Goldstone bosons in the CFL phase

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Dense Matter in Heavy Ion Collisions and Astrophysics,
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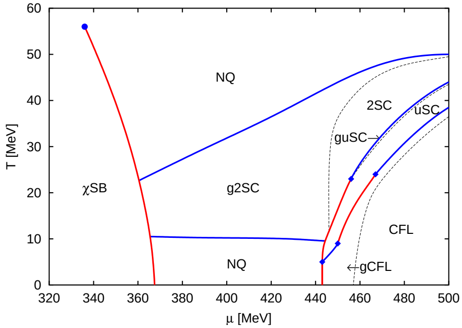
Outline

Phase diagram of neutral dense quark matter



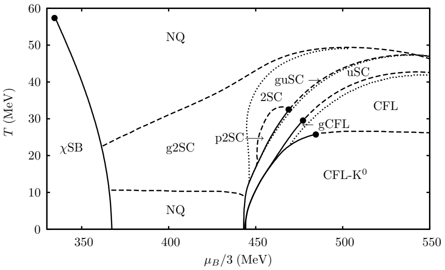
- first order phase transition
- second order phase transition

- normal quark matter: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$
- 2SC phase: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$
- uSC phase: $\Delta_{ud}, \Delta_{us} \neq 0$, $\Delta_{ds} = 0$
- CFL phase: $\Delta_{ud}, \Delta_{us}, \Delta_{ds} \neq 0$

Why are we interested in Goldstone bosons?

- chiral symmetry breaking
  - ➜ Goldstone bosons
- lightest excitations: mesons
  - effective theories predict low meson masses and meson condensation

- influence of the pseudoscalar meson condensates studied by Warringa [hep-ph/0606063]



Nambu–Jona-Lasinio model

Lagrangian

$$\mathcal{L}_{eff} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi$$
$$+ H \sum_{A=2,5,7} \sum_{A'=2,5,7} \left[\left(\bar{\psi} i\gamma_5 \tau_A \lambda_{A'} C \bar{\psi}^T \right) \left(\psi C i\gamma_5 \tau_A \lambda_{A'} \psi^T \right) \right.$$
$$\left. + \left(\bar{\psi} \tau_A \lambda_{A'} C \bar{\psi}^T \right) \left(\psi C \tau_A \lambda_{A'} \psi^T \right) \right]$$

Nambu–Jona-Lasinio model

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Four vertices in Nambu-Gorkov space

$$\Gamma_s^{ll} = \begin{pmatrix} 0 & 0 \\ i\gamma_5 \tau_A \lambda_{A'} & 0 \end{pmatrix}, \qquad \Gamma_s^{ur} = \begin{pmatrix} 0 & i\gamma_5 \tau_A \lambda_{A'} \\ 0 & 0 \end{pmatrix}$$

$$\Gamma_{ps}^{ll} = \begin{pmatrix} 0 & 0 \\ \tau_A \lambda_{A'} & 0 \end{pmatrix}, \qquad \Gamma_{ps}^{ur} = \begin{pmatrix} 0 & \tau_A \lambda_{A'} \\ 0 & 0 \end{pmatrix}$$

Ingredients

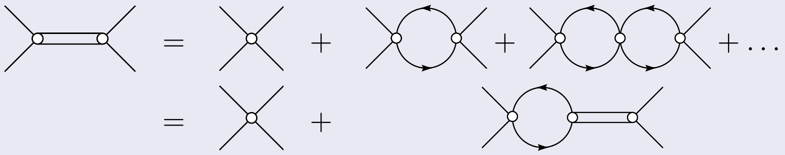
Inverse propagator in Nambu-Gorkov space

$$S^{-1} = \begin{pmatrix} \not{p} + \hat{\mu}\gamma_0 - \hat{m} & \sum_{A=2,5,7} \Delta_A \gamma_5 \tau_A \lambda_A \\ -\sum_{A=2,5,7} \Delta_A^* \gamma_5 \tau_A \lambda_A & \not{p} - \hat{\mu}\gamma_0 - \hat{m} \end{pmatrix}$$

