Dynamics of relativistic heavy ion collisions I

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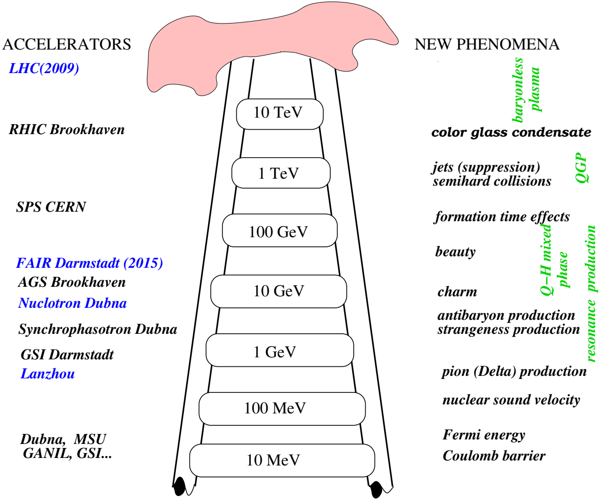
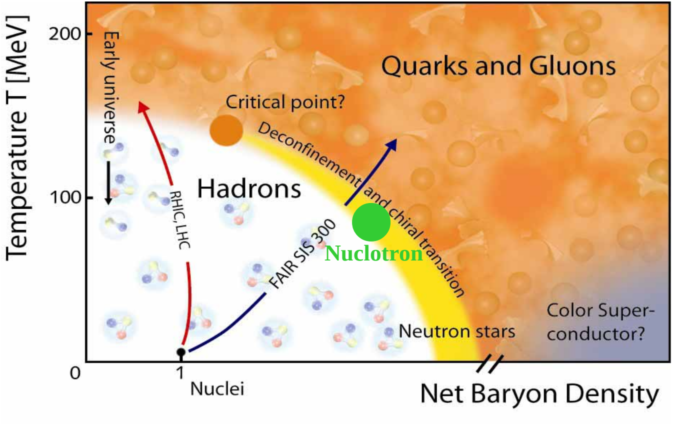
August 21, 2006

Energy stair



Phase diagram

Phases of strongly interacting matter



http://www.gsi.de/

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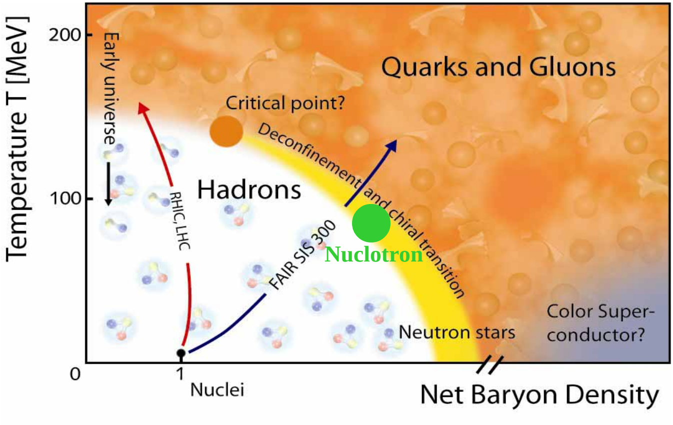

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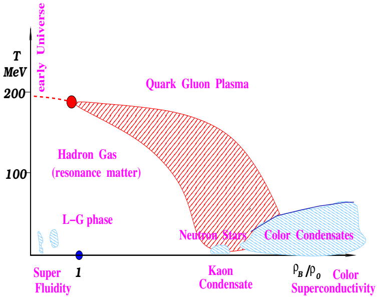
P.Crochet P.Senger A.Sorin

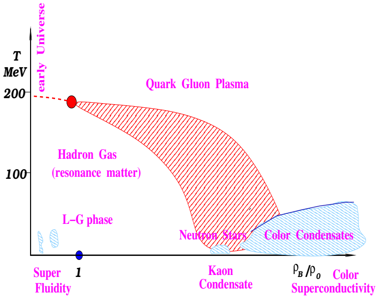
Phase diagram



Phase diagram

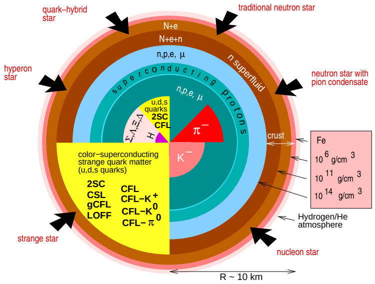
Universe expansion $\frac{T(\tau)}{\tau_0} = \frac{T_0(\tau_0)}{\tau}$

$$\tau_0 = \frac{3K \times 1.5 \cdot 10^9 \ \ years}{200 \ \ MeV} \sim \boxed{10^{-3} \ \ s}$$



