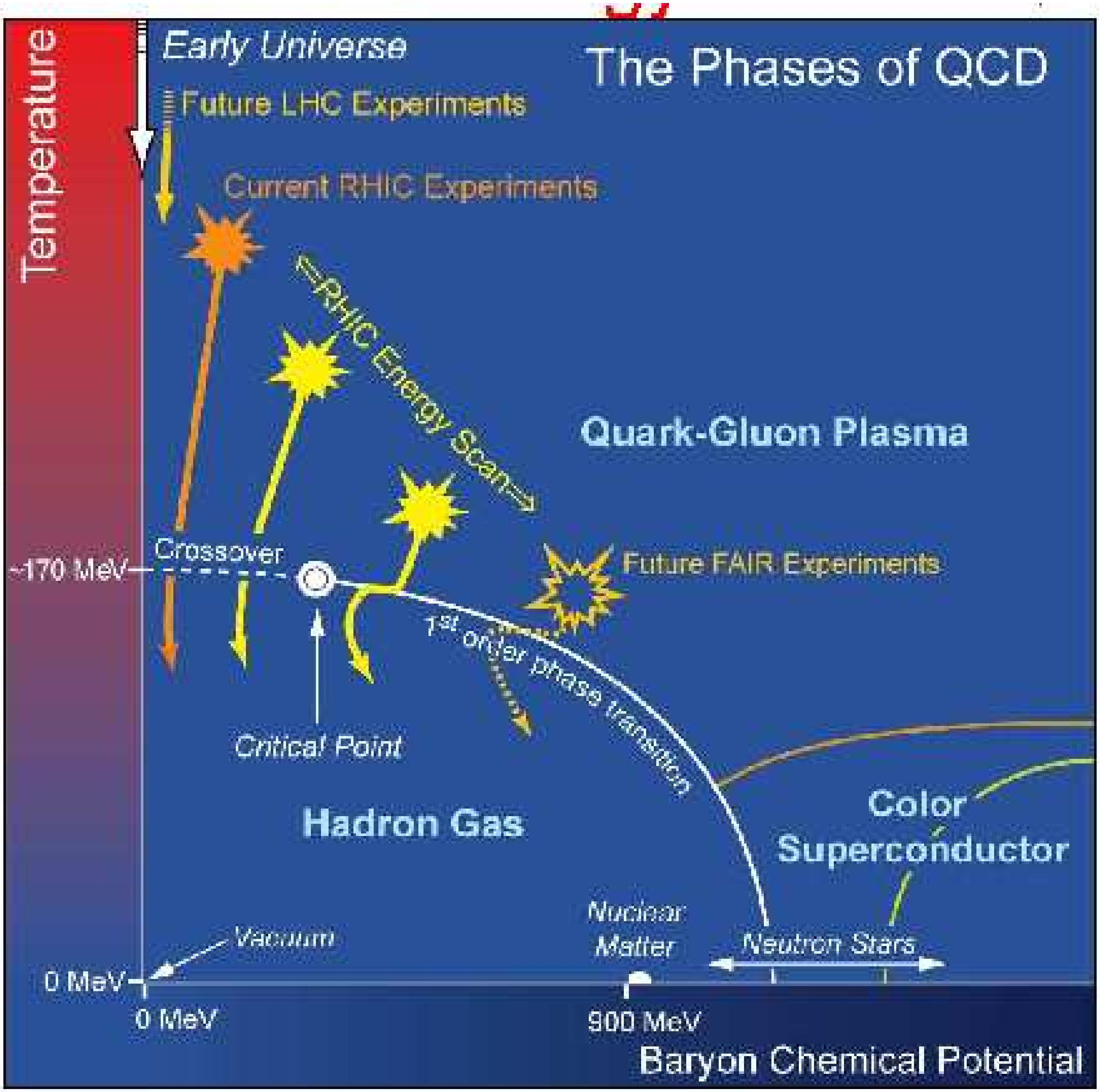
Helmholtz International Summer School

Dense Matter in Heavy Ion Collisions and Astrophysics

JINR Dubna Russia July 14–26 2008

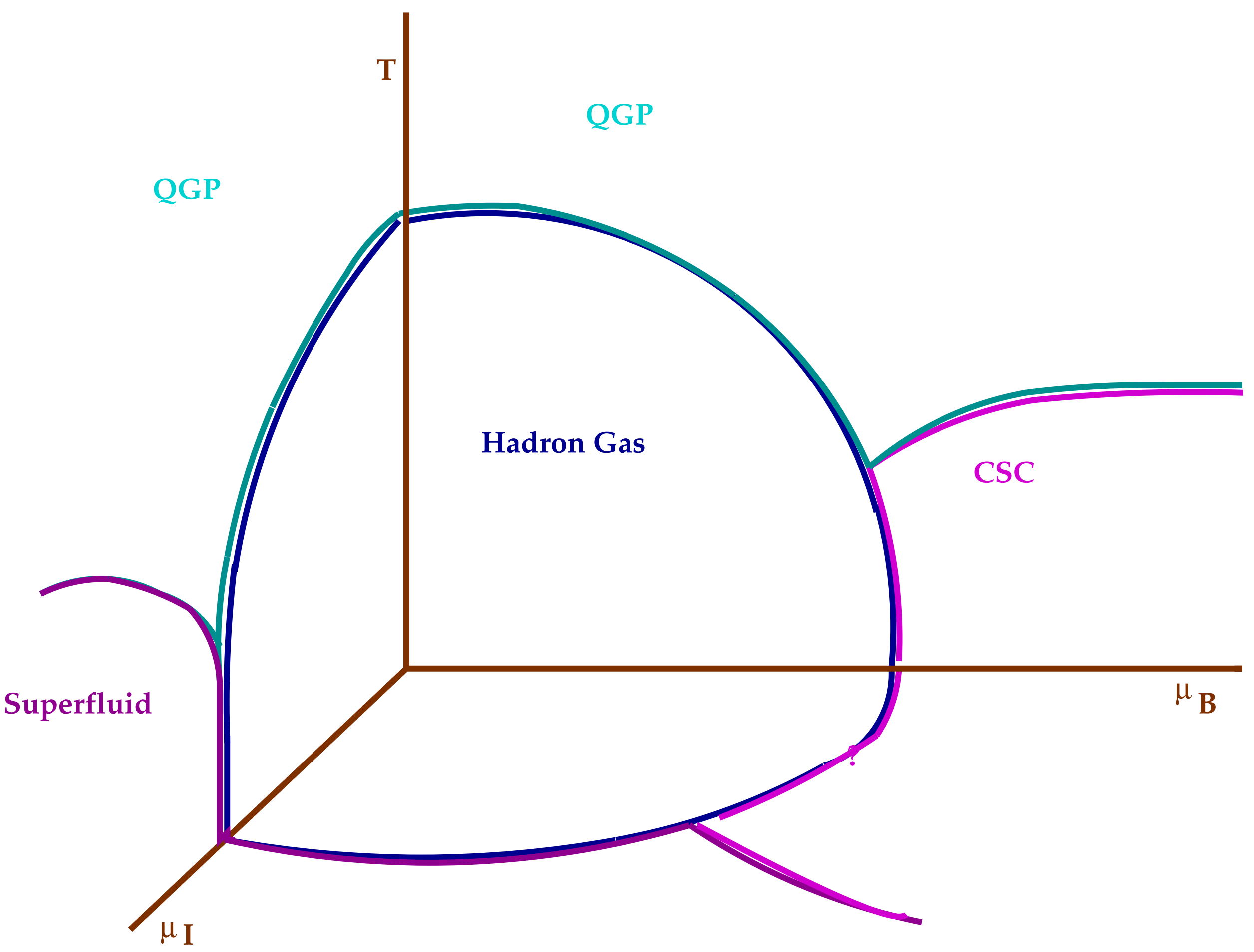
QCD at finite temperature **and density** *on the lattice*

M.P. Lombardo, INFN Laboratori Nazionali di Frascati

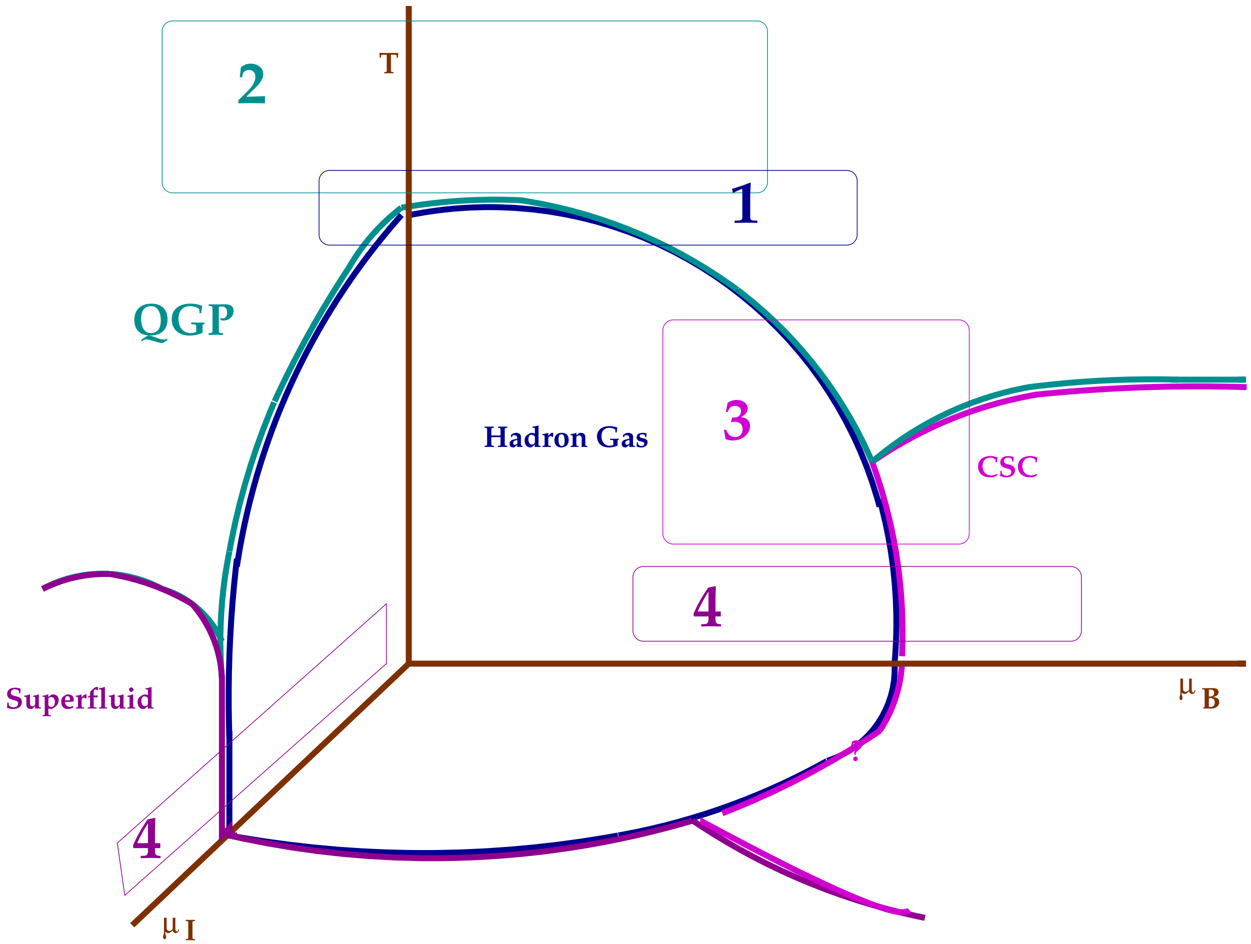


Temperature
Early Universe
The Phases of QCD
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color
Superconductor
Nuclear
Matter
Vacuum
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

