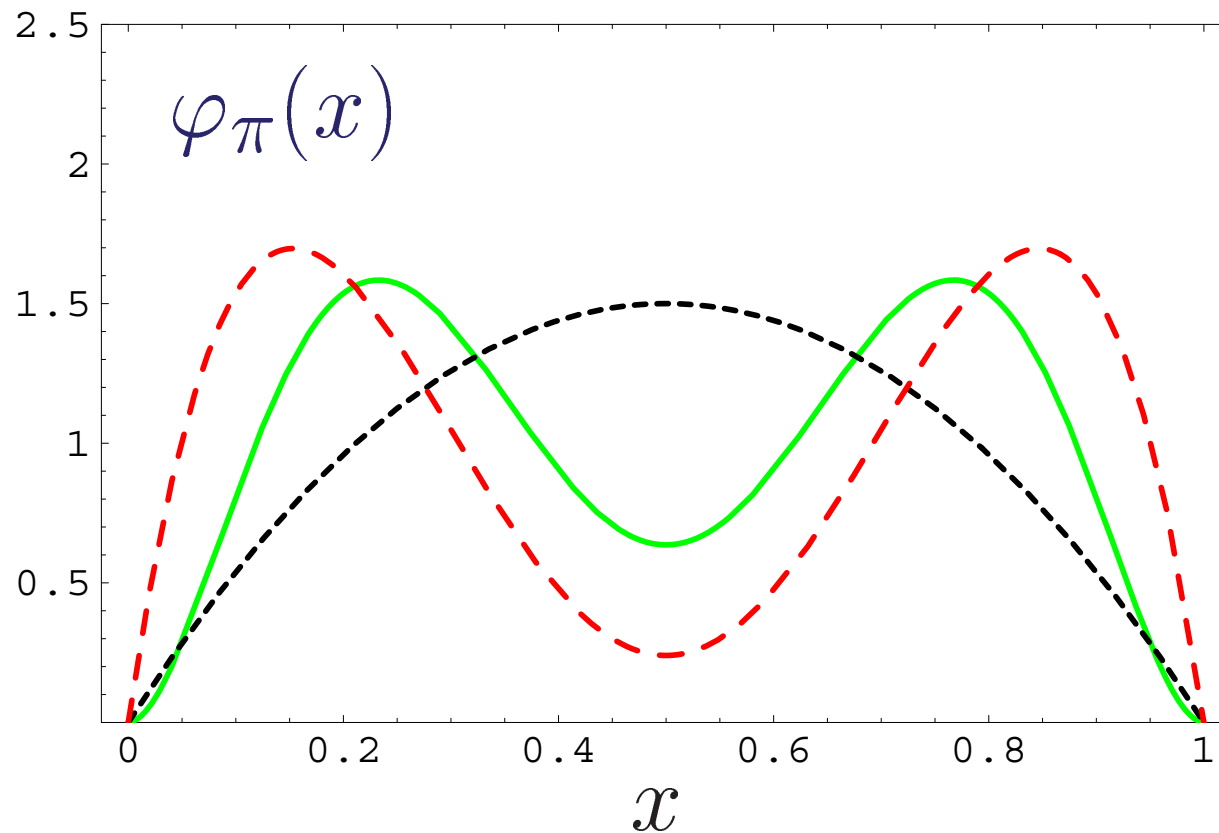
$$\varphi_\pi(x) = \varphi^{\text{as}(x)} \left[1 + a_2 C_2^{3/2}(2x-1) + a_4 C_4^{3/2}(2x-1) \right]$$



$a_2^{\text{b.f.}}$	$=$	$+0.188$
$a_4^{\text{b.f.}}$	$=$	-0.130
$\langle x^{-1} \rangle_\pi^{\text{b.f.}}$	$=$	3.17
χ^2	$\approx$	0.001
$\langle x^{-1} \rangle_\pi^{\text{NLC SR}}$	$=$	$3.30(30)$

λ_q^2 – average virtuality of vacuum quarks – *the* single parameter of NLC approach **BMS [PLB 508(2001)279]**

BMS vs CZ distribution amplitude



Curves	DAs
	CZ
	BMS
	Asymp.