Phase diagram

F.Weber, Prog.Part.Nucl.Phys. **54** (2005) 193



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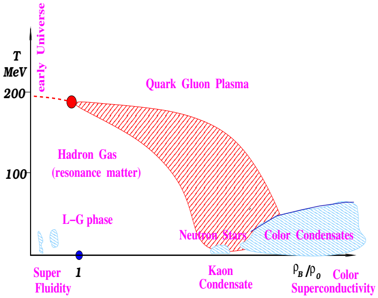

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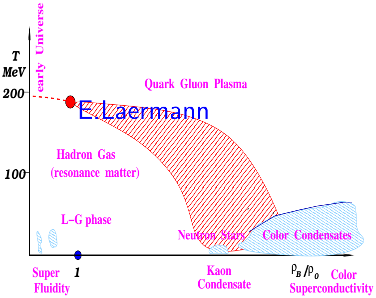
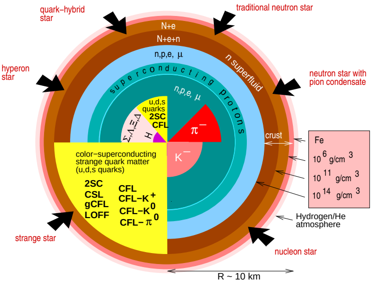


G.Röpke M.Buballa



G.Bisnovatyi-Kogan, D.Blaschke, H.Grigorian, S.Popov

General remarks

Cross section

$$\sigma \sim \int \prod_j d^3 p_j \mid < f|\mathcal{A}_n|i > |^2 \; \delta(E_f - E_i)$$

$f, i \rightarrow A, R, \rho_i, \ldots \qquad \mathcal{A}_n \rightarrow g_i, \ldots$

limiting cases:

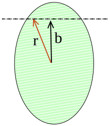
- elastic, inelastic scattering $\qquad p + A \rightarrow p' + A'$

$$\lambda = \frac{\hbar}{p} \gg 1 \qquad \psi(x) \sim \exp(\imath k z) + \mathcal{A}(\vec{q}) \; \exp(\imath \vec{k}\vec{r})/r$$

with the Glauber-Sitenko amplitude

$$\mathcal{A}(\vec{q}) = \frac{\imath}{2\pi\lambda} \int d^2 b \; \exp(\imath \vec{q}\vec{b}) \; \Gamma(\vec{b})$$

$$\Gamma(\vec{b}) = \int \mathcal{K}(r) d\vec{r} = \sum_i \eta_i(\vec{b} - \vec{r}_i)$$



General remarks

- participant-spectator model

(Fermi) phase space

$$| < f|\mathcal{A}_n|i > |^2 \simeq \text{const}$$

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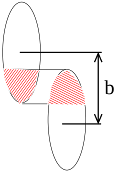

⋆ Pure state → particle ensemble → statistical consideration

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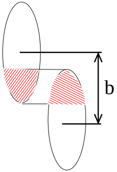

L.Turko, M.Gorenstein

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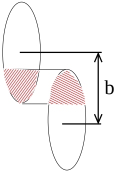

L.Turko, M.Gorenstein

⋆ Pure state → particle ensemble → statistical consideration

⋆ Adiabatic switching on the interaction ? → time evolution

N-body Liouville equation (time reversible !)

$$\frac{d\rho_N}{dt} = \frac{\partial}{\partial t}\rho_N + \frac{1}{\imath\hbar}[H, \; \rho_N] = 0$$

to solve it, justified approximations are needed

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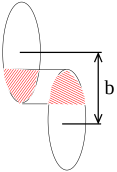

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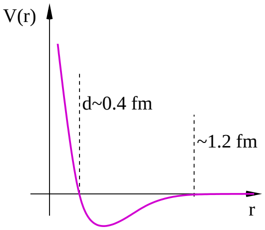
D.Voskresensky

Nuclear scales

d - repulsion NN force range
$\Lambda = \frac{1}{\sigma\rho}$ - (nucleon) mean free path
$\rho_0 \simeq 0.16\ fm^{-3}$ $\sigma \simeq 40\ mb \rightarrow \Lambda \sim$ 1.5 fm

Pauli principle
compression . . .

