

Muon g-2, Rare Decays $P \rightarrow l^+l^-$ and Transition Form Factors $P \rightarrow \gamma\gamma^*$

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Introduction

Lepton anomalous magnetic moments (status)

Rare $\pi^0 \rightarrow e^+e^-$ Decay (status)

$\pi^0 \rightarrow e^+e^-$ Decay and Dark Matter

Transition form factors and rare decays

Conclusions

Introduction

Abnormal people are looking for traces of Extraterrestrial Guests

Abnormal Educated people are looking for hints of New Physics

Cosmology tell us that 95% of matter was not described in text-books yet

Two search strategies:

1) High energy physics to excite heavy degrees of freedom.

No any evidence till now. Waiting for LHC era.

2) Low energy physics to produce Rare processes in view of huge statistics.

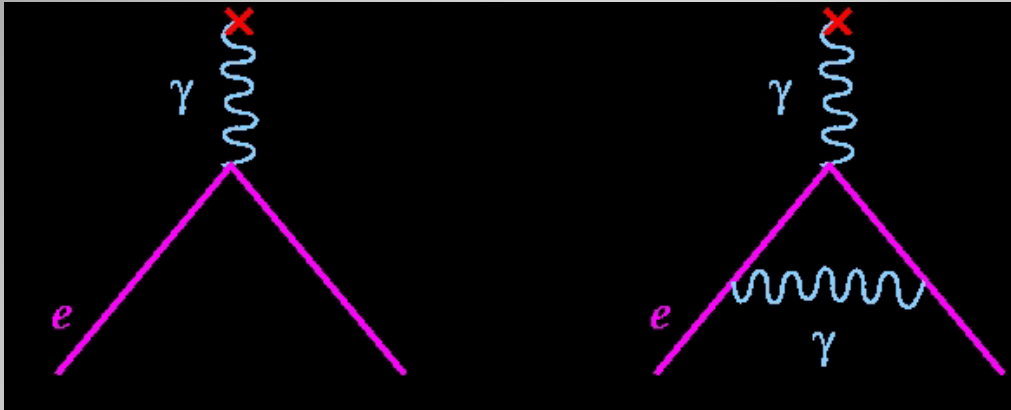
There are some rough edges of SM.

$(g-2)_\mu$ is very famous example,

$\pi_0 \rightarrow e^+e^-$ is in the list of SM test after new exp. and theor. results

That's intriguing

The lowest order radiative correction



$$\Gamma_{\mu} = e\gamma_{\mu} + a_{\ell} \frac{ie}{2m} \sigma_{\mu\nu} q_{\nu}$$

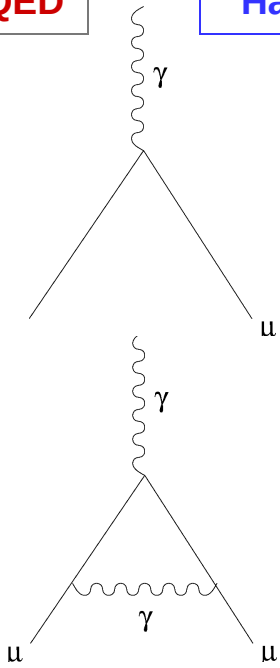
$$a_{\ell} = (g_{\ell} - 2) / 2$$

$$a = \frac{\alpha}{2\pi} = 0.001161$$

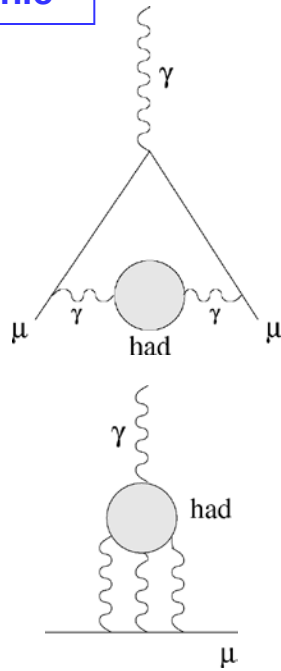
Schwinger, 1948
(Nobel prize 1965)

Magnetic Anomaly

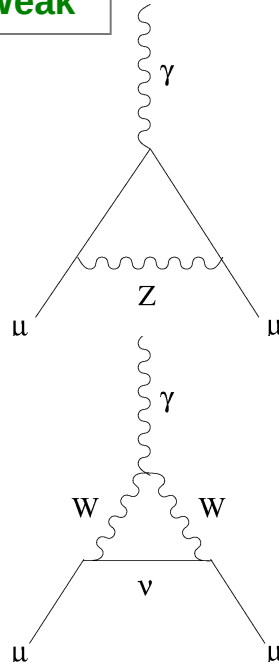
QED



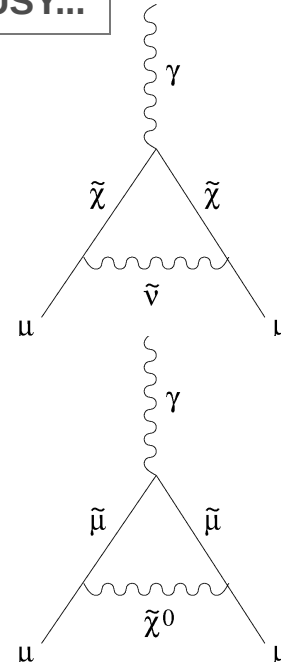
Hadronic



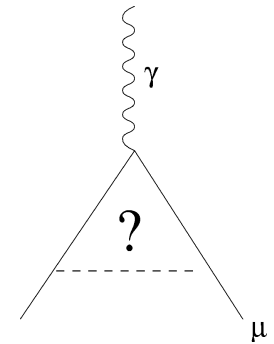
Weak



SUSY...



... or other new physics ?



