HISS: Dense Matter in HIC and Astrophysics, Dubna, 18./21. 7. 08

Condensates and Correlations in Nuclear Matter

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Problems:

- Warm Dilute Matter: Nuclear matter at subsaturation densities:

Temperature T ≤ 16 MeV = E_s/A, baryon density $n_B \leq 0.17\ \mathrm{fm}^{-3} = n_s$.

- Formation of clusters (nuclei in matter):

A = 1,2,3,4: deuterons (d), tritons (t), helions (h), alphas (α)

- Low-density, low-temperature limit:

Virial expansion, non-interacting nuclides, quantum condensates

- Transition to higher densities:

Medium effects, quasiparticles,

interpolation between Beth-Uhlenbeck and DBHF / RMF

Refs:

Particle clustering and Mott transition in nuclear matter at finite temperatures,
G. Röpke, M. Schmidt, L. Münchow, H. Schulz: NPA **399**, 587-602 (1983).

Generalized Beth-Uhlenbeck Approach for Hot Nuclear Matter,
M. Schmidt, G. Röpke, H. Schulz: Annals of Physics **202**, 57 - 99 (1990).

Outline

- Schrödinger equation with medium corrections:
  Self-energy and Pauli blocking
- Composition of the nuclear gas:
  Generalized Beth-Uhlenbeck equation
- Quantum condensates:
  Pairing and quartetting
- Composition and the EoS of nuclear matter
  (astrophysics: supernovae explosions)
- Cluster formation in dilute nuclei
  (Hoyle state and THSR wave function)
- Symmetry energy in the low-density region
  (heavy ion collisions: cluster abundances)

Low-density EoS and astrophysics

- H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,

  Progr. Theor. Phys.**100**, 1013 (1998); Nucl. Phys. **A637** 435 (1998).
- G. Röpke, A. Grigo, K. Sumiyoshi, and Hong Shen,

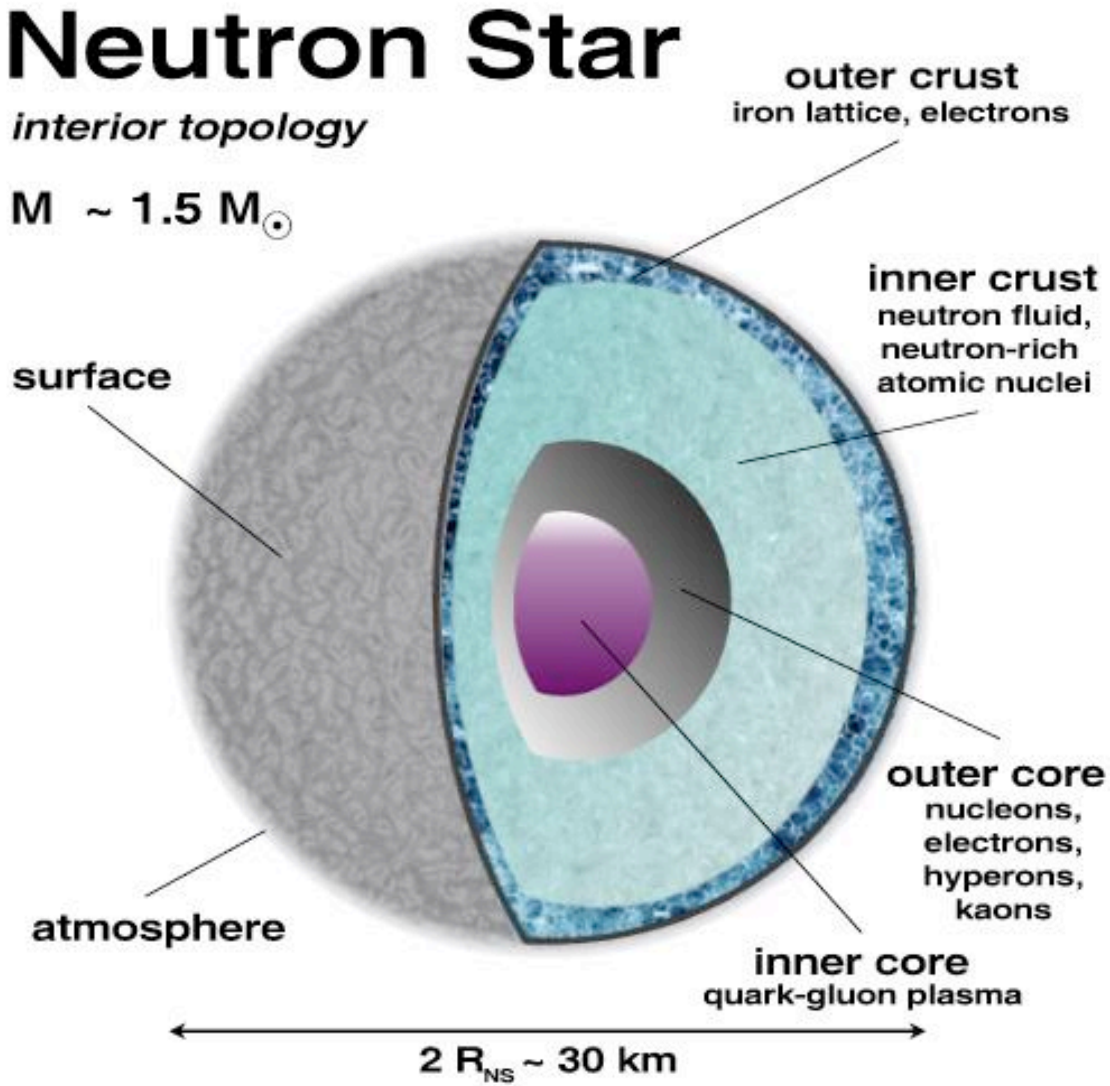
  in: Superdense QCD Matter and Compact Stars, Ed. D. Blaschke and A. Sedrakian, NATO Science Series, Springer, Dordrecht (2006), pp. 75 - 91;

  Physics of Particles and Nuclei Letters **2**, 275 (2005).
- J.M.Lattimer and F. D. Swesty,

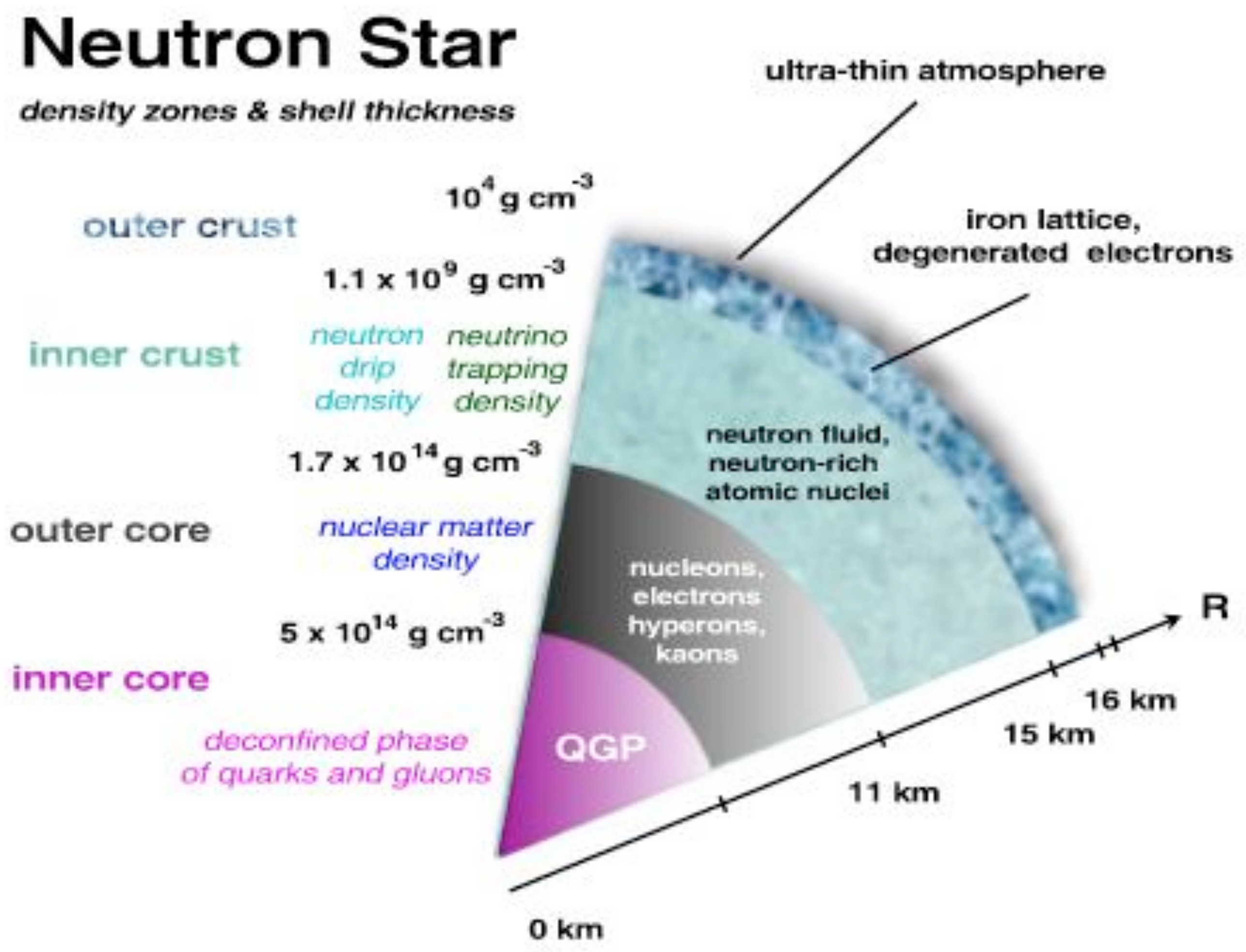
  Nucl. Phys. **A 535**, 331 (2001).
- C. J. Horowitz and A. Schwenk,

  Nucl. Phys. **A 776**, 55 (2006).

Neutron stars



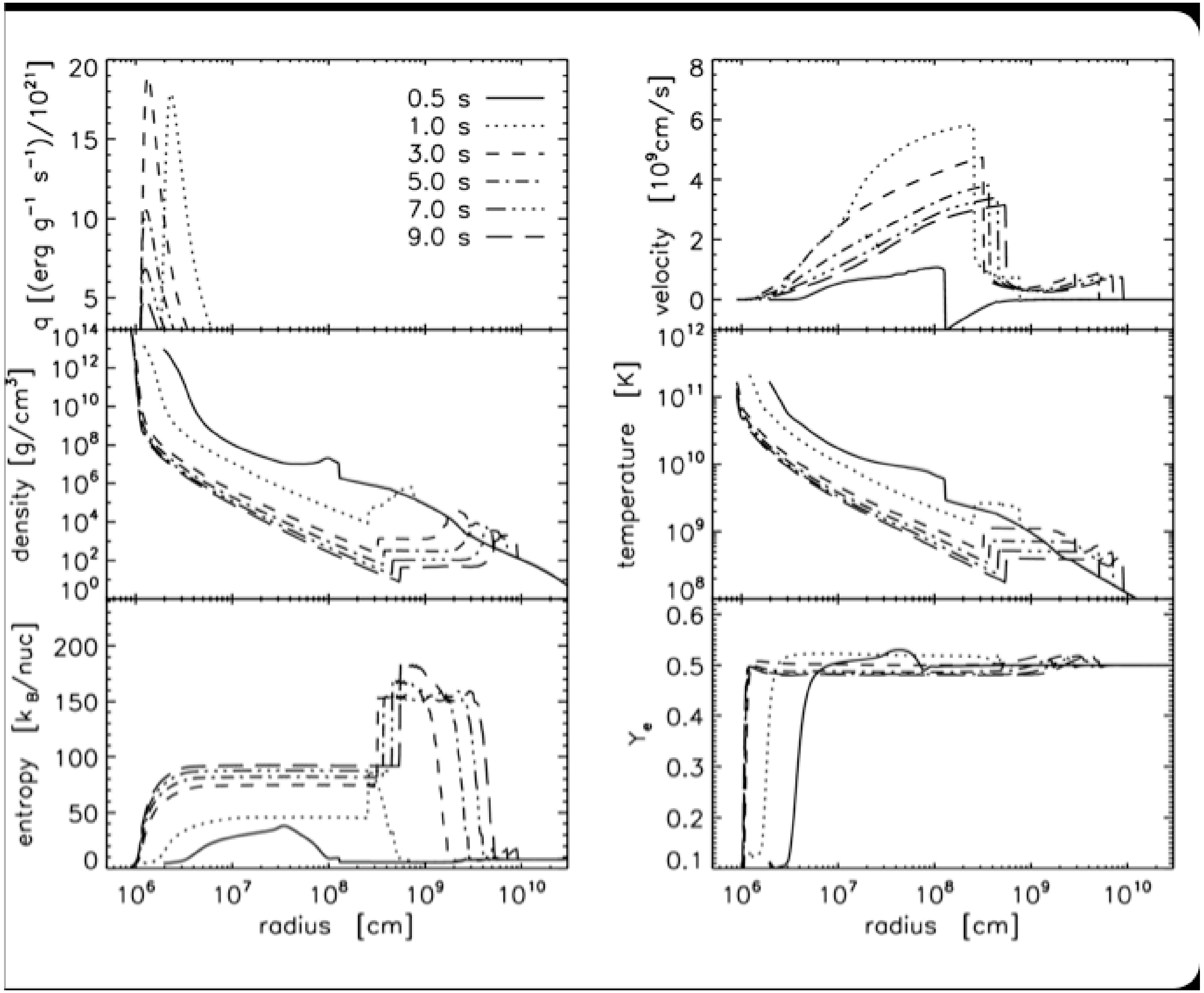
Structure of a Neutron star



Supernova

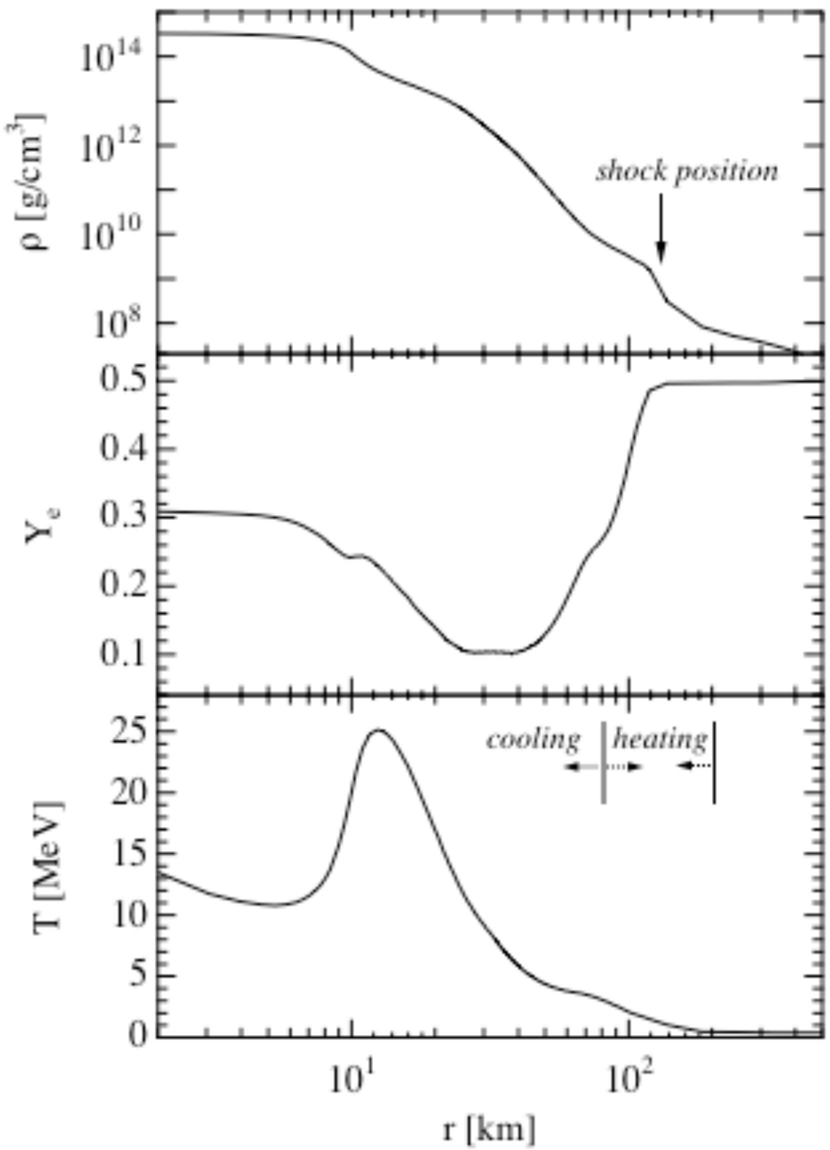
Crab nebula, 1054 China, PSR 0531+21

Supernova collapse: spherically symmetric simulations



A. Arcones
Neutrino driven winds
Talk 25. 2. 08 Ladek

Core-collapse supernovae



Density.

electron fraction, and

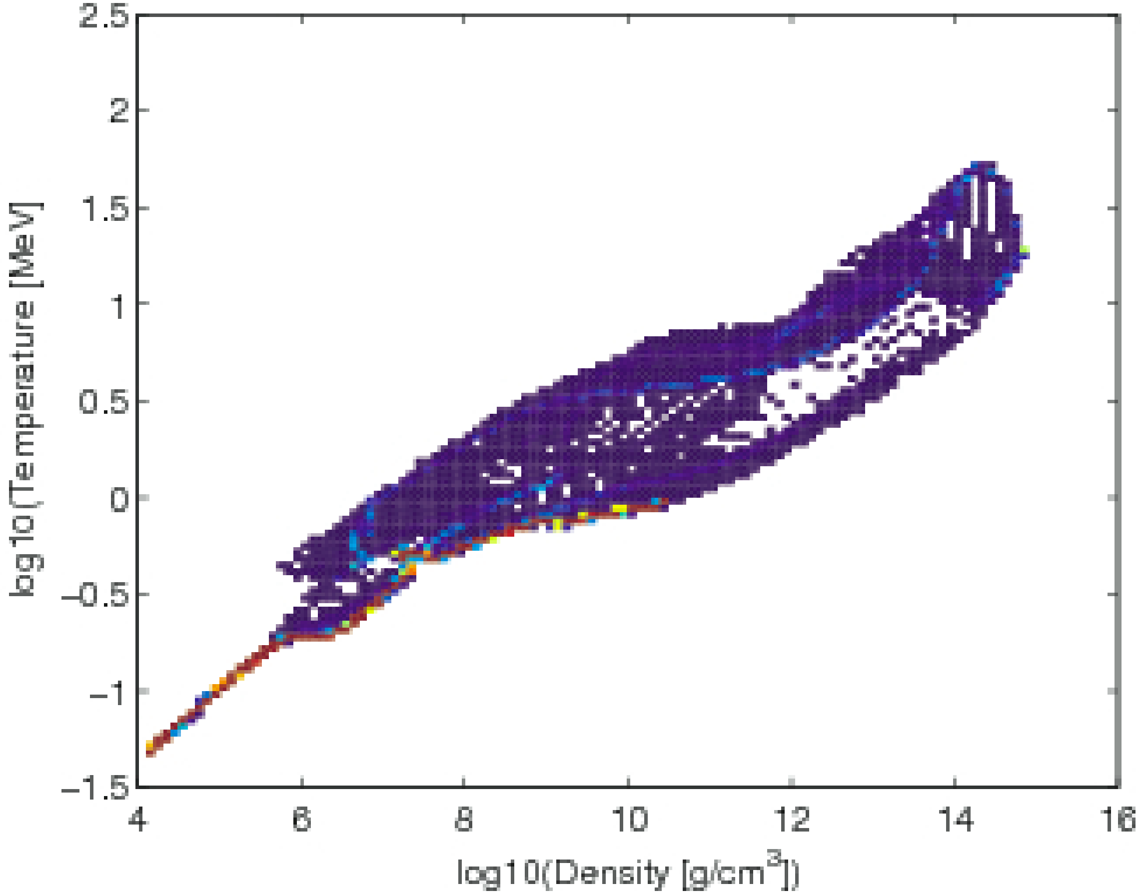
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

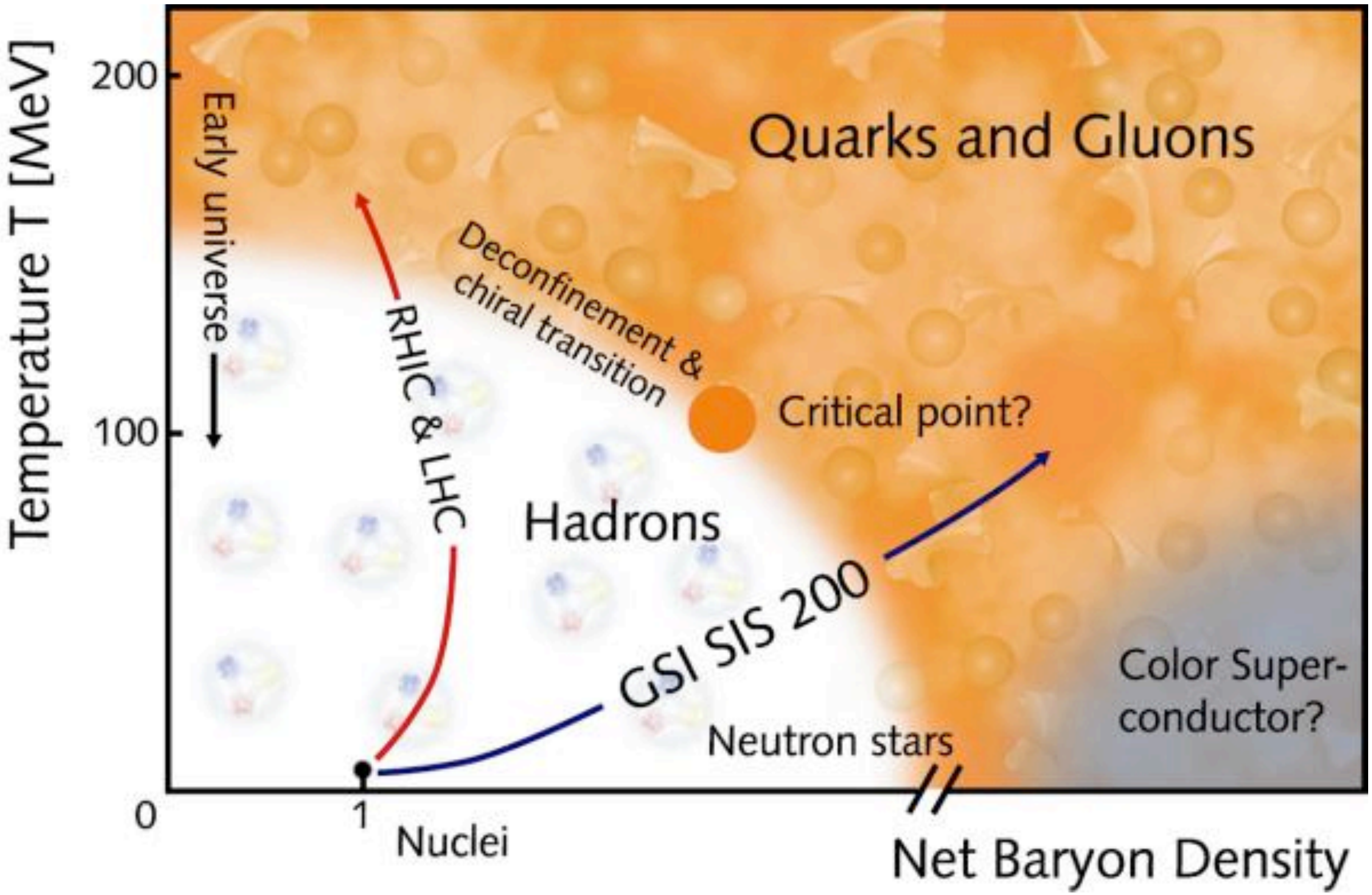
Parameter range: Explosion



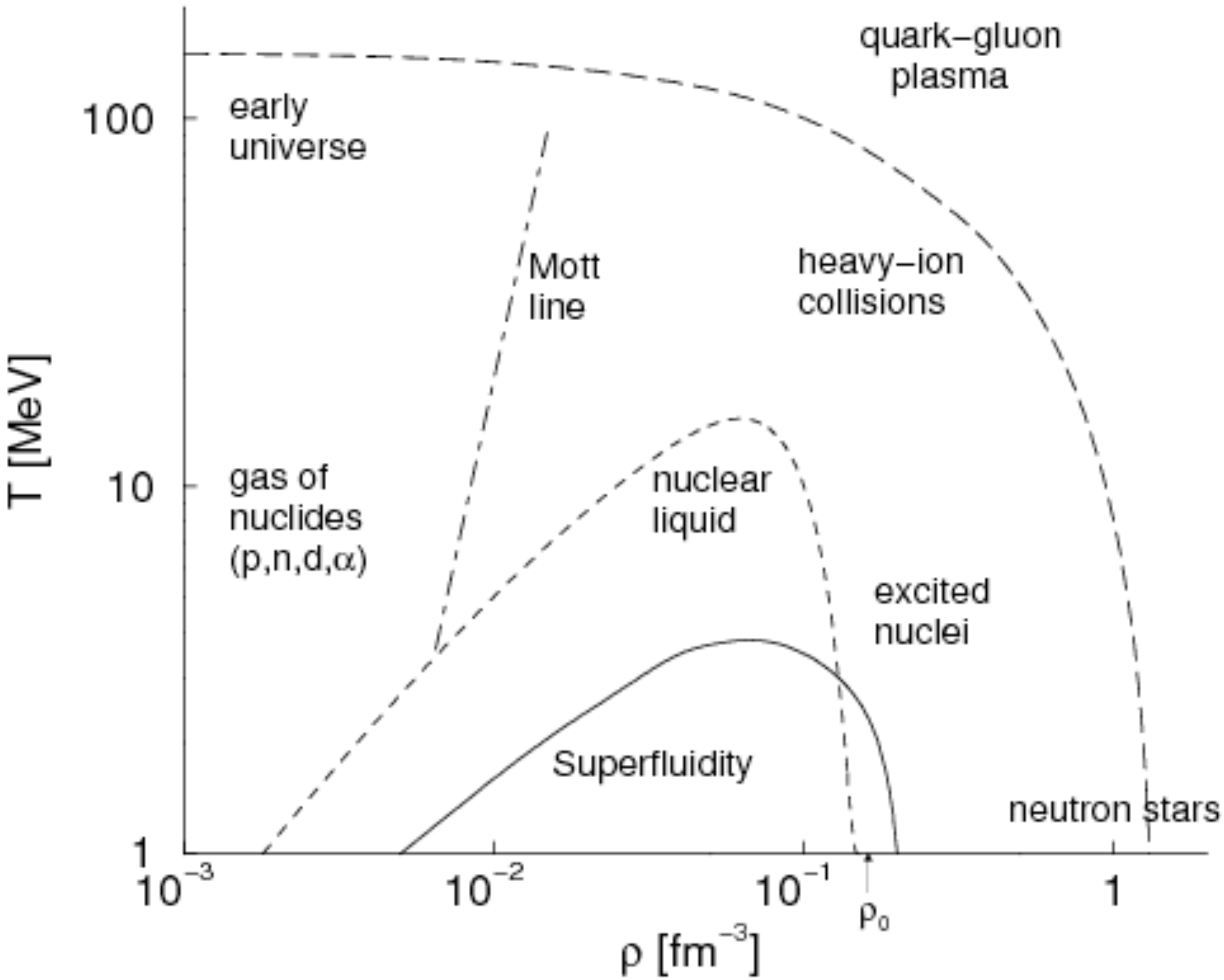
40 m_solar

T. Fischer, On the possible fate of massive progenitor stars, Talk 25.2.08 Ladek

Phase diagram: nuclear matter



Symmetric nuclear matter: Phase diagram



Correlations in low-density matter

- Ideal fermion gas of protons and neutrons?
- Formation of bound states: nuclei
- Medium modifications: quasiparticle concept
- Relativistic mean field approach, nucleons
- Cluster - mean field approach

Properties of light clusters

	binding energy	mass	spin	rms–radius
n	0	939.565 MeV/c^2	1/2	0.34 fm
p	0	938.783 MeV/c^2	1/2	0.87 fm
d	−2.225 MeV	1876.12 MeV/c^2	1	2.17 fm
t	−8.482 MeV	2809.43 MeV/c^2	1/2	1.70 fm
h	−7.718 MeV	2809.41 MeV/c^2	1/2	1.87 fm
α	−28.3 MeV	3728.40 MeV/c^2	0	1.63 fm

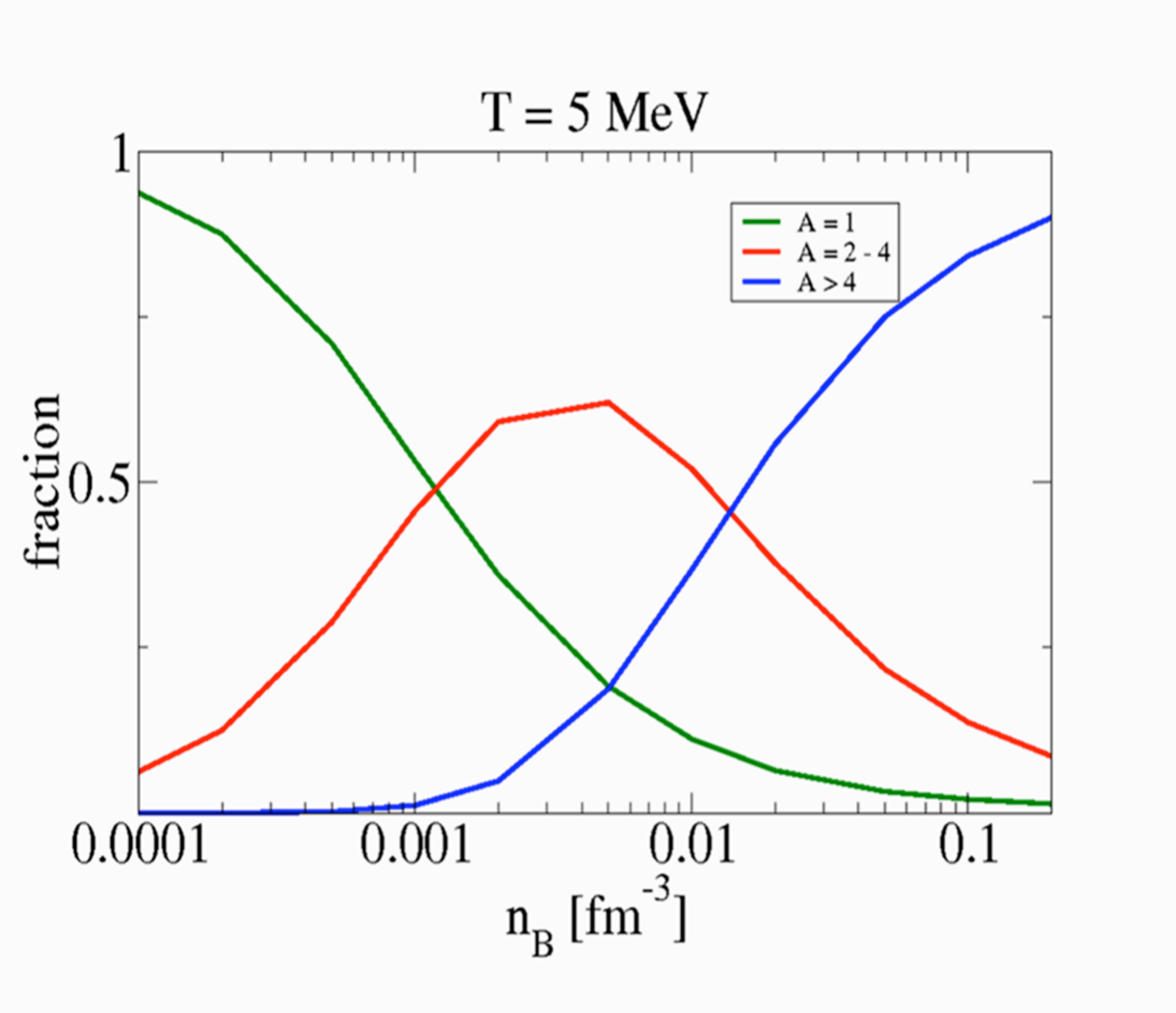
Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~K center of mass momentum

