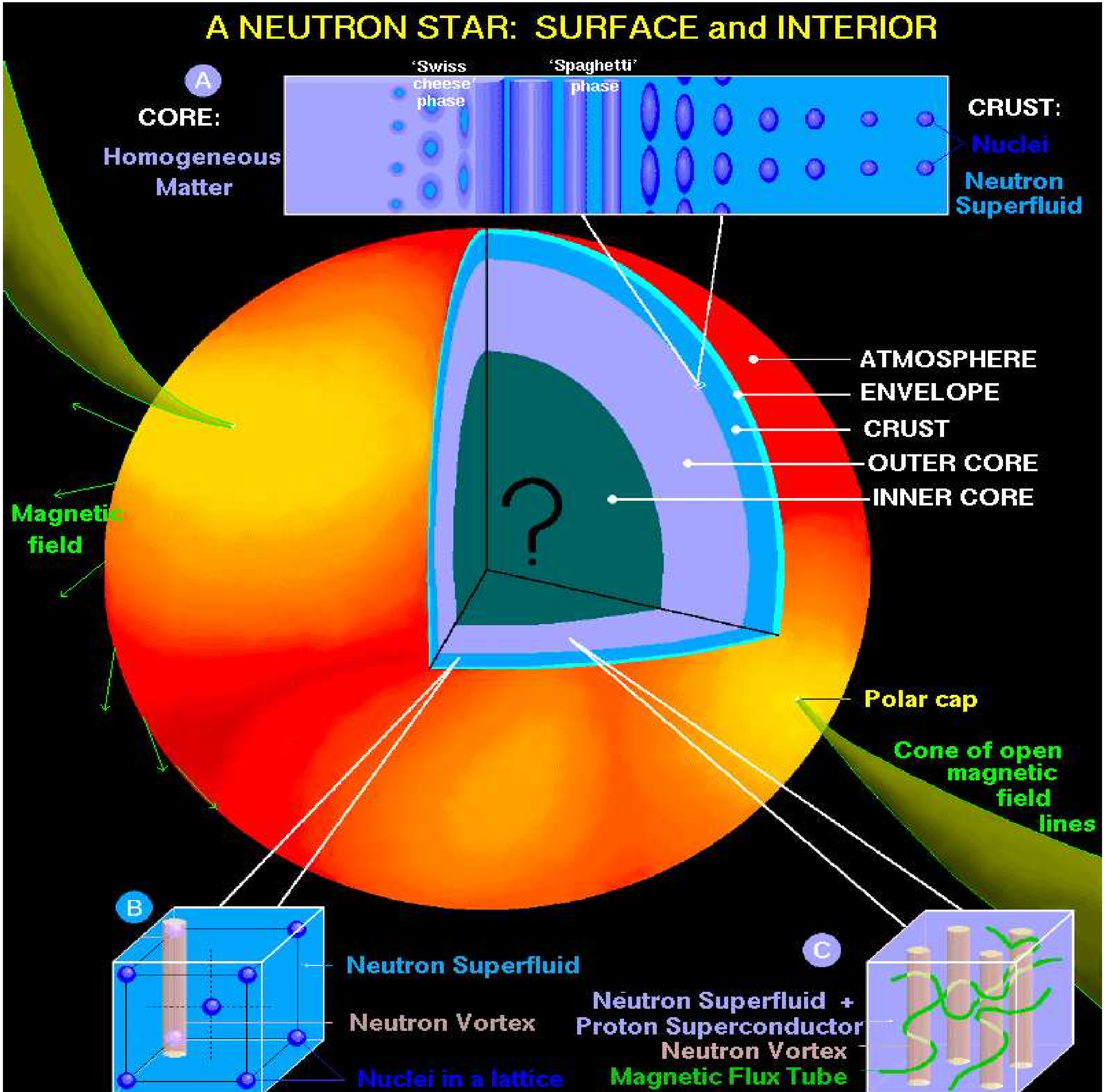
Neutron Star Structure

James M. Lattimer

lattimer@astro.sunysb.edu

Department of Physics & Astronomy

Stony Brook University



Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi p r^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$

p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Spherically Symmetric General Relativity

Static metric: $$ds^2 = e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - e^{\nu(r)}dt^2$$

Einstein's equations:

$$\begin{aligned}
8\pi\epsilon(r) &= \frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\lambda'(r)}{r}, \\
8\pi p(r) &= -\frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\nu'(r)}{r}, \\
p'(r) &= -\frac{p(r) + \epsilon(r)}{2}\nu'(r).
\end{aligned}$$

Mass: $$m(r) = 4\pi\int_0^r \epsilon(r')r'^2 dr', \qquad e^{-\lambda(r)} = 1 - 2m(r)/r$$

Boundaries:

$$\begin{aligned}
r &= 0 \qquad && m(0) = p'(0) = \epsilon'(0) = 0, \\
r &= R \qquad && m(R) = M, \quad p(R) = 0, e^{\nu(R)} = e^{-\lambda(R)} = 1 - 2M/R
\end{aligned}$$

Thermodynamics:

$$\begin{aligned}
d(\ln n) &= \frac{d\epsilon}{\epsilon + p} = -\frac{1}{2}\frac{d\epsilon}{dp}d\nu, \qquad h = \frac{d\epsilon}{dn}, \qquad \epsilon = n(m + e), \qquad p = n^2\frac{de}{dn} \\
mn(r) &= (\epsilon(r) + p(r))e^{(\nu(r) - \nu(R))/2} - n_0 e_0; \qquad p = 0 : n = n_0, e = e_0 \\
N &= \int_0^R 4\pi r^2 e^{\lambda(r)/2} n(r) dr; \qquad \mathrm{BE} = Nm - M
\end{aligned}$$

Uniform Density Fluid

$$
\begin{aligned}
m(r) &= \frac{4\pi}{3}\epsilon r^3, \qquad \beta \equiv \frac{M}{R} \\
e^{-\lambda(r)} &= 1 - 2\beta(r/R)^2, \\
e^{\nu(r)} &= \left[\frac{3}{2}\sqrt{1-2\beta} - \frac{1}{2}\sqrt{1-2\beta(r/R)^2}\right]^2, \\
p(r) &= \epsilon\left[\frac{\sqrt{1-2\beta(r/R)^2} - \sqrt{1-2\beta}}{3\sqrt{1-2\beta} - \sqrt{1-2\beta(r/R)^2}}\right], \\
\epsilon(r) &= \text{constant}; \qquad n(r) = \text{constant} \\
\frac{\text{BE}}{M} &= \frac{3}{4\beta}\left(\frac{\sin^{-1}\sqrt{2\beta}}{\sqrt{2\beta}} - \sqrt{1-2\beta}\right) \simeq \frac{3\beta}{5} + \frac{9}{14}\beta^2 + \cdots
\end{aligned}
$$

$$p_c < \infty \Longrightarrow \beta < 4/9$$

$$c_s^2 = \infty$$

Tolman VII

$$\begin{aligned}
\epsilon(r) &= \epsilon_c\left[1-(r/R)^2\right] \equiv \epsilon_c[1-x] \\
e^{-\lambda(r)} &= 1-\beta x(5-3x) \\
e^{\nu(r)} &= (1-5\beta/3)\cos^2\phi, \\
p(r) &= \frac{1}{4\pi R^2}\left[\sqrt{3\beta e^{-\lambda(r)}}\tan\phi(r)-\frac{\beta}{2}(5-3x)\right], \\
n(r) &= \frac{\epsilon(r)+p(r)}{m_b}\frac{\cos\phi(r)}{\cos\phi_1} \\
\phi(r) &= \frac{w_1-w(r)}{2}+\phi_1, \qquad \phi_1=\phi(x=1)=\tan^{-1}\sqrt{\frac{\beta}{3(1-2\beta)}}, \\
w(r) &= \ln\left[x-\frac{5}{6}+\sqrt{\frac{e^{-\lambda(r)}}{3\beta}}\right], \qquad w_1=w(x=1)=\ln\left[\frac{1}{6}+\sqrt{\frac{1-2\beta}{3\beta}}\right].
\end{aligned}$$

$$(P/\epsilon)_c=\frac{2\tan\phi_c}{15}\sqrt{\frac{3}{\beta}}-\frac{1}{3}, \qquad c_{s,c}^2=\tan\phi_c\left(\frac{1}{5}\tan\phi_c+\sqrt{\frac{\beta}{3}}\right)$$

$$\frac{\mathrm{BE}}{M}\simeq\frac{11}{21}\beta+\frac{7187}{18018}\beta^2+\cdots$$

$$p_c<\infty\Longrightarrow\phi_c<\frac{\pi}{2}, \beta<0.3862$$

$$c_{s,c}^2<1\Longrightarrow\beta<0.2698$$

Buchdahl

$$\epsilon = \sqrt{p_* p} - 5p$$

$$\begin{aligned}
e^{\nu(r)} &= (1-2\beta)(1-\beta-u(r))(1-\beta+u(r))^{-1}, \\
e^{\lambda(r)} &= (1-2\beta)(1-\beta+u(r))(1-\beta-u(r))^{-1}(1-\beta+\beta\cos Ar')^{-2}, \\
8\pi p(r) &= A^2 u(r)^2 (1-2\beta)(1-\beta+u(r))^{-2}, \\
8\pi\epsilon(r) &= 2A^2 u(r)(1-2\beta)(1-\beta-3u(r)/2)(1-\beta+u(r))^{-2}, \\
m_b n(r) &= \sqrt{p_* p(r)}\left(1-4\sqrt{\frac{p(r)}{p_*}}\right)^{3/2}, \qquad c_s^2(r) = \left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-5\right)^{-1} \\
u(r) &= \frac{\beta}{Ar'}\sin Ar' = (1-\beta)\left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-1\right)^{-1}, \\
r' &= r(1-2\beta)(1-\beta+u(r))^{-1}, \\
A^2 &= 2\pi p_*(1-2\beta)^{-1}, \qquad R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}.
\end{aligned}$$

$$p_c = \frac{p_*}{4}\beta^2, \qquad \epsilon_c = \frac{p_*}{2}\beta(1-\frac{5}{2}\beta), \qquad n_c m_b = \frac{p_*}{2}\beta(1-2\beta)^{3/2}$$

$$\frac{\mathrm{BE}}{M} = (1-\frac{3}{2}\beta)(1-2\beta)^{-1/2}(1-\beta)^{-1} \simeq \frac{\beta}{2}+\frac{\beta^2}{2}+\cdots$$

$$c_{s,c}^2 < 1 \Longrightarrow \beta < 1/6$$

Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$\begin{aligned} p(\epsilon) = 0, \qquad & \epsilon \leq \epsilon_o \\ p(\epsilon) = \epsilon - \epsilon_o, \qquad & \epsilon \geq \epsilon_o \end{aligned}$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$\begin{aligned} \frac{dy}{dx} &= 4\pi x^2 (1+q) \\ \frac{dq}{dx} &= -\frac{(y + 4\pi q x^3)(1+2q)}{x(x-2y)} \end{aligned}$$

The solution with the maximum central pressure and mass and the minimum radius:

$$\begin{aligned} q_{max} &= 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825 \\ p_{max} &= 307 \left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.2 \left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ M}_\odot, \quad R_{min} = 2.825 \frac{GM_{max}}{c^2}. \end{aligned}$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \qquad P_{min} = 0.82 \left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ ms}$$

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)

- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

Koranda, Stergioulas & Friedman (1997)

- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

Lattimer & Prakash (2004)

- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Another Kind of Star – Self-Bound

Finite, large surface energy density

Lake's solution

$$
\begin{aligned}
e^{\nu(r)} &= \frac{(1-\frac{5}{2}\beta(1-\frac{1}{5}x)^2}{(1-\beta)}, \\
e^{\lambda(r)} &= \frac{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}-2(1-\beta)^{2/3}\beta x}, \\
4\pi p R^2 &= \frac{\beta}{1-\frac{5}{2}\beta(1-\frac{1}{5}x)}\left[1-(1-\beta)^{2/3}\frac{(1-\frac{5}{2}\beta(1-x))}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}}\right], \\
4\pi\epsilon R^2 &= 3(1-\beta)^{2/3}\beta\frac{1-\frac{5}{2}\beta(1-\frac{1}{3}x)}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{5/3}}, \\
m &= \frac{(1-\beta)^{\frac{2}{3}}Mx^{3/2}}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}} \\
c_s^2 &= \frac{(2-5\beta+3\beta x)}{5(2-5\beta+\beta x)^3}\left[\frac{(2-5\beta+3\beta x)^{5/3}}{2^{2/3}(1-\beta)^{2/3}}+(2-5\beta)^2-5\beta^2 x\right], \\
\frac{\epsilon_{surf}}{\epsilon_c} &= \left(1-\frac{5}{3}\beta\right)\left(1-\frac{5}{2}\beta\right)^{2/3}(1-\beta)^{-5/3}.
\end{aligned}
$$

$$0.30 < c_{s,c}^2 < 0.44, \qquad 0.265 < \frac{\epsilon_{surf}}{\epsilon_c} < 1$$

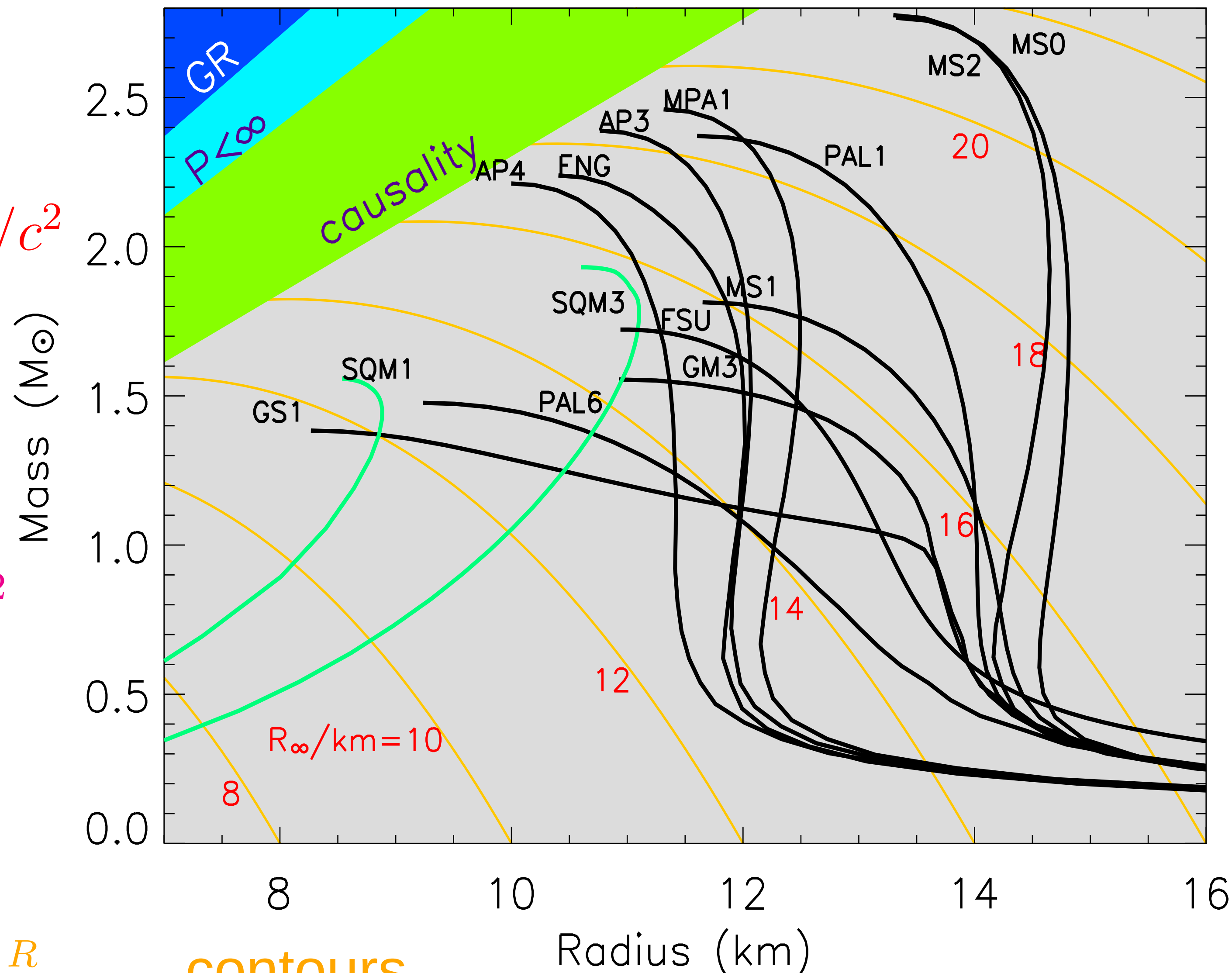
Mass-Radius Diagram and Theoretical Constraints

GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R \gtrsim 2.9GM/c^2$

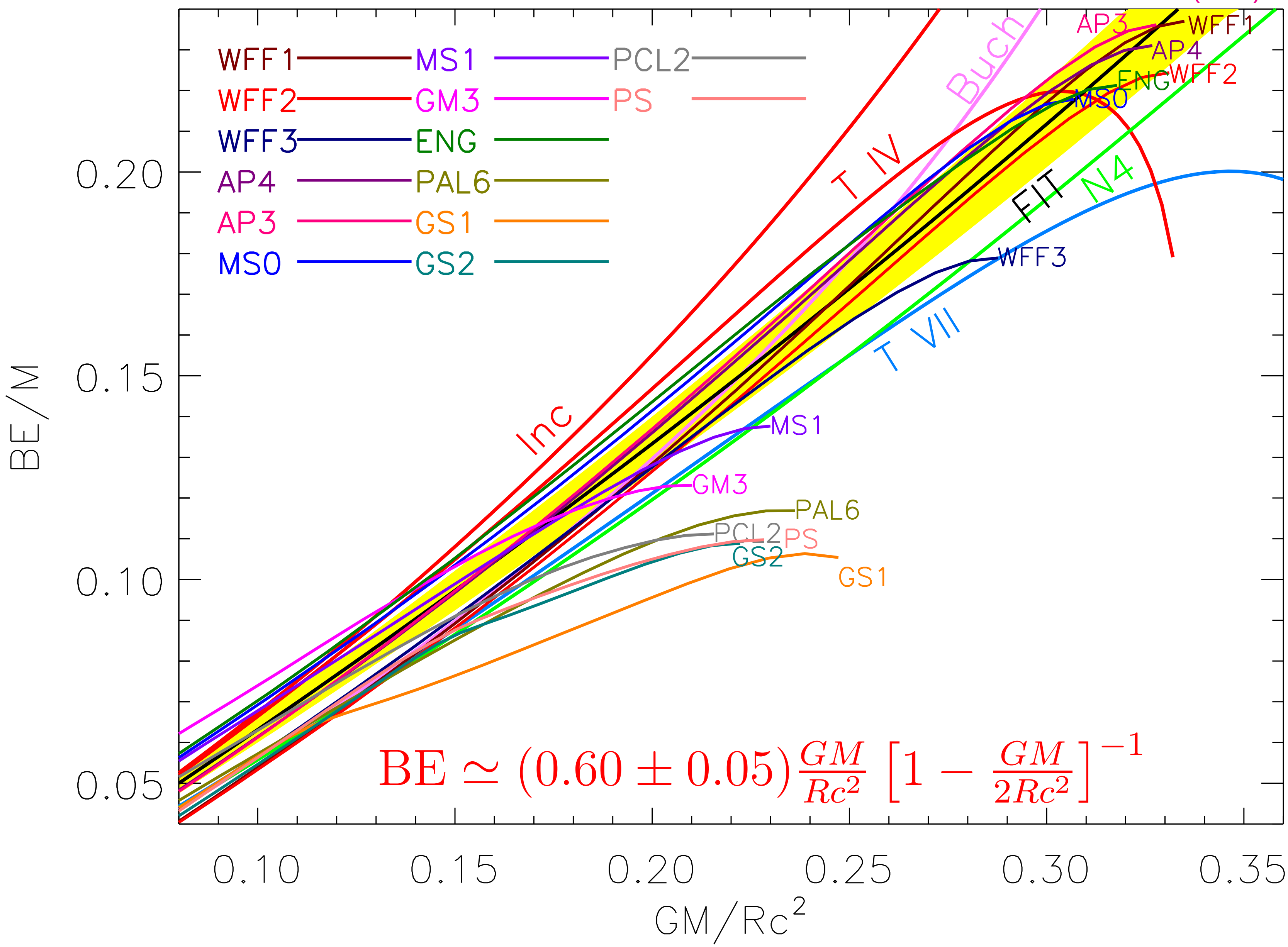
— normal NS
— SQS



— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours

$\mathrm{BE}(M, R)$

Lattimer & Prakash (2001)



$$\mathrm{BE} \simeq (0.60 \pm 0.05) \frac{GM}{Rc^2} \left[1 - \frac{GM}{2Rc^2}\right]^{-1}$$

Moment of Inertia

$$
\begin{aligned}
I &= -\frac{2c^2}{3G}\int_0^R r^2\omega(r)\frac{dj(r)}{dr}dr \\
&= \frac{8\pi}{3c^4}\int_0^R r^4\left[\epsilon(r)+p(r)\right]e^{\lambda(r)}j(r)\omega(r)dr.
\end{aligned}
$$

where

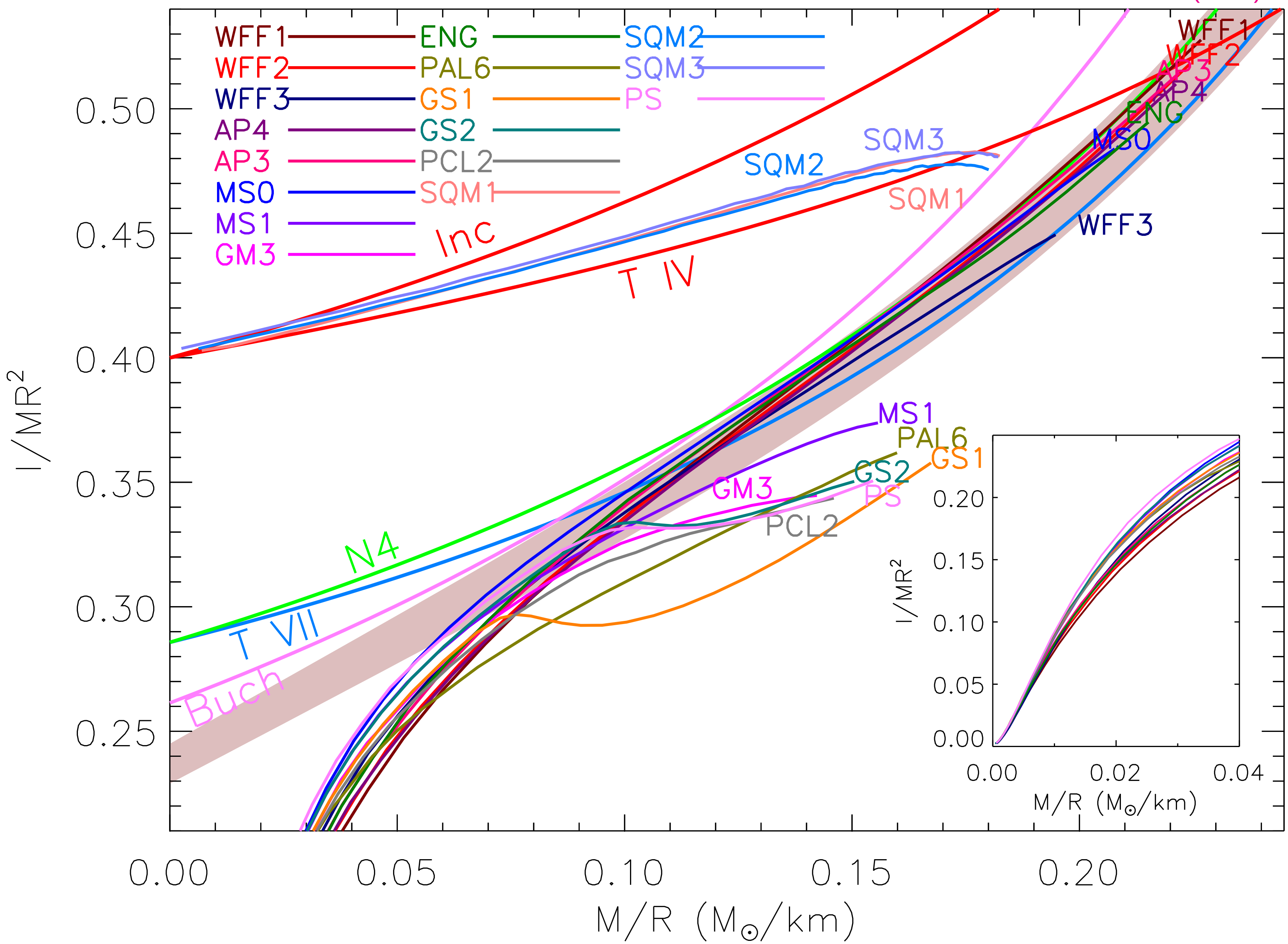
$$
\begin{aligned}
j(r) &= e^{-(\lambda(r)+\nu(r))/2}; \\
\frac{d}{dr}\left[r^4 j(r)\frac{d\omega(r)}{dr}\right] &= -4r^3\omega(r)\frac{dj(r)}{dr}; \\
j(R)=1, \qquad \frac{dj(R)}{dr} &= 0, \qquad \omega(R)=1-\frac{2GI}{R^3c^2}, \\
\frac{dj(0)}{dr} &= \frac{d\omega(0)}{dr}=0.
\end{aligned}
$$