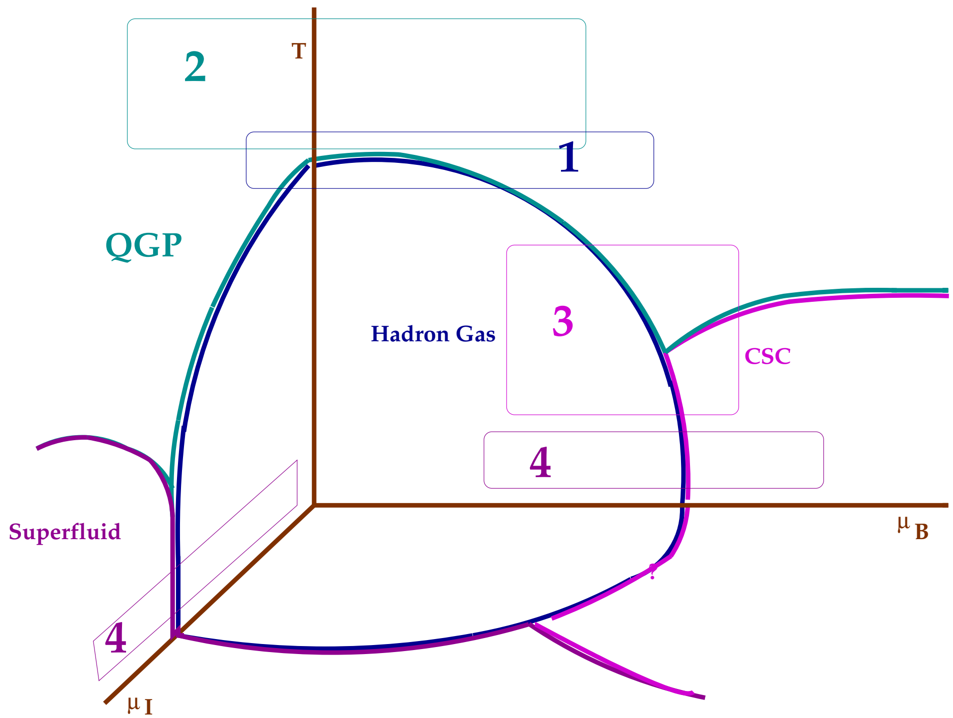
LATTICE FIELD THEORY : FIRST PRINCIPLE APPROACH TO
THE QCD PHASE DIAGRAM



PLAN

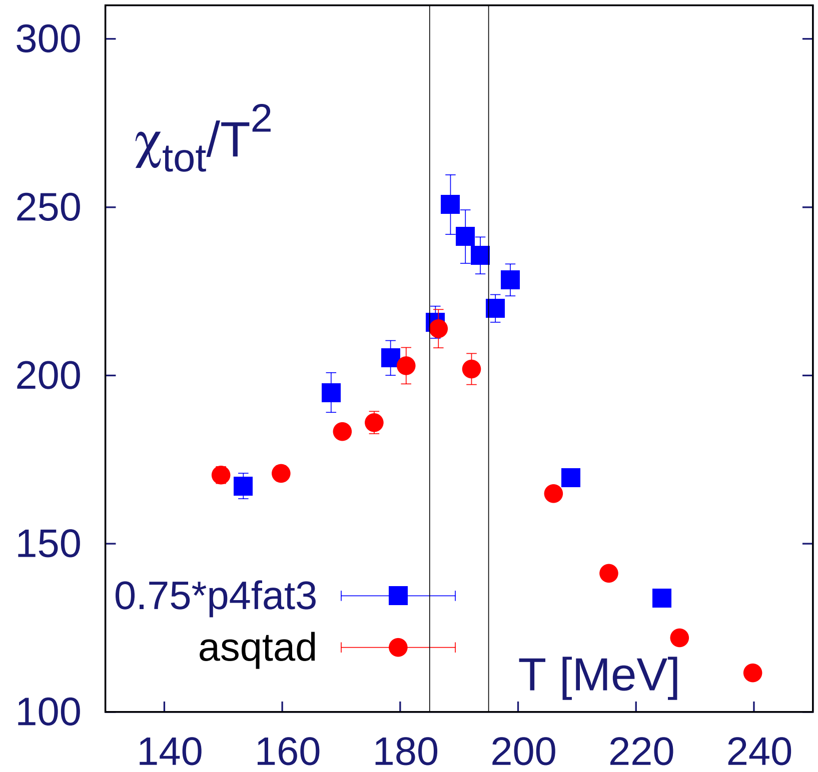


1. Critical line and Critical Point
2. EoS and Critical Behaviour
3. Towards FAIR
4. Cold and Dense Matter : OCD–like models

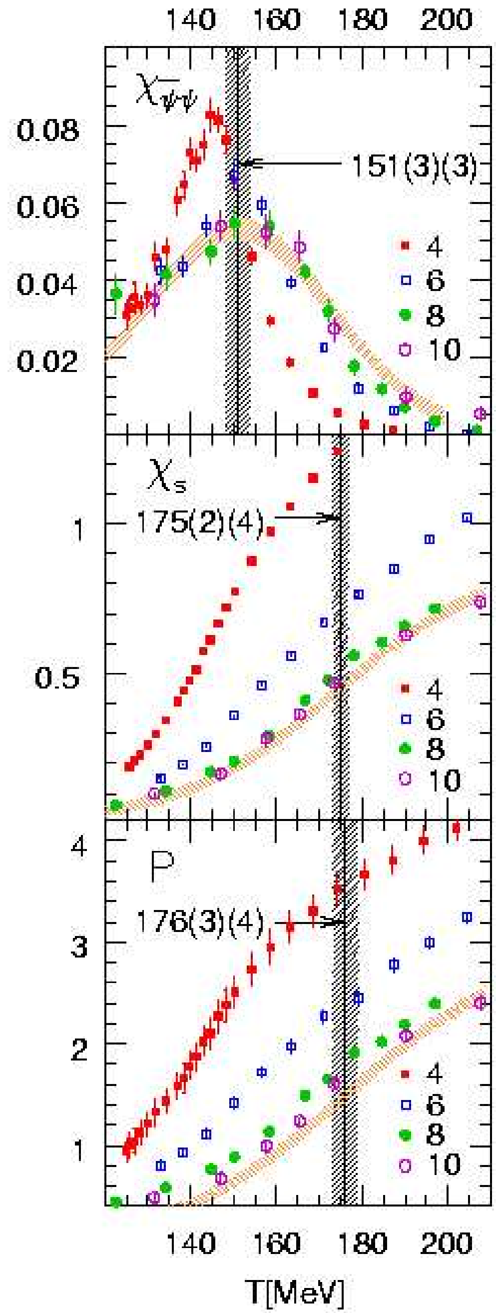


1. T_c, THE CRITICAL LINE AND THE CRITICAL POINT

T_c:
RBC-BIELEFELD



T_c:
WUPPERTAL-JUELICH



T_c AT $\mu_B = 0$: STATUS AS OF QM2008

RBC-BIELEFELD :

$T_c = 190(5)$ MeV

WUPPERTAL-JUELICH :

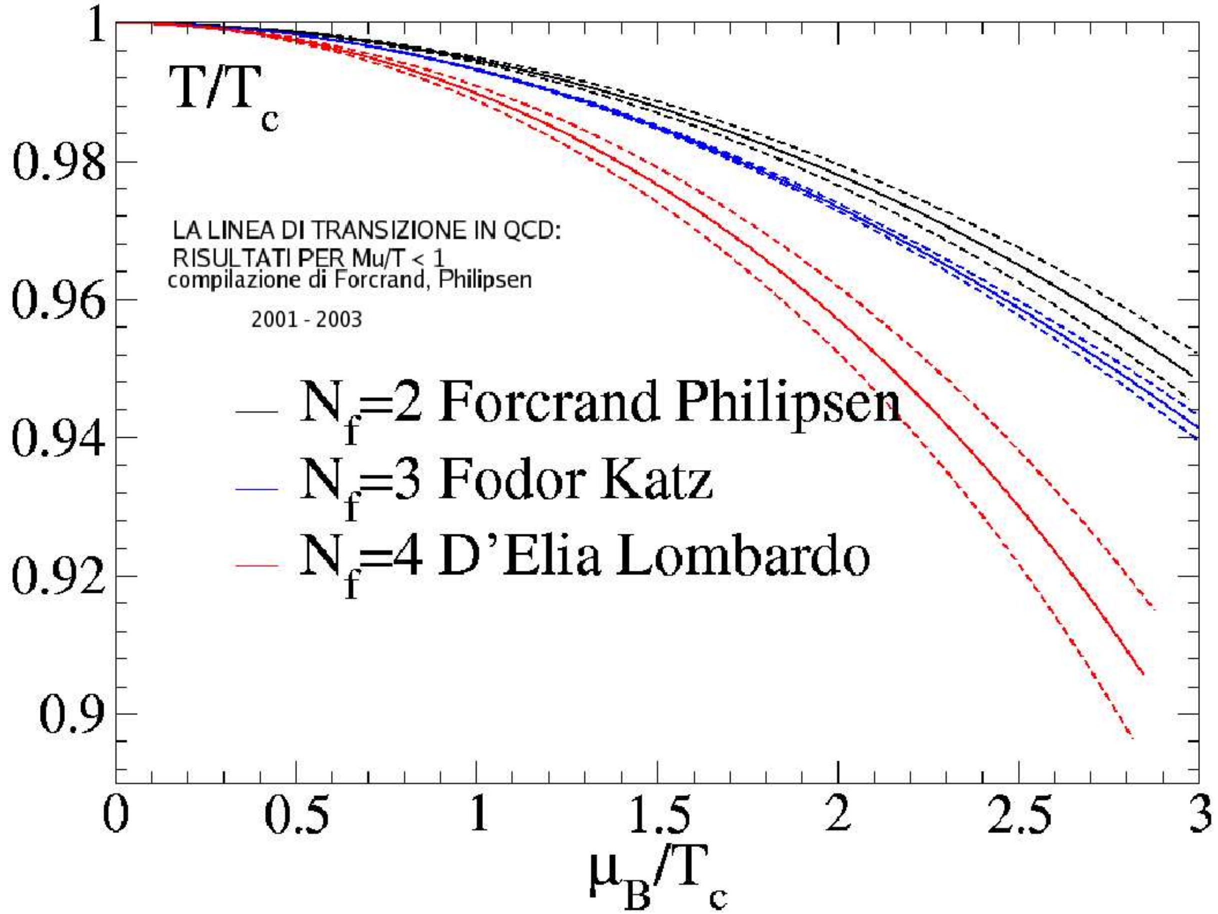
$T_c = 175(5)$ MeV (Glue)

$T_c = 151(6)$ MeV (Fermions)

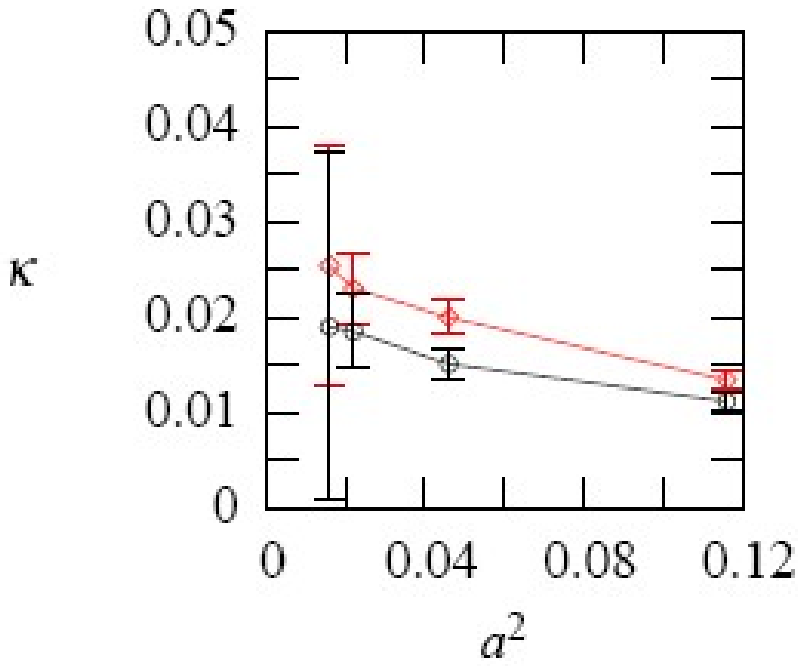
THE CRITICAL LINE IN THE μ_B, T PLANE AT SMALL μ_B

