QCD Sum Rules for D Mesons at Finite Density

Thomas Hilger
Forschungszentrum Dresden-Rossendorf
TU Dresden

$$\rho, \omega \sim \mathbf{\bar{u}u \mp \bar{d}d} : \quad \mathbf{m_q \langle \bar{q}q \rangle}, \quad \mathbf{\ldots} \quad \mathbf{HADES}$$

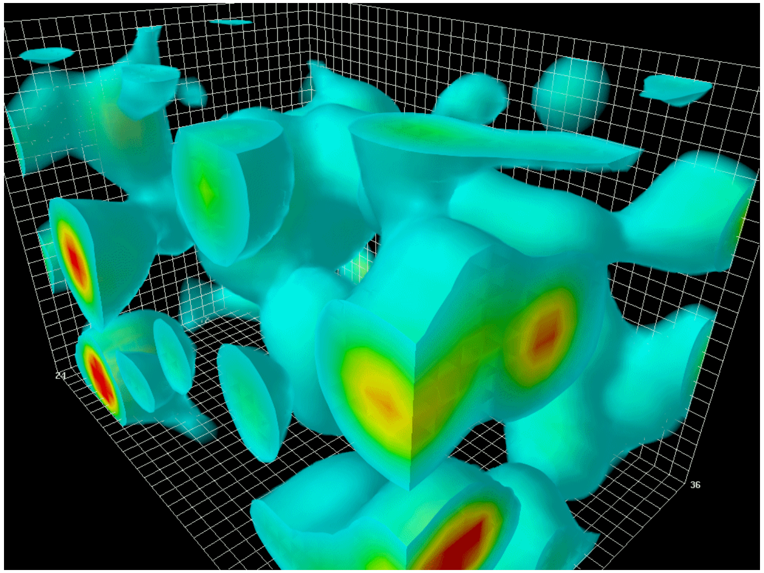
$$\mathbf{D \sim \bar{c}d} : \quad \mathbf{m_c \langle \bar{q}q \rangle}, \quad \mathbf{\langle \bar{c}c \rangle = ?} \quad \mathbf{\ldots} \quad \mathbf{CBM\ @\ FAIR}$$

$$\langle \frac{\alpha_\mathbf{s}}{\pi} \mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu} \rangle$$

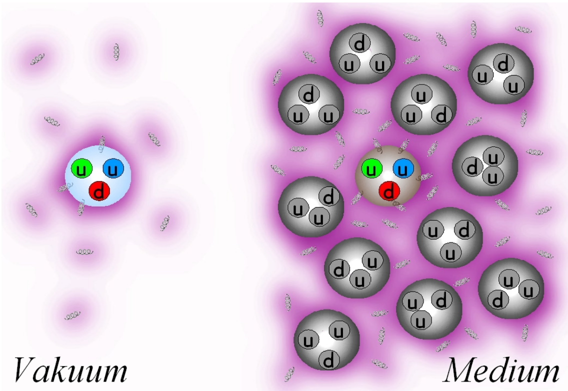
Current-Current Correlation Function

$$\Pi(q) = i \int d^4x\, e^{iqx} \langle\Omega| T\left[j(x) j^\dagger(0)\right] |\Omega\rangle$$

- $a|\Omega\rangle \neq 0$
- state of minimum energy



[Leinweber:http://www.physics.adelaide.edu.au/ dleinweb/]



particle	interpolating field $j(x)$
D^+-meson	$i\bar{d}(x)\gamma_5 c(x)$
D^--meson	$i\bar{c}(x)\gamma_5 d(x)$
ρ-meson	$\frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)$
ω-meson	$\frac{1}{2}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)$
nucleon	$\epsilon^{abc}\left[u_a^T C\gamma_\mu u_b\right]\gamma_5\gamma^\mu d_c$

Perturbative Approach

$$\mathbf{\Pi(q) = i \int d^4x\, e^{iqx} \frac{\langle 0|T\left[j(x)j^\dagger(0)S\right]|0\rangle}{\langle 0|S|0\rangle}}$$

