Helmholtz International Summer School

***Dense Matter in Heavy Ion Collisions and Astrophysics***

**JINR Dubna Russia July 14–26 2008**

# *QCD at finite temperature* **and density** *on the lattice*

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# Thermodynamics Basics

## Thermodynamical Ensembles

- Microcanonical: E, N, V fixed
- Canonical: T, N, V fixed
- Grancanonical : T, $\mu$, V fixed

In a GrandCanonical Ensemble define:

$$\hat{\rho} = e^{-(H-\mu\hat{N})/T}$$
$$\mathcal{Z} = Tr\hat{\rho}$$

$\rho$ e $\mathcal{Z}$ determine the system's state:

$$< O >= TrO\hat{\rho}/\mathcal{Z}$$

$$\begin{aligned} P &= T\frac{\partial ln\mathcal{Z}}{\partial V} \\ N &= T\frac{\partial ln\mathcal{Z}}{\partial \mu} \\ S &= \frac{\partial Tln\mathcal{Z}}{\partial T} \\ E &= -PV + TS + \mu N \end{aligned}$$

*Bosons and fermions*

- 1 bosonic degree of freedom

$$\hat{H} = 1/2\omega a^\dagger a + 1/2\omega = \omega(\hat{N} + 1/2)$$

$$\begin{aligned} \mathcal{Z} &= Tre^{-\beta(H-\mu\hat{N})} = Tre^{-\beta(\omega-\mu)\hat{N})} \\ &= \sum_0^\infty Tre^{-\beta(\omega-\mu)n} = (1-e^{-\beta(\omega-\mu)n})^{-1} \end{aligned}$$

$$N = (e^{\beta(\omega-\mu)} - 1)^{-1}$$

- 1 fermionic degree of freedom

$$\hat{H} = 1/2\omega a^\dagger a - 1/2\omega = \omega(\hat{N} - 1/2)$$

$$\begin{aligned} \mathcal{Z} &= Tre^{-\beta(H-\mu\hat{N})} = Tre^{-\beta(\omega-\mu)\hat{N})} \\ &= \sum_0^1 Tre^{-\beta(\omega-\mu)n} \end{aligned}$$

$$N = (e^{\beta(\omega-\mu)} + 1)^{-1}$$

*Example I : Relativistic free fermionic gas*

Non interacting particles: $\mathcal{Z}$

$$\mathcal{Z} = \prod \mathcal{Z}^{\rangle}$$

$$ln\mathcal{Z} = d^3x \int d^3p(ln(e^{-\beta(\omega-\mu)} + 1) + ln(e^{-\beta(\omega+\mu)} + 1))$$

Note! Relativistic particles!

- Fermions $(+\mu\ )$ and antifermions $(-\mu)$.
- $\omega = \sqrt{(p^2 + m^2)}$.
- Relativistic chemical potential

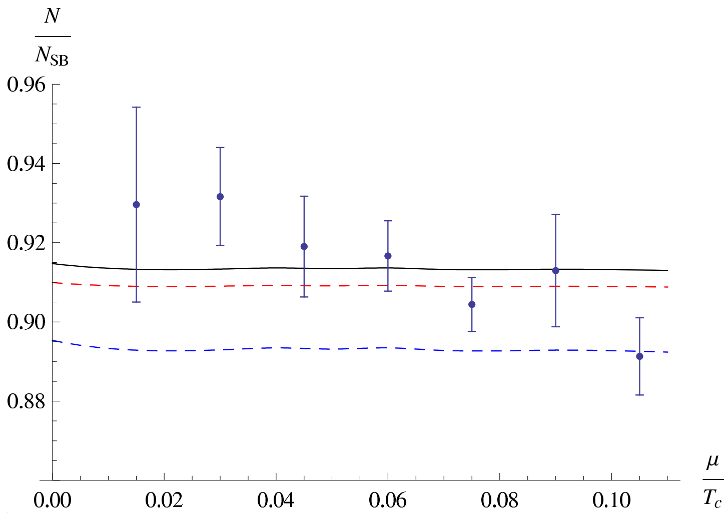
$\mathcal{Z}$ can be exactly computed in the chiral limit $m = 0$:

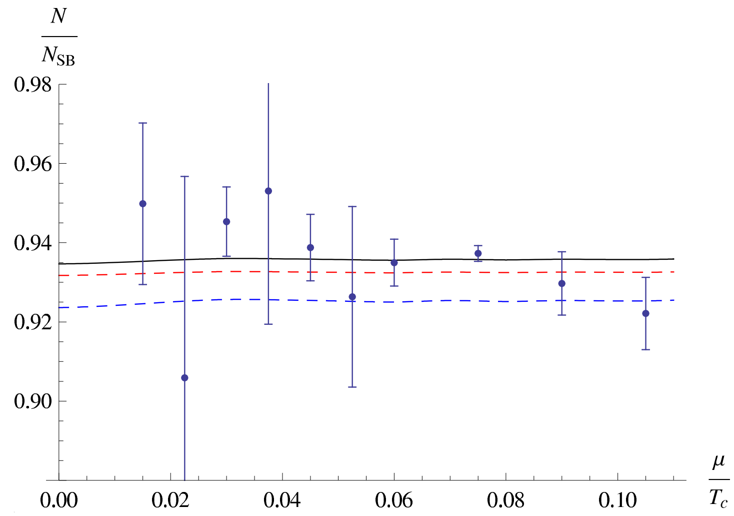
$$\begin{aligned} TlnZ &= V\mu^4/(12\pi^2) + V\mu^2T^2/6 + 7V\pi^2T^4/180 \\ n &= 4V\mu^3/12\pi^2 + 2V\mu T^2/6 \end{aligned}$$

Ideally, this should be the state of the quarks at very high temperatures!

## APPROACH TO FREE FIELD : ANALYTIC RESULTS VS. LATTICE DATA

**Based on A. Vuorinen, 2004**




$T = 1.5T_c$ $T = 3.5T_c$

**D'Elia, Di Renzo, MpL, Vuorinen, in progress**

## *More observables : Response Functions*

**Susceptibilities**

$$\chi_{j_u,j_d}(T) = \left. \frac{\partial^{(j_u+j_d)} p(T,\mu_u,\mu_d)}{\partial \mu_u^{j_u}\, \partial \mu_d^{j_d}} \right|_{\mu_u=\mu_d=0} .$$

Test for fluctuations.

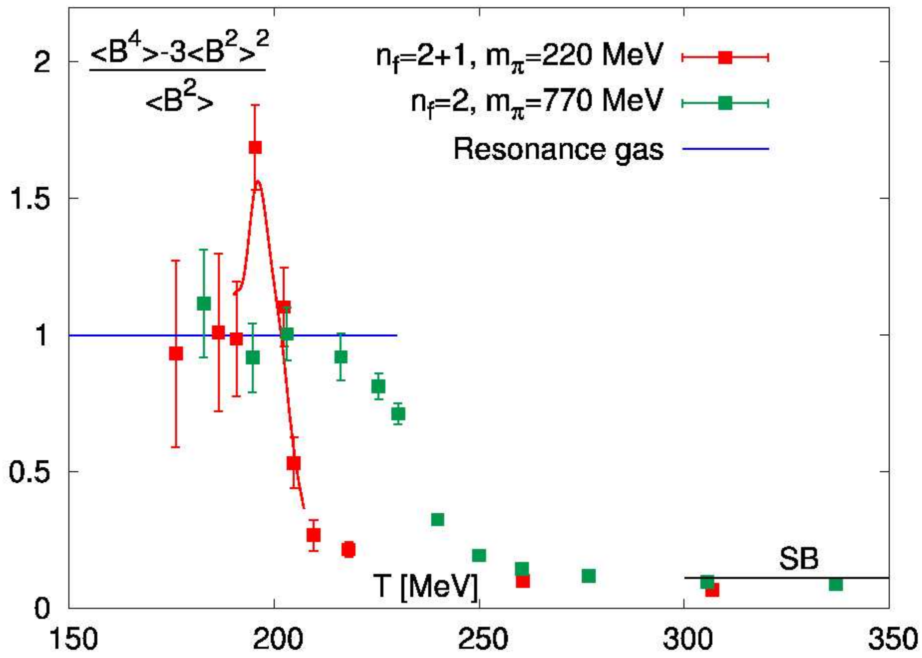
Taylor coefficients of the excess pressure:

$$\Delta p(T,\mu_u,\mu_d) \equiv p(T,\mu_u,\mu_d) - p(T,\mu_u=0,\mu_d=0)$$

$$\Delta p(T,\mu_u,\mu_d) = \sum_{j_u,j_d} \chi_{j_u,j_d}(T) \frac{\mu_u^{j_u}}{j_u!} \frac{\mu_d^{j_d}}{j_d!} ,$$

containing information about baryon density effects in the EoS.

## Susceptibilities Towards Free Field




RBC-Bielefeld Collaboration, 2008

*Example II : Interacting Fermions*

3 d Gross Neveu Model

$$L = \bar{\psi}(\not\partial + m)\psi - g^2/N_f[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$$

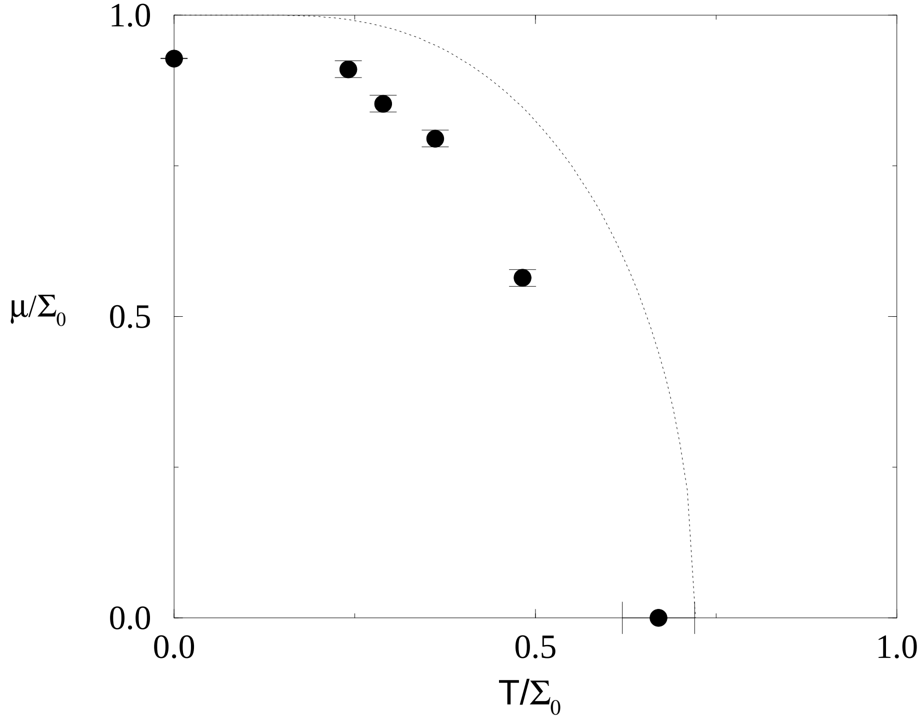
Global Chiral Invariance

$$\begin{aligned} \psi_i &\rightarrow e^{i\alpha\gamma_5}\psi_i \\ \bar{\psi}_i &\rightarrow \bar{\psi}_i e^{i\alpha\gamma_5} \end{aligned}$$

Basic Properties

- A $T = \mu = 0$ and $g$ 'large', spontaneous symmetry breaking, Goldstone mechanism.
- Rich particle spectrum
- Amenable to a lattice study at $T, \mu \neq 0$ !!!!

*Phase diagram of the 3d Gross Neveu model*




da S. Hands, 1998

Mean Field Solution vs Exact Lattice Results

- Grand Canonical Formalism at finite T, $\mu$
- Basic Observables : number density, susceptibilities
- Free Fermions : Exact Solution
- Simple model with interacting fermions : mean field solution
- Phase Diagram at nonzero T and $\mu$ of a purely fermionic model
- Simple calculations : can reproduce limiting behaviour and give generic information; in general inaccurate

*Lattice QCD*
=
*first principles calculations from the*
*QCD Lagrangian*

$$\mathcal{L} = \mathcal{L}_{YM} + \bar{\psi}(i\gamma_\mu D_\mu + m + \mu\gamma_0)\psi$$

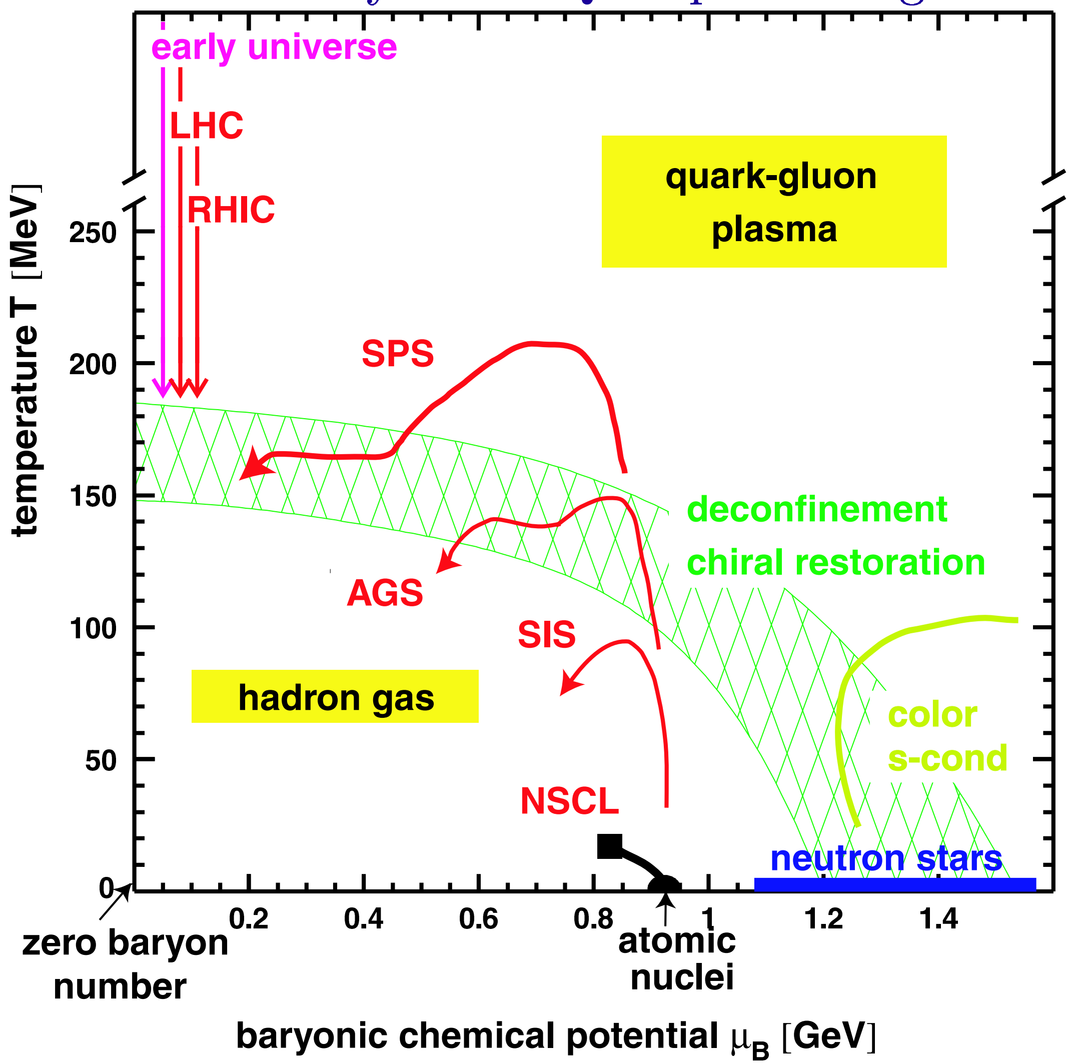
A vaste phase space to be explored:

*Real baryon chemical potential, temperature, isospin chemical potential.*
*And also: Imaginary chemical potential, number of color and number of flavor, bare masses*

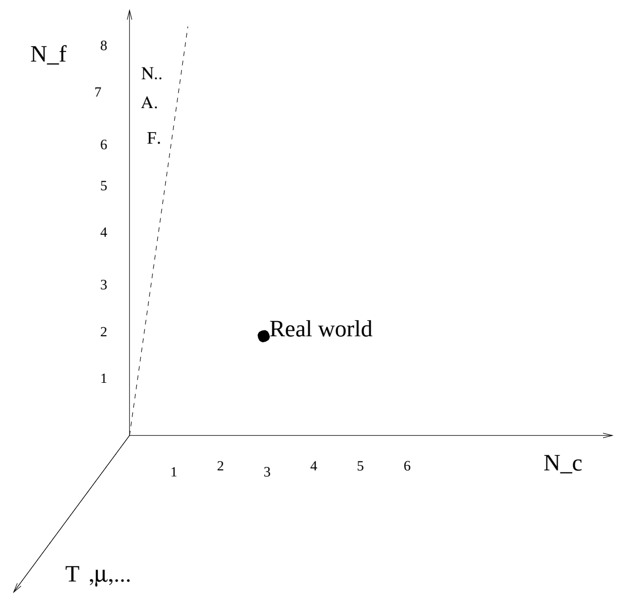
to address phenomenological issues as well as to study more theoretical questions

*Phenomenological challenges:*

ab initio study of the QCD phase diagram



Theoretical questions can (or, rather, should ?) be addressed in a larger phase space:



- *High T*
  - Chiral symmetry pattern : order disorder, light baryons
  - Deconfinement/screening: string breaking via recombination with light pairs
  - Instanton molecules
- *High $\mu$*
  - Chiral symmetry pattern : instability at the Fermi surface
  - Deconfinement/screening : string breaking via recombination with real particles
  - Instanton chains

Differences at high T and high $\mu$ in the *gauge dynamics* provide futher motivation to study nonzero $\mu$ on a lattice.

# *Lattice QCD at Finite Temperature and Density*

## *Lecture I*

## *I Formulation*

## *II Calculational Schemes*

*II.1 Effective Fermionic Models - Analytic approaches*

*II.2 Effective Gluonic Models - Numerical approaches*

## *III QCD at Finite Baryon Density: Methods*

*III.1 Derivatives*

*III.2 Reweighting*

*III.3 Expanded Reweighting*

*III.4 Imaginary Chemical Potential*

## *IV Results - Discussion : Tomorrow's lecture*

## I Formulation

### GranCanonical Formalism and Path Intergral

⋆ Chemical Potential for Conserved Charge $\hat{N}$

$$\begin{aligned} \hat{\rho} &= e^{-(H-\mu\hat{N})/T} \\ \mathcal{Z} &= Tr\hat{\rho} = \int d\phi d\psi e^{-S(\phi,\psi)} \end{aligned} \tag{1}$$

⋆ Temperature: Reciprocal of Imaginary Time

$$S(\phi,\psi) = \int_0^{1/T} dt \int d^d x \mathcal{L}(\phi,\psi)$$

*with boundary conditions for fermions and bosons*

$$\begin{aligned} \phi(t=0,\vec{x}) &= \phi(t=1/T,\vec{x}) \\ \psi(t=0,\vec{x}) &= -\psi(t=1/T,\vec{x}) \end{aligned}$$

⋆ *$\mathcal{Z}$ = partition function of a statistical system in d+1 dimension, where T is the reciprocal of the imaginary time.*

⋆ *Thermodynamics and spectrum properties are treated on the same footing.*

## Lattice QCD at T,$\mu \neq 0$

⋆ **Temperature**: as in the continuum $T = 1/N_t * a$

⋆ **Density**

In the continuum: $L(\mu) = L_0 + \mu J_0 \;\; J_0 = \bar{\psi}\gamma_0\psi \rightarrow N - \bar{N} = \int J_0$

On the lattice:

$$L(\mu) = \bar{\psi}_x \gamma_0 e^{\mu a} \psi_{x+\hat{0}} - \bar{\psi}_{x+\hat{0}} \gamma_0 e^{-\mu a} \psi_x$$
$$J_0 = -\partial_\mu L = \bar{\psi}_x \gamma_0 e^{\mu a} \psi_{x+\hat{0}} + \bar{\psi}_{x+\hat{0}} \gamma_0 e^{-\mu a} \psi_x$$

*Time Forward propagation enhanced by* $e^{\mu a}$

*Time Backward propagation discouraged by* $e^{-\mu a}$

*Particles–antiparticles asymmetry!*

# More on The Lattice (digression)

Path integral is a regulated on a four dimensional lattice

- *Gauge fields: link variables $U_\mu(x)$ for parallel trasport of field $\mathcal{A}$ from $x$ to $x+\hat{\mu}a$*

  $x \longrightarrow x+\hat{\mu}a$
  $U_\mu(x)$

$$U_{x,\mu} = \mathrm{P}\exp\left( ig \int_x^{x+\hat{\mu}a} dx^\mu \, A_\mu(x) \right)$$

- *Gauge invariants and Yang Mill Action:*

$$\begin{aligned} W^{(1,1)}_{n,\mu\nu} &= 1 - \frac{1}{3}\mathrm{Re}\,\square_{n,\mu\nu} \\ &= \mathrm{Re}\ \mathrm{Tr}\ U_{n,\mu} U_{n+\hat{\mu},\nu} U^\dagger_{n+\hat{\nu},\mu} U^\dagger_{n,\nu} \\ &= \frac{g^2 a^4}{2} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{O}(a^6) \end{aligned}$$

- Lattice Yang Mill Action

$$\beta S_G = \beta \sum_{\substack{n \\ 0 \le \mu < \nu \le 3}} W^{(1,1)}_{n,\mu\nu} \quad \rightarrow \quad \int d^4x \, \mathcal{L}_{YM} \; + \; \mathcal{O}(a^2)$$

$\beta = 6/g(a)^2$.