Composition of nuclear matter



Virial expansion

- excited nuclei
- resonances
- scattering phase shifts (no double counting)
- virial expansions
- quantum statistical approach

Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

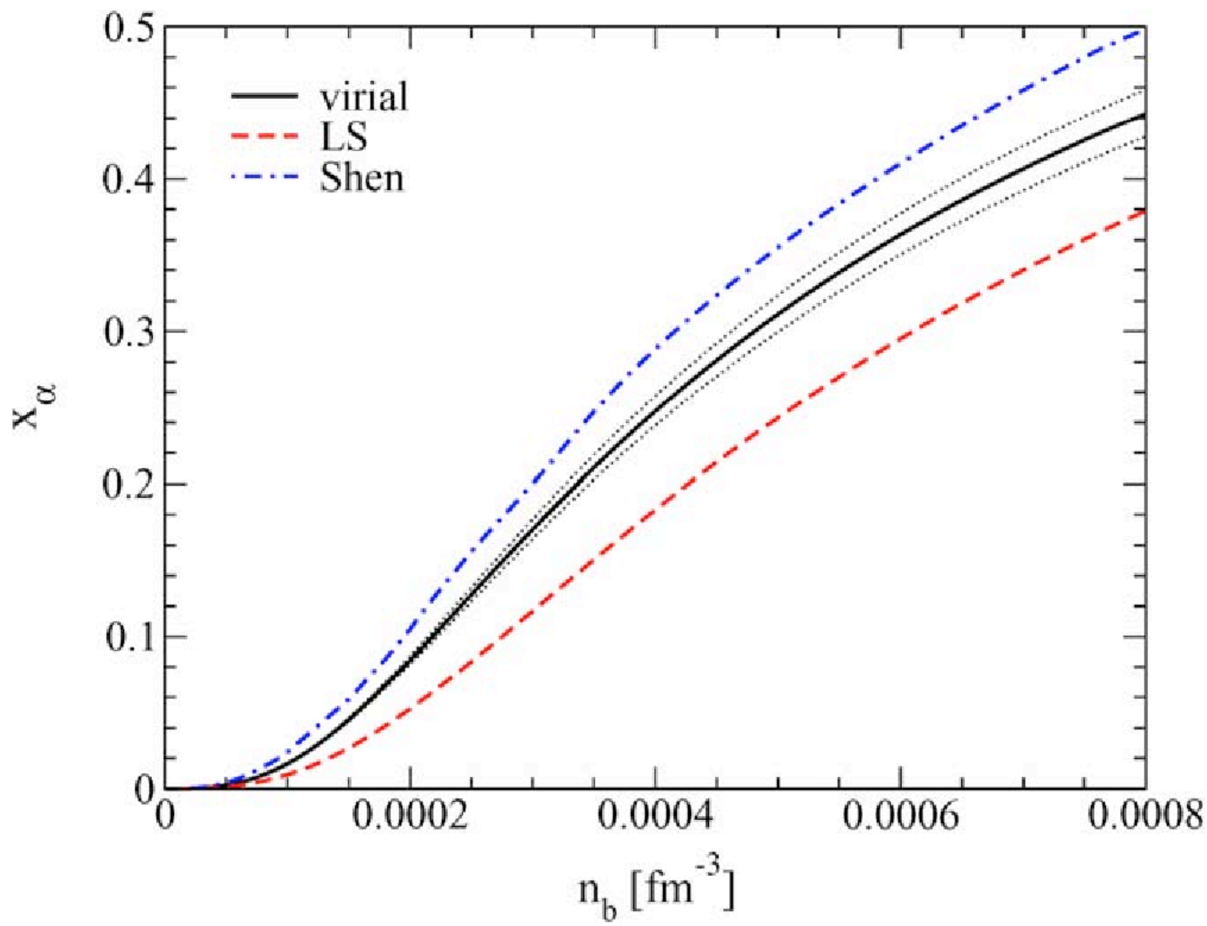
Beth-Uhlenbeck formula

$$
\begin{aligned}
n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\
&+ \dots
\end{aligned}
$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

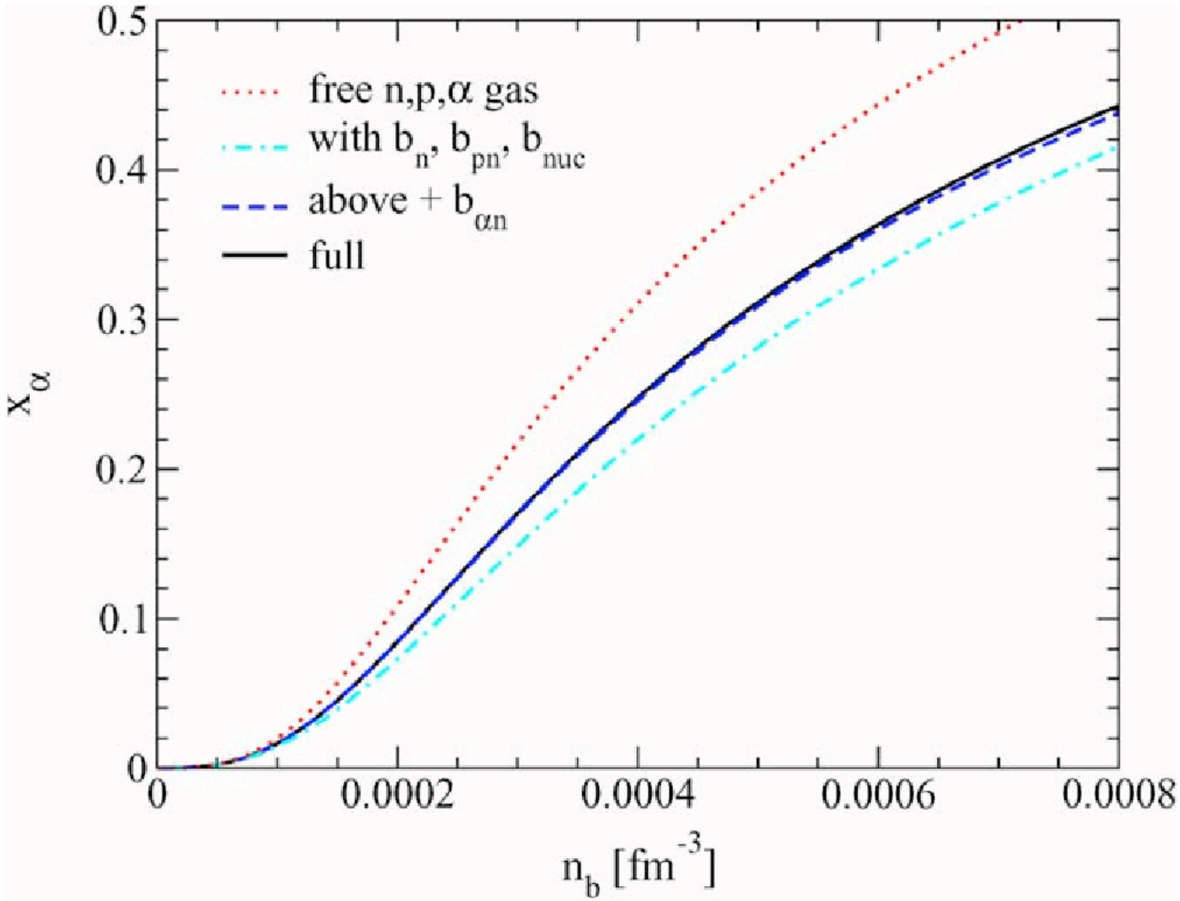
Alpha-particle fraction in the low-density limit

symmetric matter, T=4 MeV



Horowitz & Schwenk (2006), Lattimer & Swesty, (2001),Shen et al. (1998))

alpha - fraction at T = 4 MeV



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Nucleon-nucleon interaction

- general form:

$$V_\alpha(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}(p)\lambda_{\alpha ij} w_{\alpha j}(p') \qquad \text{uncoupled}$$

and

$$V_\alpha^{LL'}(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}^{L}(p)\lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \qquad \text{coupled}$$

p, p'		in- and outgoing relative momentum
α	...	channel
N	...	rank
$\lambda_{\alpha ij}$	.	coupling parameter
L, L'		orbital angular momentum

Separable nucleon-nucleon interaction

- examples:
  - Yamaguchi potential

    Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954)

    rank = 1, uncoupled, only S-waves
  - PARIS-potential, PESTN

    J. Haidenbauer and W. Plessas, Phys. Rev. **C 30**, 1822 (1984)

    consideration of partial waves up to $L = 2$
  - BONN-potential, BESTN

    Plessas et al., Few-Body Syst. Suppl. 7, 251 (1994)

    consideration of partial waves up to $L = 3$

Many-particle theory

- equilibrium correlation functions

  e.g. equation of state $n(\beta,\mu) = \frac{1}{\Omega_0}\sum_1 \langle a_1^\dagger a_1 \rangle$

  density matrix $\langle a_1^\dagger a_{1'} \rangle = \int \frac{d\omega}{2\pi}\, e^{-i\omega t} f_1(\omega) A(1,1',\omega)$

- Spectral function

$$A(1,1',\omega) = \mathrm{Im}\left[G(1,1',\omega+i\eta) - G(1,1',\omega-i\eta)\right]$$

- Matsubara Green function

$$G(1,1',iz_\nu), \qquad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \cdots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega-\mu)}+1}, \quad \Omega_0 - \text{volume}$$

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1,\omega) = \frac{2\mathrm{Im}\,\Sigma(1, \omega + i0)}{[\omega - E(1) - \mathrm{Re}\,\Sigma(1,\omega)]^2 + [\mathrm{Im}\,\Sigma(1, \omega + i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

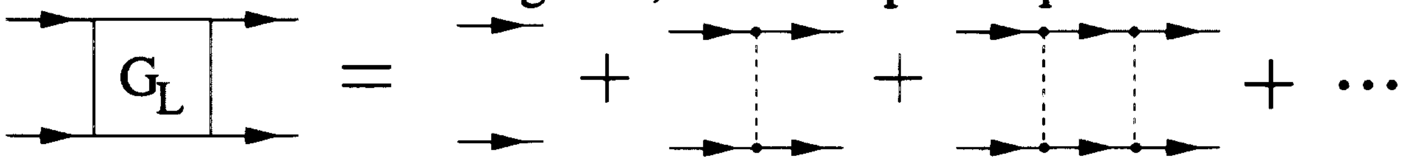
- Expansion for small $\mathrm{Im}\,\Sigma(1,\omega+i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\rm quasi}(1))}{1-\frac{\rm d}{{\rm d}z}{\rm Re}\;\Sigma(1,z)|_{z=E^{\rm quasi}-\mu_1}} - 2{\rm Im}\;\Sigma(1,\omega+i\eta)\frac{\rm d}{{\rm d}\omega}\frac{P}{\omega+\mu_1-E^{\rm quasi}(1)}$$

quasiparticle energy $E^{\rm quasi}(1) = E(1) + {\rm Re}\;\Sigma(1,\omega)|_{\omega=E^{\rm quasi}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation



Medium effects

Quasiparticle approximation

Lagrangian: non-linear sigma

$$
\begin{aligned}
\mathcal{L} \;=\; & \bar{\psi}_i[i\gamma_\mu\partial^\mu - m_i - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho_a^\mu]\psi_i \\
& +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 \\
& -\frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho_a^\mu \;.
\end{aligned}
$$

TM1 parameters

- Shen et al. (1998)

Parameter	Numerical	value
m_p	938.783	MeV
m_n	939565	MeV
m_σ	511.19777	MeV
m_ω	783.0	MeV
m_ρ	770.0	MeV
g_σ	10.02892	
g_ω	12.61394	
g_ρ	4.63219	
g_2	-7.23247	fm^{-1}
g_3	0.61833	
c_3	71.30747	

Quasiparticle energies

- Skyrme
- relativistic mean field (RMF)

$$e_i(k) = \sqrt{(m_i c^2 - S(n_B, \delta, T))^2 + \hbar^2 c^2 k^2} + V_i(n_B, \delta, T)$$

$$n_i = \frac{1}{\pi^2} \int_0^\infty dk \, \frac{k^2}{\exp[e_i(k)/k_B T - \mu_i/k_B T] + 1}$$

Single particle modifications

- effective mass

$$m_i^* = m_i + g_\sigma \sigma_0 \,,$$

- energy shift

$$E_p(k;T,\mu_p,\mu_n) = \sqrt{k^2 + {m_p^*}^2} + g_\omega \omega_0 + g_\rho \rho_0 \,,$$
$$E_n(k;T,\mu_p,\mu_n) = \sqrt{k^2 + {m_n^*}^2} + g_\omega \omega_0 - g_\rho \rho_0$$

DD-RMF (Typel, 2007)

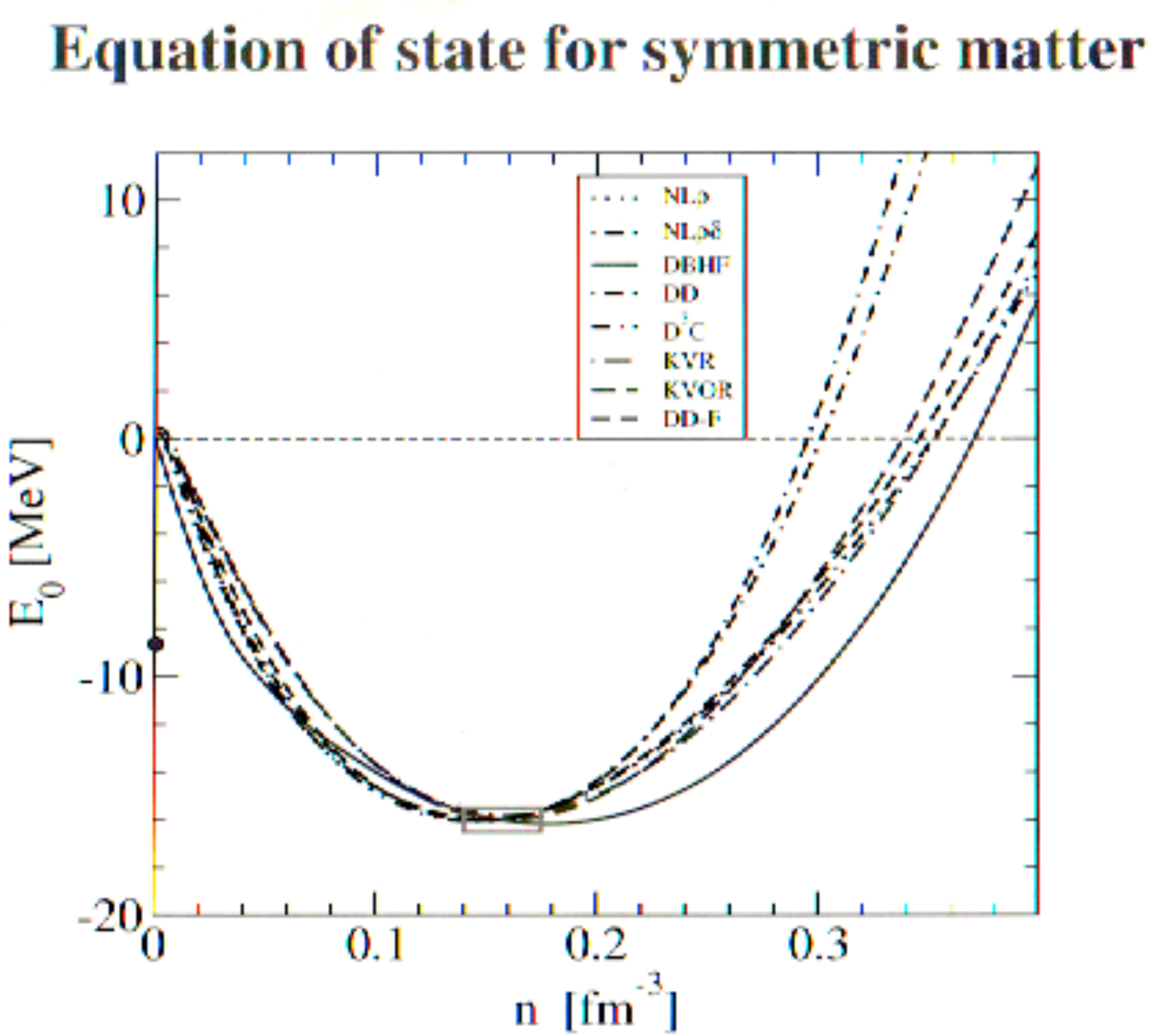
- scalar field

$$
\begin{aligned}
S(n_B,\delta,T) &= n_B[(4524.13-6.926T)-14.5157/4\delta^2+0.833943/16\delta^4-9.00693/64\delta^6] \\
&+n_B^2[-19190.7-2426.57/4\delta^2-317.732/16\delta^4-1547.38/64\delta^6] \\
&+n_B^3[62169.5+2521.29/4\delta^2+3470.28/16\delta^4] \\
&+n_B^4[-91005.1+3984.82/4\delta^2-9148.6/16\delta^4];
\end{aligned}
$$

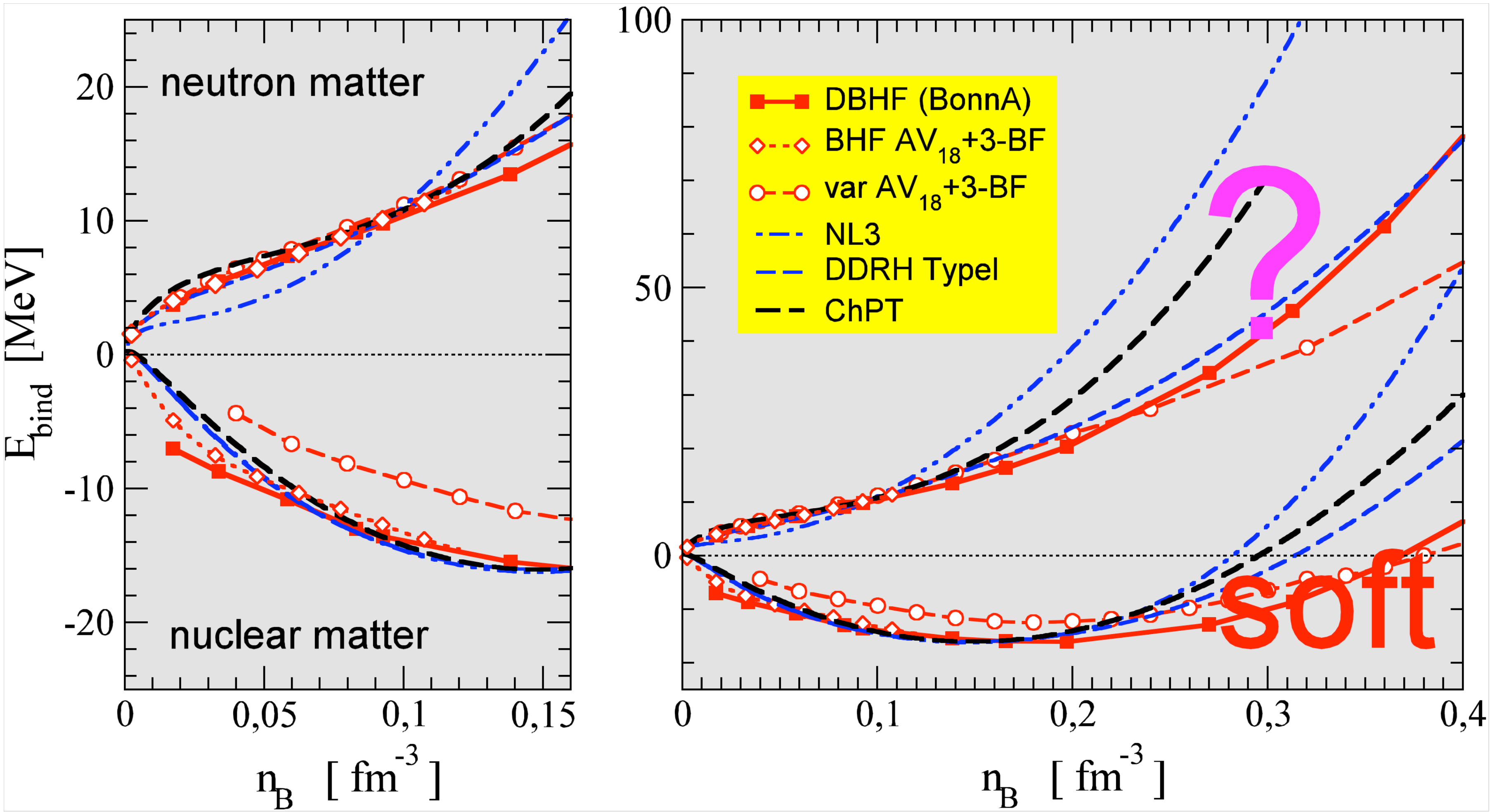
- vector fields

$$
\begin{aligned}
V_p(n_B,\delta) &= V_n(n_B,-\delta)=n_B[3462.24+946.705/2\delta-0.334508/4\delta^2] \\
&+n_B^2[-11312.4-6246.21/2\delta-6353.53/4\delta^2-0.099478/8\delta^3] \\
&+n_B^3[20806.1+18717.6/2\delta+29298./4\delta^2-0.490543/8\delta^3] \\
&+n_B^4[352.371-24887.2/2\delta-39807.4/4\delta^2-0.346218/8\delta^3].
\end{aligned}
$$

Quasiparticle approximation for nuclear matter



RMF and DBHF



C. Fuchs et al. (2007)

Different approximations

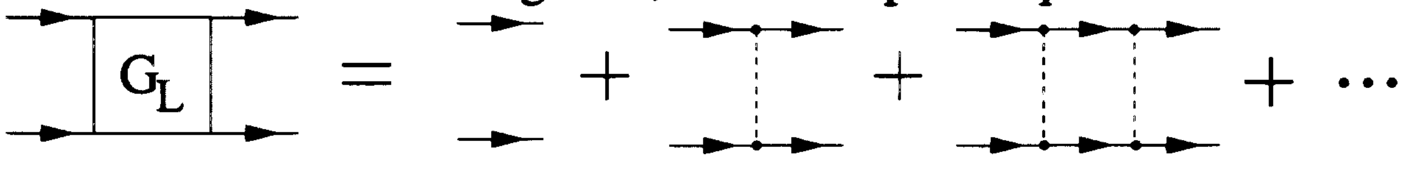
- **Expansion for small Im $\Sigma(1, \omega + i\eta)$**

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{\mathrm{d}}{\mathrm{d}z}\text{Re }\Sigma(1,z)|_{z=E^{\text{quasi}}-\mu_1}} - 2\text{Im }\Sigma(1,\omega+i\eta)\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{P}{\omega+\mu_1-E^{\text{quasi}}(1)}$$

quasiparticle energy $E^{\text{quasi}}(1) = E(1) + \text{Re }\Sigma(1,\omega)|_{\omega=E^{\text{quasi}}}$

- **chemical picture: bound states $\hat{=}$ new species**

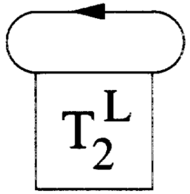
summation of ladder diagrams, Bethe-Salpeter equation



Different approximations

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$ 

$$n(\beta,\mu) = \sum_{1} f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k \; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2\sin^2\delta_n(k) \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}k} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Effective wave equation for the deuteron in matter

$$\left(\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}\right)\Psi_{n,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2')$$

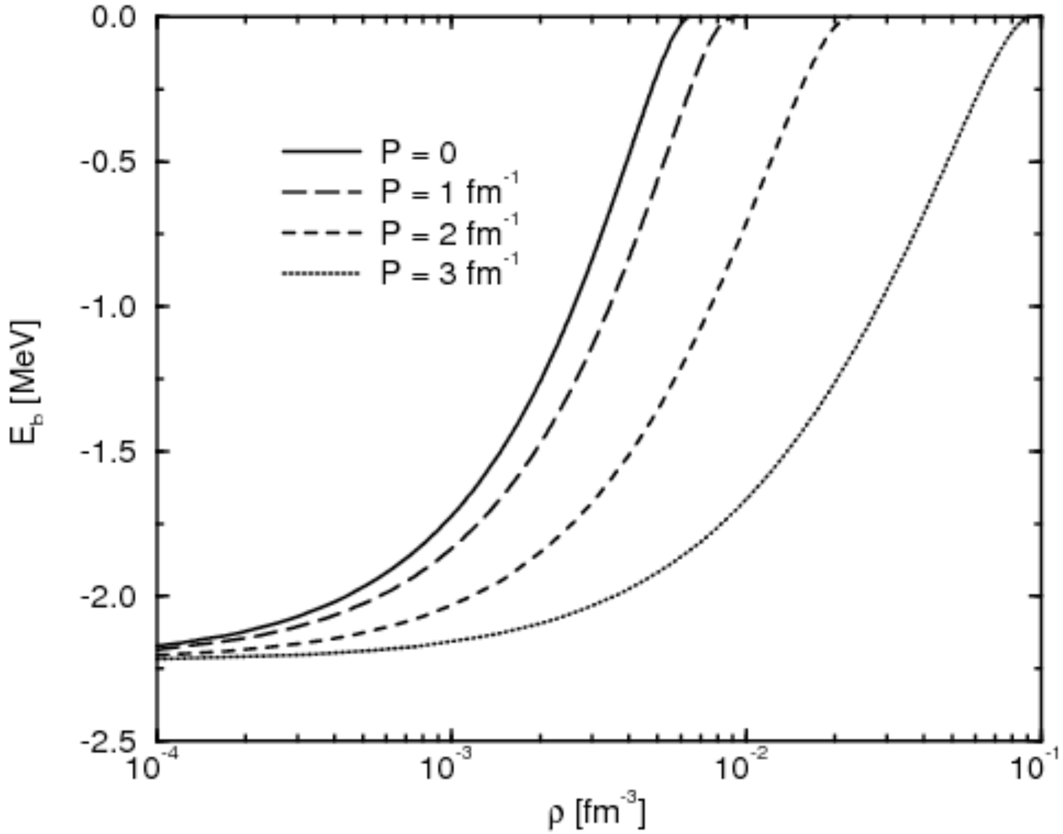
$$=E_{n,P}\Psi_{n,P}(p_1,p_2)$$

Pauli-blocking

Fermi distribution function

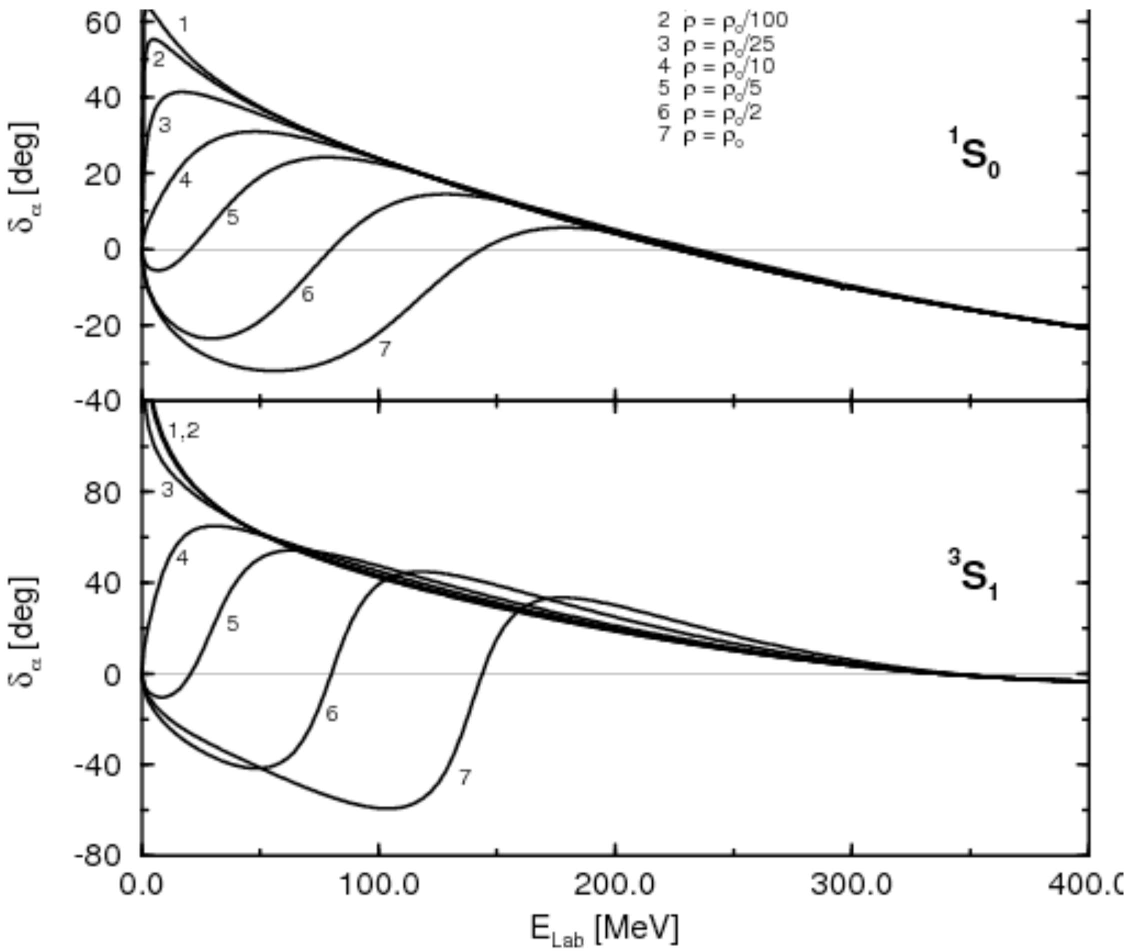
$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Deuterons in nuclear matter



