Many-body theories of strongly correlated systems

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- Lecture I: Many-body theories nuclear systems
  - Real-time GF technique - quantum kinetic equations
  - The two-body problem and equations of state
  - The three-body problem
  - Alpha particle condensation
- Lecture II: Neutrino radiation from dense matter
  - Classifications of the reactions
  - Self-energies and loop-expansions
  - The role of pairing correlations
  - Neutrino rates from color superconducting matter

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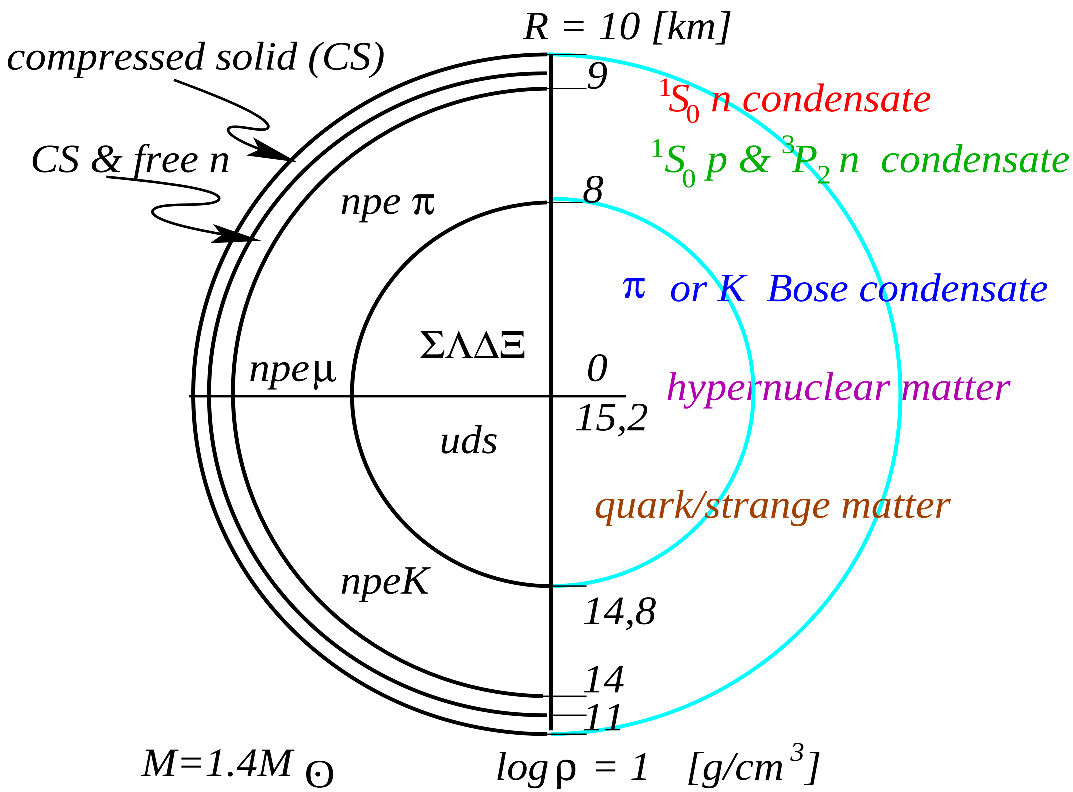