L - "macroscopic" length, 2-8 fm



units	d	Λ	L	d/Λ	$Kn = \Lambda/L$
air (10^{-8} cm)	1	10^5	10^8	10^{-3}	10^{-3}
liquid (10^{-8} cm)	1	2-10	10^8	0.1-0.5	10^{-7}
nuclei (0.4/1.2 fm)	1	1.5-2	2-8	0.2-0.6	1-0.2

kinetics $\Leftarrow$ $\boxed{d \ll \Lambda \ll L}$ $\Rightarrow$ *hydrodynamics*

For nuclear case (intermediate energies) : $d < \Lambda < L$

Molecular dynamics

- A-body problem in a classical picture

[Quantum] Molecular Dynamics

$$\dot{\vec{x}}_i = \frac{\partial}{\partial \vec{p}_i} H(i = 1, \dots A)$$

$$\dot{\vec{p}}_i = -\frac{\partial}{\partial \vec{x}_i} H(i = 1, \dots A)$$

with $H = -\sum \nabla^2_{p_i} + \sum_{i>k} V_{ik}$

nuclear stability

$V \rightarrow V^{Pauli}(p)$

NN-scattering ?

- *A*-body problem in a classical picture

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- Fermionic Molecular Dynamics

$$\sum \mathcal{A}_{\mu\nu}\ \dot{q}^{\nu} = -\frac{\partial}{\partial q^{\mu}}\ H$$

$q = \{\vec{p}, \vec{x}, s \dots\}$

wave packet

$$\text{with} \quad \mathcal{A}_{\mu\nu} = \frac{\partial^2 \mathcal{L}_0}{\partial \dot{q}^{\mu}\ \partial q_{\nu}} - \frac{\partial^2 \mathcal{L}_0}{\partial \dot{q}^{\nu}\ \partial q_{\mu}}$$

$$\text{CMD limit: } \mathcal{A}_{\mu\nu} \Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

BBGKY-Hierarchy

- Non-relativistic kinetic models

$$H = T + V = \sum \epsilon_i a_i^\dagger a_i + \sum V(ij, i'j')\ a_i^\dagger a_j^\dagger a_{i'} a_{j'}$$

n-particle density :
$\rho_n(x_1, x_2, \ldots x_n) = \mathcal{V}^{n-N} \int dx_{n+1} \ldots dx_N\ \rho(x_1 \ldots x_N)$

$$\begin{aligned} i\hbar \frac{\partial \rho_1(1)}{\partial t} &= [T_1, \rho_1(1)] + Tr_{(2)}[V_{12}, \rho_2(1,2)] \\ i\hbar \frac{\partial \rho_2(1,2)}{\partial t} &= [(T_1 + T_2 + V_{12}), \rho_2(1,2)] \\ &+ Tr_{(3)}[(V_{13} + V_{23}), \rho_3(1,2,3)] \\ \ldots\ldots\ldots\ldots\ &\ldots\ \ldots\ldots\ldots\ldots \end{aligned}$$

$\rho_1 \Rightarrow f^W(\vec{p}, \vec{x}, t) =< n(\vec{p}, \vec{x}) >_t$
with $n(\vec{p}, \vec{x}) = \int \frac{d^3k}{(2\pi\hbar)^3}\ e^{\imath \vec{k}\vec{x}}\ a^\dagger_{\vec{p}-\vec{k}/2} a_{\vec{p}+\vec{k}/2}$
and $\frac{1}{\Delta\mu} \int f^W(\vec{p}, \vec{x}, t)\ d\mu = f(\vec{p}, \vec{x}, t) + O(\frac{\hbar}{\Delta\mu})$

approximations are needed $\rho_2(1,2) = \rho_1(1)\ \rho_1(2)$

Vlasov-Boltzmann terms

Generalized kinetic equation :

$$\boxed{\frac{\partial f(\vec{p},\vec{x},t)}{\partial t} = -D(f) + C(ff)}$$

- Driving Vlasov term (classical limit)