$T = T_c - K\mu_B^2$

$K \propto 1/N_f$



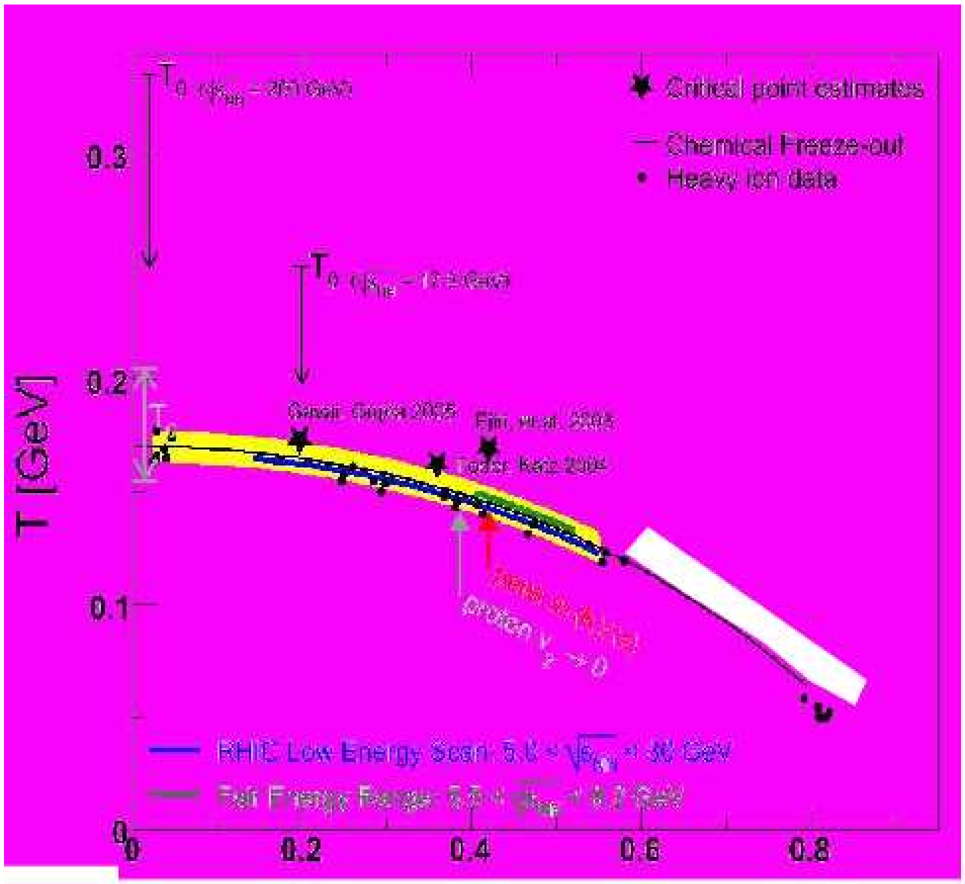
TOWARDS THE CONTINUUM LIMIT: CURVATURE K OF THE CRITICAL LINE IN THE μ_B-T PLANE

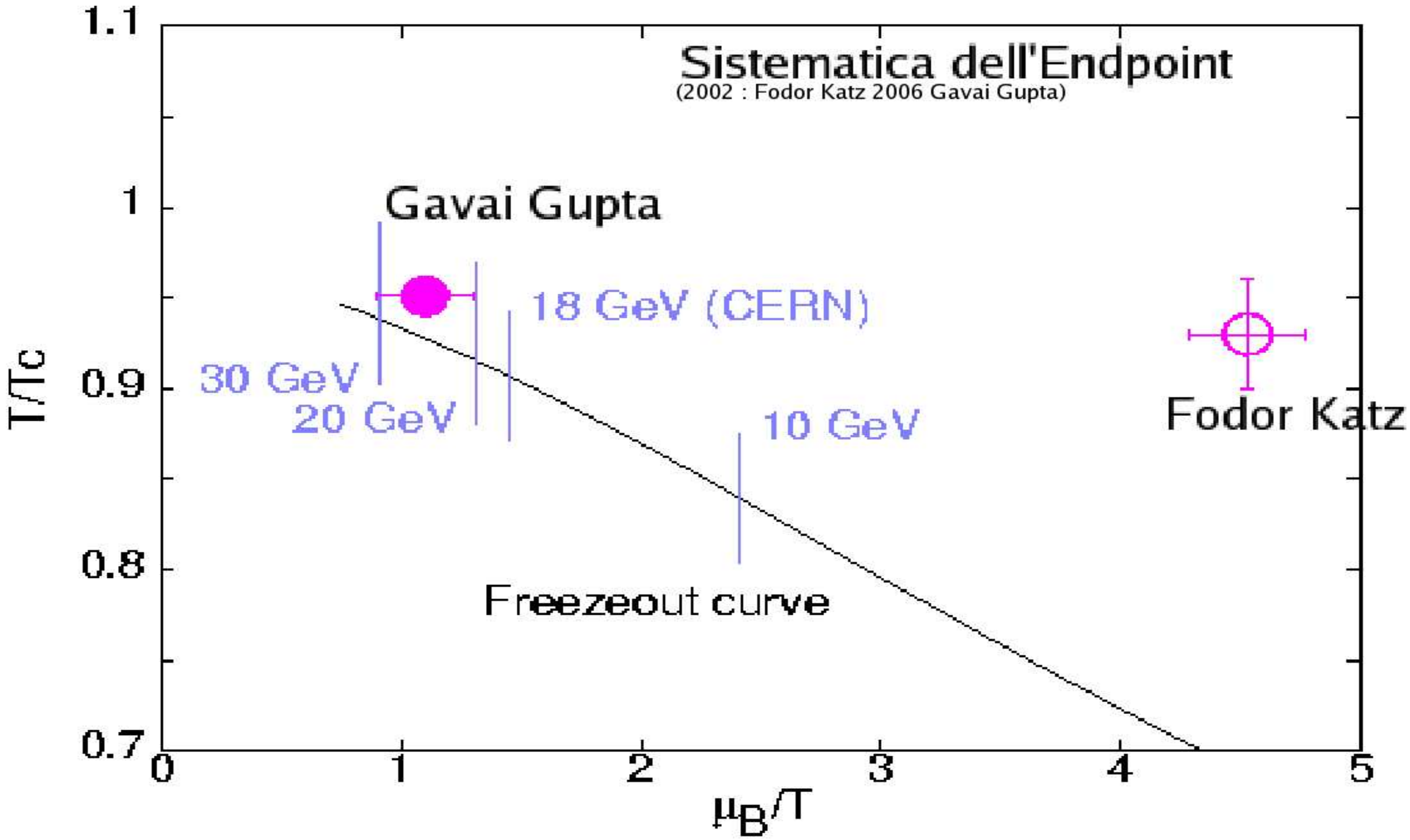


Results for Nf=2+1

from gluonic and fermionic susceptibilities
Fodor, Guse, Katz, Kálmán K. Szabó, QM2008

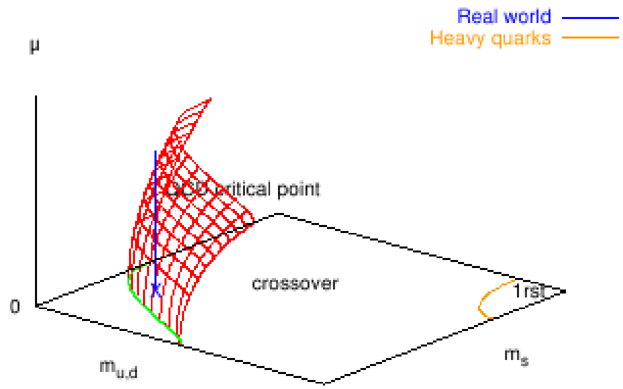
THE CRITICAL ENDPOINT OF QCD

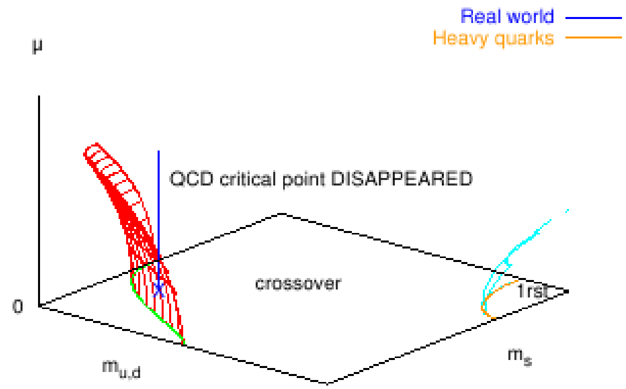



Sistematica dell'Endpoint
(2002 : Fodor Katz 2006 Gavai Gupta)
Gavai Gupta
18 GeV (CERN)
30 GeV
20 GeV
10 GeV
Fodor Katz
Freezeout curve
T/Tc
μB/T
1.1
1
0.9
0.8
0.7
0
1
2
3
4
5

CHALLENGING THE CRITICAL ENDPOINT OF QCD

Forcrand-Philipsen : strategy





Scenario I or Scenario II ? To decide, measure slope K in

$$\frac{m_c(\mu)}{m_c(0)} = 1 + K\left(\frac{\mu}{T}\right)^2 + \dots$$

$K > 0$ **: Scenario I , critical endpoint at small μ_B**
$K < 0$ **: Scenario II, NO critical endpoint at small μ_B: favored at QM2006**

Forcrand-Philipsen Lat07 : improve and confirm previous results

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{T}\right)^2 + \ldots$$

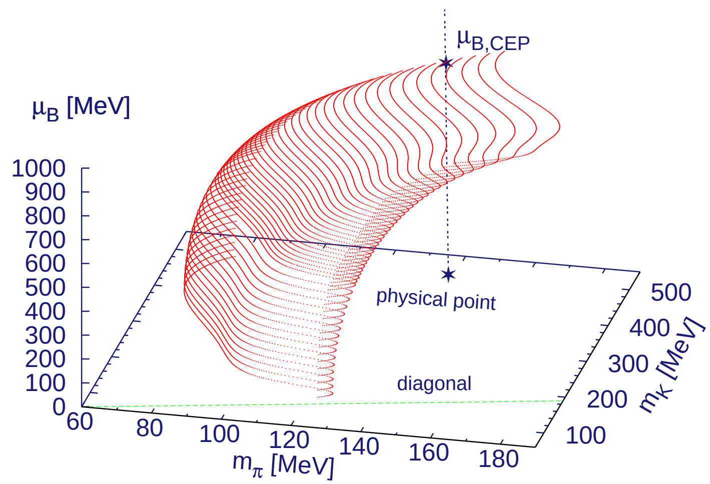
Kogut-Sinclair 2007 : QCD at finite isospin density
NB : $\mu_I \simeq \mu_B$ **at** $T \simeq T_c$ **Toublan, Kogut, Sinclair 2004**

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.(1)\left(\frac{\mu_I}{T}\right)^2 + \ldots$$

Current Results:

- **Confirm unusual scenario for** $N_t = 4$
- **Suggest NO critical endpoint for** $\mu_B < 600 MeV$
- **See talk by Philippe de Forcrand**

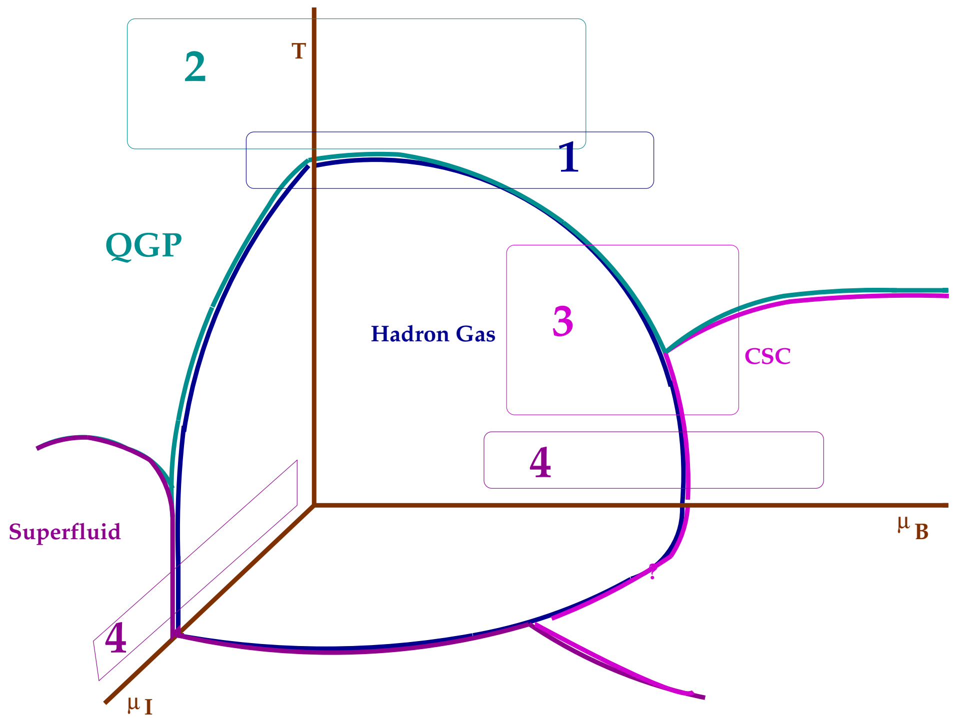
$SU(3)_L \times SU(3)_R$ NJL Model



Kovács and Szép, 2006 compute critical surface of the $SU(3)_L \times SU(3)_R$ chiral quark model at non-zero baryon density.

