JINR Dubna

# Dynamics of the phase transition in nuclear matter

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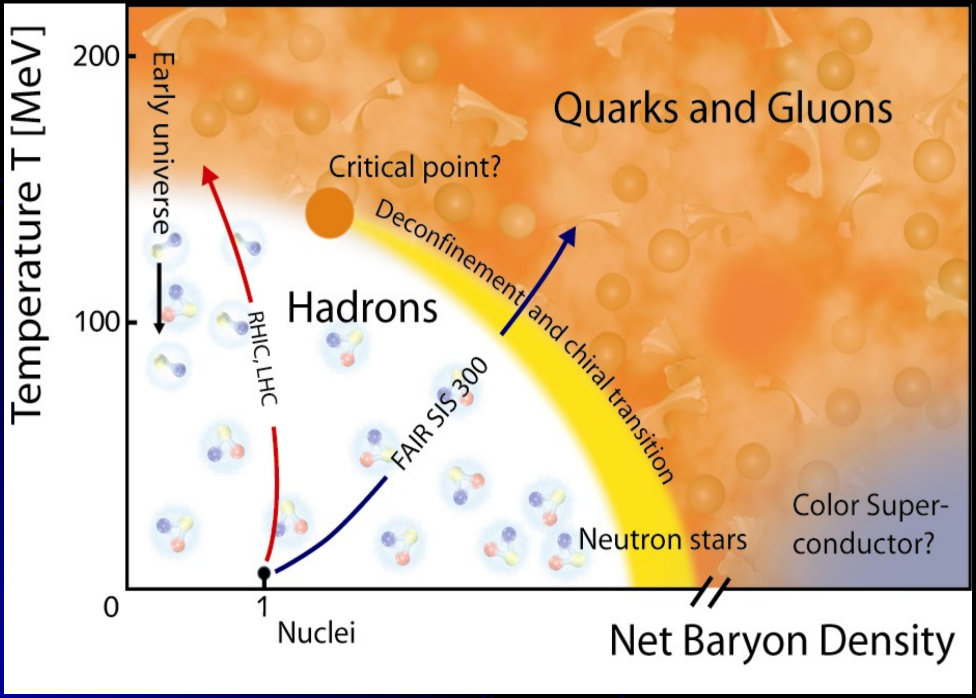
# Introduction

Hydrodynamics as applied to heavy-ion collisions gives us possibilities for extracting information about global properties of compressed and hot nuclear matter. In this talk I consider a hydrodynamic approach to description of evolution of system produced in heavy-ion central collisions.

Nuclear matter may be in different phase states and suffers a phase transition.

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How can this phenomena influence on hydrodynamics of expanding fireball ?

The main goal of our research is to investigate the first order phase transitions in mixed phase region and apply obtained knowledge to hydrodynamic description of system.

In the first steps of the research we use the simple model of first order phase transition in nuclear matter with zero chemical potential. This model let us get to know, what we should expect in more complicated cases.

It based on the  developed 1+1 dimensional hydrodynamic model with dynamics of phase transition. Assuming an existence of hadron, quark-gluon and mixed phases in nuclear matter we describe the phase transition taking into account thermal nonequilibrium in the system.

# The relaxation approximation for phase transition in nuclear matter

Conservation laws: $\partial_\nu T^{\mu\nu} = 0,$ (1)

$$\partial_\mu N_B^\mu = 0.$$

In perfect fluid approximation $T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu},$

$$N_B^\mu = n_B u^\mu.$$

Mixed phase is a mixture of hadron and quark-gluon phases.

If both phases have the same collective velocity $u^\mu$, we can use following expressions for each of them:

$$T_i^{\mu\nu} = (\varepsilon_i + P_i) u^\mu u^\nu - P_i g^{\mu\nu},$$

$N_{B\,i}^{\mu} = n_{B\,i} u^\mu,$ where $i = I, II$ defines the phase.

$$T^{\mu\nu} = \lambda T_I^{\mu\nu} + (1-\lambda) T_{II}^{\mu\nu},$$

$$N_B^\mu = \lambda N_{B\,I}^{\mu} + (1-\lambda) N_{B\,II}^{\mu}; \quad \lambda = V_I / V$$

We assume the existence of mechanical equilibrium for mixed phase $(P_I = P_{II} = P)$

and consider the system with zero chemical potentials $(\mu_{B\,I} = \mu_{B\,II} = 0)$

Then we have $T^{\mu\nu} = (\lambda\varepsilon_I + (1-\lambda)\varepsilon_{II} + P)u^\mu u^\nu - g^{\mu\nu}P$ (2)

$$\varepsilon = \lambda\varepsilon_I + (1-\lambda)\varepsilon_{II} \quad (3)$$

Rewrite equation (1) as a set of equations, taking into account (2) and (3) :

$$u^\nu \partial_\nu \varepsilon + (\varepsilon + P)\partial_\nu u^\nu = 0,$$
$$(\varepsilon + P)u^\mu \partial_\mu u_\nu + u_\nu u^\mu \partial_\mu P - \partial_\nu P = 0. \quad (4)$$

To simulate thermal non-equilibrium we define $\lambda$ in relaxation time approximation:

$$\Gamma \partial_t \lambda = \lambda_{eq} - \lambda. \quad (5)$$

Here $\lambda_{eq}$ is given by the Gibbs conditions: $P_I = P_{II}, T_I = T_{II}, \mu_I = \mu_{II}.$ (6)