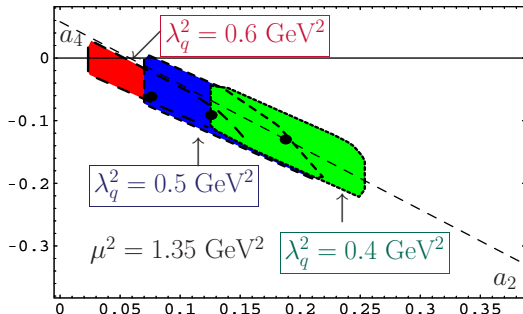
BMS bunch is 2-humped, but end-point suppressed!

Pion distribution amplitudes in NLC QCD SRs

$$\varphi_{\pi}^{(2)}(x; \mu_F^2) = \psi_0(x) + a_2(\mu_F^2) \psi_2(x) + a_4(\mu_F^2) \psi_4(x) + \dots$$

$$\varphi_{\pi}^{(2)} \Leftrightarrow \{a_n\}; \quad \text{partial waves: } \psi_n(x) = 6x\bar{x} C_n^{(3/2)}(x - \bar{x}) \quad (\text{Gegenbauer harmonics})$$

BMS estimates for a_2 , a_4 from NLC model



- ▶ **Green rectangle** forms **BMS “bunch”** of DAs, $\psi_0 + a_2\psi_2 + a_4\psi_4$
- ▶ ψ_0 – Asymptotic (Asy) DA
- ▶ **Chernyak-Zhitnitsky (CZ)** DA, $\psi_0 + a_2\psi_2$, $a_2(1) \approx 0.56$

NLO and NNLO amplitudes.

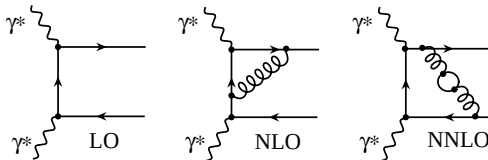
Collinear factorization Theorem [Efremov&Radyushkin 1978]

$$F^{\gamma^* \gamma^* \pi}(Q^2, q^2) \sim \left(T_0(Q^2, q^2; x) + a_s^1 T_1(Q^2, q^2; \mu_F^2; x) \right. \\ \left. + a_s^2 T_2(Q^2, q^2; \mu_F^2; \mu_R^2; x) + \dots \right) \otimes \varphi_\pi^{(2)}(x; \mu_F^2) \\ - \delta_{tw4}^2 \cdot T_0^2(Q^2, q^2; x) \otimes \varphi_\pi^{(4)}(x)$$

T_i calculable in pQCD, $a_s(\mu_R^2) = \alpha_s/(4\pi)$

$\mu_R^2 = \mu_F^2$ simplifies and $\mu_F^2 = \langle Q^2 \rangle$ minimizes radiative corrections

$\delta_{tw4}^2 = (0.19 \pm 0.02) \text{ GeV}^2$ - twist-4 scale parameter



NNLO: $a_s^2 T_2 \rightarrow a_s^2 \beta_0 T_{2\beta_0}$ [Melić&Müller&Passek PRD (2003)]

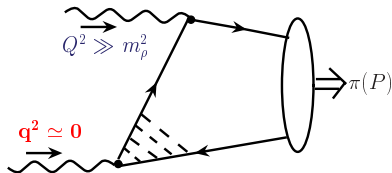
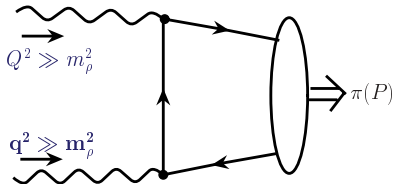
This contribution gives sign and size of NNLO effect following Brodsky-Lepage-Mackenzie prescription

Light Cone Sum Rules (LCSR)

Why Light Cone Sum Rules (LCSR)?

