

Contributions of semi-hadronic states $P\gamma; S\gamma$ to amm of muon.

Extraction of mesons form factors

A.I.Ahmadov,^{*} E.A.Kuraev,[†] and M.K.Volkov[‡]

JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

We calculate the contribution of semi-hadronic states with pseudoscalar(scalar) meson accompanied with real photon as an intermediate state of a heavy photon to the anomalous magnetic moment of muon on the level of a few units 10^{-10} . Both the theoretical and experimental approaches are used to calculate the relevant cross sections in e^+e^- annihilation. The measurement of these processes can shed the light to the problem of mesons form-factors first discussed in the single pion production in the scattering channel.

I. INTRODUCTION

One of modern precise test of Standard Model is the measurement of anomalous magnetic moment of muon (amm) a_μ . The SM contributions are usually split into three parts:

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr}$$

The main part of contribution to a_μ^{hadr} of order 510 (units 10^{-10} implied) arise from $\pi_+\pi_-$ channel annihilation of e^+e^- pair. A "semi-hadronic" process, which takes into account the contributions of light-by-light type block with scalar and pseudo-scalar mesons.

Below we will consider the annihilation channels

$$e^+e^- \rightarrow \gamma^* \rightarrow P\gamma; S\gamma, P = \pi_0, \eta, \eta'; S = \sigma, f_0, a_0. \quad (1)$$

Besides the check of validity of low-energy hadronic models predictions, we want to stress

^{*}Electronic address: ahmadov@theor.jinr.ru

[†]Electronic address: kuraev@theor.jinr.ru

[‡]Electronic address: volkov@theor.jinr.ru

the attention on the possibility to measure the mesons form-factors, describing the conversion of a heavy photon (with the square momentum $q^2 > 0$) to meson and a real photon.

Contributions to amm are:

$$a_\mu^{P(S)\gamma} = \frac{1}{4\pi^3} \int_{m_{P(S)}}^{\infty} ds \sigma^{e\bar{e} \rightarrow P(S)\gamma}(s) \cdot K^{(1)}\left(\frac{s}{M_\mu^2}\right). \quad (2)$$

The the analytic form of the kernel $K(\rho)$ is:

$$\begin{aligned} K^{(1)}(\rho) &= \int_0^1 \frac{x^2(1-x)dx}{x^2 + (1-x)\rho}; \\ K(\rho) &= \frac{1}{2} - \rho + \frac{1}{2}\rho(\rho-2) \ln \rho - \frac{(\rho^2 - 4\rho + 2)\rho}{2\sqrt{\rho(\rho-4)}} \\ &\quad \ln \frac{\sqrt{\rho} + \sqrt{\rho-4}}{\sqrt{\rho} - \sqrt{\rho-4}}; \\ \rho &= \frac{s}{M_\mu^2}; \quad K^{(1)}(\rho)|_{\rho \gg 1} = \frac{1}{3\rho}. \end{aligned} \quad (3)$$

We will use the matrix element of sub-process $\gamma^*(q, \mu) \rightarrow P(p)\gamma(k, \nu)$

$$M^{\gamma^* \rightarrow P\gamma} = \frac{\alpha}{\pi f_\pi} F_P(q^2) \varepsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta \epsilon^\mu(k) \epsilon^\nu(q), \quad (4)$$

with initial condition $F(0) = 1$. We remind the current algebra expression for the pion decay width

$$\Gamma^{\pi_0 \rightarrow 2\gamma} = \alpha^2 M_\pi^3 / (64\pi^3 f_\pi^2) \approx 7.1 eV$$

.

The similar expression for $\gamma^*(q, \mu) \rightarrow S(p)\gamma(k)$:

$$M^{\gamma^* \rightarrow S\gamma} = \frac{\alpha}{\pi f_\pi} F_S(q^2) (g_{\mu\nu} \cdot kq - q_\nu \cdot k_\mu) \epsilon^\mu(k) \epsilon^\nu(q). \quad (5)$$