- Lattice fermions

  *Simply:*

  $\psi(x) \rightarrow \psi(n)$ *!*

  $\partial_\mu \psi_f(x) = (\psi(n+\hat{\mu}) - \psi_(n-\hat{\mu}))/2a,$

  *[doubling problem and chiral symmetry: staggered fermions, Wilson fermions, chiral fermions]*

$\mu\bar{\psi}\gamma_0\psi$ on the lattice
*Naive discretisation:*

$$\begin{aligned}
\phi_{LATT}(n_1, n_2, n_3, n_4) &= \phi(n_1 a, n_2 a, n_3 a, n_4 a) \\
\Delta_\mu \phi_{LATT}(n_1, n_2, n_3, n_4) &= \\
(\phi(n_1 a, (n_\mu + 1)a, n_3 a, n_4 a) &- \phi(n_1 a, (n_\mu a, n_3 a, n_4 a))/2a
\end{aligned}$$

*Problems with free fermions: the internal energy* $\epsilon$ *diverges in the continuum limit* $a \rightarrow 0$

$$L = \bar{\psi}_x \gamma_\mu \psi_{x+\mu a} + m\bar{\psi}_x \psi_x + \mu\bar{\psi}_x \gamma_0 \psi_x$$

$$\epsilon \propto \frac{\mu^2}{a^2} \rightarrow_{a \rightarrow 0} \infty$$

Elegant solution : $\mu$ is an external field in the 0th direction

$$\bar{\psi}\gamma_\mu A_\mu \psi \longleftrightarrow i\mu\bar{\psi}\gamma_0\psi$$

- *External fields live on lattice link. (cfr. electrodynamics:* $A \rightarrow \theta = e^(iA)$ *)*

- $L(\mu) = \bar{\psi}_x \gamma_0 e^{\mu a} \psi_{x+\hat{0}} - \bar{\psi}_{x+\hat{0}} \gamma_0 e^{-\mu a} \psi_x$

- *Simple intepretation*
    - *Time Forward propagation enhanced by* $e^{\mu a}$
    - *Time Backward propagation discouraged by* $e^{-\mu a}$

  *Particles–antiparticle asymmetry!*

- $\lim_{a \to 0} J_0 = -\partial_\mu L = \bar{\psi}_x \gamma_0 e^{\mu a} \psi_{x+\hat{0}} + \bar{\psi}_{x+\hat{0}} \gamma_0 e^{-\mu a} \psi_x = \mu \bar{\psi} \gamma_0 \psi$

*Via an unitary transformation for the field*

$$L(\mu) = L(0)$$

*+ boundary conditions*
*Explicit dependence on fugacity*

## Lattice QCD Thermodynamics at a Glance

$$
\begin{aligned}
\mathcal{L}_{QCD} &= 6/g^2 \mathrm{Tr}\, U_{n,\mu} U_{n+\hat{\mu},\nu} U^{\dagger}_{n+\hat{\nu},\mu} U^{\dagger}_{n,\nu} \\
&+ \sum_{i=1}^{3} (\bar{\psi}_x \gamma_i U_i(x) \psi_{x+\hat{i}} - \bar{\psi}_{x+\hat{i}} \gamma_i U_i^{\dagger}(x) \psi_x) \\
&+ \bar{\psi}_x \gamma_0 e^{\mu} U_0(x) \psi_{x+\hat{0}} - \bar{\psi}_{x+\hat{0}} \gamma_0 e^{-\mu} U_0^{\dagger}(x) \psi_x \\
&+ m \bar{\psi} \psi
\end{aligned}
$$

Imaginary time

and

Inverse
Temperature

d-dimensional space

## II Computational Schemes

$$\mathcal{Z} = \int d\phi d\psi e^{-S(\phi,\psi)}; (\phi, \psi) = \int_0^{1/T} dt \int d^d x \mathcal{L}(\phi, \psi)$$
$$\mathcal{L}_{QCD} = \mathcal{L}_{YM} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

*Two options:*

1. *Integrate out gluons first:*

$$\mathcal{Z}(T, \mu, \bar{\psi}, \psi, U) \simeq \mathcal{Z}(T, \mu, \bar{\psi}, \psi) \rightarrow$$

   *effective approximate fermion models*

2. *Integrate out fermions exactly as $S$ is bilinear in $\psi, \bar{\psi}$*
   $S = S_{YM}(U) + \bar{\psi}M(U)\psi$

$$\mathcal{Z}(T, \mu, U) = \int dU e^{-(S_{YM}(U) - log(det M))}$$

$$\mathcal{Z}(T,\mu,U) = \int dU e^{-(S_{YM}(U) - log(detM))}$$

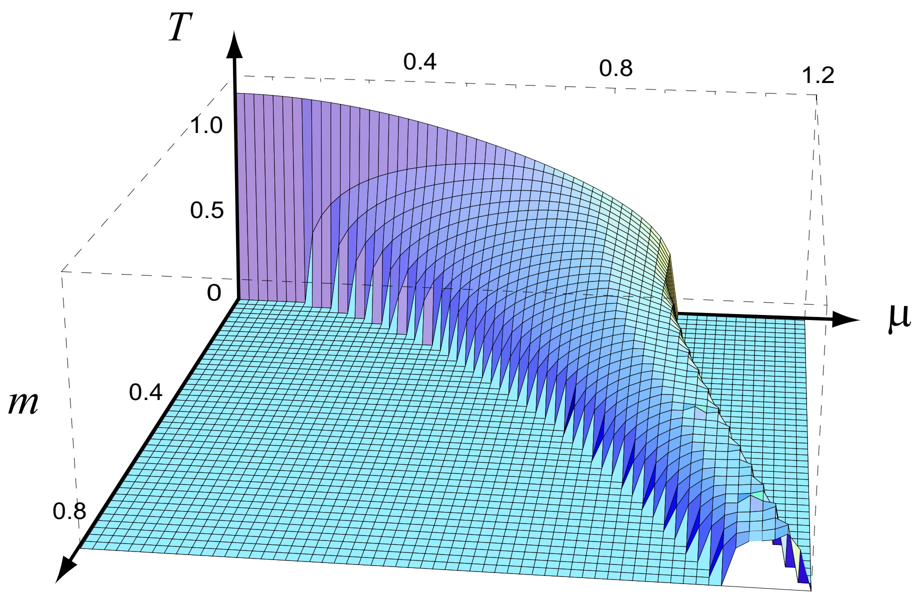
*starting point for numerical calculations*

*Lattice Strong Coupling Calculations:*

*Starting point : Yang Mill Action decouples*

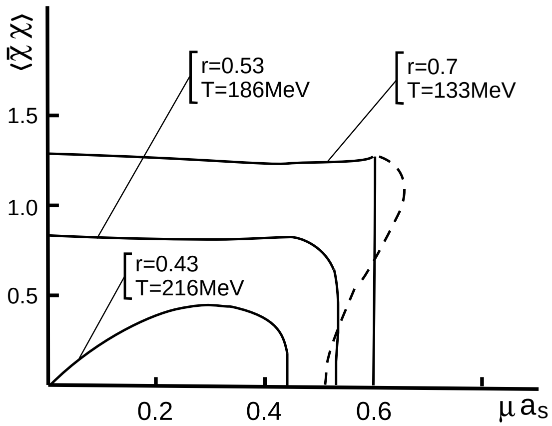
*at* $g = \infty \rightarrow \int dU$ *exact*