- **S-matrix operator in interaction picture**

$$\mathbf{S = I - i \int_{-\infty}^{+\infty} H_{int}(x) d^4x + \ldots}$$

- **free equations of motion**

$$\mathbf{i\left[P^0_\mu, \Psi_\alpha(x)\right] = \partial_\mu \Psi_\alpha(x)}$$

- **ground state** $|0\rangle$ **annihilated by all annihilation operators**

Analytic Properties of Π(q)

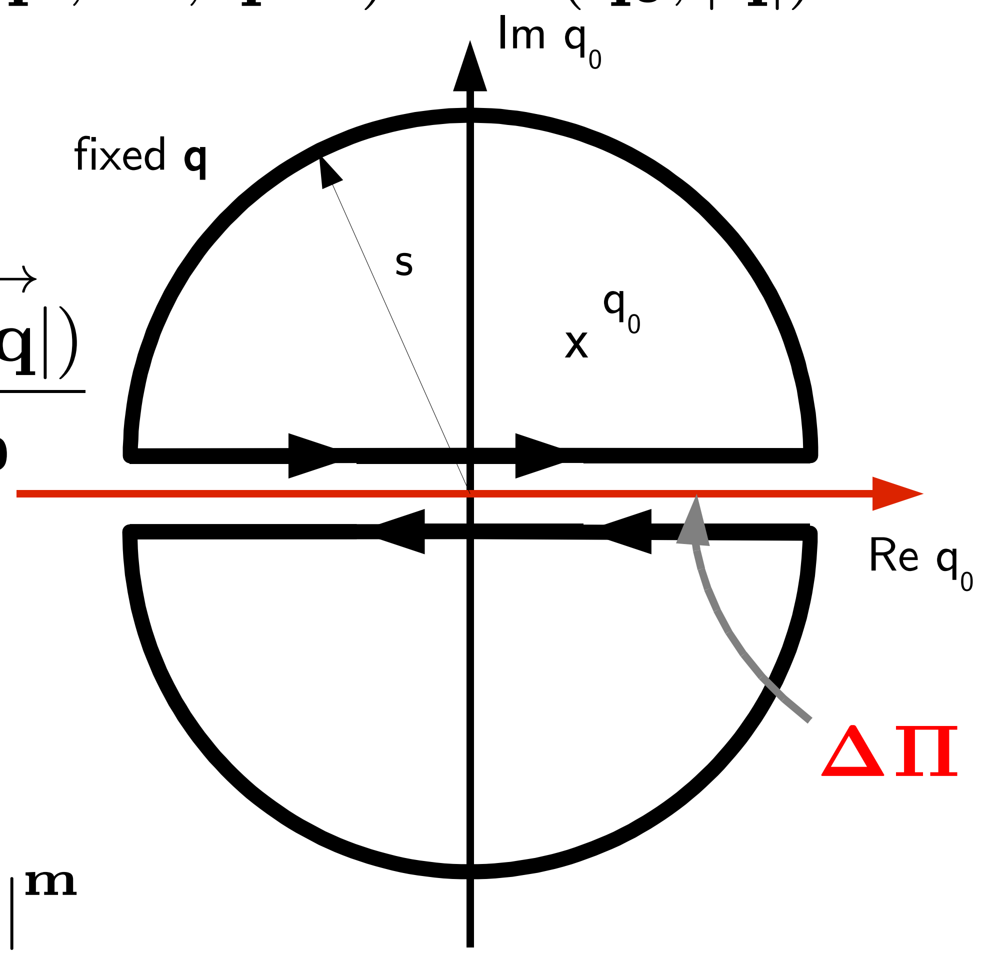
Lehmann-representation: poles at the entire real axis

in-medium: $\mathbf{\Pi(q_\mu, v_\nu) = \Pi(q^2, v^2, q \cdot v) \equiv \Pi(q_0, |\vec{q}|)}$

dispersion relation:

$$\mathbf{\Pi(q_0, |\vec{q}|) = \frac{1}{\pi} \int_{-\infty}^{+\infty} ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0}}$$