Total cross sections of creation $P\gamma$, $S\gamma$ in electron-positron annihilation are:

$$\sigma_{theor}^{e\bar{e} \rightarrow p\gamma} = \frac{\alpha^3 |F(s)|^2}{24\pi^2 f_\pi^2} \left(1 - \frac{M_P^2}{s}\right)^3, \quad f_\pi = 93 MeV, \quad (6)$$

$$\sigma^{e\bar{e} \rightarrow \gamma s} = \frac{\alpha^3}{24 f_\pi^2 \pi^2} |F_s(s)|^2 \left(1 - \frac{M_s^2}{s}\right)^3. \quad (7)$$

We remind here the general advantage of describing the annihilation processes

$$e^+e^- \rightarrow \pi_0\gamma; 2\pi_0\gamma, \sigma\gamma, a_0\gamma, f_0\gamma$$

in absence of initial state radiation.

When considering the relevant contribution to a_μ we can omit the effect of final state radiation corrections.

Taking into account the effects of vacuum polarization in paper [1] was inferred

$$\begin{aligned}\sigma(e^+e^- \rightarrow \pi_0\gamma)(s) &= \sigma_{pt}\left(\frac{M_\omega^2}{M_\omega^2 - s}\right)^2, \\ \sigma_{pt}(s) &= \frac{8\pi\alpha}{3m_\pi^3}\Gamma(\pi_0 \rightarrow 2\gamma)\left(1 - \frac{m_\pi^2}{s}\right)^3.\end{aligned}\tag{8}$$

which help to estimate the contribution out the experimentally accessed region $0.6 < \sqrt{s} < 1.03\text{GeV}$. It was obtained [4]

$$\begin{aligned}a_\mu(\pi_0\gamma, 0.6 < \sqrt{s} < 1.03\text{GeV})^{exp} &= (4.5 \pm 0.15) \times 10^{-10}; \\ a_\mu(\eta\gamma, 0.69 < \sqrt{s} < 1.33\text{GeV})^{exp} &= (0.73 \pm 0.03) \times 10^{-10}.\end{aligned}\tag{9}$$

The contribution from the region below the experimentally accessible region is

$$a_\mu(\pi_0\gamma, \sqrt{s} < 0.6\text{GeV}) = (0.13 \pm 0.01) \times 10^{-10}.\tag{10}$$

The contribution of radiative processes with charged pions production and 2 neutral ones was found [11] to be

$$\begin{aligned}a_\mu(e^+e^- \rightarrow \pi^+\pi^-\gamma, \sqrt{s} < 1.2\text{GeV}) &= (38.6 \pm 1.0) \times 10^{-11}; \\ a_\mu(e^+e^- \rightarrow \pi^+\pi^-\gamma, \sqrt{s} > 1.2\text{GeV}) &= (4 \pm 1.0) \times 10^{-11}.\end{aligned}\tag{11}$$

Total contribution $\pi_0\gamma; \eta\gamma; 2\pi\gamma$ is

$$a_\mu(e^+e^- \rightarrow \text{had}\gamma) = (93 \pm 1.0) \times 10^{-11}.\tag{12}$$

The 2 pions radiative events presumably are connected with $\sigma\gamma$ channel.

The convergence of the loop momentum integral for a_μ provide the application of such a low-energy models as Nambu-Iona-Lasinio (NJL) one [2] for description of processes of conversion of a virtual photon to light mesons and in particular to mesons and a real photons. For process $e^+e^- \rightarrow \sigma\gamma; \eta\gamma$; S. Narison had obtained [3]:

$$a_\mu(\sigma(600)\gamma) = (0.1 - 1.0) \times 10^{-10}.\tag{13}$$

We see that the application of QCD sum rules and using the PDG data are in conflict.

In NJL approach we obtain for $e^+e^- \rightarrow \sigma(600)\gamma; e^+e^- \rightarrow f_0(980)\gamma; e^+e^- \rightarrow a_0(980)\gamma$:

$$a_\mu(\sigma(600)\gamma) = (6.17 \pm 0.15) \times 10^{-14}, 0.6 < \sqrt{s} < 4GeV, \quad (14)$$

$$a_\mu(f_0(980)\gamma) = (2.55 \pm 0.25) \times 10^{-13}, 0.980 < \sqrt{s} < 4GeV, \quad (15)$$

$$a_\mu(a_0(980)\gamma) = (2.33 \pm 0.10) \times 10^{-14}, 0.980 < \sqrt{s} < 4GeV. \quad (16)$$

Here both mechanisms quark loops and mesons loops are become relevant. Namely in $f_0\gamma$ main contributions arises from s quarks and kaon loops which partly compensate each other $\Gamma_{f_0 \rightarrow 2\gamma} = 0.33KeV$.