*Work on two color QCD by Y. Nishida, K. Fukushima, and T. Hatsuda*



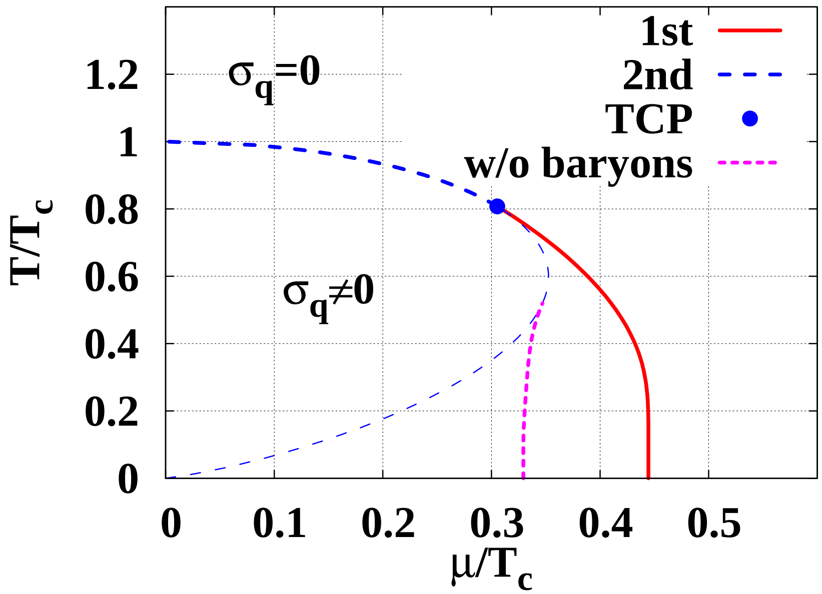

*Initial studies by F. Karsch, U. Wolff, and others.*

*Work from the 80's on three color :*
*B. Petersson and collaborators, P. Damgaard, F. Karsch and many others.*




$$\mathcal{L} = G\frac{1}{8N_c^2}\left[(\bar{\psi}\tau^-\psi)^2 + (\bar{\psi}\tau^-\gamma_5\psi)^2\right]$$

*CSC/CFL Phase from Strong Coupling?*




**The phase diagram of QCD at strong coupling** *Kawamoto et al, 2005.*

## II.2 Importance Sampling and The Positivity Issue

$$\mathcal{Z}(T,\mu,U) = \int dU e^{-(S_{YM}(U) - log(\det M))}$$

$$M^{\dagger}(\mu_B) = -M(-\mu_B)$$

$\det M > 0 \rightarrow$ *Importance Sampling*

- $\mu = 0 \rightarrow \det M$ *is real*
  *Particles-antiparticles symmetry*
- *Imaginary* $\mu \neq 0 \rightarrow \det M$ *is real*
  *(Real) Particles-antiparticles symmetry*
- *Real* $\mu \neq 0$ *Particles-antiparticles asymmetry*
  $\rightarrow \det M$ *is complex in QCD*
- QCD with a real baryon chemical potential:
  use information from the accessible region

$$Re\mu = 0, Im\mu \neq 0$$

## III QCD AT FINITE BARYON DENSITY-METHODS

QCD and a Complex $\mu_B$

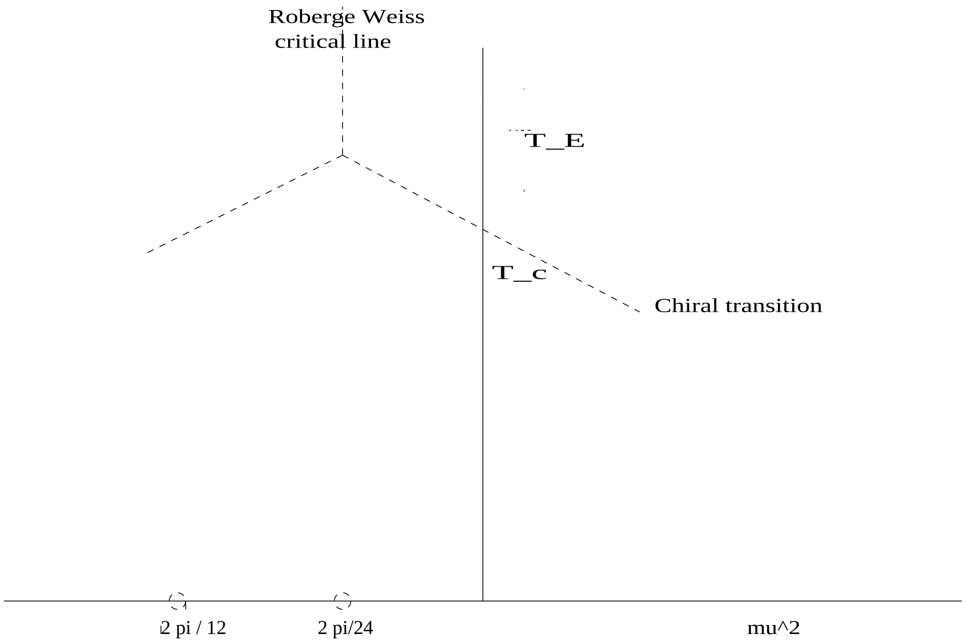
*A map: complex* $\mu \rightarrow$ *complex* $\mu^2$.

$\mathcal{Z}(\mu^2)$ *is real valued for real* $\mu^2$

Analogy with statistical models in external fields

The Phase Diagram in the T, $\mu_B^2$ Plane *Region accessible to simulations:* $\mu^2$ *real* $\leq 0$.

- $\mu = 0$ *Derivatives, Reweighting, Expanded reweighting*
- $\mu^2 \leq 0$ *Imaginary chemical potential*


Roberge Weiss
critical line
T_E
T_c
Chiral transition
2 pi / 12
2 pi/24
mu^2

## The Roberge and Weiss analysis

$$\mathcal{Z}(\nu) = Tre^{-\beta H + i\beta\nu N} = e^{-\beta H + i\theta N}$$

1. *$\mathcal{Z}(\theta)$ has a periodicity $2\pi$ anyway.*
2. *If only color singlet are allowed, then $N = 0$ mod $(N_c)$ and periodicity becomes $2\pi/N_c$*