Conventional Scenario realized in NJL, $\mu_B^{CEP} \sim 900$ MeV.

Qualitative difference between QCD and NJL at *small* μ_B.: Details of the dynamics important.



2. EOS AND CRITICAL BEHAVIOUR

LATTICE OBSERVABLES FOR THERMODYNAMICS

Number Density : accessible at imaginary chemical potential.

$$n_{u,d}(T,\mu_u,\mu_d,m_u,m_d) = \frac{\partial p(T,\mu_u,\mu_d)}{\partial \mu_{u,d}}; p(T,\mu_u,\mu_d) = \frac{T}{V}\ln Z(T,\mu_u,\mu_d)$$

Susceptibilities: accessible at at $\mu = 0$

$$\chi_{j_u,j_d}(T) = \left.\frac{\partial^{(j_u+j_d)} p(T,\mu_u,\mu_d)}{\partial \mu_u^{j_u}\,\partial \mu_d^{j_d}}\right|_{\mu_u=\mu_d=0} .$$

Test for fluctuations.

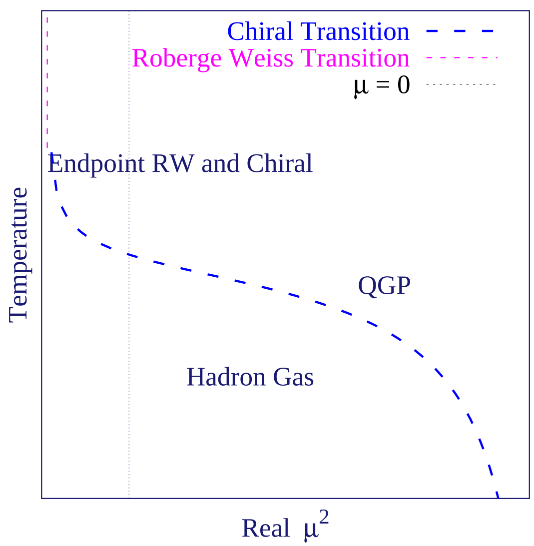
Taylor coefficients of the excess pressure:

$$\Delta p(T,\mu_u,\mu_d) \equiv p(T,\mu_u,\mu_d) - p(T,\mu_u=0,\mu_d=0) = \sum_{j_u,j_d} \chi_{j_u,j_d}(T)\frac{\mu_u^{j_u}}{j_u!}\frac{\mu_d^{j_d}}{j_d!} ,$$

containing information about baryon density effects in the EoS.

THERMODYNAMICS AND CRITICAL LINES in THE T μ^2 PLANE

Three regimes for thermodynamics:



- **Low Temperature,**

  away from critical lines:

  Hadron Gas QM2006

  $n(T, \mu) = K(T)\sin(N_c\mu/T)$

- **In the critical region:**

  $p(T, \mu)$

  $= b(T)|t + a(T)(\mu^2 - \mu_c^2)|^{(2-\alpha)}$

  implying

  $n(T, \mu) =$

  $A(T)\mu(\mu^{c2} - \mu^2)^{(2-\alpha)}$

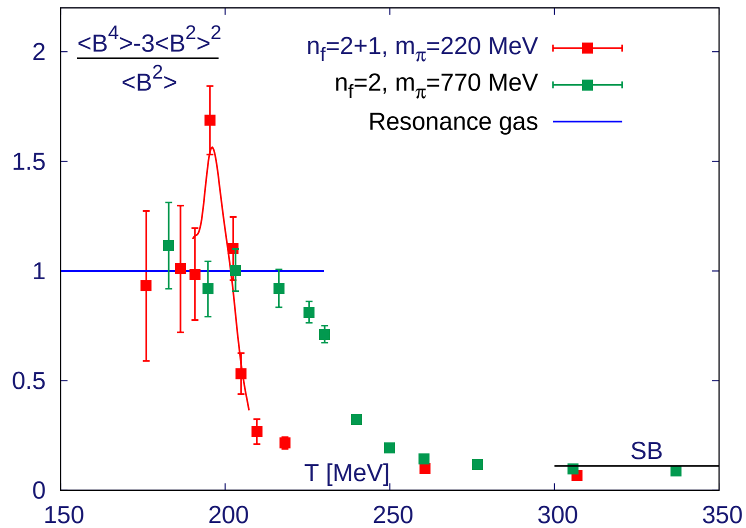
- **High Temperature,**

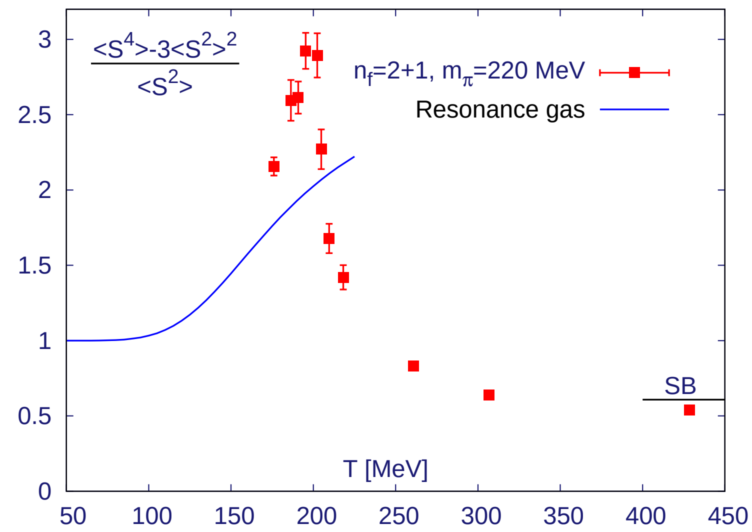
  away from critical line

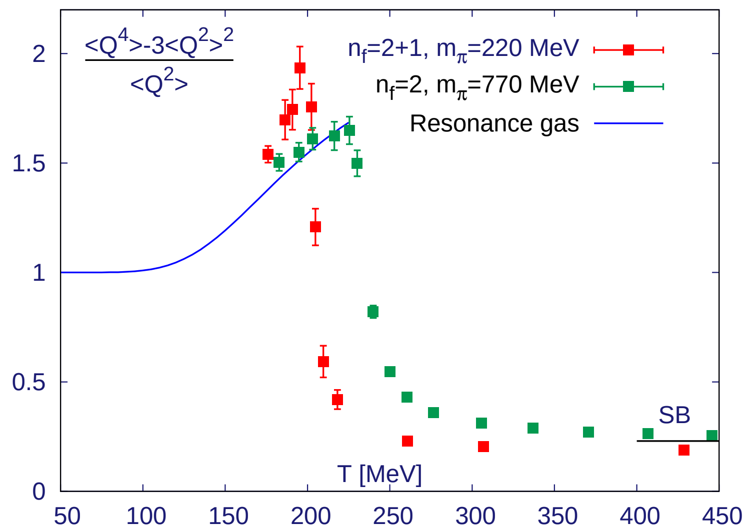
  Approach to Free Field

  $n(T, \mu) \longrightarrow n_{SB}(T, \mu)$

HADRON GAS; CRITICAL BEHAVIOUR; SB from SUSCEPTIBILITIES







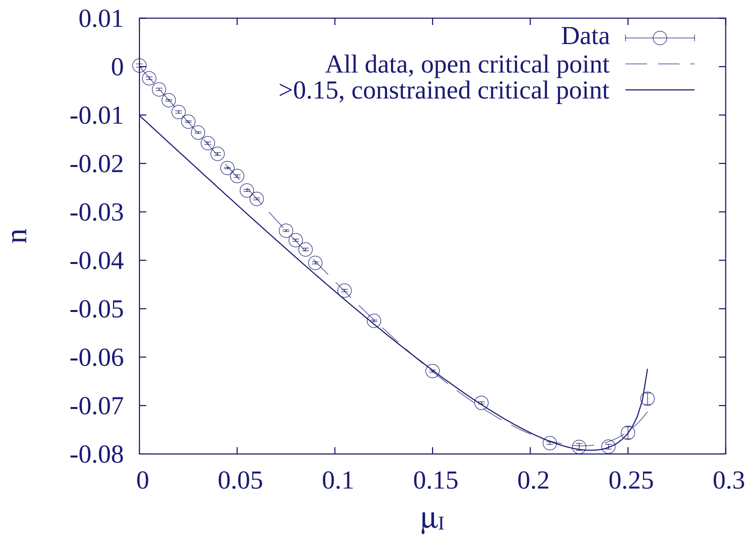
2+1 Flavor, $m_\pi = 220$ MeV

RBC-Bielefeld QM2008; courtesy C. Schmidt

CRITICAL BEHAVIOR AND THERMODYNAMICS AT THE ENDPOINT OF THE RW TRANSITION

Critical behavior at imaginary μ

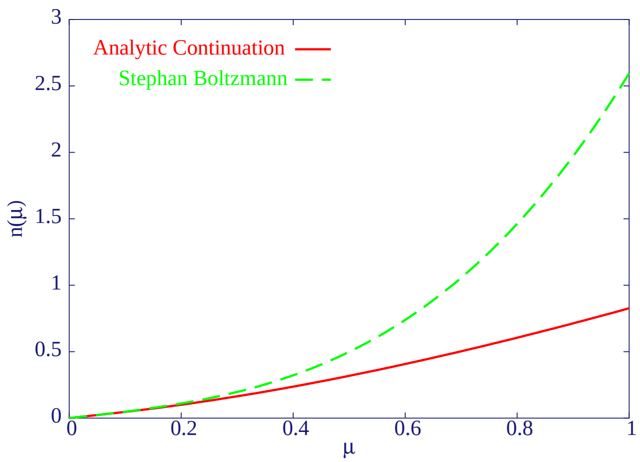
$$n(\mu_I) = A(T)\mu_I(\mu^{c2} - \mu_I^2)^{(2-\alpha)}$$



Continued to real μ..