$$\mathbf{+ \, polynomials}$$



restriction: $\mathbf{|\Pi(q_0)| \overset{|q_0| \to \infty}{\leq} |q_0|^m}$

for some arbitrary but finite and fixed m

Operator Product Expansion

$$\mathbf{T[A(x)B(y)] = \sum_i C_i(x-y) O_i}$$

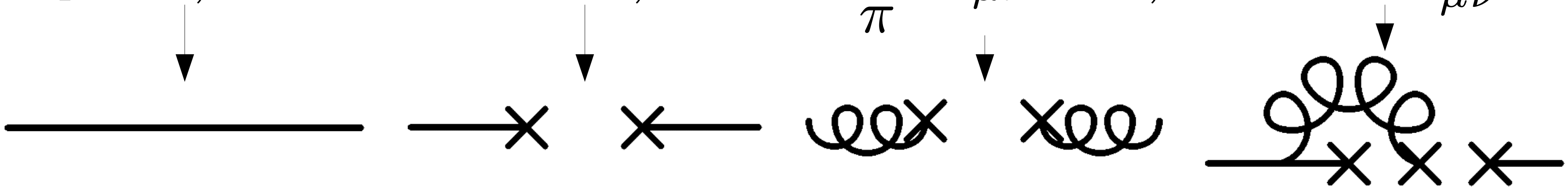
expansion at operator level! $\Rightarrow$ **state independent Wilson coefficents**

$$\mathbf{\langle \Omega | T[A(x)B(y)] | \Omega \rangle = \sum_i C_i(x-y) \langle \Omega | O_i | \Omega \rangle}$$

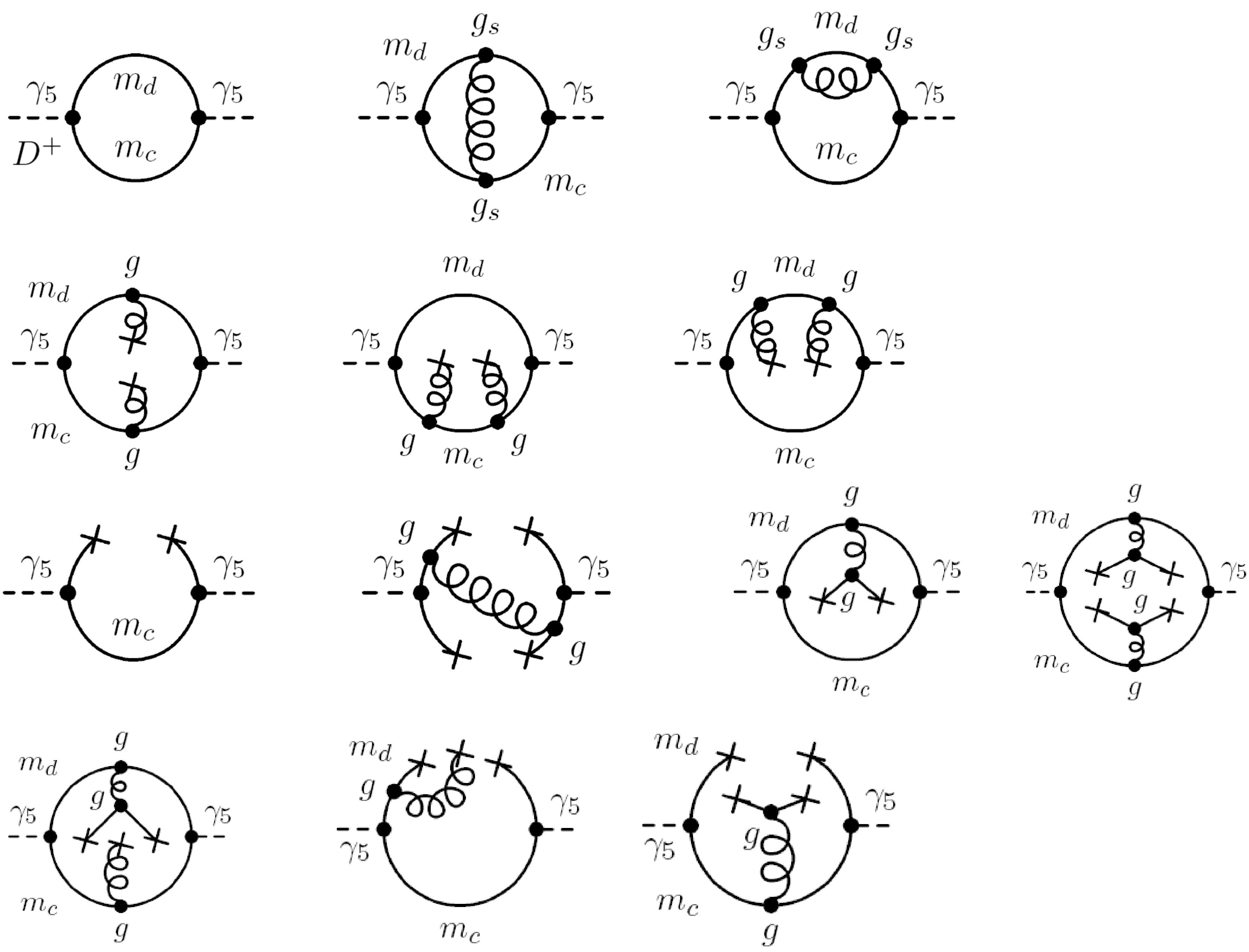
condensates: parameters characterizing QCD