(Hartree approximation, no exchange terms)

$$D(\vec{p},\vec{x},t) = \frac{\vec{p}}{m}\frac{\partial}{\partial\vec{x}}f(\vec{p},\vec{x},t) - \frac{\partial}{\partial\vec{x}}U(x)\ \frac{\partial}{\partial\vec{p}}f(\vec{p},\vec{x},t)$$

with an effective potential

$$U(x) = \int \frac{d^3x_1\ d^3p_1}{(2\pi\hbar)^3}\ V(\vec{x}-\vec{x}_1)\ f(\vec{p}_1,\vec{x}_1,t)$$

phenomenologically (Skyrme) $U(x) = -a\rho + b\rho^2$

Boltzmann terms

- Collision term

$\vec{p} + \vec{p}_2 \Rightarrow \vec{p}_1' + \vec{p}_2'$, no correlation and retardation effects

$$\begin{aligned} C(\vec{p}, \vec{x}, t) &= \int \frac{d^3 p_2 d^3 p_1' d^3 p_2'}{(2\pi\hbar)^6} \left| T_2(\vec{p}\vec{p}_2; \vec{p}_1'\vec{p}_2') - T_2(\vec{p}\vec{p}_2; \vec{p}_2'\vec{p}_1') \right|^2 \\ &\times \delta(E_p + E_{p_2} - E'_{p_1} - E'_{p_2}) \, \delta(\vec{p} + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \\ &\times \left[f_{p_1'} f_{p_2'} (1 - f_p)(1 - f_{p_2}) - f_p f_{p_2} (1 - f_{p_1'})(1 - f_{p_2'}) \right] \end{aligned}$$

$\Uparrow$ gain $\Uparrow$ loss

no exchange, no im-medium effects, ladder approximation for T_2

$$\begin{aligned} C(\vec{p}, \vec{x}, t) &= \int \frac{d^3 p_2 d^3 p_2'}{(2\pi\hbar)^3} \, \delta(\vec{p} + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \, v_{12} \, \frac{d\sigma^{el}}{d\Omega} \\ &\times \left[f_{p_1'} f_{p_2'} (1 - f_p)(1 - f_{p_2}) - f_p f_{p_2} (1 - f_{p_1'})(1 - f_{p_2'}) \right] \end{aligned}$$

$$\boxed{\left\{ \frac{\partial}{\partial t} + \frac{\vec{p}}{m} \frac{\partial}{\partial \vec{x}} + \frac{\dot{\vec{p}}}{m} \frac{\partial}{\partial \vec{p}} \right\} f(\vec{p}, \vec{x}, t) = C(\vec{p}, \vec{x}, t)} \quad + \delta C$$

BUU $\Rightarrow$ events generators; $f \ll 1 \Rightarrow$ Boltzmann equation

account for fluctuations $\Rightarrow$ B-Langevin equation random force $\Uparrow$

Relativistic kinetic equations

- the Walecka $\sigma - \omega$ model : $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$

$$\mathcal{L}_0 = \bar{\psi}(i\gamma_\mu\partial^\mu - m_N)\psi + \frac{1}{2}(\partial_\mu\sigma\,\partial^\mu\sigma - m_S\,\sigma^2) - \frac{1}{4}F_{\mu\nu}\,F^{\mu\nu} + \frac{1}{2}m_V^2\,\omega_\mu\omega^\mu;$$

$$\mathcal{L}_{int} = g_S\bar{\psi}\psi\sigma - g_V\bar{\psi}\gamma^\mu\psi\omega_\mu \qquad \text{with } F^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu.$$

Equations of motion

$$(\partial_\mu\partial^\mu + m_S^2)\,\sigma = g_S\bar{\psi}\psi \qquad \text{Klein-Gordon}$$

$$\partial F^{\mu\nu} + m_V^2 = g_V\bar{\psi}\gamma^\nu\psi \qquad \text{Proka}$$

$$\gamma^\mu(i\partial_\mu + g_V\omega_\mu) - (m_N - g_S\sigma)\psi = 0 \qquad \text{Dirac}$$

In the mean-field approximation

$$\sigma_0 = \frac{g_S}{m_S^2} < \bar{\psi}\psi > \equiv \frac{g_S}{m_S^2}\rho_s$$

$$\omega_0 = \frac{g_V}{m_V^2} < \bar{\psi}\gamma_0\psi > \equiv \frac{g_V}{m_V^2}\rho_B$$

$$\boxed{\left[p_\mu\partial^\mu - m_N^*\dot{p}^\nu\frac{\partial}{\partial p^\nu}\right] f(p,x) = C^{rel}(p,x)}$$

with $m_N^*\dot{p}^\nu = g_V p_\mu F^{\mu\nu} + m_N^*(\partial^\nu m_N^*)$ and quasiparticle parameters:
$m_N^* = m_N - g_S\,\sigma_0$ - eff. mass, $p_\mu \to p_\mu - g_V\,\omega_\mu$ - kinetic momentum