$$n(\mu) = A(T)\mu(\mu^{c2} + \mu^2)^{(2-\alpha)}$$

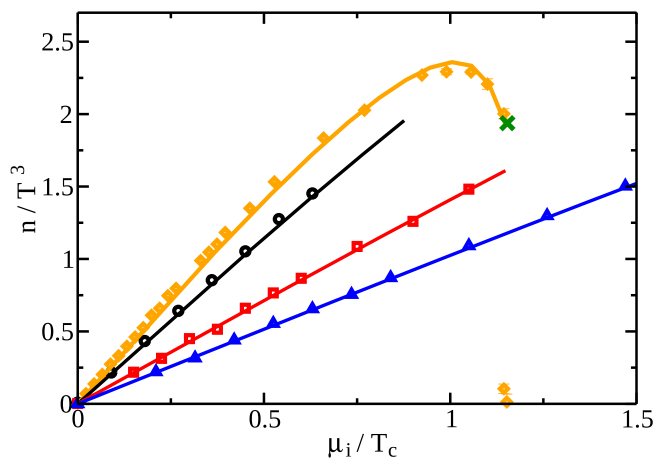
$$n_{SB}(\mu) = A\mu + B\mu^3) \rightarrow \alpha = 1$$

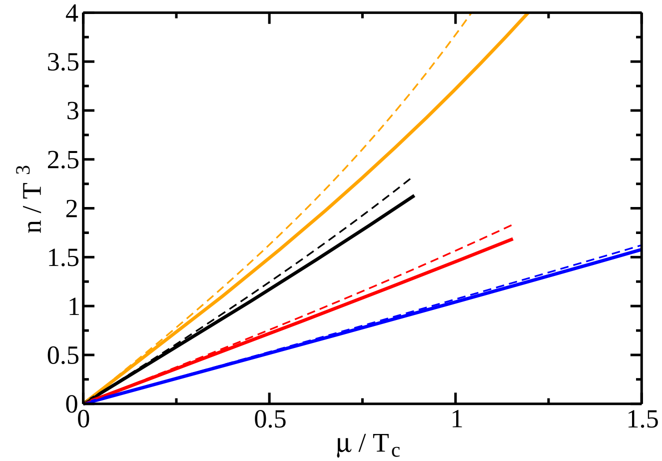


D'Elia, Di Renzo, Lombardo, 2007, QM2008

CRITICAL BEHAVIOR, THERMODYNAMICS, QUASIPARTICLE MODELS
Kämpfer, Bluhm Proposal
amenable to an easy comparison with lattice data.

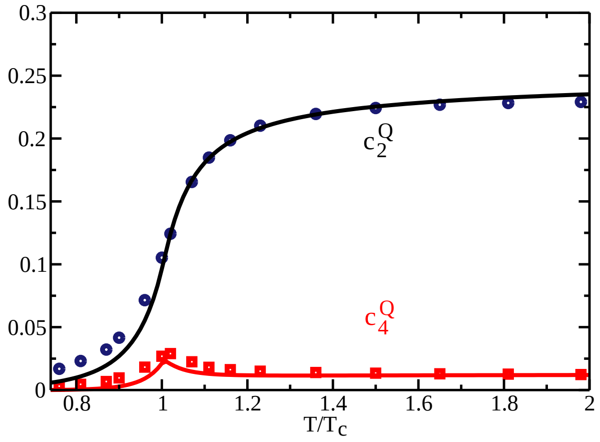
I. Quasiparticlemodel vs Imaginary Chemical Potential Lattice Data, and analytic continuation to Real Chemical Potential

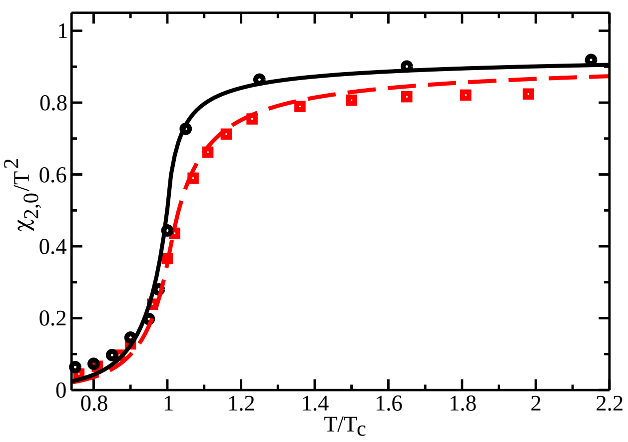




Kämpfer, Bluhm , 2007, QM2008

II. Quasiparticlemodel vs Taylor Coefficients





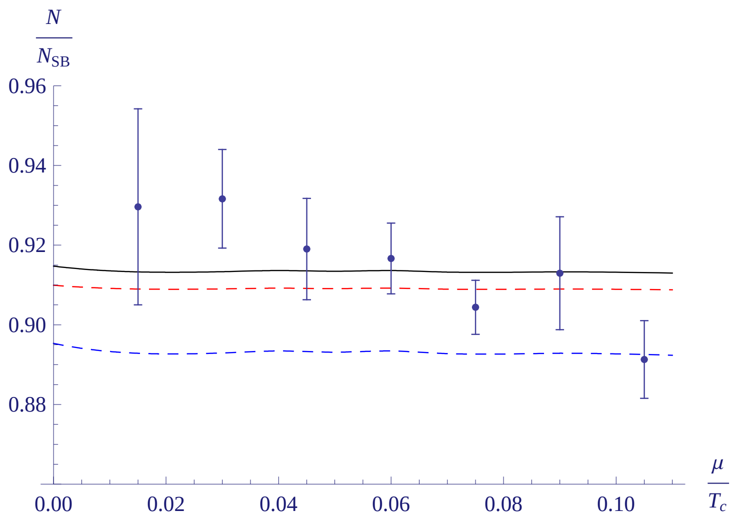
Crucial ingredients:

- **Explicit dependence of the self-energy parts on** $\mu_i = \mu_{u,d}$ **and** T
- **Implicit dependence via the effective coupling** $G^2(T, \mu_u, \mu_d)$**.**

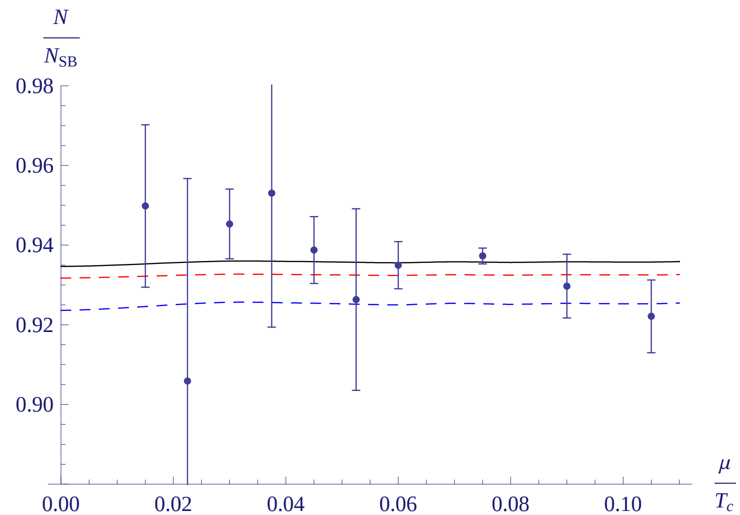
$$\omega_i^2 = k^2 + m_i^2 + \Pi_i, \quad \Pi_i = \frac{1}{3}\left(T^2 + \frac{\mu_i^2}{\pi^2}\right) G^2(T, \mu_u, \mu_d)\,.$$

Kämpfer-Bluhm : see Poster

APPROACH TO SB : ANALYTIC RESULTS VS. LATTICE DATA
Based on A. Vuorinen, 2004



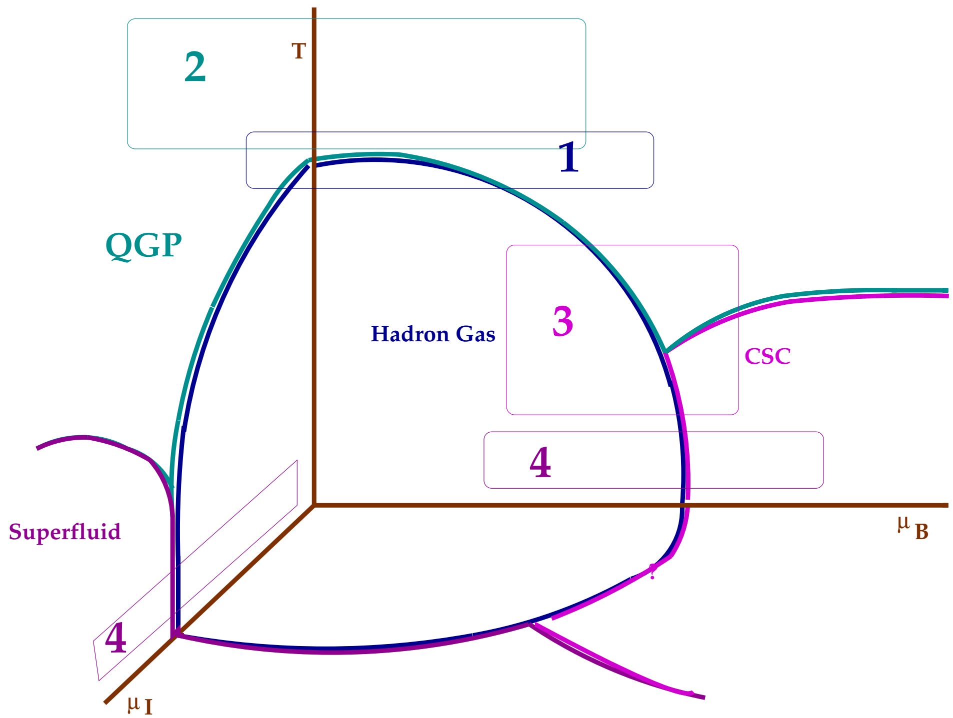
$T = 1.5T_c$



$T = 3.5T_c$

D'Elia, Di Renzo, Lombardo, Vuorinen, in progress

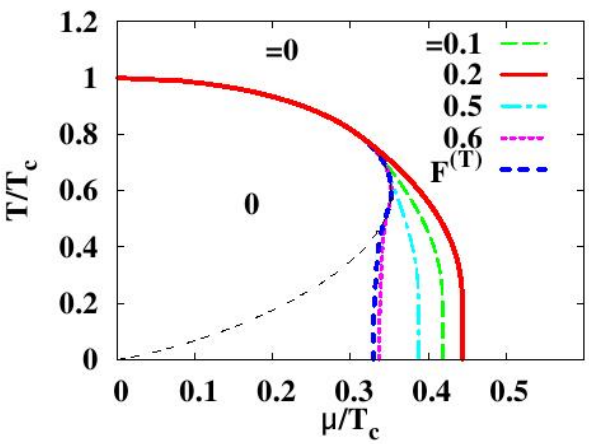
Lattice Cutoff Effects : Karsch, Laermann, 2008