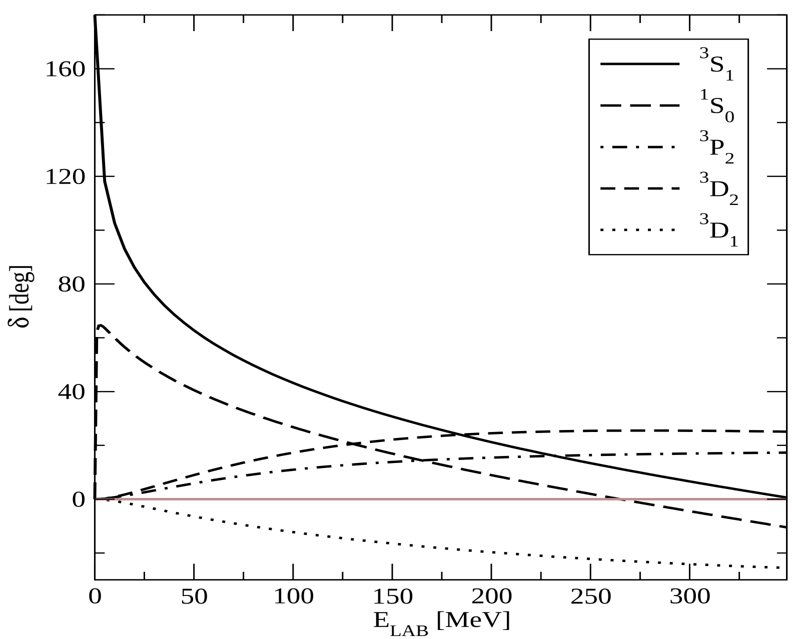
Measured parameters: Mass, radius, rotation frequency and its first, second derivatives and anomalies, magnetic fields, surface composition, thermal/non-thermal radiation, gravity waves

Density-temperature plane

$\rho \leq 3\rho_0$, $\rho_0 = 0.16$ fm^{-3}, $T \sim 0 - 60$ MeV

- Free-space interaction is well constrained (phase-shifts below $E \sim$ 350 MeV)

pulsive force



- Complexity:
  1. pair-correlations
  2. bound states and their Bose condensation
  3. neutron rich exotic nuclei
  4. strangeness (stable hypernuclear matter)
  5. strong fields
  6. . . .

A short introduction to Real-Time GF

Equation of motion for Heisenberg operators

$$i\partial_t \psi(x) = \left[\psi(x), \mathcal{H}\right], \tag{1}$$

Path-ordered correlation function

$$\boldsymbol{G}(1,1') = -i\langle \mathcal{P}\psi(1)\psi^\dagger(1')\rangle$$

Equations of motion for PO correlation function

$$\boldsymbol{G}(1,1') = \boldsymbol{G}_0(1,1') - i\int_C d2d3d4\; \boldsymbol{G}_0(1,1')\; V(12;34)\; \boldsymbol{G}_2(34,1'2^+)$$

$$\boldsymbol{G}_0(1)^{-1}\boldsymbol{G}(1,1') = \boldsymbol{\delta}(1-1') - i\int_C d2d3d4\; V(12;34)\; \boldsymbol{G}_2(34,1'2^+),$$

Contour self-energies decouple the hierarchy as

$$\boldsymbol{\Sigma}(1,3)\boldsymbol{G}(3,1') = -i\int_C d2d4\; V(12;34)\; \boldsymbol{G}_2(34,1'2).$$

“Prototype” kinetic equation

$$\left[\boldsymbol{G}_0^*(1') - \boldsymbol{G}_0(1)\right]\boldsymbol{G}(1,1') = \int_C d2\left[\boldsymbol{G}(1,2)\;\boldsymbol{\Sigma}(2,1') - \boldsymbol{\Sigma}(1,2)\;\boldsymbol{G}(2,1')\right].$$

Six propagators on the contour

$$G^{<}(1,1') = -i\langle\psi(1)\psi^{\dagger}(1')\rangle, \quad G^{>}(1,1') = i\langle\psi^{\dagger}(1')\psi(1)\rangle.$$

$$\begin{aligned} G^{c}(1,1') &= \theta(t_1 - t_1')G^{>}(1,1') + \theta(t_1' - t_1)G^{<}(1,1'), \\ G^{a}(1,1') &= \theta(t_1' - t_1)G^{>}(1,1') + \theta(t_1 - t_1')G^{<}(1,1'), \end{aligned}$$

$$\begin{aligned} G^{R}(1,1') &= \theta(t_1 - t_1')[G^{>}(1,1') - G^{<}(1,1')], \\ G^{A}(1,1') &= \theta(t_1' - t_1)[G^{<}(1,1') - G^{>}(1,1')]. \end{aligned}$$