T=10 MeV, P: center of mass momentum

Scattering phase shifts in matter



Generalized Beth-Uhlenbeck formula

- quantum gas
- medium modifications

$$n(T, \mu) = n_1(T, \mu) + n_2(T, \mu)$$

contribution of free quasiparticles

$$n_1(T, \mu) = \frac{1}{V} \sum_p \frac{1}{e^{(E^{\mathrm{HF}}(p)-\mu)/k_B T} + 1}$$

($E^{\mathrm{HF}}(p)$: quasiparticle energy)

contribution of two-particle correlations

$$n_2(T, \mu) = n_2^{\mathrm{bound}}(T, \mu) + n_2^{\mathrm{scatt}}(T, \mu)$$

Generalized Beth-Uhlenbeck formula

$$n_2^{\mathrm{bound}}(T,\mu) = \frac{1}{V}\sum_{nP}\frac{1}{e^{(E_{nP}^{\mathrm{mean\,field}}-2\mu)/k_BT}-1},$$

$$n_2^{\mathrm{scatt}}(T,\mu) = \frac{1}{V}\sum_{\alpha P}\int_0^\infty\frac{dE}{2\pi}\frac{1}{e^{(E_{\alpha P}^{\mathrm{mean\,field}}+E-2\mu)/k_BT}-1}\sin^2(\delta_\alpha(E))\frac{d}{dE}\delta_\alpha(E)$$

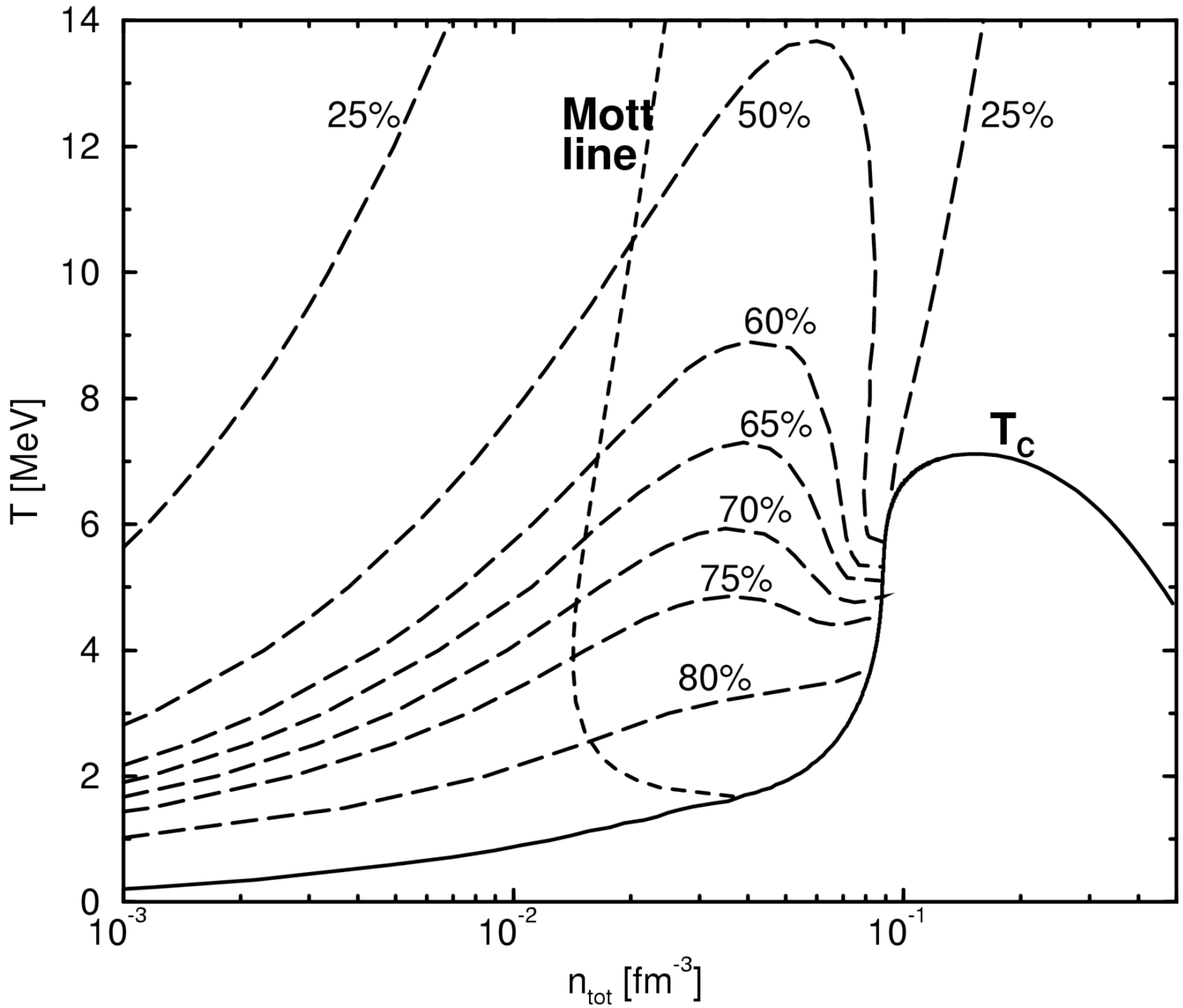
Composition: (ionization degree)

correlated part $\frac{n_2(T,\mu)}{n(T,\mu)}$

Bose distribution function: Quantum condensates

Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

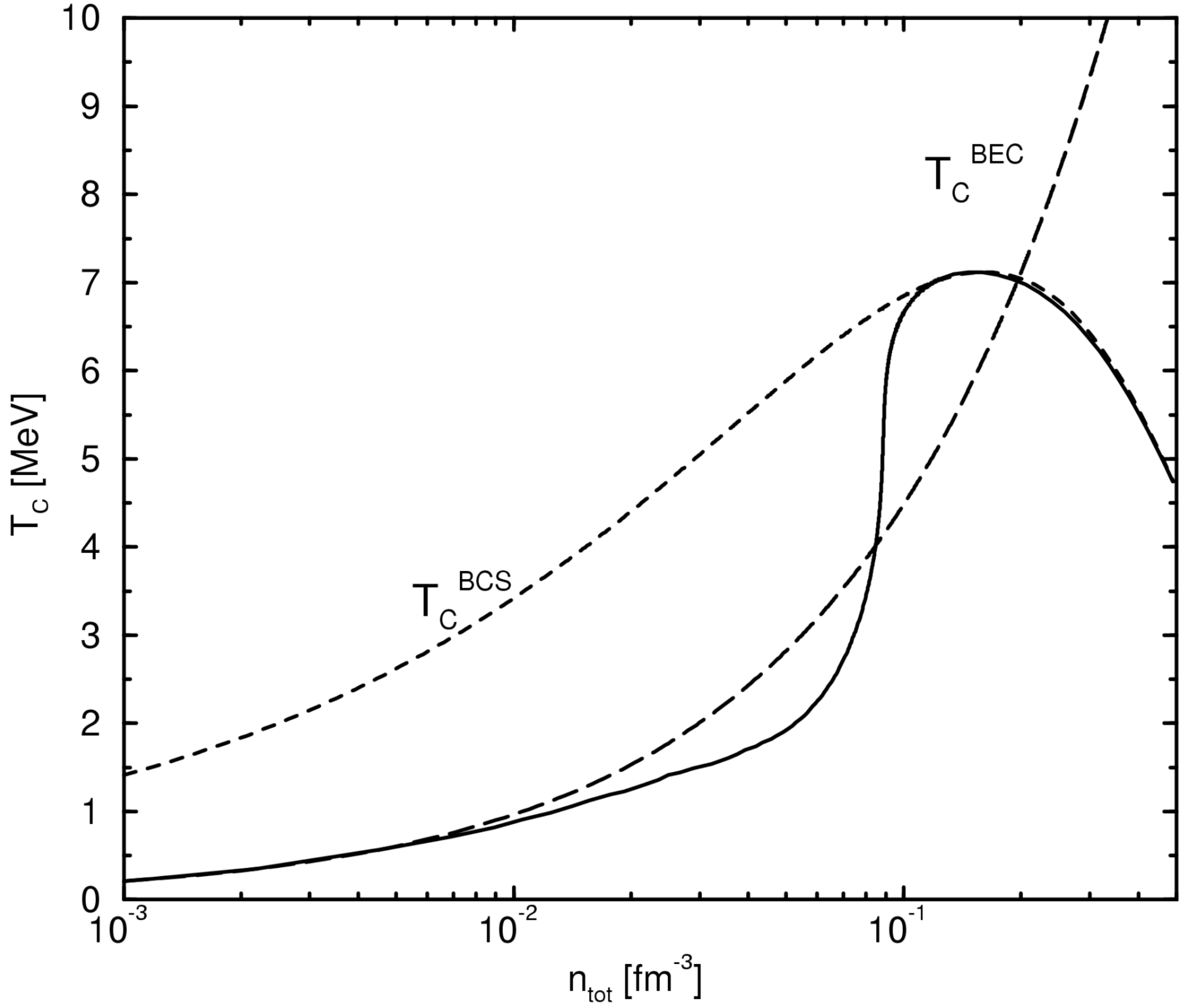


H. Stein et al.,
Z. Phys. **A351**, 259 (1995)

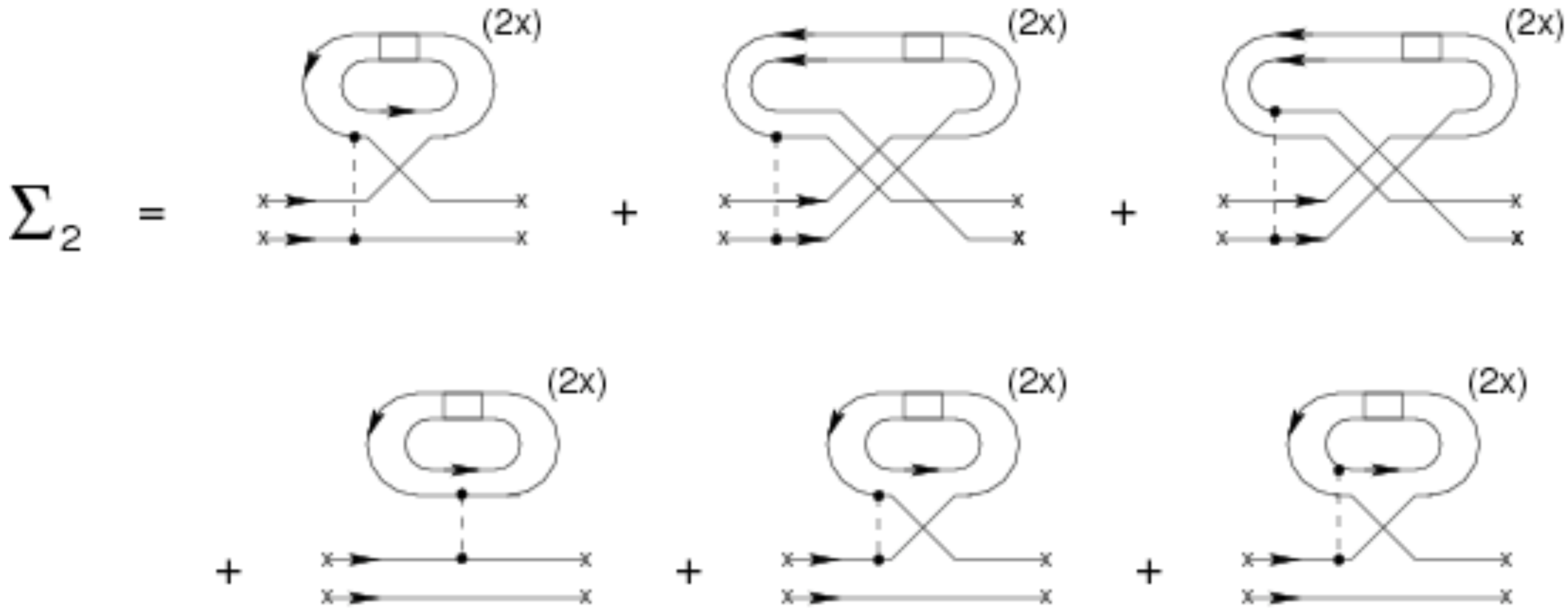
Quantum condensate

Bose-Einstein-
Condensation
of deuterons
(BEC)

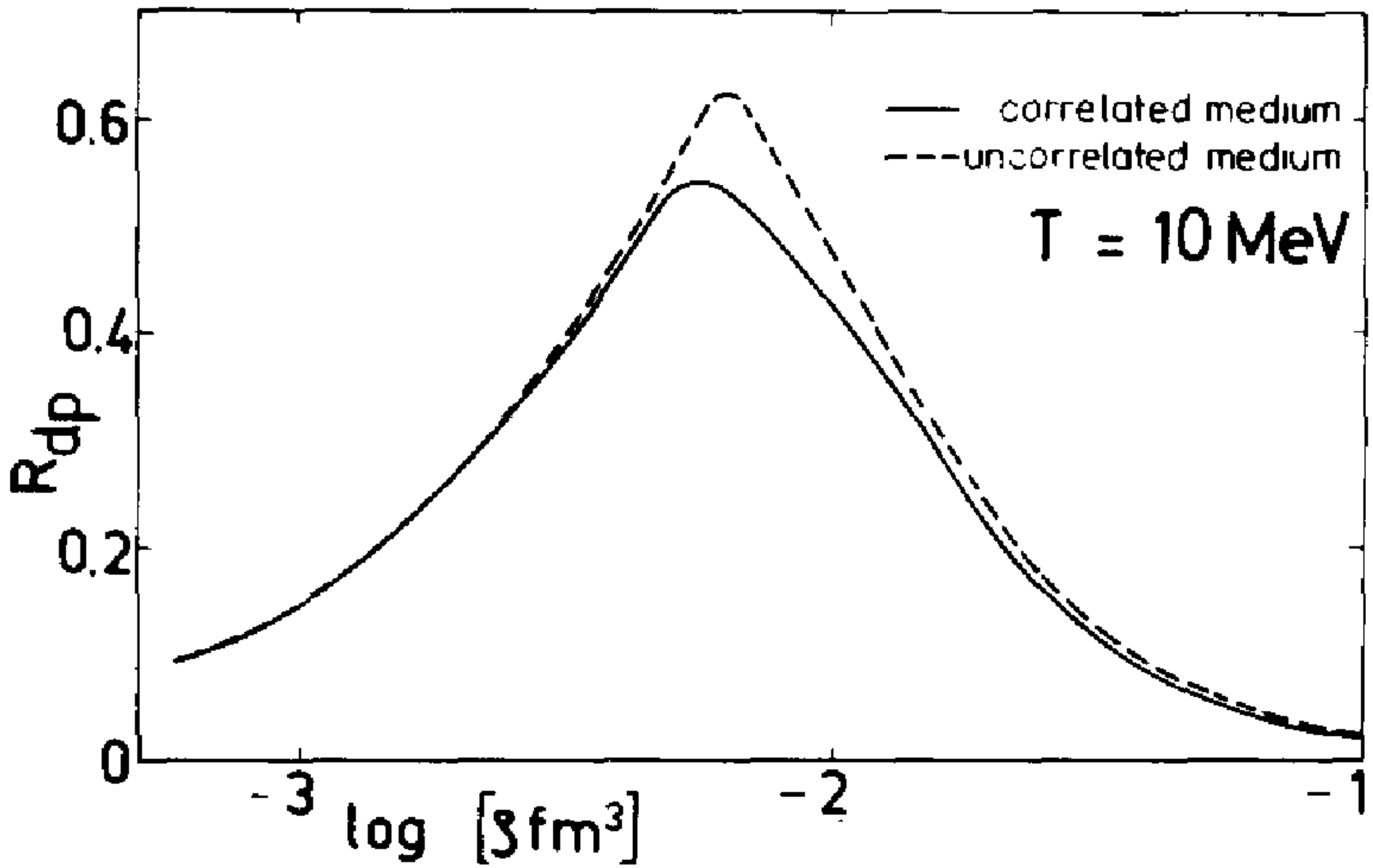
Bardeen-Cooper
Schrieffer
pairing
(BCS)



Correlations in the medium

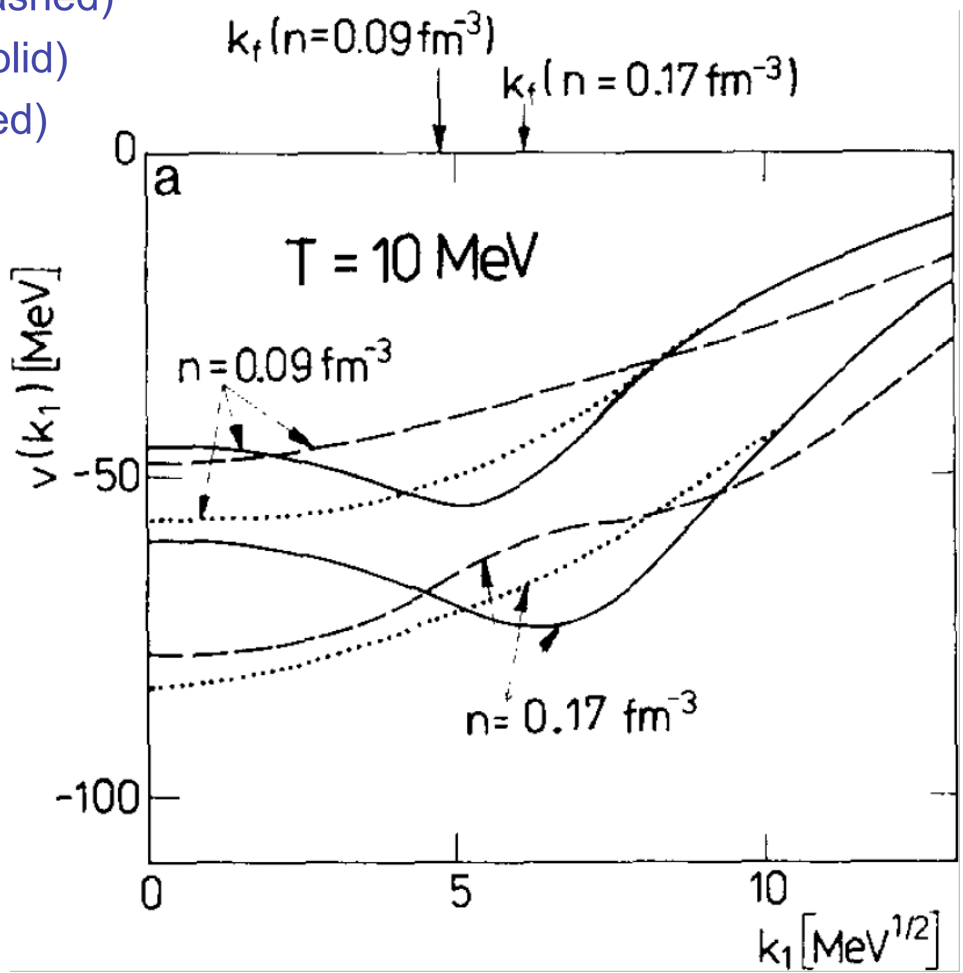


Account of two-particle correlations in the medium

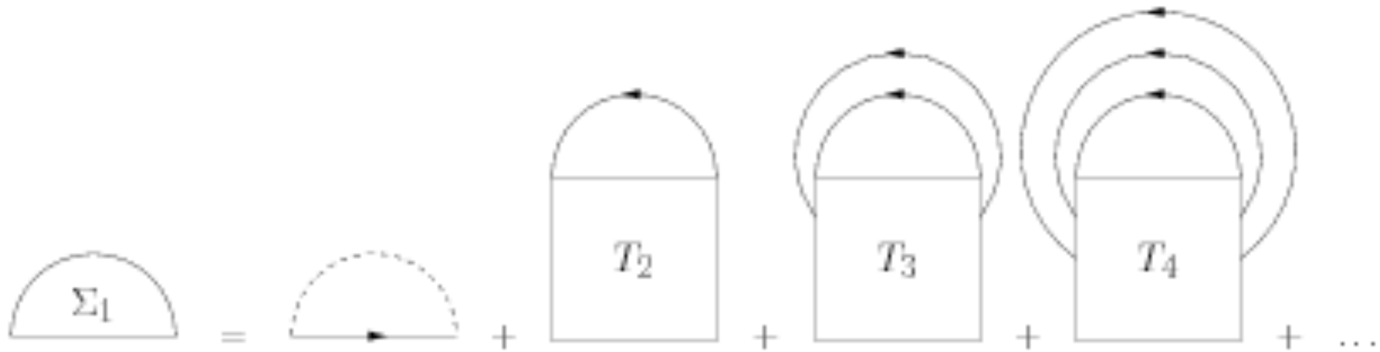


Quasiparticle energy shift

- Brueckner-Bethe-Goldstone (dashed)
- generalized Beth-Uhlenbeck (solid)
- incomplete Pauli-blocking (dotted)



Cluster decomposition of the self-energy



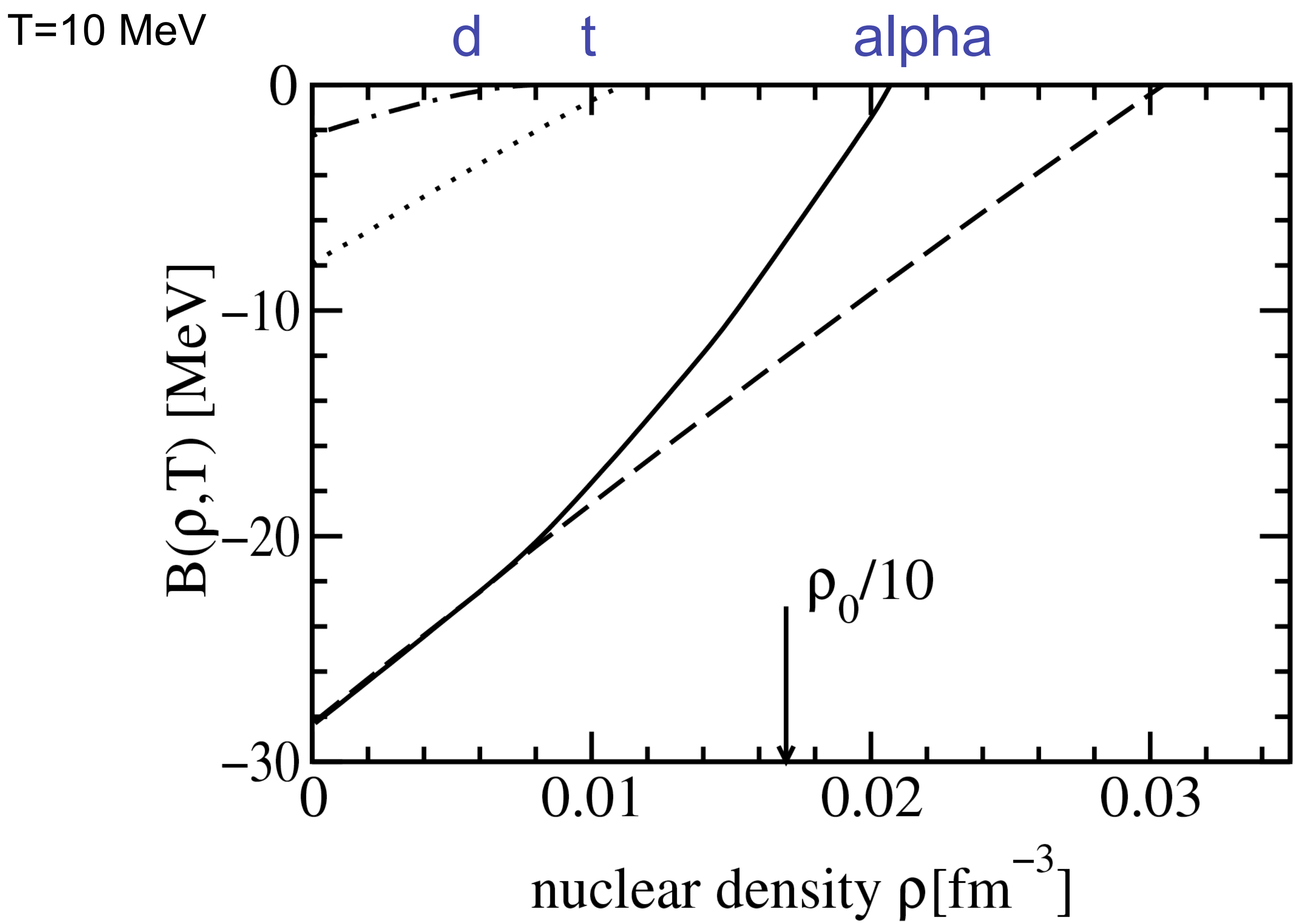
Few-particle Schoedinger equation in a dense medium

Four-particle Schrödinger equation with medium effects

$$
\begin{aligned}
&[E^{\mathrm{HF}}(p_1) + E^{\mathrm{HF}}(p_2) + E^{\mathrm{HF}}(p_3) + E^{\mathrm{HF}}(p_4)]\ \psi_{nP}(p_1, p_2, p_3, p_4) \\
&+ \sum_{p'_1 p'_2 p'_3 p'_4} \Big\{ [1 - f(p_1) - f(p_2)]\ V(p_1 p_2, p'_1 p'_2) \delta_{p_3 p'_3} \delta_{p_4 p'_4} \\
&\qquad + [1 - f(p_1) - f(p_3)]\ V(p_1 p_3, p'_1 p'_3) \delta_{p_2 p'_2} \delta_{p_4 p'_4} \\
&\qquad + \text{permutations} \Big\}\ \psi_{nP}(p'_1, p'_2, p'_3, p'_4) \\
&= E_{nP}\ \psi_{nP}(p_1, p_2, p_3, p_4)
\end{aligned}
$$

In-medium shift of binding energies of clusters

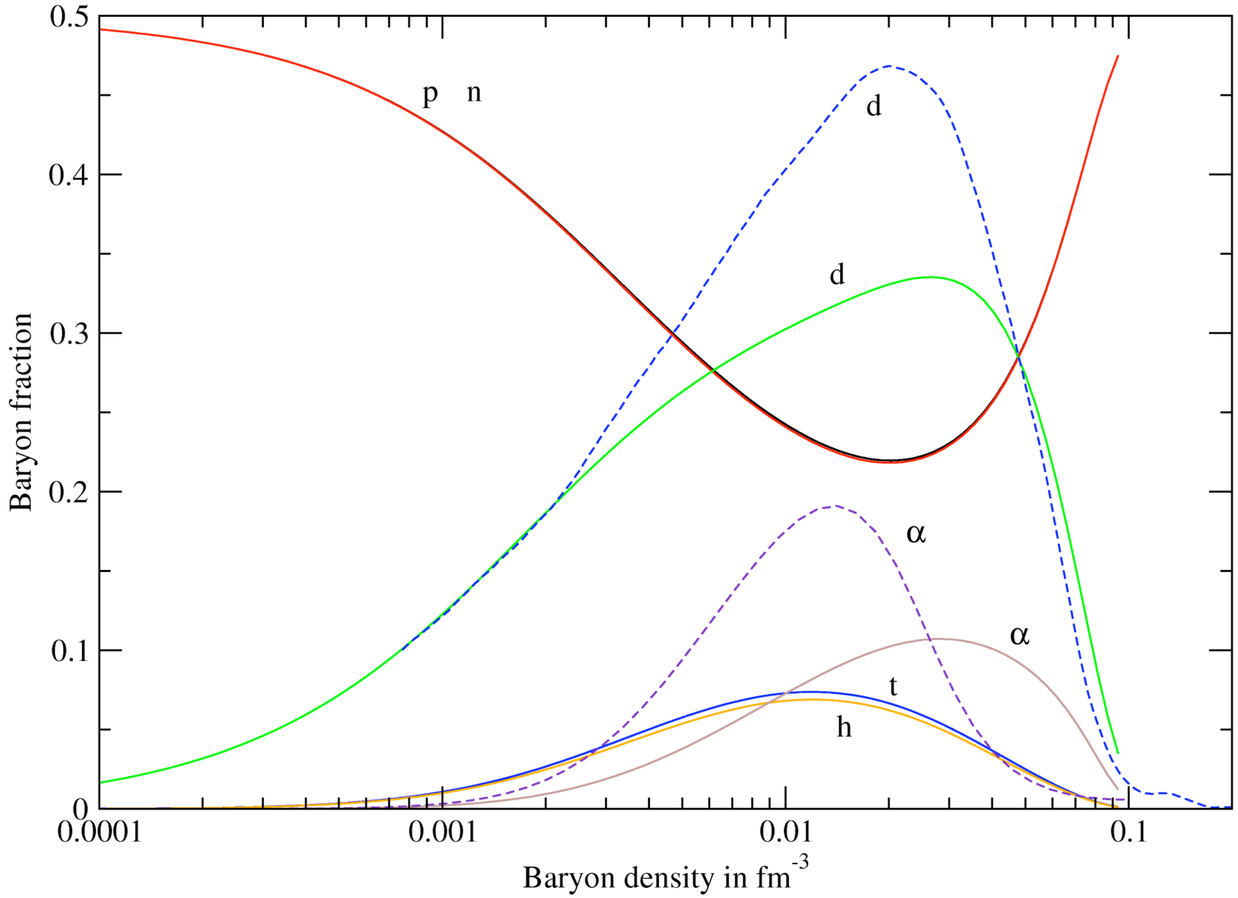
A. Sedrakian et al., PRC (2006), M. Beyer et al,, Phys.Lett.B **488**, 247 (2000)



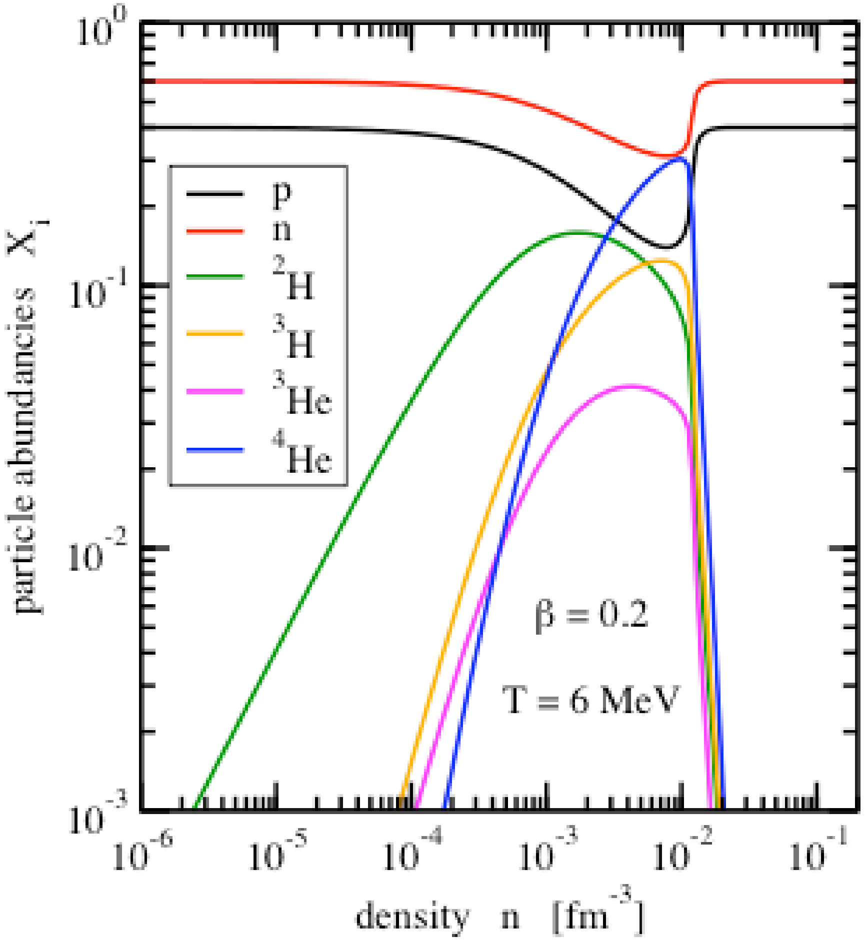
Composition of symmetric nuclear matter

T=10 MeV

G.Ropke, A.Grigo, K. Sumiyoshi, Hong Shen, Phys.Part.Nucl.Lett. **2**, 275 (2005)