Combining these:

$$
I=\frac{c^2}{6G}R^4\frac{d\omega(R)}{dr}
$$

$I(M,R)$

Lattimer & Prakash (2001)



$$I \simeq (0.237 \pm 0.008) M R^2 \left[1 + 4.2 \frac{M\ \mathrm{km}}{R\ \mathrm{M}_\odot} + 90 \left(\frac{M\ \mathrm{km}}{R\ \mathrm{M}_\odot} \right)^4 \right]$$

Maximum Possible Density in Stars

The scaling from the maximally compact EOS yields

$$\epsilon_{c,max} = 3.026 \left(\frac{4.1\ \mathrm{M}_\odot}{M_{max}}\right)^2 \epsilon_s \simeq 13.7 \times 10^{15} \left(\frac{\mathrm{M}_\odot}{M_{max}}\right)^2 \mathrm{g\ cm}^{-3}.$$

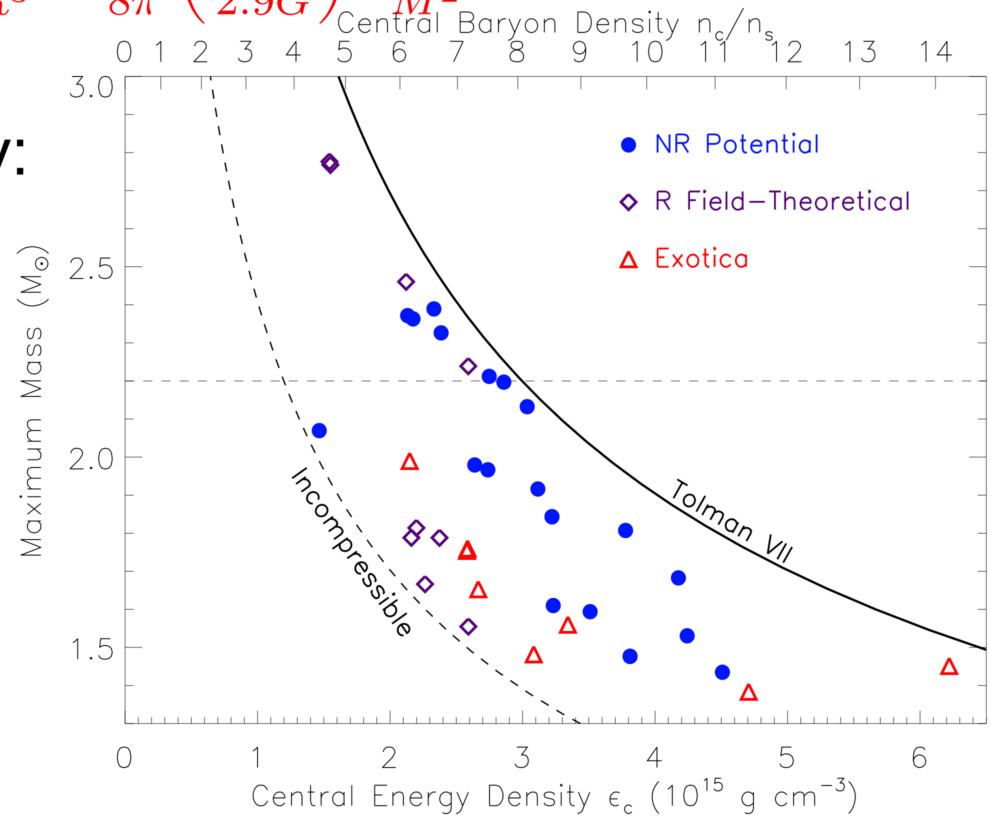
A virtually identical result arises from combining the maximum compactness constraint ($R_{min} \simeq 2.9GM/c^2$) with the Tolman VII relation

$$\epsilon_{c,VII} = \frac{15}{8\pi}\frac{M}{R^3} = \frac{15}{8\pi}\left(\frac{c^2}{2.9G}\right)^3 \frac{1}{M^2}$$

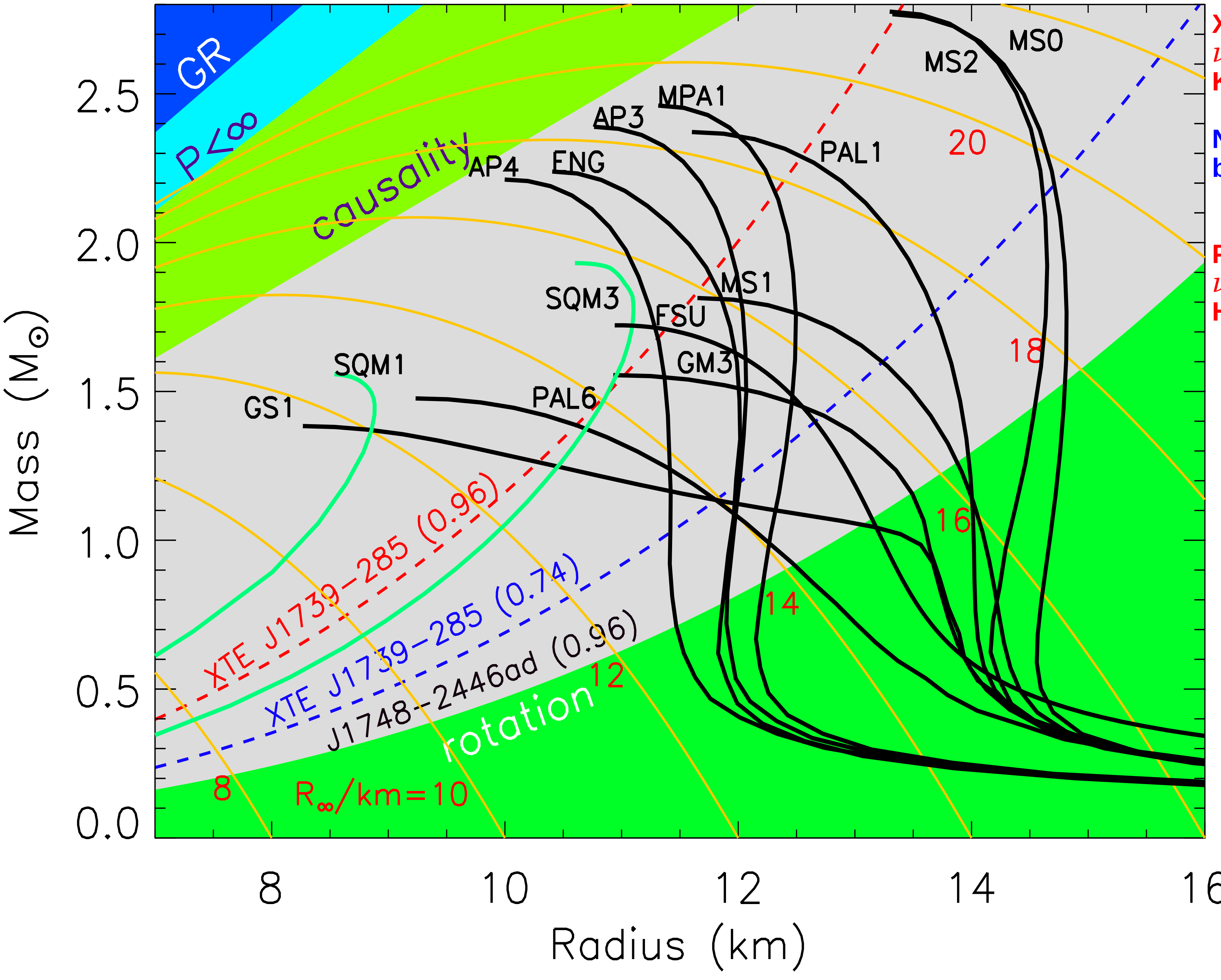
Maximum possible density:

$2.2\ \mathrm{M}_\odot \Rightarrow$

$\epsilon_{max} < 2.8 \times 10^{15} \mathrm{g\ cm}^{-3}$



Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

Newtonian Roche model for rotation

(*c.f.*, Shapiro & Teukolsky 1983)

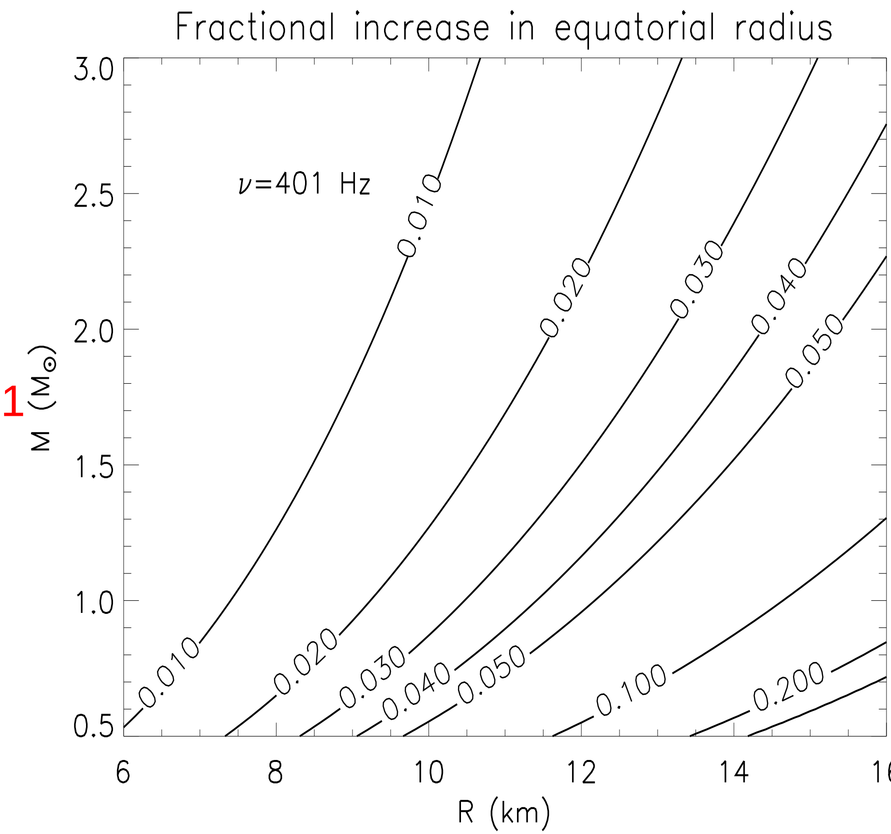
$\rho^{-1}\nabla P = \nabla h = -\nabla(\Phi_G + \Phi_c), \qquad \Phi_G \simeq -GM/r, \qquad \Phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$

Bernoulli integral:

$H = h + \Phi_G + \Phi_c = -GM/R_p$

Enthalpy $h = \int_0^p \rho^{-1} dp = \mu_n(\rho) - \mu_n(0)$ in beta equilibrium

Numerical calculations show R_p is nearly constant for arbitrary rotation



Evaluate at equator: $\frac{\Omega^2 R_{eq}^3}{2GM} = \frac{R_{eq}}{R_p} - 1$

Mass-shedding limit $\Omega^2_{shed} = \frac{GM}{R_{eq}^3} : \frac{R_{eq}}{R_p} = \frac{3}{2}$

GR: Cook, Shapiro & Teukolsky (1994): 1.43–1.51

$$\Omega_{shed} = \left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{GM}{R_p^3}} = 0.544\sqrt{\frac{GM}{R_p^3}}$$

GR: Lattimer & Prakash (2005): $0.570 \pm 3\%$

Shape: $\frac{\Omega^2 R^3 \sin^2\theta}{2GM} = \frac{R}{R_p} - 1$

$$\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos\left[\frac{1}{3}\cos^{-1}\left(1 - 2\left(\frac{\Omega\sin\theta}{\Omega_{shed}}\right)^2\right)\right]$$

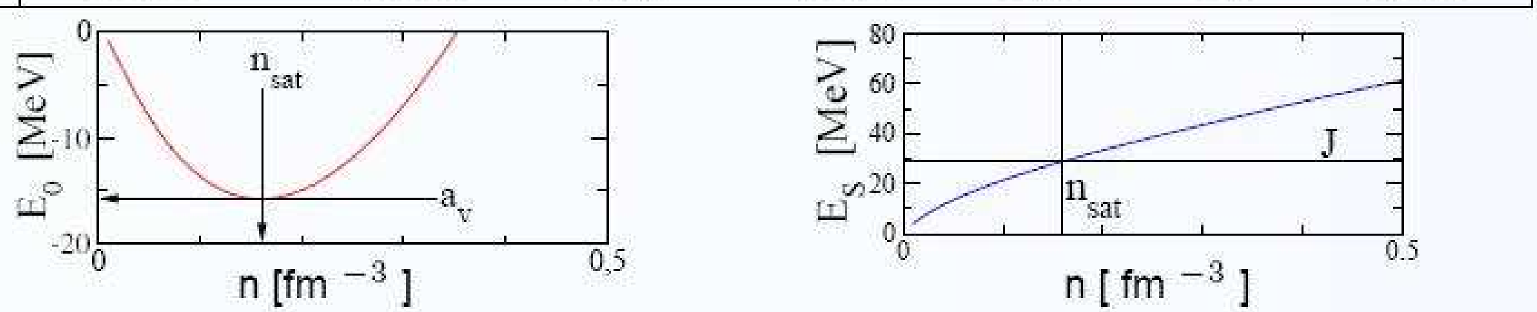
Limit: $\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos\left[\frac{1}{3}\cos^{-1}(1 - 2\sin^2\theta)\right] = \frac{\sin(\theta)}{3\sin(\theta/3)}.$

Exploring the Limits - The EoS beyond saturation

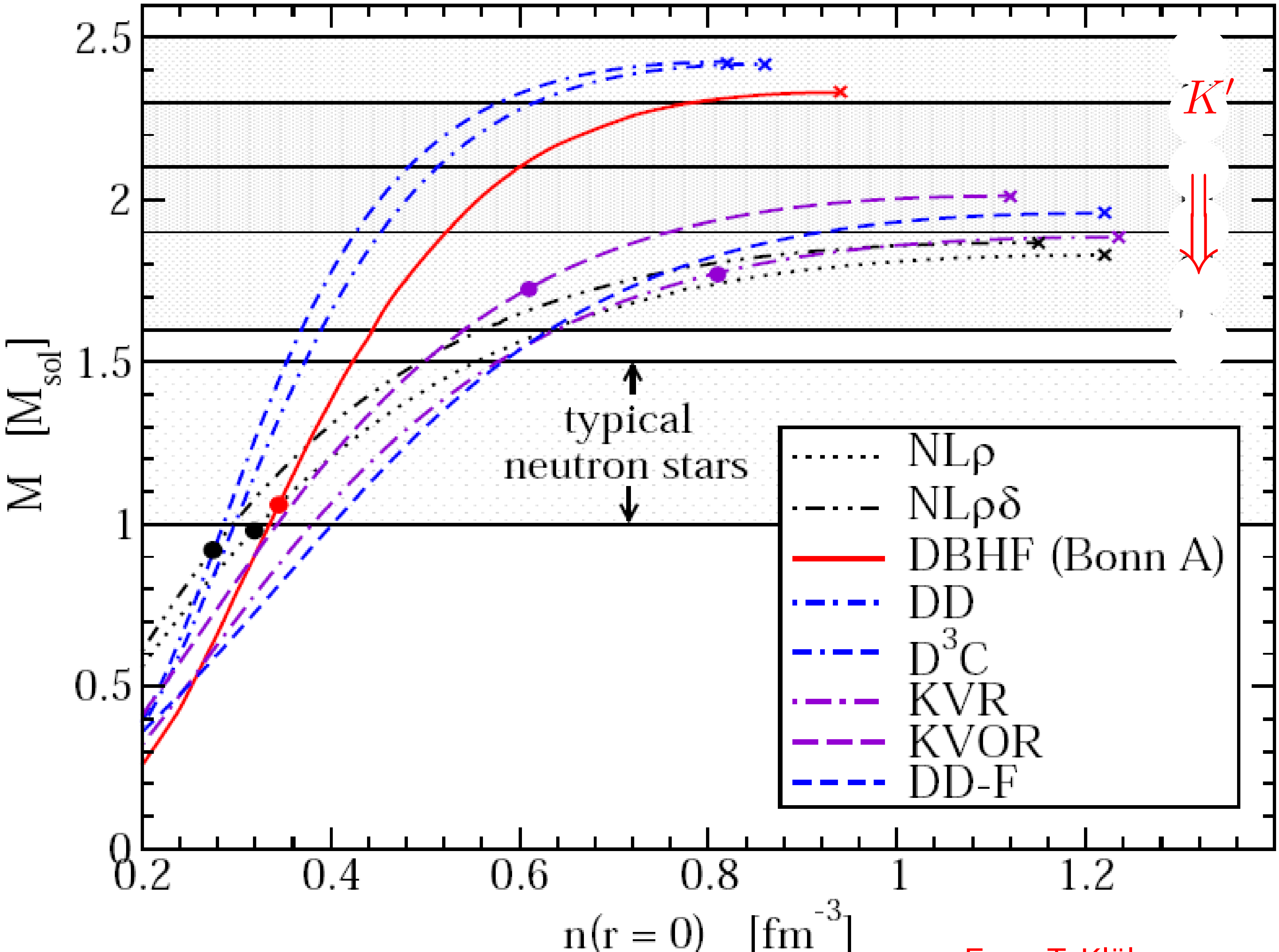
$$E(n,\beta) = E_0(n) + \beta^2 E_S(n) \approx a_V + \frac{K}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + \ldots + \beta^2\left(J + \frac{L}{3}\epsilon + \ldots\right) + \ldots$$

$$\epsilon = (n - n_{sat})/n \qquad \beta = (n_n - n_p)/(n_n + n_p)$$

	$n_{\rm sat}$	a_V	K	K'	J	L	m_D/m
	[fm^{-3}]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	
exp.	0.16 ± 0.01	-16 ± 1	$200 - 300$		$25 - 35$		$0.6 - 1.0$
NLρ	0.1459	−16.062	203.3	576.5	30.8	83.1	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	576.5	31.0	92.3	0.603
DBHF	0.1779	−16.160	201.6	507.9	33.7	69.4	0.684
DD	0.1487	−16.021	240.0	−134.6	32.0	56.0	0.565
D^{3}C	0.1510	−15.981	232.5	−716.8	31.9	59.3	0.541
KVR	0.1600	−15.800	250.0	528.8	28.8	55.8	0.800
KVOR	0.1600	−16.000	275.0	422.8	32.9	73.6	0.800
DD-F	0.1469	−16.024	223.1	757.8	31.6	56.0	0.556



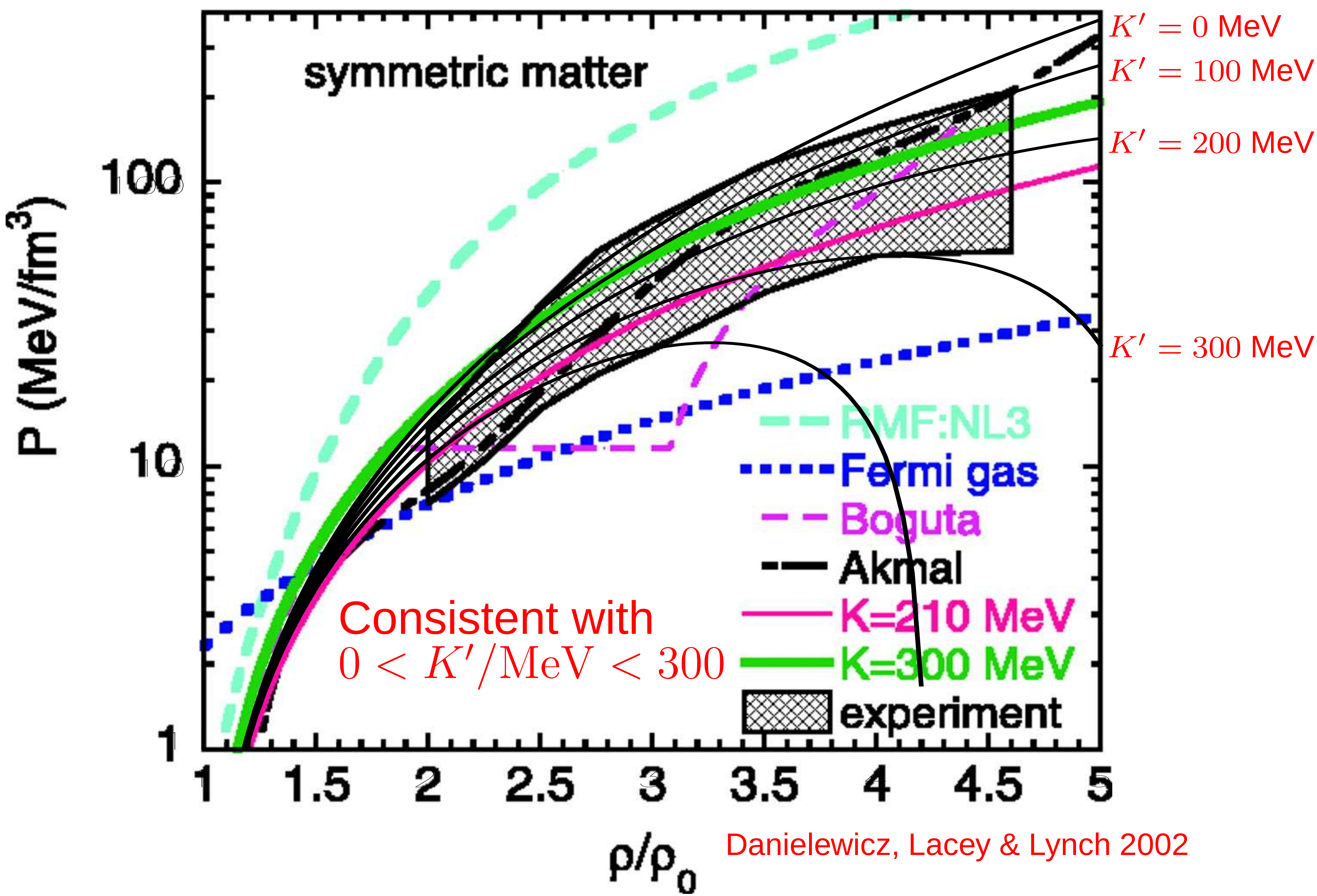
T. Klähn et al., Phys.Rev.C **74**, 035802 (2006)