Lepton Anomalies

- Electron anomaly is measured extremely accurately.
QED test.
- It is the best for determining α
- For a lepton L, Mass Scale contributes to a_L as (m_L^2 / Λ^2)
- Tau anomaly is difficult to measure since its fast decay
- Muon anomaly is measured to 0.5 parts in a million (ppm) *SM test.*
- Thus muon AMM leads to a $(m_\mu/m_e)^2 \sim 40\,000$ enhancement of the sensitivity to New Physics versus the electron AMM, the muon anomaly is sensitive to ≥ 100 GeV scale physics.

Electron AMM

To measurable level a_e arises entirely from virtual electrons and photons

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \text{ [0.24 ppb] Harvard 2008}$$

$$a_e^{\text{SM}} = a_e(\text{QED}) + a_e(\text{hadron}) + a_e(\text{weak}),$$

$$a_e(\text{QED}) = \sum_{n=1}^5 C_{2n} \left(\frac{\alpha}{\pi} \right)^n + \dots$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling $a \equiv e^2/(4\pi)$

$$\alpha^{-1} = 137.035\,999\,084(51) \text{ [0.37 ppb]}$$

Das ist fantastisch!

QED is at the level of the best theory ever built to describe nature

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Tau anomaly

- **Tau** due to its highest mass is the best for searching for **New Physics**,
- But **Tau** is short living particle, so the **precession method** is not perspective
- The best existing limits (see S. Eidelman, M. Passera

$$-0.052 < a_{\tau}^{\text{Exp}} < 0.013$$

are obtained at OPAL and L3 (LEP, CERN) from the high energy process

$$e^+e^- \rightarrow e^+e^- \tau^+\tau^- ,$$

- While the **SM** estimate is

$$a_{\tau}^{\text{SM}} = 1.17721(5) \cdot 10^{-3}$$

Lepton Anomalies

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Anomalous magnetic moment of muon

From BNL E821 experiment (1999-2006)

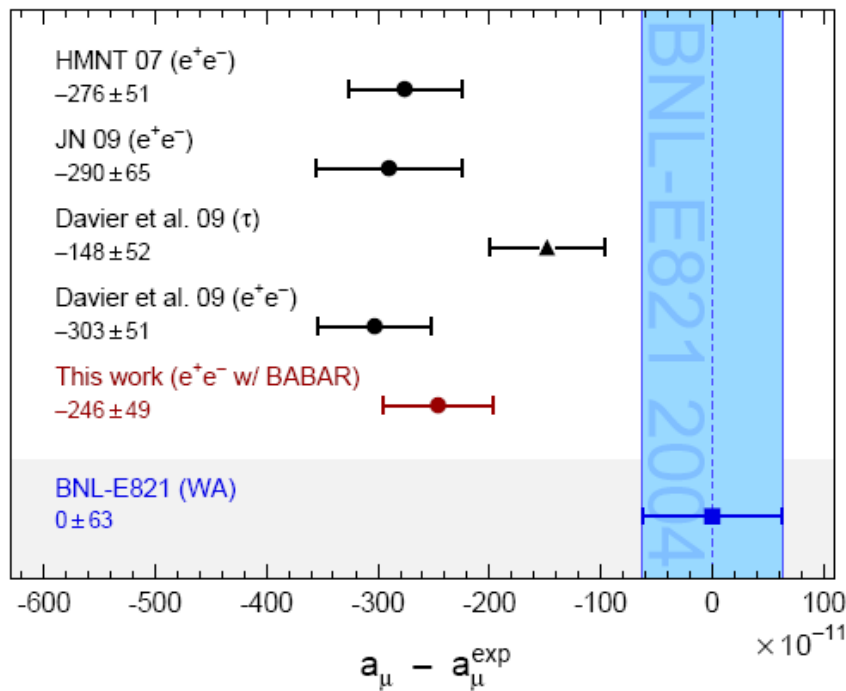
New proposals for
BNL, FNAL, JPARC

$$a_{\mu}^{\text{BNL}} = 11\,659\,208.0(6.3) \cdot 10^{-10}$$

Standard Model

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{Hadr}} = 11\,659\,177.3(5.0) \cdot 10^{-10}$$

predicts the result which is **3.1 σ** below the experiment (since 2006)

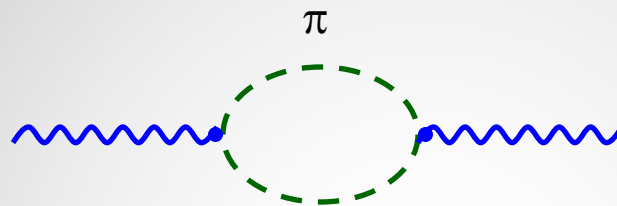
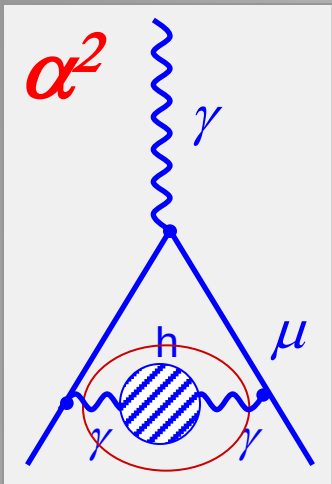


1.9 σ

3.1 σ with new BABAR (2009)