- Δ_A and μ_8 derived from self-consistent solution of the CFL gap equation + neutrality conditions

Bethe-Salpeter equation

Bethe-Salpeter equation



$$i\hat{T} = i\hat{V} + i\hat{V}(-i\hat{J})i\hat{T}$$

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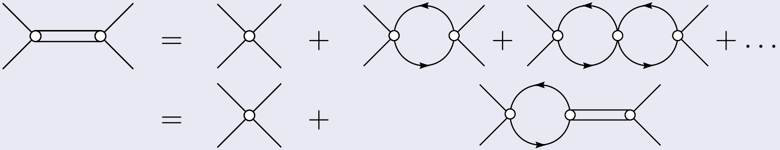

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$$\hat{V} = \Gamma_i V_{ij} \Gamma_j^\dagger \qquad \hat{T} = \Gamma_i T_{ij} \Gamma_j^\dagger$$

$$\Gamma_i^\dagger \hat{J} \Gamma_j = J_{ij}$$

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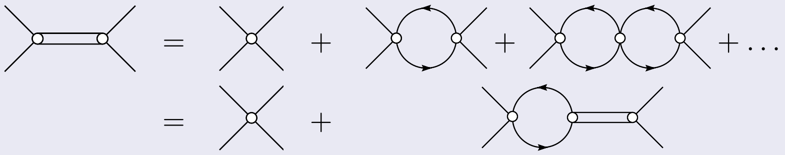

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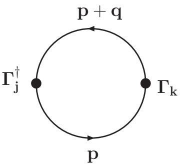
T-matrix

$$T = V + VJT$$

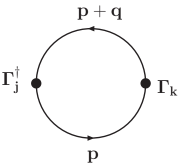
$$\Longrightarrow \quad T = (\mathbb{1} - VJ)^{-1} V$$

Polarization function



$$-iJ_{\Gamma_j^\dagger \Gamma_k}(q) = -\int \frac{d^4p}{(2\pi)^4}\frac{1}{2}\mathrm{Tr}[\Gamma_j^\dagger iS(p+q)\Gamma_k iS(p)]$$

Polarization function



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- applying Matsubara formalism and restriction to $\vec{q} = 0$

$$-iJ_{\Gamma_j^\dagger \Gamma_k}(q) \overset{\vec{q}=0}{=} \frac{i}{2} T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \mathrm{Tr}[\Gamma_j^\dagger iS(i\omega_n + i\omega_m, \vec{p})\Gamma_k iS(i\omega_n, \vec{p})]$$

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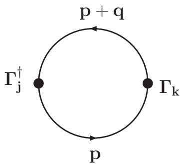

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- only a few vertex-combinations non-vanishing
  - ➜ J can be transformed into block-diagonal structure

Structure of T

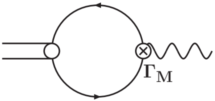
- choose the basis in which J is block-diagonal
  - ➜ V is diagonal
  - ➜ T is block-diagonal

➜ problem can be decomposed into smaller blocks:

- six 2x2 blocks
  (correspond to π^+, π^-, K^+, K^-, K^0, and $\bar{K}^0$)
- one 6x6 block
  (corresponds to π^0, η, and η')

“Mesons”

- T couples to external meson



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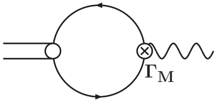
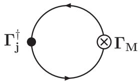


$\Longmapsto$

- we have to calculate which diquark vertices contribute to the loop



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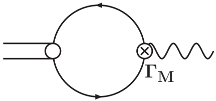