RBUU $\Rightarrow$ events generators

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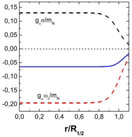

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RBUU $\Rightarrow$ events generators

Steps towards higher energies

- Relativistic Boltzmann equation $(\psi, \sigma, \omega \Rightarrow 0; f \ll 1)$

$$(p_\mu \partial^\mu)\ f_i(x,p_i) = \sum_j C^{rel}(x,p_i) + \sum_r R_{r\to i}$$

$$= -f_i(x,p_i)\sum_j \int d\omega_j f_j(x,p_j) Q_{ij}\sigma^{ij} + \sum_{kj}\int d\omega_k d\omega_j \Phi(p_j p_k \mid x, p_i, \tau_f)$$

$$+ \sum_r \int d\omega_{k'} d\omega_r\ f_r(x,p_r)\ \Gamma^{r\to i+k'}\ \delta(p_r - p_i - k')$$

hadron production rate

$$\Phi(p_j p_k \mid x, p_i, \tau_f) = \int dx'\ \underbrace{f_k(x',p_i) f_j(x',p_j) Q_{ij}\sigma^{ij}}_{collision\ rate}\ \underbrace{\phi(x' \mid x, p_i, \tau_f)}_{transition\ prob.}$$

with $d\omega = d^3p/E,\ \ Q_{ij} = (p_i p_j)^2 - p_i^2 p_j^2 = \mid v_i - v_j \mid E_i E_j$

transition probability for a finite formation time

$$\phi(x' \mid x, p, \tau_f) = \frac{1}{\sigma}\frac{d\sigma}{d\omega}\ \theta(t - t' - \tau_f)\ \delta^{(3)}(\vec{x} - \vec{x}' - \frac{\vec{p}}{E}(t - t'))\ F(\tau_f)$$

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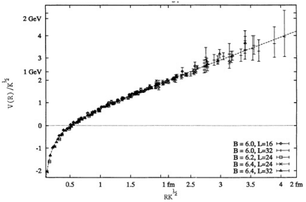
- ⋆ multiple particle production $\Rightarrow$ coupled set of equations for stable hadrons and resonances $\{h_i\}$; new flavors
- ⋆ finite formation time $\theta(t - \tau_f)$, $\tau_f = (E/m)\tau_f^0$ with $\tau_f^0 \sim 1$ fm; $\Rightarrow$ memory (retarded) effect (non-Markovian process)

Strings

- new degrees of freedom (QCD) : quark/gluons , strings, formation of color rope

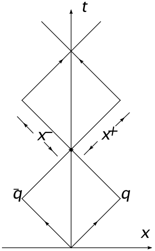
⋆ Hadron as a string

G.S.Bali, K.Schilling, Phys.Rev.D **46** (1992) 2636



$$V_{q\bar{q}} = -\frac{\alpha_{eff}}{r} + \kappa r$$

yo-yo mode



$$H_{yo-yo} = |\, p_1 \,| + |\, p_2 \,| + \kappa \,|\, x_1 - x_2 \,|$$

$$\frac{dp_{1,2}}{dt} = \pm\kappa \; , \; \frac{dx_{1,2}}{dt} = \pm 1$$

$$x^+ = \frac{p^+}{\kappa} = \frac{E+p}{\kappa} \; , \; x^- = \frac{p^-}{\kappa} = \frac{E-p}{\kappa} , \; S = \frac{p^+ p^-}{\kappa^2} = \frac{E^2 - p^2}{\kappa^2} = \frac{m^2}{\kappa^2}$$

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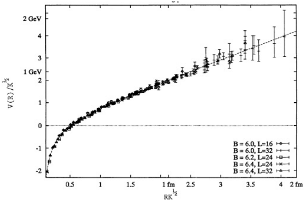

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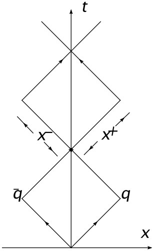
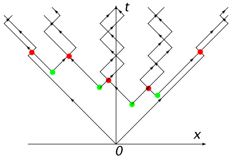