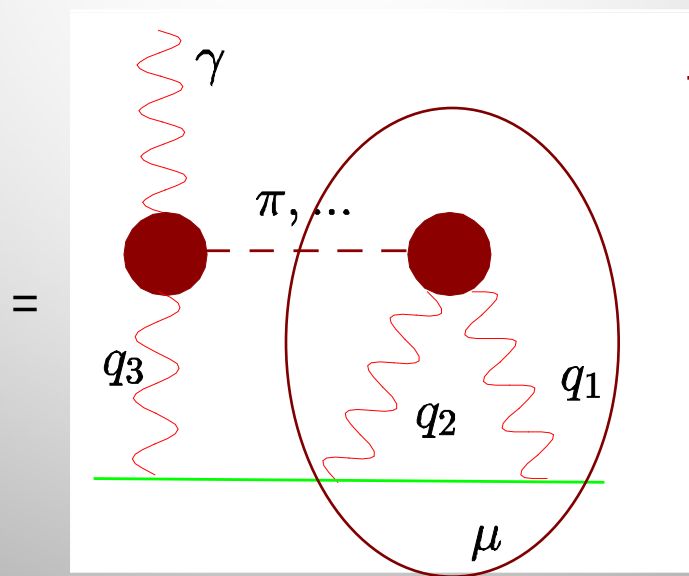
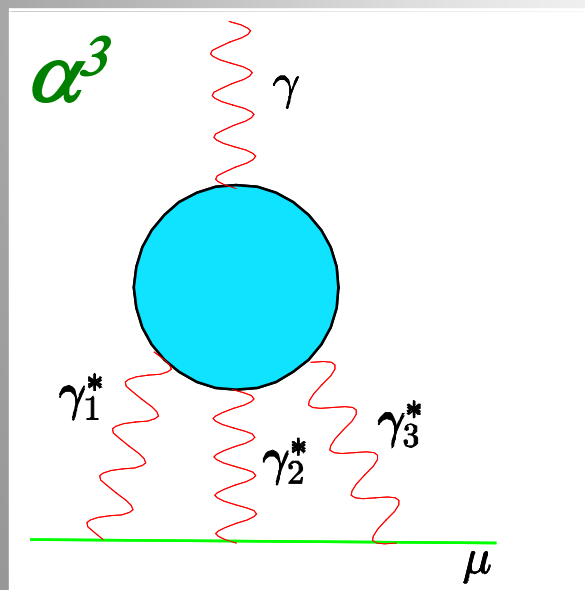
M. Davier et al., 2009

Hadronic corrections



Charge pion form factor

LbL to g-2

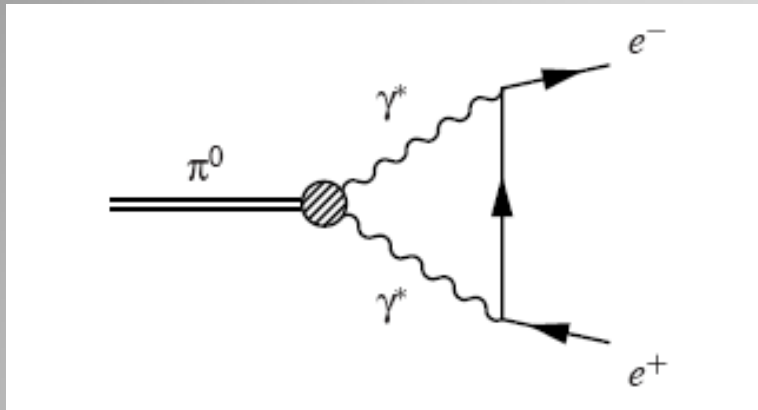


Transition pion form factor

+ ...

Rare Pion Decay $\pi^0 \rightarrow e^+e^-$ from KTeV

PRD (2007)



One of the simplest
process for THEORY

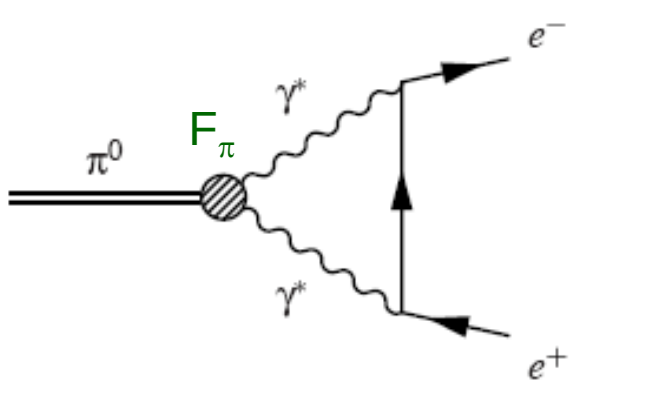
Lowest order diagram

From KTeV E799-II EXPERIMENT at Fermilab experiment (1997-2007)

$$B_{\pi \rightarrow e^+e^-}^{\text{KTeV}} = (7.48 \pm 0.29 \pm 0.25) \cdot 10^{-8} \text{ 99-00' set,}$$

The result is based on observation of 794 candidate $\pi^0 \rightarrow e^+e^-$ events using $K_L \rightarrow 3\pi_0$ as a source of tagged π^0 s.

Classical theory of $\pi^0 \rightarrow e^+e^-$ decay



Drell (59'), Berman, Geffen (60'),
Quigg, Jackson (68')

Bergstrom, et.al. (82') Dispersion Approach
Savage, Luke, Wise (92') χ PT

$$R(\pi^0 \rightarrow e^+e^-) = \frac{B(\pi^0 \rightarrow e^+e^-)}{B(\pi^0 \rightarrow \gamma\gamma)} = 2\beta(m_\pi^2) \left(\frac{\alpha}{\pi} \frac{m_e}{m_\pi} \right)^2 \left[\underbrace{(\text{Re } \mathcal{A}(m_\pi^2))^2}_{\text{blue}} + \underbrace{(\text{Im } \mathcal{A}(m_\pi^2))^2}_{\text{red}} \right]$$

$$\beta(q^2) = \sqrt{1 - 4 \frac{m_e^2}{q^2}}$$

$$\mathcal{A}(q^2) = \frac{2i}{q^2} \int \frac{d^4k}{\pi^2} \frac{q^2 k^2 - (qk)^2}{(k^2 + i\varepsilon) \left((k-q)^2 + i\varepsilon \right) \left((k-p)^2 - m_e^2 + i\varepsilon \right)} \underbrace{F_\pi(k^2, (k-q)^2)}_{\text{red}}$$

$$\text{Im } \mathcal{A}(q^2) = \frac{\pi}{2\beta(q^2)} \ln \left(\frac{1 - \beta(q^2)}{1 + \beta(q^2)} \right)$$