M [M_sol]
n(r = 0) [fm^-3]
typical
neutron stars
NLρ
NLρδ
DBHF (Bonn A)
DD
D^3C
KVR
KVOR
DD-F
K′

From T. Klähn

Flow Constraint From Heavy Ions



Danielewicz, Lacey & Lynch 2002

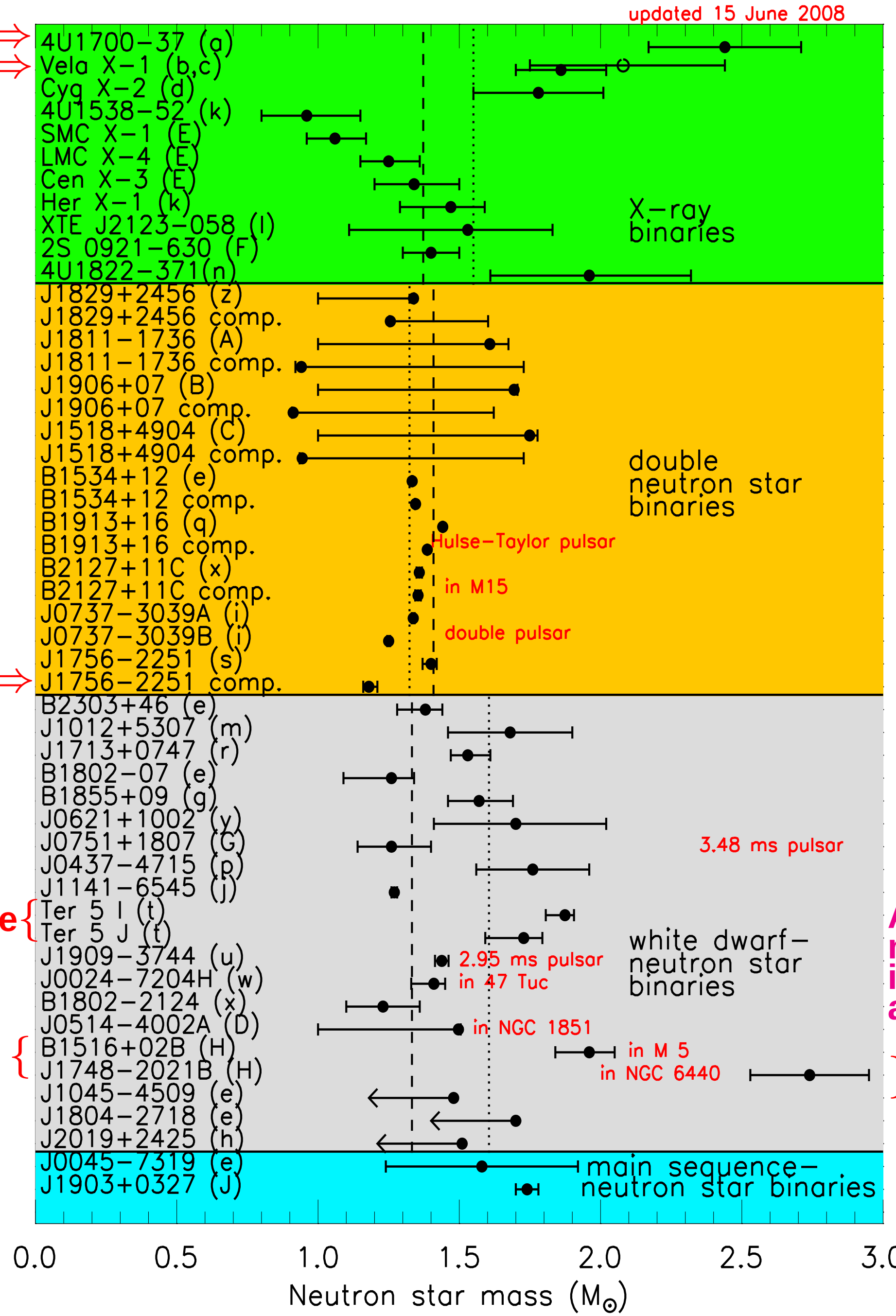
Observed Masses

Black hole? ⇒
Firm lower mass limit? ⇒

$M \simeq 1.18$ M$_\odot$ ⇒

$M > 1.68$ M$_\odot$, **95% confidence** {

Freire et al. 2007 {



Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.015 M$_\odot$ larger

} **w.d. companion?**
statistics?

Polytropes

Polytropic Equation of State: $p = Kn^\gamma$
n is number density, γ is polytropic exponent.
Hydrostatic Equilibrium in Newtonian Gravity:

$$\frac{dp(r)}{dr} = -\frac{Gm(r)n(r)}{r^2}, \qquad \frac{dm(r)}{dr} = 4\pi n r^2$$

Dimensional analysis:

$$M \propto n_c R^3, \quad p \propto \frac{M^2}{R^4}, \quad R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

When $\gamma \sim 2$: $$R \propto K^{1/2} M^0 \propto p_f^{1/2} n_f^{-1} M^0$$

General Relativistic analysis using Buchdahl's solution:

$$R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}, \quad \left.\frac{d\ln R}{d\ln p}\right|_{n,M} = \frac{1}{2}\frac{(1-\beta)(2-\beta)}{(1-3\beta+3\beta^2)}\frac{1-10\sqrt{p/p_*}}{1+2\sqrt{p/p_*}}.$$

For $M = 1.4$M$_\odot$, $R = 14$ km, $n = 1.5n_s$, $\epsilon = 1.5 m_b n_s \simeq 3\times 10^{-4}$ km^{-2}:

$$\beta = 0.148, \quad p_* = 0.00826, \quad p/p_* = 0.00221.$$

$$\left.\frac{d\ln R}{d\ln p}\right|_{n,M} \simeq 0.234$$

Neutron Star Matter Pressure and the Radius

$$p \simeq K n^{\gamma}$$

$$\gamma = d \ln p / d \ln \; n \sim 2$$

$$R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

$$R \propto p_f^{1/2} n_f^{-1} M^0$$

$$(1 < n_f/n_s < 2)$$

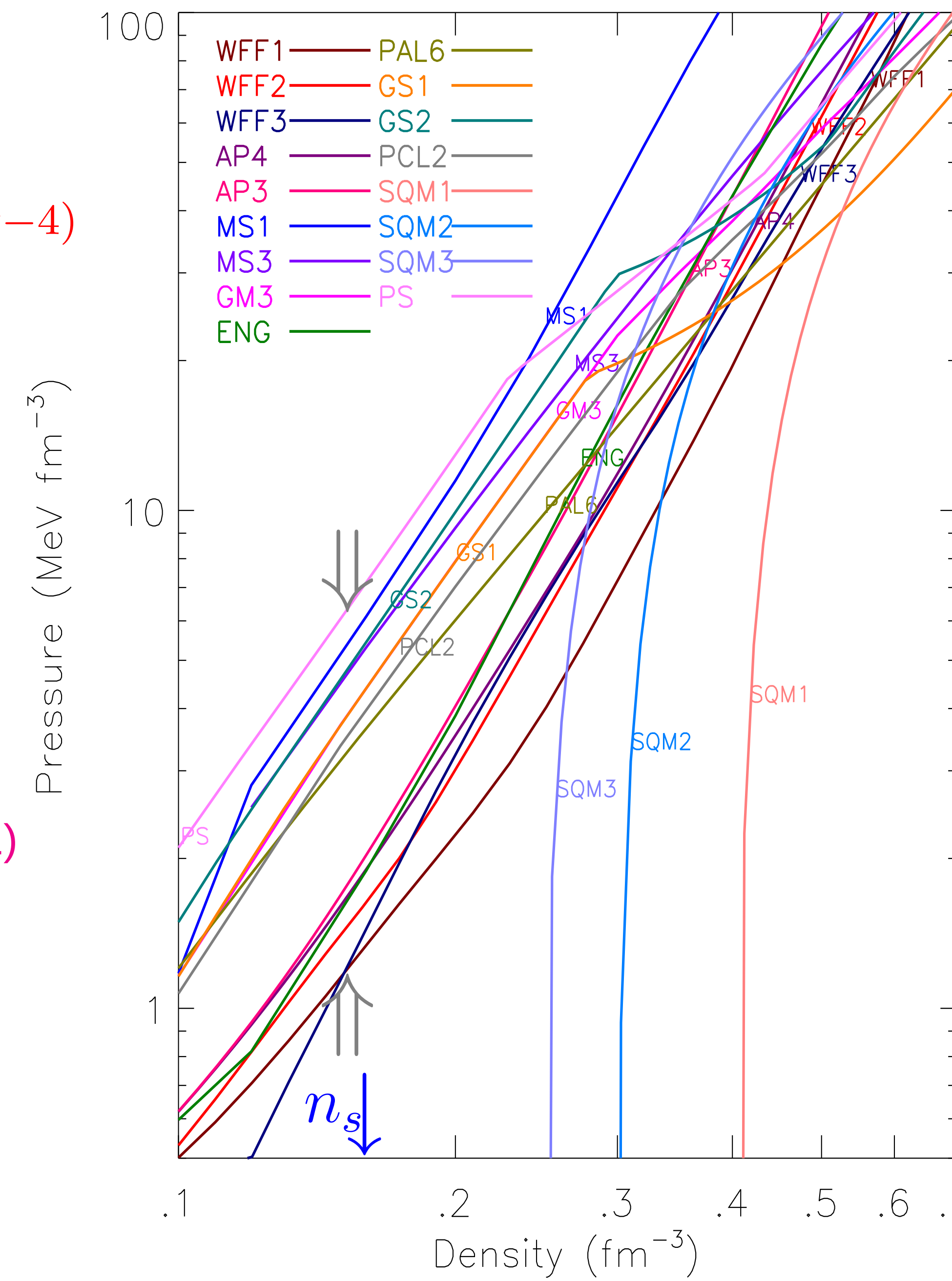
Wide variation:

$$1.2 < \frac{p(n_s)}{\mathrm{MeV\ fm^{-3}}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

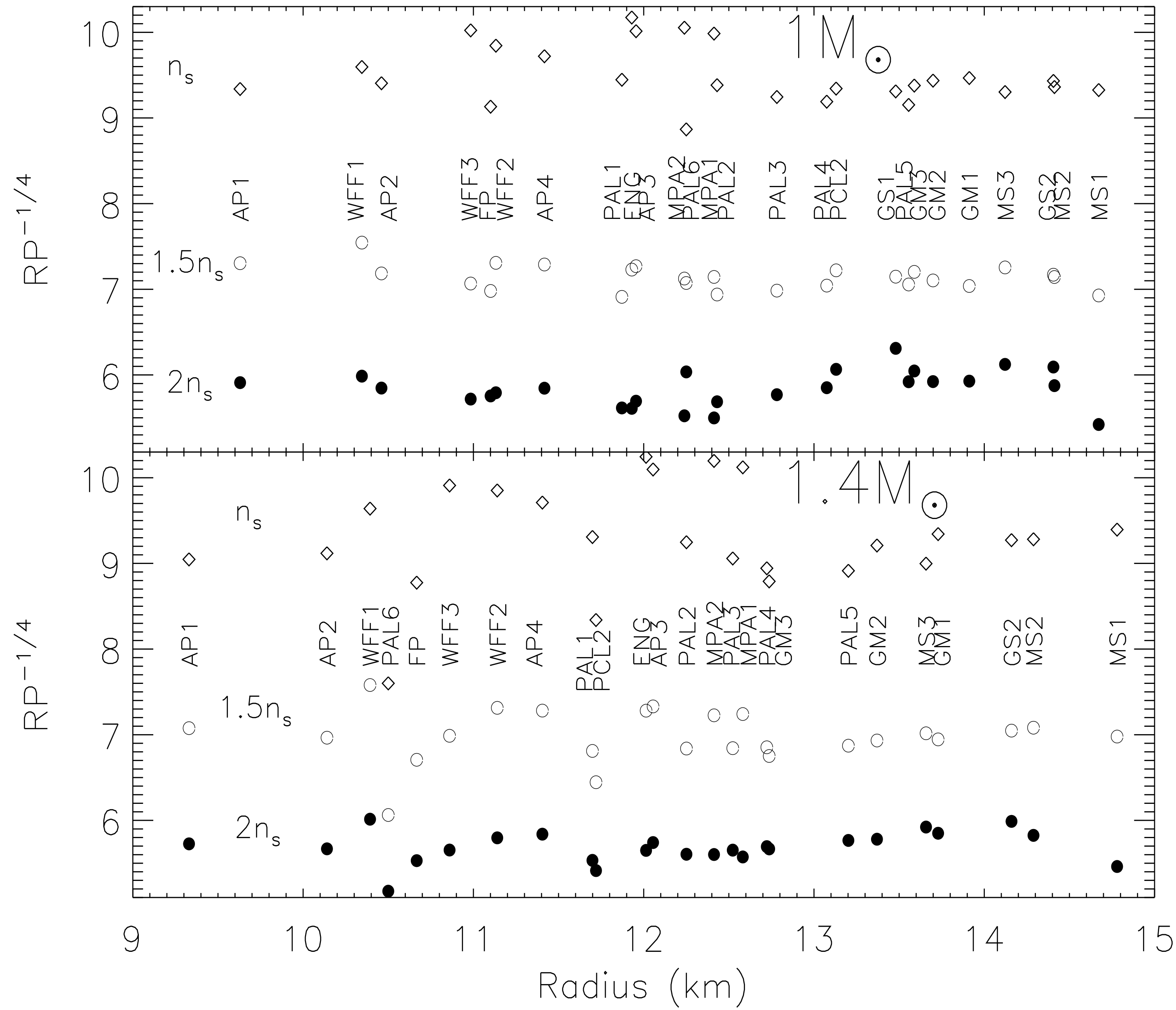
$$R \propto p_f^{1/4} n_f^{-1/2}$$

$$p_f = n^2 dE_{sym}/dn$$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

The Pressure of Neutron Star Matter

Expansion of cold nucleonic matter energy near n_s and isospin symmetry $x = 1/2$:

$$
\begin{aligned}
E(n,x) &\simeq E(n,1/2) + E_{sym}(n)(1-2x)^2 + \frac{3\hbar c}{4}x(3\pi^2 nx)^{1/3}\,, \\
P(n,x) &\simeq n^2\left[\frac{dE(n,1/2)}{dn} + \frac{dE_{sym}}{dn}(1-2x)^2\right] + \frac{\hbar c}{4}nx(3\pi^2 nx)^{1/3}\,, \\
\mu_e &= \hbar c(3\pi^2 nx)^{1/3}\,, \qquad E(n,1/2) \simeq -B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2.
\end{aligned}
$$

Beta Equilibrium:

$$\left(\frac{\partial E}{\partial x}\right)_n = \mu_p - \mu_n + \mu_e = 0\,.$$

$$
\begin{aligned}
x_\beta &\simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3\,, \\
P_\beta &= \frac{Kn^2}{9n_0}\left(\frac{n}{n_s}-1\right) + n^2(1-2x_\beta)^2\frac{dE_{sym}}{dn} + E_{sym}nx_\beta(1-2x_\beta)
\end{aligned}
$$

$E_{sym}(n_s) \equiv S_v \simeq 30$ MeV, $\hbar c \simeq 200$ MeV/fm, $\qquad n \rightarrow n_s \Longrightarrow$

$$x_\beta \rightarrow 0.04\,, \qquad P_\beta \rightarrow n_s^2 \left.\frac{dE_{sym}}{dn}\right|_{n_s}\,.$$

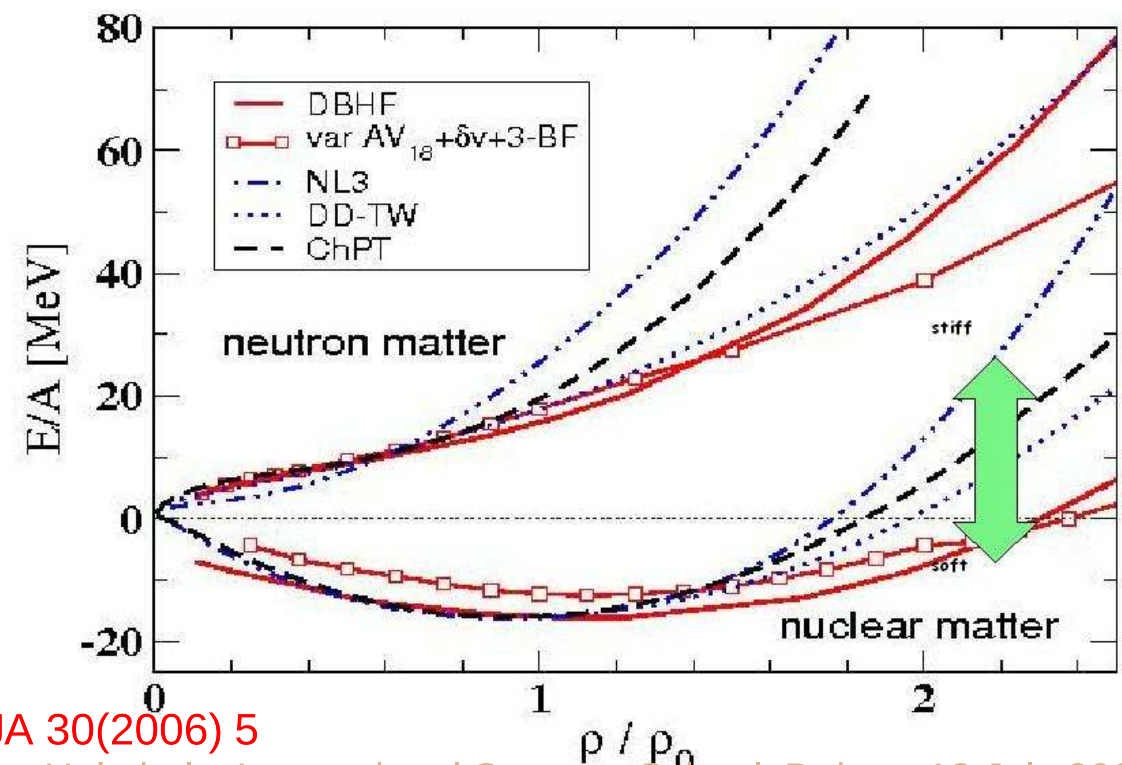
Nuclear Symmetry Energy

The density dependence of $E_{sym}(n)$ is crucial. Some information is available from nuclei (for $n < n_s$). Heavy ion collisions have potential for constraining it for $n > n_s$. It is common to expand $E_{sym}(n)$ as

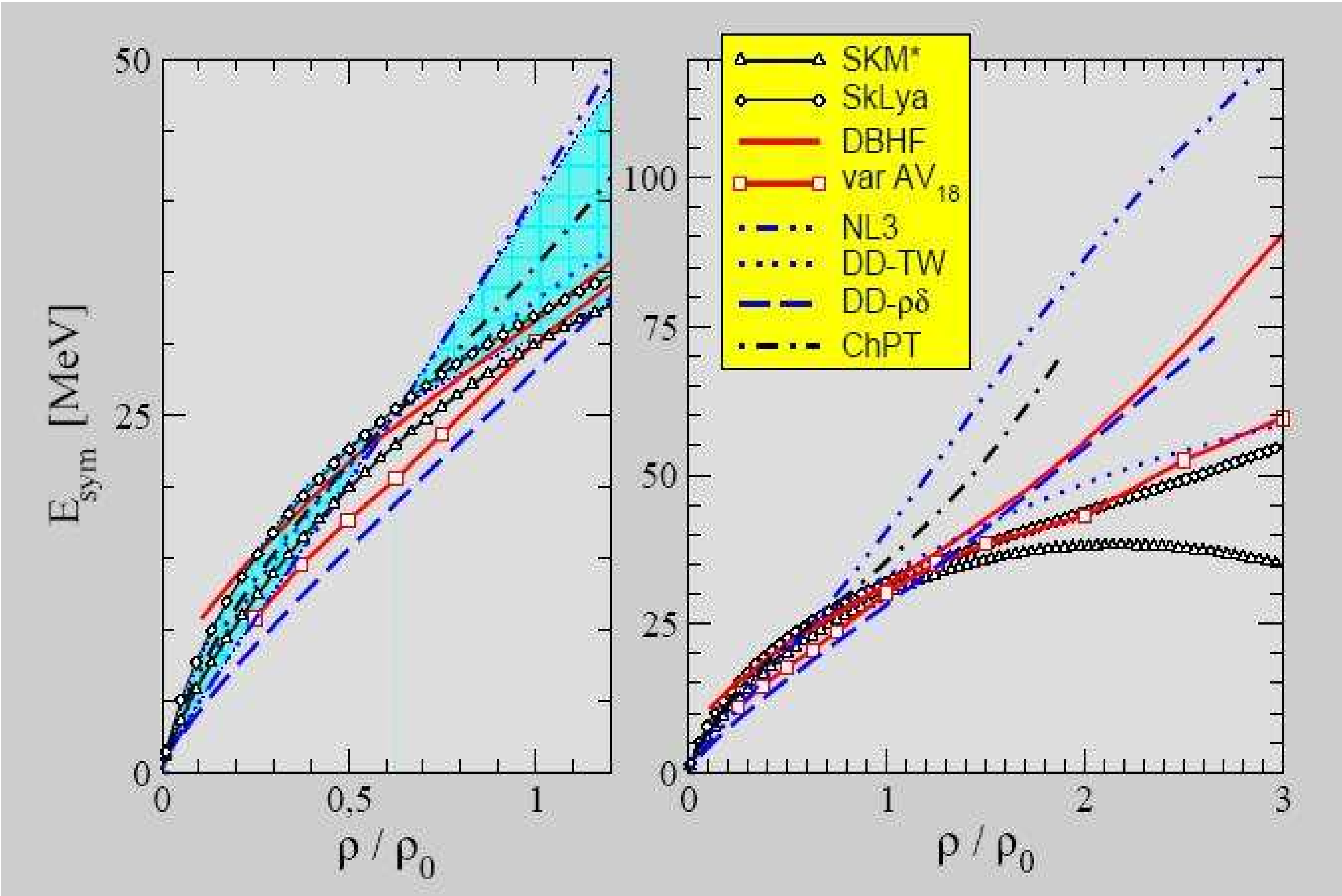
$$E_{sym}(n) \simeq J + \frac{L}{3}\left(\frac{n}{n_s} - 1\right) + \frac{K_{sym}}{18}\left(\frac{n}{n_s} - 1\right)^2 + \cdots$$

$$J = E_{sym}(n_s), \qquad L = 3n_s\left(\frac{\partial E_{sym}}{\partial n}\right)_{n_s}, \qquad K_{sym} = 9n_s^2\left(\frac{\partial^2 E_{sym}}{\partial n^2}\right)_{n_s}$$

Almost no information is available for K_{sym}.