Obviously in mixed phase $\lambda$ changes in the range from 0 to 1

If the relaxation time $\Gamma=0$ then $\lambda = \lambda_{eq}$ and phase transition proceeds in a way

defined by the Gibbs conditions (6). (Maxwell construction)

**Hadron EoS:**

$$P_I = \alpha T_I^4, \alpha = g_\pi \frac{\pi^2}{90}, g_\pi = 3.$$

$$\varepsilon_I = T_I \frac{\mathrm{d}P_I}{\mathrm{d}T_I} - P_I = 3\alpha T_I{}^4.$$

$$P_I = \frac{\varepsilon_I}{3}. \qquad (7)$$

**Quark-gluon EoS:**

$$P_{II} = \beta T_{II}^4 - B, \beta = g_g \frac{\pi^2}{90}, g_g = 16.$$

$$B = 0.1946 \ \mathrm{GeV/fm^3}.$$

$$\varepsilon_{II} = T_{II} \frac{\mathrm{d}P_{II}}{\mathrm{d}T_{II}} - P_{II} = 3\beta T_{II}{}^4 + B.$$

$$P_{II} = \frac{\varepsilon_{II}}{3} - \frac{4}{3}B. \qquad (8)$$

Gibbs conditions (6) define the temperature of phase transition

$$T_{pt} = \sqrt[4]{\frac{B}{\beta - \alpha}} \approx 0.18\,\mathrm{GeV}.$$

$$\varepsilon_I(T_{pt}) = \frac{3\alpha B}{\beta - \alpha} \approx 0.1347 \ \mathrm{GeV/fm}^3,$$

$$\lambda_{eq}(\varepsilon) = \frac{\varepsilon_{II}(T_{pt}) - \varepsilon}{4B}. \qquad (9)$$

$$\varepsilon_{II}(T_{pt}) = \frac{3\beta B}{\beta - \alpha} + B \approx 0.9131\,\mathrm{GeV/fm}^3.$$

We solve system (4) completed by (7) or (8) in case of pure hadron or quark-gluon phase, respectively.

In the mixed phase case we solve system (4) completed by relaxation equation (5) and expression (9) for $\lambda_{eq}$.

# Application to Bjorken expansion with phase transition

$$\frac{d\varepsilon}{d\tau} + \frac{D}{\tau}(\varepsilon + P) = 0, \quad \text{assuming } D = 1.$$

$$\Gamma \frac{d\lambda}{d\tau} = \lambda_{eq} - \lambda. \qquad (10)$$

Initial conditions:

$\tau_0 = 1 \text{fm/c}$

$\varepsilon(\tau_0) = 2.75 \text{ GeV/fm}^3$

($\varepsilon(\tau_0)$ accords to $E_{lab} = 158 \text{ A GeV}$)