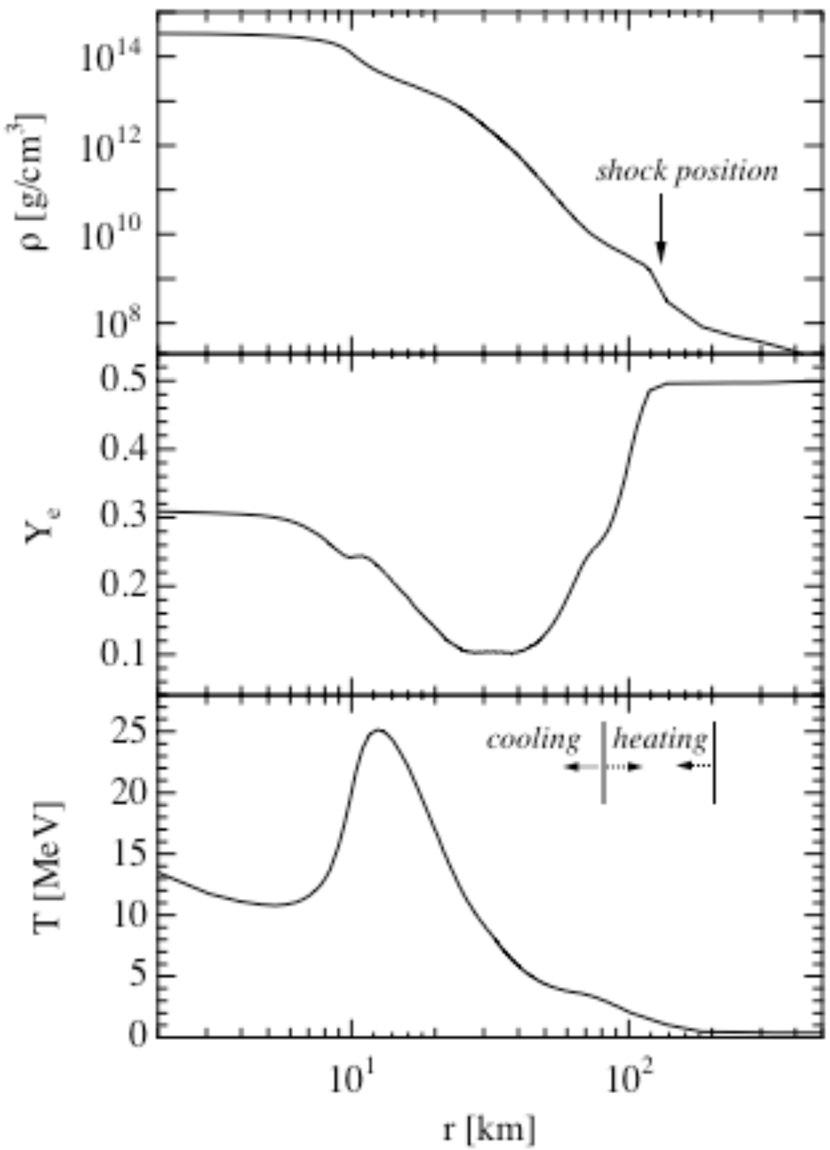


Light Cluster Abundances



S. Typel, 2007

Core-collapse supernovae



Density.

electron fraction, and

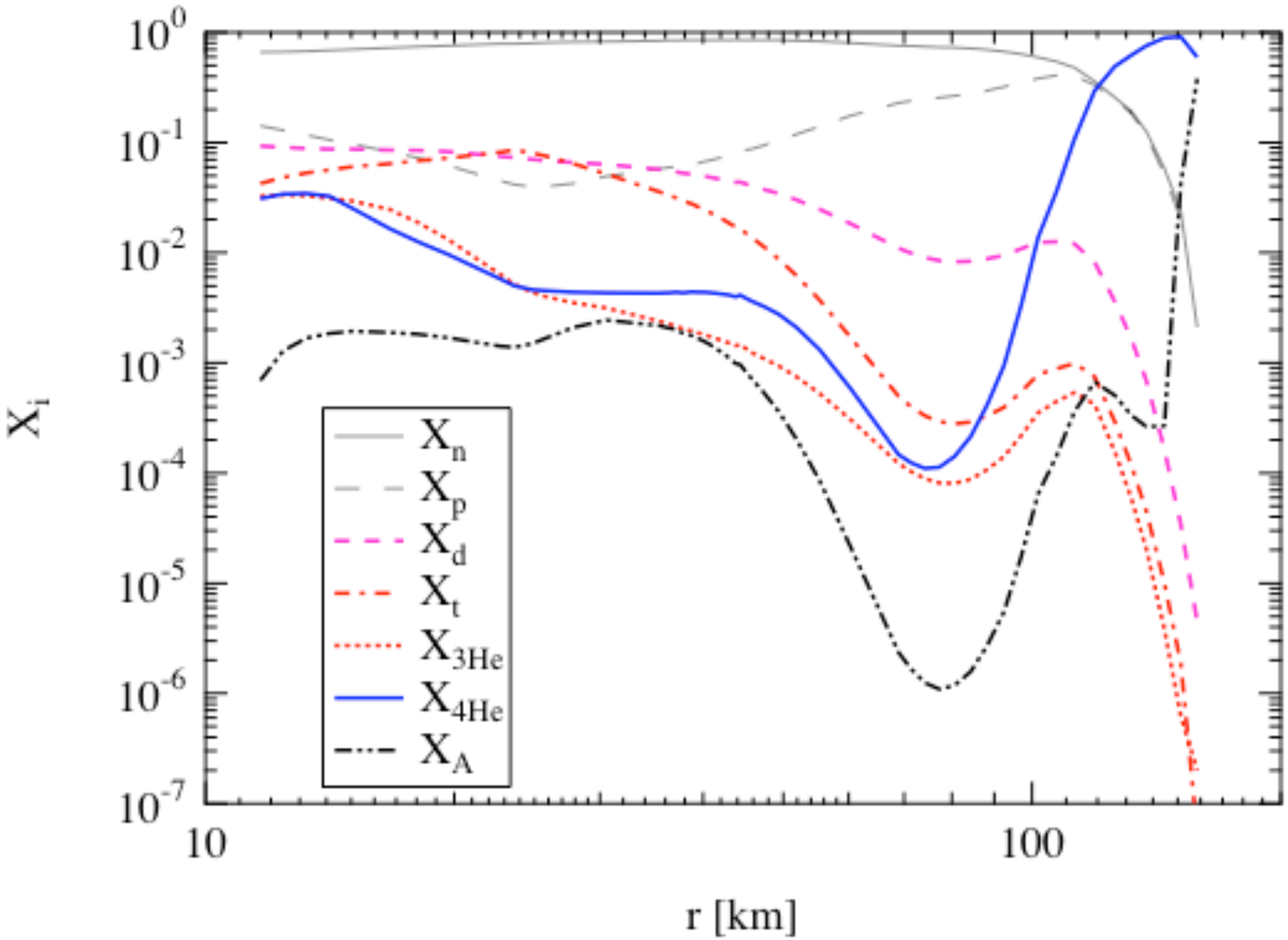
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

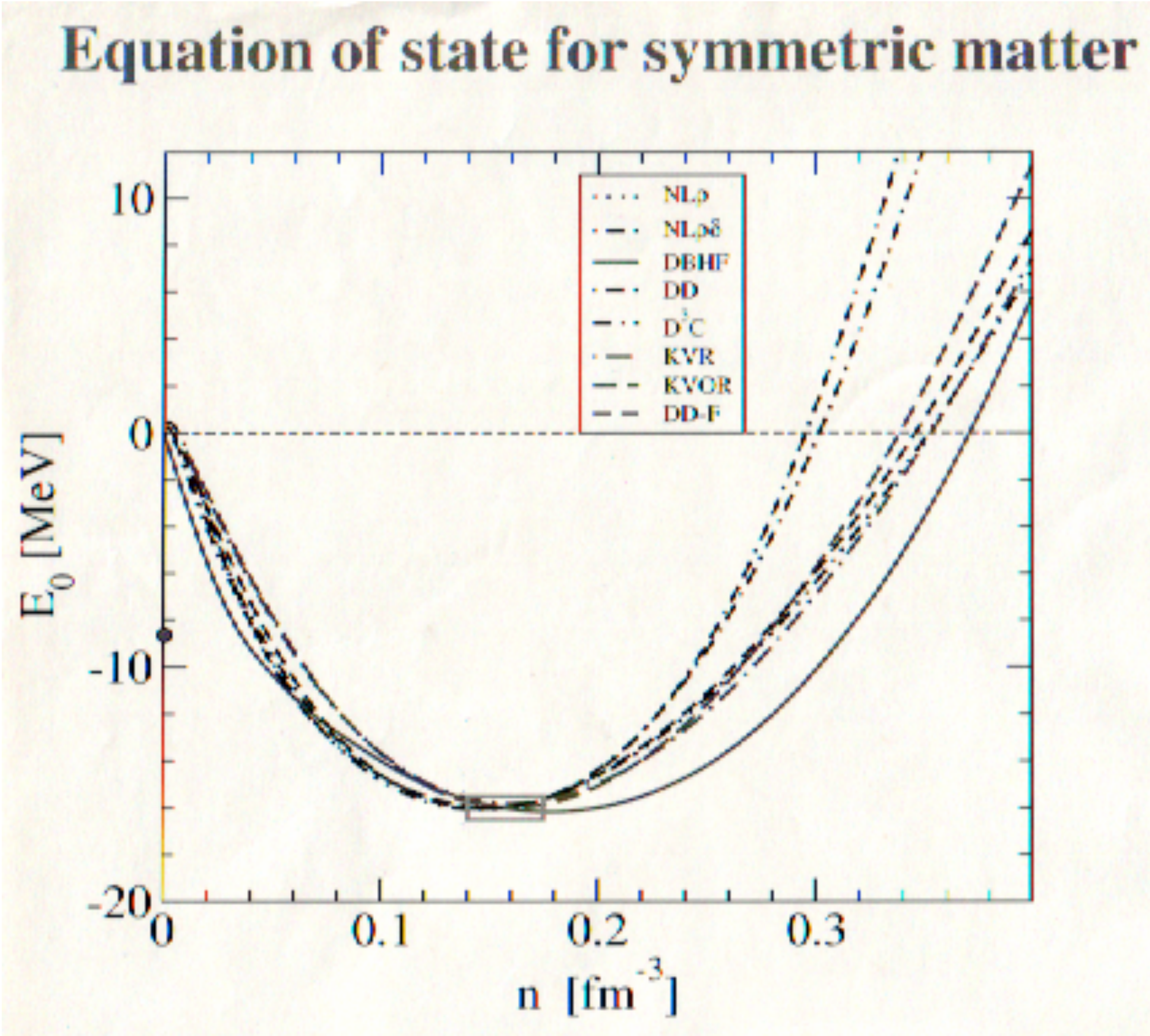
Composition of supernova core



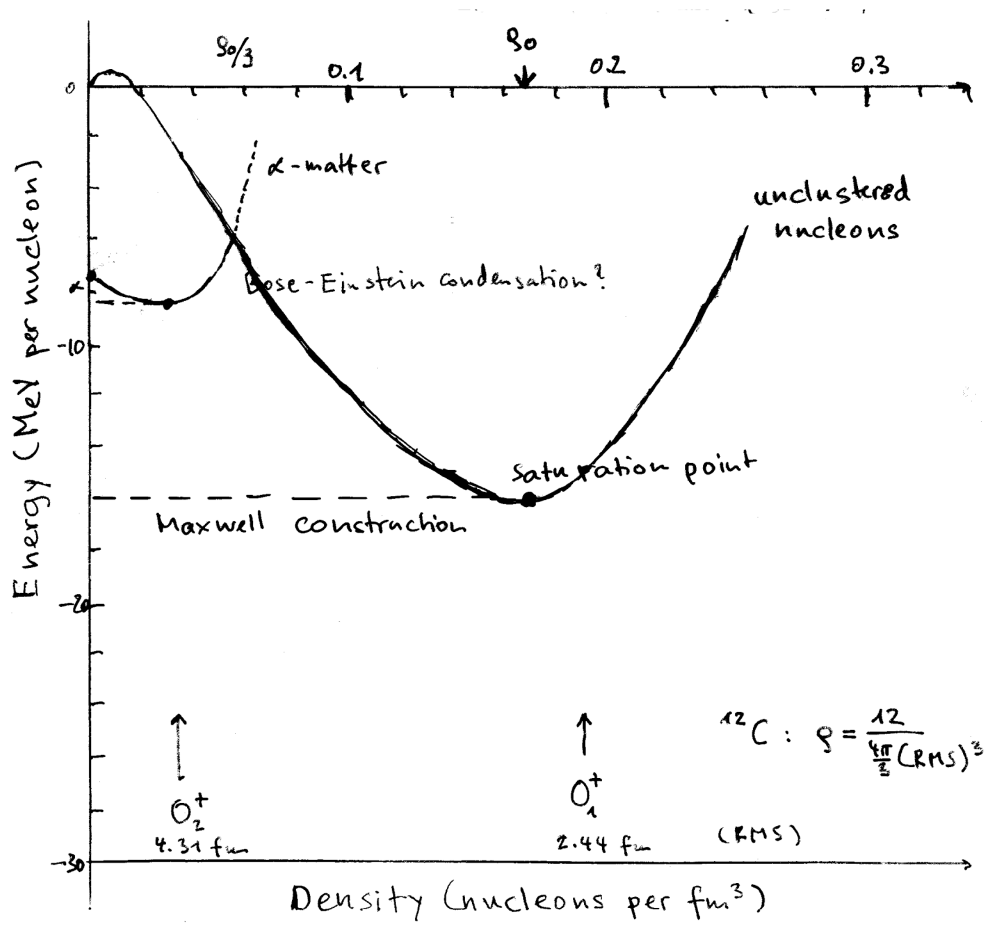
Mass fraction X of light clusters for a post-bounce supernova core

K. Sumiyoshi, G. Roepke
PRC, 2008

Quasiparticle approximation for nuclear matter



Low-density limit: alpha matter?



Break

- Shown today:
  - correlations
  - composition
  - medium effects
  - astrophysical relevance

- Monday's lecture:
  - quantum condensates
  - pairing and quartetting
  - Hoyle state and low-density isomers
  - low density limit of symmetry energy

HISS: Dense Matter in HIC and Astrophysics, Dubna, 18./21. 7. 08

Condensates and Correlations in Nuclear Matter

Gerd Röpke
Universität Rostock

Problems:

- Warm Dilute Matter: Nuclear matter at subsaturation densities:

Temperature $T \leq 16$ MeV $= E_s/A$, baryon density $n_B \leq 0.17$ fm^{-3} $= n_s$.

- Formation of clusters (nuclei in matter):

A = 1,2,3,4: deuterons (d), tritons (t), helions (h), alphas (α)

- Low-density, low-temperature limit:

Virial expansion, non-interacting nuclides, quantum condensates

- Transition to higher densities:

Medium effects, quasiparticles,

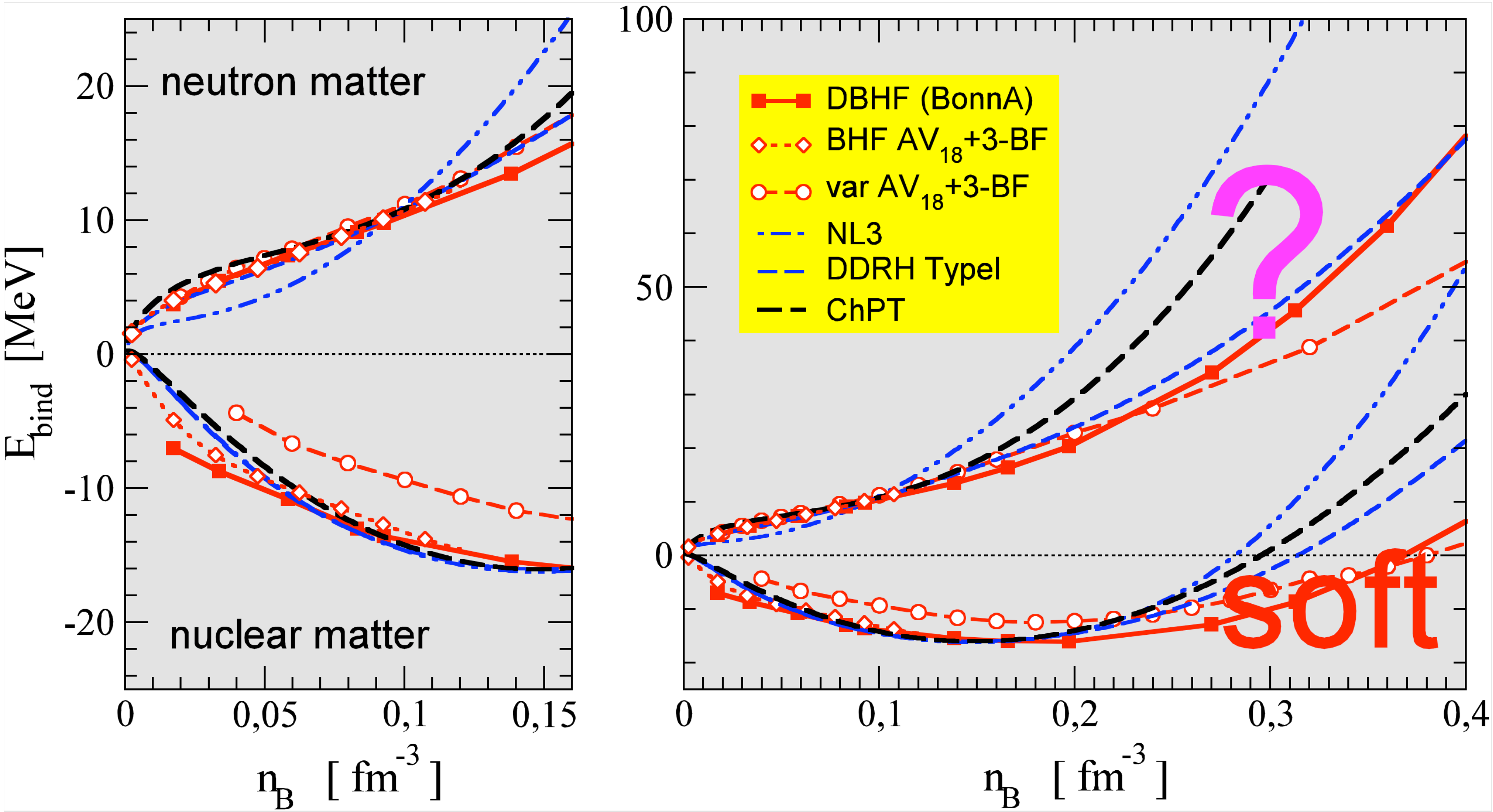
interpolation between Beth-Uhlenbeck and DBHF / RMF

Refs:

Particle clustering and Mott transition in nuclear matter at finite temperatures,
G. Röpke, M. Schmidt, L. Münchow, H. Schulz: NPA **399**, 587-602 (1983).

Generalized Beth-Uhlenbeck Approach for Hot Nuclear Matter,
M. Schmidt, G. Röpke, H. Schulz: Annals of Physics **202**, 57 - 99 (1990).

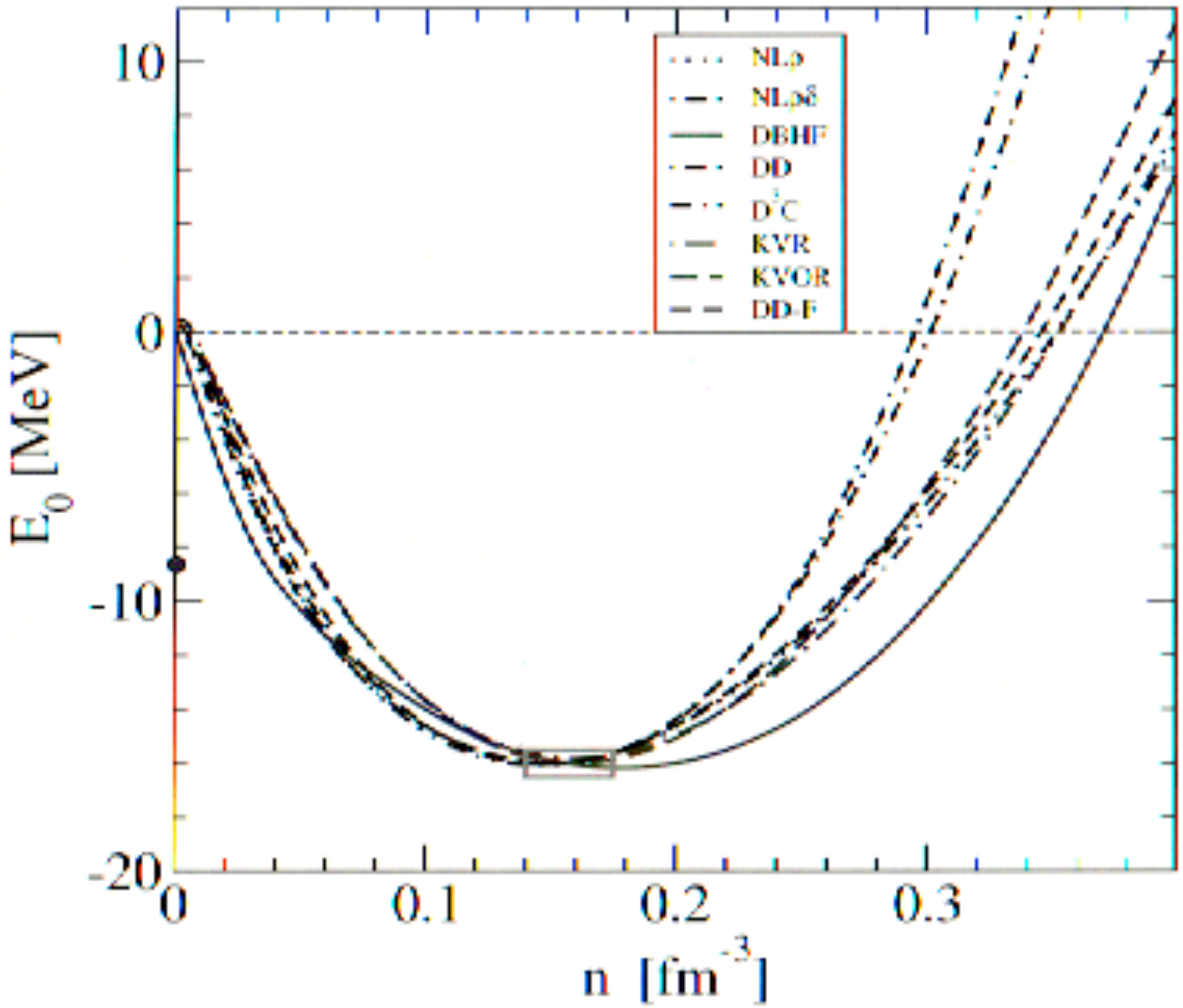
RMF and DBHF



C. Fuchs et al. (2007)

Quasiparticle approximation for nuclear matter

Equation of state for symmetric matter



Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

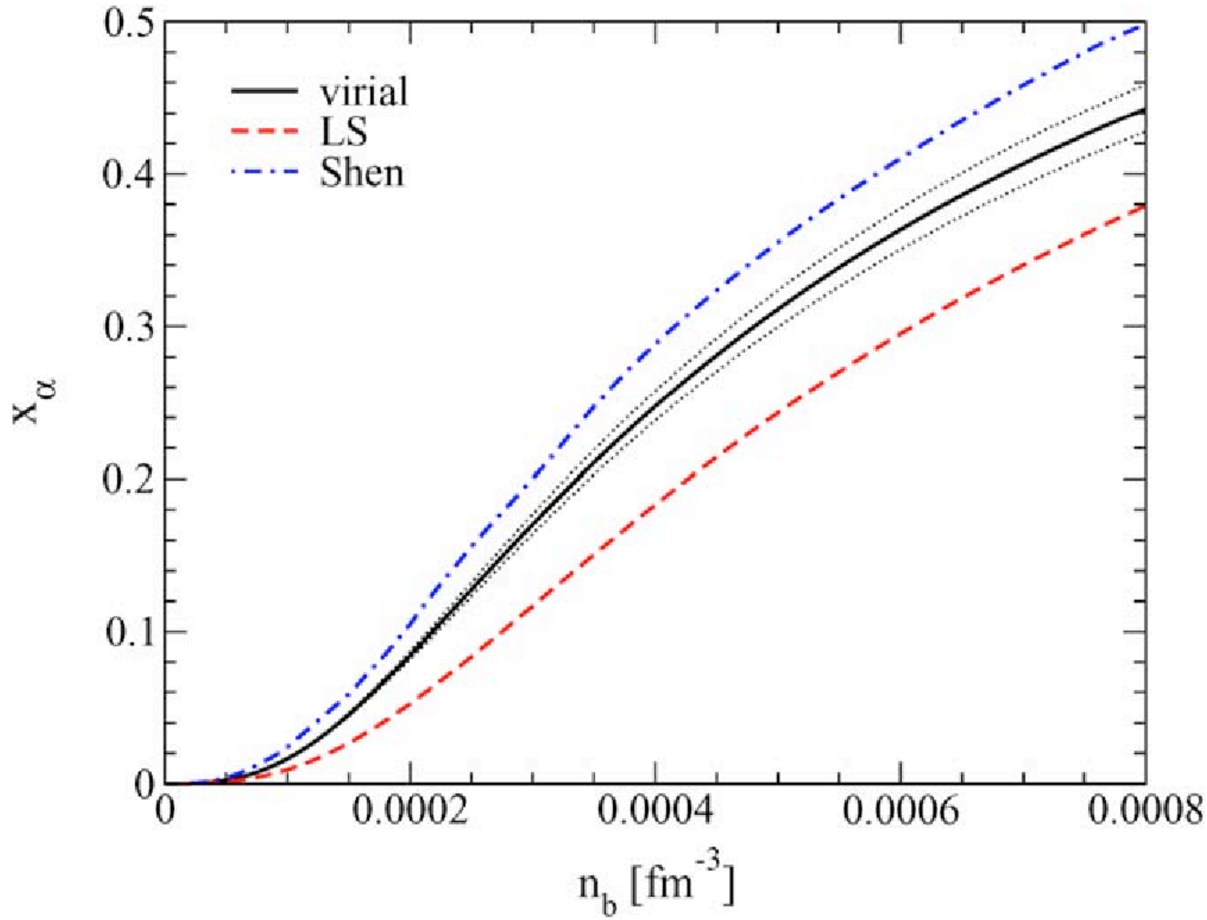
Beth-Uhlenbeck formula

$$\begin{aligned} n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\ &+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\ &+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\ &+ \dots \end{aligned}$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Alpha-particle fraction in the low-density limit

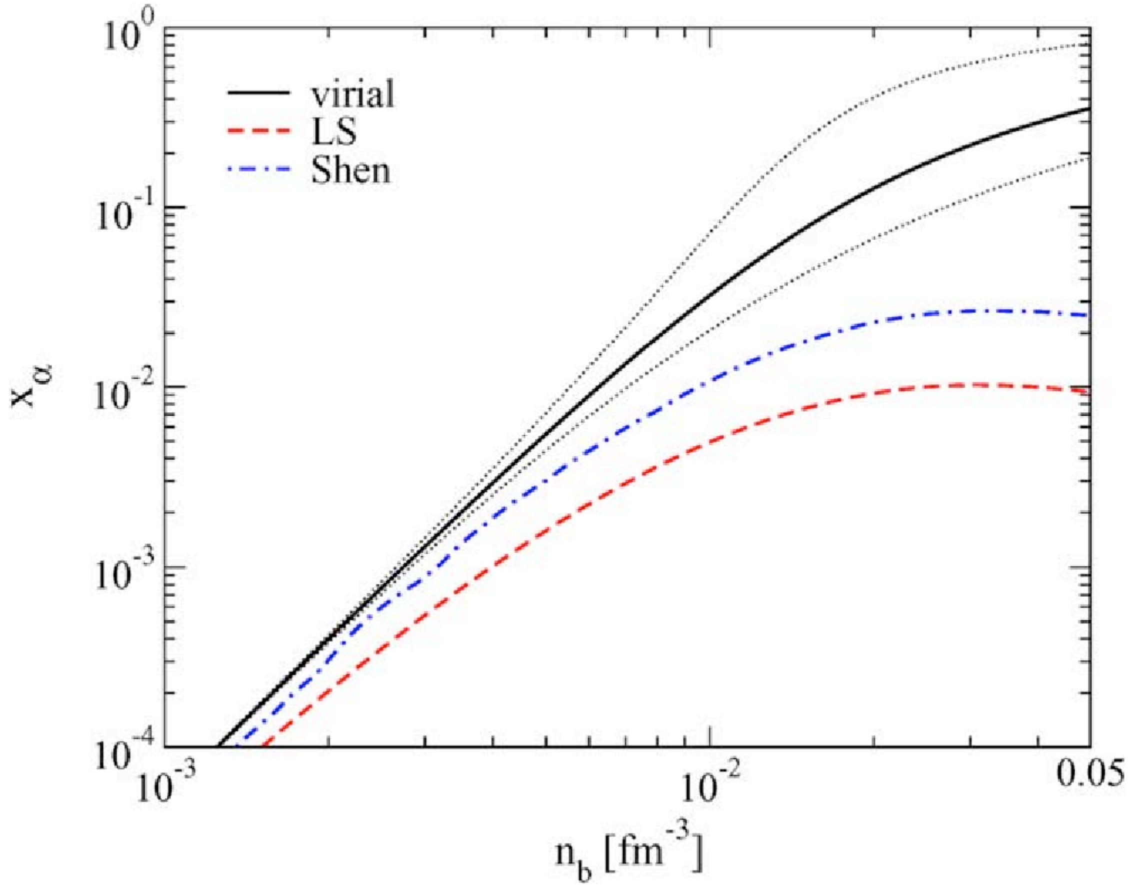
symmetric matter, T=4 MeV



Horowitz & Schwenk (2006), Lattimer & Swesty, (2001),Shen et al. (1998))