Quasi-classical kinetic equation

$$i\{\Re G^{-1}(p,x), G^{<}(p,x)\}_{P.B.} + i\{\Sigma^{<}(p,x), \Re G(p,x)\}_{P.B.} = \Sigma^{<}(p,x)G^{>}(p,x) - \Sigma^{>}(p,x)G^{<}(p,x),$$

the Kadanoff-Baym (KB) ansatz,

$$-iG^{<}(p,x) = a(p,x)f(p,x), \quad iG^{>}(p,x) = a(p,x)\left[1 - f(p,x)\right]$$

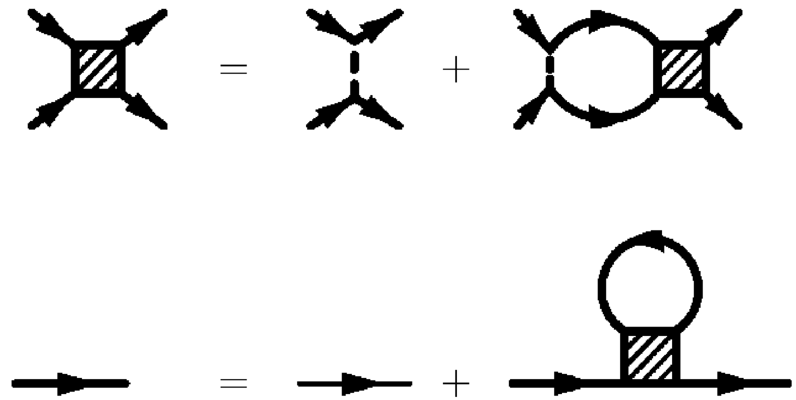
$$\left[\partial_t + \partial_{\boldsymbol{p}}\epsilon(p,x)\partial_{\mathbf{r}} + \partial_{\mathbf{r}}\epsilon(p,x)\partial_{\boldsymbol{p}}\right] f(\boldsymbol{p},x) = \int \frac{d\omega}{(2\pi)}\left[\Sigma^{>}(p,x)G^{<}(p,x) - G^{>}(p,x)\Sigma^{<}(p,x)\right].$$

(some) advantages of RTGF

- Boltzmann equation is recovered in the quasiparticle limit $a(p) \propto \delta(\omega - \epsilon_p)$; more general framework (off-shell transport)
- valid for arbitrary non-equilibrium situations
- Particle-hole symmetry of the self-energies must be preserved (violated, e.g., in Bruckner type calculations)
- Very convenient diagrammatic extension of the Feynman rules to finite temperatures

for further reading, see W. Botermans and R. Malfliet, Phys. Rep. 1990

Self-consistent solution of coupled equations



$$\boldsymbol{T}(12;34) = \boldsymbol{V}(12;34) + i\int_C d5d6\ \boldsymbol{V}(12;34)\ \boldsymbol{G}(35)\ \boldsymbol{G}(46)\ \boldsymbol{T}(56;34)$$

$$\boldsymbol{\Sigma}(1,2) = i\int_C d3d4\ \boldsymbol{T}(12;34)\boldsymbol{G}(43^+).$$