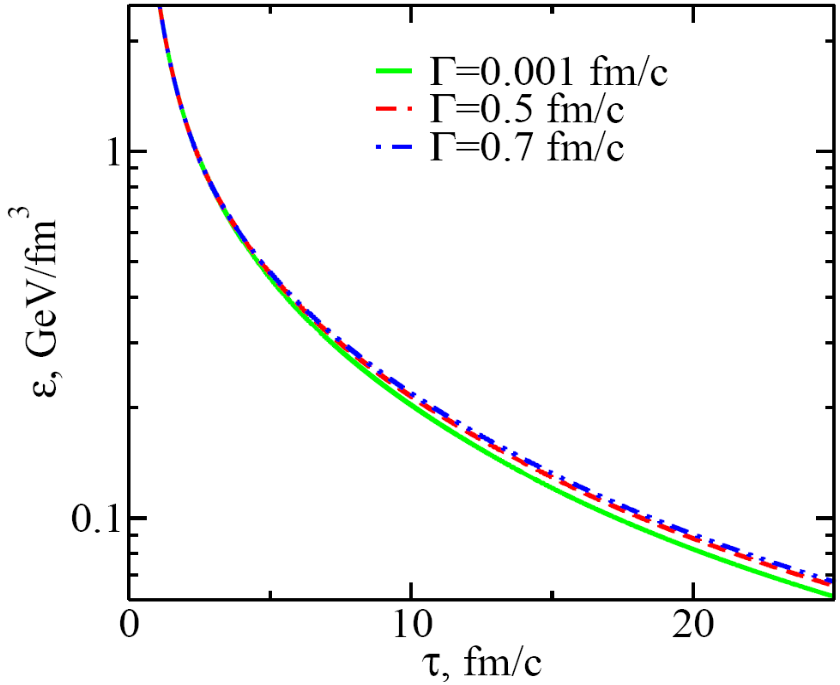



***Fig.1***: Energy density vs. the proper time for various values of the relaxation time $\Gamma$

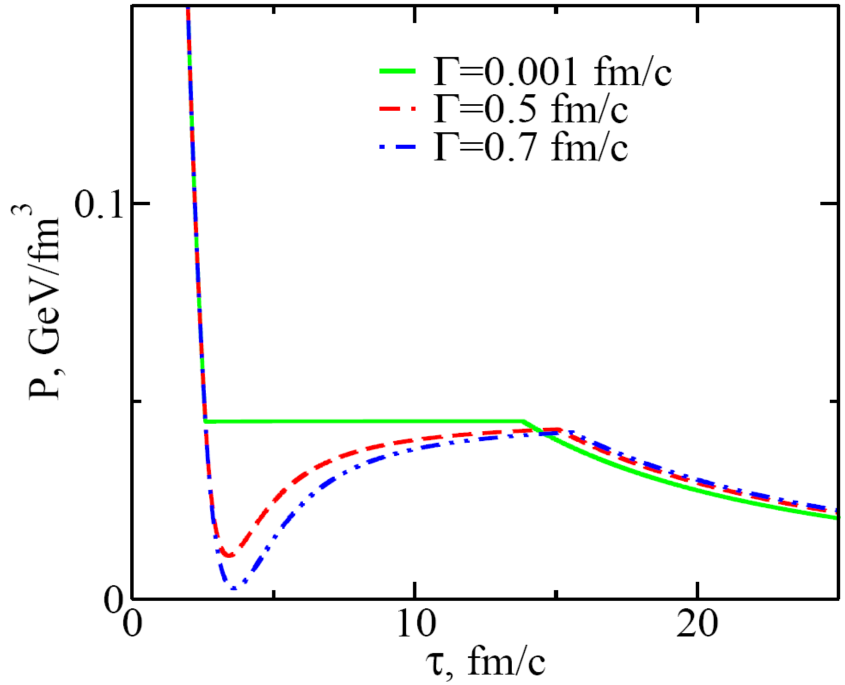



***Fig.2***: Pressure dependence on the proper time for various relaxation times $\Gamma$

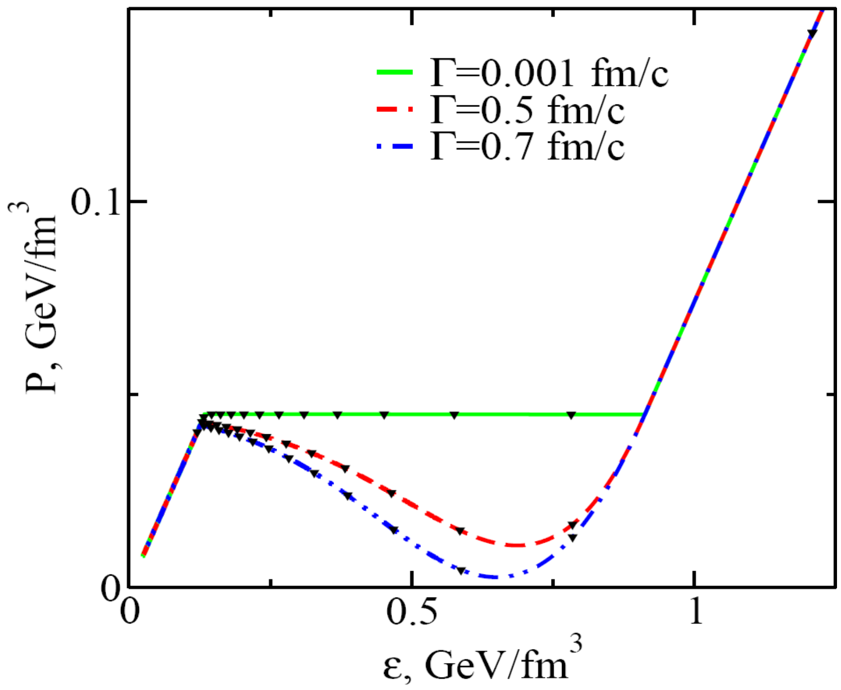



*Fig. 3*: Pressure as a function of energy density for various relaxation times $\Gamma$. (Effective EoS) Black triangles illustrate the proper time steps. The value of step is equal to 1 fm/c.

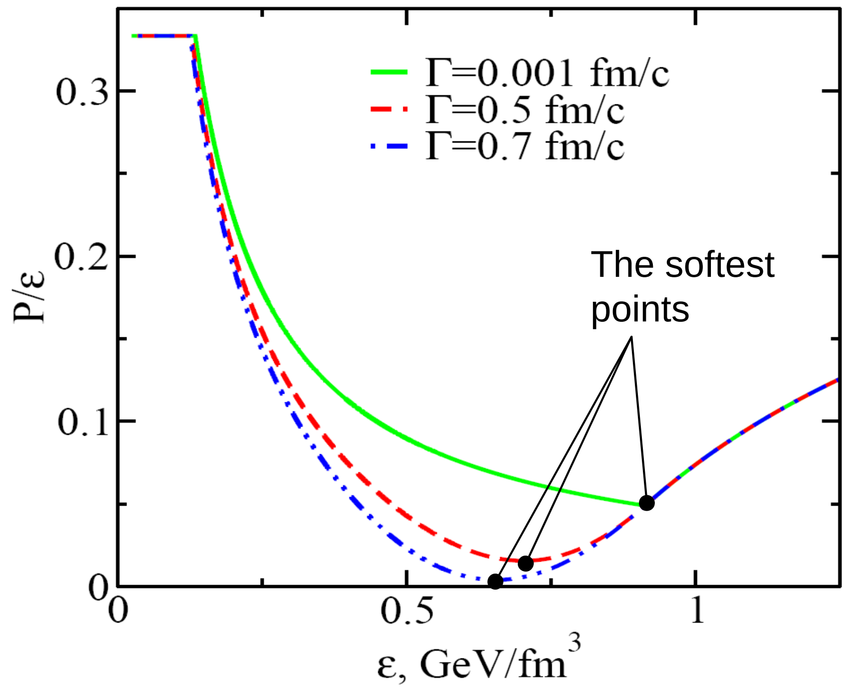



*Fig. 4*: Ratio $P/\varepsilon$ as a function of energy density for various values of the relaxation time $\Gamma$

Fig.3 illustrate that in case $\Gamma \to 0$ EoS obtained by solving (10) corresponds to Maxwell construction.