For the case $a_0\gamma$ main contribution arises from u, d quark loops and kaon loop and they as well partly compensate each other resulting the width $\Gamma_{a_0 \rightarrow 2\gamma} = 0.29KeV$.

For the case $\sigma\gamma$ Main contributions arises from light quarks and from the pion loop with constructive interference, resulting $\Gamma_{\sigma \rightarrow 2\gamma} = 4.3KeV$.

We see that the version with applying the QCD sum rules is confirmed by NJL model but in contradiction with real CMD data [4].

Analog of semi hadronic contributions is the light by light (L-b-L) scattering mechanism with intermediate states with scalar and pseudo scalar mesons.

Convergence of different recent model calculations lead to the result (see [10], A. Nyfeller talk)

$$a_\mu^{L-b-L} = (10.5 \pm 2.6) \times 10^{-10}. \quad (17)$$

We put below the definite contributions (we follow the paper [7]):

$$\begin{aligned} \pi_0 : 81.8 \times 10^{-11}; \eta : 5.62 \times 10^{-11}; \eta' : (8 \pm 1.7) \times 10^{-11}; \\ \sigma(600) : 11.67 \times 10^{-11}; a_0(980) : 0.62 \times 10^{-11}, \end{aligned} \quad (18)$$

with the total contribution

$$a_\mu^{LbL} = (107.74 \pm 16.81) \times 10^{-11}. \quad (19)$$

In the paper [8] the new method of taking into account the hadronic contributions in leading (LO) and non-leading (NLO) approximations was suggested. Namely the to-day

accepted method consists in rather cumbersome procedure of extracting the "bare" hadronic cross section $\sigma^0(s)$, eliminating the effect of a moving coupling constant. For this aim the factor $(\alpha/\alpha(s))^2$ must be introduced using the experimentally measured cross section:

$$\sigma^0(s) = (\alpha/\alpha(s))^2 \sigma_{exp}(s). \quad (20)$$

This procedure imply the using the another experimental program of precise measuring of a coupling constant $\alpha(s)$, which is as complicate as the problem of $g - 2$ ones.

We suggest to use $\sigma_{exp}(s)$ directly. It implies some modification of the non-leading contribution to the kernel K . The relevant formula reads as

$$a_\mu^{hadr} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma^{exp}(s) [K^{(1)}(s) + \frac{\alpha}{\pi} \bar{K}^{(2)}(s)] ds, \quad (21)$$

with (we use the notations of a paper of F. Krause [9]):

$$\bar{K}^{(2)}(s) = K^{(2a)}(s) - K^{(2b)}(s)|_{m_f=M_\mu}. \quad (22)$$

Expansion on the powers of $\rho = \frac{s}{M_\mu^2}$ is:

$$\begin{aligned} \bar{K}^{(2)}(s) = & \frac{2}{\rho} [a_1 + b_1 L + \frac{1}{\rho} (a_2 + b_2 L + c_2 L^2) + \\ & \frac{1}{\rho^2} (a_3 + b_3 L + c_3 L^2) + \frac{1}{\rho^3} (a_4 + b_4 L + c_4 L^2)] + \\ & O((\frac{1}{\rho})^5), \end{aligned} \quad (23)$$

with $L = \ln \frac{s}{M_\mu^2}$ and

$$\begin{aligned} a_1 &= 0.76; & b_1 &= -0.638; \\ a_2 &= -6.25; & b_2 &= -2.25; & c_2 &= 0.3; \\ a_3 &= 2.3; & b_3 &= -8.13; & c_3 &= 2.76; \\ a_4 &= 5.2; & b_4 &= -32.17; & c_4 &= 15.67. \end{aligned} \quad (24)$$

We remind the to-day values of hadronic contributions in leading and next-to leading approximations ([10], T. Tuebner talk)(units 10^{-10} implied):

$$\begin{aligned} a_\mu(LO, had) &= 689.41 \pm 3.61_{exp} \pm 1.82_{rad}; \\ a_\mu(NLO, had) &= -9.79 \pm 0.06_{exp} \pm 0.03_{rad}. \end{aligned} \quad (25)$$