Equation of state

internal energy

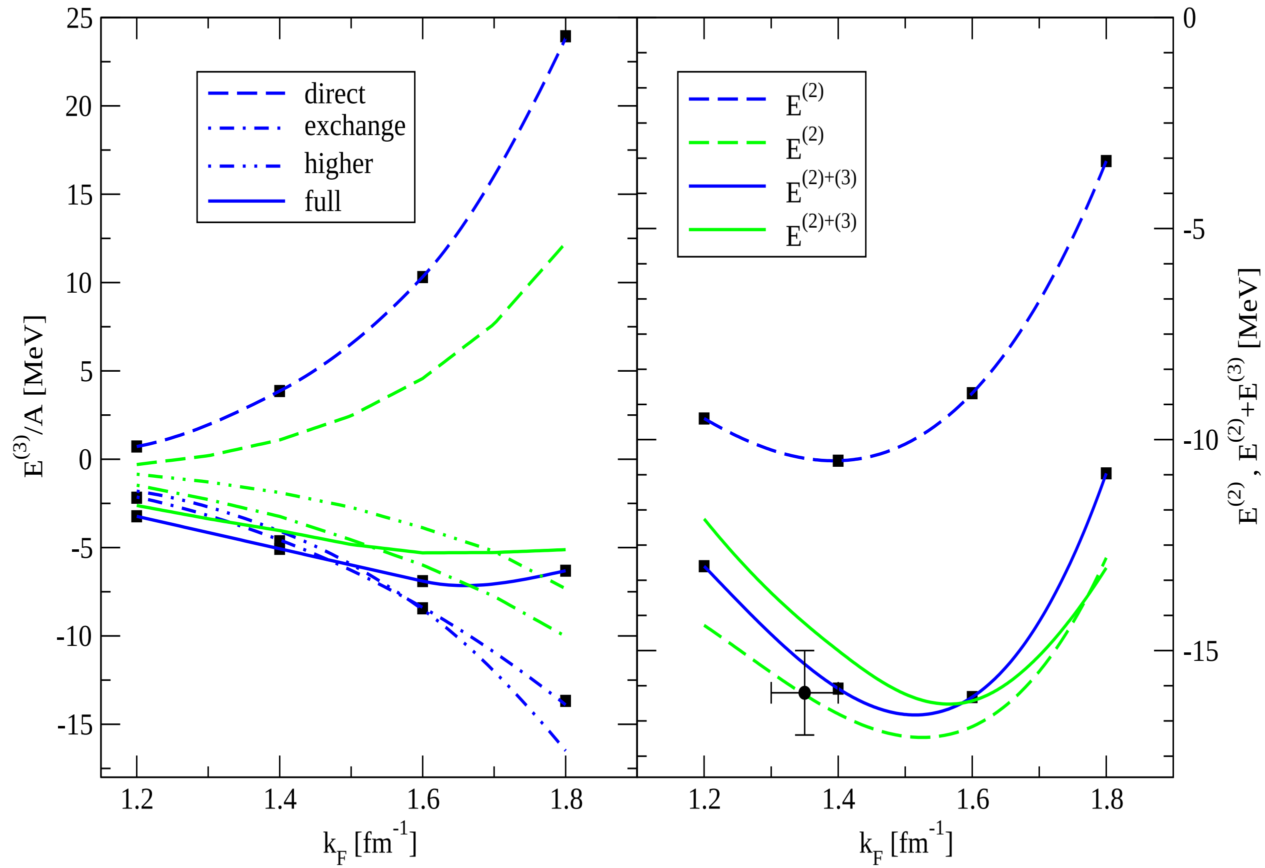
$$E = g \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \left[\omega + \frac{p^2}{2m} + \mathrm{Re}\Sigma(p) \right] a(p) f_F(\omega), \tag{2}$$

free-energy

$$F = E - \beta^{-1} S, \tag{3}$$

entropy

$$S = g \int \frac{d^4p}{(2\pi)^4} A(p) \left\{ f_F(\omega) \ln f_F(\omega) + [1 - f_F(\omega)] \ln [1 - f_F(\omega)] \right\}. \tag{4}$$


direct
exchange
higher
full
$E^{(3)}/A$ [MeV]
k_F [fm^{-1}]
$E^{(2)}$
$E^{(2)}$
$E^{(2)+(3)}$
$E^{(2)+(3)}$
$E^{(2)}$, $E^{(2)}+E^{(3)}$ [MeV]
k_F [fm^{-1}]

- Saturation problem
- Particle-hole symmetries are broken
- Need three-body forces
- Relativistic kinematics and dynamics (?)
- Density function theories that fit the properties

Many possible solutions, no definitive answer yet

NS observations have predictive power

Charge neutrality and weak equilibrium

equilibrium with respect to weak interactions

$$n \rightarrow p + e^- + \bar{\nu}_e, \qquad p + e^- \rightarrow n + \nu_e, \quad \mu_e = \mu_n - \mu_p. \tag{5}$$

charge neutrality

$$n_p = n_e, \quad Y_p = Y_e = \frac{8}{3\pi^2 n} S_2^3 \left(\frac{1}{2} - Y_p\right)^3 \tag{6}$$