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

Nuclear Mass Formula

Bethe-Weizsäcker (neglecting pairing and shell effects)

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2.$$

Myers & Swiatecki introduced the surface asymmetry term:

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 - S_s (N-Z)^2/A^{4/3}.$$

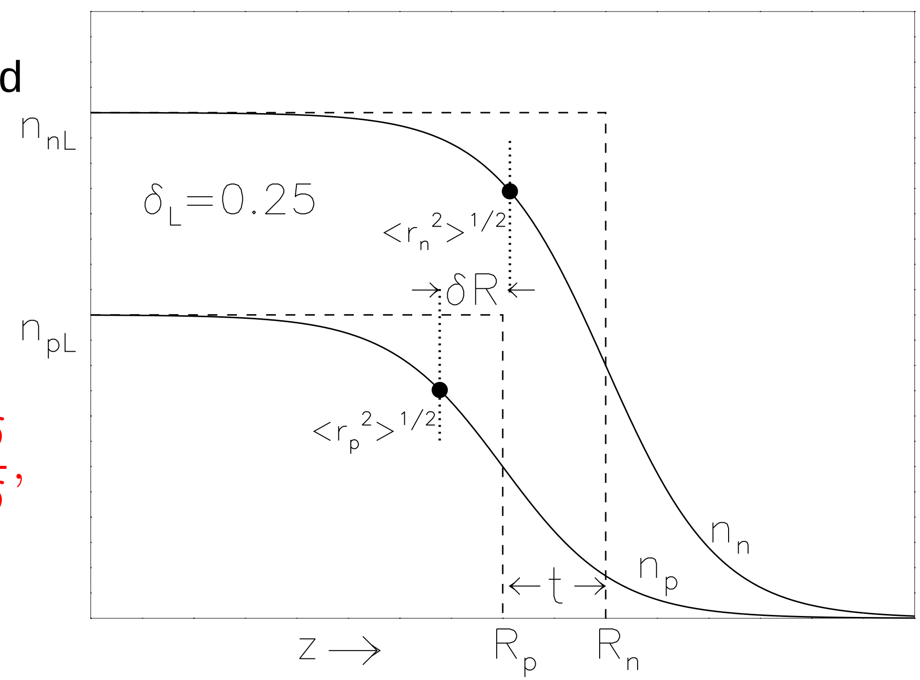
Droplet extension: consider the neutron/proton asymmetry of the nuclear surface.

$$E(A,Z) = (-a_v + S_v\delta^2)(A - N_s) + a_s A^{2/3} + a_C Z^2/A^{1/3} + \mu_n N_s.$$

N_s is the number of excess neutrons associated with the surface, $I = (N-Z)/(N+Z)$, $\delta = 1 - 2x = (A - N_s - 2Z)/(A - N_s)$ is the asymmetry of the nuclear bulk fluid, and μ_n is the neutron chemical potential. From thermodynamics,

$$N_s = -\frac{\partial a_s A^{2/3}}{\partial \mu_n} = \frac{S_s}{S_v}\frac{\delta}{1-\delta} = A\frac{I-\delta}{1-\delta},$$

$$\delta = I\left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1},$$



$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2\left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1}.$$

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

Fitting binding energies results in a strong correlation between S_v and S_s, but not definite values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

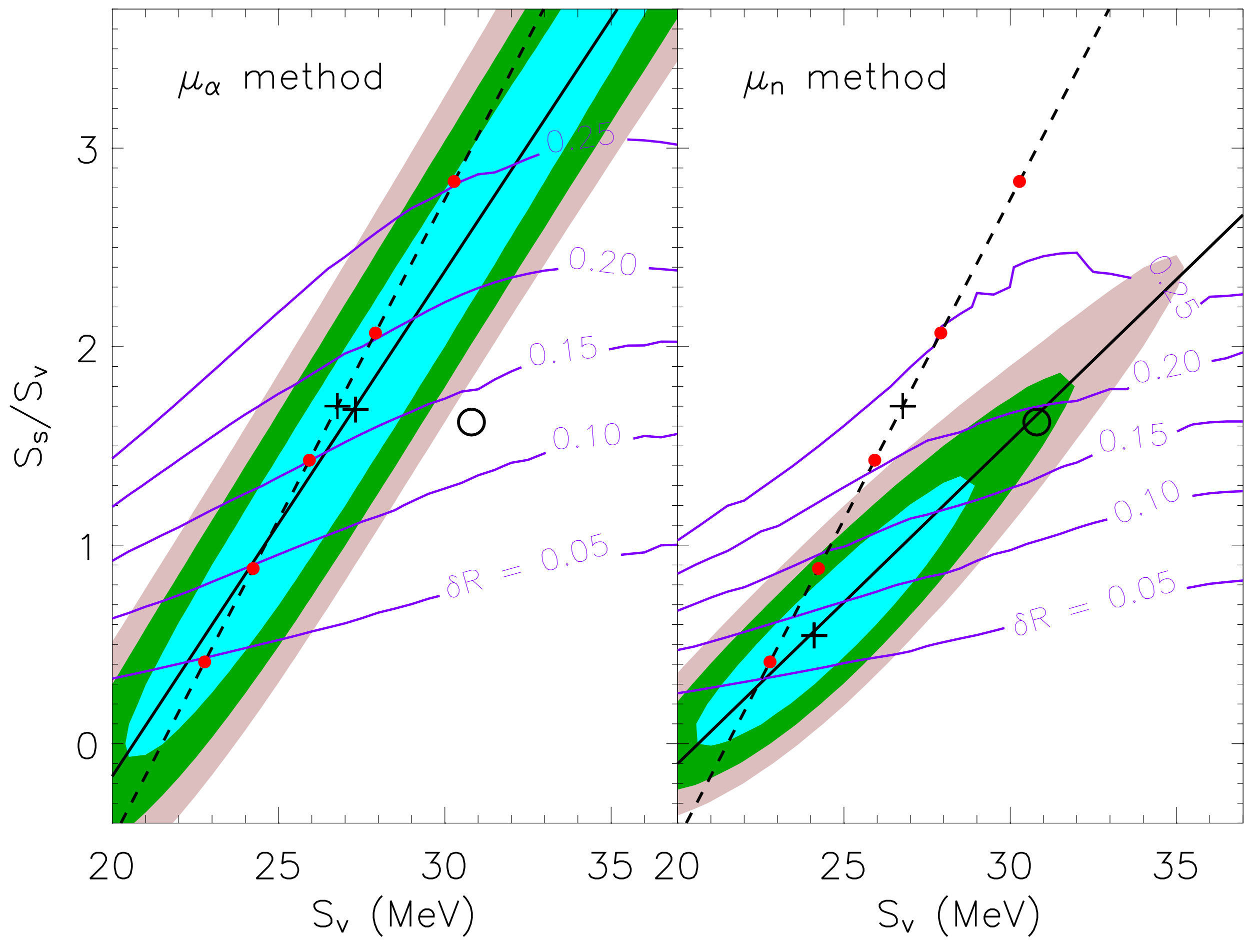
Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)

Crosses: Best fits

Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)



Schematic Dependence

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2}n'^2\,, \quad H_B \simeq n\left[-B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2\right] + E_{sym}(1-2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2}n'^2 = \frac{K}{18}n\left(1-\frac{n}{n_s}\right)^2\,, \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2\sigma_0, \qquad S_s = 4\pi r_0^2\sigma_\delta$

$$\sigma_0 = \int_{-\infty}^{+\infty}[H-\mu_0 n]dz = \int_0^{n_s}(H_B-\mu_0 n)\frac{dn}{n'} = \frac{4}{45}\sqrt{QKn_s^3}$$

$$t_{90-10} = \int_{0.1n_s}^{0.9n_s}\frac{dn}{n'} = 3\sqrt{\frac{Qn_s}{K}}\int_{0.1}^{0.9}\frac{du}{\sqrt{u}(1-u)} \simeq 9\sqrt{\frac{Qn_s}{K}}$$

$$\sigma_\delta = S_v\sqrt{\frac{Q}{2}}\int_0^{n_s} n\left(\frac{S_v}{E_{sym}}-1\right)(H_B-\mu_0 n)^{-1/2}dn$$

$$= \frac{S_v t_{90-10} n_s}{3}\int_0^1\frac{\sqrt{u}}{1-u}\left(\frac{S_v}{E_{sym}}-1\right)du$$

$$E_{sym} \simeq S_v\left(\frac{n}{n_s}\right)^p \Longrightarrow \int \rightarrow 0.28\ (p=\frac{1}{2})\,,\ 0.93\ (p=\frac{2}{3})\,,\ 2.0\ (p=1)$$

$$E_{sym} \simeq S_v + \frac{L}{3}\left(\frac{n}{n_s}\right) \Longrightarrow \int \rightarrow 2-\sqrt{\frac{3S_v}{L}-1}\tan^{-1}\sqrt{\left(1+\frac{S_v}{3L}\right)^{-1}} \simeq 1+\frac{L}{3S_v}$$

Schematic Dependence

$$\frac{S_s}{S_v} \simeq \frac{t_{90-10}}{r_0} \int \simeq 2.05 \int \Longrightarrow 0.57 \qquad 1.91 \qquad 4.1\,.$$

For Pb^{208}:

$$\delta R = \sqrt{\frac{3}{5}}(R_n - R_p) \simeq \frac{t_{90-10}}{6} \frac{A-2Z}{Z(1-Z/A)} \int \Longrightarrow 0.05 \qquad 0.16 \qquad 0.35$$

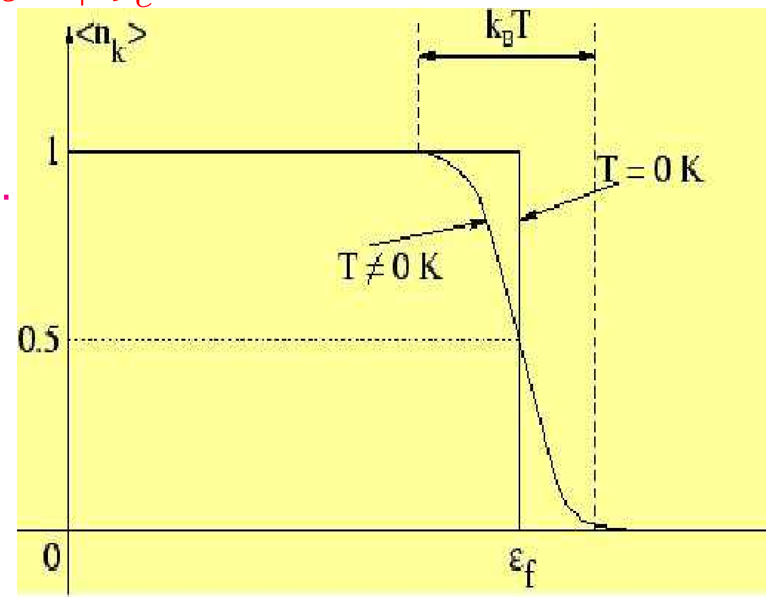
PREX experiment (E06002) at Jefferson Lab to measure the neutron radius of lead to about 1% accuracy (current accuracy is about 5%) using the parity violating asymmetry in elastic scattering due to the weak neutral interaction. Requires corrections for Coulomb distortions (Horowitz).

PREX Workshop August 17–19, 2008

Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \to p + e^- + \nu_e\,, \qquad p \to n + e^+ + \bar{\nu}_e$$



Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \geq 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.
With muons ($n > 2n_s$), $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.

$$(n,p) + n \to (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \to (n,p) + n + e^+ + \bar{\nu}_e$$

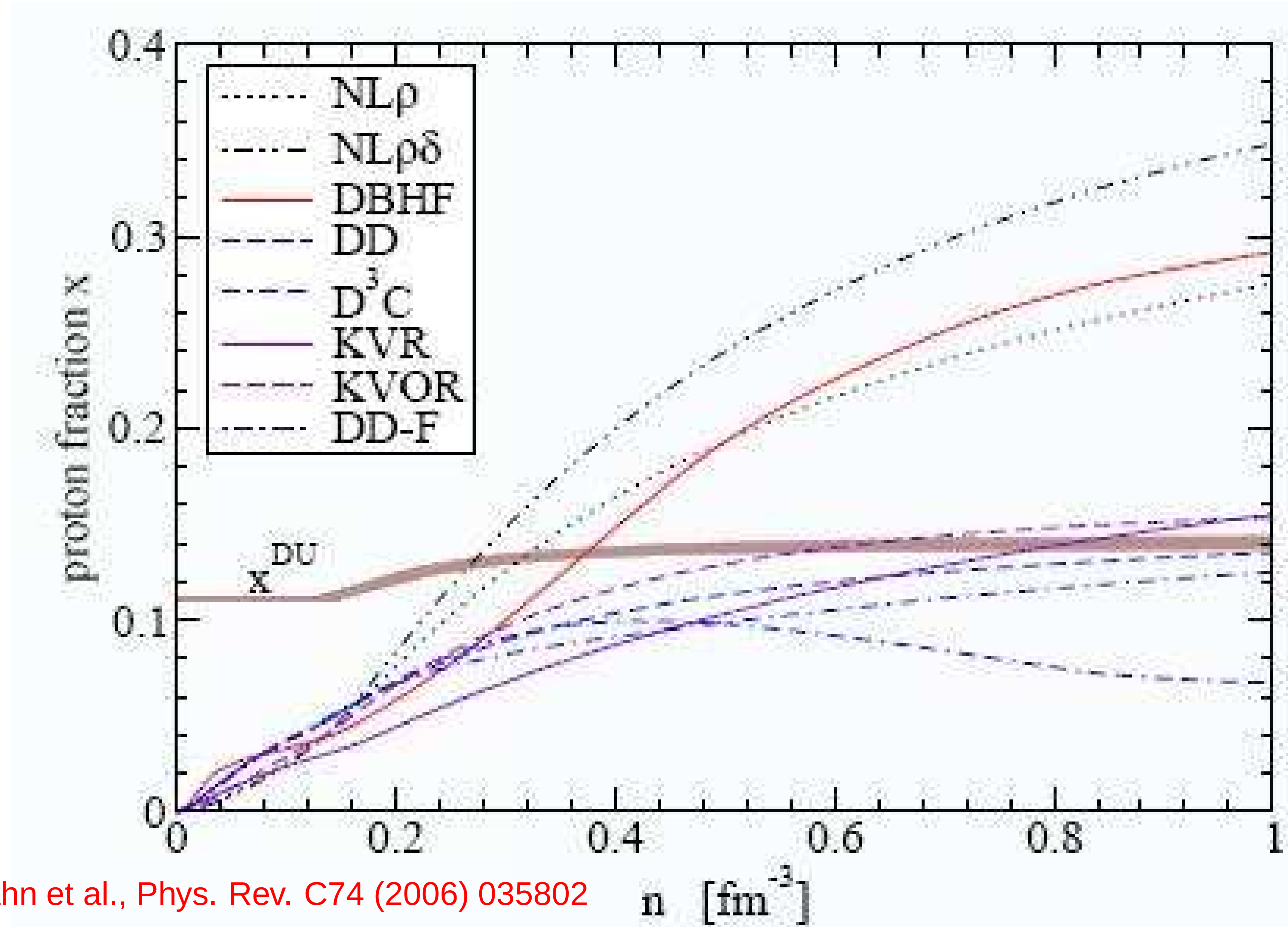
Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA}\,.$$

Beta equilibrium composition:

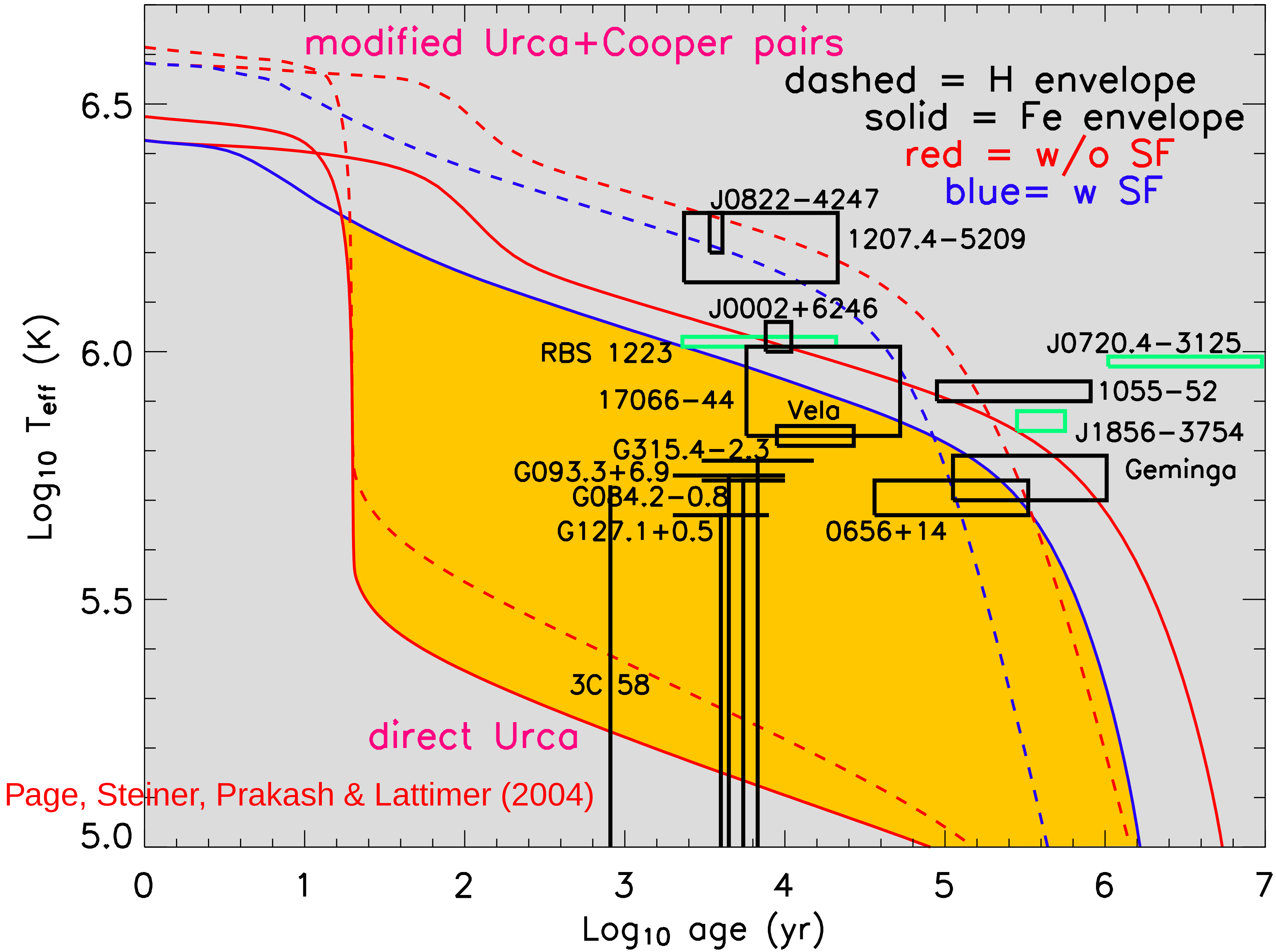
$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04 \left(\frac{n}{n_s}\right)^{0.5-2}\,.$$

Direct Urca Threshold



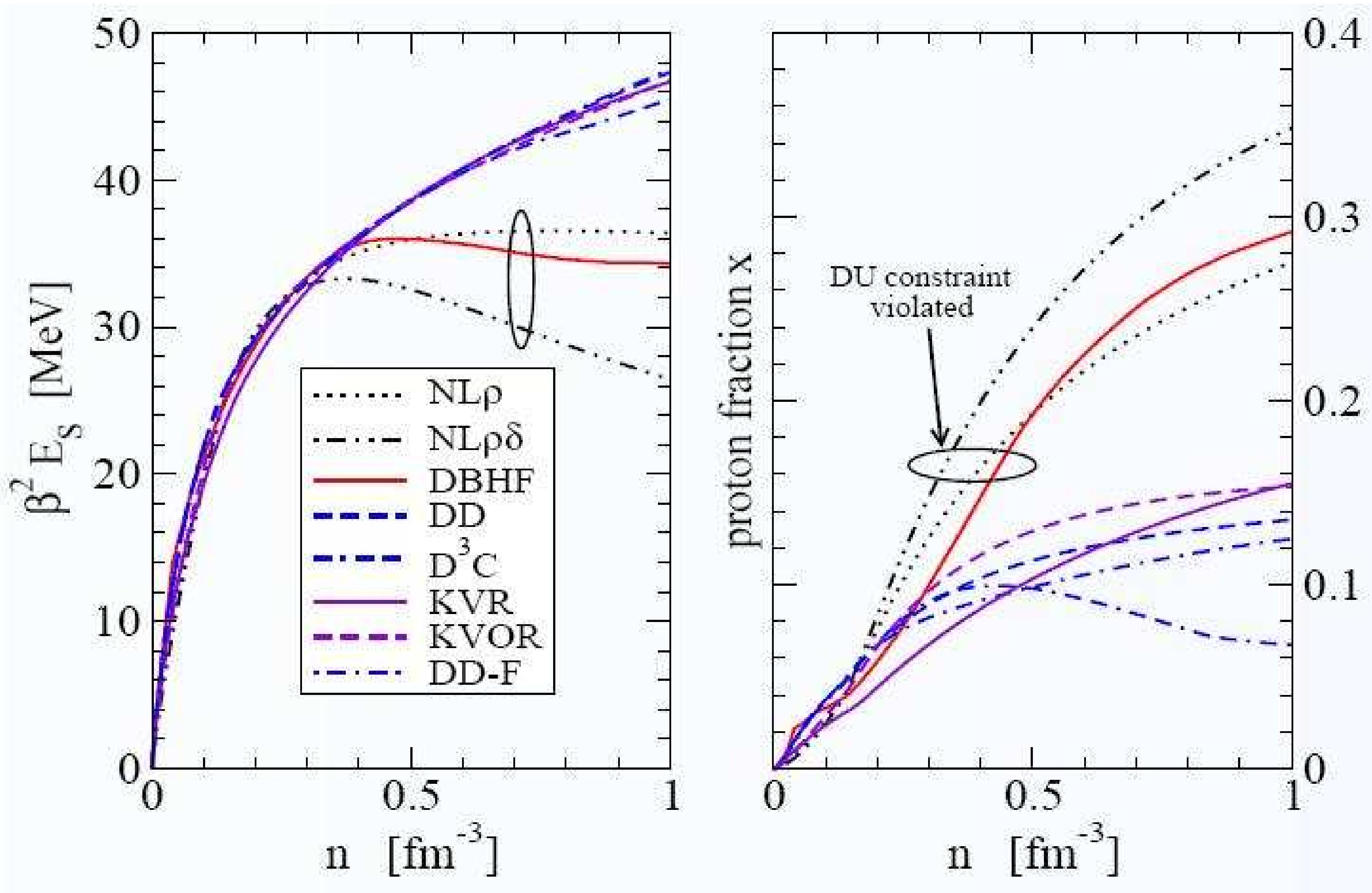
Klähn et al., Phys. Rev. C74 (2006) 035802

Neutron Star Cooling



Direct Urca Constraint?