String breakup



$$H_{yo-yo} = |\, p_1 \,| + |\, p_2 \,| + \kappa \,|\, x_1 - x_2 \,|$$

$$\frac{dp_{1,2}}{dt} = \pm\kappa \;, \quad \frac{dx_{1,2}}{dt} = \pm 1$$

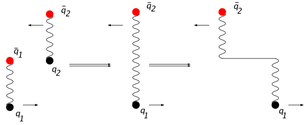
$$x^+ = \frac{p^+}{\kappa} = \frac{E+p}{\kappa} \;, \quad x^- = \frac{p^-}{\kappa} = \frac{E-p}{\kappa}, \quad S = \frac{p^+p^-}{\kappa^2} = \frac{E^2-p^2}{\kappa^2} = \frac{m^2}{\kappa^2}$$

String interaction

- Particularities of space-time evolution

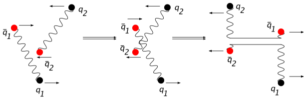
CLASSICAL STRING THEORY

⋆ string fusion



time ⇒

⋆ string rearrangement

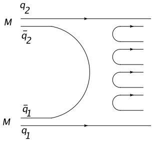


⋆ leading particle effect

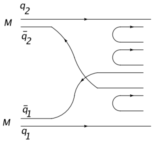
⋆ color rope formation

DUAL TOPOL. MODEL

⋆ planar diagram

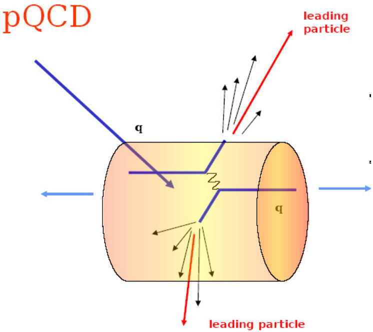


⋆ cylindrical diagram



Jets

- jet production



Hard parton-parton collision (true two-body scattering)

$p_\perp \gg \Lambda_{QCD}, \quad r_\perp \sim \hbar c/p_\perp$

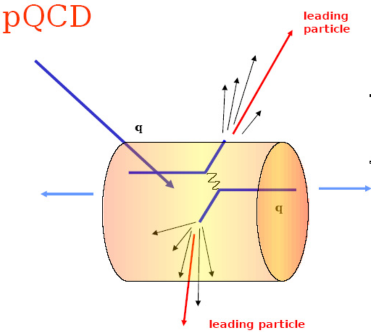
$$\frac{d\sigma}{dp_\perp} = \frac{C}{p_\perp^n}$$

RHIC physics

HIJING code

Jets

- jet production



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RHIC physics
HIJING code

- solution ⇒ Monte Carlo Methods:
  event generators ⇒ UrQMD, QGSM, HSD ...

- quark-gluon transport (color → dynamical degree of freedom, in the quasi-classical limit → $(p\partial_x - g\,pF\partial_p)W$)

⋆ Nambu-Jona-Losinio model   V.Yudichev

Basic kinetic idea

HIC $\Rightarrow$ subsequent collisions between quasiparticles (Boltzmann-like equations)

Physics : What is a quasiparticle ?

non-relativistic

$$\left(\frac{\partial}{\partial t} + \vec{v}\vec{\bigtriangledown}_x + \frac{d\vec{p}}{dt}\vec{\bigtriangledown}_p\right) f(\vec{p},\vec{x},t) = C(\vec{p},\vec{x},t)$$
$$\Uparrow \frac{d\vec{p}}{dt} = -\vec{\bigtriangledown}_x \frac{d\vec{p}}{dt} U(\vec{r},t)$$

(p-h)
$N + V(r)$
free N

relativistic – QHD

$$(p_\mu \partial^\mu + m^* \dot{p}_\mu \partial^\mu) f(p,x) = C^{rel}(p,x)$$
$$m^* \dot{p}_\mu = g_V p_\nu F^{\mu\nu} + m^*(\partial_x^\mu m^*) + \text{field eqs.}$$

hadrons $+\psi$
(Walecka-like)

Boltzmann : $(p_\mu \partial^\mu) f(p,x) = C^{rel}(p,x)$

resonances
strings
color ropes
jets

non-abelian fields (color) – QCD

$p, x \Rightarrow p, x, \mathcal{Q}$

flow term +source term

extreme case: free rescatt. of quarks and gluons

quarks/gluons