Symmetry energy

$$E_S = S_2\alpha^2 + S_4\alpha^4 + O(\alpha^6), \tag{7}$$

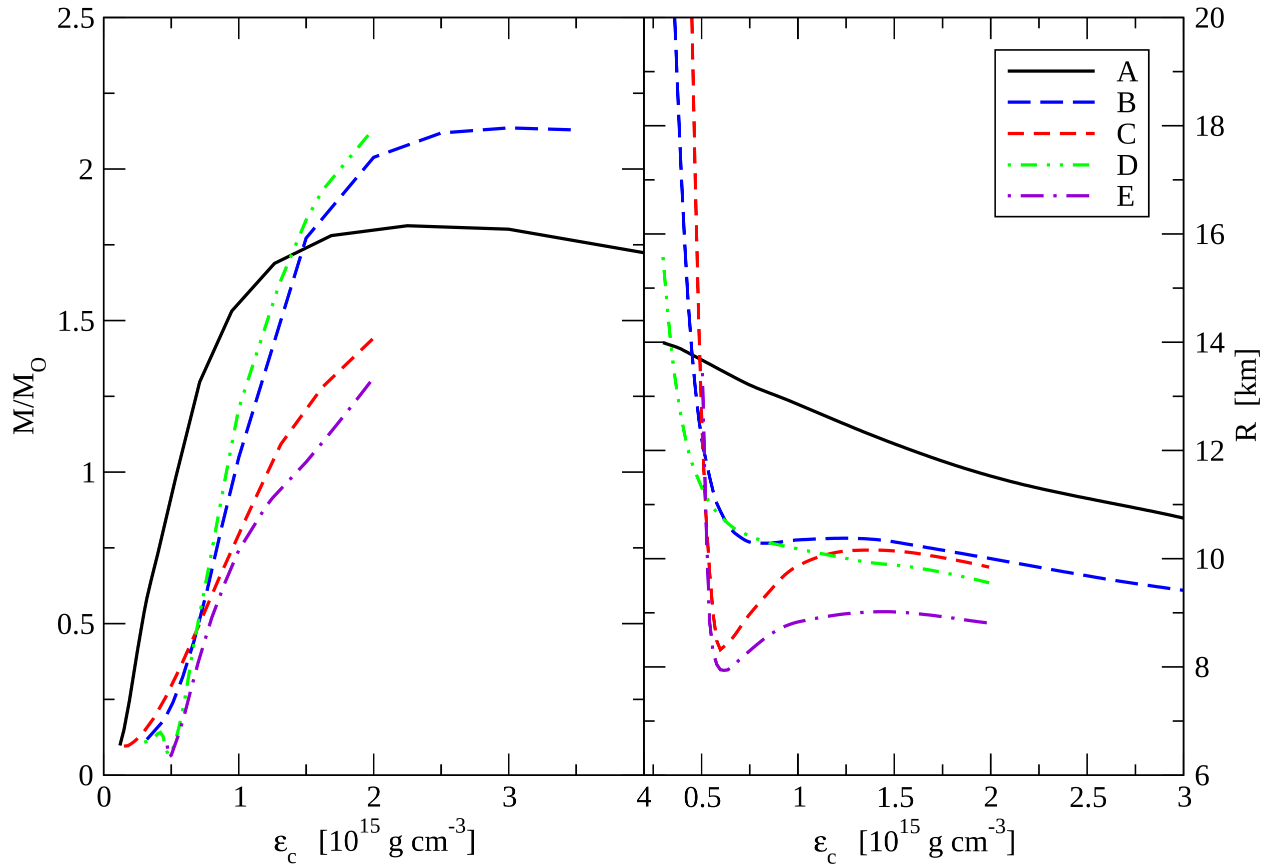
$\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