Apparently, some/most neutron stars don't have accelerated cooling. If direct Urca doesn't occur for these stars, the direct Urca density threshold is above $2 - 3n_s$, ruling out too-rapid density-dependence for $E_{sym}(n)$. Also, hyperon threshold density is high. However, suppression of accelerated cooling by superfluidity could invalidate this.



Klähn et al., Phys. Rev. C74 (2006) 035802

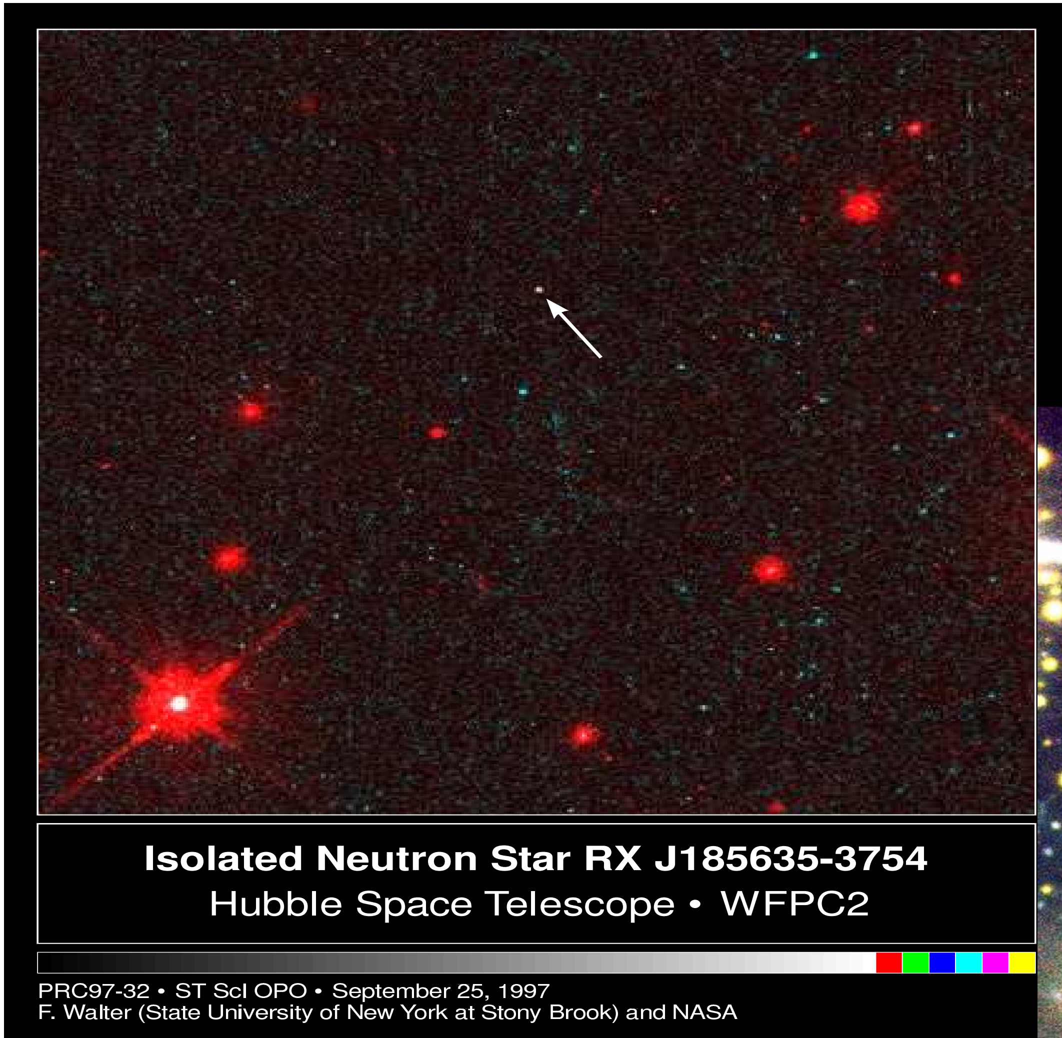
Radiation Radius

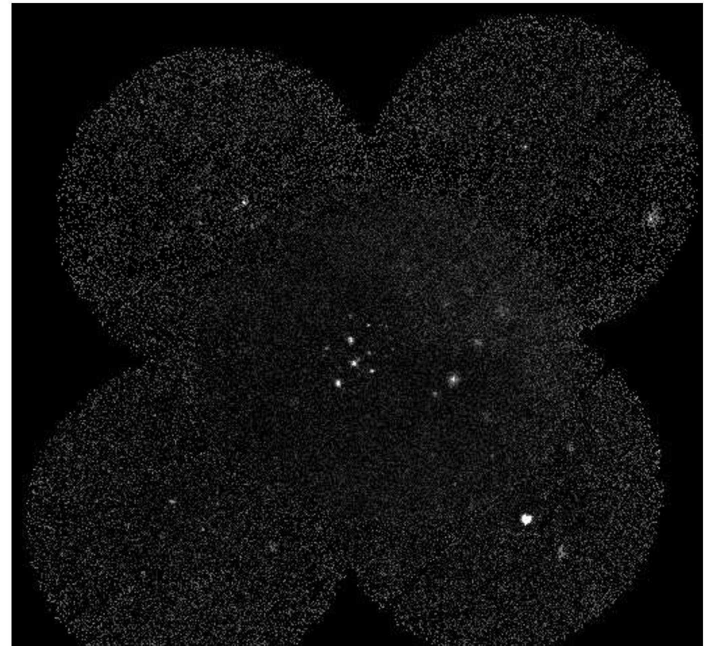
- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

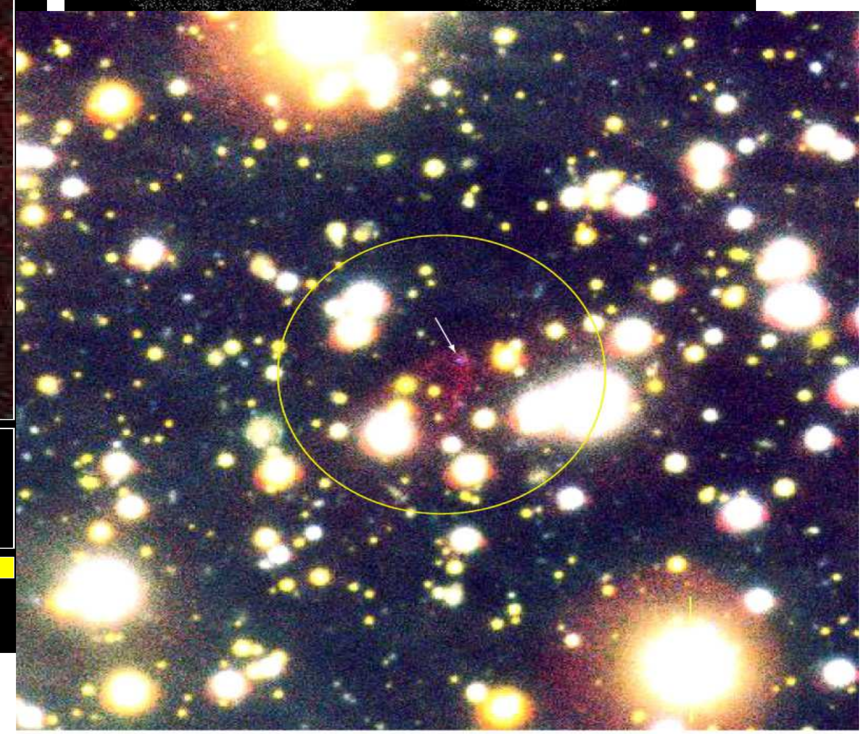
$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

RX J1856-3754

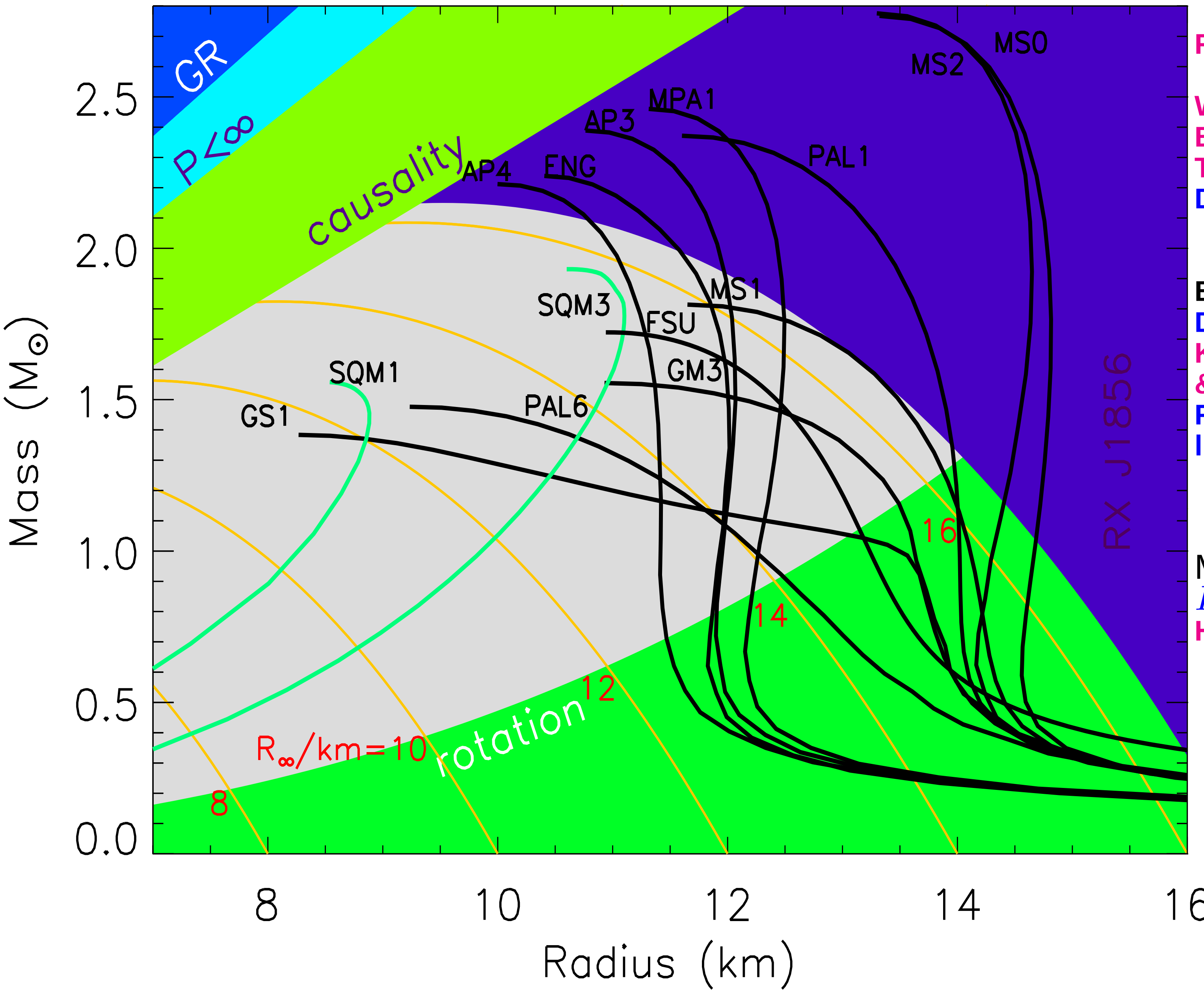






A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)
ESO PR Photo 23b/00 (11 September 2000)
©European Southern Observatory

Radiation Radius: Nearby Neutron Star



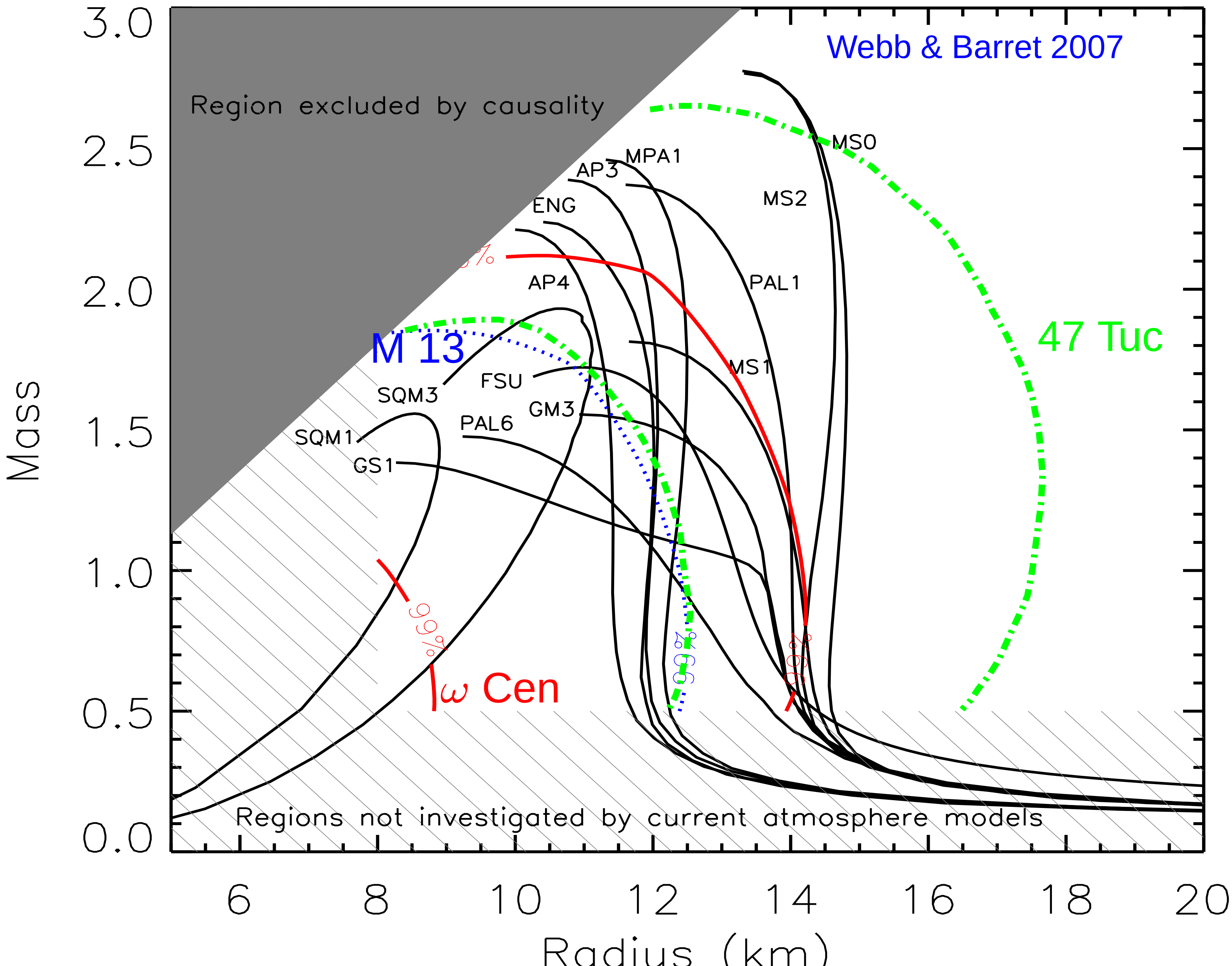
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk
& Anderson 2002
Raises R_∞
limit to 19.5 km

Magnetic H atmosphe
$R_\infty \approx 17$ **km**
Ho et al. 2007

Radiation Radius: Globular Cluster Sources



The Neutron Star Crust

Hydrostatic equilibrium in the crust:

$$\frac{dp(r)}{m_b n} = \frac{d\mu}{m_b} = -\frac{GM}{r^2 - 2GMr/c^2} dr.$$

$$\frac{\mu_t - \mu_0}{m_b c^2} = \frac{1}{2} \ln \left[\frac{r_t(R - 2GM/c^2)}{R(r_t - 2GM/c^2)} \right].$$

Defining $\ln \mathcal{H} = 2(\mu_t - \mu_0)/m_b c^2$,

$$\frac{\Delta}{R} \equiv \frac{R - R_t}{R} = \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)(1-2\beta)\frac{R^2}{2M}.$$

Crustal moment of inertia

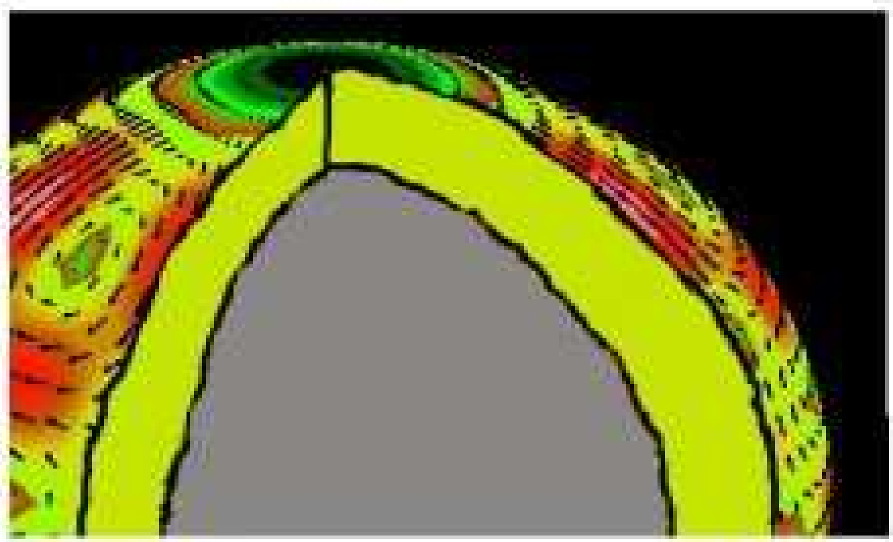
$$\Delta I = \frac{8\pi}{3c^4} \int_{R-\Delta}^{R} r^4 (\epsilon + p) e^{-\lambda} j\omega dr \simeq \frac{8\pi\omega(R)}{3Mc^2} \int_{p(R-\Delta)}^{0} r^6 dp$$

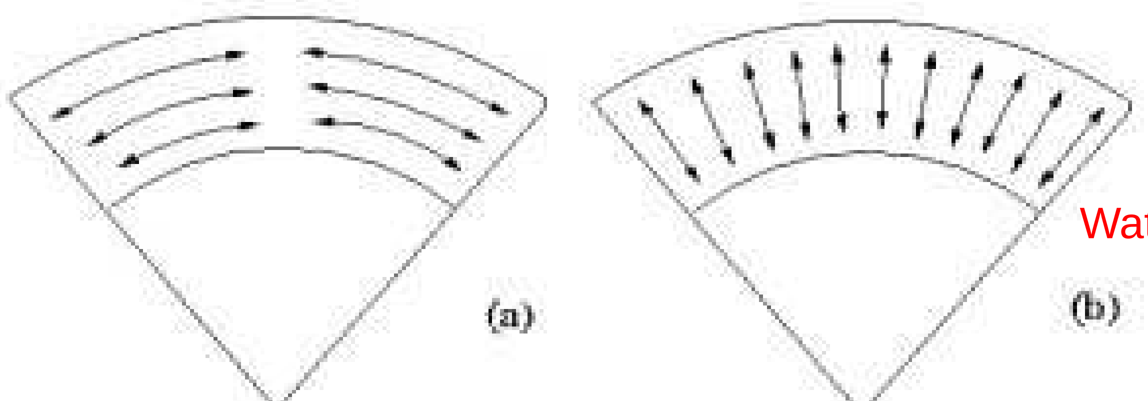
Approximately, $\int_{p(R-\Delta)}^{0} r^6 dp \simeq R^6 p_t e^{-4.8\Delta/R}$. $\quad p_t < 0.65$ MeV fm^{-3}.

$$\frac{\Delta I}{I} \simeq \frac{8\pi R^4 p_t}{3M^2c^2} \left(\frac{MR^2}{I} - 2\beta \right) e^{-4.8\Delta/R}$$

Giant Flares in Soft Gamma-Ray Repeaters (SGRs)

Quasi-periodic oscillations observed following giant flares in three soft gamma-ray repeaters (Israel et al. 2005; Strohmayer & Watts 2005, 6; Watts & Strohmayer 2006) which are believed to be highly magnetized neutron stars (magnetars).
Fields decay and twist, becoming periodically unstable. Eventually, the field lines snap and shift, launching starquakes and bursts of gamma-rays. Torsional shear modes are much easier to excite than radial modes.