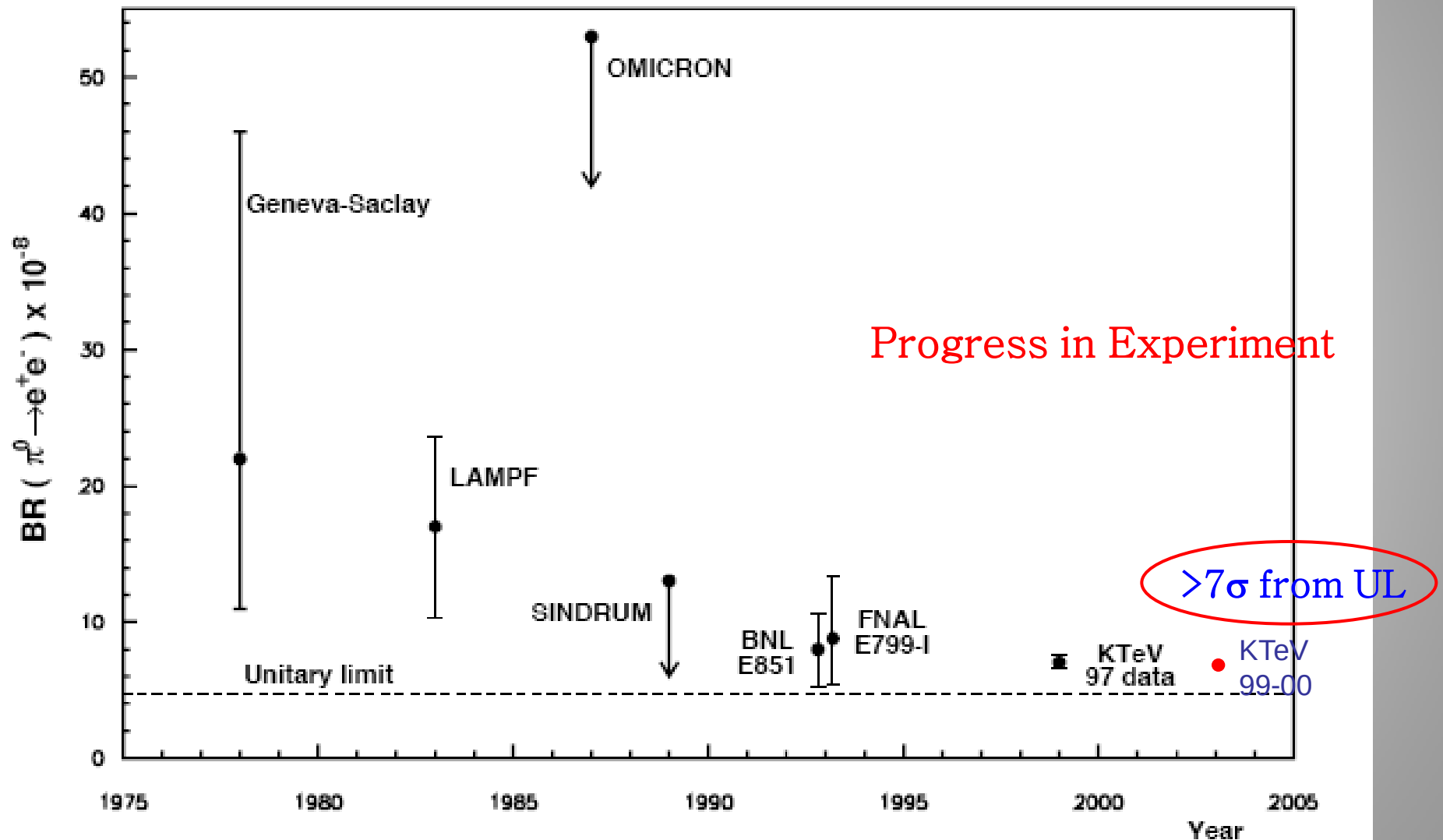
**The Imaginary part is Model
Independent;
Unitary limit**

From condition

$$|\mathcal{A}|^2 \geq (\text{Im } \mathcal{A})^2$$

one has the *unitary limit*

$$R(\pi^0 \rightarrow e^+e^-) \geq R^{\text{unitary}}(\pi^0 \rightarrow e^+e^-) = 4.75 \cdot 10^{-8}.$$



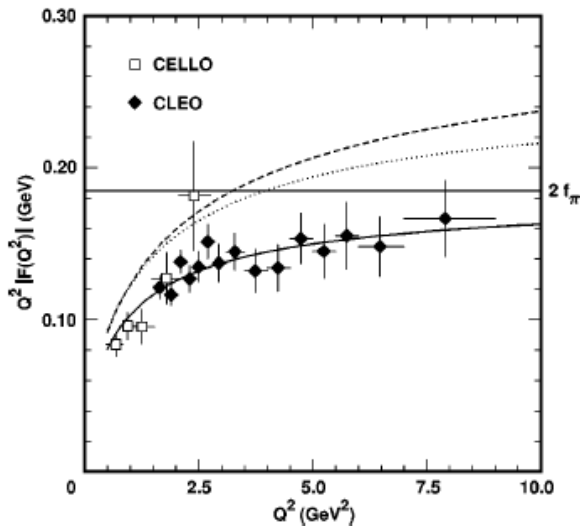
Still no intrigue

Theory contrasts KTeV experiment: π^0 anomaly

$$\text{Re } A(m_\pi^2) = \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12} + 3\ln\left(\frac{m_e}{\mu}\right) + \chi_P(\mu) + \mathcal{O}\left(\frac{m_e^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right)$$