collection of EOS

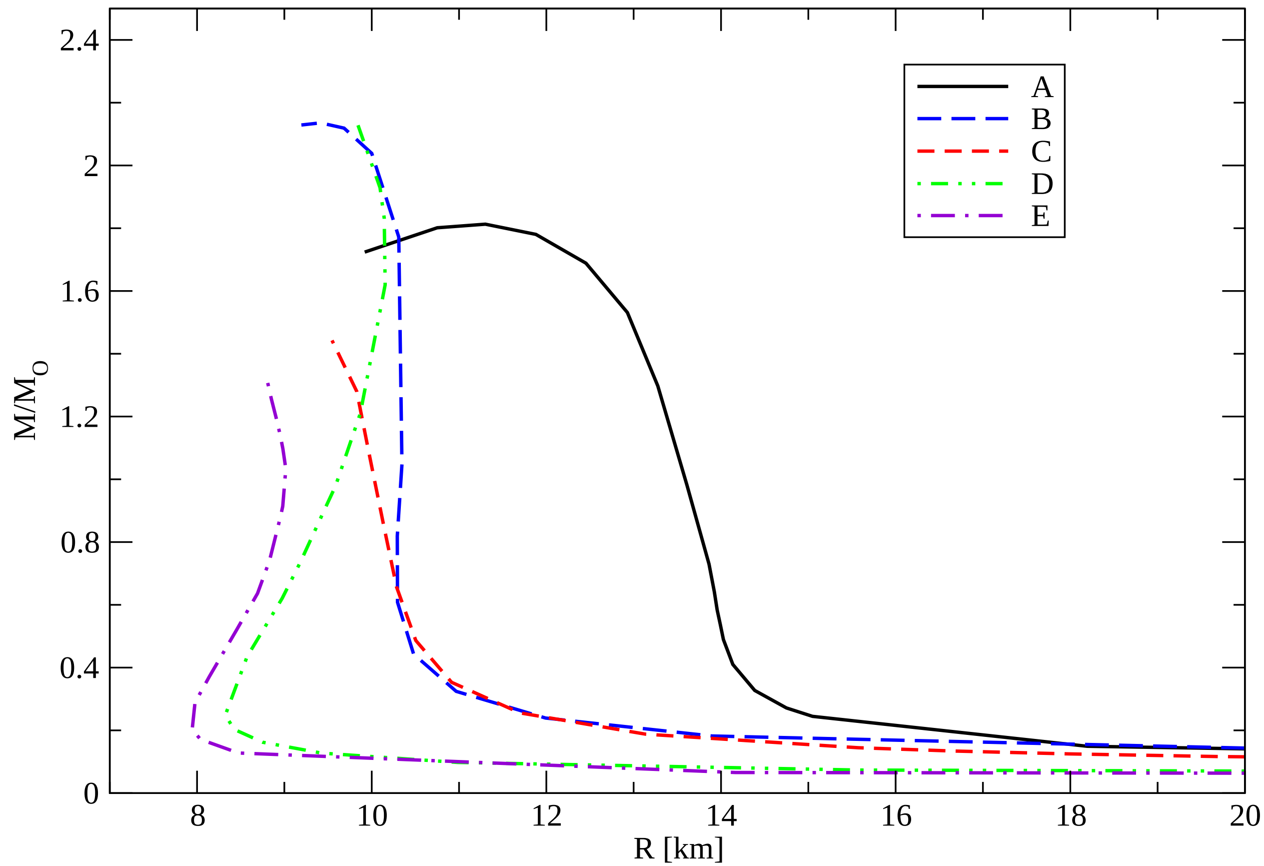
EoS	method	composition	forces
A	RMF	$npeH\pi$	contact 2B
B	variational	npe	realistic 2B + 3B
C	DBHF	npe	realistic 2B
D	BBG	npe	realistic 2B + 3B
E	BBG	$npeH$	realistic 2B + 3B

Table 1:

Mass and radius as a function of the central density



Mass radius relationship



The three-body problem in background medium

.... as an example of efficient application of the RTG formalism

Foundations:

- Skornyakov and Ter-Martirosian, 1956 (contact interactions)
- Faddeev, 1960 (for arbitrary, finite range interactions)
- Bethe, 1965 (nuclear matter problem, hole-line expansion)
- Alt, Sandhas et al 1969 (alternative forms of Faddeev equations)