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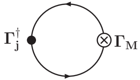

meson vertices

$$\Gamma_{\pi^\pm} = \frac{i}{2}\gamma_5 \begin{pmatrix} \tau_1 \pm i\tau_2 & 0 \\ 0 & \tau_1 \mp i\tau_2 \end{pmatrix}$$

$$\Gamma_{K^\pm} = \frac{i}{2}\gamma_5 \begin{pmatrix} \tau_4 \pm i\tau_5 & 0 \\ 0 & \tau_4 \mp i\tau_5 \end{pmatrix}$$

$$\Gamma_{\overset{(-)}{K^0}} = \frac{i}{2}\gamma_5 \begin{pmatrix} \tau_6 \pm i\tau_7 & 0 \\ 0 & \tau_6 \mp i\tau_7 \end{pmatrix}$$

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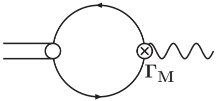

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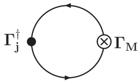

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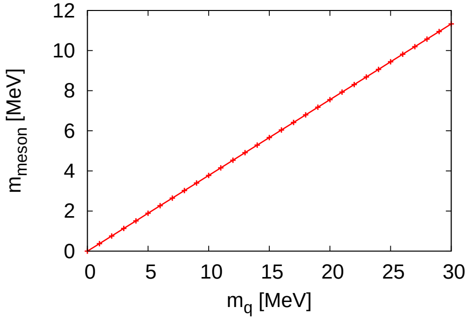
non-vanishing combinations

- $\Gamma_{\pi^+} \longrightarrow \Gamma^{ll}_{75}, \Gamma^{ur}_{57}$
- $\Gamma_{\pi^-} \longrightarrow \Gamma^{ll}_{57}, \Gamma^{ur}_{75}$
- $\Gamma_{K^+} \longrightarrow \Gamma^{ll}_{72}, \Gamma^{ur}_{27}$
- $\Gamma_{K^-} \longrightarrow \Gamma^{ll}_{27}, \Gamma^{ur}_{72}$
- $\Gamma_{K^0} \longrightarrow \Gamma^{ll}_{52}, \Gamma^{ur}_{25}$
- $\Gamma_{\bar{K}^0} \longrightarrow \Gamma^{ll}_{25}, \Gamma^{ur}_{52}$

Meson mass for equal quark masses

$\pi^{\pm}, K^{\pm}, K^0, \bar{K}^0$



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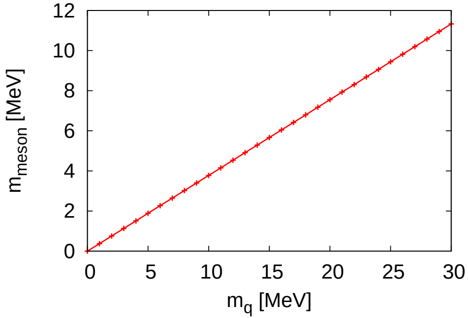

- prediction from EFT:

$$m_M = a m_q$$

$$a = \sqrt{\frac{8A}{f_\pi^2}}$$

$$A = \frac{3\Delta^2}{4\pi^2}$$

$$f_\pi^2 = \frac{21 - 8\ln 2}{18}\frac{\mu^2}{2\pi^2}$$

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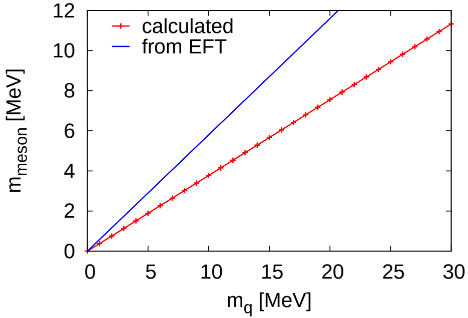

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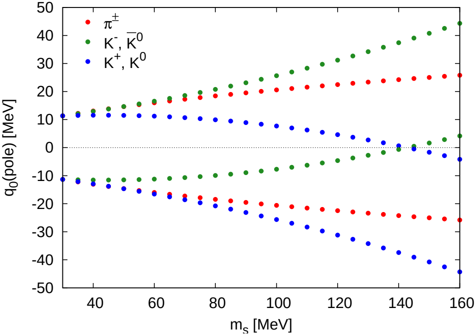
$$\left.\begin{aligned} a_{\text{fit}} &= 0.38 \\ a_{\text{EFT}} &= 0.58 \end{aligned}\right\}$$ fit 35 % smaller