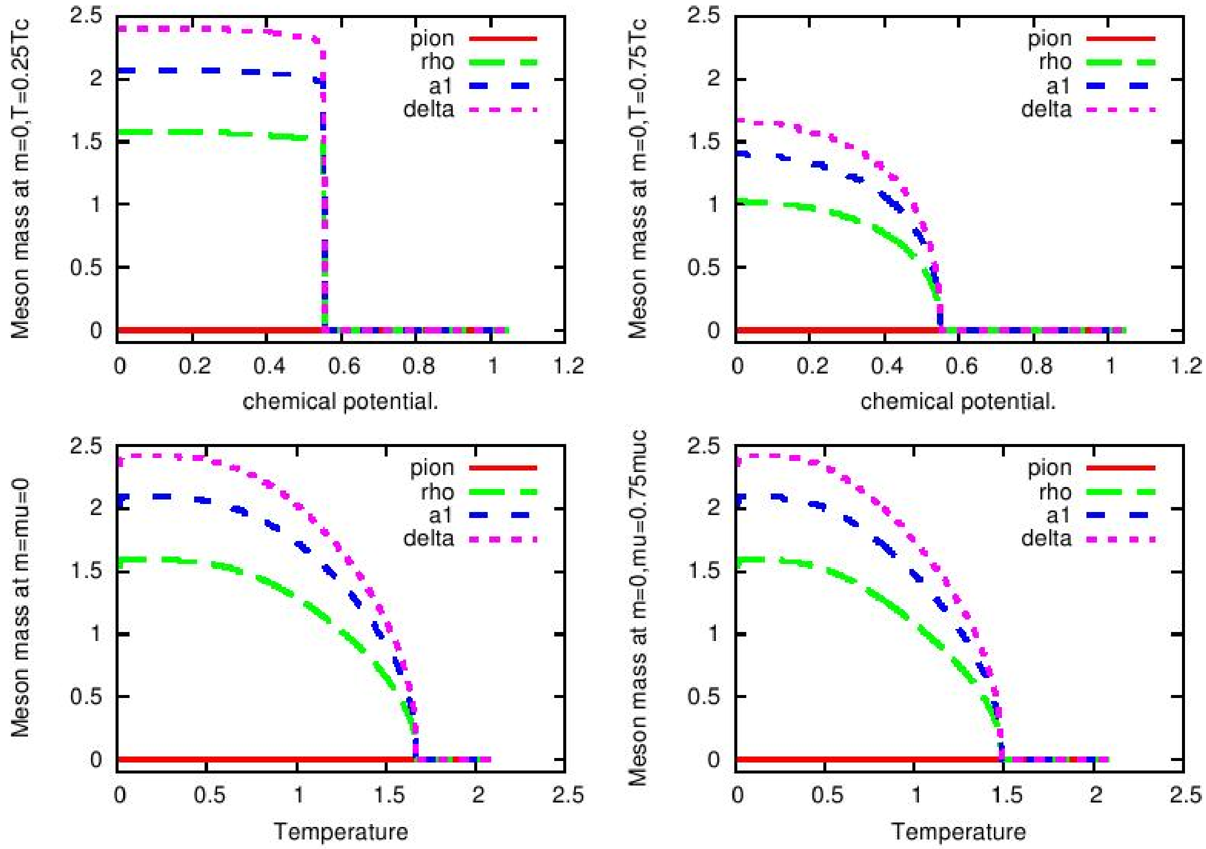
3. TOWARDS FAIR

STRONG COUPLING

New Results on The Mass Spectrum



Kawamoto, Miura 2007

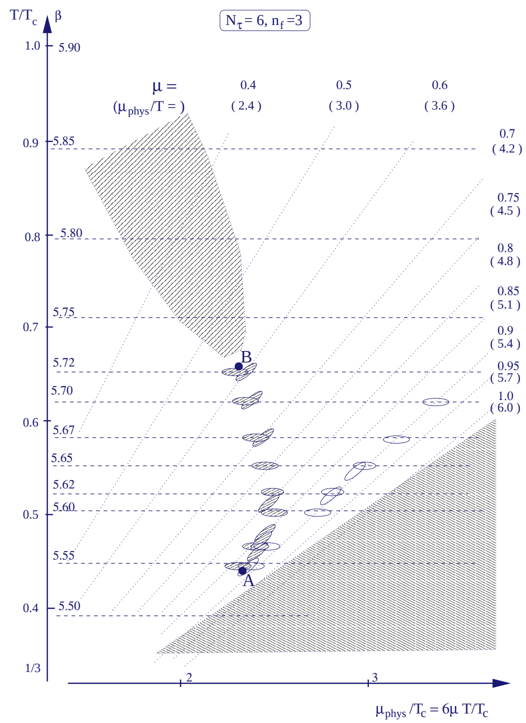


HEAVY QUARK EFFECTIVE MODEL

Double limit: $M \to \infty, \; \mu \to \infty, \; \zeta \equiv \exp(\mu - \ln M)$ **: Fixed**

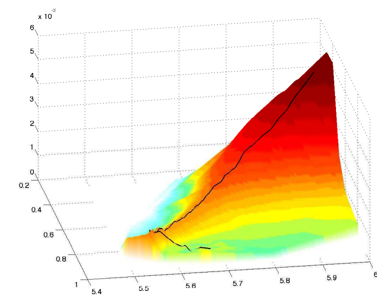
Evolved 'quenched approximation' in the presence of charged matter

T



μ_B

Polyakov Loop



Results for $N_f = 3$:

- **Identified phase transition**
- **Indentified Ridge in the T, μ plane**
- **Studies of diquark in progress**

Di Pietro, Feo, Seiler, Stamatescu 2008

CANONICAL FORMALISM

$$\mathcal{Z}_{\rm C}(T,N) =$$

$$\frac{3}{2\pi}\int_0^{2\pi T} d(\mu_I/T) e^{-Ni\mu_I/T}\mathcal{Z}_{\rm GC}(T,i\mu_I)$$

Forcrand and Kratovchila, 2006,2007

Canonical partition a la Hasenfratz-Toussaint:

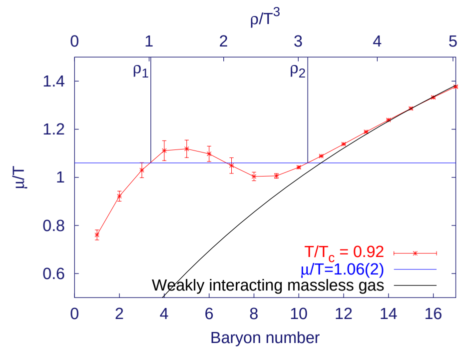
$$\frac{Z_C(B,\beta)}{Z_{GC}(\beta_0=\beta,\mu=i\mu_{I_0})}$$

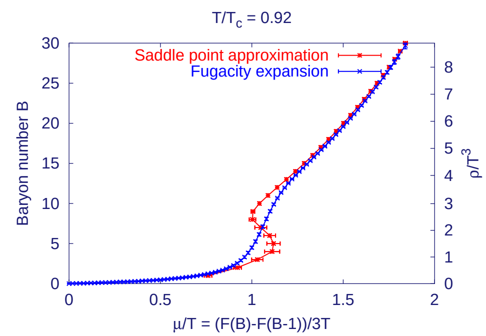
$$= \langle \frac{1}{2\pi}\int_{-\pi}^{\pi} d\left(\frac{\mu_I}{T}\right)\ e^{-i3B\frac{\mu_I}{T}}\frac{\det(U;i\mu_I)}{\det(U;i\mu_{I_0})}\rangle_{\beta_0,i\mu_{I_0}}$$

$$\equiv \langle \frac{\hat{Z}_C(U;B)}{\det(U;i\mu_{I_0})}\rangle_{\beta_0,i\mu_{I_0}}$$

Results for $N_f = 4$:

- **First order line-coexistence region**
- **Critical μ and Critical densities $\longrightarrow$**

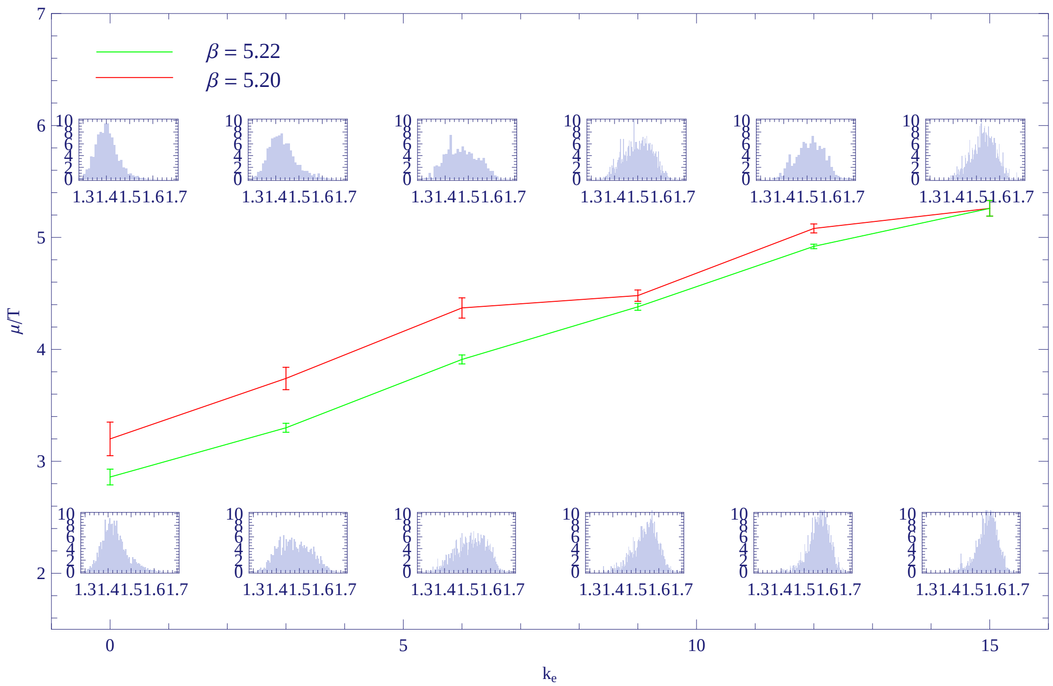




CANONICAL APPROACH II : HISTOGRAMS
Proposed by Ejiri,2007

Alternative search for a critical point in $N_f = 2$ QCD, large masses

Alexandru, Li, Liu, 2007 :

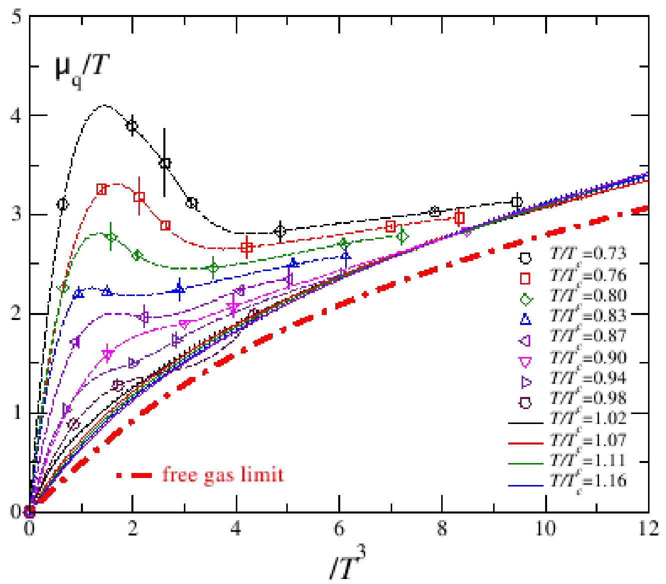


CANONICAL APPROACH III : Grand Partition Function via a Taylor Expansion.

Ejiri, 2007,2008

$$\frac{\mathcal{Z}_{\rm GC}(T,\mu_q)}{\mathcal{Z}_{\rm GC}(T,0)} \equiv \left\langle e^{\left[N_{\rm f} N_t V \sum_{n=1}^{\infty} D_n \left(\frac{\mu_q}{T}\right)^n\right]} \right\rangle_{(T,\mu_q=0)},$$

$$\frac{\mathcal{Z}_{\rm C}(T,\bar\rho V)}{\mathcal{Z}_{\rm GC}(T,0)} \text{ at large V} \approx \left\langle \exp\left[V\left(N_{\rm f} N_t \sum_{n=1}^{\infty} D_n z_0^n - \bar\rho z_0\right)\right] e^{-i\alpha/2} \sqrt{\frac{1}{V|D''(z_0)|}} \right\rangle_{(T,\mu=0)}$$



Results for $N_f = 2$:

- **Qualitative change at $T/T_c \simeq 0.8$**
- **Indication of First Order Line**

Ejiri 2008

DENSITY OF STATES

Luo, Azcoiti et al,, Ambjorn et al., Anagnostopoulos and J. Nishimura,

$< O >= \int d\phi \, \langle O f(U) \rangle_\phi \, \rho(\phi) \Big/ \int d\phi \, \langle f(U) \rangle_\phi \, \rho(\phi)$, ϕ **fixed**