The experimental conditions prefer $q^2 \rightarrow 0$

For $Q^2 \gg m_\rho^2$, $q^2 \ll m_\rho^2$ pQCD factorization valid only in leading-twist approximation; hence, higher twists become important. Reason: if $q^2 \rightarrow 0$, one needs to take into account interaction of real photon at long distances of order of $O(1/\sqrt{q^2})$



pQCD is OK

photon behaves like a hadron

LCSR effectively accounts for long-distance effects of real photon using [Khodjamirian, EJPC (1999)]:

- ▶ **dispersion relation in variable q^2**
- ▶ **quark-hadron duality in vector channel.**

Dispersion relation for $F^{\gamma^* \gamma \pi}$

Main goal: spectral density ρ

$$F^{\gamma^* \gamma^* \pi}(Q^2, q^2) = \int_0^\infty ds \frac{\rho^{\text{ph}}(Q^2, s)}{s + q^2}$$

$$\rho^{\text{ph}} = \theta(s_0 - s) \rho^{\text{phen}}(Q^2, s) + \theta(s - s_0) \rho^{\text{PT}}(Q^2, s)$$

$$\rho^{\text{PT}}(Q^2, s) = \frac{\text{Im}}{\pi} \left[F^{\gamma^* \gamma^* \pi}(Q^2, -s) \right]$$

$$\rho^{\text{phen}}(Q^2, s) = \sqrt{2} f_\rho F^{\gamma^* V \pi}(Q^2) \cdot \delta(s - m_V^2) \Big|_{V=\rho, \omega}$$

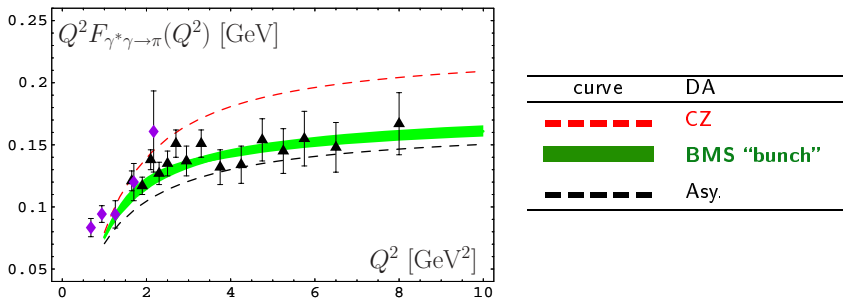
Quark-hadron duality in vector channel for $F^{\gamma^* V \pi}$ [Khodjamirian 1999]:

$$\begin{aligned} F^{\gamma \gamma^* \pi}(Q^2, q^2 \rightarrow 0) &= \frac{1}{\pi} \int_{s_0}^\infty \frac{\text{Im} F^{\gamma^* \gamma^* \pi}(Q^2, -s)}{s} ds \\ &+ \frac{1}{\pi} \int_0^{s_0} \frac{\text{Im} F^{\gamma^* \gamma^* \pi}(Q^2, -s)}{m_\rho^2} e^{(m_\rho^2 - s)/M^2} ds \end{aligned}$$

$s_0 \simeq 1.5 \text{ GeV}^2$ – effective threshold in vector channel,

M^2 – Borel parameter (0.5 – 0.9 GeV^2).

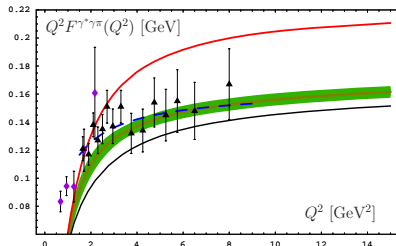
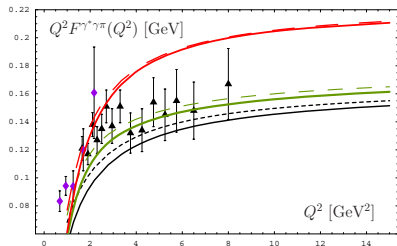
NLO LCSR vs. CELLO (◆) & CLEO (▲) data