Dependence on the strange quark mass

$m_u = m_d = 30\,\text{MeV}$



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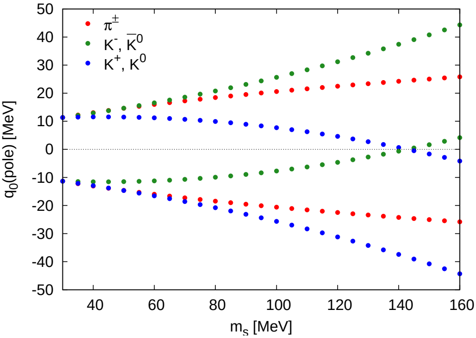

- prediction from EFT

[Bedaque, Schaefer, Nucl.Phys. A697 (2002)]

$$q_{\text{meson}}(pole) = -\mu_{\text{meson}} + m_{\text{meson}}$$

$$q_{\pi^\pm}(pole) = \mp \frac{m_d^2 - m_u^2}{2\mu} + \sqrt{\frac{4A}{f_\pi^2} m_s (m_u + m_d)}$$

$$q_{K^\pm}(pole) = \mp \frac{m_s^2 - m_u^2}{2\mu} + \sqrt{\frac{4A}{f_\pi^2} m_d (m_u + m_s)}$$

$$q_{K^0,\bar{K}^0}(pole) = \mp \frac{m_s^2 - m_d^2}{2\mu} + \sqrt{\frac{4A}{f_\pi^2} m_u (m_d + m_s)}$$

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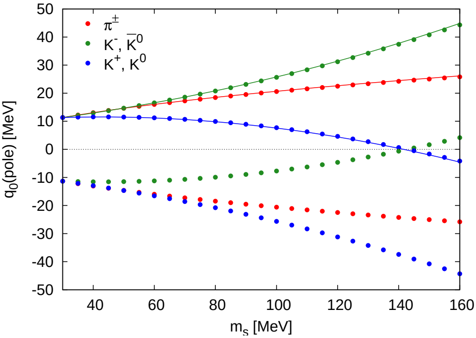

$$\sqrt{\frac{4A}{f_\pi^2}} \longrightarrow \frac{a_{\text{fit}}}{\sqrt{2}}$$

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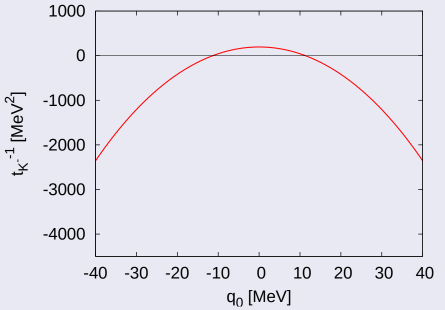
Kaon propagator

$m_u = m_d = 30\,\text{MeV}$

Inverse K^- propagator for $m_s = 30\,\text{MeV}$



- comparison with a free boson propagator with chemical potential μ_{K^-} and mass m_{K^-}

$$t_{K^-}^{-1} = \frac{(q_0 + \mu_{K^-})^2 - m_{K^-}^2}{-g^2}$$

$$\mu_{K^-} = -\frac{m_s^2 - m_u^2}{2\mu}$$

$$m_{K^-} = \sqrt{\frac{a_{\text{fit}}}{\sqrt{2}} m_d (m_u + m_s)}$$

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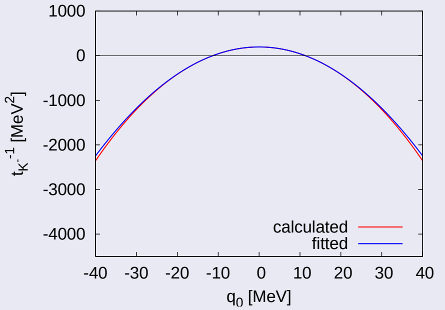