Keeping in mind the recent data of BNL experiment of direct measurement a_μ :

$$a_\mu = 116592089(63) \times 10^{-11} (0.5 ppm), \quad (26)$$

we have for difference of experimental and theoretical predictions:

$$a_\mu^{exp} - a_\mu^{TH} = (31.6 \pm 7.9) \times 10^{-10} \approx 4\sigma. \quad (27)$$

This 4 standard deviation difference can be considered as the place for a "new physics" predictions in lowest order of perturbation theory:

$$a_\mu^{SUSY} = \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{M_\mu^2}{M_{SUSY}^2} \text{tg } \beta, \text{tg } \beta = 10. \quad (28)$$

Some information can be extracted from the annihilation channels about meson form factors.

We remind the problem related with the neutral pion form factor measured in Cornell Laboratory in the experiments $e^+e^- \rightarrow e^+e^-\pi_0$, where this quantity was measured in the scattering channel $q^2 < 0$. It was found the clear resemblance with the low energy QCD predictions-namely form factor exhibits the power-like growth with $Q^2 = -q^2$.

In Cornell experiments the differential cross section of a process

$$\begin{aligned} e^+(p_+)e^-(p_-) &\rightarrow e^+(p'_+)e^-(p'_-)\pi_0(q_0), \\ s + (p_+ + p_-)^2 &\gg Q^2 = -(p_+ - p'_+)^2 = \bar{q}^2 \gg m_\pi^2, \end{aligned} \quad (29)$$

was measured in so called Weiszacker-Williams approximation

$$\begin{aligned} \frac{d\sigma^{ee \rightarrow ee\pi^0}}{d\bar{q}^2} &= \frac{|F(\bar{q}^2)|^2}{32\pi^3 \bar{q}^2 f_\pi^2} \int_{\frac{\bar{q}^2 + m_\pi^2}{s}}^1 \frac{d\beta_1}{\beta_1} \left(\ln \frac{\bar{q}^2}{m_e^2 \beta_1^2} - 1 \right) \approx \\ &\frac{|F(\bar{q}^2)|^2}{32\pi^3 \bar{q}^2 f_\pi^2} \times [\ln \frac{\bar{q}^2}{m_e^2} (L_q - 1) + L_q^2], L_q = \ln \frac{s}{\bar{q}^2}. \end{aligned} \quad (30)$$

Investigation of relevant squares of module of meson form-factors in the annihilation channel is in progress.

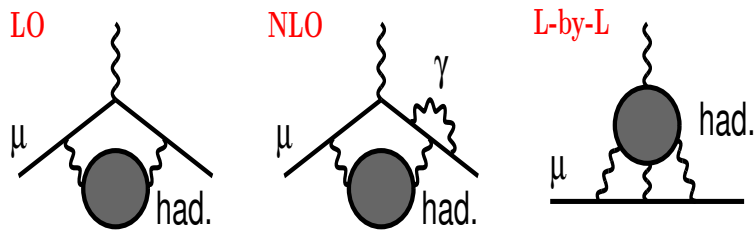
We are grateful to Yu.M. Bystritskii for help.

-
- [1] N. Achasov and A. Kiselev, Phys.Rev. D 65,097302 (2002).
 - [2] M. Volkov et. al, hep-ph 0901.1981.
 - [3] S. Narison, Phys. Lett. B 568(2003),231-236.
 - [4] R. Akhmetishin et. al, Phys. Lett. B 605(2005),26-36.
 - [5] A.Nyffeler, Nucl.Phys. Proc.Suppl. **131**,162 (2004)
 - [6] A.Czarnecki and W.J.Marciano, Phys.Rev.D **64**, 013014 (2001)
 - [7] E. Bartosh,et. al,Nucl. phys. B632 (2002),330.
 - [8] Yu. Bystritskii et. al, ZhETP, Pisma, 83,N2 p 57 (2006).
 - [9] B. Krause, Phys. Lett. B **390**, 392(1997).
 - [10] International Workshop on e^+e^- collisions from Φ to Ψ , 2009.
 - [11] F. Troconiz and F. Yndurain, Phys.Rev. D 65,093001, (2002).