We can see also that the evolution of system delays with increasing of proper time.

# Application to 1+1 D hydrodynamic model in the Cartesian coordinates

In case of 1+1 hydrodynamic model system (4) is reduced to

$$v\partial_t P + \partial_x P + \gamma^2(\varepsilon + P)(\partial_t v + v\partial_x v) = 0,$$
$$\partial_t \varepsilon + v\partial_x \varepsilon + \gamma^2(\varepsilon + P)(v\partial_t v + \partial_x v) = 0;$$

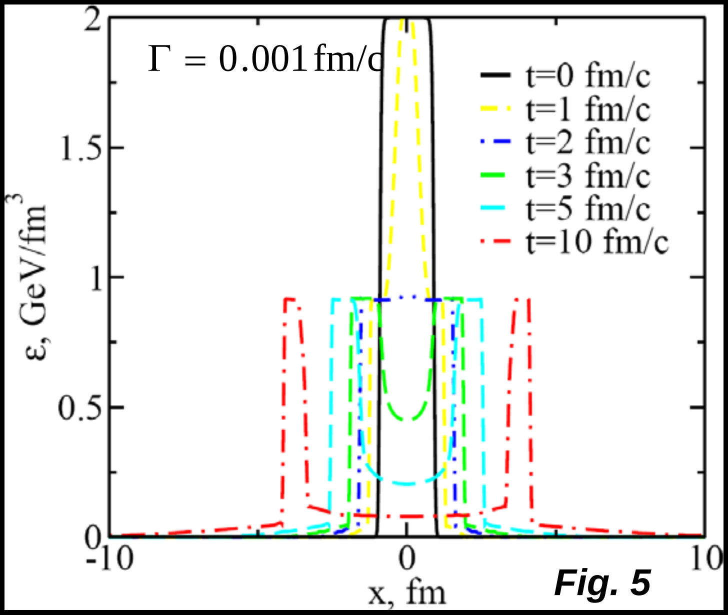
where $v = \frac{u^x}{u^t}, \gamma = \frac{1}{\sqrt{1-v^2}}.$

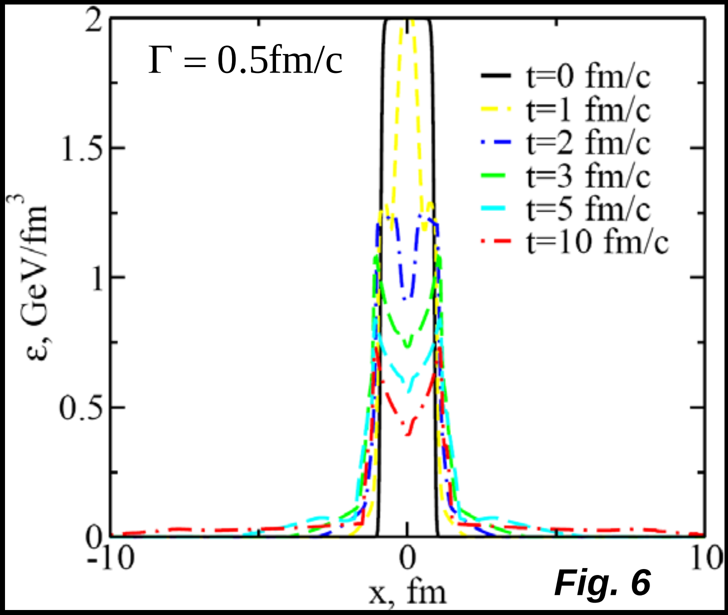
Initial conditions:

$$t_0 = 0 \text{ fm/c},$$
$$\varepsilon(x, t_0) = \frac{\varepsilon_0 \left[1 - \tanh(k_1|x - k_2|)\right]^2}{4},$$
$$v(x, t_0) = \operatorname{sgn}(x)\frac{1 + \tanh(k_1|x - k_2|)}{2}.$$

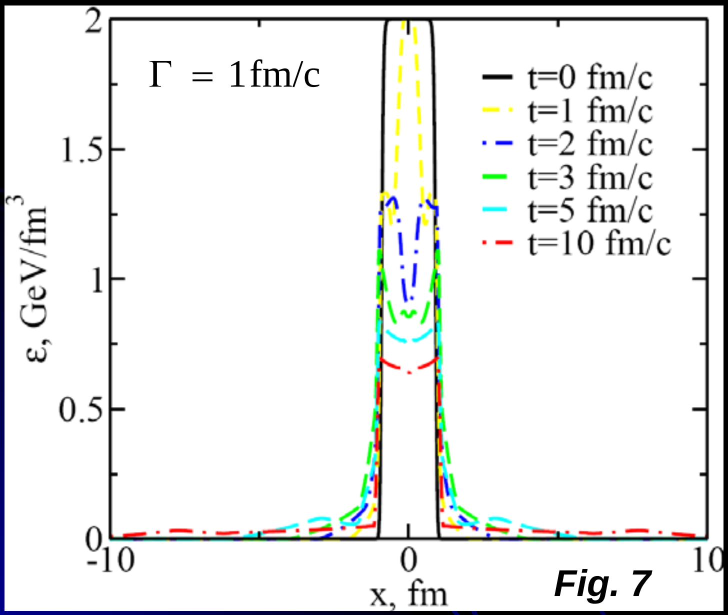
$\varepsilon_0 = 2\,\text{GeV/fm}^3$ accords to $E_{lab} = 40A\,\text{GeV}$