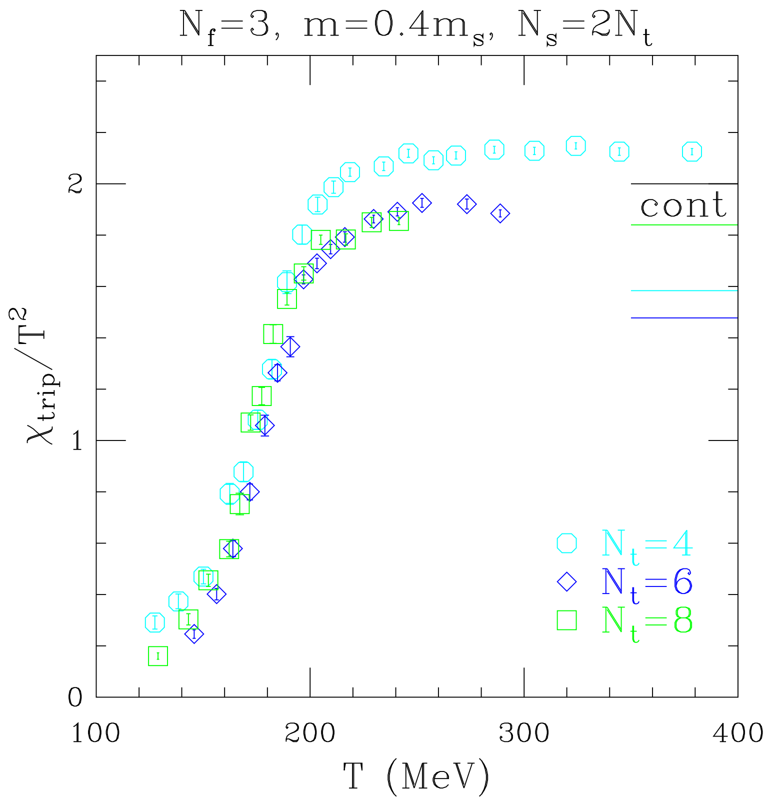
*However (Roberge Weiss (1986))*

*$\mathcal{Z}(\theta)$ has always period $2\pi/N_c$*

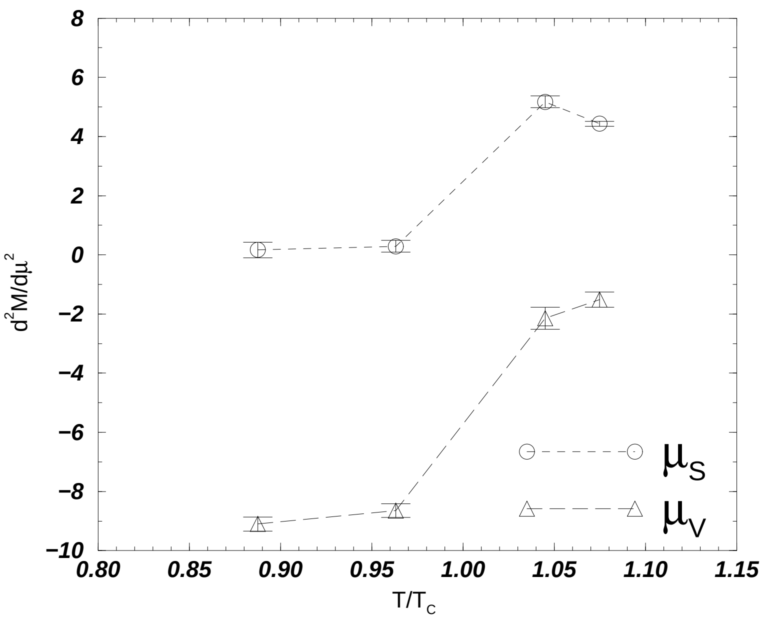
*The imaginary chemical potential changes the preferred vacuum for the Polyakov loop from $\phi_P = 0$ to one of its $Z_3$ images*

*The strong coupling analysis shows that periodicity is smooth at low temperature, and p.t. theory suggests that it is sharp at high $T$*

## III.1 Derivatives at $\mu = 0.0$




*MILC*




*QCD-TARO*

## III.2 Reweighting from $\mu = 0$

Ian Barbour's proposal, *or*

*The Glasgow method:*

$\mathcal{Z}(\mu)$ *can be computed using simulations at* $\mu = 0$*:*

$$\mathcal{Z} = \left\langle \frac{|M(\mu)|}{|M(\mu = 0)|} \right\rangle_{\mu=0}$$

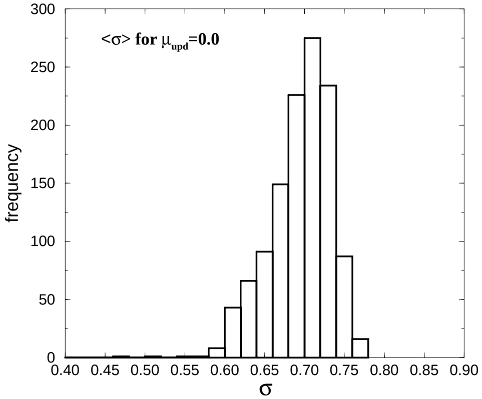
$$\mathcal{Z} = \frac{\int [dU][dU^\dagger]|M(\mu)|e^{-S_g[U,U^\dagger]}}{\int [dU][dU^\dagger]|M(\mu = 0)|e^{-S_g[U,U^\dagger]}}$$

*Neeeds overlap between*

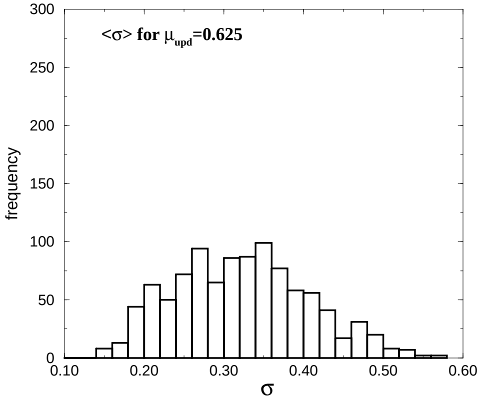
simulation ensemble *at* $\mu = 0$

target ensemble *at* $\mu \neq 0$

*At* $T = 0$ *the Glasgow procedure fails because of a poor overlap.*



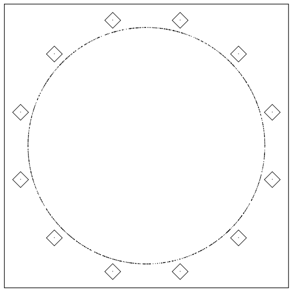

Broken phase



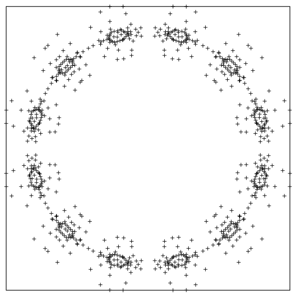

Symm. Phase

*Distributions of the* $<\sigma>=<\bar{\psi}\psi>$ *fields*

Example of successful reweighting at $\mu \neq 0$ : no conceptual problems
*1-dim* $SU(3)$ *can be exactly solved (Bilic, Demeterfi, 1988)*



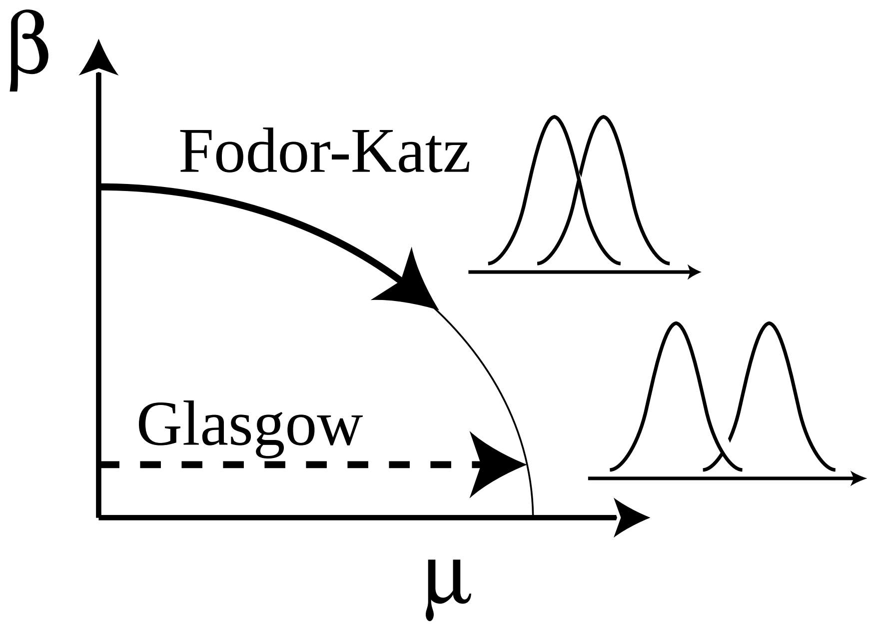
Exact Z's zeros in the Imaginary, Real $\mu$ plane (diamonds) and the cloud of zeros obtained from reweighting with a very poor statistics



Z's zeros from an high statistics reweighting : OK

Z.Fodor and F.Katz's proposal *:*

Multiparameter reweighting *use fluctuations around* $T_c$ *to explore the critical region*