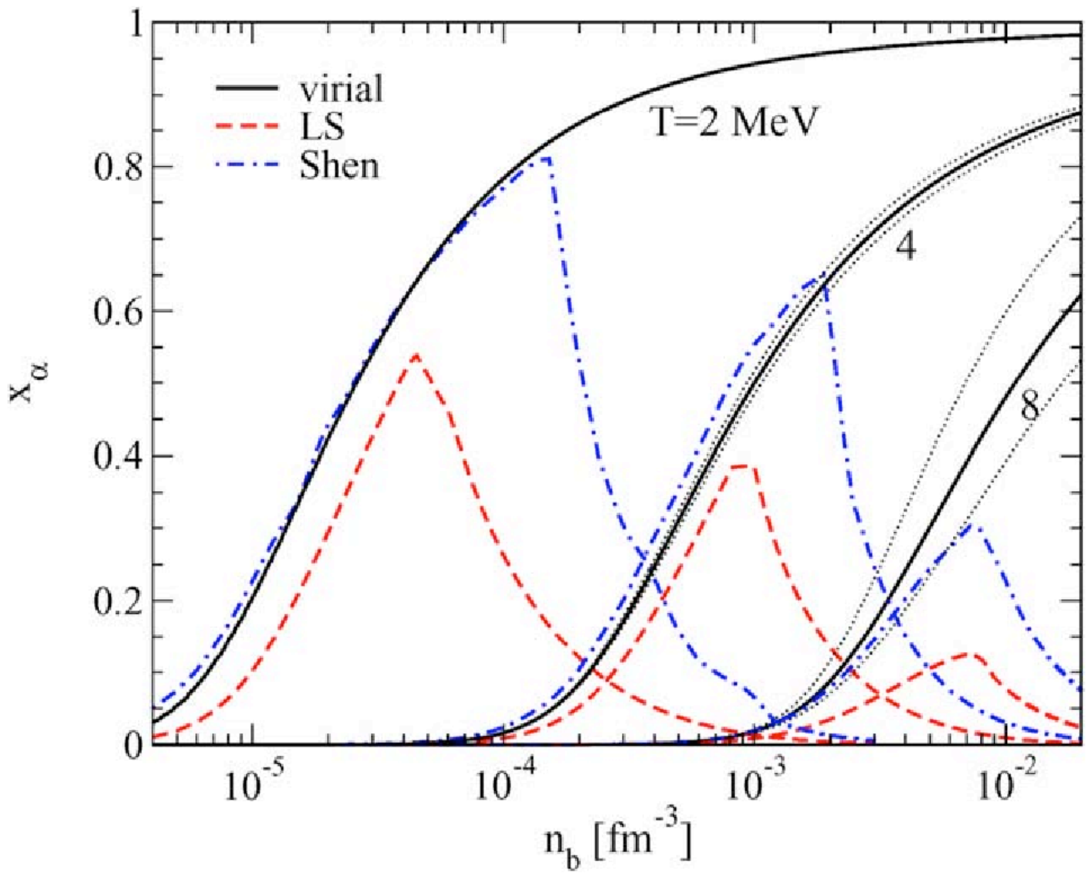
Alpha-particle fraction in the low-density limit

symmetric matter, T=20 Mev

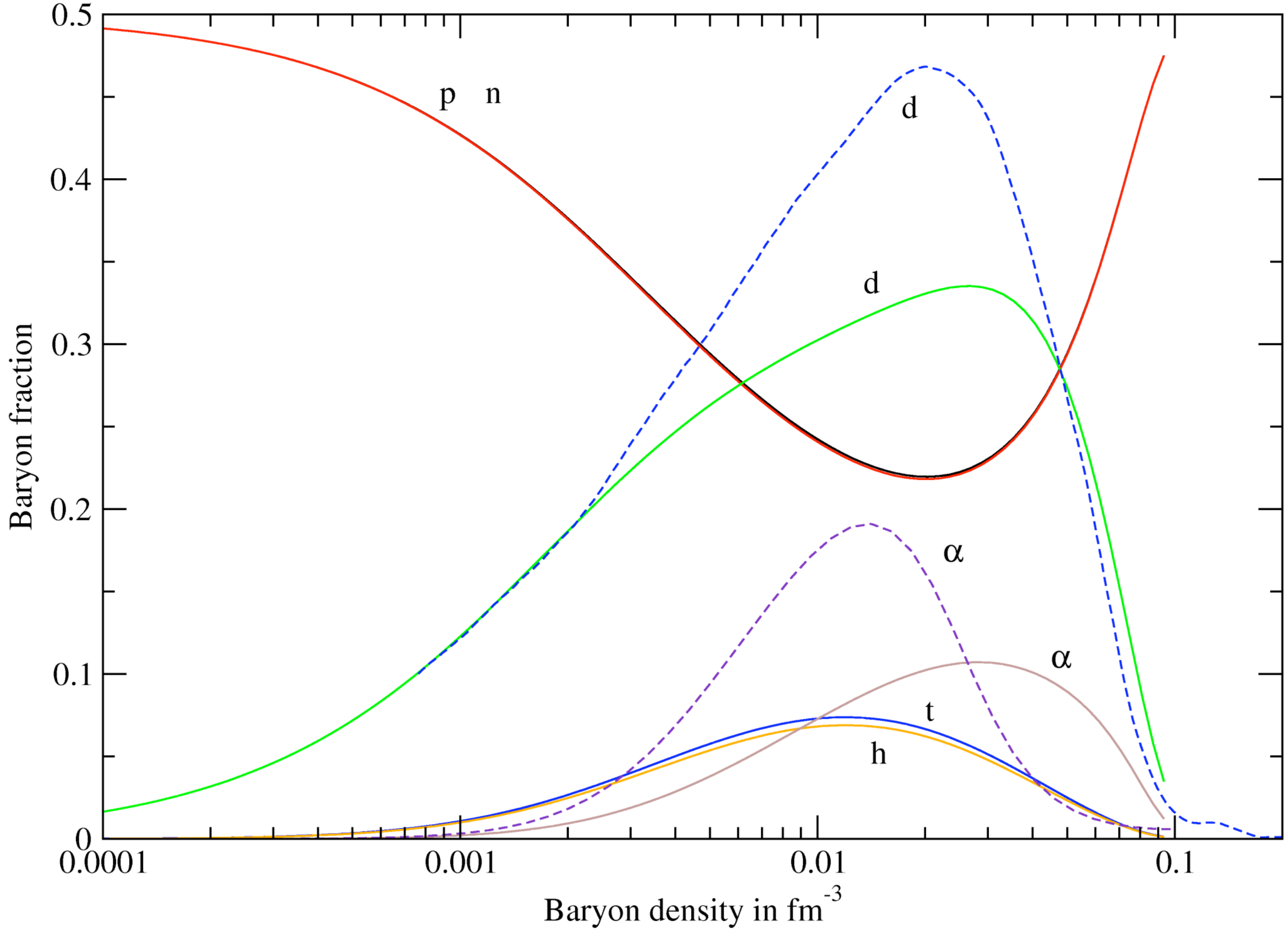


Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



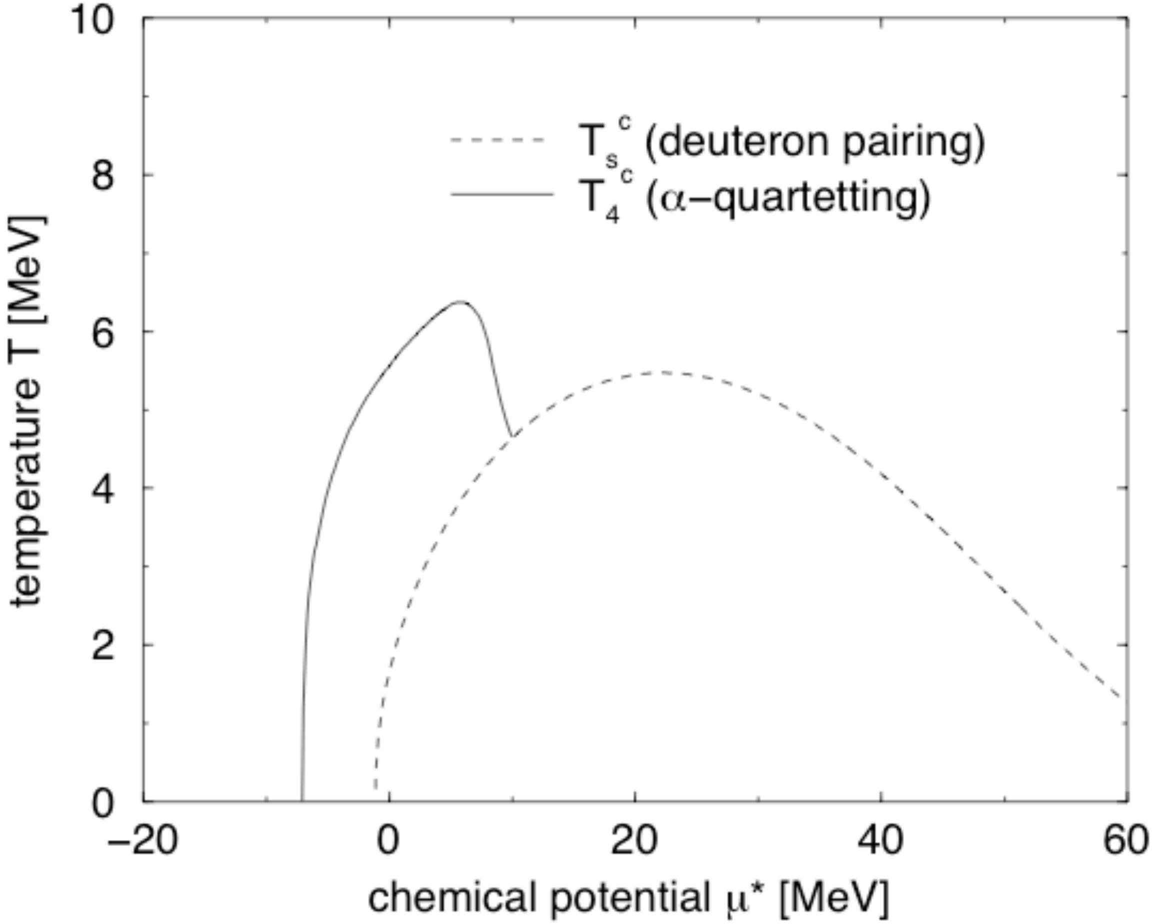
Composition of symmetric nuclear matter



T=10 MeV

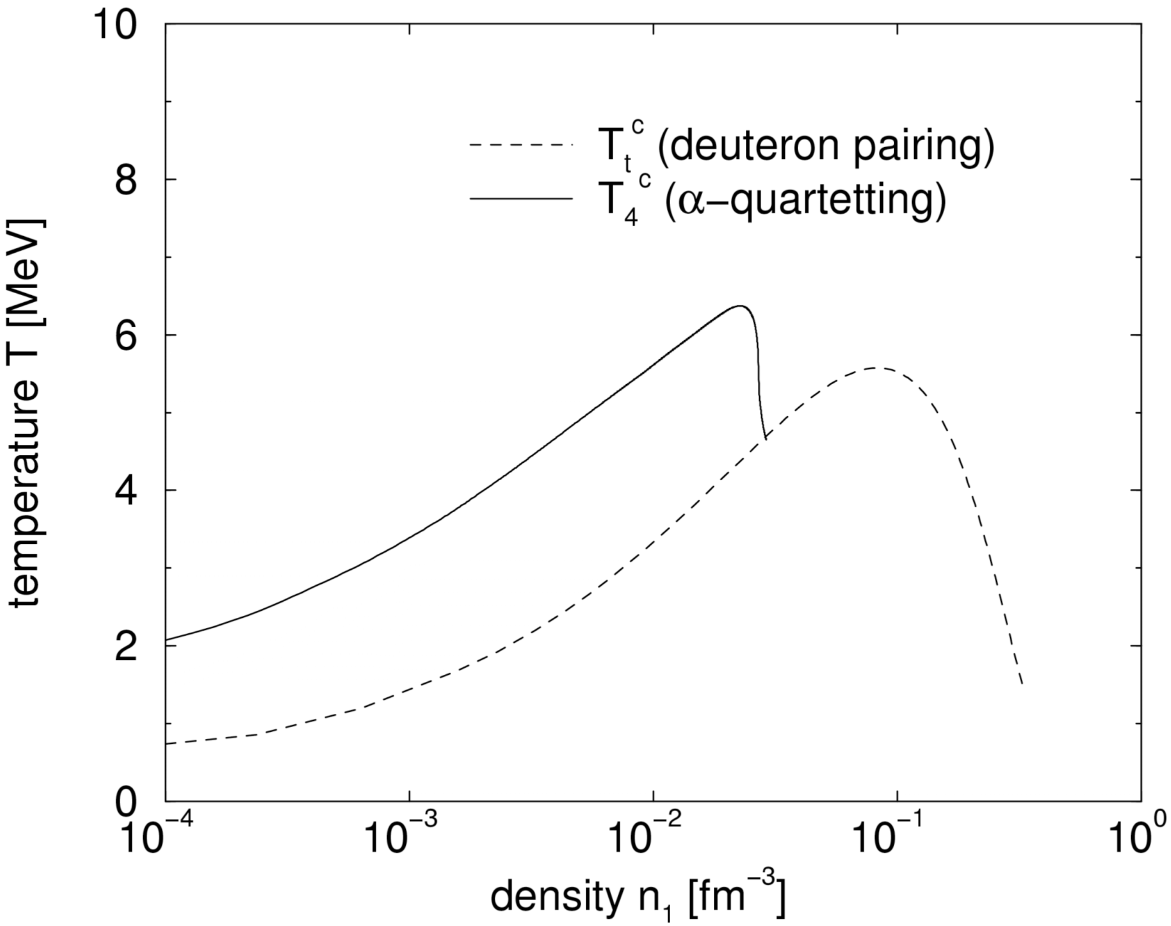
α-cluster-condensation

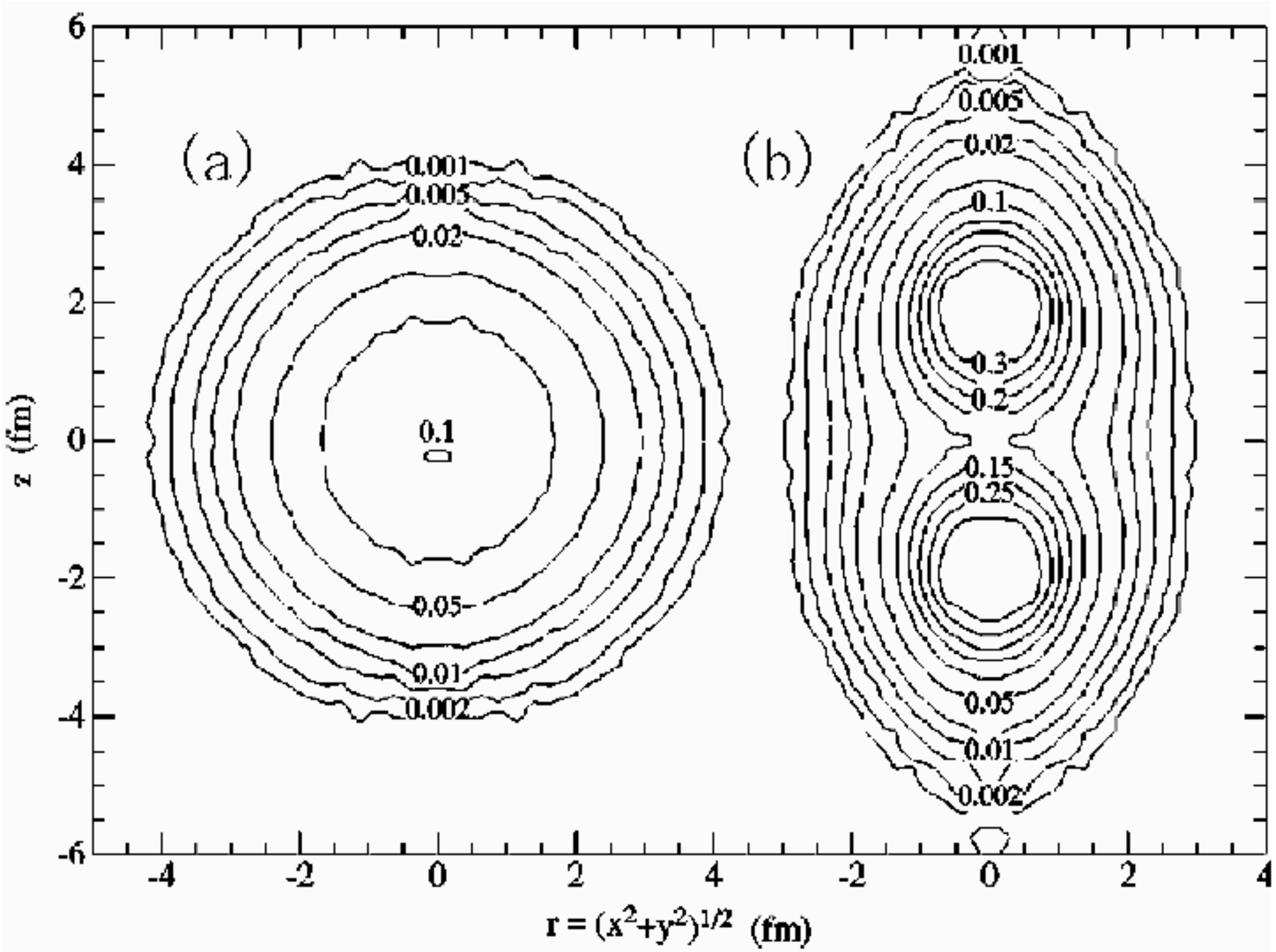
(quartetting)



α-cluster-condensation

(quartetting)





(a)
(b)
0.001
0.005
0.02
0.1
0.05
0.01
0.002
0.3
0.2
0.15
0.25
z (fm)
r = (x^2+y^2)^1/2 (fm)

Alpha-condensate (quartetting) in 4n symmetric nuclei

- A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,
  Phys. Rev. Lett. **87**, 192501 (2001).

- G. Röpke, A. Schnell, P. Schuck, and P. Nozieres,
  Phys. Rev. Lett. **80**, 3177 (1998).

- Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,
  Phys. Rev. C **67,** 051306(R) (2003); Eur. Phys. J. **A 28**, 259 (2006).

- T. Yamada, P. Schuck,
  Phys. Rev. C **69,** 024309 (2004).

Self-conjugate 4n nuclei

^{12}C:

0^+ state at 0.39 MeV above the 3α threshold energy:
α cluster interact predominantly in relative S waves, gaslike structure

α-particle condensation in low-density nuclear matter ($\rho \leq \rho_0/5$)

$n\alpha$ cluster condensed states
– a general feature in $N = Z$ nuclei?

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, $\ldots$

Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\mathrm{thr}} = nE_\alpha$

Variational ansatz

$$|\Phi_{n\alpha}\rangle = \left(C_\alpha^\dagger\right)^n |\mathrm{vac}\rangle$$

α- particle creation operator

$$C_\alpha^\dagger = \int d^3R e^{-\vec{R}^2/R_0^2}$$

$$\times \int d^3r_1 \ldots d^3r_4 \phi_{0s}(\vec{r}_1 - \vec{R}) a^\dagger_{\sigma_1\tau_1}(\vec{r}_1) \ldots \phi_{0s}(\vec{r}_4 - \vec{R}) a^\dagger_{\sigma_4\tau_4}(\vec{r}_4)$$

with

$$\phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)}$$

Variational ansatz

total $n\alpha$ wave function

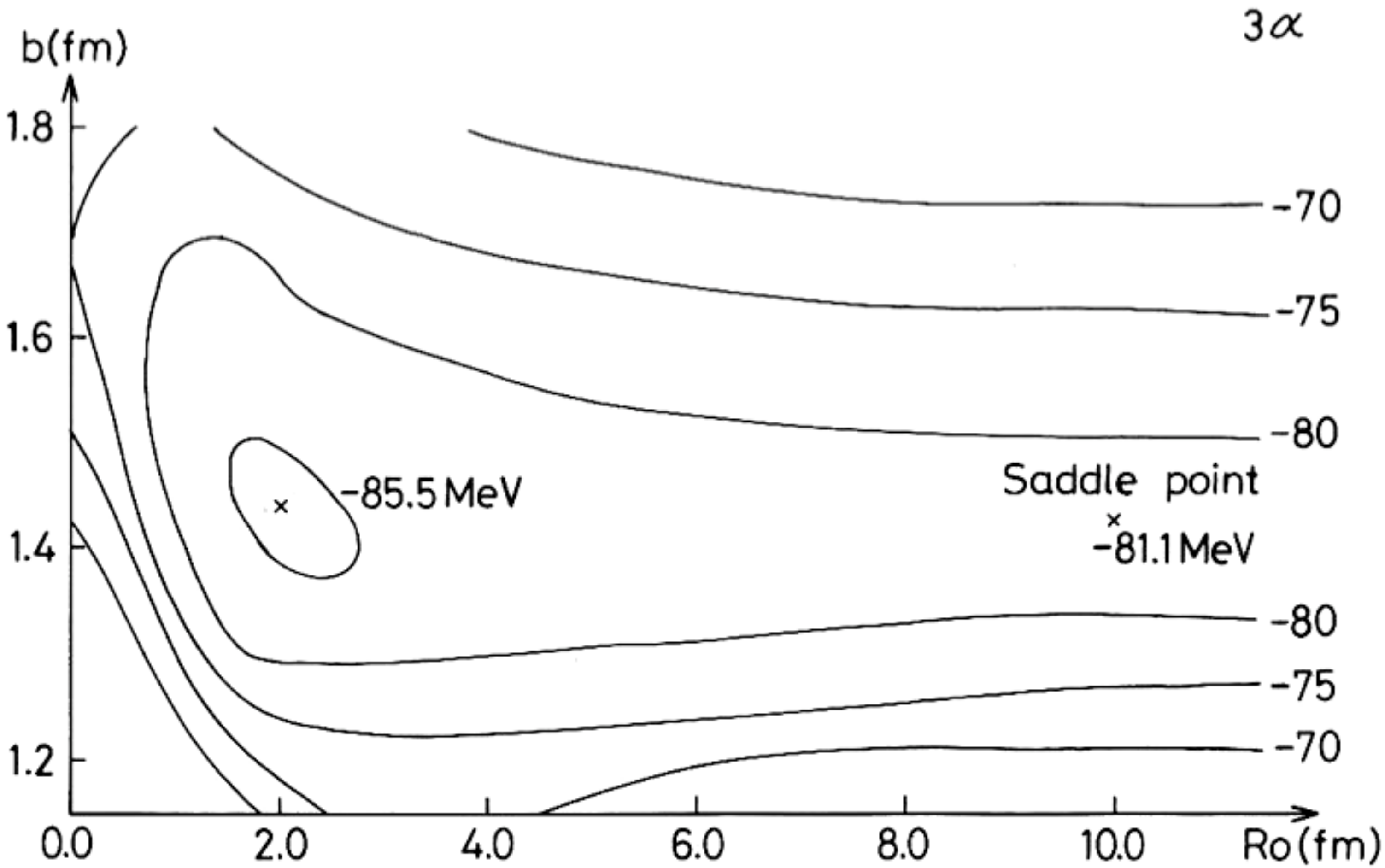
$$\langle \vec{r}_1 \sigma_1 \tau_1 \ldots \vec{r}_{4n} \sigma_{4n} \tau_{4n} | \Phi_{n\alpha} \rangle$$

$$\propto \mathcal{A} \left\{ e^{-\frac{2}{B^2}(\vec{X}_1^2 + \ldots \vec{X}_n^2)} \phi(\alpha_1) \ldots \phi(\alpha_n) \right\}$$

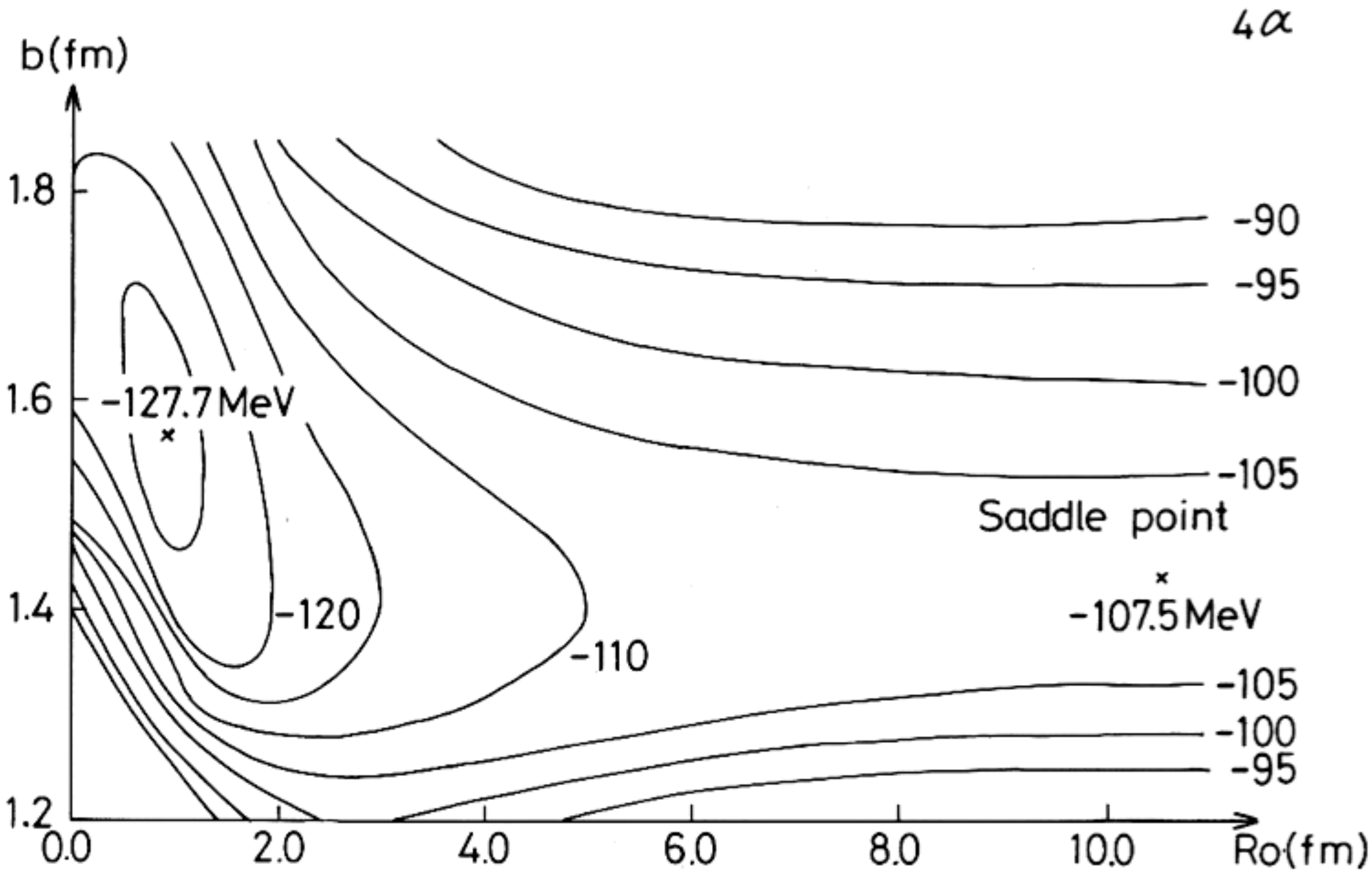
where $B^2 = (b^2 + 2R_0^2)$, $\vec{X}_i = \frac{1}{4} \Sigma_n \vec{r}_{in}$,
$\phi(\alpha_i) = e^{-\frac{1}{8b^2} \Sigma_{m>n}^4 (\vec{r}_{im} - \vec{r}_{in})^2}$ - internal α wave function

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL **87**, 192501 (2001)

3 alpha variational energy



4 alpha variational energy



Results

		E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{thr}$ (MeV)	$(E - E_{n\alpha}^{thr})_{exp}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{exp}$ (fm)
^{12}C	$k=1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
	$k=2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
	$E_{3\alpha}^{thr}$	-82.5	-84.89				
^{16}O	$k=1$	-124.8	-127.62 (0_1^+)	-14.8	-14.44	2.59	2.73
		$(-128.0)^*$		$(-18.0)^*$			
	$k=2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
	$k=3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
	$E_{4\alpha}^{thr}$	-110.0	-113.18				
^8Be				-0.17	+0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{thr} = nE_\alpha$ denotes the threshold energy for the decay into α-clusters, the values marked by * correspond to a refined mesh.