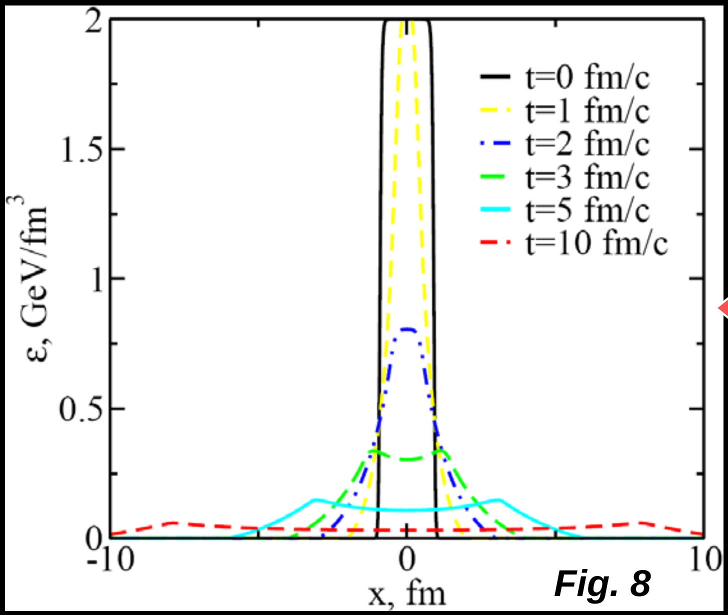
$k_1$ and $k_2$ are coefficients which are taken in agreement with initial radius of system at $E_{lab} = 40A\,\text{Gev}.$




**Phase transition slows down evolution of the system.**




pure hadron phase

Space distributions of energy density at fixed times **t .**
Fig. 5,6,7 correspond to phase transitions with various values of the relaxation time $\Gamma$ .
Fig. 8 illustrates the expansion of fireball without phase transition.