Radiative corrections contribute up to **-17%** at low/moderate Q^2

- ▶ **BMS "bunch"** describes rather well all data above $Q^2 \gtrsim 1.5 \text{ GeV}^2$;
- ▶ Low- Q^2 CELLO data **excludes** Asy DA
- ▶ high- Q^2 CLEO data **excludes** CZ DA

NNLO _{β_0} LCSR vs. NLO LCSR



Dashed lines – NLO form factor

Solid lines – [NNLO _{β_0}] plus [Improved Breit-Wigner ρ^{phen} model]

- ▶ Combined effect: -7% at $Q^2 \simeq 2 \text{ GeV}^2$, -2% at $Q^2 \geq 6 \text{ GeV}^2$
- ▶ **Uncertainty** owing to uncalculated NNLO remainder expected to be less -10%

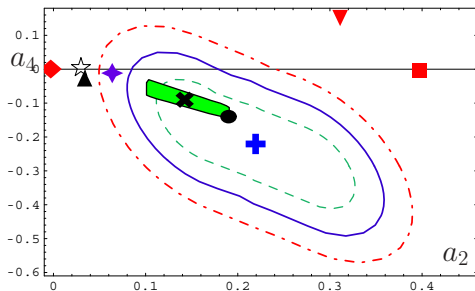
CLEO data on $F_{\text{LCSR}}^{\gamma^* \gamma \pi} \Rightarrow$ Pion DA in plane (a_2, a_4)

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus (\mu^2 = Q^2)$

with **20%** uncertainty of $\delta_{\text{Tw-4}}^2$: $\delta_{\text{Tw-4}}^2 = 0.19$ (4) GeV^2

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4 \text{ GeV}^2$

1 σ - green line; 2 σ - blue line; 3 σ -dashed-dotted red line.



at $\mu_{\text{SY}}^2 = 5.76 \text{ GeV}^2$

+ = best-fit BMS,

● = SY points

◆ = Asymptotic DA

■ = CZ DA, **▼** = Braun-Filyanov DA

X = BMS model

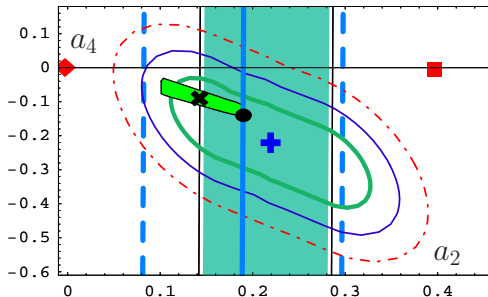
Even with 20% uncertainty in twist-4

CZ, BF DA excluded at least at 4 σ -level!

As DA — at 3 σ -level.

BMS DA and most of BMS “bunch” — inside 1 σ -domain.

Recent Lattice data vs. CLEO analysis and Pion DA



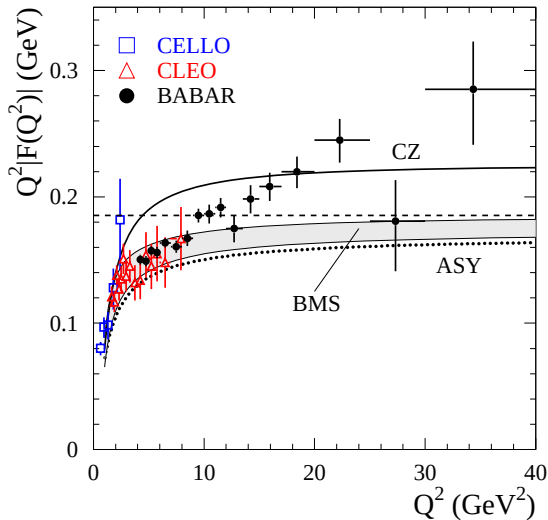
Comparison of the CLEO-data constraints on $F^{\gamma^* \gamma \pi}(Q^2)$ in the (a_2, a_4) plane at the scale $\mu_{SY}^2 = 5.76 \text{ GeV}^2$ in terms of error regions around the BMS best-fit point $+$
 1σ — green line; 2σ — blue line; 3σ —dashed-dotted red line.