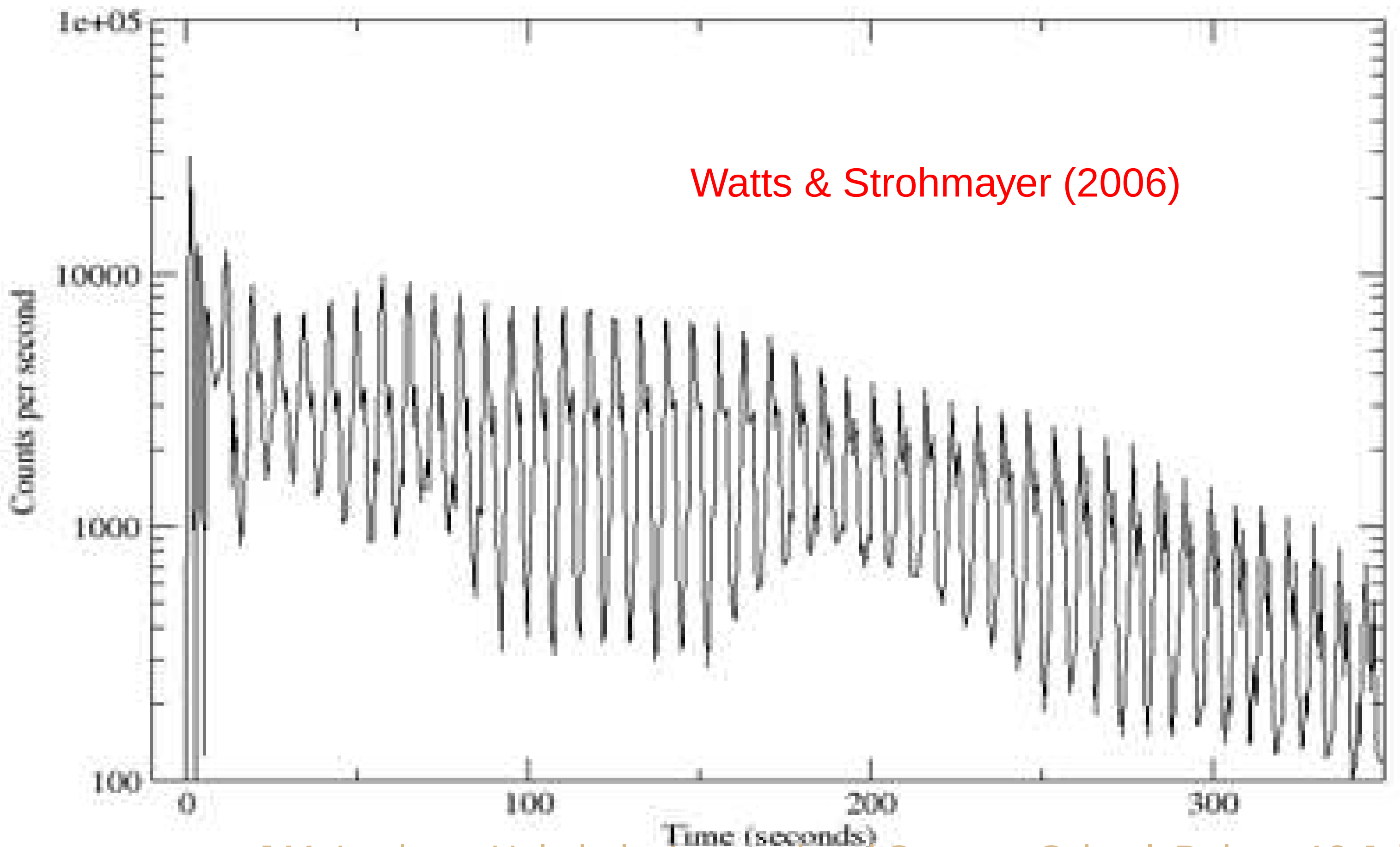




Watts & Reddy (2006)

Observations

Typical frequencies observed: 28-29 Hz, 50-150 Hz, 625 Hz
(SGRs 1806-20, 1900+14, 0526-66)
Frequencies of fundamental mode (28-29 Hz) agree well with expected torsional mode of neutron star crust (Duncan 1998)



Analysis–Samuelsson & Andersson

Include crust elasticity in a relativistic context of axial oscillations; calculate both fundamental and overtone frequencies using the Cowling approximation: neglect dynamical nature of spacetime (i.e., perturbations of gravitational field). Ignores magnetic fields and superfluidity and is thus insensitive to core structure.

$$\begin{aligned} F'' + A'F' + BF &= 0 \\ e^A &= r^4 e^{(\nu-\lambda)/2}(\epsilon + p)v_r^2 \\ B &= e^{\lambda} v_r^{-2}[e^{-\nu}\omega^2 - v_t^2(\ell-1)(\ell+2)r^{-2}] \\ F'(R) &= F'(R_c) = 0 \end{aligned}$$

F: amplitude of fluid oscillations

ν, λ: metric functions: at surface $e^{\nu} \simeq e^{-\lambda} \simeq 1 - 2\beta$

v_r, v_t: radial/transverse shear velocities

Analytical approximation:

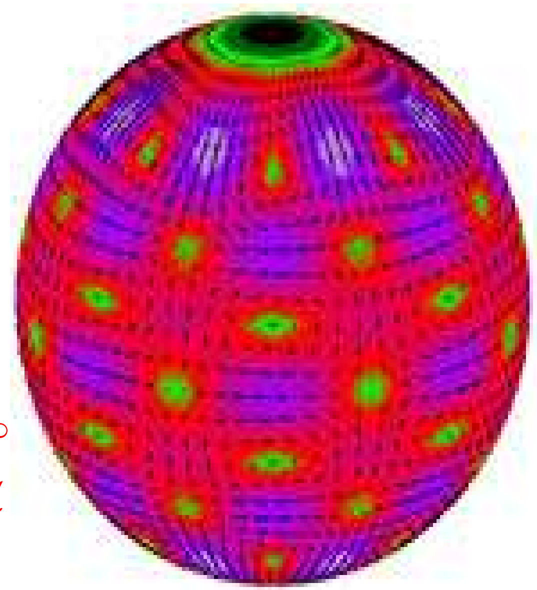
$\omega >> v_t/r;\ v_r \simeq v_t \simeq v$

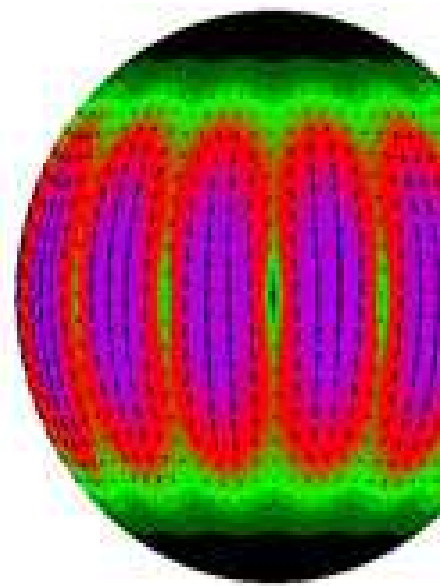
$F'(R) = F'(R_t) = 0 \Rightarrow \int_{R_t}^{R} \sqrt{B} dr = n\pi$

$\omega^2 - \frac{n\pi v \omega}{\Delta} e^{(\nu-\lambda)/2} - \frac{v^2(\ell-1)(\ell+2)}{2RR_t} e^{\nu} = 0$

$n = 0: \quad \omega_{0,\ell} \simeq e^{\nu/2} v \sqrt{\frac{(\ell-1)(\ell+2)}{2RR_c}} \sim v/R_\infty$

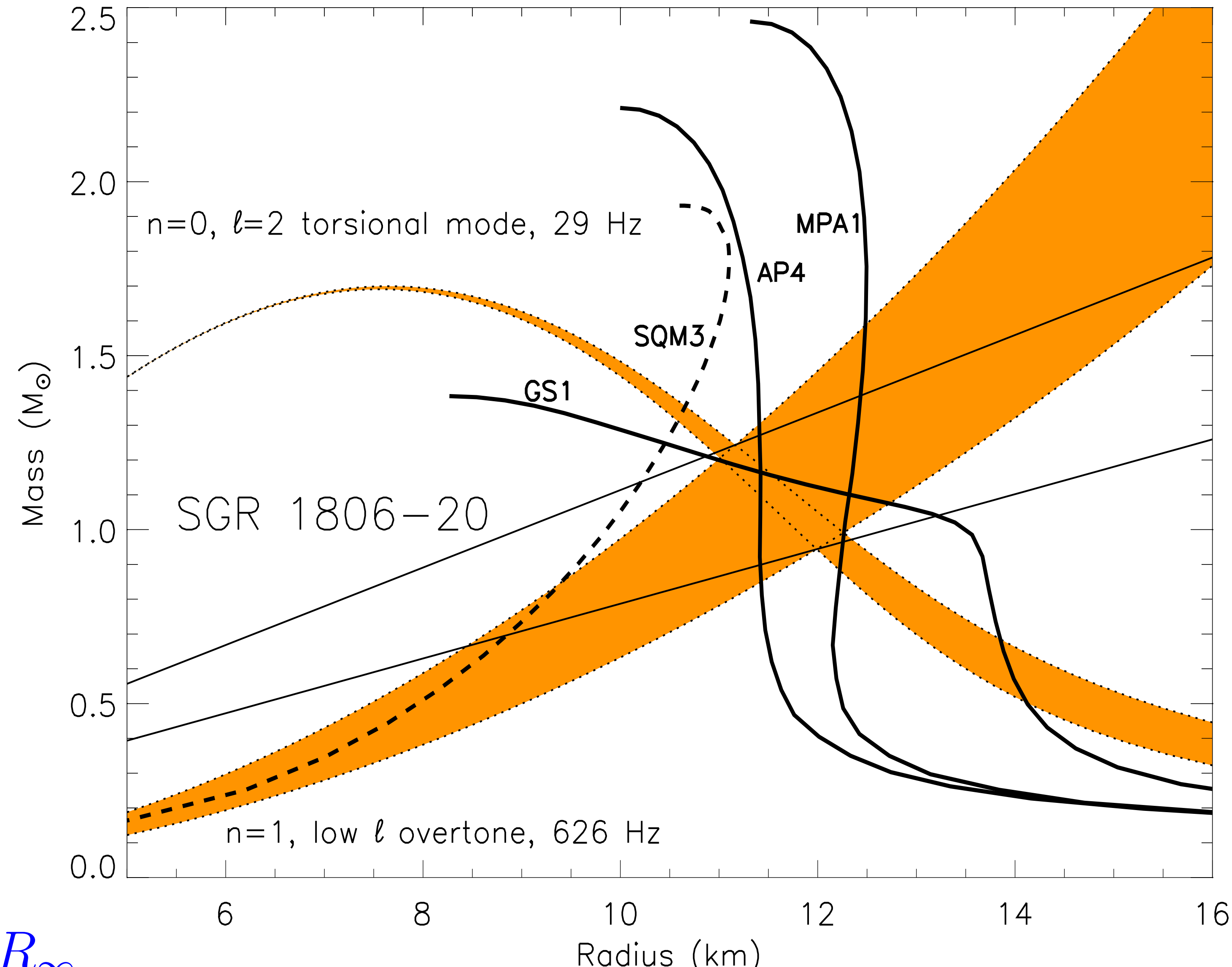
$n > 0: \quad \omega_n \simeq (1-2\beta) n\pi v \Delta^{-1} \propto \frac{Mv}{R^2} \ln \mathcal{H}$





Watts & Strohmayer (2006)

Neutron Star Seismology



$$f_{n=0} \sim v_t / R_\infty$$
$$f_{n>0} \sim v_r(1-2\beta)/\Delta \sim (v_r R^2 \ln \mathcal{H})/M$$

Strohmayer & Watts (2005)
Samuelsson & Andersson (2006)
Lattimer & Prakash (2006)

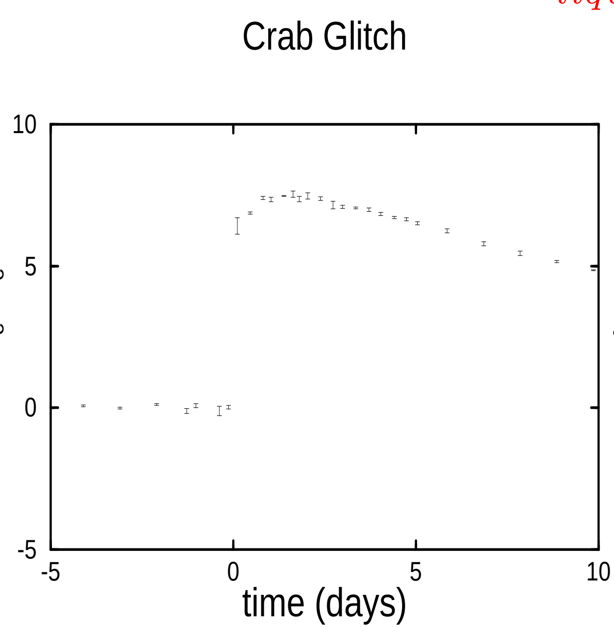
Pulsar Glitches

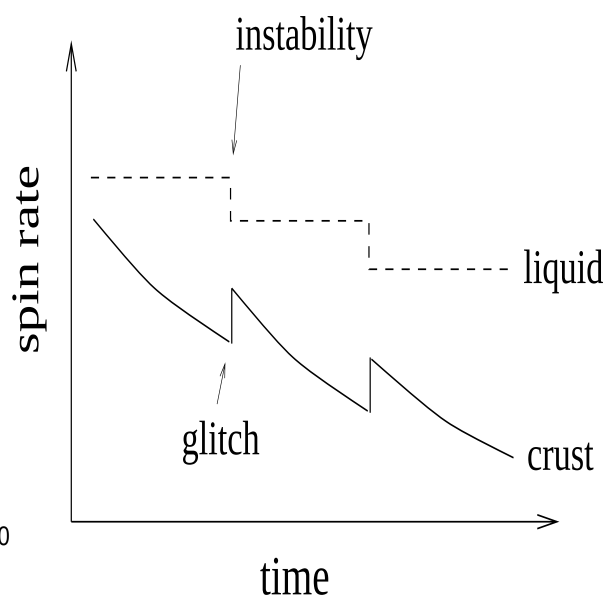
Pulsars occasionally undergo glitches, when the spin rate "hiccups".
Each glitch changes the angular momentum of the star by $\Delta J = I_{liquid}\Delta\Omega$.
The glitches are stochastic, but total angular momentum transfer is regular.

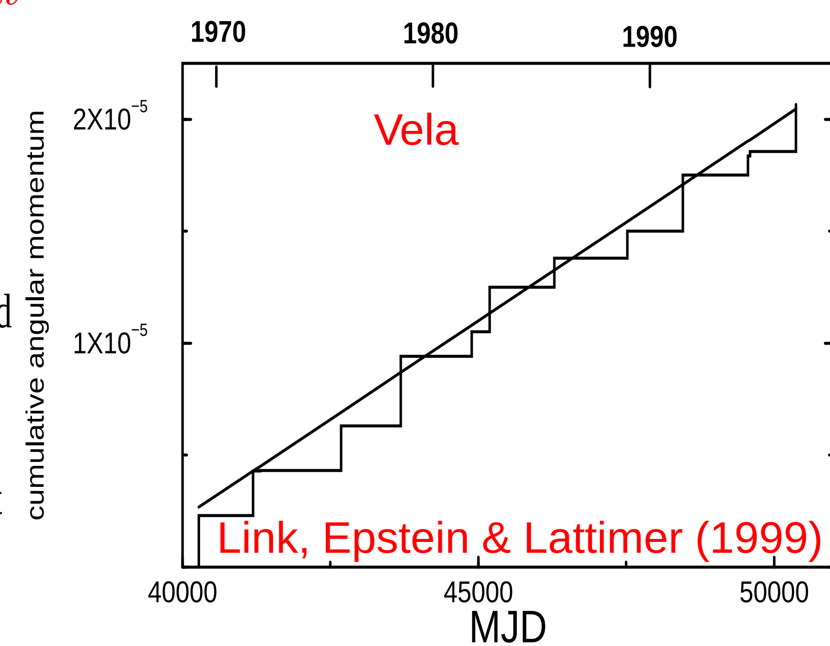
$$J(t) = I_{liquid}\bar{\Omega}\sum\frac{\Delta\Omega}{\Omega}, \qquad \dot{J} = I_{liquid}\bar{\Omega}\frac{d(\Delta\Omega/\Omega)}{dt}.$$

A leading model is that as the crust slows due to pulsar's dipole radiation, the interior acquires an excess differential rotation. The acquired excess is limited: $\dot{J} \le \dot{\Omega} I_{crust}$.

$$\frac{I_{crust}}{I_{liquid}} = \frac{I_{crust}}{I - I_{crust}} \ge \frac{\bar{\Omega}}{\dot{\Omega}}\frac{d\sum(\Delta\Omega/\Omega)}{dt} \simeq 0.014.$$

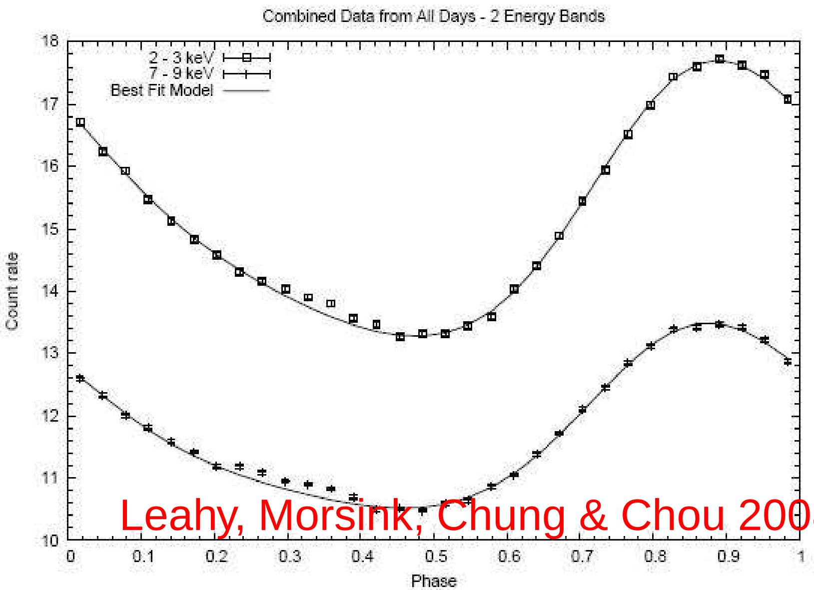




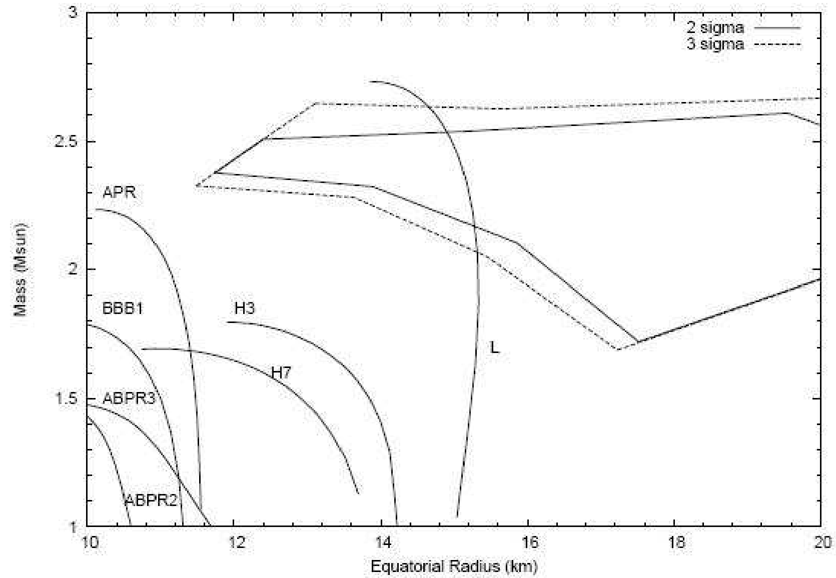


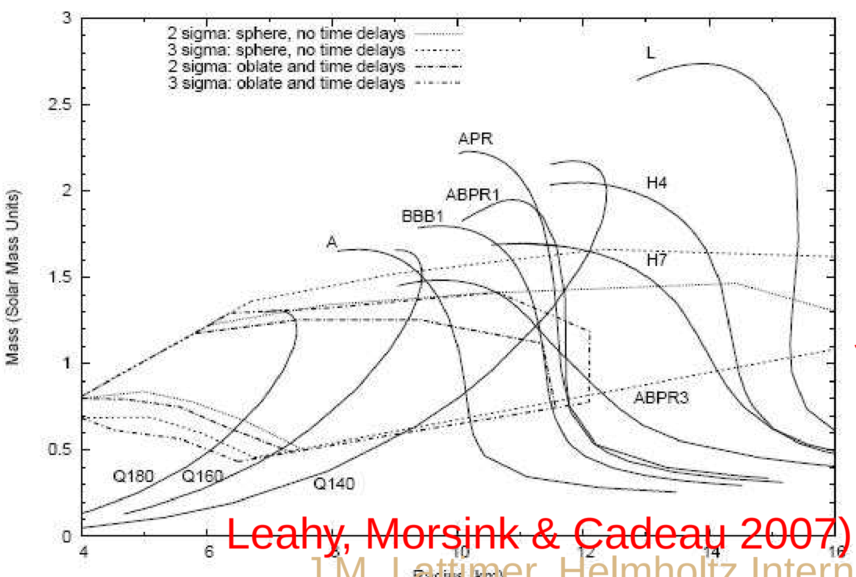
Constraints From X-Ray Pulse Shapes

Pulse profiles for accretion-powered pulsations from XTE J1814 from June 5-27, 2003 (excluding bursts). Stellar spin frequency is 314 Hz. Analysis yields two χ^2 minima, the smaller being excluded by its unphysical nature.



Leahy, Morsink, Chung & Chou 2008)





SAX J1808.4-3658

Leahy, Morsink & Cadeau 2007)

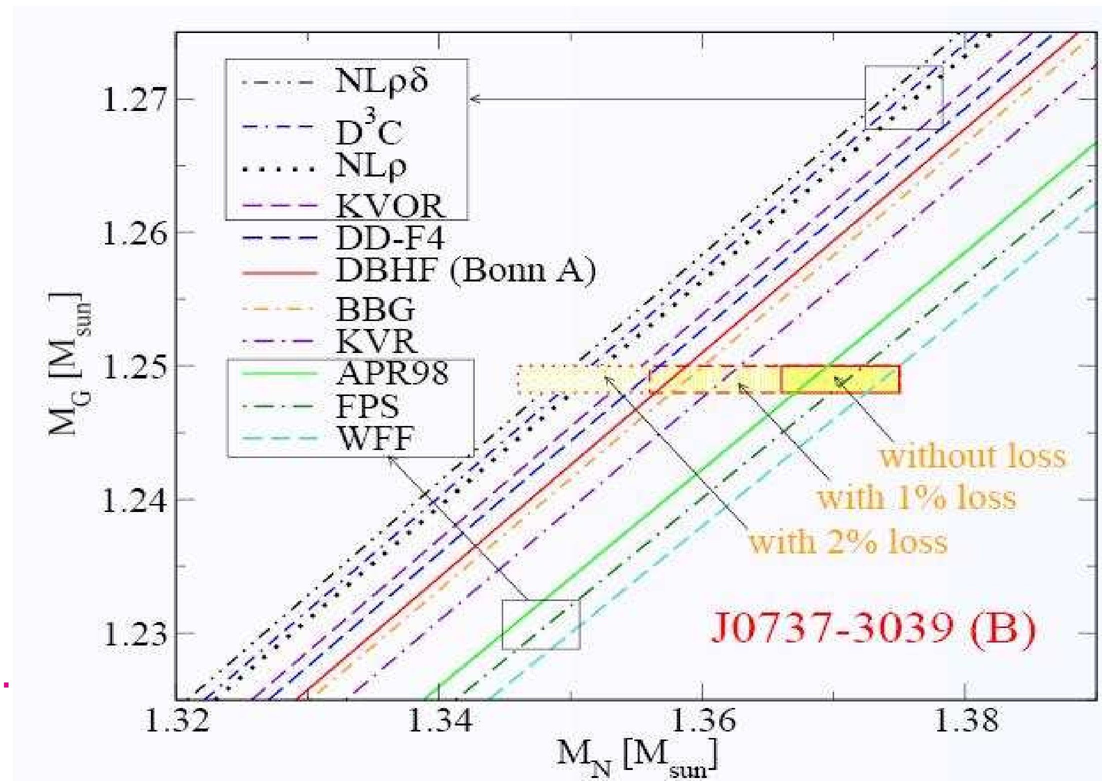
Binding Energy Constraints

Neutrino emission from supernova

Although ν_e in stellar core are degenerate with typical energies ~ 300 MeV, they must diffuse to surface ($\lambda_\nu \sim 10$ cm). Their energies are thermalized, creating neutrinos of all flavors in roughly equal proportions. Emitted neutrinos have typical energy $E_\nu \sim 10$ MeV and contain 99.7% of neutron star binding energy. From the number and energies of observed neutrinos, the total binding energy can be obtained. Accuracy $\sim 1/\sqrt{N} \sim 0.01$ for a galactic-center supernova ($d = 8$ kpc).

Low-mass neutron stars

The smallest measured mass so far: PSR J0737-3039B, with $M_G = 1.249 \pm 0.001$ M$_\odot$.
If progenitor is ONeMg white dwarf destabilized by e^- captures on Mg and Ne, supernova expected with little mass loss (Podsiadlowski et al. 2005). Critical explosion mass: $1.366 < M_N < 1.375$ M$_\odot$.
BE$= 0.12$ M$_\odot$, $\beta = 0.15$, $R = 12.4$ km.



Other Observable Quantities

- **Redshift** $z = (1 - 2GM/Rc^2)^{-1/2} - 1$

  Possible lines from active X-ray bursters XTE J1814-338 $z < 0.38$, 4U1820-30 $0.20 < z < 0.30$, EXO 0748-676 $z \simeq 0.345$ (possibly Fe XXV and XXVI Cottam, Paerels & Mendez 2002).