$$\mathbf{\Pi(q) \Rightarrow \Pi_{OPE}(q) = \sum_i \tilde{C}_i(q) \langle O_i \rangle}$$

$\mathbf{O_i = 1}$, $\quad \mathbf{\bar{\Psi}\Psi}$, $\quad \frac{\alpha_s}{\pi}\mathbf{G_{\mu\nu}G^{\mu\nu}}$, $\quad \mathbf{\bar{\Psi}\sigma^{\mu\nu}G^A_{\mu\nu}t^A\Psi}$



$+$ **additional condensates in medium, e.g.** $\quad \langle \mathbf{\Psi^+\Psi} \rangle$



γ_5 m_d γ_5 D^+ m_c
g_s m_d γ_5 γ_5 m_c g_s
m_d g_s g_s γ_5 γ_5 m_c
g m_d γ_5 γ_5 m_c g
m_d γ_5 γ_5 g m_c g
g m_d g γ_5 γ_5 m_c
γ_5 γ_5 m_c
g γ_5 γ_5 g
g m_d γ_5 γ_5 g g m_c
g m_d γ_5 g γ_5 g m_c g
g m_d g γ_5 γ_5 m_c g
m_d g γ_5 γ_5 m_c
m_d γ_5 γ_5 g m_c g

QCD Sum Rules*

$$\mathbf{\Pi(q_0, |\vec{q}|) = \frac{1}{\pi} \int_{-\infty}^{+\infty} ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0}}$$

QCD structure via OPE

splitting in hadronic parts

$$\mathbf{\Pi_{OPE}(q_0, |\vec{q}|) = \frac{1}{\pi} \left(\overbrace{\underbrace{\int_{s_0}^{\infty} + \int_{-\infty}^{-s_0}} + \int_{-s_0}^{s_0}} \right) ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0}}$$

semi-local quark hadron duality OPE ←

hadronic properties via optical theorem

$\mathbf{\Delta\Pi = Im\Pi^{\mu}_{\mu}}$ **→ observable,**
e.g. Dilepton emission rate or $\mathbf{R = \frac{\sigma_{e^+e^- \to hadrons}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \propto Im\Pi^{\mu}_{\mu}(s)}$

[*Shifman, Vainshtein, Zakharov: Nucl.Phys.B147(1979)]

OPE for D-Mesons

B $\left[\Pi^e(\omega^2)\right](M^2)$

$$= \frac{1}{\pi}\int_{m_c^2}^{\infty} ds e^{-s/M^2} \mathrm{Im}\Pi_{D^+}^{per}(s)$$

$$+ e^{-m_c^2/M^2}\left(-m_c\langle \bar{d}d\rangle + \frac{1}{2}\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\langle \bar{d}g\sigma \mathsf{G} d\rangle + \frac{1}{12}\langle \frac{\alpha_s}{\pi}G^2\rangle\right.$$