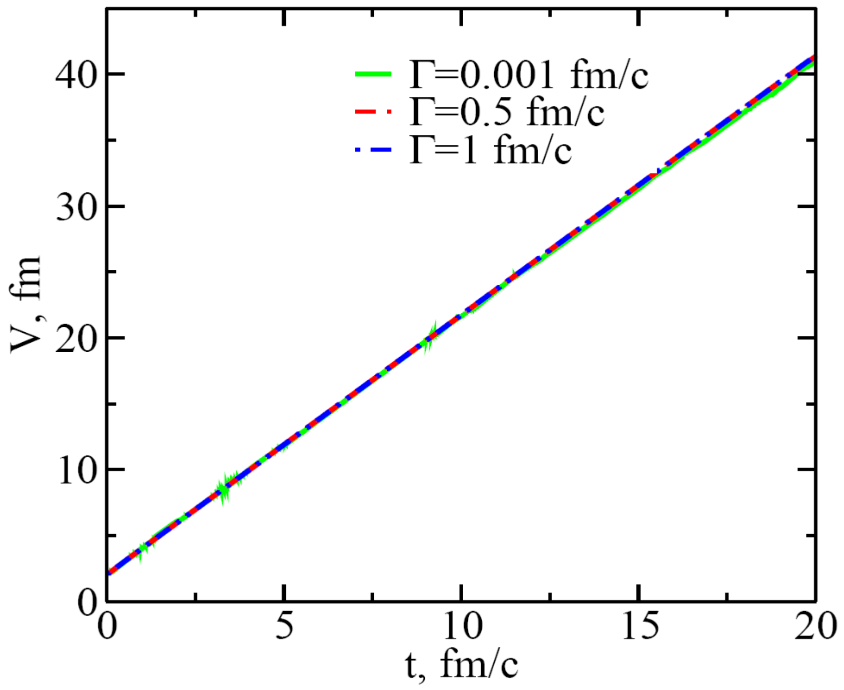



***Fig. 9***: time-dependence of one dimensional volume of whole system for various relaxation times $\Gamma$

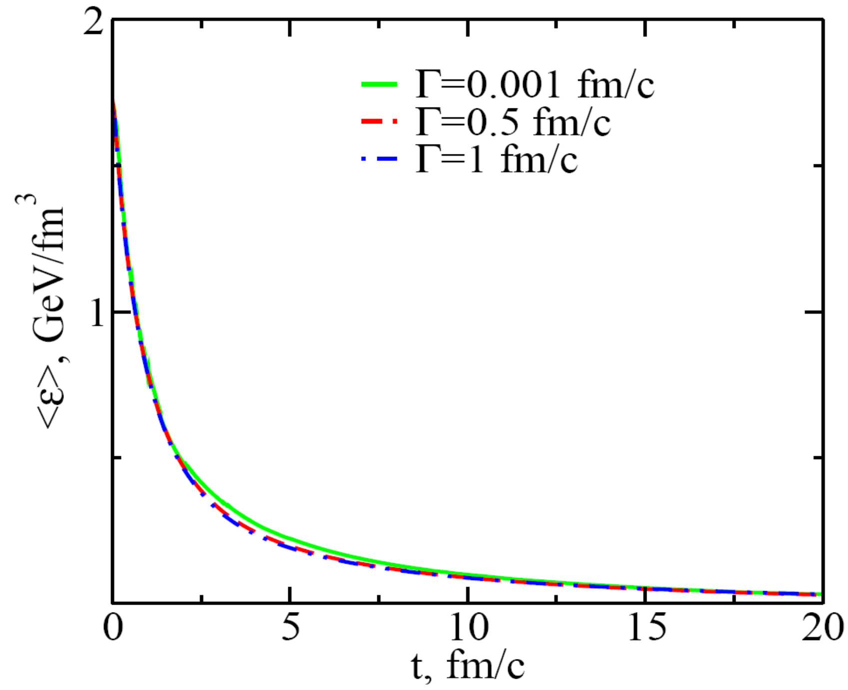
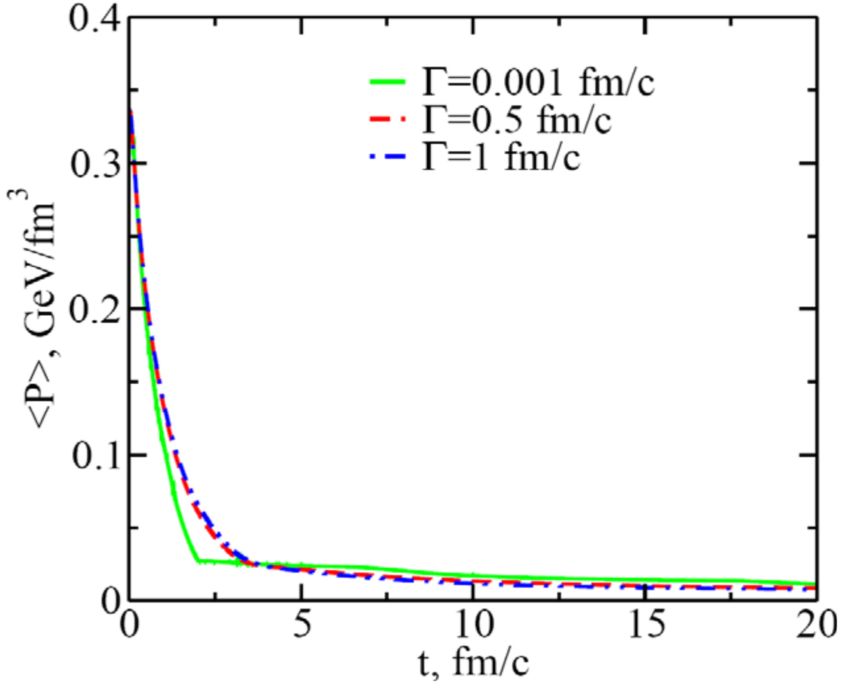



***Fig. 10***: dependence of 1D volume-averaged energy density on time for various values of the relaxation time $\Gamma$

The total volume of expanding system weakly depends on relaxation time (Fig. 9)



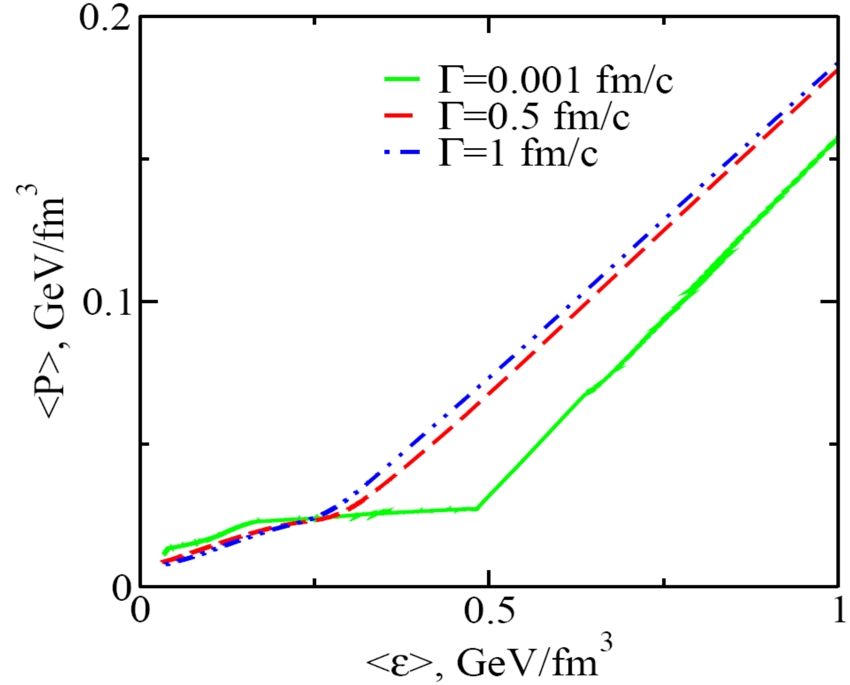




***Fig.11***: dependence of 1D volume-averaged pressure on time for various relaxation times $\Gamma$

***Fig. 12***: 1D volume-averaged pressure as a function of 1D volume-averaged energy density for various values of the relaxation time $\Gamma$

When relaxation time $\Gamma$ goes to 0, effective EoS is similar to Maxwell construction (Fig. 12).