I. Recent developments in $g - 2$

- **QED**: Predictions consolidated, further work (numerical five-loop) ongoing, big surprises very improbable, error formidably small: $a_\mu^{\text{QED}} = 116584718.08(15) \cdot 10^{-11}$ ✓
- **EW**: reliable two-loop predictions, accuracy fully sufficient: $a_\mu^{\text{EW}} = (154 \pm 2) \cdot 10^{-11}$ ✓
- **Hadronic contributions**: uncertainties completely dominate Δa_μ^{SM} !

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



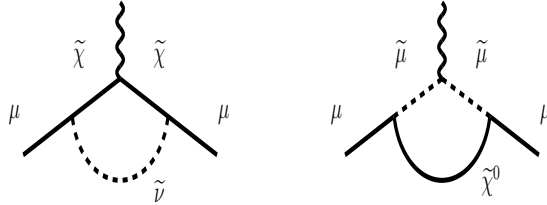
- Hadronic contributions from low γ virtualities not calculable with perturbative QCD
- For VacPol contributions use of *dispersion relations*, with exp. data for $\sigma_{\text{had}}^0(s)$:

$$a_\mu^{\text{had,LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} ds \sigma_{\text{had}}^0(s) K(s), \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.63 \dots 1)$$

SUSY contributions in a_μ ?

$$a_\mu^{\text{SUSY}, 1\text{-loop}} \simeq \frac{\alpha}{8\pi \sin^2 \theta_W} \tan \beta \operatorname{sign}(\mu) \frac{m_\mu^2}{M_{\text{SUSY}}^2}$$

They mainly come from:



SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$, but

- no chargino at LEP
- so far no light Higgs
- $\tilde{\tau}$ prob. not LSP
- + limits from direct searches
- SPS 1a' in 1σ band from $g-2$

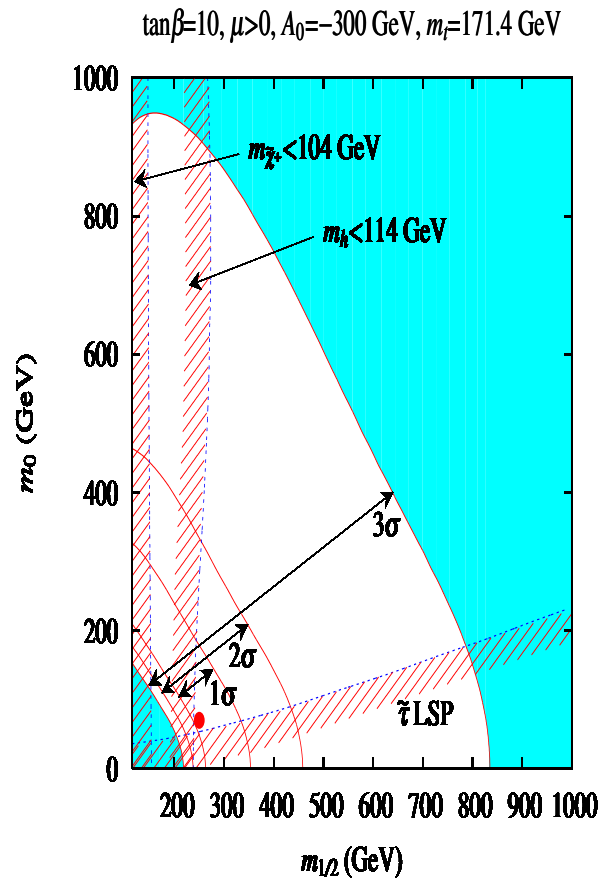


Рис. 2: ааа

a_μ^{SM} : the QED contribution

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857408 (27) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$$+ 24.05050959 (42) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;

Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell '08

$$+ 130.805 (8) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... ; Kinoshita & Nio '04, '05;

Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$$+ 663 (20) (\alpha/\pi)^5 \quad \text{In progress}$$