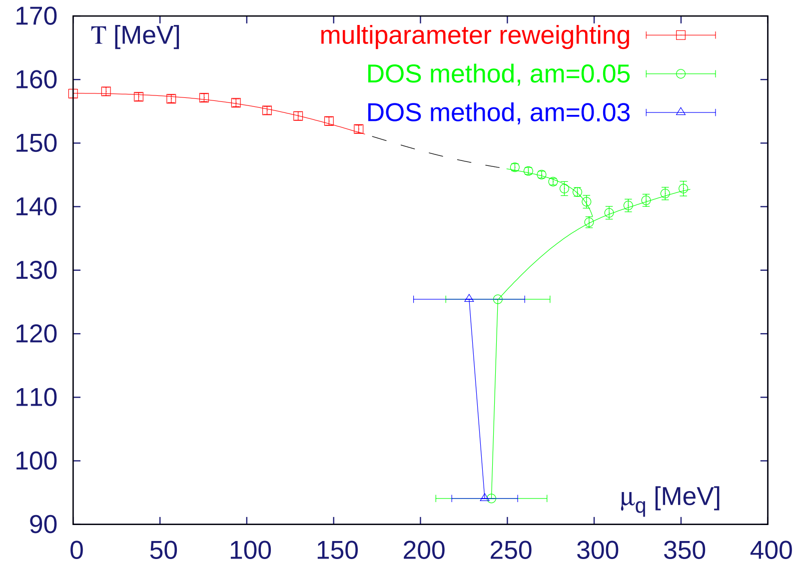
Density of states – ρ –
<u>constrained</u> partition function:

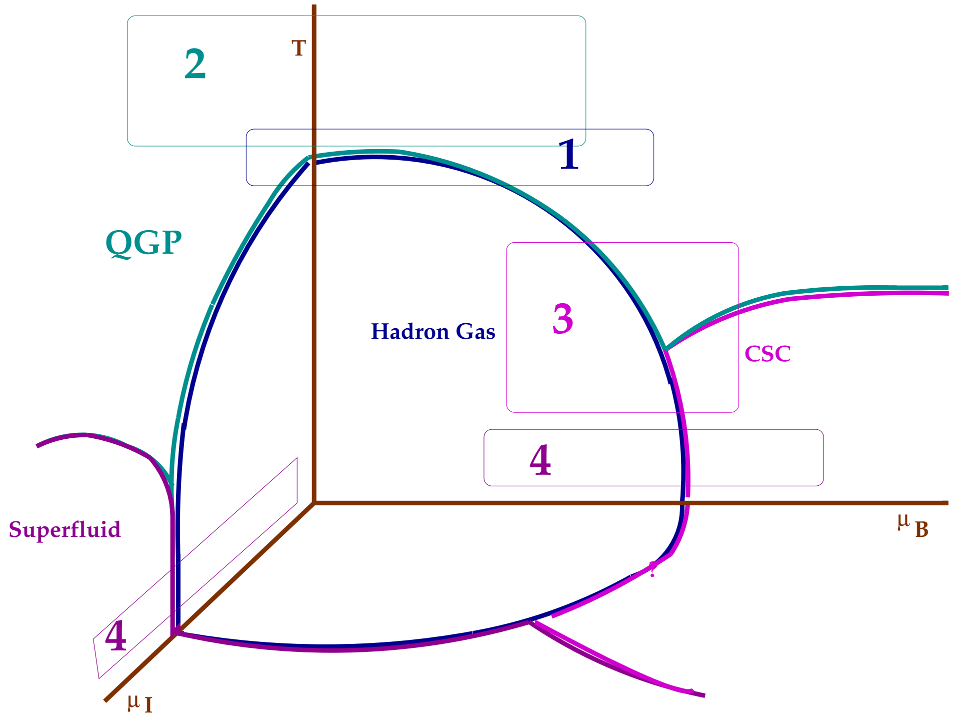
$\rho(x) = \int \mathcal{D}U \, g(U) \, \delta(\phi - x).$

Results for $N_f = 4$

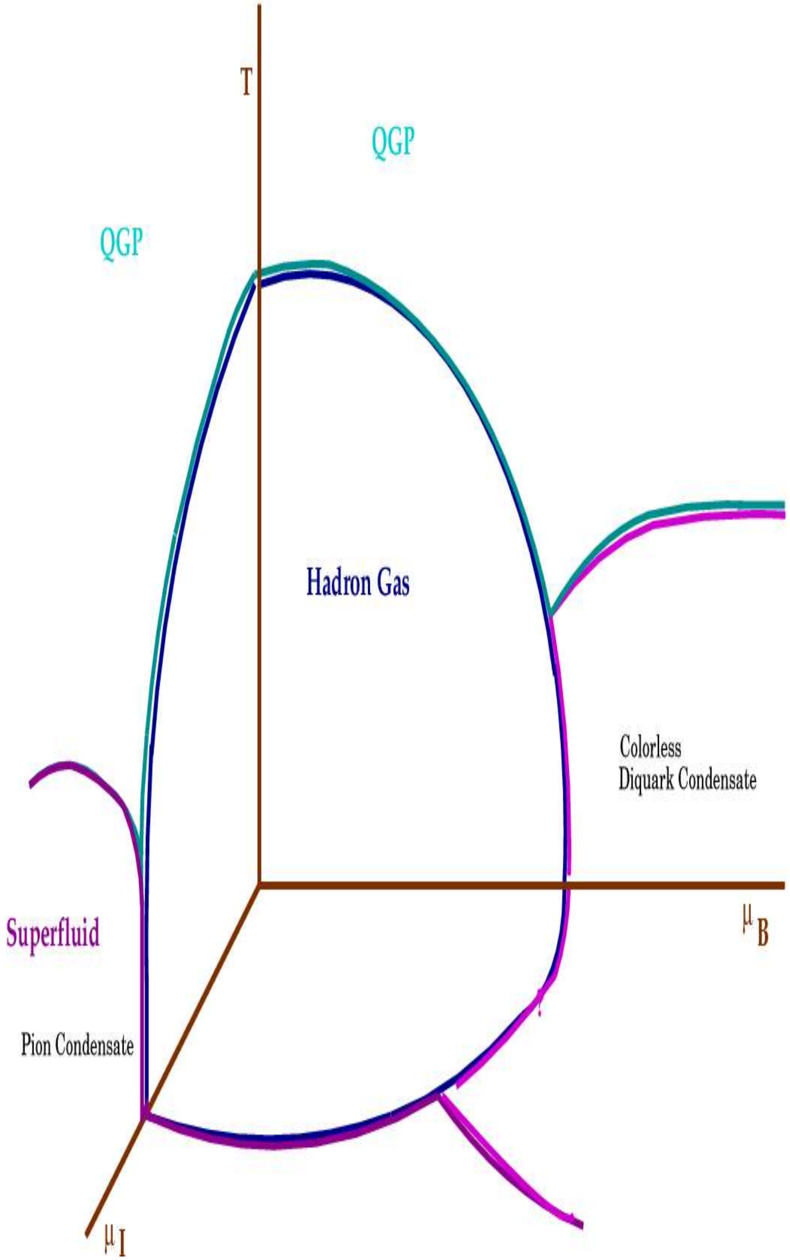
- **Signal of two phase transition lines**
- **Indication for a triple point**

Fodor, Katz, Schmidt 2007





4. COLD AND DENSE MATTER



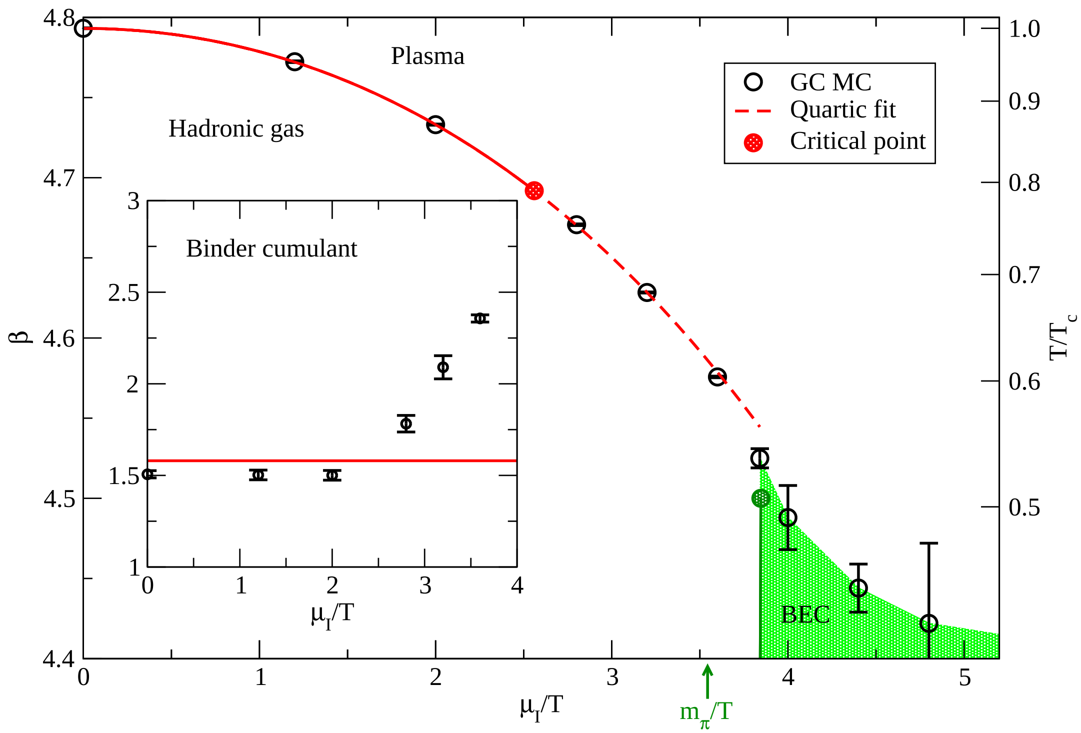
TWO COLOR QCD

- **Symmetry :** $qq \longleftrightarrow \bar{q}q$
- **Symmetry :** $\mu_B \longleftrightarrow \mu_I$
- **At T=0** $\mu_B^c = \mu_I^c = m_\pi/2$
- **Diquark Condensate for** $\mu_B > \mu_B^c$
- **Pion Condensate for** $\mu_B > \mu_I^c$
- **No sign problem**

QCD at $\mu_I \neq 0$ and TWO COLOR QCD at $\mu_B \neq 0$
No sign problem, many studies!

- Low Temperature, Fermionic Sector
  - Strong coupling analysis [Dagotto, Karsch, Moreo,Wollf, 1986]
  - Laboratory for diquark condensation [Alford,Rajagopal and Wilczek; Rapp, Schäfer, Shuryak and Velkovsky, 1998]
  - Numerical studies of symmetries and spectrum [Hands, Kogut, Morrison, MpL 1998]
  - Lattice studies of diquark condensation [Hands, Kogut, Morrison, Sinclair;1998; Aloisio, Azcoiti, di Carlo, Galante, 1999]
  - RMT analysis [Akemann, Splittorff, Toublan, Verbaarschot,2001]
  - Studies of the Dirac spectrum [Bittner, Markum, Pullirsch, MpL, 2001]
  - $\mu_C(T=0) = m_\pi/2$: amenable to χPT analysis.
    [Kogut, Stephanov, Toublan, Verbaarschot and Zhitnitsky, 2001; Kogut, Toublan, Sinclair 2002]
  - EoS and Gluon Condensate from χPT [Metlitsky,Toublan,Zhitnisky, 2005,2006]
  - Model studies of the vector sector [Lenaghan, Sannino, Splittorff 2002]
  - Vector spectroscopy on the lattice [Alles, D'Elia, Pepe, MpL, 2001; Muroya,Nakamura,Nonaka, 2003]
  - Test Bed for Imaginary μ [Cea, Cosmai, Giudice, D'Elia, Papa, 2006, 2007]
- High T , Gluon Sector finite μ transition from hadron gas to QGP, similar to the T=0 transition
  - Critical Line [Kogut, Sinclair, 2003-2008]
  - Topological Susceptibility [Alles, D'Elia, Pepe, MpL 2002]
  - Polyakov Loop [Muroya,Nakamura, 2004; Alles, D'Elia, MpL 2006]
  - Order Parameter for Cofinement [D'Elia Conradi 2007]
  - Critical Line [Forcrand, Stephanov, Wenger, 2007]
- Low T : Chiral Symmetry , Confinement, BEC Phase
  - Confinement/Deconfinenement,Screening [Hands, Kim, Skullerud 2006–2008]
  - Glueballs spectrum, screening [Lombardo, Paciello, Petrarca, Taglienti, 2007]
  - BEC Phase, interface BEC/QGP [Forcrand, Stephanov, Wenger, 2007]