Decay of α-condensate at high densities

i) The bound state dissolves into free nucleon states

Occupied phase space cannot be used to form bound states

Pauli blocking $V(12, 1'2')[1 - f(1) - f(2)]$

α-like bound state only below $\rho_0/10$

α-condensate survives to ≈ 0.3 fm^{-3}

Decay of α-condensate at high densities

ii) Reduction of the condensate due to repulsive interaction

Penrose, Onsager 1956:
Off-diagonal long range order (ODLRO)
density matrix $\rho(\vec{r},\vec{r}')$ at $|\vec{r}-\vec{r}'| \to \infty$:

$$\rho(\vec{r},\vec{r}') = \Psi(\vec{r})\Psi(\vec{r}') + \gamma(|\vec{r}-\vec{r}'|)$$

hard core: known solution (Rosenbluth)

Application to liquid ^{4}He:
filling factor $\approx$ 28 %, condensate $\approx$ 8 %

"excluded" volume for α particles $\approx 20\ \mathrm{fm}^3$

Estimation of condensate fraction in zero temperature α-matter

$$n_0 = \frac{\langle \Psi | a_0^\dagger a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

destruction of the BEC of the ideal Bose gas:
thermal excitation, but also correlations

"excluded" volume for α-particles ≈ 20 fm^3
at nucleon density $\rho = 0.048$ fm^{-3} filling factor ≈ 28 %
(liquid ^{4}He: 8 % condensate),
destruction of the condensate at $\approx \rho_0/3$

Estimation of condensate fraction in zero temperature α-matter

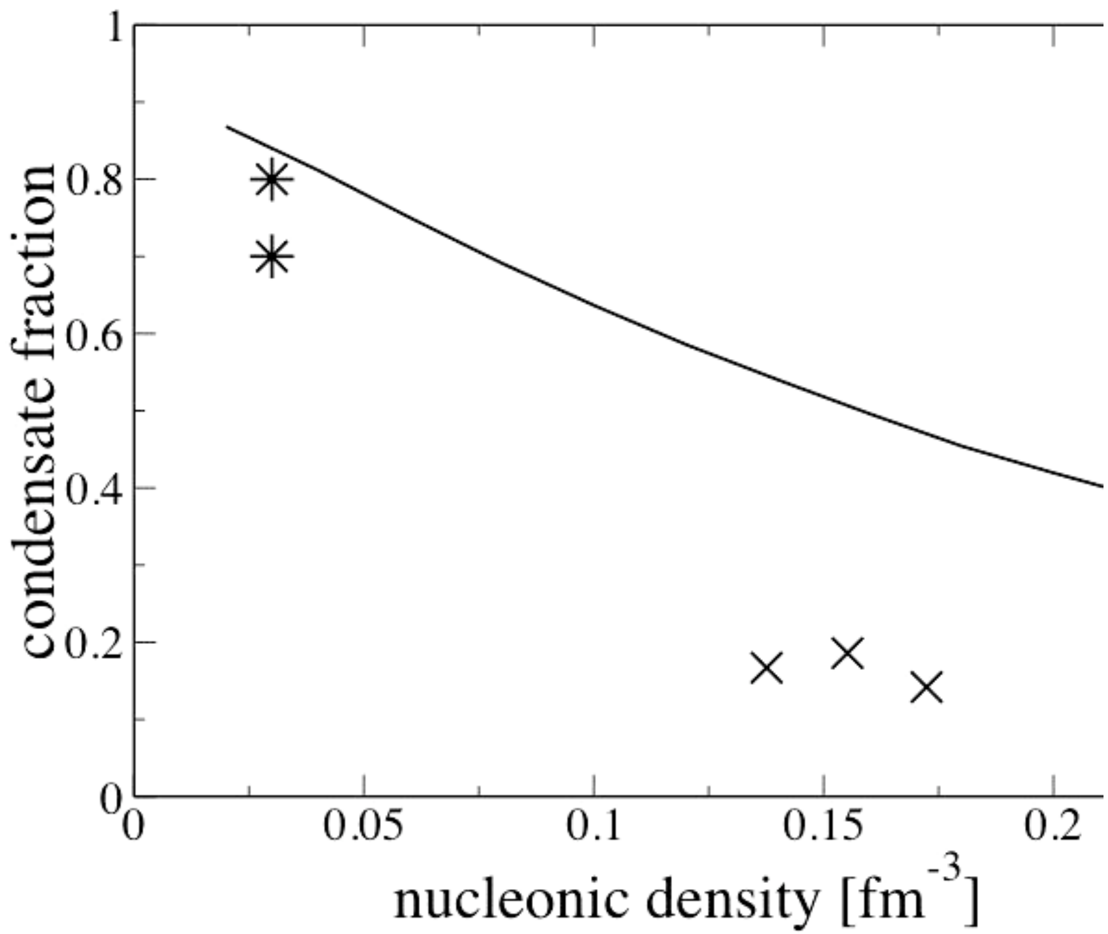
α-cluster condensate in ^{12}C, ^{16}O:
resonating group method
occupation numbers of α-orbits in ^{12}C

	RMS radii	S-orbit	D-orbit	G-orbit
0_1^+ (g.s.)	2.44 fm	1.07	1.07	0.82
0_2^+	4.31 fm	2.38	0.29	0.16

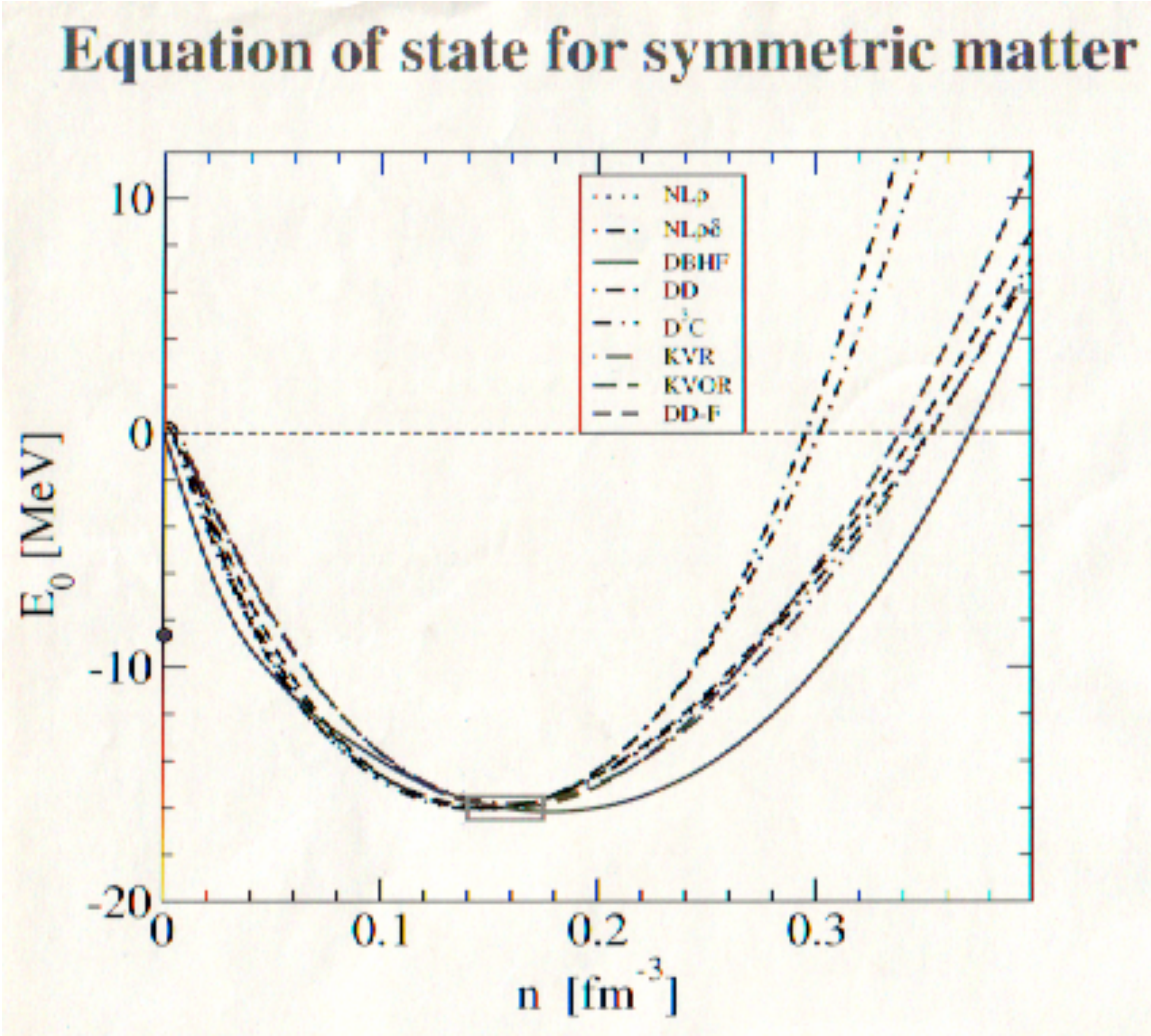
80 % condensate at 1/8 nuclear matter density

T. Yamada, P. Schuck: (2.16 − normal)/3 ≈ 60%

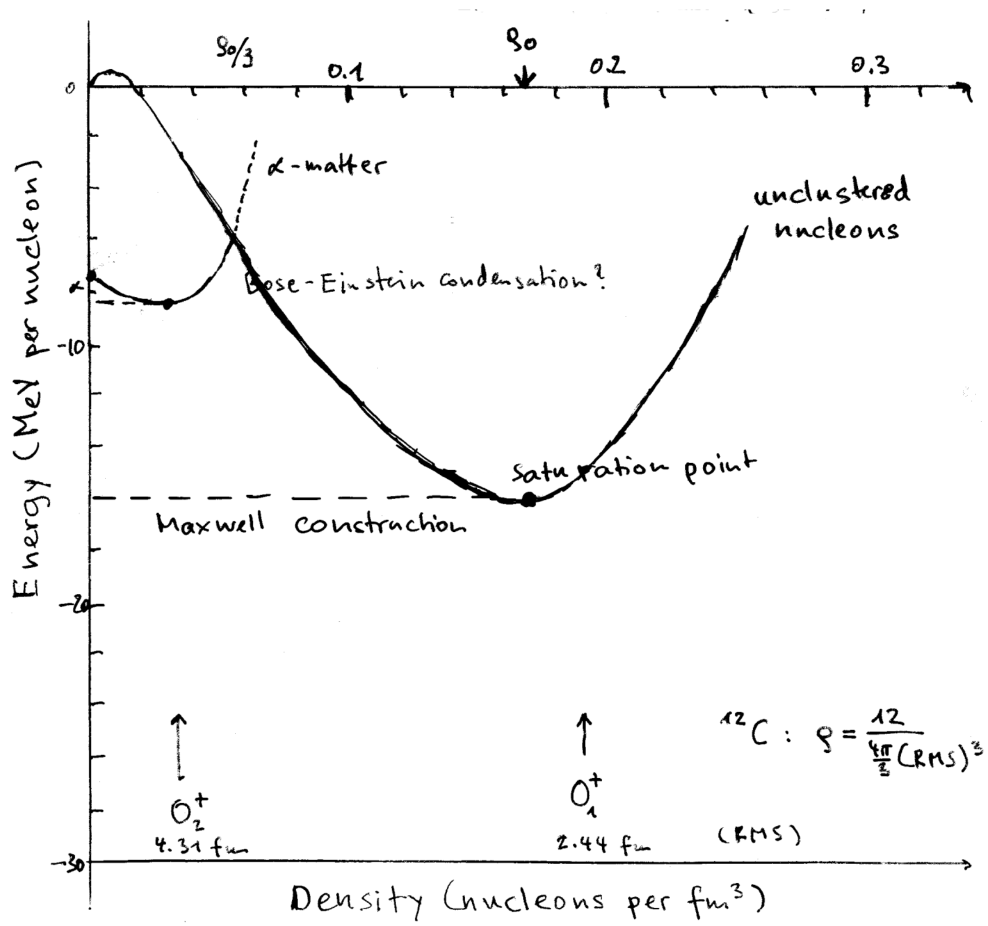
Suppresion of condensate fraction



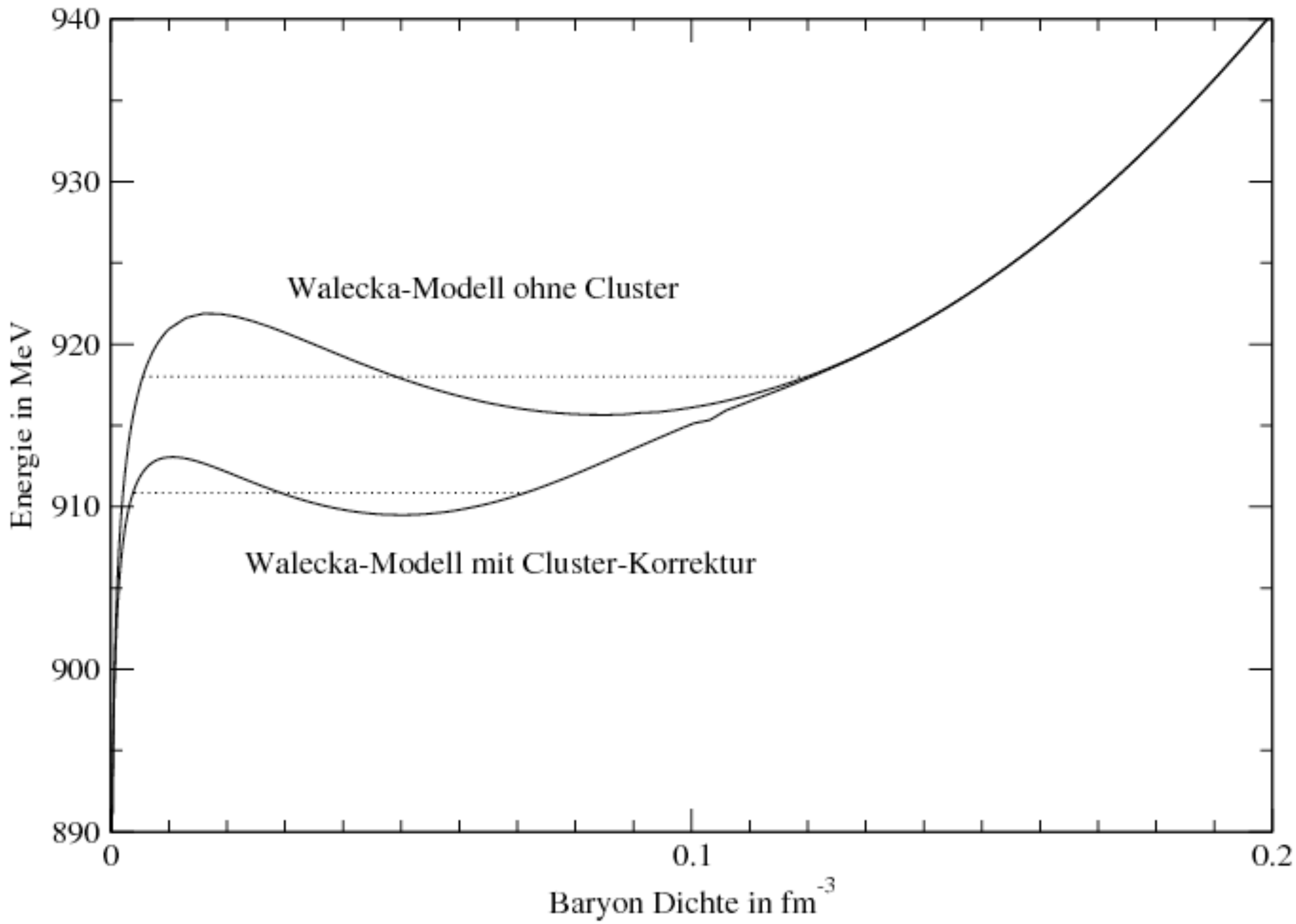
Quasiparticle approximation for nuclear matter



Low-density limit: alpha matter?



Influence of cluster formation on the equation of state



T=10 Mev

Symmetry energy of a low density nuclear gas

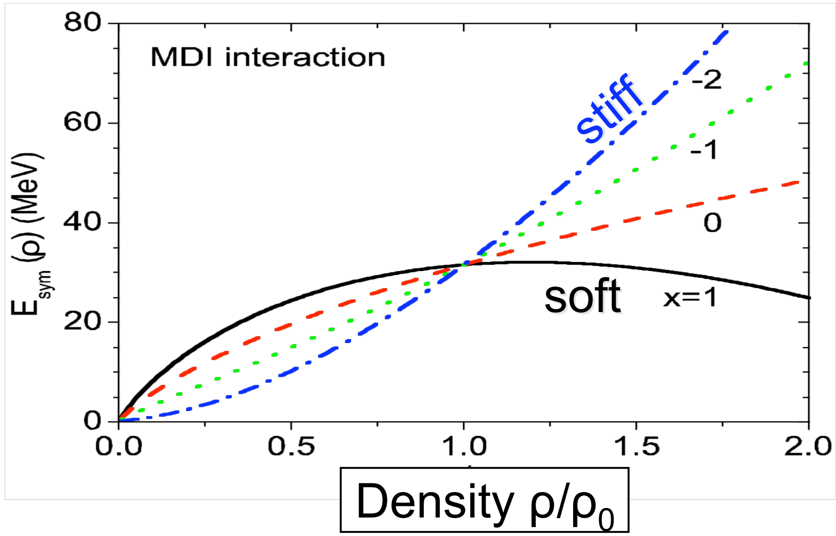
- L. W. Chen, C. M. Ko, and B. A. Li,
  Phys. Rev. Lett. **94**, 032701 (2005).

- T. Klähn *et al*.,
  Phys. Rev. C **74**, 035802 (2006).

- C. J. Horowitz and A. Schwenk,
  Nucl. Phys. **A 776**, 55 (2006).

- S. Kowalski *et al.*,
  Phys. Rev. C **75**, 014601 (2007).

Symmetry energy and single nucleon potential used in the IBUU04 transport model



The x parameter is introduced to mimic various predictions by different microscopic Nuclear many-body theories using different Effective interactions

Single nucleon potential within the HF approach using a modified Gogny force:

$$U(\rho,\delta,\vec{p},\tau,x) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma(1 - x\delta^2) - 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}$$

$$+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3p'\frac{f_\tau(r,p')}{1+(p-p')^2/\Lambda^2} + \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3p'\frac{f_{\tau'}(r,p')}{1+(p-p')^2/\Lambda^2}$$

$$\tau,\tau' = \pm\frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma+1}, A_u(x) = -96 - \frac{2Bx}{\sigma+1}, K_0 = 211MeV$$

The momentum dependence of the nucleon potential is a result of the non-locality of nuclear effective interactions and the Pauli exclusion principle

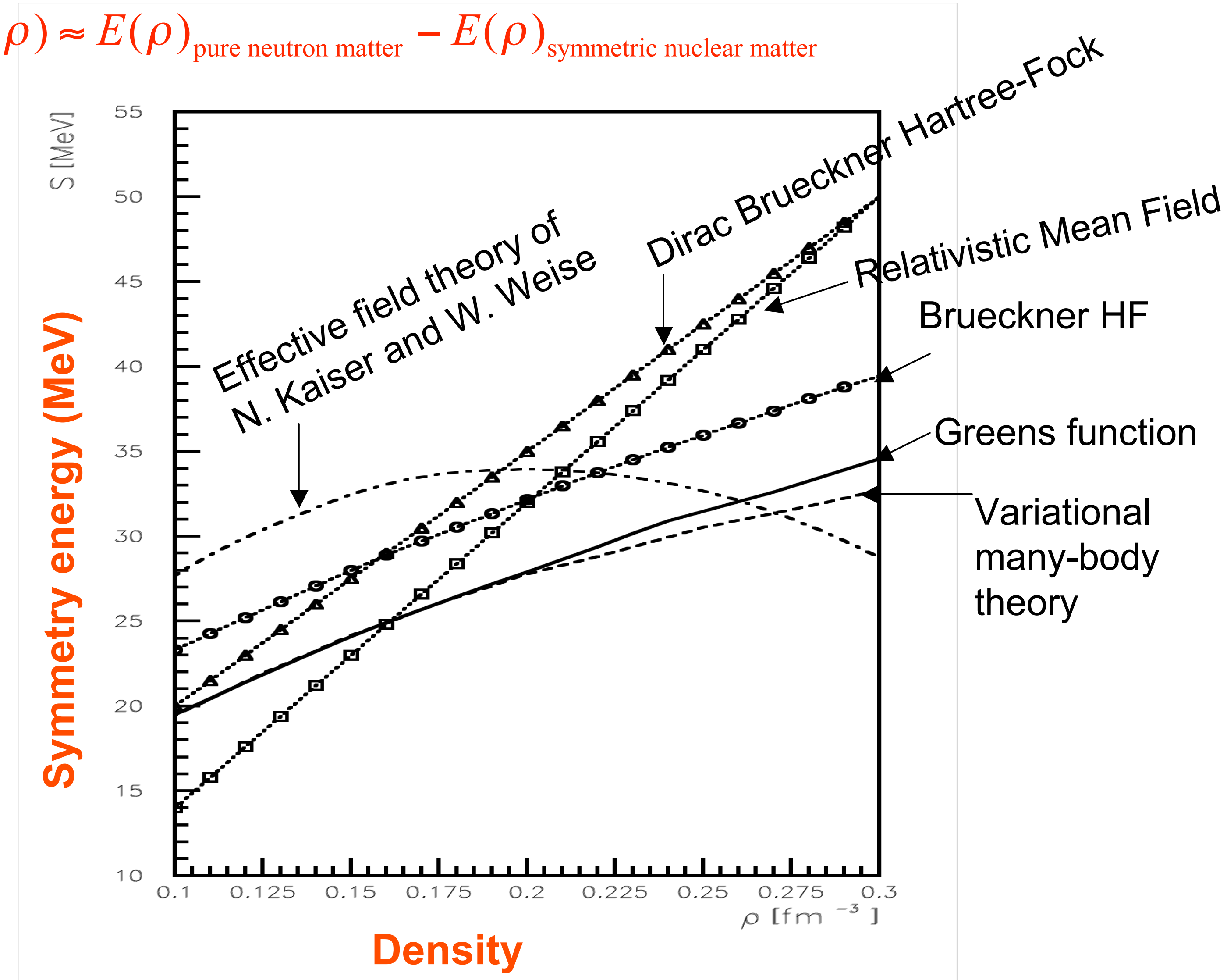
C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC **69**, 034614; NPA **735**, 563 (2004).

E_{sym} (ρ) predicted by microscopic many-body theories

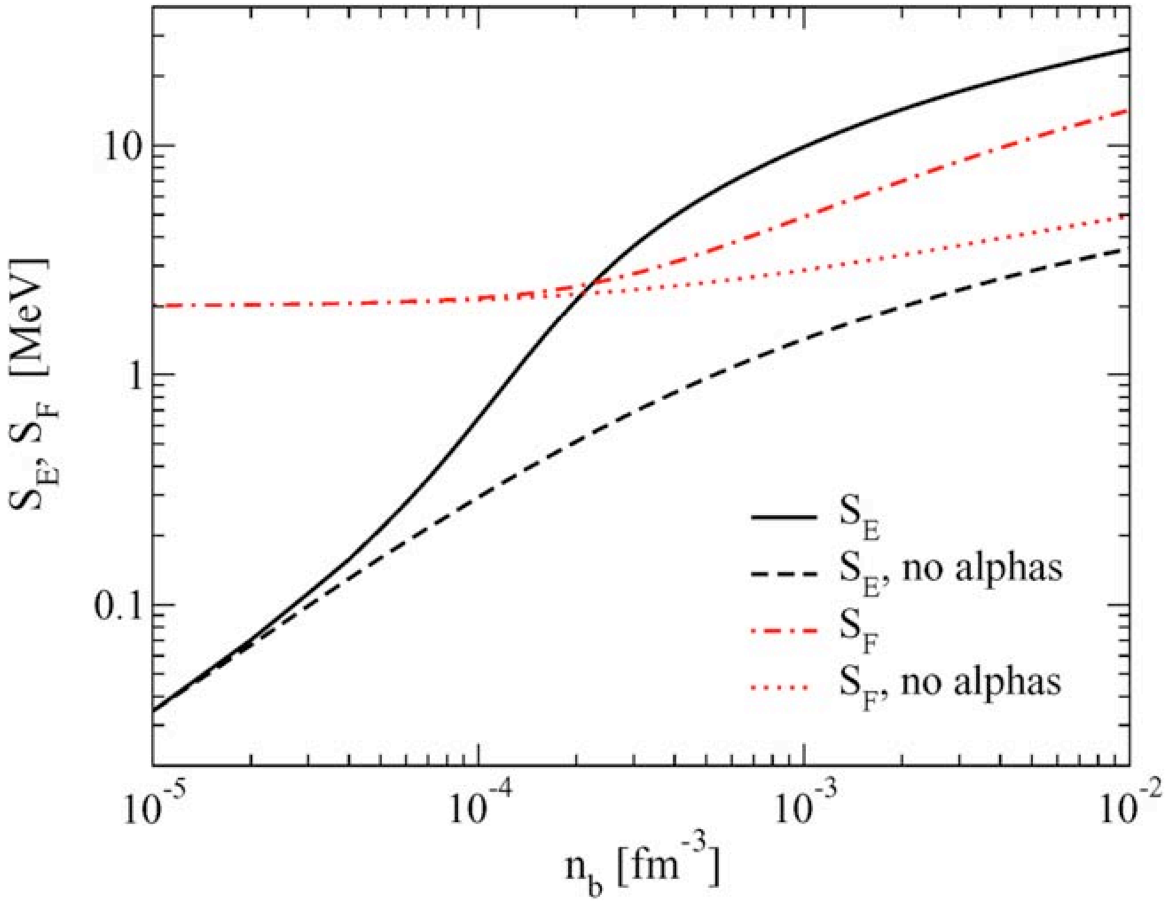
EOS: $E(\rho,\delta) = E_0(\rho,0) + E_{sym}(\rho)\delta^2 + o(\delta^4)$, where $\delta \equiv (\rho_n - \rho_p)/\rho$

$$E_{sym}(\rho) \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$



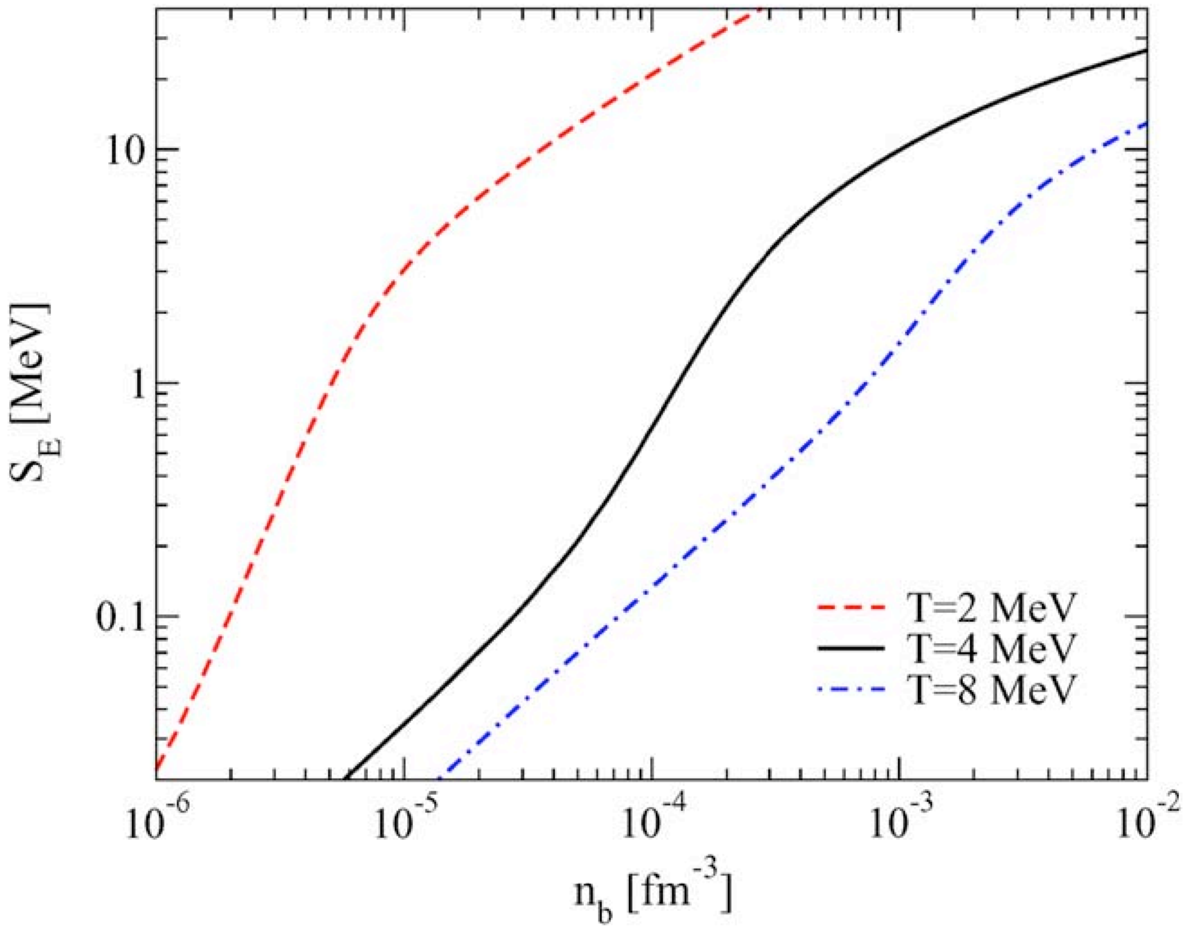
A.E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier and V. Rodin, Phys. Rev. C68 (2003) 064307