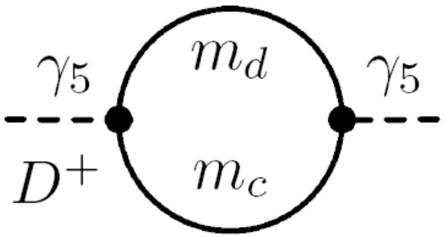
$$+ \left[\left(\frac{7}{18} + \frac{1}{3}\ln\frac{\mu^2 m_c^2}{M^4} - \frac{2\gamma_E}{3}\right)\left(\frac{m_c^2}{M^2} - 1\right) - \frac{2}{3}\frac{m_c^2}{M^2}\right]\langle \frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right)\rangle$$

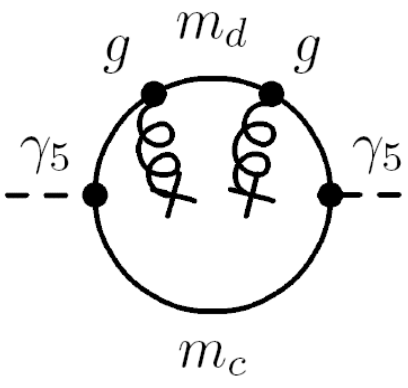
$$\left. +2\left(\frac{m_c^2}{M^2} - 1\right)\langle d^\dagger i D_0 d\rangle + 4\left(\frac{m_c^3}{2M^4} - \frac{m_c}{M^2}\right)\left[\langle \bar{d}D_0^2 d\rangle - \frac{1}{8}\langle \bar{d}g\sigma \mathsf{G} d\rangle\right]\right)$$

B $\left[\Pi^o(\omega^2)\right](M^2)$

$$= e^{-m_c^2/M^2}\left(\langle d^\dagger d\rangle - 4\left(\frac{m_c^2}{2M^4} - \frac{1}{M^2}\right)\langle d^\dagger D_0^2 d\rangle - \frac{1}{M^2}\langle d^\dagger g\sigma \mathsf{G} d\rangle\right)$$

Mass-Logarithms





- **mass logarithms** $(\ln \mathbf{m^2})$ **of light quarks appear**
- **remnants of large distance behaviour**
- **to perform a consistent seperation of scales**

  $\rightarrow$ **absorption into condensates**

$$\Pi^{G^2}(q) = \langle:\frac{\alpha_s}{\pi}G^2:\rangle \left(-\frac{1}{24}\frac{1}{q^2-m_c^2} - \frac{1}{12}\frac{m_c}{m_d}\frac{1}{q^2-m_c^2} - \frac{1}{24}\frac{m_c^2}{(q^2-m_c^2)^2}\right)$$
$$+ \langle:\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right):\rangle \left(q^2 - 4\frac{(vq)^2}{v^2}\right) \left(-\frac{1}{6q^2}\frac{1}{q^2-m_c^2} - \frac{1}{6q^2}\frac{m_c^2}{(q^2-m_c^2)^2}\right.$$
$$\left. - \frac{1}{9q^2}\left(\frac{m_c^2}{(q^2-m_c^2)^2} + \frac{1}{q^2-m_c^2}\right)\ln\left(\frac{m_d^2}{m_c^2}\right) - \frac{2}{9q^2}\left(\frac{m_c^2}{(q^2-m_c^2)^2} + \frac{1}{q^2-m_c^2}\right)\ln\left(-\frac{m_c^2}{q^2-m_c^2}\right)\right)$$

Absorption of Divergences

- **def. of physical condensate:**

$$\langle\mathbf{\Omega}|\bar{\mathbf{\Psi}}\mathbf{O}\left[\mathbf{D}_\mu\right]\mathbf{\Psi}|\mathbf{\Omega}\rangle = \langle\mathbf{\Omega}|{:}\bar{\mathbf{\Psi}}\mathbf{O}\left[\mathbf{D}_\mu\right]\mathbf{\Psi}{:}|\mathbf{\Omega}\rangle$$

$$-\mathbf{i}\int \mathbf{d^4p}\langle\mathbf{\Omega}|\mathbf{Tr}\left[\mathbf{O}\left(-\mathbf{ip}_\mu - \mathbf{i}\tilde{\mathbf{A}}_\mu\right)\mathbf{S_\Psi(p)}\right]|\mathbf{\Omega}\rangle$$