$$\chi_P(\mu) = -\frac{5}{4} - \frac{3}{2} \left[\int_0^{\mu^2} dt \frac{F_\pi(t, t) - 1}{t} + \int_{\mu^2}^{\infty} dt \frac{F_\pi(t, t)}{t} \right]$$

$$m_e^2 \ll m_\pi^2 \ll \Lambda^2 \approx m_\rho^2$$



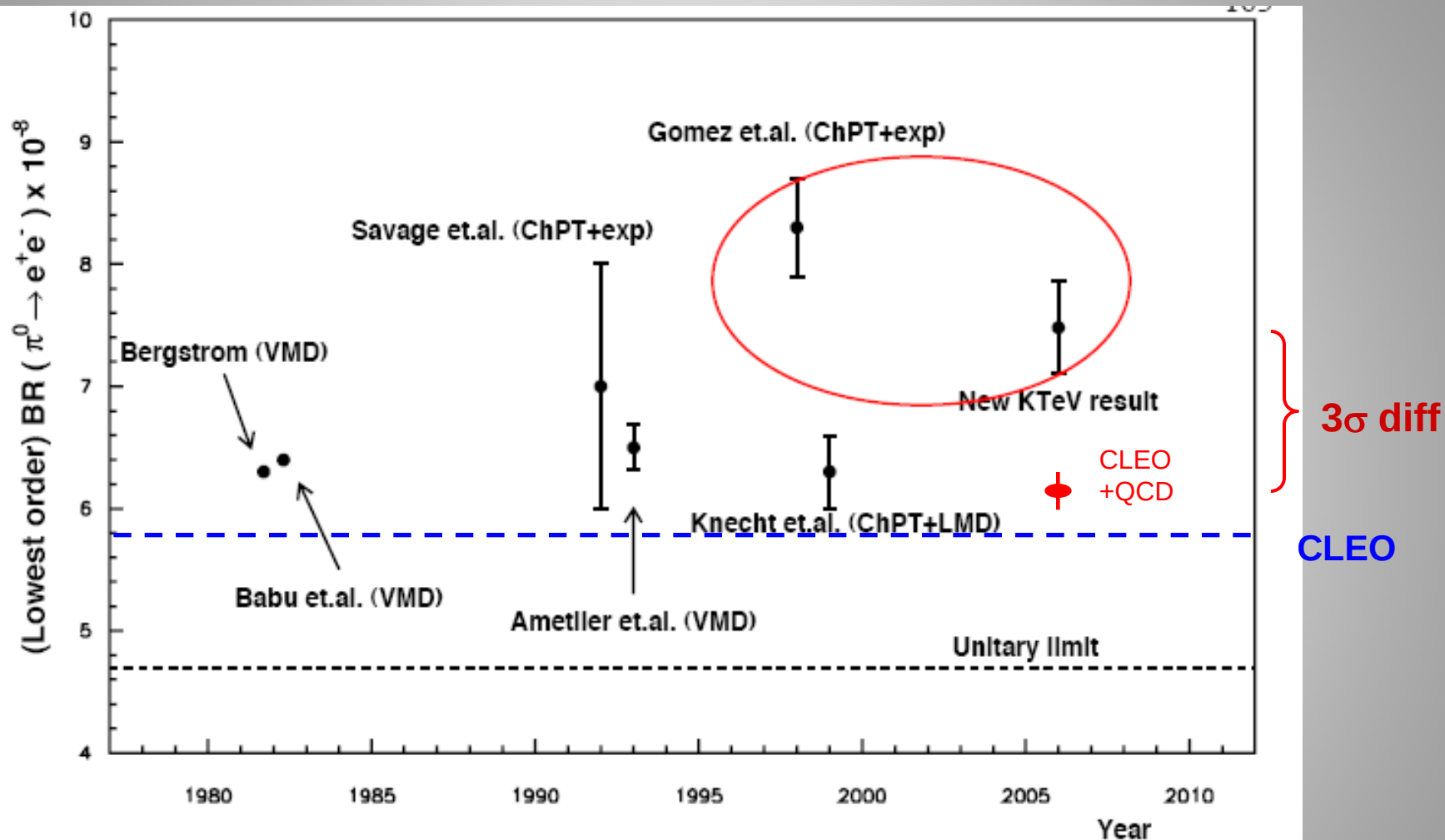
Basing on

- 1) CLEO data for $F_\pi(t, 0)$ and,
 - 2) QCD constraints for $F_\pi(t, t)$
- one gets the branching

$$B_{\pi \rightarrow e^+ e^-}^{\text{theory}} = (6.2 \pm 0.1) \cdot 10^{-8}$$

Which is **3.3 σ below data!!**

$$B_{\pi \rightarrow e^+ e^-}^{\text{KTeV}} = (7.48 \pm 0.38) \cdot 10^{-8}$$



What is next? It would be very desirable if **Others** will confirm KTeV result
 Also, Pion transition FF need to be more accurately measured.

Possible explanations of the π^0 anomaly

1) Radiative corrections

KTeV used in their analysis the results from Bergstrom 83'.

A.D., Kuraev, Bystriksky, Secansky (EJPC 08') confirmed Numerics.