Symmetry energy and symmetry free energy

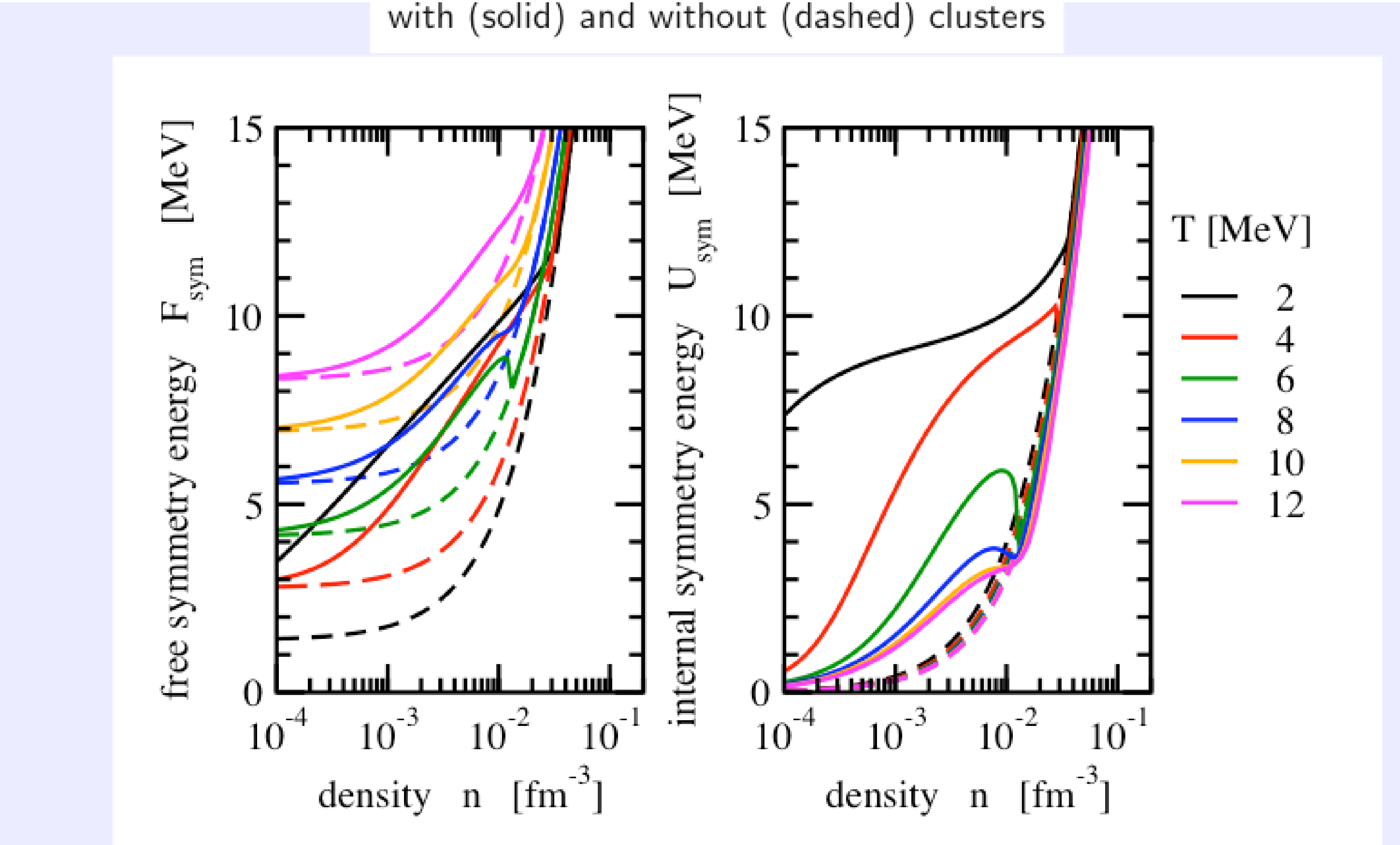


T=4MeV

Symmetry energy for T=2,4,8 MeV

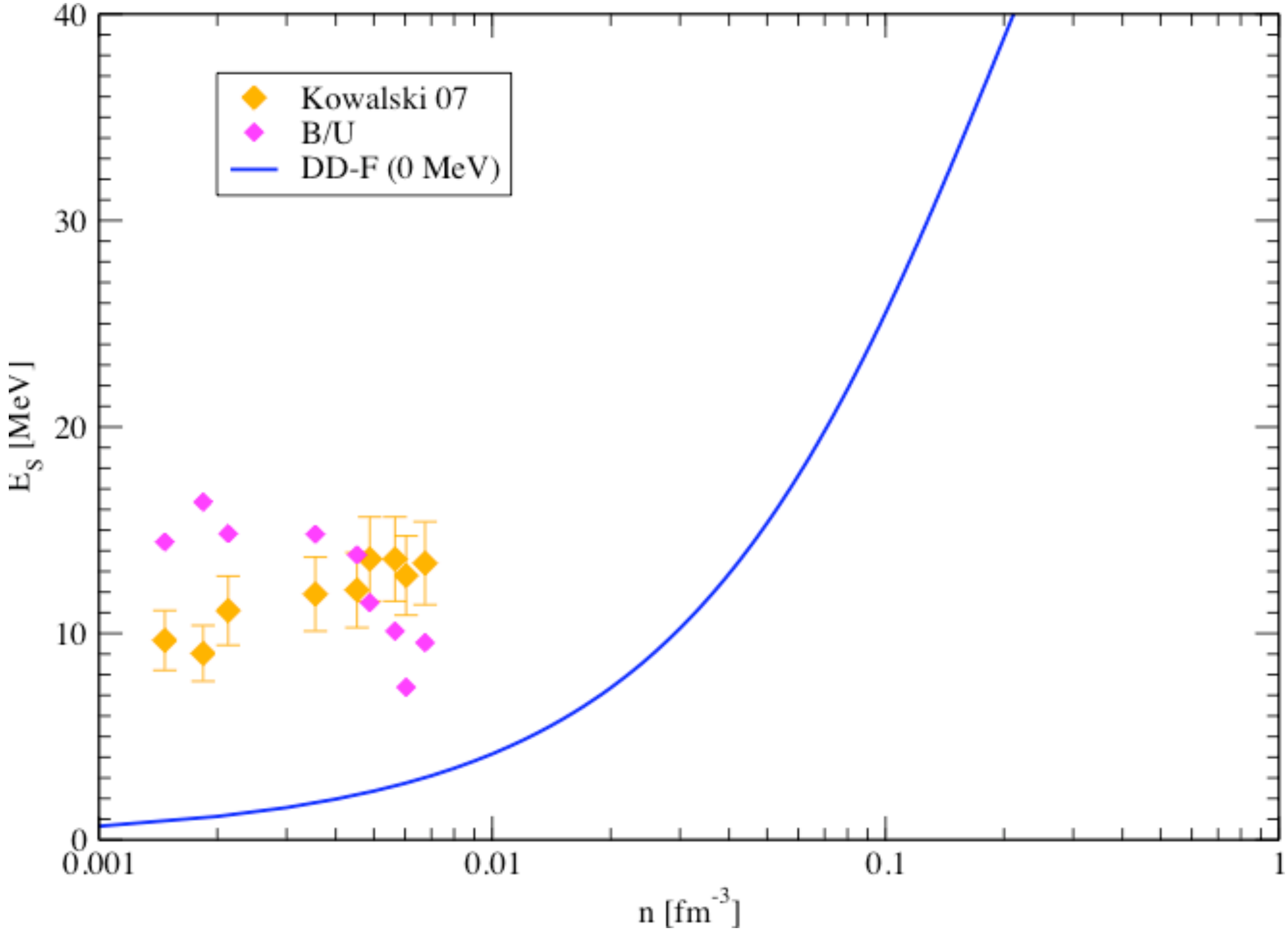


Free and Internal Symmetry Energy



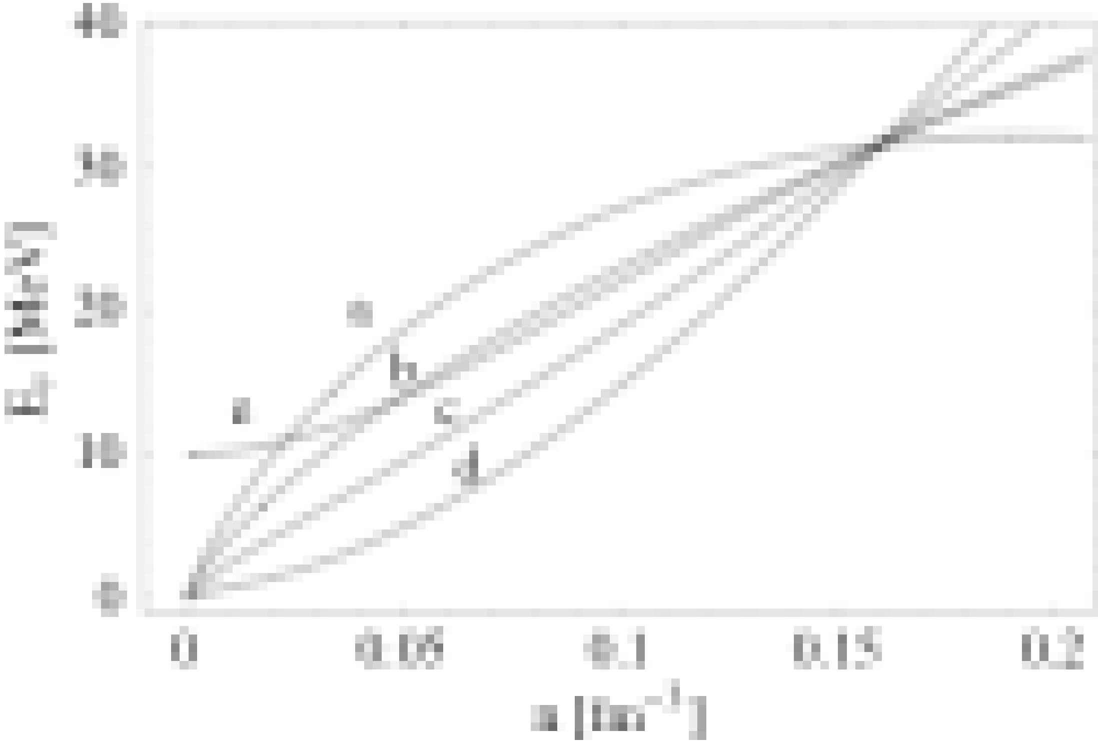
S. Typel, Talk 07

Symmetry energy



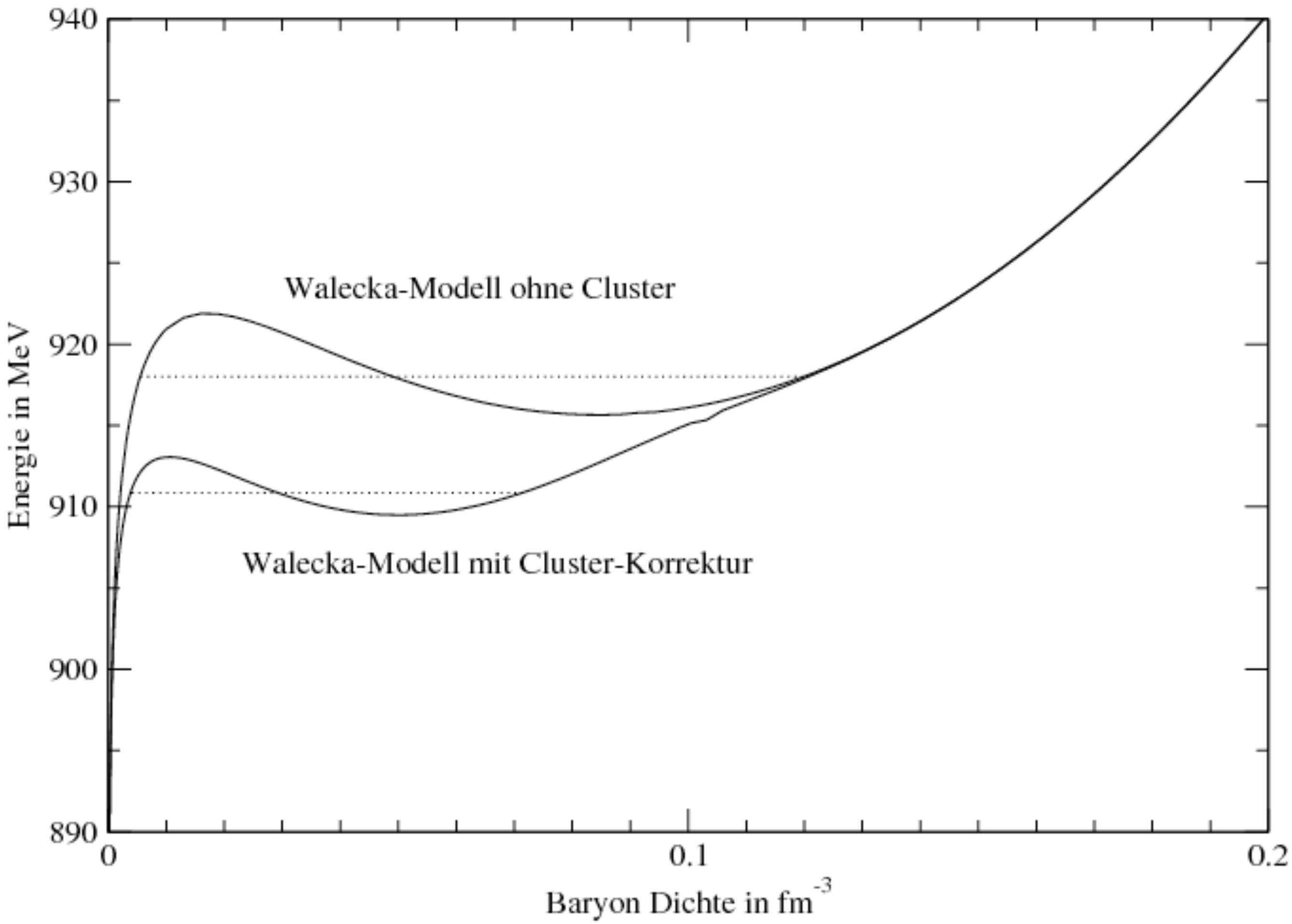
Approximations to the symmetry energy

a-d - Chen,Ko,Li '05
e - Kowalski et.al '06



S. Kubis,
Neutron stars with
non-homogeneous
core,
Talk 26.2.08, Ladek

Influence of cluster formation on the equation of state



T=10 MeV

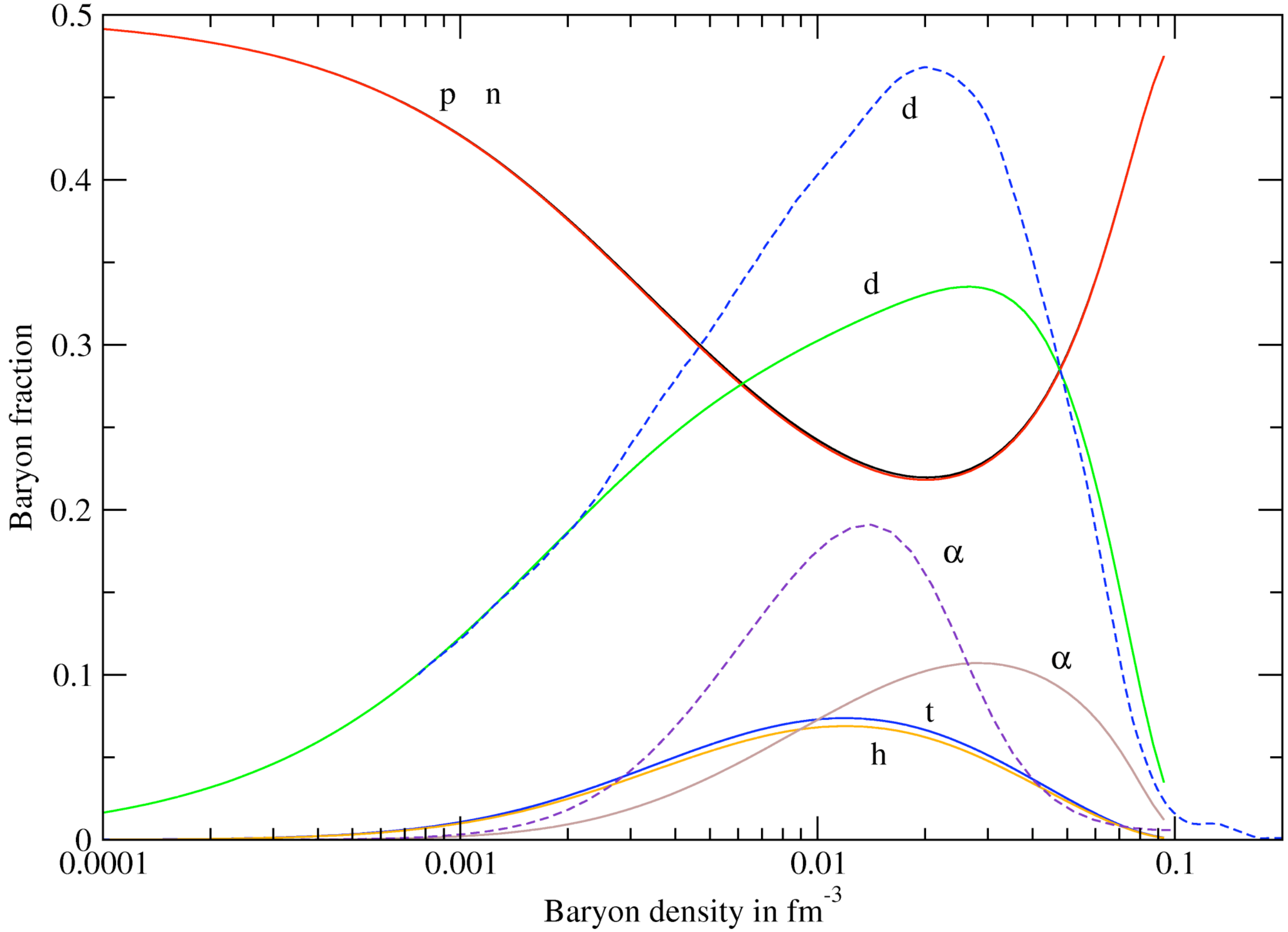
Chemical potential of symmetric nuclear matter.

Inclusion of cluster formation shifts down the chemical potential.

The region of thermodynamical instability is reduced.

G.R., A. Grigo (2003)

Composition of symmetric nuclear matter



T=10 MeV

Low-density EoS and astrophysics

- H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,

  Progr. Theor. Phys.**100**, 1013 (1998); Nucl. Phys. **A637** 435 (1998).
- G. Röpke, A. Grigo, K. Sumiyoshi, and Hong Shen,

  in: Superdense QCD Matter and Compact Stars, Ed. D. Blaschke and A. Sedrakian, NATO Science Series, Springer, Dordrecht (2006),

  pp. 75 - 91;

  Physics of Particles and Nuclei Letters **2**, 275 (2005).
- J.M.Lattimer and F. D. Swesty,

  Nucl. Phys. **A 535**, 331 (2001).
- C. J. Horowitz and A. Schwenk,

  Nucl. Phys. **A 776**, 55 (2006).

Outline

- Schroedinger equation with medium corrections:

  Self-energy and Pauli blocking

- Composition of the nuclear gas:

  Generalized Beth-Uhlenbeck equation

- Quantum condensates:

  Pairing and quartetting

- Alpha clustering in 4n nuclei
- Composition and the EoS of symmetric matter
- Symmetry energy in the low-density region

Summary

- The low-density limit of the nuclear matter EoS can be rigorously treated and has to be considered as benchmark.
- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self- energy and Pauli blocking is included. An interpolation between low and high densities is possible.
- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation, and the behavior of the symmetry energy.

Thanks for attention

Problems:

- Nuclear matter at subsaturation densities:

Temperature T < 30 MeV, baryon density n_b < 0.17 fm^-3

- Formation of clusters (nuclei in matter):

A = 1,2,3,4: deuterons (d), tritons (t), helions (h), alphas (a)

- Low-density, low-temperature limit:

Virial expansion, non-interacting nuclides, quantum condensates

- Transition to higher densities:

Medium effects, quasiparticles,
interpolation between Beth-Uhlenbeck and DBHF / RMF

Two-particle states in matter

Two-particle Schrödinger equation with medium effects

$$[E^{\rm HF}(p_1) + E^{\rm HF}(p_2)]\ \psi_{nP}(p_1, p_2)$$
$$+[1 - f(p_1) - f(p_2)] \sum_{p_1' p_2'} V(p_1 p_2, p_1' p_2')\ \psi_{nP}(p_1', p_2')$$
$$= E_{nP}\ \psi_{nP}(p_1, p_2)$$

$$E^{HF}(p_1) = E(p_1) + \sum_{p_2} V(p_1 p_2, p_1 p_2)_{ex}\ f(p_2)$$

The Equation of State of neutron-rich nuclear matter

symmetry energy

isospin asymmetry $\delta=(\rho_n-\rho_p)/\rho$

$$E(\rho_n,\rho_p) \quad E_0(\rho_n=\rho_p)+E_{sym}(\rho)\delta^2+O(\delta^4)$$

EOS of isospin-symmetric nuclear matter with equal numbers of protons and neutrons

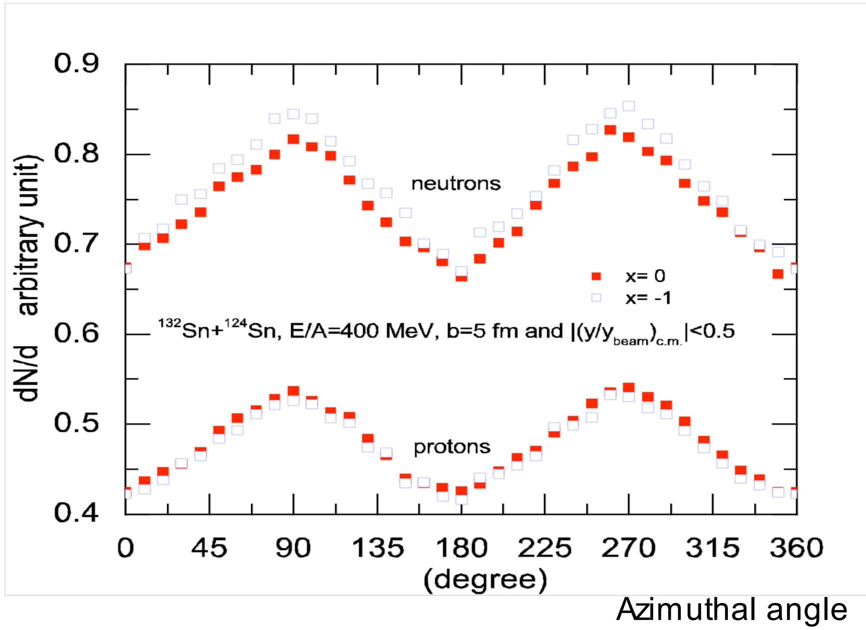
Energy per nucleon in isospin-asymmetric nuclear matter with different numbers of protons and neutrons

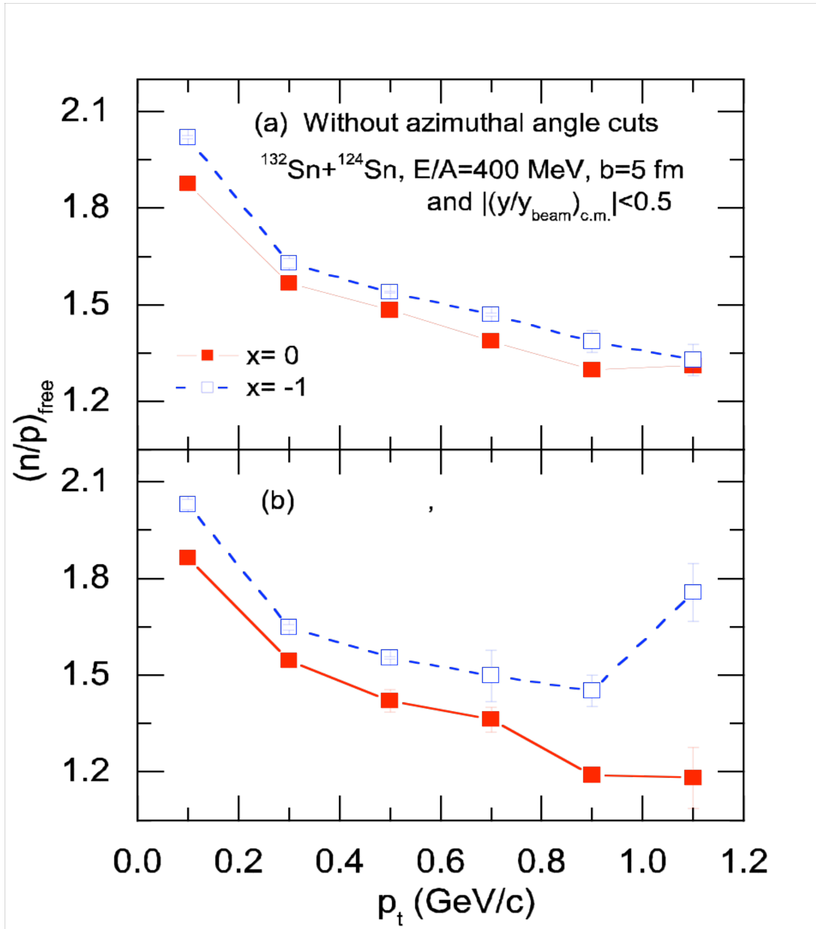
- This parabolic approximation is valid up to pure neutron matter as shown by all existing many-body theories

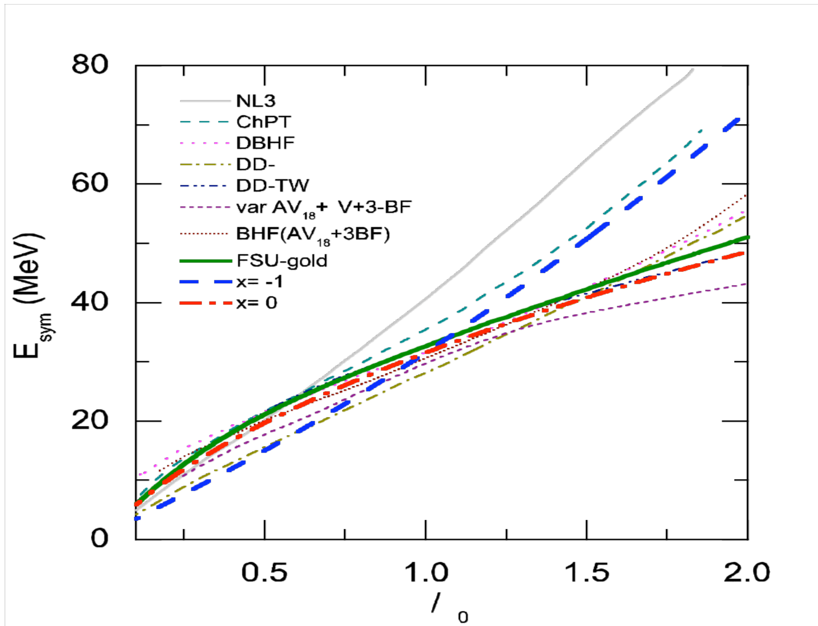
- The EOS of isospin-symmetric nuclear matter is relatively well determined after almost 30 years of hard work by many people in the nuclear physics community

- Besides the possible phase transition at high densities, the symmetry energy $E_{sym}(\rho)$ is the most uncertain term in the EOS of neutron-rich matter

Squeeze-out of neutrons perpendicular to the reaction plane







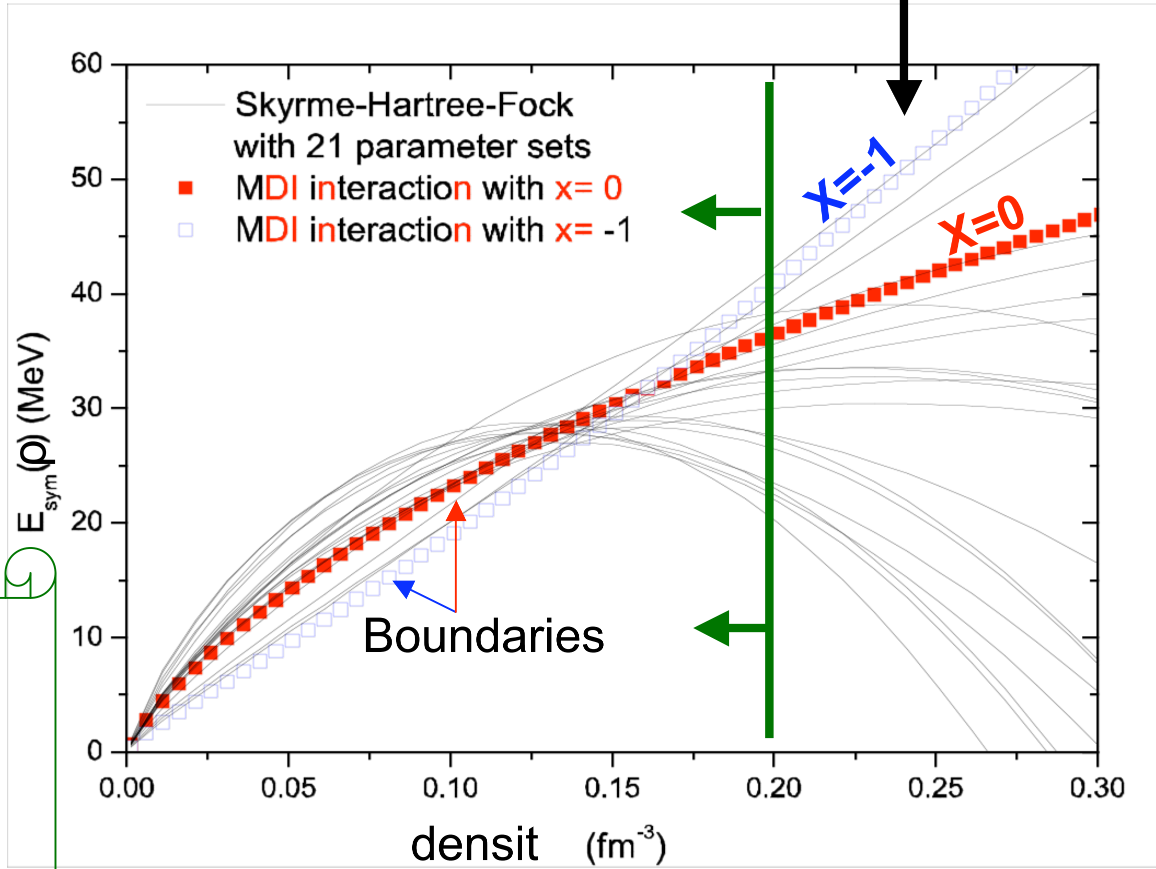
Constraining nuclear effective interactions within Skyrme Hartree-Fock

$$E_{\rm sym}(\rho)=\frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}-\frac{1}{8}t_0(2x_0+1)\rho$$

$$-\frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1}+\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}\left[-3t_1x_1+(4+5x_2)t_2\right]\rho^{5/3}$$

$E_{\rm sym}$ in the high density region is still not constrained !

ρ



y

Most of the 21 widely used Skyrme interactions for nuclear structure studies are ruled out !

Only 5 survived !

L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C72, 064309 (2005).