  Gravitational light-bending analysis of emissions from Her X-1:

  $0.247 < z < 0.268 \quad (1.29 < M/\mathrm{M}_\odot < 1.59 \Rightarrow 10.1 < R/\mathrm{km} < 13.1)$

  Permits determination of M and R if both z and R_∞ known.

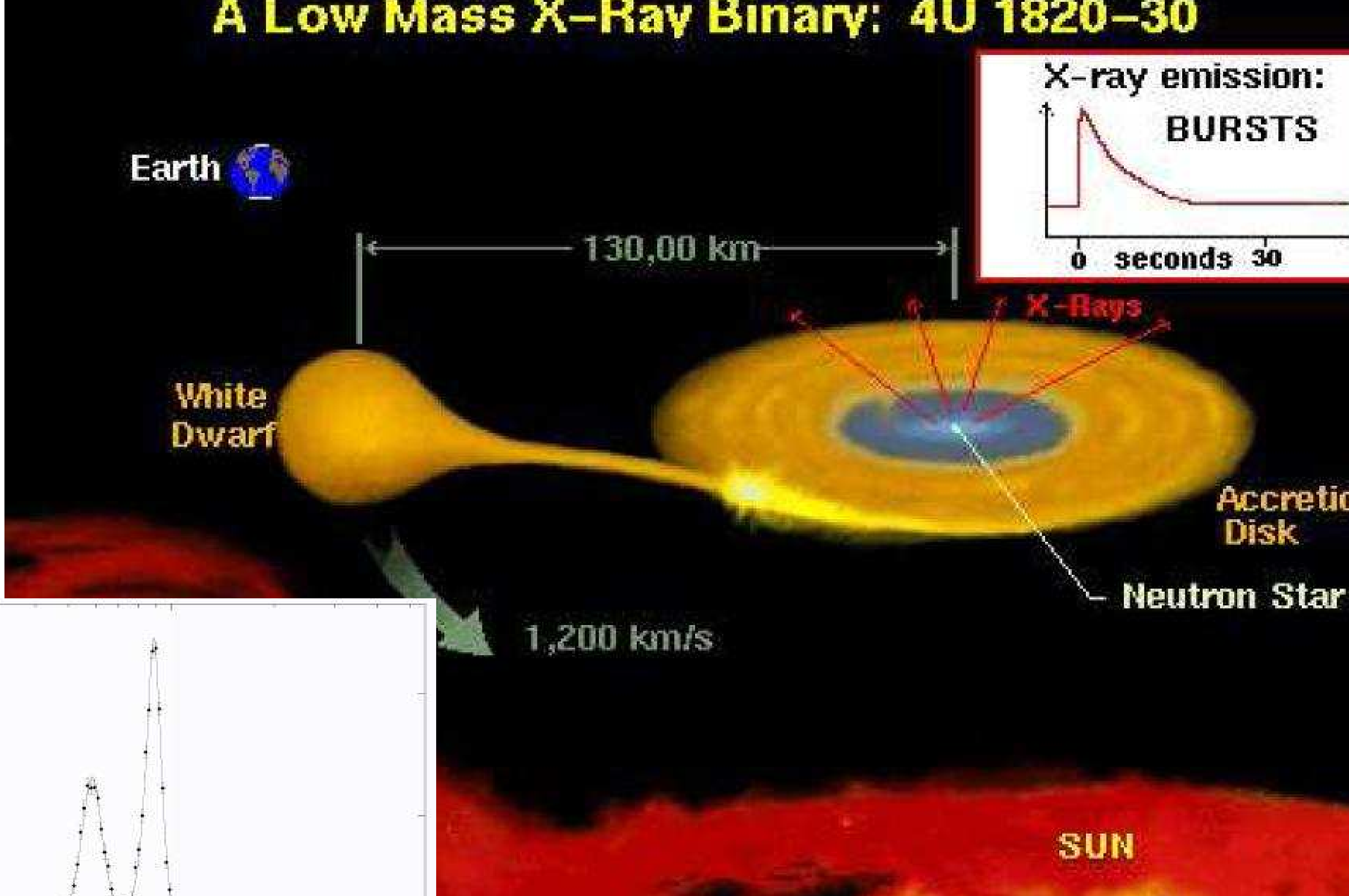
  $R = R_\infty(1+z)^{-1}, \quad M = R_\infty c^2(1+z)^{-1}[1-(1+z)^{-2}]/2G$

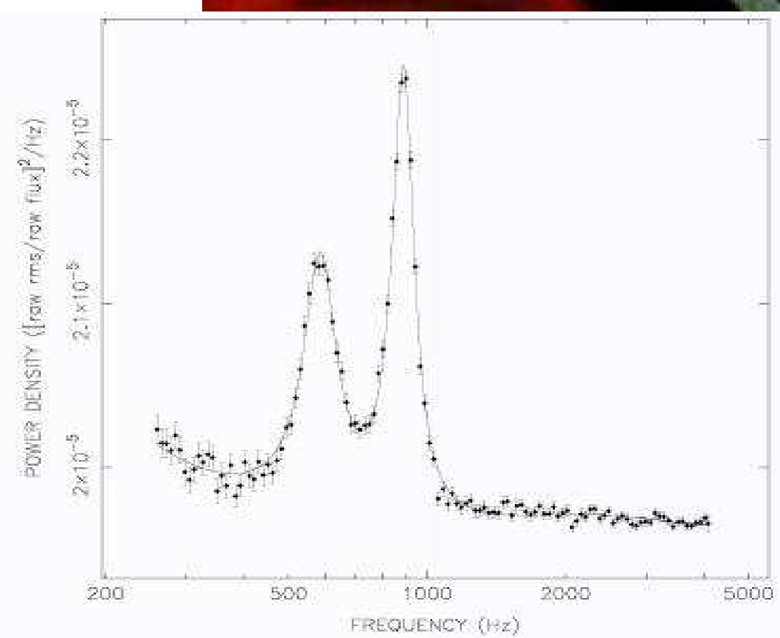
- **QPOs**

  Accreting matter in low mass X-ray binaries emits X-rays due to conversion of gravitational energy into thermal energy, $kT \simeq GM/R$. Fourier analysis of the X-rays often show quasi-periodic frequency peaks, including a high-frequency peak around 1300 Hz. It is thought that this peak could be associated with the orbital frequency $\omega = \sqrt{GM/R_A^3}$ of the inner radius of accretion disk (Lamb & Miller 2000). In GR, the innermost circular stable orbit about a non-rotating star has a radius $R_{ISCO} = 6GM/c^2$.

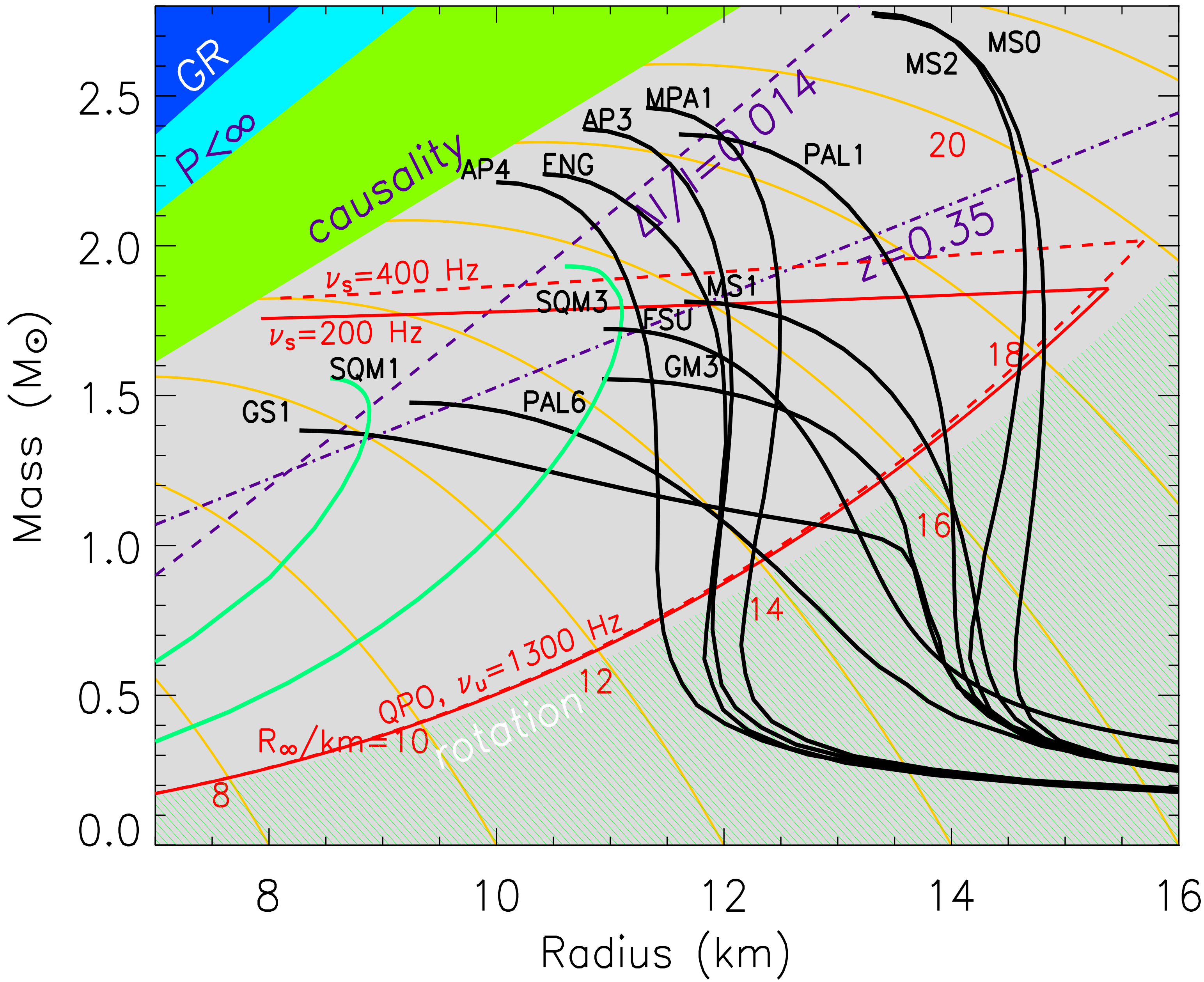
  From $R < R_A < R_{ISCO}$,

$$M < \frac{c^3}{6^{3/2}G\omega} = 2.2\left(\frac{1000\ \mathrm{Hz}}{\nu}\right)\ \mathrm{M}_\odot, \qquad R < \frac{c}{6^{1/2}\omega} = 19.5\left(\frac{1000\ \mathrm{Hz}}{\nu}\right)\ \mathrm{km}.$$





Main figure courtesy D. Page; inset R. van der Klies 2000


GR
P<∞
causality
ΔI/I=0.014
z=0.35
νs=400 Hz
νs=200 Hz
QPO, νu=1300 Hz
R∞/km=10
rotation
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
MS1
SQM3
FSU
GM3
SQM1
PAL6
GS1
20
18
16
14
12
8
Mass (M⊙)
Radius (km)
0.0
0.5
1.0
1.5
2.0
2.5
8
10
12
14
16

Moment of Inertia

- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2} \vec{L} \times \vec{S}_A$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2} \vec{L} \times \vec{S}_A$
- Precession Period:

$$P_p = \frac{2(M_A + M_B)ac^2}{GM_B(4M_A + 3M_B)} P(1 - e^2) \simeq 74.9 \text{ years}$$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2} \vec{L} \times \vec{S}_A$
- Precession Period:

$$P_p = \frac{2(M_A + M_B)ac^2}{GM_B(4M_A + 3M_B)} P(1 - e^2) \simeq 74.9 \text{ years}$$

- Precession Amplitude $\propto \vec{S}_A \times \vec{L}$:

$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|} \sin\theta \simeq \frac{I_A(M_A + M_B)}{a^2 M_A M_B} \frac{P}{P_A} \sin\theta \simeq (3.6 - 7.2) \sin\theta \times 10^{-5}$$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2}\vec{L} \times \vec{S}_A$
- Precession Period:

$$P_p = \frac{2(M_A + M_B)ac^2}{GM_B(4M_A + 3M_B)} P(1 - e^2) \simeq 74.9 \text{ years}$$

- Precession Amplitude $\propto \vec{S}_A \times \vec{L}$:

$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|} \sin\theta \simeq \frac{I_A(M_A + M_B)}{a^2 M_A M_B} \frac{P}{P_A} \sin\theta \simeq (3.6 - 7.2) \sin\theta \times 10^{-5}$$

- Delay in Time-of-Arrival:

$$\Delta t = \left(\frac{M_B}{M_A + M_B} \right) \frac{a}{c} \delta_i \cos i \approx 0.4 - 4.0 \ \sin\theta \ \mu\text{s}$$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2}\vec{L}\times\vec{S}_A$
- Precession Period:

$$P_p = \frac{2(M_A+M_B)ac^2}{GM_B(4M_A+3M_B)}P(1-e^2) \simeq 74.9 \text{ years}$$

- Precession Amplitude $\propto \vec{S}_A \times \vec{L}$:

$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|}\sin\theta \simeq \frac{I_A(M_A+M_B)}{a^2 M_A M_B}\frac{P}{P_A}\sin\theta \simeq (3.6-7.2)\sin\theta \times 10^{-5}$$

- Delay in Time-of-Arrival:

$$\Delta t = \left(\frac{M_B}{M_A+M_B}\right)\frac{a}{c}\delta_i \cos i \approx 0.4 - 4.0\ \sin\theta\ \mu\text{s}$$

- Periastron Advance $\propto \vec{S}_A \cdot \vec{L}$: $A_P/A_{PN} =$

$$\frac{2\pi I_A}{P_A}\left(\frac{2+3M_B/M_A}{3M_A^2+3M_B^2+2M_AM_B}\right)\sqrt{\frac{M_A+M_B}{Ga}}\cos\theta \simeq (2.2-4.3)\times 10^{-4}\cos\theta$$

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2}\vec{L}\times\vec{S}_A$
- Precession Period:

$$P_p = \frac{2(M_A+M_B)ac^2}{GM_B(4M_A+3M_B)}P(1-e^2) \simeq 74.9 \text{ years}$$

- Precession Amplitude $\propto \vec{S}_A \times \vec{L}$:

$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|}\sin\theta \simeq \frac{I_A(M_A+M_B)}{a^2 M_A M_B}\frac{P}{P_A}\sin\theta \simeq (3.6-7.2)\sin\theta\times 10^{-5}$$

- Delay in Time-of-Arrival:

$$\Delta t = \left(\frac{M_B}{M_A+M_B}\right)\frac{a}{c}\delta_i\cos i \approx 0.4-4.0\ \sin\theta\ \mu\text{s}$$

- Periastron Advance $\propto \vec{S}_A\cdot\vec{L}$: $A_P/A_{PN} =$

$$\frac{2\pi I_A}{P_A}\left(\frac{2+3M_B/M_A}{3M_A^2+3M_B^2+2M_AM_B}\right)\sqrt{\frac{M_A+M_B}{Ga}}\cos\theta \simeq (2.2-4.3)\times 10^{-4}\cos\theta$$

- Moment of Inertia – Mass – Radius

$$I \simeq (0.237\pm 0.008)MR^2\left[1+4.2\frac{M\text{ km}}{R\text{ M}_\odot}+90\left(\frac{M\text{ km}}{R\text{ M}_\odot}\right)^4\right]$$

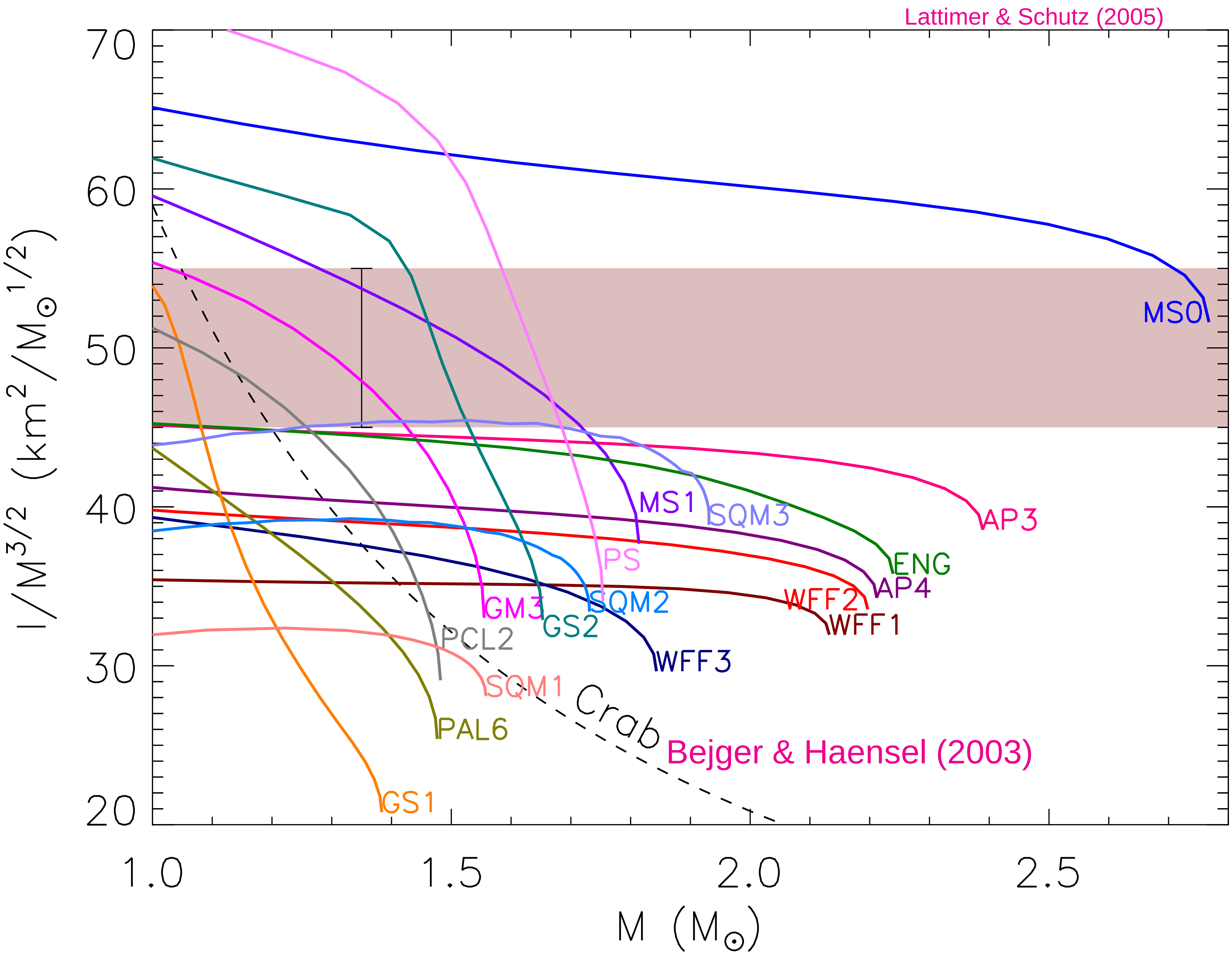
Comparison of Binary Pulsars

References	PSR 0707-3039 a, b, c	PSR 1913+16 d	PSR 1534+12 e, f
a/c **(s)**	**2.93**	**6.38**	**7.62**
P **(h)**	**2.45**	**7.75**	**10.1**
e	**0.088**	**0.617**	**0.274**
M_A **($M_\odot$)**	1.337 ± 0.005	1.4414 ± 0.0002	1.333 ± 0.001
M_B **($M_\odot$)**	1.250 ± 0.005	1.3867 ± 0.0002	1.345 ± 0.001
T_{GW} **(M yr)**	**85**	**245**	**2250**
i	$87.9 \pm 0.6°$	$47.2°$	$77.2°$
P_A **(ms)**	**22.7**	**59**	**37.9**
θ_A	$13° \pm 10°$	$21.1°$	$25.0° \pm 3.8°$
ϕ_A	$246° \pm 5°$	$9.7°$	$290° \pm 20°$
P_{pA} **(yr)**	**74.9**	**297.2**	**700**
$\delta t_a/I_{A,80}$ $(\mu$**s)**	0.7 ± 0.6	**11.2**	7.9 ± 1.1
$A_{pA}/(A_{1PN} I_{A,80})$	$3.4^{+0.2}_{-0.1} \times 10^{-5}$	1.0×10^{-5}	$1.15^{+0.04}_{-0.03} \times 10^{-5}$
A_{2PN}/A_{1PN}	5.2×10^{-5}	4.7×10^{-5}	2.3×10^{-5}

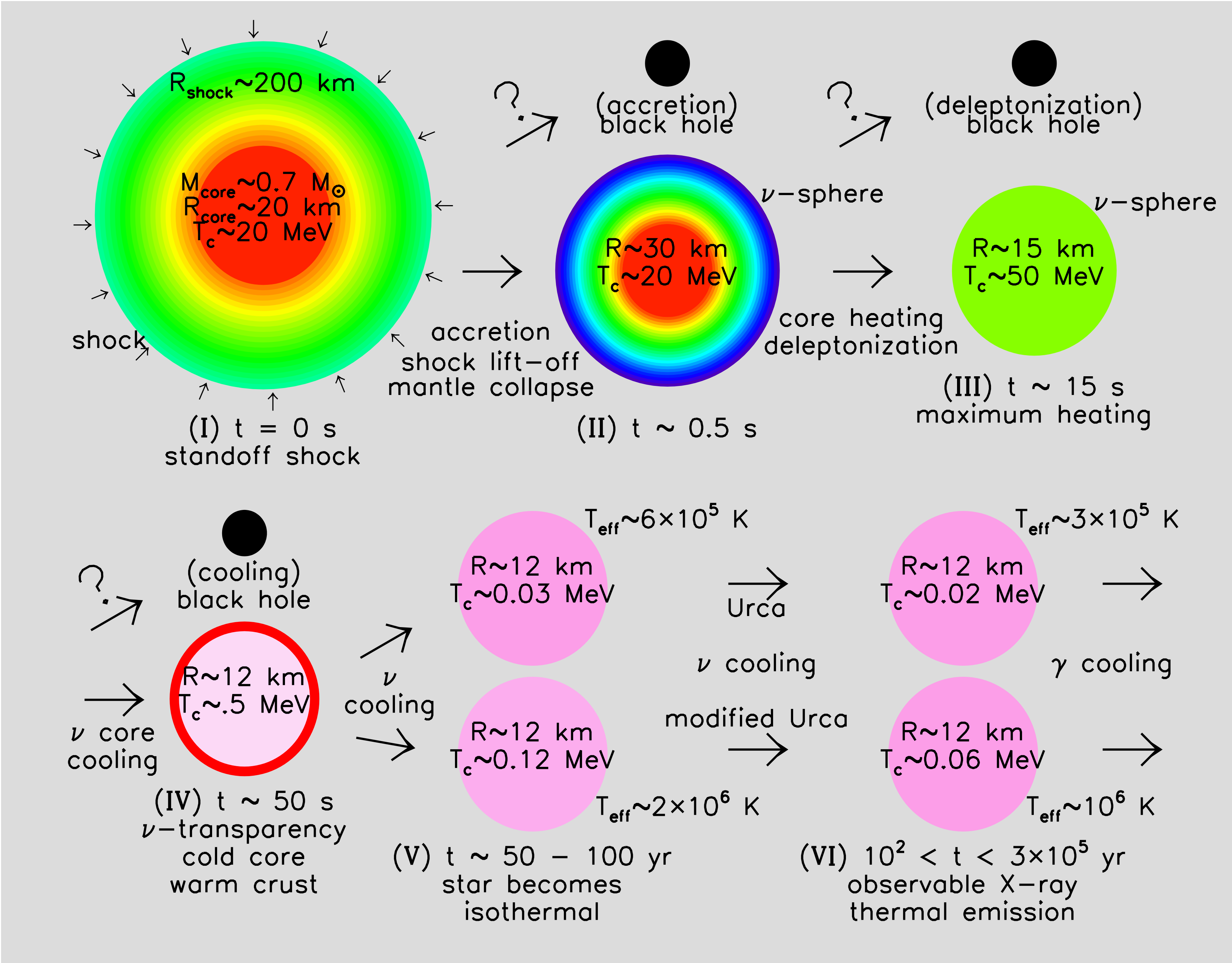
a: Lyne et al. (2004); b: Solution 1, Jenet & Ransom (2004); c: Coles et al. (2004)

d: Weisberg & Taylor (2002, 2004); e: Stairs et al. (2002, 2004); f: Bogdanov et al. (2002)