Recent lattice simulations, denoted by vertical dashed lines [LAT2006] and solid ones [PoS LAT2007] are also shown together with predictions obtained from NLC QCD SR (slanted green rectangle)

Do the new BaBar data pose a puzzle?

New data [June 2009] on pion-photon transition form factor

by BaBar Collaboration [B. Aubert, Phys. Rev. D 80, 052002 (2009)]:

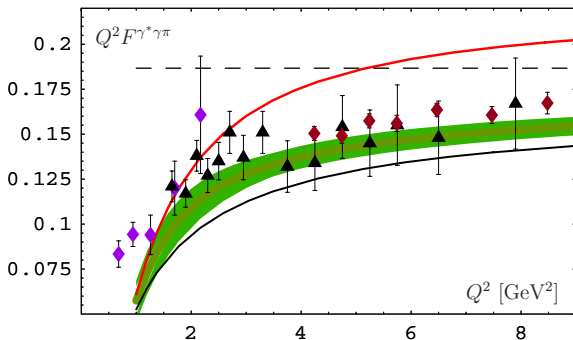


These authors claimed:

“BaBar data contradicts most models for the pion DA ...”

New data [June 2009] on pion-photon transition form factor

NNLO predictions and *all* data below 9 GeV²

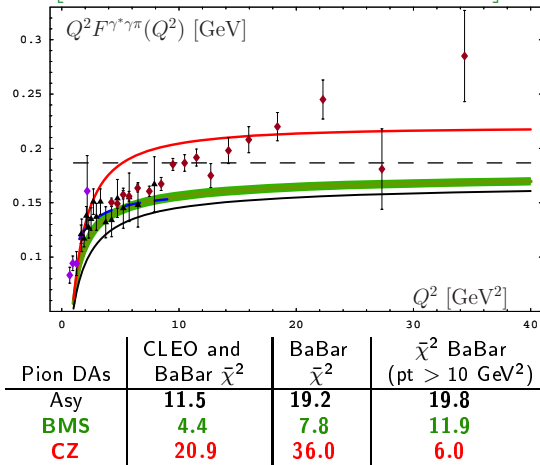


Observation: BMS “bunch” still provides comparatively best agreement with these data

More theoretical uncertainties pending [Mikhailov&Stefanis, work in progress]

Is the new BaBar data a kind of puzzle?

[Mikhailov&Stefanis, NPB 821, 291 (2009)]



The high- Q^2 BaBar data points—growing with Q^2 —contradict **all** pion DA models that have a finite projection on Gegenbauer polynomials, i.e., vanish at the endpoints 0, 1.

Is the new BaBar data a kind of puzzle?

'QCD like' Dipole fit, Λ, b – adjusted parameters

$$Q^2 F^{\gamma^* \gamma \pi} = \frac{Q^2}{2\sqrt{2}f_\pi} \left[\frac{\Lambda^2}{\Lambda^2 + Q^2} + b \left(\frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 \right]$$

	Cello&CLEO	BaBar
Cello&CLEO	0.48	7.8
BaBar	10.8	1.8

This is a hint that the Cello and CLEO and the BaBar data cannot be fitted simultaneously with the same accuracy.