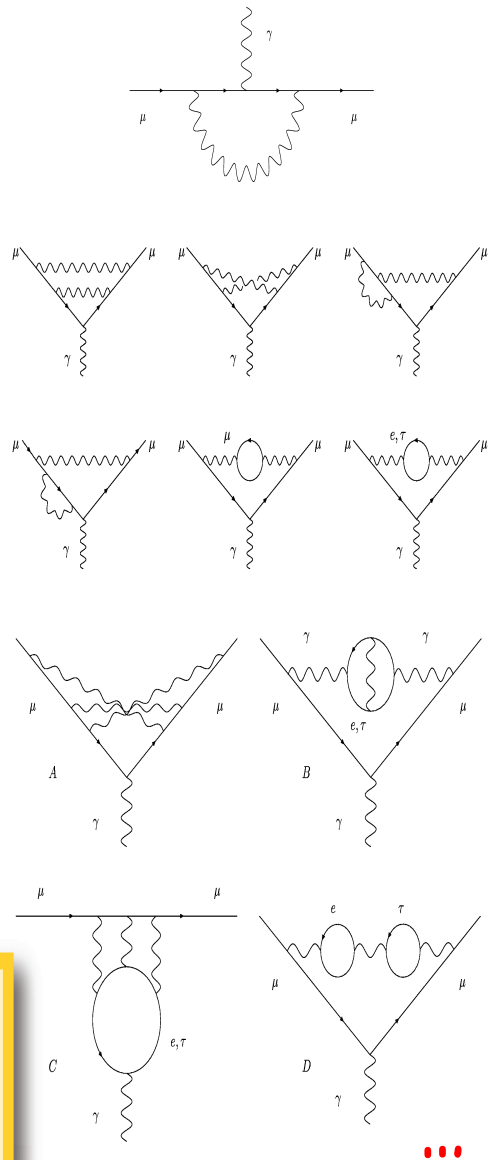
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim, ..., Kataev, Kinoshita & Nio March '06.

Adding up, we get:

$$a_\mu^{\text{QED}} = 116584718.08 (14)(04) \times 10^{-11}$$

from coeffs, mainly from 5-loop unc  from new $\delta\alpha$ ('08) 

with $\alpha=1/137.035999084(51)$ [0.37 ppb]



a_μ^{SM} : the hadronic higher-order (HHO) contributions

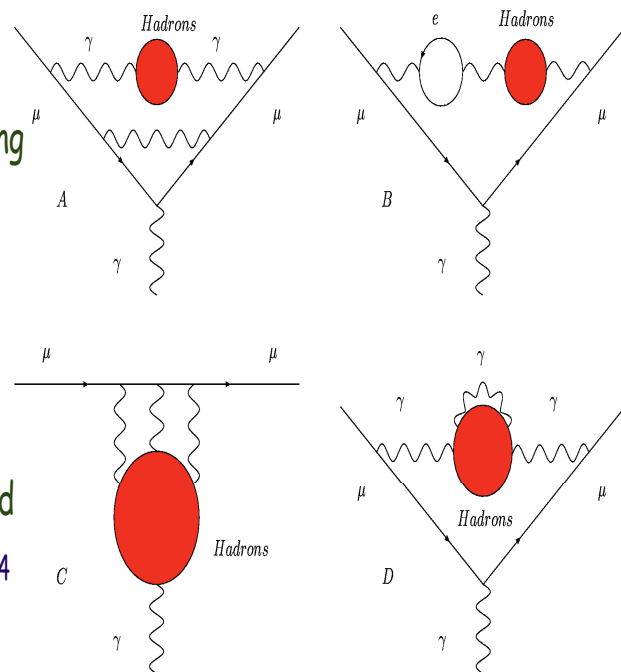
● Vacuum Polarization

$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{\text{HHO}(\text{vp})} = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. '03 & '06

Shifts by $\sim -3 \times 10^{-11}$ if tau data are used instead of the e^+e^- ones Davier & Marciano '04



● Light-by-Light

Recent values of the contribution of the hadronic LBL diagrams:

$$\begin{aligned} a_\mu^{\text{HHO}(\text{lb})} &= +105 (26) \times 10^{-11} && \text{Prades, de Rafael, Vainshtein '09} \\ &= +116 (39) \times 10^{-11} && \text{Jegerlehner & Nyffeler '09} \end{aligned}$$

This contribution will likely become the ultimate limitation of the SM prediction.