EOS Constraint

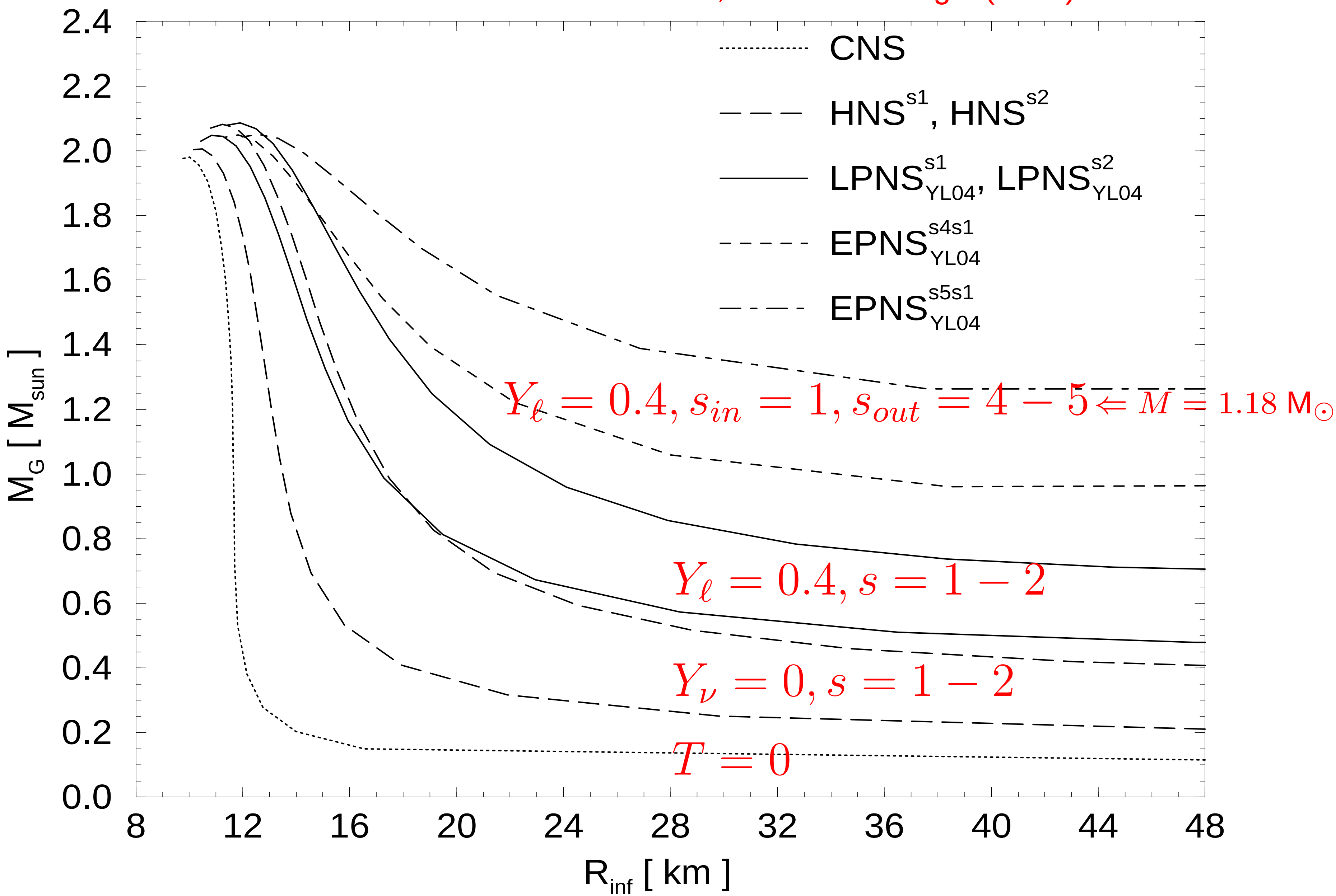


Proto-Neutron Stars



Effective Minimum Masses

Strobel, Schaab & Weigel (1999)



Proto-Neutron Star Evolution

$$\begin{aligned}
\frac{dP}{dr} &= -\frac{G(M+4\pi r^3 P)(\rho + P/c^2)}{r(r-2GM/c^2)} \\
\frac{dM}{dr} &= 4\pi r^2 \rho \\
\frac{dN}{dr} &= \frac{4\pi r^2 n}{\sqrt{1-2GM/rc^2}} \\
\frac{d\phi}{dP} &= -\frac{1}{P+\rho c^2} \\
\frac{dY_\nu}{d\tau} &= -e^{-\phi}\frac{\partial(4\pi r^2 F_\nu e^\phi)}{\partial N} + S_\nu \\
\frac{dY_e}{d\tau} &= -S_\nu \\
\frac{dU}{d\tau} &= -P\frac{d(1/n)}{d\tau} - e^{-2\phi}\frac{\partial L_\nu e^{2\phi}}{\partial N}
\end{aligned}$$

In the diffusion approximation, fluxes are driven by density gradients:

$$\begin{aligned}
F_\nu &= -\int_0^\infty \frac{c}{3}\left(\lambda_\nu \frac{\partial n_\nu(E_\nu)}{\partial r} - \lambda_{\bar\nu}\frac{\partial n_{\bar\nu}(E_\nu)}{\partial r}\right) dE_\nu, \\
L_\nu &= -\int_0^\infty 4\pi r^2 \sum_i \frac{c\lambda_E^i}{3}\frac{\partial \epsilon_i(E_\nu)}{\partial r} dE_\nu.
\end{aligned}$$

λ_ν and λ_E^i's are mean free paths for number and energy transport, respectively. $n_\nu(E_\nu)$ and $\epsilon_i(E_\nu)$ are the number and energy density of species $i = e, \mu$ at neutrino energy E_ν.

We can combine with the first law of thermodynamics to obtain the rate of change of the total lepton number and the entropy:

$$
\begin{aligned}
n\frac{dY_L}{dt} = n\frac{dY_e}{dt} + \frac{dY_\nu}{dt} &= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 F_\nu \\
nT\frac{ds}{dt} &= -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n\sum_{n,p,e,\nu}\mu_i\frac{dY_i}{dt}.
\end{aligned}
$$

There are two main sources of opacity:

1. ν-nucleon absorption. Affects only $e-$types.
2. Neutrino-electron scattering. Inelastic scattering affects all types of neutrinos.

Mean free paths for these processes are approximately:

1. $\lambda_\nu \simeq \lambda_{\bar\nu} \simeq 5$ cm, $\lambda_\nu \propto E_\nu^{-2}$;
2. $\lambda_E^i \simeq 100$ cm, $\lambda_E^i \propto T^{-1} E_\nu^{-2}$.

Proto-Neutron Stars – Analytic Analysis

Neutrino fluid $n_\nu(E_\nu) = \frac{E_\nu^2}{2\pi^3(\hbar c)^3} f_\nu(E_\nu), \qquad f_\nu(E_\nu) = \left[1 + e^{(E_\nu - \mu_\nu)/T}\right]^{-1}$

Diffusion approximation

$$F_\nu = -\frac{c}{3}\int_0^\infty \left[\lambda_\nu(\partial n_\nu(E_\nu)/\partial r) - \lambda_{\bar\nu}(\partial n_{\bar\nu}(E_\nu)/\partial r)\right] dE_\nu$$

$$L_\nu = -4\pi r^2 \int_0^\infty \sum_i (c\lambda_E^i/3)(\partial\epsilon_i(E_\nu)/\partial r) dE_\nu, \quad \epsilon_\nu(E_\nu) = E_\nu n_\nu$$

Evolution equations $n(\partial Y_L/\partial t) = -r^{-2}(\partial r^2 F_\nu/\partial r)$

$$nT(\partial s/\partial t) = -(4\pi r^2)^{-1}(\partial L_\nu/\partial r) - n\sum_j \mu_j (dY_j/dt)$$

Number transport dominated by degenerate electron neutrino absorption

$$\lambda_\nu \simeq \lambda_0(T_0/T)\left[1 + (E_\nu - \mu_\nu)/(\pi T)\right]^{-1}, \quad \lambda_0 \simeq 50\ \mathrm{cm}, \quad T_0 \simeq 10\ \mathrm{MeV}$$

Energy transport dominated by all-flavor neutrino scattering

$$\lambda_E^i \simeq \lambda_C(T_0^3/TE_\nu^2)(n_s/n)^{1/3}, \quad \lambda_C \simeq 0.2\ \mathrm{km}$$

Degenerate neutrinos

$$\frac{\partial f_\nu}{\partial r} \simeq \frac{f_\nu(1-f_\nu)}{T}\frac{\partial \mu_\nu}{\partial r} \simeq \delta(E_\nu - \mu_\nu)\frac{\partial \mu_\nu}{\partial r}$$

Deleptonization of Proto-Neutron Star

Energy transport dominated by degenerate electron neutrinos propagating through degenerate matter.

Number transport $\frac{\partial n Y_L}{\partial t} = \frac{c\lambda_0}{18\pi^2(\hbar c)^3}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{T_0}{T}\right)^2\frac{\partial \mu_\nu^3}{\partial r}\right]$

$$\frac{\partial Y_L}{\partial Y_\nu} \simeq \left(\frac{\partial Y_L}{\partial Y_\nu}\right)_0 \simeq 5, \quad 6\pi^2 n Y_\nu (\hbar c)^3 = \mu_\nu^3, \quad \mu_\nu^3 = \mu_{\nu,0}^3 \psi(x)\phi(t)$$

Dimensionless radius $x = r x_1/R$

Eigenvalue equation $\frac{\tau_D}{\phi}\frac{d\phi}{dt} = \frac{1}{x^2\psi}\frac{\partial}{\partial x}\left[x^2\frac{\partial\psi}{\partial x}\right] = -1$

Solutions $\psi = \sin(x)/x, \quad \phi = \exp(-t/\tau_D), \quad x_1 = \pi$

$$\tau_D = \frac{3}{c\lambda_0}\left(\frac{R}{x_1}\right)^2\left(\frac{T_c}{T_0}\right)^2\left(\frac{\partial Y_L}{\partial Y_\nu}\right)_0 \simeq 16 \text{ s}, \qquad \tau_D \propto M^{4/3}$$

$R \simeq 20$ km, $\quad T_c \simeq 20$ MeV

Neutrino number flux $F_\nu(x,t) = -\frac{c\lambda_0}{18\pi^2}\left(\frac{\mu_{\nu,0}}{\hbar c}\right)^3\left(\frac{T_0}{T_c}\right)^2\frac{x_1}{R}\frac{\partial\psi(x)}{\partial x}\phi(t)$

Outer boundary $x_1 \partial\psi/\partial x_{x_1} = -1 \qquad F_\nu(R,t) = \frac{c\lambda_0}{18\pi^2 R}\left(\frac{\mu_{\nu,0}}{\hbar c}\right)^3\left(\frac{T_0}{T_c}\right)^2\phi(t)$

Initial flux $F_\nu(R,0) = 2.5\times 10^{42}$ neutrinos cm^{-2} s^{-1}

This ignores the larger initial neutrino flux originating from the hot, shocked mantle, about 10^{39} cm-2 s^{-1}.

Energy Transport During Deleptonization

Beta equilibrium

$$\sum_j \mu_j dY_j = (-\mu_n + \mu_p + \mu_e - \mu_\nu) dY_e + \mu_\nu dY_L = \mu_\nu dY_L$$

Energy transport

$$nT \frac{\partial s}{\partial t} = -\frac{1}{4\pi r^2} \frac{\partial L_\nu}{\partial r} - n \sum_j \mu_j \frac{\partial Y_j}{\partial t} \simeq -n\mu_\nu \frac{d\partial Y_L}{\partial t}$$

$s \simeq aT, \qquad a \simeq 0.05 \text{ MeV}^{-1}$

$$\frac{s}{a}\frac{ds}{dt} \simeq -\mu_\nu \frac{\partial Y_L}{\partial t} \simeq -\left(\frac{\partial Y_L}{\partial Y_\nu}\right)_0 \left(\frac{Y_\nu}{Y_{\nu,0}}\right)^{1/3} \mu_{\nu,0} \frac{\partial Y_\nu}{\partial t}$$

$$s_f^2 - s_i^2 \simeq \frac{3a}{2} \left(\frac{\partial Y_L}{\partial Y_\nu}\right)_0 \mu_{\nu,0} Y_{\nu,0}$$

$s_i \sim 1, \qquad s_f \sim 2.5; \qquad\qquad s_f \propto M^{1/6}$

Core Cooling

Following deleptonization, $\mu_\nu << T$, $\int_0^\infty E_\nu f_\nu dE_\nu \simeq \pi^2 T^2/12$, $\partial\mu_\nu/\partial r \sim 0$

Neutrino luminosity $L_\nu = -4\pi r^2 \frac{c\lambda_C}{6}\left(\frac{T_0}{\hbar c}\right)^3 \frac{\partial T}{\partial r}$

Energy transport $nT\frac{\partial s}{\partial t} = -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n\sum_j \mu_j \frac{dY_j}{dt} \simeq -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r}$

$$T\frac{\partial s}{\partial T}\frac{dT}{dt} = \frac{c\lambda_C}{6n_s}\left(\frac{n_s}{n_*}\right)^{4/3}\left(\frac{T_0}{\hbar c}\right)^3 \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right]$$

n_* is density in core, T_* is initial core temperature following deleptonization.

Baryon dominated specific heat: $s \simeq a(n_s/n_*)^{2/3}T$, $a \simeq 0.1$ MeV^{-1}

$T = T_*\psi(x)\phi(t)$

Separable solution $\tau_C\frac{\partial\phi}{\partial t} = \frac{1}{\psi^2 x^2}\frac{\partial}{\partial x}\left[x^2\frac{\partial\psi}{\partial x}\right] = -1$

$x_1^2 \simeq 19, -x_1(\partial\psi/\partial x)_{x=x_1} \simeq 0.56,$ $n = 2$ Lane-Emden solution

$$\tau_C = \frac{6an_sT_*R^2}{c\lambda_C x_1^2}\left(\frac{\hbar c}{T_0}\right)^3\left(\frac{n_*}{n_s}\right)^{2/3} \propto M^{4/3}$$

$\phi(t) = 1 - t/\tau_C$

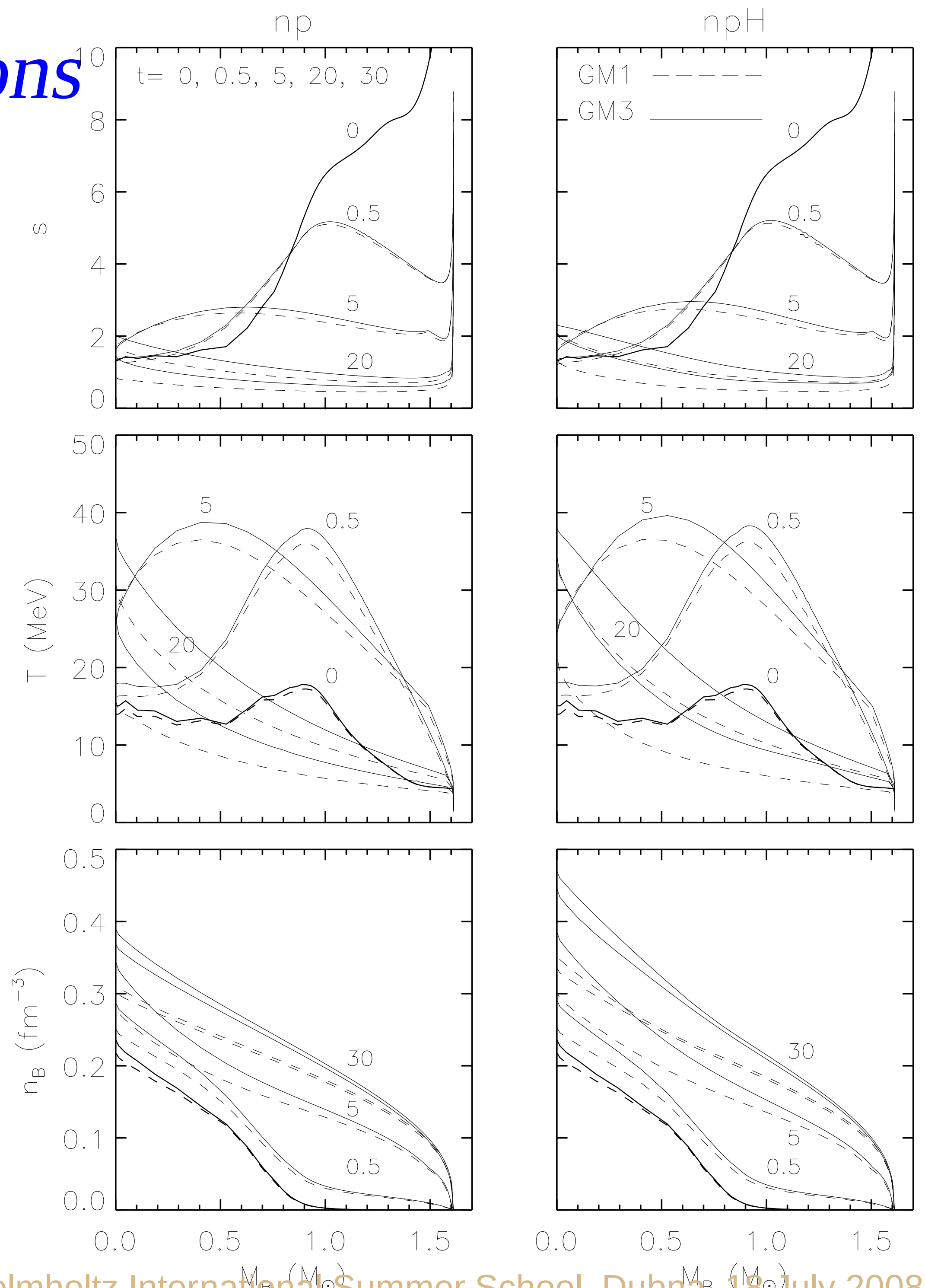
Emergent Luminosity

$$L_\nu(R,t) = -4\pi RT_*\frac{c\lambda_C}{6}\left(\frac{T_0}{\hbar c}\right)^3\left(x\frac{\partial\psi}{\partial x}\right)_{x_1}\phi(t) = \frac{cF_3(0)}{2(\hbar c)^3}RT_e(t)^4$$

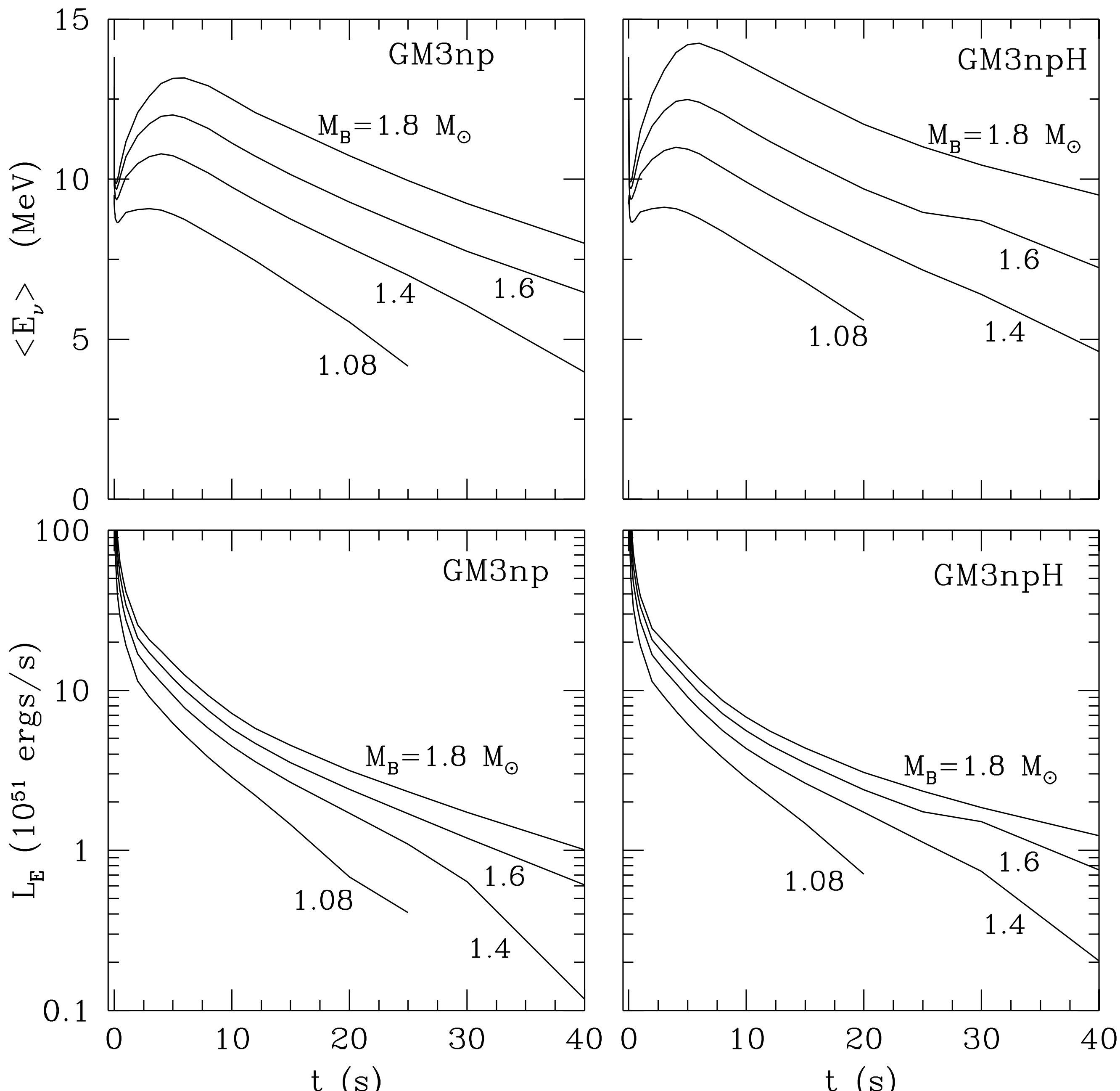
$T_* = 50$ MeV, $n_* = 4n_s \Rightarrow \tau_C \simeq 18$ s, $L_\nu(R,0) \simeq 11$ bethe s^{-1},

$T_e(0) \simeq 5.5$ MeV, and $< E_\nu > \simeq 17$ MeV.

Model Simulations



Model Simulations



Model Signal