2) Mass corrections: tiny, but visible for η and η'

A.D., M. Ivanov, S. Kovalenko (ZhETPh Lett 08' and PLB 09)

Dispersion approach and χ PT are corrected by power corrections $(m_\pi/m_\rho)^n$

3) New physics

Kahn, Schmidt, Tait PRD 08' Low mass dark matter particles

Chang, Yang 08' Light CP-odd Higgs in NMSSM

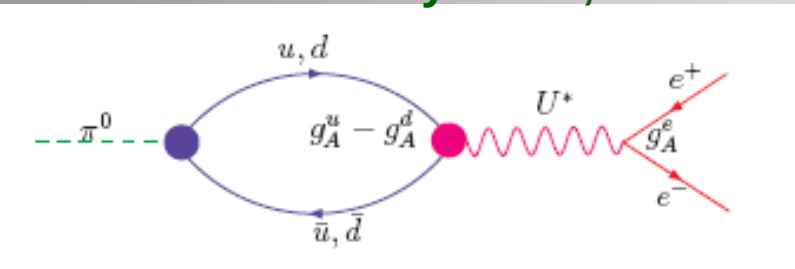
4) Experiment wrong

Waiting for new results from

KLOE, NA48, WASA@COSY, BESIII, ...

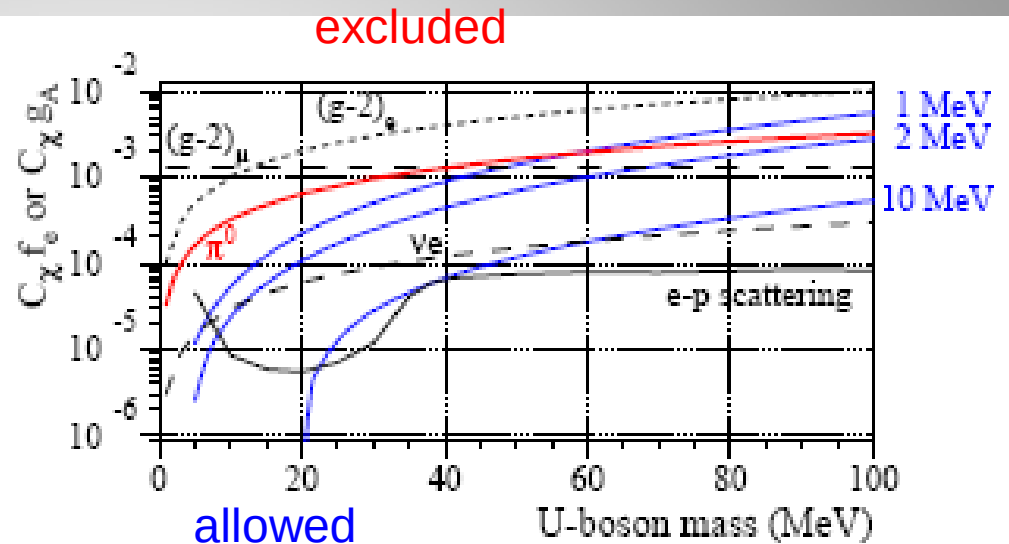
Enhancement in Rare Pion Decays from a Model of MeV Dark Matter (Boehm&Fayet)

was considered by Kahn, Schmitt and Tait (PRD 2008)



$$M_{U^*} \sim 10 \text{ MeV}$$

$$\mathcal{L} \supset U_\mu \{ \bar{u} \gamma^\mu (g_V^u + \gamma_5 g_A^u) u + \bar{d} \gamma^\mu (g_V^d + \gamma_5 g_A^d) d + \bar{e} \gamma^\mu (g_V^e + \gamma_5 g_A^e) e \},$$



The anomalous 511 keV γ -ray signal from Galactic Center observed by INTEGRAL/SPI (2003) is naturally explained

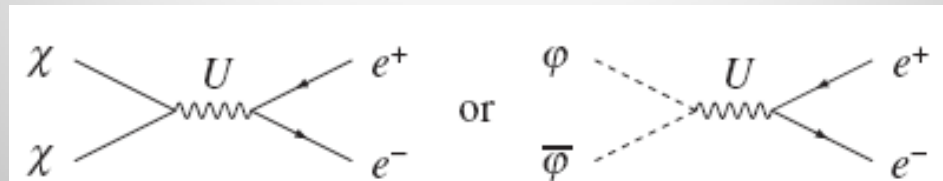


FIG. 1. Dark matter annihilations into e^+e^- pairs [12,13]. The first diagram corresponds to the pair annihilation of spin- $\frac{1}{2}$ LDM particles χ (which may be self-conjugate, or not); and the second one to the case of spin-0 particles ϕ .

Rare decay $\pi^0 \rightarrow e^+e^-$ as a sensitive probe of light CP-odd Higgs in Next-to-Minimal SuperSymmetric Model (NMSSM)

(Qin Chang, Ya-Dong Yang, PLB, 2009)

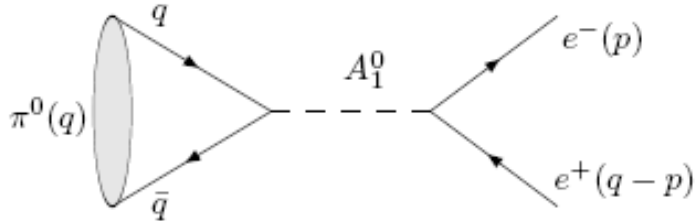
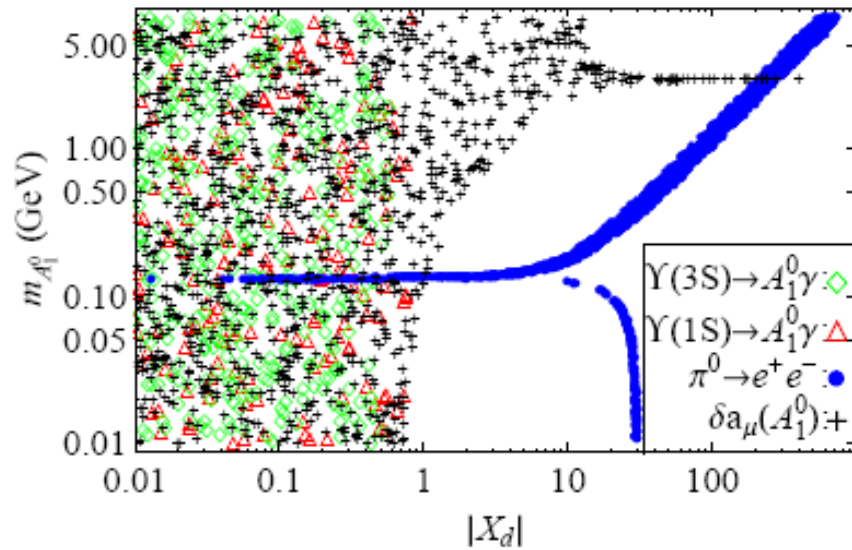