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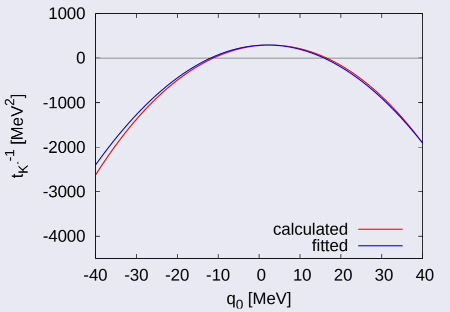
Kaon propagator

$m_u = m_d = 30\,\text{MeV}$

Inverse K^- propagator for $m_s = 60\,\text{MeV}$



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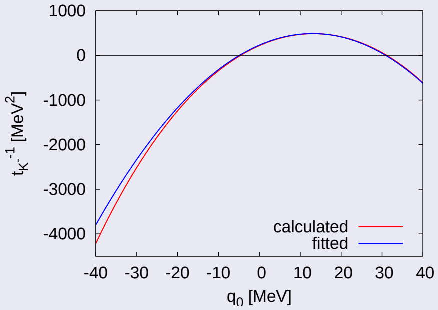
Kaon propagator

$m_u = m_d = 30\,\text{MeV}$

Inverse K^- propagator for $m_s = 120\,\text{MeV}$



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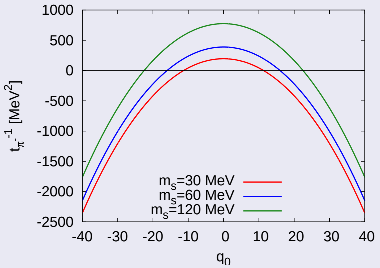
Meson propagators

Inverse π^- propagators



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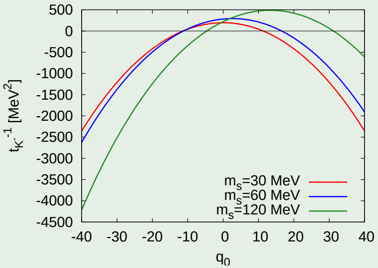
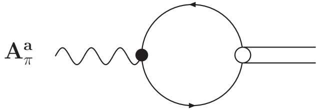


Pion decay constant



$$A^0_{\pi^+} = \frac{i}{2\sqrt{2}}\gamma^0\gamma_5 \begin{pmatrix} \tau_1 - i\tau_2 & 0 \\ 0 & \tau_1 + i\tau_2 \end{pmatrix}$$

Calculate loop at $q_0 = q_0(pole)$

$$f_\pi q_0 = \int \frac{d^4p}{(2\pi)^4}\frac{1}{2}\mathrm{Tr}\left[A^0_{\pi^+} S(p+q)(g_1\Gamma^{ll}_{75} + g_2\Gamma^{ur}_{57})S(p)\right]$$

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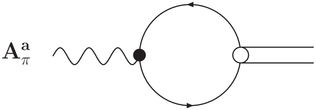

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- for $\mu = 500\,$MeV, $T = 0$, and $m_u = m_d = m_s = 30\,$MeV we find

$$f_\pi \approx 95.1\,\text{MeV}$$

- comparison with effective theory:

$$f_\pi = \sqrt{\frac{21 - 8\ln 2}{36}}\frac{\mu}{\pi} \approx 104.3\,\text{MeV}$$

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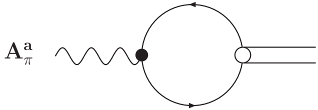

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  $f_\pi \approx 95.1\,$MeV   9 % smaller than in EFT

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Summary & Outlook

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- description of mesons with diquark loops
- good qualitative agreement with low-energy effective theory

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Outlook

- wider range of chemical potentials
- finite temperature
- include additional interactions
    - quark-antiquark interactions
- mesons in the new CFL+meson groundstate