THE PHASE DIAGRAM FOR A FINITE ISOSPIN DENSITY



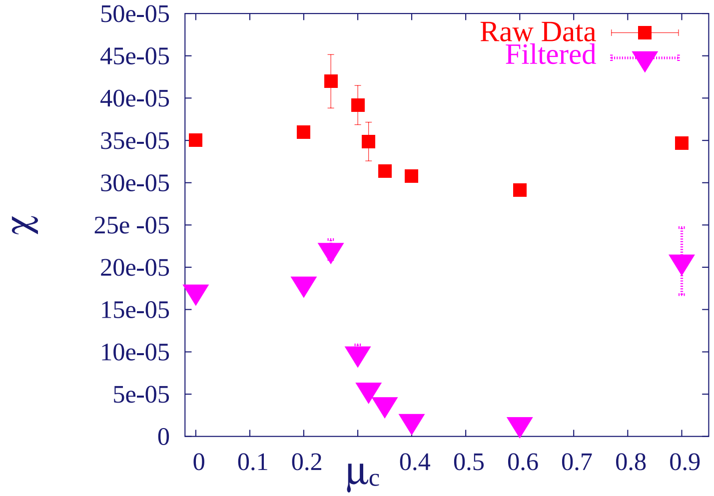
Forcrand, Stephanov, Wenger, 2007

GLUONIC OBSERVABLES IN THE BEC PHASE of QC$_2$D

0^{++} Glueball : lighter in the BEC phase

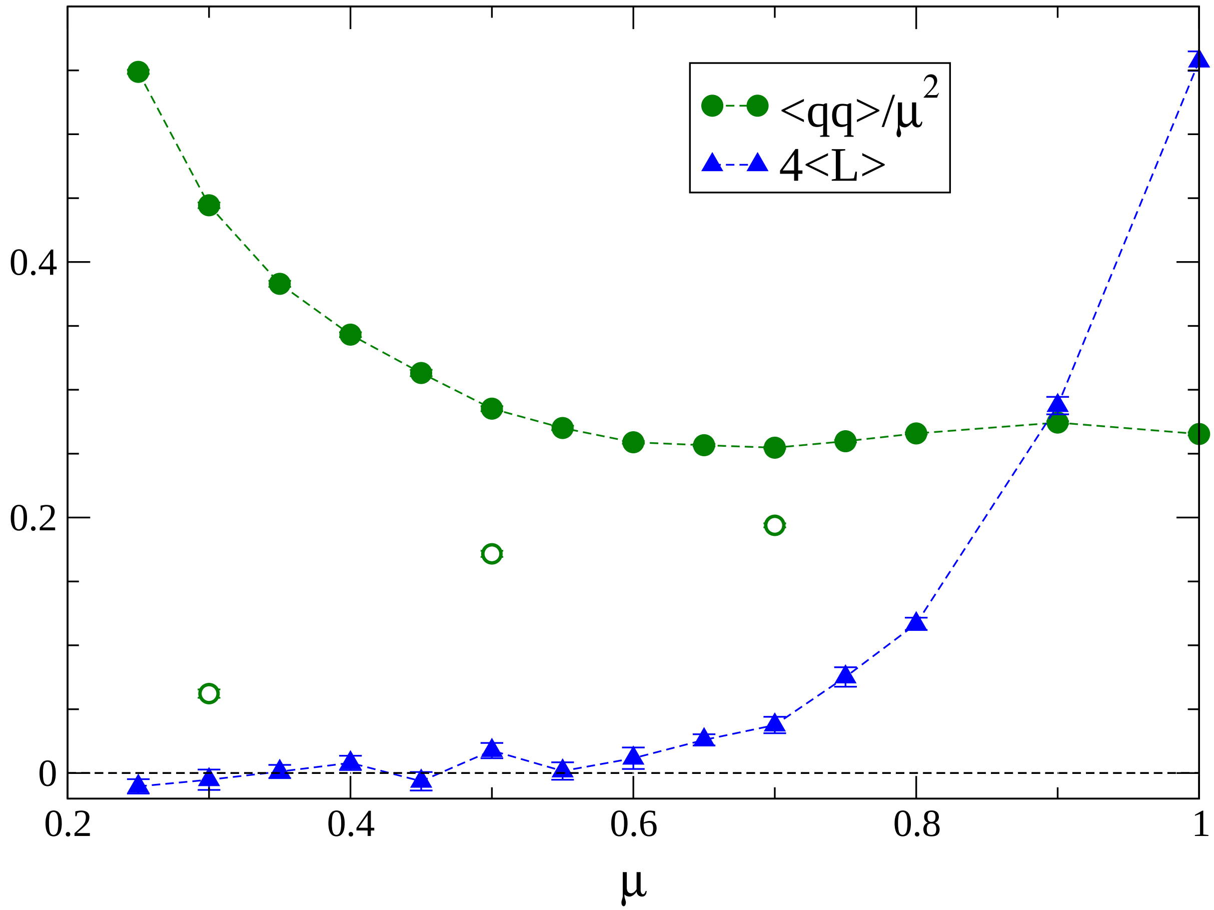
Susceptibility: $\chi = < P^2 > - < P >^2$ peaks at μ_c

Normal Phase	
m_π/m_ρ	m_0^{++}/m_ρ
0.40	1.07
0.42	1.26
BEC	
0.64	0.80
0.80	0.23



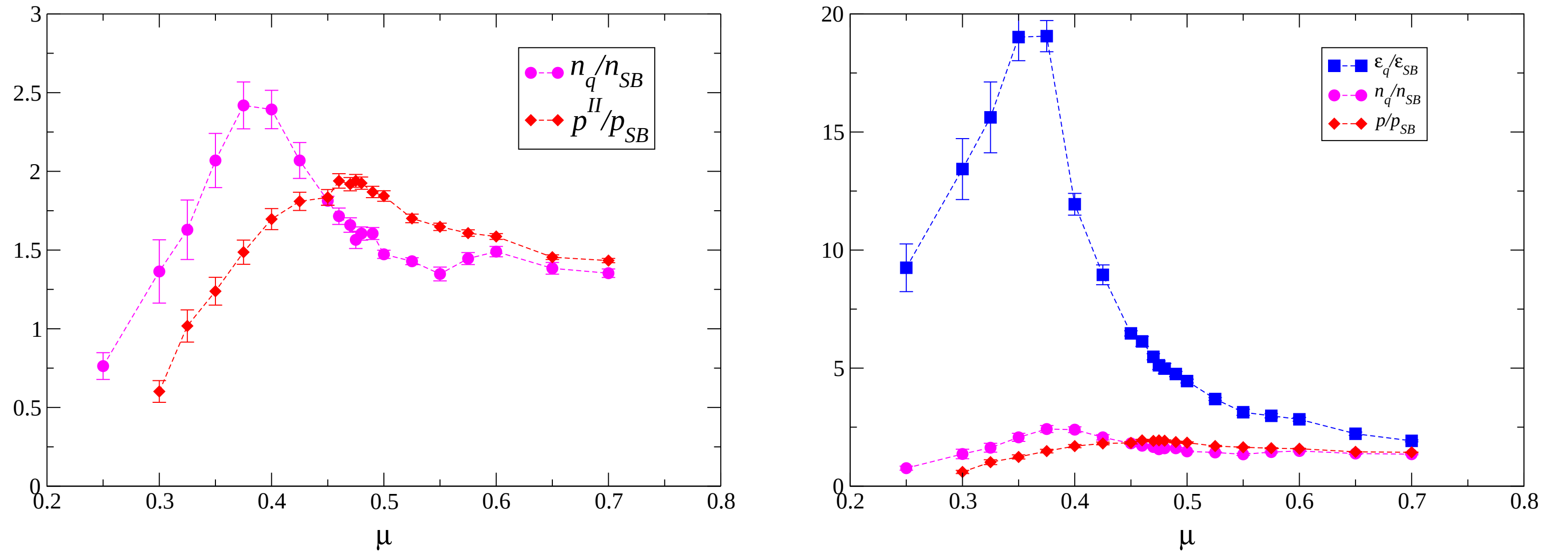
Lombardo,Petrarca,Paciello,Taglienti,2007

BEC AND CONFINEMENT: deconfinement deep in the BEC phase

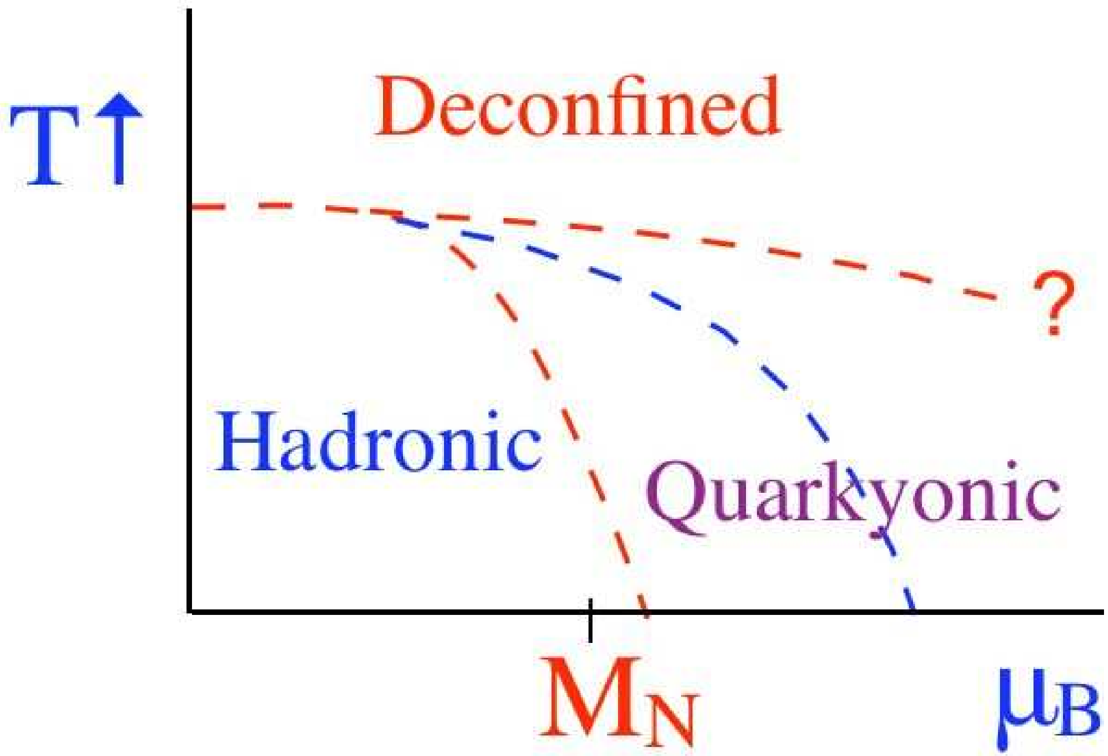


Hands, Kim, Skullerud 2006,2007

DECONFINEMENT IN THE BEC PHASE – BEC/BCS crossover



Hands, Kim, Skullerud 2008; preliminary, courtesy S. Hands



From Small $N_c = 2$ to
Large N_c : Proposed Phase Diagram
McLerran, Pisarski, 2007

QCD AT FINITE TEMPERATURE AND DENSITY : SUMMARY

T_c **known within 20 % .**
Fate of the critical endpoint of QCD???

Small μ **: Mature calculations. Critical line towards continuum**
Mass spectrum depends on μ_B

Critical line at lower temperature, larger μ **'bends down'**
First evidences of a triple point in the T,μ_B **plane**

Cold Phases studied at $\mu_I \neq 0$ **/ Q**C_2**D at** $\mu_B \neq 0$
Examples of interesting patterns confinement/chiral symmetry.

Critical region needs exact dynamics, models do not suffice
Qualitative difference critical surface QCD / NJL , HRG fails around T_c