Figure 1: Relevant Feynman diagram within NMSSM.



They show the combined constraints from $Y \rightarrow \gamma A_1^0$, a_μ , $\pi^0 \rightarrow e^+e^-$ and $\pi^0 \rightarrow \gamma\gamma$ can not be resolved simultaneously with a very light A_1^0 ($m_{A_1^0} \simeq 135$ MeV)

Also there is no consistency with anomaly In $\Sigma^+ \rightarrow p \mu^+ \mu^-$ observed by HyperCP Coll

Other $P \rightarrow l^+l^-$ decays

A.D., M. Ivanov, S. Kovalenko (Phys Lett 2009)

TABLE I: Values of the branchings $B(P \rightarrow l^+l^-)$ obtained in our approach and compared with the available experimental results.

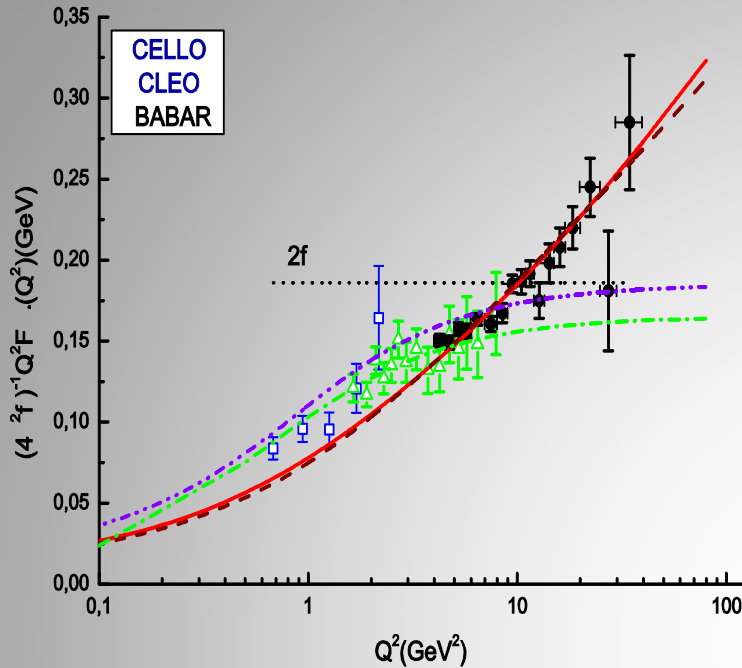
R_0	Unitary bound	CLEO bound	CLEO+OPE	This work	Experiment	BES III (1 year)
$R_0(\pi^0 \rightarrow e^+e^-) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.12	6.26	7.49 ± 0.38 [1]	WASA@COSY
$R_0(\eta \rightarrow \mu^+\mu^-) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.12 ± 0.27	4.64	5.8 ± 0.8 [20, 21]	0.08
$R_0(\eta \rightarrow e^+e^-) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.09	5.24	$\leq 2.7 \cdot 10^4$ [22]	$0.7 \cdot 10^2$
$R_0(\eta' \rightarrow \mu^+\mu^-) \times 10^7$	≥ 1.35	$\leq 1.44 \pm 0.01$	1.364 ± 0.010	1.30		0.8
$R_0(\eta' \rightarrow e^+e^-) \times 10^{10}$	≥ 0.36	$\geq 1.121 \pm 0.004$	1.178 ± 0.014	1.86		$0.7 \cdot 10^3$

$\pi \rightarrow ee$ will be available soon from WASA@COSY

Mass power corrections are visible for $\eta(\eta')$ decays

BESIII for one year will get for $\eta, \eta' \rightarrow ll$ the limit $0.7 \cdot 10^{-7}$

BABAR, PRD 09



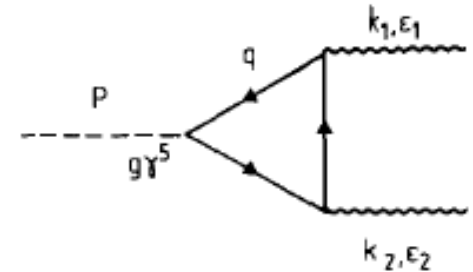
$$F_{\pi\gamma\gamma^*}^{\text{CLEO}}(Q^2, 0) = \frac{1}{4\pi^2 f_P} \frac{1}{1 + Q^2/\Lambda_P^2}$$