First attempts to understand the “BaBar effect”

A possible scenario to explain the BaBar data was proposed by [Dorokhov arXiv:0905.3577], [Polyakov JETP Lett. (2009)] [Radyushkin arXiv:0906.0323], [H.-n. Li, S. Mishina PRD80 (2009)]:

“practically flat DA”: $\varphi_\pi \approx 1$ that generates the logarithmic growth in Q^2 and destroys the collinear factorization

	Cello& CLEO	BaBar
Cello& CLEO	1.22	15.8
BaBar	3.5	1.8

σ, m^2 – adjusted parameters for fit

$$Q^2 F^{\gamma^* \gamma \pi} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{1}{x} \left[1 - \exp \left(-\frac{x Q^2}{\bar{x} 2 \sigma} \right) \right] dx$$

$$\text{or} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{\varphi_\pi(x, Q^2)}{x + m^2/Q^2} dx \quad \text{with}$$

$$\varphi_\pi(x, \mu_0) = 1.3 - 0.3 \cdot 6x(1-x)$$

Conclusions

- ▶ BaBar data contradict the **collinear factorization formula and the “counting rules”**
- ▶ However, BaBar prefer to shy away from this conclusion.
They claimed instead agreement with QCD collinear factorization, provided one uses CZ-like pion DA.
- ▶ Provided the BaBar data are correct, **they constitute a challenge for QCD and may trigger important investigations**
- ▶ Great opportunity for other Collaborations, e.g., Belle, to solidify/disprove the BaBar effect

Buck up slides

NLO hard amplitudes and NLO evolution

NLO (last editions):

[Bakulev&MS&Stefanis(2003)],

[Melić&Müller&Passek-Kumerički(2003)]

$$T_1(x; Q^2, q^2) \otimes \varphi(x) = T_0(Q^2, q^2; y) \otimes \left\{ C_F \mathcal{T}^{(1)}(y, x) + \mathbf{L}(y) \cdot \mathbf{V}^{(0)}(y, x) \right\} \otimes \varphi(x; \mu_F^2)$$
$$\mathcal{T}^{(1)} = \left[-3 \mathbf{V}^b + \mathbf{g} \right] (x, y)_+ - 3\delta(x - y), \quad \mathbf{L}(y) \equiv \ln \left[(Q^2 y + q^2 \bar{y}) / \mu_F^2 \right]$$

$$\mathbf{g}(x, y) = -2 \frac{\theta(y > x)}{y - x} \ln(1 - x/y) + (x \rightarrow \bar{x}, y \rightarrow \bar{y})$$

$\varphi(x; \mu^2) \rightarrow \varphi(x; Q^2)$ evolves according to **NLO ERBL [79-80]** equation:

$$\mu^2 \frac{d}{d\mu^2} \varphi(x; \mu^2) = \left(a_s \mathbf{V}^{(0)}(x, y) + a_s^2 \mathbf{V}^{(1)}(x, y) \right) \otimes \varphi(y; \mu^2)$$

$$\mathbf{V}^{(0)} \otimes \psi_n = 2 C_F \mathbf{v}(n) \cdot \psi_n$$

Eigen functions: $\psi_n(x) = 6x\bar{x} C_n^{(3/2)}(x - \bar{x})$ – Gegenbauer harmonics

NNLO amplitude and coefficient functions

β_0 -part of NNLO: $T_2 \otimes \varphi \rightarrow \beta_0 \cdot T_\beta \otimes \varphi$, at $\mu_R^2 = \mu_F^2$
[Melić&Müller&Passek-Kumerički(2003)]

$$\begin{aligned} a_s^2 \beta_0 T_\beta \otimes \varphi = a_s^2 \beta_0 T_0 \otimes \left\{ C_F \mathcal{T}_\beta^{(2)} - C_F \mathbf{L}(\mathbf{y}) \cdot \mathcal{T}^{(1)} \right. \\ \left. + \mathbf{L}(\mathbf{y}) \cdot \left(\mathbf{V}_\beta^{(1)} \right)_+ - \frac{1}{2} \mathbf{L}^2(\mathbf{y}) \cdot \mathbf{V}^{(0)} \right\} \otimes \varphi. \end{aligned}$$

The origins of these terms:

$\sim \mathbf{V}^{(0)}$ – 1-loop ERBL-evolution together with RG- a_s one, while

$\sim \left(\mathbf{V}_\beta^{(1)} \right)_+$ – as the β_0 -part of 2-loop ERBL kernel;

$\mathcal{T}^{(1)}$ – from RG a_s evolution.