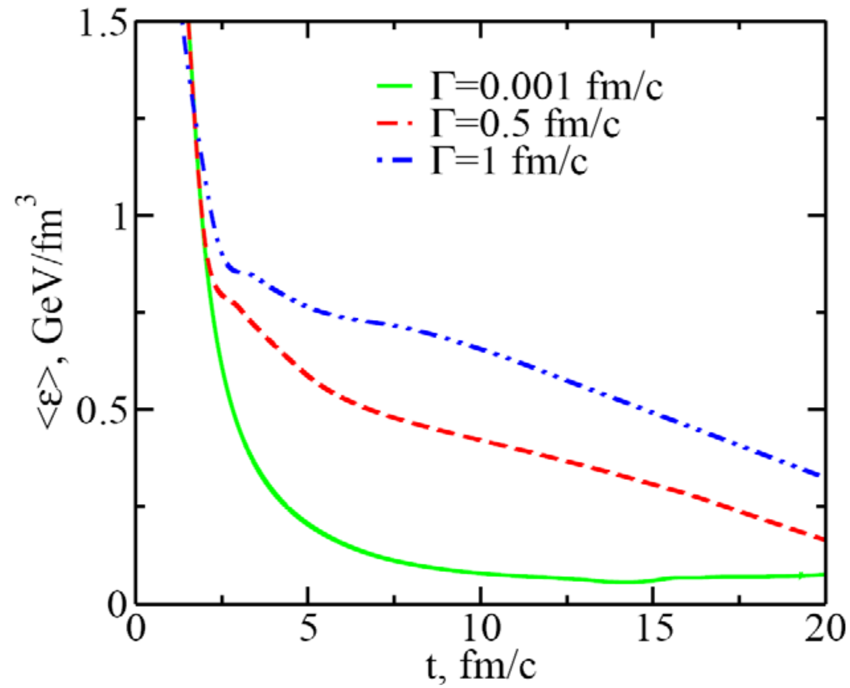



*Fig. 13*: little 1D volume-averaged energy density vs. time for various relaxation times $\Gamma$

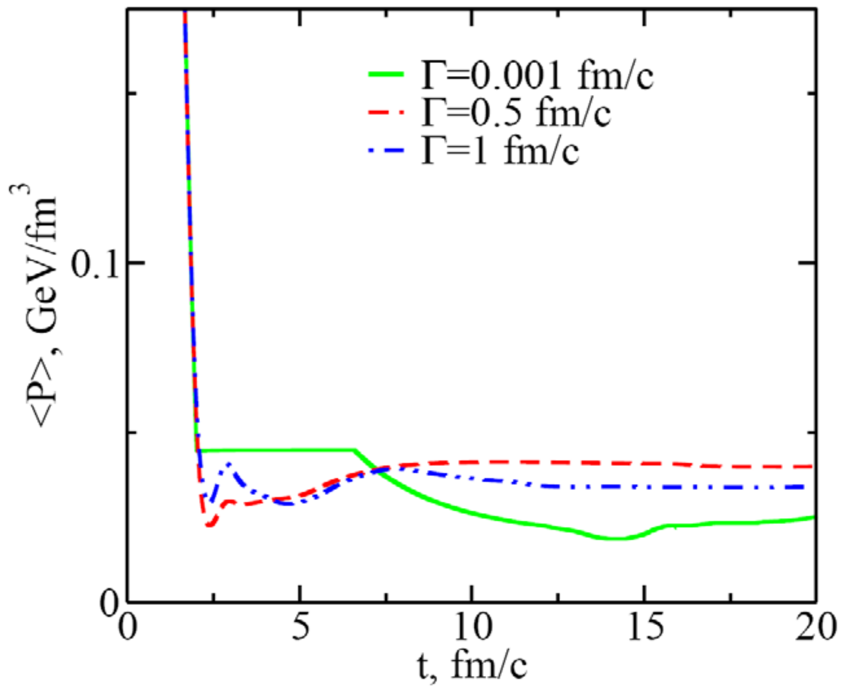
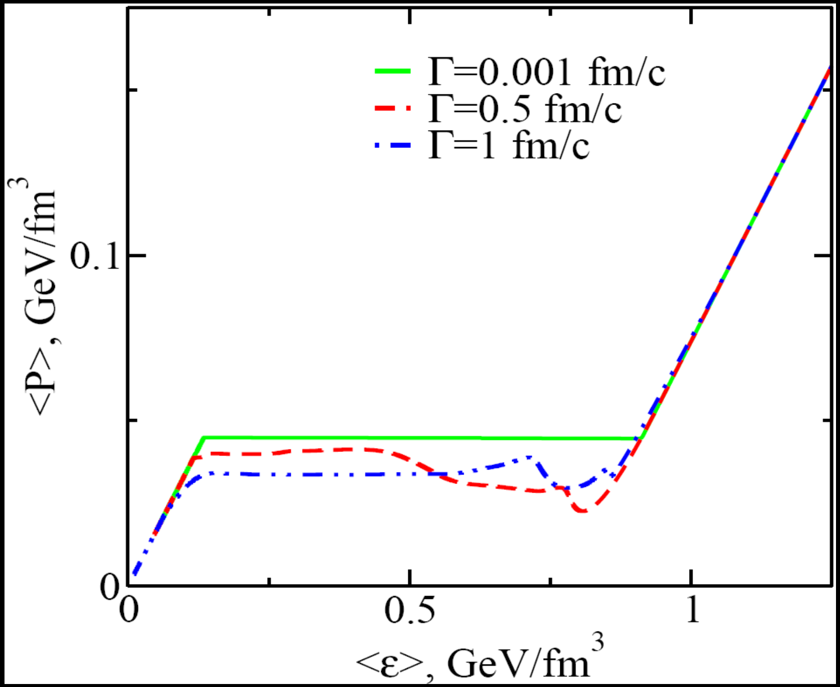



*Fig. 14*: little 1D volume-averaged pressure vs. time for various relaxation times $\Gamma$




It reproduces Maxwell construction for **Γ**=0.001 fm/c more precisely than effective EoS from Fig.12

***Fig. 15***: little 1D volume-averaged pressure as a function of little 1D volume-averaged energy density for various values of the relaxation time **Γ** .