CLEO

$$\Lambda_\pi = 776 \pm 22 \text{ MeV}$$

$$F_{\pi\gamma\gamma^*}(t, 0) = \frac{m_\pi^2}{m_\pi^2 + t} \frac{1}{2 \arcsin^2\left(\frac{m_\pi}{2M_Q}\right)} \left\{ 2 \arcsin^2\left(\frac{m_\pi}{2M_Q}\right) + \frac{1}{2} \ln^2 \frac{\beta_Q + 1}{\beta_Q - 1} \right\}$$

$$M_Q = 135 \text{ MeV}$$



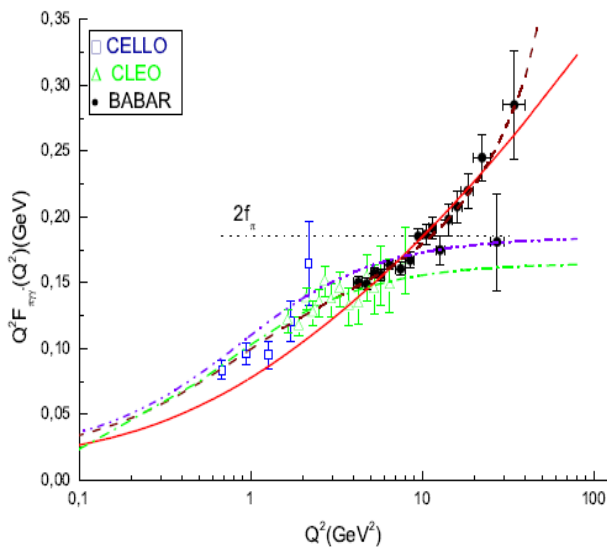
A.E. Dorokhov, arXiv:0905.4577

$$F_{\pi\gamma\gamma^*}(Q^2, 0) = \frac{2}{3} \frac{f_\pi}{Q^2} \cdot \int_0^1 \frac{dx}{x} \left[1 - \exp\left(-\frac{xQ^2}{2\sigma(1-x)}\right) \right]$$

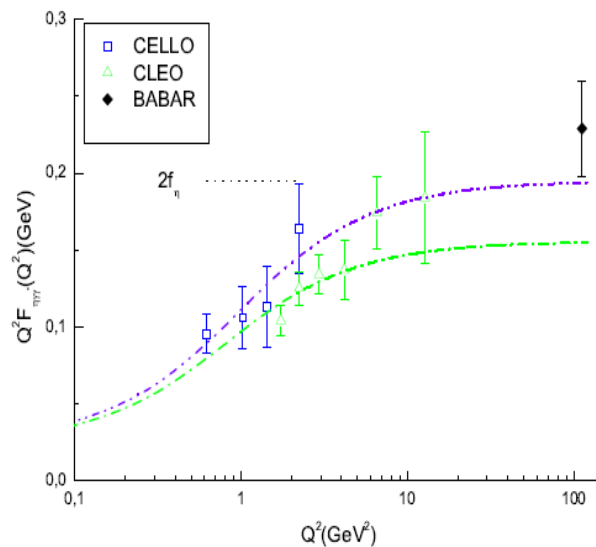
A.V. Radyushkin, arXiv:0906.0323

$$\sigma = 0.48 \text{ GeV}^2$$

$$\pi \rightarrow \gamma\gamma^*$$



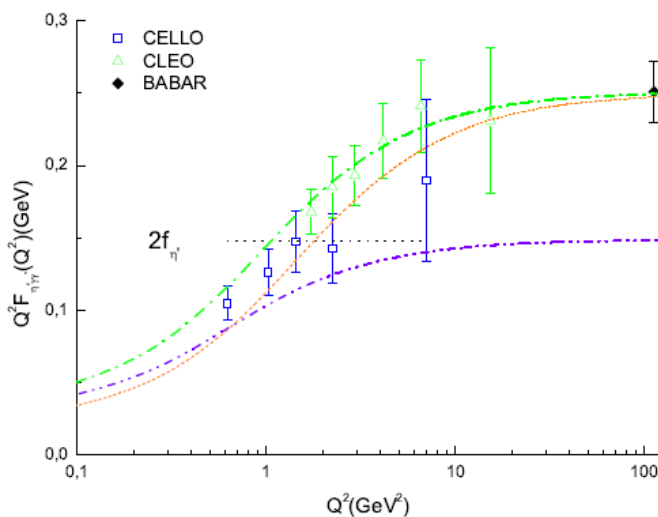
$$\eta \rightarrow \gamma\gamma^*$$



Nyffeler, Yegelehner 09

$$F_{\pi\gamma\gamma^*}(Q^2, 0) = \frac{f_\pi}{3M_V^2 M_{V'}^2} \cdot \frac{h_1 Q^4 + h_5 Q^2 + h_7}{(Q^2 + M_V^2)(Q^2 + M_{V'}^2)},$$

$$\eta' \rightarrow \gamma\gamma^*$$



$$F_{\pi\gamma\gamma^*}^{\text{CLEO}}(Q^2, 0) = \frac{1}{4\pi^2 f_P} \frac{1}{1 + Q^2/\Lambda_P^2}$$

$$\Lambda_\pi = 776 \pm 22 \text{ MeV}$$

$$\Lambda_\eta \approx 866 \text{ MeV}$$

$$\Lambda_{\eta'} = 859 \pm 28 \text{ MeV}$$

$$\Lambda_{\eta_c} = 2.92 \pm 16 \text{ GeV}$$

Other $P \rightarrow l^+l^-$ decays

After BABAR 09

Values of the branchings $B(P \rightarrow l^+l^-)$ obtained in our approach and compared with the available experimental results.

R_0	Unitary bound	BABAR bound	BABAR+OPE	[9]	Experiment
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Summary

1) The processes $P \rightarrow l+l-$ as like as muon $g-2$ are good for test of SM.

2) Long distance physics is fixed phenomenologically. New measurements of the transition form factors at low momenta are welcome.

Radiative and mass corrections are well under control.

3) At present there is 3.3σ disagreement between SM and KTeV experiment for $\pi^0 \rightarrow e^+e^-$

KLOE, WASA@COSY, BESS III are interested in new measurements

4) If effect found persists it might be evidence for the SM extensions with low mass (10-100 MeV) particles (Dark Matter, NSSM)