These terms together form the exponential RG-solution:

$$\exp \left\{ \int^{\mathbf{L}} \mathbf{V}(a_s(L)) dL \right\}$$

$\mathcal{T}_\beta^{(2)}$ – the coefficient function – **original, the most cumbersome part**

This contribution gives the sign and size of NNLO effect following to BLM prescription

NLO Spectral density $\rho^{(1)}$

$$\rho^{(1)}(Q^2, s) = \frac{\text{Im}}{\pi} [(T_1 \otimes \varphi_\pi)(Q^2, -s)] , \quad s \geq 0$$

$$\rho_n^{(1)}(x, \mu_F^2) \text{ for Gegenbauer harmonic } \psi_n, \quad \boxed{x = Q^2/(s + Q^2)}$$

The general case **[M&Stefanis NPB(2009)]** in press:

$$\begin{aligned} \bar{\rho}_n^{(1)}(x; \mu_F^2) = & C_F \left\{ -3 [1 - \mathbf{v}^a(n)] + \frac{\pi^2}{3} - \ln^2 \left(\frac{\bar{x}}{x} \right) + 2 \mathbf{v}(n) \ln \left(\frac{\bar{x}}{x} \frac{Q^2}{\mu_F^2} \right) \right\} \psi_n(x) \\ & - C_F 2 \left[\sum_{l=0,2,\dots}^n (\mathbf{G}_{nl} + \mathbf{v}(n) \cdot b_{nl}) \psi_l(x) \right] \end{aligned}$$

$\mathbf{G}_{nl}$ (originates from $\mathbf{g}$), b_{nl} – calculable triangular matrices

The partial case $\bar{\rho}_0^{(1)}$ **[Schmedding&Yakovlev (2000)]**:

$$\bar{\rho}_0^{(1)}(x) = C_F \left[-5 + \frac{\pi^2}{3} - \ln^2 \left(\frac{\bar{x}}{x} \right) \right] \psi_0(x)$$

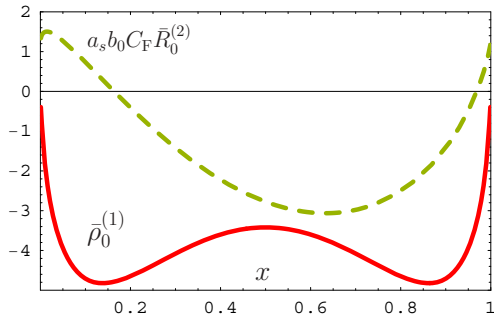
Conclusion: The spectral density and $F^{\gamma\gamma^*\pi}$ can be obtained for any numbers of Gegenbauer harmonics

NNLO _{β_0} Spectral density

$$\rho^{(2)}(Q^2, s) = \frac{\text{Im}}{\pi} \left[(T_2 \otimes \varphi_\pi)(Q^2, -s) \right], \quad s \geq 0$$

$$x = Q^2/(s + Q^2), \quad \text{put } \mu_R^2 = \mu_F^2$$

$$\bar{\rho}_n^{(2)} \rightarrow b_0 C_F \left[\bar{R}_n^{(2)} \left(x; \frac{\bar{x}}{x} \frac{Q^2}{\mu_F^2} \right) \right].$$

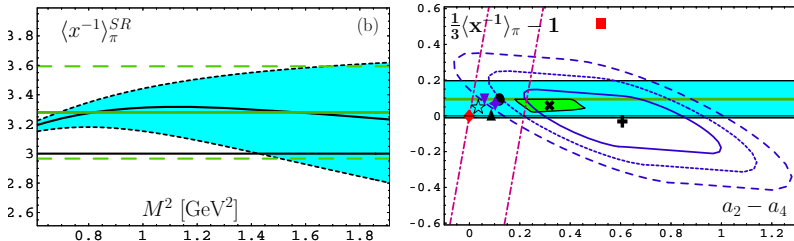


The dashed green line shows $a_s(\mu_F^2) \bar{\rho}_0^{(2)} = a_s(\mu_F^2) b_0 C_F \bar{R}_0^{(2)}(x, \bar{x}/x)$ at the typical CLEO reference scale $Q^2 = \mu_F^2 = \langle Q^2 \rangle = (2.4 \text{ GeV})^2$, whereas the solid red line represents $\bar{\rho}_0^{(1)}(x)$