Pauli-blocking

- Wellenfunktion des Bindungszustandes
- Impulsverteilung der Nukleonen:
  kann durch eine Wahrscheinlichkeitsverteilung im Impulsraum dargestelt werden
- endliche Dichte der umgebenden Nukleonen
- kann durch eine Fermiverteilung im Impulsraum dargestellt werden
- Pauli-Prinzip: Zustände dürfen nicht mehrfach besetzt werden
- Wechselwirkung wird weniger effektiv,
  wenn der Impulsraum schon besetzt ist, es können sich keine Bindungszustände ausbilden

Few-particle Schoedinger equation in a dense medium

Two-particle Schrödinger equation with medium effects

$$[E^{\underline{\mathrm{HF}}}(p_1) + E^{\underline{\mathrm{HF}}}(p_2)]\ \psi_{nP}(p_1, p_2)$$
$$+[1 - \underline{f(p_1) - f(p_2)}] \sum_{p_1' p_2'} V(p_1 p_2, p_1' p_2')\ \psi_{nP}(p_1', p_2')$$
$$= E_{nP}\ \psi_{nP}(p_1, p_2)$$

$$E^{HF}(p_1) = E(p_1) + \sum_{p_2} V(p_1 p_2, p_1 p_2)_{ex}\ \underline{f(p_2)}$$

Nuclear matter: Hamiltonian

atomic nuclei

neutron stars

- non-relativistic approach

$$H = \sum_{1} E(1) a_1^\dagger a_1 + \frac{1}{2} \sum_{121'2'} V(121'2') a_{1'}^\dagger a_{2'}^\dagger a_2 a_1$$

$$1 = \{\boldsymbol{p_1}, \sigma_1, \tau_1\}$$

- kinetic energy $E(1) = \frac{p_1^2}{2m_1}$

Thermodynamic relations

<u>equation of state:</u>
given temperature T and chemical potential μ,
density $n = n(T, \mu)$

<u>thermodynamic potentials</u>

$$\begin{aligned} p(T,\mu) &= \int_{-\infty}^{\mu} n(T,\mu')d\mu' \\ f(T,n) &= \int_{0}^{n} \mu(T,n')dn' \end{aligned}$$

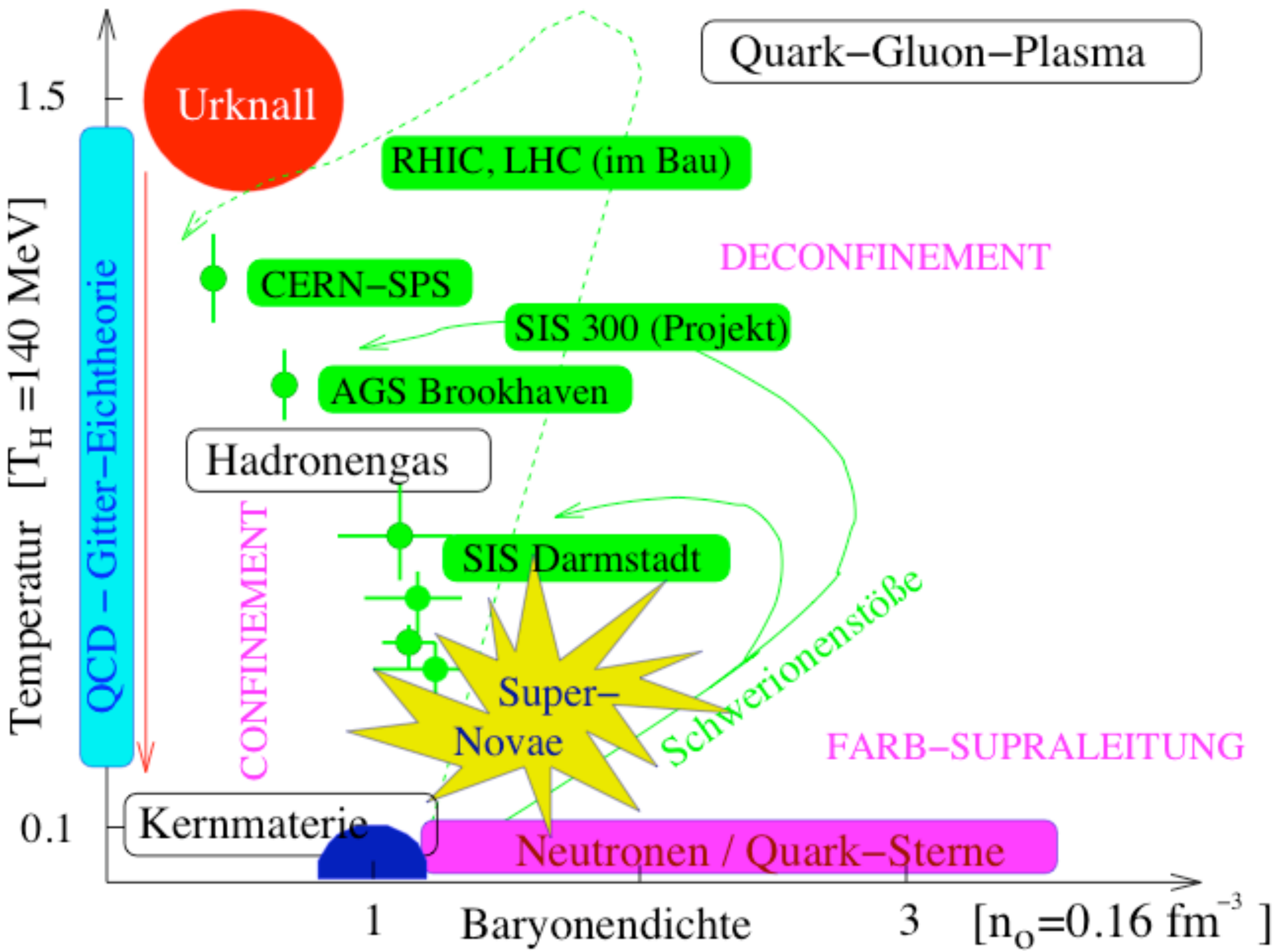
thermodynamic stability?

$$n(T,\mu) = \frac{1}{V}\sum_{p}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}f(\omega)A(\vec{p},\omega)$$

Pairing und α-Cluster in Atomkernen

- Bindungsenergien der Atomkerne: gerade oder ungerade Anzahl von Protonen bzw. Neutronen
- Virtuelle alpha-Cluster in der äußeren Schicht der Transurane
- Bose-Einstein-Kondensate in angeregten n-alpha-Kernen
- Dünne heiße Materie in Schwerionenstößen

Phasendiagramm Kernmaterie



Zusammenfassung

- Bildung von Clustern (Bindungszuständen)
  in niederdichten Systemen

- Auflösung der Cluster bei hohen Dichten
  (Abschirmung oder Pauli-blocking)

- Metall-Isolator-Übergang: Phasenübergang?
  Transporteigenschaften, optische Eigenschaften

- Quantenkondensate: Übergang von der Bose-Einstein-
  Kondensation gebundener Fermionen zum
  Bardeen-Cooper-Schrieffer-Pairing der Fermionen

Summary

- Bound states demand a quantum statistical description. Properties of Coulomb systems are strongly influenced by bound state formation.
- Medium effects change the bound state properties. Dissolution of bound states at high densities (Mott effect). Instead of a chemical picture, the spectral function should be used as an appropriate concept.
- As a consequence of the dissolution of bound states, a crossover from Bose-Einstein condensation to Cooper pairing occurs at low temperatures.
- The physics of the Mott transition from a nonmetallic to a metal-like state remains unclear. Reflecting on concepts of condensed matter, hopping should be considered as an important process.

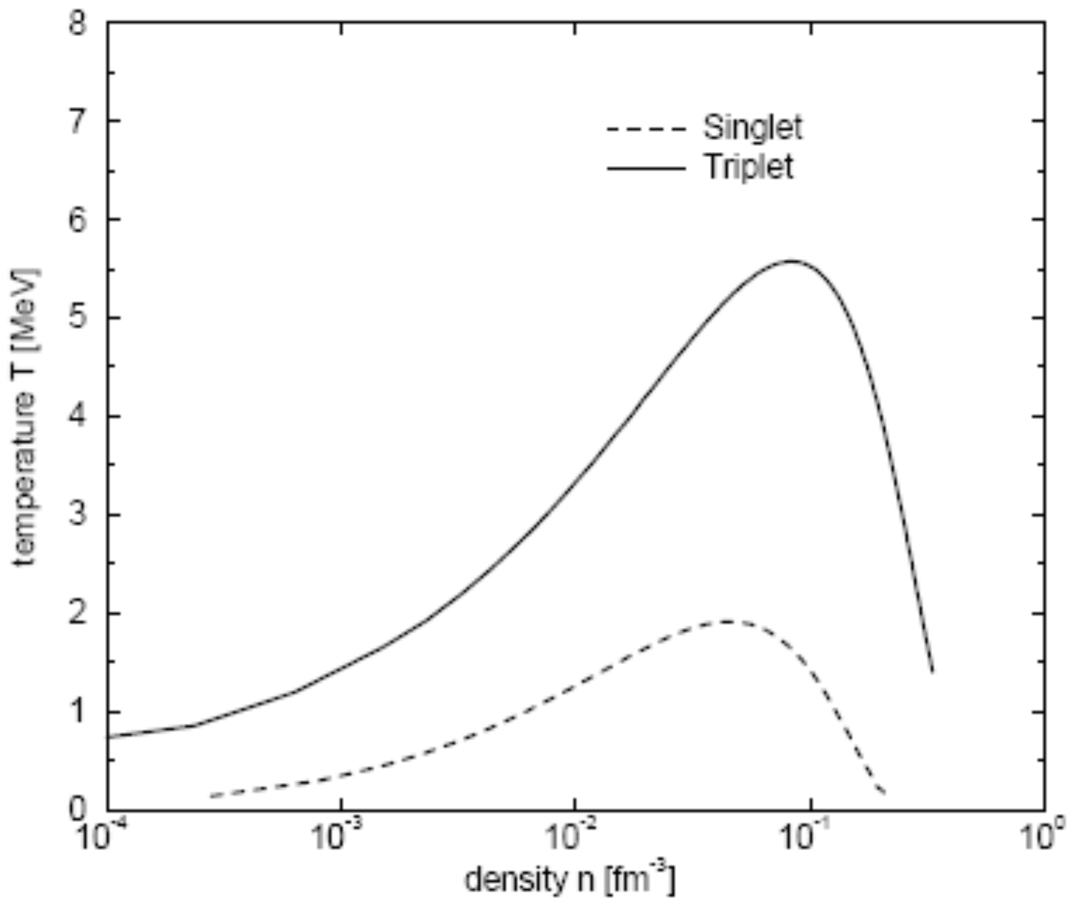
Danke

Nuclear matter: Hamiltonian

- interaction $V(121'2')$: empirical approach
  - long-range attraction
  - short-range repulsion
  - spin-isospin dependent
    coupled orbital momenta
- bound states, continuum correlations
- in-medium scattering, effective interaction
- superfluidity
- phase transitions
- example of a quantum liquid

Deuteron-Kondensation

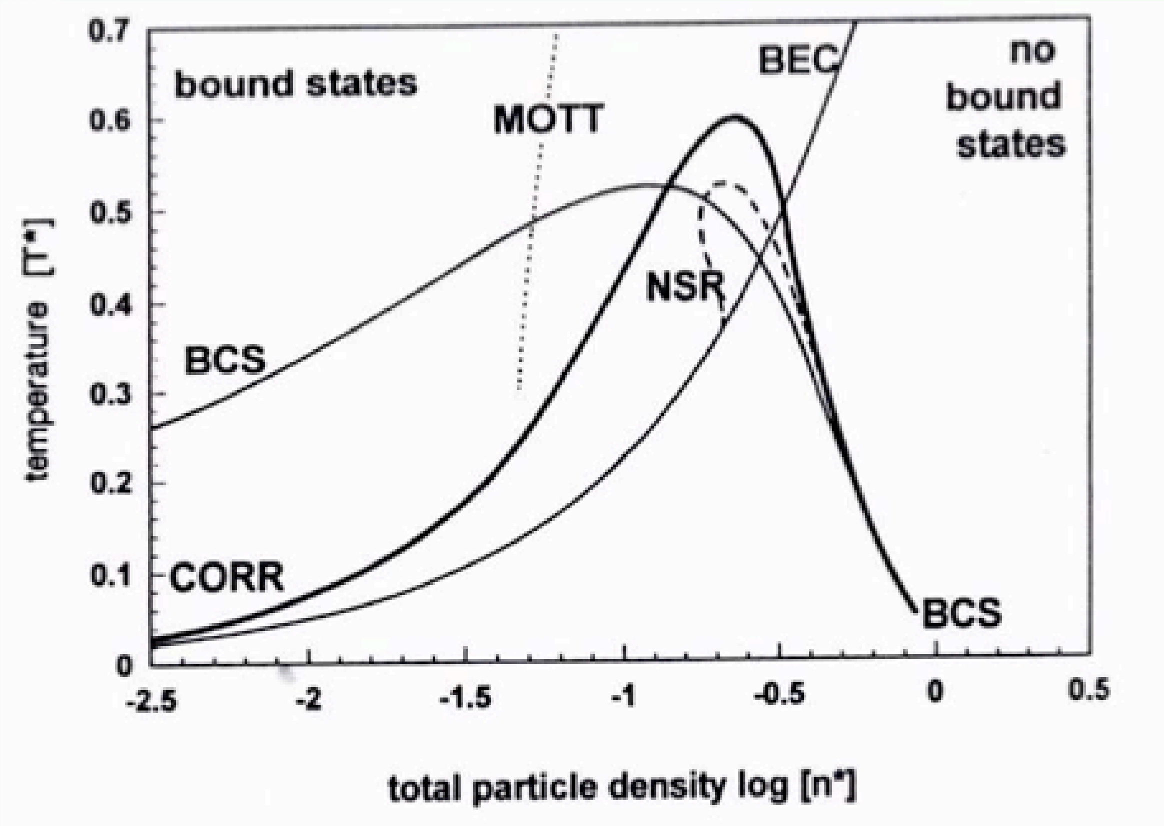
(Bose-Einstein-Kondensation --- BCS-Pairing)



Crossover from BEC to BCS

Phase transition to the superfluid state

fermionic model system with separable interaction, $T^* = T/E_0, n^* = n(\hbar^2/mE_0)^{3/2}$



NSR[a]: blocking by single-particle distribution function

thick line[b]: including the interaction with the correlated component of the medium

a P. Nozieres, S. Schmitt-Rink, J. Low Temp. Phys. **59**, 159 (1985)

b G. Röpke, Ann. Phys. (Leipzig) **3**, 145 (1994)

Bindungszustände und kondensierte Phase

Coulombsysteme (QuantenElektroDynamik)

Vielteilchensystem	elementare Teilchen	Bindungszustände	hohe Dichte
Ionenplasma	Elektronen, Ionen	Atome, Moleküle	flüssiges Metall
Halbleiterplasma	Elektronen, Löcher	Exzitonen, Biexzitonen	Elektron-Loch-Flüssigkeit

Stark wechelwirkende Systeme (QuantenChromoDynamik)

Vielteilchensystem	elementare Teilchen	Bindungszustände	hohe Dichte
Vielnukleonen-system	Protonen, Neutronen	Atomkerne	Kernmaterie
Quark-Gluon-System	Quarks	Hadronen	Quark-Gluon-Plasma

Abschirmung der Coulombwechselwirkung

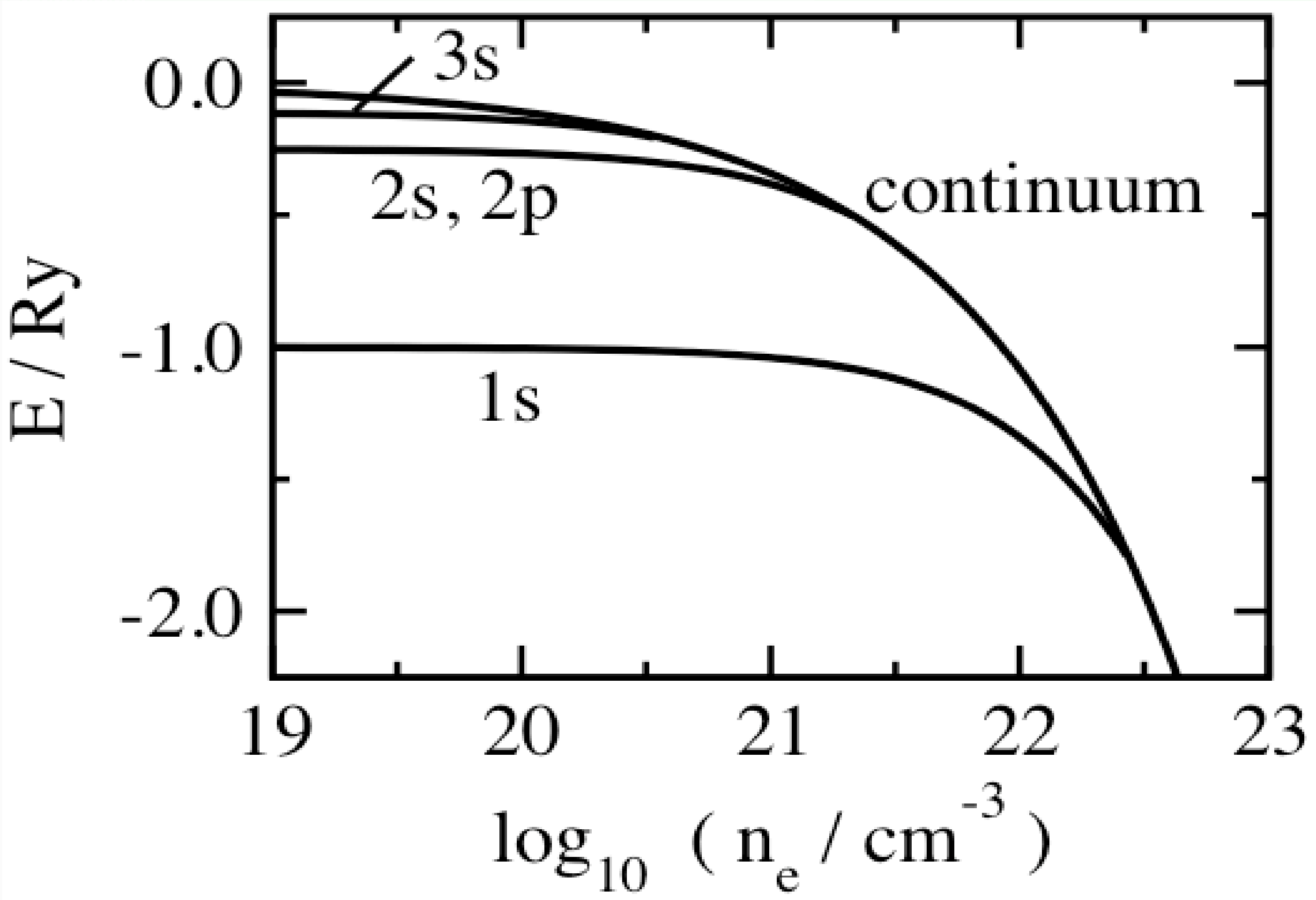
- Ladung (Atomkern, positive Ladung)

- umgebendes Plasma wird polarisiert:
  gleichnamige Ladungen werden abgestoßen
  ungleichnamige Ladungen werden angezogen

- eine Polarisationswolke bildet sich (Raumladung)

- Abschwächung des elektrischen Feldes

Shift of binding energy and Mott effect [a]



Dissolution of bound states at increasing density: screening

[a] D. Kremp et al., Quantum Statistics of Nonideal Plasmas, Berlin 2005

Mott-Effekt und Ionisationsgrad in Wasserstoffplasmen

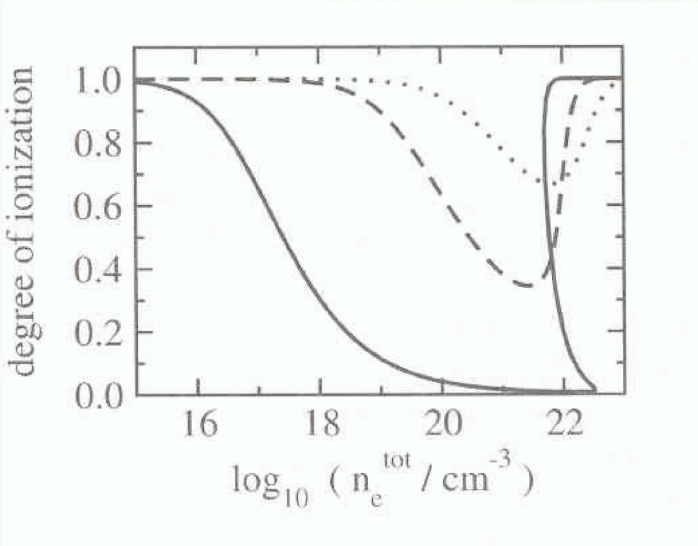
- two-particle partition function $\rightsquigarrow$ Planck-Larkin form
- quasiparticle shifts for the one-and two-particle energies
- effective binding energies $\rightsquigarrow$ lowering of the ionization energy ΔE

$$\frac{1-\alpha_{\rm ion}}{\alpha_{\rm ion}^2} = n_e \lambda_e^3 Z_b^{\rm PL} \exp(\beta \Delta E) \; , \; \Delta E = \Delta_e + \Delta_p - \Delta_H$$

dotted line: 50000 K

dashed line: 30000 K

solid line: 15000 K



[a] see D. Kremp, M. Schlanges, W.D. Kraeft, Quantum Statistics of Nonideal Plasmas (Springer, Berlin, 2005)

Mikroskopische Beschreibung

- Quantenphysik:
  - Bindungszustände (Cluster)
  - Schrödingergleichung
- Statistische Physik:
  - Einfluss der Umgebung
- Relativistische Beschreibung:
  - bei hohen Energien

Effektive Wellengleichung

- Schrödingergleichung

$$-\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial \vec{r}^{\,2}}\Psi_n(\vec{r}) + V_{eff}(\vec{r})\Psi_n(\vec{r}) = E_n\Psi_n(\vec{r})$$

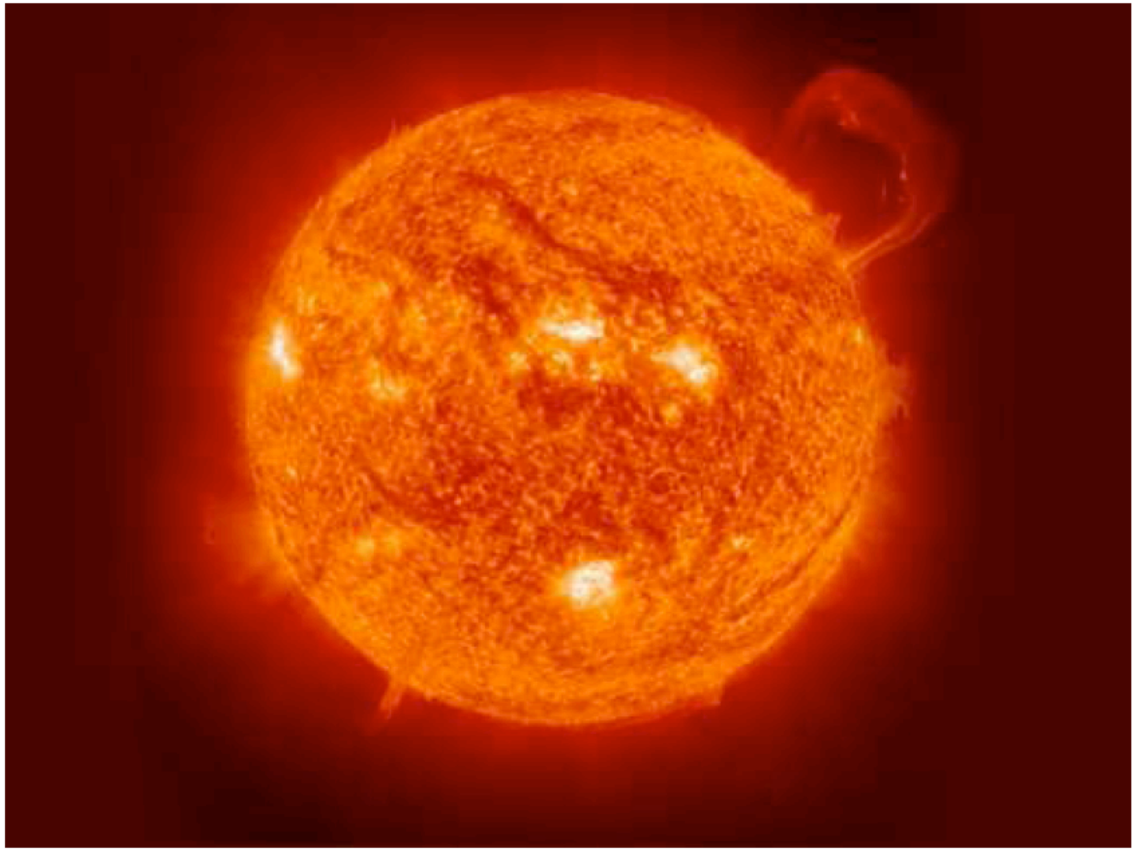
- Effektive Wechselwirkung

$$V_{eff}(\vec{r}) = -\frac{e^2}{4\pi\varepsilon_0 r}e^{-\kappa r}$$

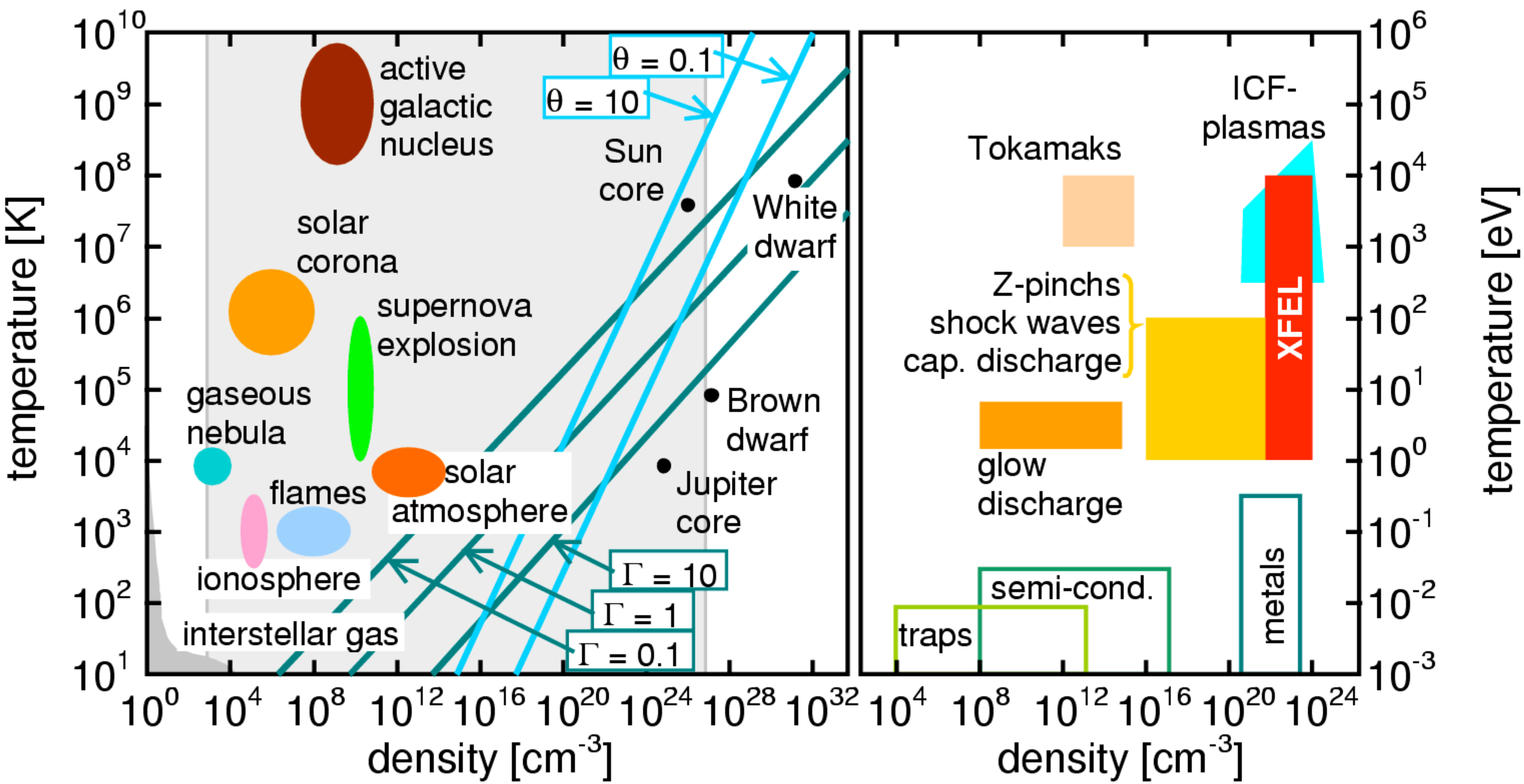
- Abschirmparameter

$$\kappa = \sqrt{\rho e^2/\varepsilon_0 k_B T}$$

The sun: plasma state



Phase diagram: Coulomb systems



Results

	E_{exc} (MeV)	Γ (MeV)
0_2^+	6.06	
0_3^+	11.26	2.6
0_4^+	12.05	1.6×10^{-3}
0_5^+	14.0	4.8
0_6^+	14.03	2.0×10^{-1}

$(^{12}C + \alpha)_S$

$(^{12}C + \alpha)_D$

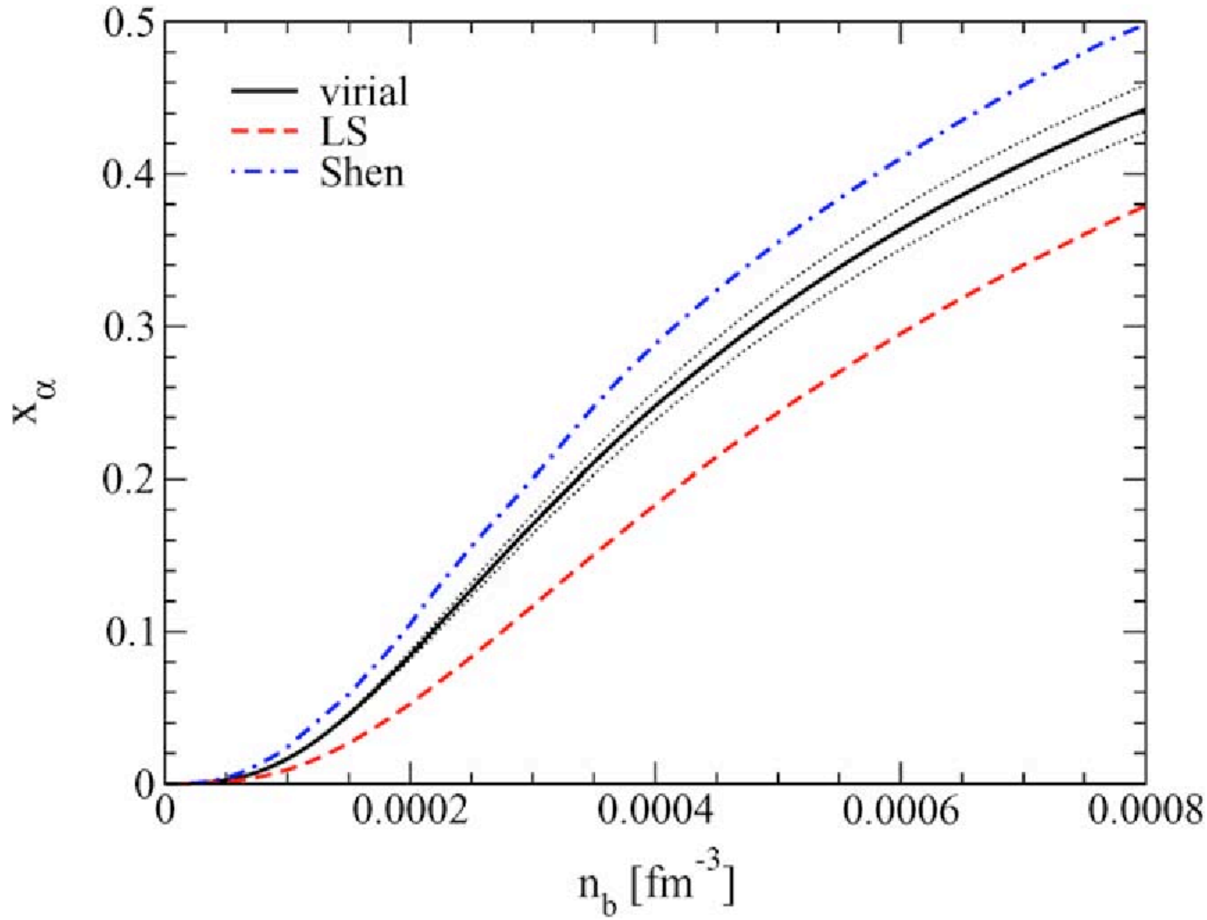
α cluster, $k=2$

α condensate, $k=3$

Tabelle 2: Observed excitation energies E_{exc} and widths Γ of the lowest five 0^+ excited states in ^{16}O

Alpha-particle fraction in the low-density limit

symmetric matter, T=4 MeV



Horowitz & Schwenk (2006), Lattimer & Swesty, (2001),Shen et al. (1998))