Obtained as implicit function by using time-dependencies $\langle \varepsilon \rangle(t)$ (Fig.13) and $\langle P \rangle(t)$ (Fig.14).

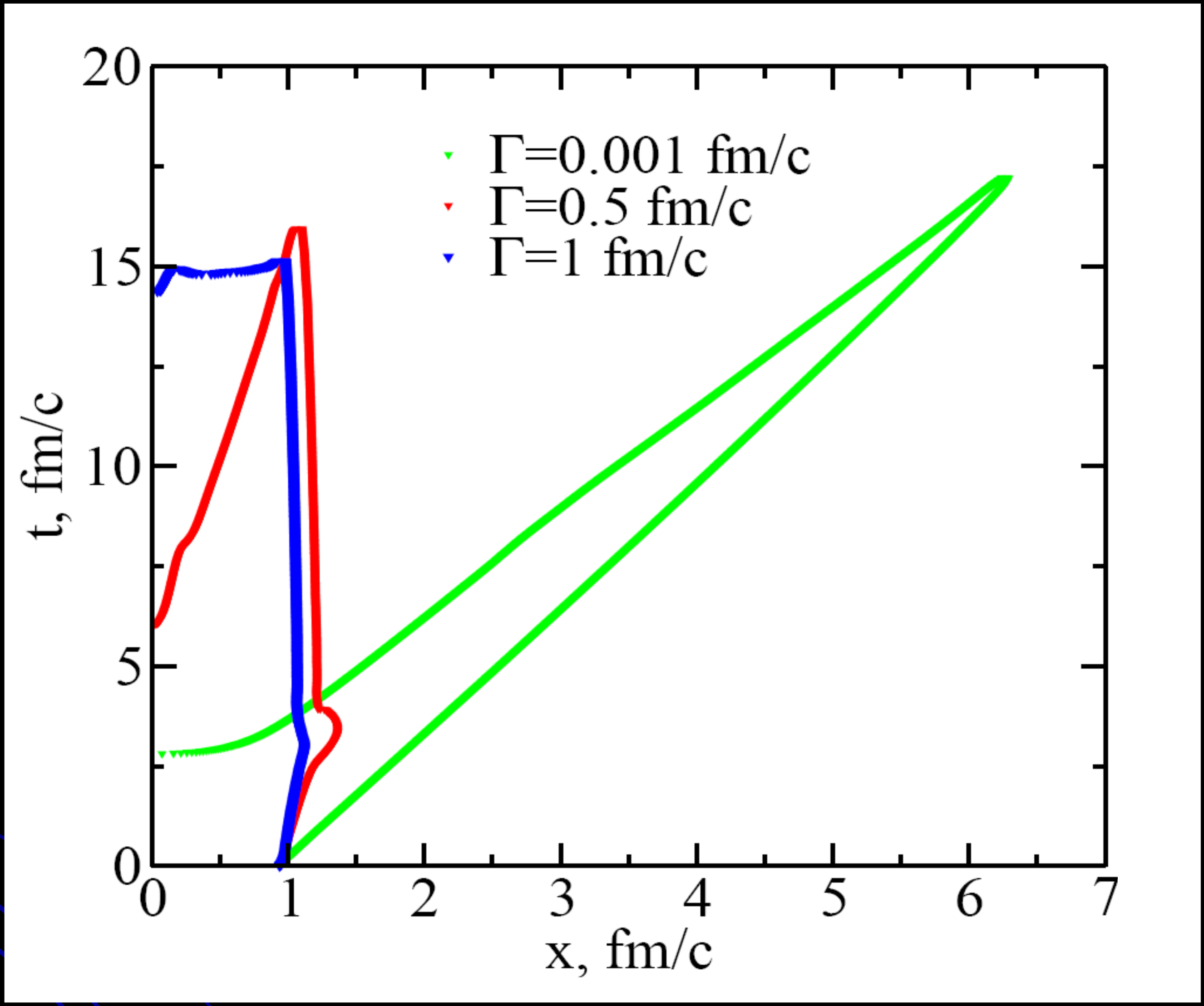



***Fig.16***: Freeze out profiles for various values of the relaxation time $\Gamma$.

Freeze out energy density $\varepsilon_{fr} = 0.5 \mathrm{GeV/fm^3}$

# Summary

- Dynamical model of the first order phase transition in the relaxation time approximation has been constructed.
- Existence of phase transition delays evolution of system.
- The experimental observables which may be sensitive to the dynamics of phase transition: Dileptons and direct photons (dileptons from mixed phase, see V.D. Toneev's lecture).
- **Finite** baryon chemical **potential**, **3D** calculations with a **realistic EoS** are to be done.