Independent estimate of **Pion DA** inverse moment $\langle x^{-1} \rangle_\pi$

BMS [Ann. Phys.(Leipzig)13(2004) 629]:

processing of CLEO data is evolved to normalization scale $\mu^2 \simeq 1 \text{ GeV}^2$



$$\lambda_q^2 = 0.4 \text{ GeV}^2, \quad \frac{1}{3}\langle x^{-1} \rangle_\pi^{SR} - 1 = 0.1 \pm 0.1$$

Good agreement of a theoretical “tool” of different origin with CLEO data

Conclusion

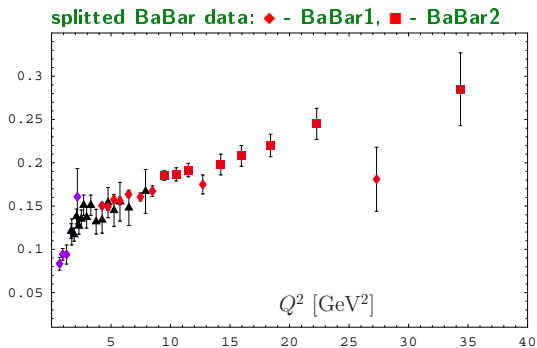
- ▶ **NLO LCSR** produces **constraints** on pion DA parameters ($\mathbf{a_2, a_4}$) in conjunction with CLEO data. The same can be done for any number of parameters $\{\mathbf{a_n}\}$.
- ▶ **QCD SR** with **NLC** for pion DA gives **admissible sets** of DAs and, independently, the inverse moment $\langle \mathbf{x^{-1}} \rangle_{\pi}^{SR}$
- ▶ **This NLO LCSR constraints agree well** with:
the bunch of pion DAs and $\langle \mathbf{x^{-1}} \rangle_{\pi}^{SR}$,
and recent **lattice** data.
- ▶ **NNLO LCSR** correction does no change $F^{\gamma^* \gamma \pi}(Q^2)$ significantly.
The common effect of NNLO_{β_0} and improved hadron model leads:
from **-7% at $Q^2 \simeq 2 \text{ GeV}^2$** , to **$-2\%$ at $Q^2 \geq 6 \text{ GeV}^2$**

We thank **A. P. Bakulev** for collaboration and help in the preparation of this report.

Looks tempting to consider BaBar data as **two different experiments**:
BaBar1 and BaBar2

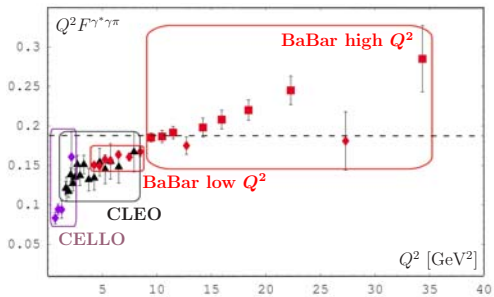
“Flat DA” fit

	◆ BaBar1	■ BaBar2
BaBar1	3.3	0.33
BaBar2	3.5	0.26



This gives a hint that the BaBar data is not in good agreement with the
 “flat description”

Looks tempting to consider BaBar data as **two different experiments**:



Conclusions

- ▶ This is a hint that the “old data” – Cello and CLEO and the “new data” – BaBar **cannot be fitted simultaneously.**
- ▶ **From the theoretical side, it is not possible to describe these data with the inclusion of higher radiative corrections– as we have shown at the NNLO level.**