- **result in $\overline{\mathbf{MS}}$ up to** $O(\alpha_s)$

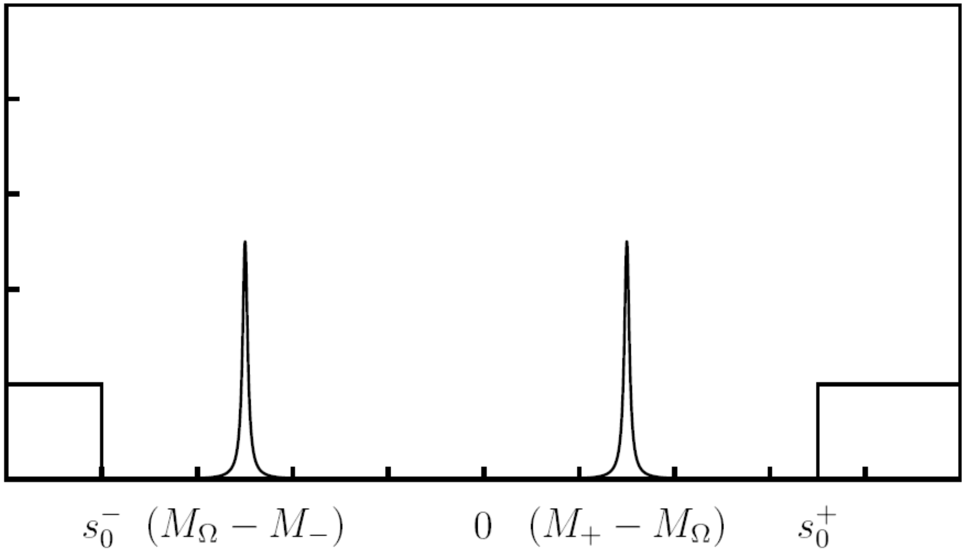
$$\langle\bar{\mathbf{q}}\mathbf{q}\rangle = \langle:\bar{\mathbf{q}}\mathbf{q}:\rangle + \frac{\mathbf{3}}{\mathbf{4}\pi^{\mathbf{2}}}\mathbf{m_q^3}\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} + \mathbf{1}\right) - \frac{\mathbf{1}}{\mathbf{12m_q}}\langle:\frac{\alpha_\mathbf{s}}{\pi}\mathbf{G^2}:\rangle + \ldots$$

$$\langle\bar{\mathbf{q}}\mathbf{g}\sigma\mathbf{G^At^Aq}\rangle = \langle:\bar{\mathbf{q}}\mathbf{g}\sigma\mathbf{G^At^Aq}:\rangle - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m_q}\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}}\langle:\frac{\alpha_\mathbf{s}}{\pi}\mathbf{G^2}:\rangle + \ldots$$

$$\langle\bar{\mathbf{q}}\gamma_\mu\mathbf{iD}_\nu\mathbf{q}\rangle = \langle:\bar{\mathbf{q}}\gamma_\mu\mathbf{iD}_\nu\mathbf{q}:\rangle + \frac{\mathbf{9}}{\mathbf{4}\pi^{\mathbf{2}}}\mathbf{m_q^4g}_{\mu\nu}\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} + \frac{\mathbf{5}}{\mathbf{12}}\right) - \frac{\mathbf{g}_{\mu\nu}}{\mathbf{48}}\langle:\frac{\alpha_\mathbf{s}}{\pi}\mathbf{G^2}:\rangle$$
$$+\frac{\mathbf{1}}{\mathbf{18}}\left(\mathbf{g}_{\mu\nu} - \mathbf{4}\frac{\mathbf{v}_\mu\mathbf{v}_\nu}{\mathbf{v^2}}\right)\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} - \frac{\mathbf{1}}{\mathbf{3}}\right)\langle:\frac{\alpha_\mathbf{s}}{\pi}\left(\frac{\mathbf{(vG)^2}}{\mathbf{v^2}} - \frac{\mathbf{G^2}}{\mathbf{4}}\right):\rangle + \ldots$$