*Picture taken from*

S. Muroya, A. Nakamura, C. Nonaka and T. Takaishi

## III.3 Taylor Expanded Reweighting

*Bielefeld-Swansea*

*Taylor expansion of the reweighting factor as a power series in* $\lambda = \mu/T$*, and similarly for any operator.*

*Computationally convenient: simplifies calculation of determinant.*

*Expectation values are then given by*

$$\langle \mathcal{O} \rangle_{(\beta,\mu)} = \frac{\langle (\mathcal{O}_0 + \mathcal{O}_1 \lambda + \mathcal{O}_2 \lambda^2 + \ldots) \exp(\mathcal{R}_1 \lambda + \mathcal{R}_2 \lambda^2 + \ldots - \Delta S_g) \rangle_{\lambda=0,\beta_0}}{\langle \exp(\mathcal{R}_1 \lambda + \mathcal{R}_2 \lambda^2 + \ldots - \Delta S_g) \rangle_{\lambda=0,\beta_0}}.$$

## III.IV Imaginary Baryon Chemical Potential

*Bridge between Canonical and GranCanonical ensembles*

*A. Hasenfratz, D. Toussaint, M. Alford, A. Kapustin, F. Wilczek, ...*

$$\mathcal{Z}(\mu) = \sum \mathcal{Z}(\mathcal{N}) e^{\beta \mu_B N)}$$

$$\mathcal{Z}(\rangle \nu) = Tr e^{-\beta(H - i\nu_B N)}$$

$$\mathcal{Z}(\mathcal{N}) = \frac{\beta}{2\pi} \int_0^{2\pi/\beta} d\nu \mathcal{Z}(i\nu) e^{-i\beta\nu N}$$

*Idea: $\mu = 0$ fluctuations allow the exploration of $N_b \neq 0$ hence tell us about $\mu \neq 0$*

*Note: same argument suggests Glasgow reweighting might work*

*Practical Strategy:*

$\mathcal{Z}(\mu, T)$ *must be*

1. *analitic*

2. *non trivial*

*Rule of thumb:*

$\chi(T, \mu) = \partial\rho(\mu, T)/\partial\mu = \partial^2 log Z(\mu, T)/\partial\mu^2 > 0$

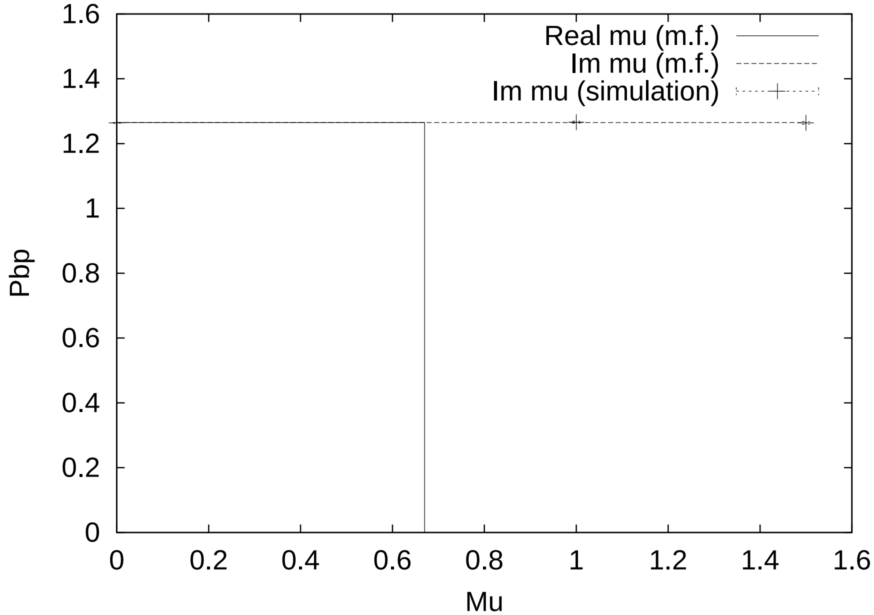
## $\mu$ Imm.: Lessons from $g = \infty$

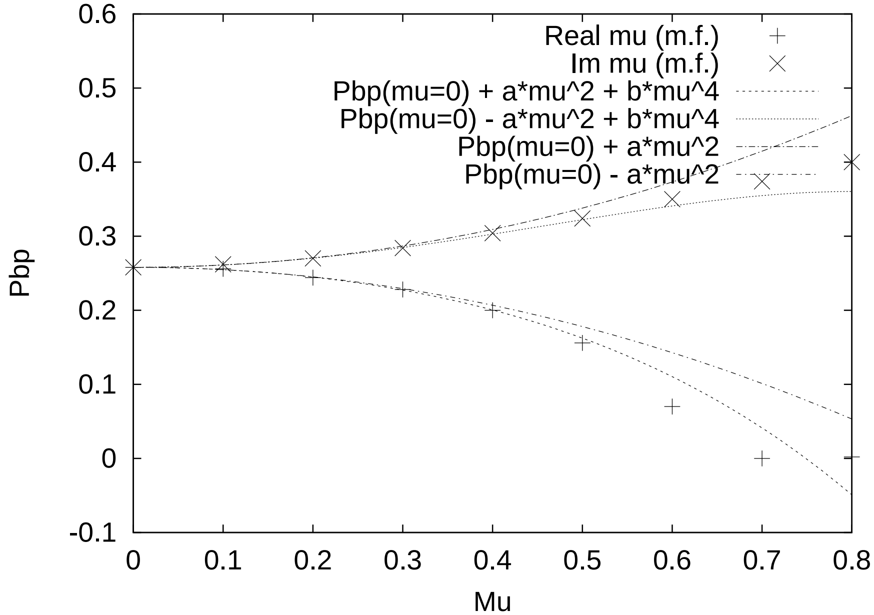
$$S_{QCD} = S_{YM} + S_F \to g \to \infty = S_F$$

$$\mathcal{Z} = (\int V_{eff}(< \bar{\psi}\psi > d < \bar{\psi}\psi >)^{V_s}$$

$$V_{eff}(< \bar{\psi}\psi >, \mu) = 2cosh(rN_tN_c\mu) + sinh[(N_t+1)N_c < \bar{\psi}\psi >]/sinh(N_t < \bar{\psi}\psi >)$$