- The three-body equation for the $\mathcal{T}$-matrix

$$\mathcal{T} = \mathcal{V} + \mathcal{V}\,\mathcal{G}\,\mathcal{V} = \mathcal{V} + \mathcal{V}\,\mathcal{G}_0\,\mathcal{T}, \tag{8}$$

where the interaction $\mathcal{V} = V_{12} + V_{23} + V_{13}$

- Reformulate the problem: $\mathcal{T} = \mathcal{T}^{(1)} + \mathcal{T}^{(2)} + \mathcal{T}^{(3)}$

$$\mathcal{T}^{(k)} = \mathcal{V}_{ij} + \mathcal{V}_{ij}\mathcal{G}_0\mathcal{T} \quad ijk = 123,\,231,\,312. \tag{9}$$

Define: $\mathcal{T}_{ij} = \mathcal{V}_{ij} + \mathcal{V}_{ij}\mathcal{G}_0\mathcal{T}_{ij}$ and eliminate the potentials

- Non-singular three-body equations

$$\mathcal{T}^{(k)} \;=\; \mathcal{T}_{ij} + \mathcal{T}_{ij}\mathcal{G}_0\left(\mathcal{T}^{(i)} + \mathcal{T}^{(j)}\right). \tag{10}$$

Three-body propagator in background medium

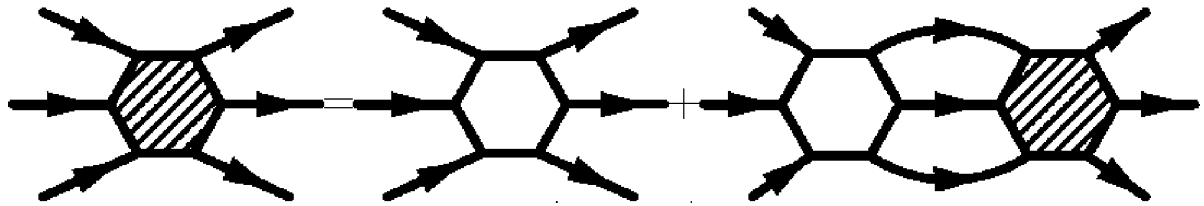
- The time structure of the three-body T-matrix

$$\mathcal{T}^{R\,(1)}(t,t') = \mathcal{T}^{R}_{23}(t,t') + \int \left[\, \mathcal{T}^{R\,(2)}(t,\bar{t}) + \mathcal{T}^{R\,(3)}(t,\bar{t}) \right] \mathcal{G}^{R}_{0}(\bar{t},t'')\mathcal{T}^{R}_{23}(t'',t')d\bar{t}dt'',$$

- Possible particle-hole channels

$$\mathcal{G}^{R}_{0}(t_1,t_2) = \theta(t_1-t_2) \begin{cases} G^{>}G^{>}G^{>}(t_1,t_2) - (> \leftrightarrow <) & (3p) \\ G^{>}G^{>}G^{<}(t_1,t_2) - (> \leftrightarrow <) & (2ph \\ G^{>}G^{<}G^{<}(t_1,,t_2) - (> \leftrightarrow <) & (p2h \\ G^{<}G^{<}G^{<}(t_1,t_2) - (> \leftrightarrow <) & (3h) \end{cases}$$

Particle-hole content of the T-matrix



- 3-particle – 3-hole scattering T-matrix

$$\mathcal{T}^{R\,(1)} = \mathcal{T}^{R}_{23} + \int \left[\mathcal{T}^{R\,(2)} + \mathcal{T}^{R\,(3)} \right] \frac{Q_3(\Omega')}{\Omega - \Omega' + i\eta} \mathcal{T}^{R}_{23}(\Omega') d\Omega',$$

- 3-body Pauli-blocking: $\bar{f}_F = 1 - f_F$

$$Q_3(p_\alpha, p_\beta, p_\gamma) = \bar{f}_F(p_\alpha)\bar{f}_F(p_\beta)\bar{f}_F(p_\gamma) - f_F(p_\alpha) f_F(p_\beta) f_F(p_\gamma).$$

p_α are spanned in terms of Jacobi coordinates.