$$\langle\bar{\mathbf{q}}\mathbf{iD}_\mu\mathbf{iD}_\nu\mathbf{q}\rangle = \langle:\bar{\mathbf{q}}\mathbf{iD}_\mu\mathbf{iD}_\nu\mathbf{q}:\rangle - \frac{\mathbf{m_q^5}}{\mathbf{2}\pi^{\mathbf{2}}}\mathbf{g}_{\mu\nu}\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} + \mathbf{1}\right) - \frac{\mathbf{m_q}}{\mathbf{16}}\mathbf{g}_{\mu\nu}\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} - \frac{\mathbf{1}}{\mathbf{3}}\right)\langle:\frac{\alpha_\mathbf{s}}{\pi}\mathbf{G^2}:\rangle$$
$$+\frac{\mathbf{m_q}}{\mathbf{36}}\left(\mathbf{g}_{\mu\nu} - \mathbf{4}\frac{\mathbf{v}_\mu\mathbf{v}_\nu}{\mathbf{v^2}}\right)\left(\ln\frac{\mu^{\mathbf{2}}}{\mathbf{m_q^2}} + \frac{\mathbf{2}}{\mathbf{3}}\right)\langle:\frac{\alpha_\mathbf{s}}{\pi}\left(\frac{\mathbf{(vG)^2}}{\mathbf{v^2}} - \frac{\mathbf{G^2}}{\mathbf{4}}\right):\rangle + \ldots$$

Analyzing the Sum Rules



$$\Delta\Pi(s) = \begin{cases} \pi \sum_\pm \pm F_\pm \delta(s \pm (M_\pm - M_\Omega)) \\ \mathrm{Im}\Pi^{per}(s) \end{cases}$$

$\rightarrow$ **pole + continuum**

upon Borel transformation

$$f \equiv \int_{s_0^-}^{s_0^+} ds\, s \Delta\Pi e^{-s^2/M^2} = \sum_\pm m_\pm F_\pm e^{-m_\pm^2/M^2}$$

$$g \equiv \int_{s_0^-}^{s_0^+} ds\, \Delta\Pi e^{-s^2/M^2} = \sum_\pm \pm F_\pm e^{-m_\pm^2/M^2}$$

$$\frac{dm_\pm}{dM} = 0$$

$$\Delta m = \frac{1}{2}\frac{gf' - fg'}{f^2 + gg'} = \frac{1}{2}(m_+ - m_-)$$

$$m = \sqrt{\Delta m^2 - \frac{ff' + (g')^2}{f^2 + gg'}} = \frac{1}{2}(m_+ + m_-)$$

for small densities

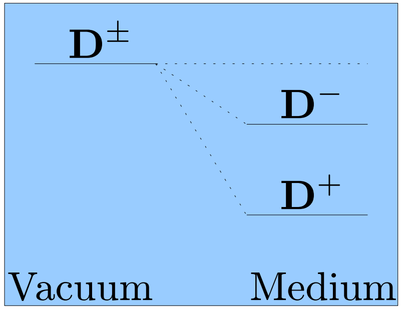
$$\Delta m(n) \approx -\frac{1}{2}\frac{\frac{dg}{dn}\Big|_0 m^2(0) + \frac{dg'}{dn}\Big|_0}{f(0)} n$$

$$m(n) \approx m(0) - \frac{1}{2m(0)}\frac{\frac{df}{dn}\Big|_0 m^2(0) + \frac{df'}{dn}\Big|_0}{f(0)} n$$

- **mass splitting driven by odd OPE**
- **mass shift determined by even OPE**
- **complicated dependence on vacuum mass**

Numerical Results

- **$D^{\pm}$-pattern at nuclear saturation density $n = 0.17 \mathrm{fm}^{-3}$ in linear density approximation for the condensates:**
  $\Delta m \approx -50$ **MeV (robust)**
  $\delta m \approx -50$ **MeV (not robust)**



- **strong dependence of the mass splitting on the chiral condensate and the odd mixed quark-gluon condensate**