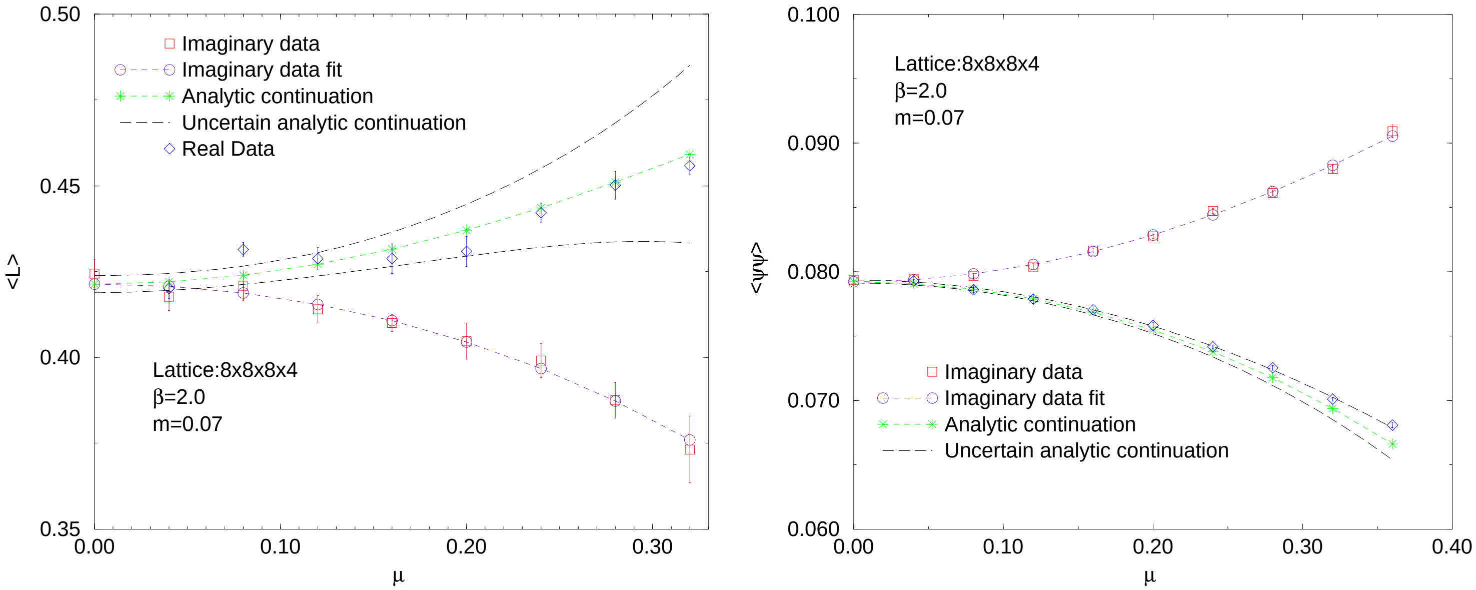
$$V_{eff}(< \bar{\psi}\psi >, i\mu) = 2cos(rN_tN_c\mu) + sinh[(N_t+1)N_c < \bar{\psi}\psi >]/sinh(N_t < \bar{\psi}\psi >)$$




$<\bar{\psi}\psi>$ as a function of real and imaginary $\mu$ , for $T \simeq 0$ and $T \simeq T_c$

# Two color QCD as a testbed for Imaginary $\mu_B$




P. Giudice, Tesi di Laurea in Fisica, Universita della Calabria; Advisor A. Papa

*courtesy of the Author*

## Gross Neveu Model

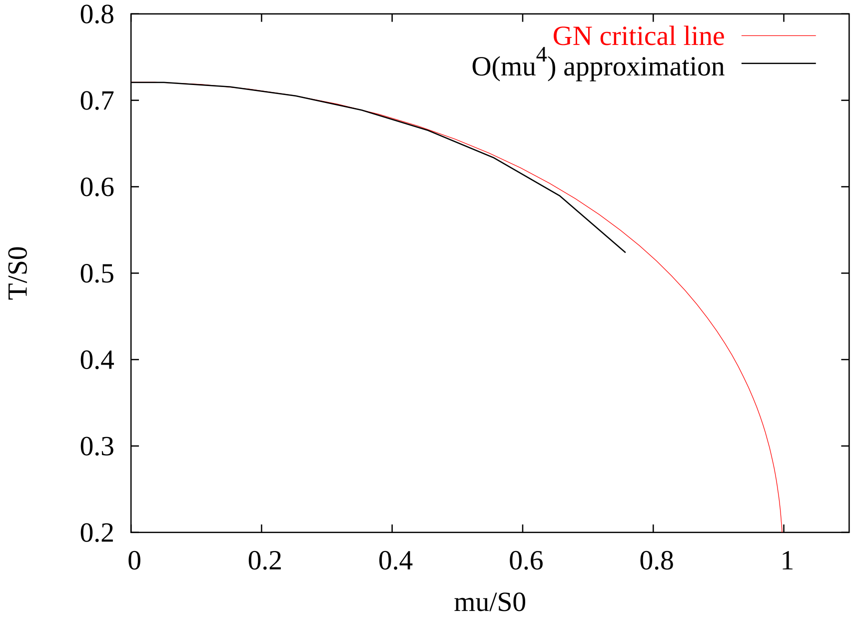
*The critical line:*

$$1 - \mu/\Sigma_0 = 2T/\Sigma_0 ln(1 + e^{-\mu/T})$$

*Reduces to:*

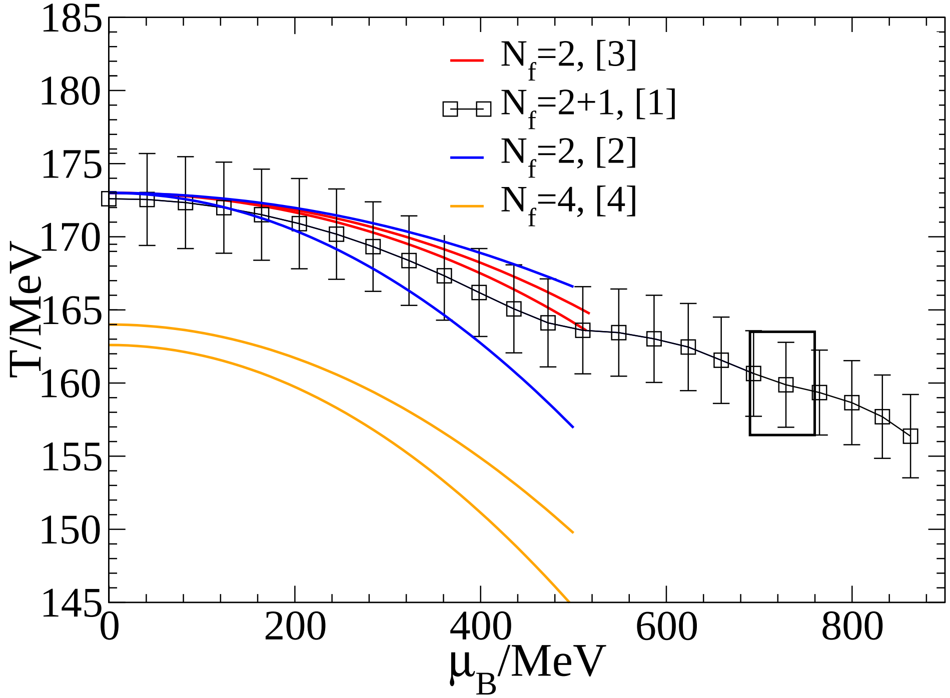
$$T(T - T_c) + \mu^2/(8ln2) = 0$$

*Second order approximation good up to* $\mu \simeq T_c$


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.2
0.4
0.6
0.8
1
T/S0
mu/S0
GN critical line
O(mu^4) approximation

## *From O. Philipsen and E. Laermann*

*Ann. Rev. Nucl. Part. Phys. 2003*





1. *Fodor Z and Katz SD, JHEP 0203:014 (2002).*
2. *Allton CR et al., Phys. Rev. D 66:074507 (2002).*
3. *de Forcrand P and Philipsen O, Nucl. Phys. B642:290 (2002).*
4. *D'Elia M and Lombardo MP,Phys. Rev. D 1:074507 (2003).*

# Lattice QCD at Finite Temperature and Density

## Lecture I

### I Formulation

### II Calculational Schemes

II.1 Effective Fermionic Models - Analytic approaches

II.2 Effective Gluonic Models - Numerical approaches

### III QCD at Finite Baryon Density: Methods

III.1 Derivatives

III.2 Reweighting

III.3 Expanded Reweighting

III.4 Imaginary Chemical Potential

### IV Results - Discussion : Tomorrow's lecture