Bound states in background medium

- Bound state wave-function

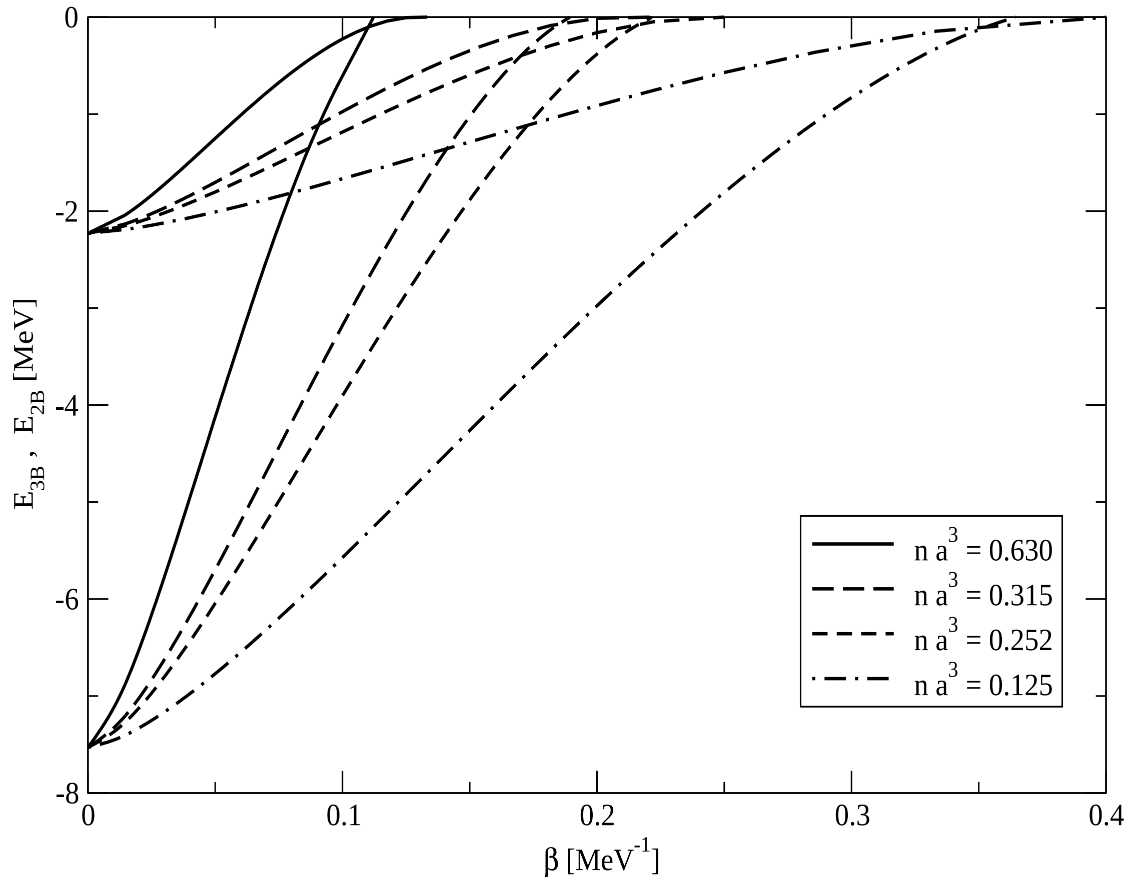
$$\Psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)}; \quad \psi^{(k)} = \mathcal{G}_0 T_{ij}(\psi^{(i)} + \psi^{(j)}). \quad (11)$$

- Need the channel T-matrix

$$T^R(\vec{p}, \vec{p}'; \vec{P}, E)$$
$$= V(\vec{p}, \vec{p}') + \int \frac{d\vec{p}''}{(2\pi)^3} V(\vec{p}, \vec{p}'') G_0^R(\vec{p}'', \vec{P}, E) T^R(\vec{p}'', \vec{p}'; \vec{P}, E)$$

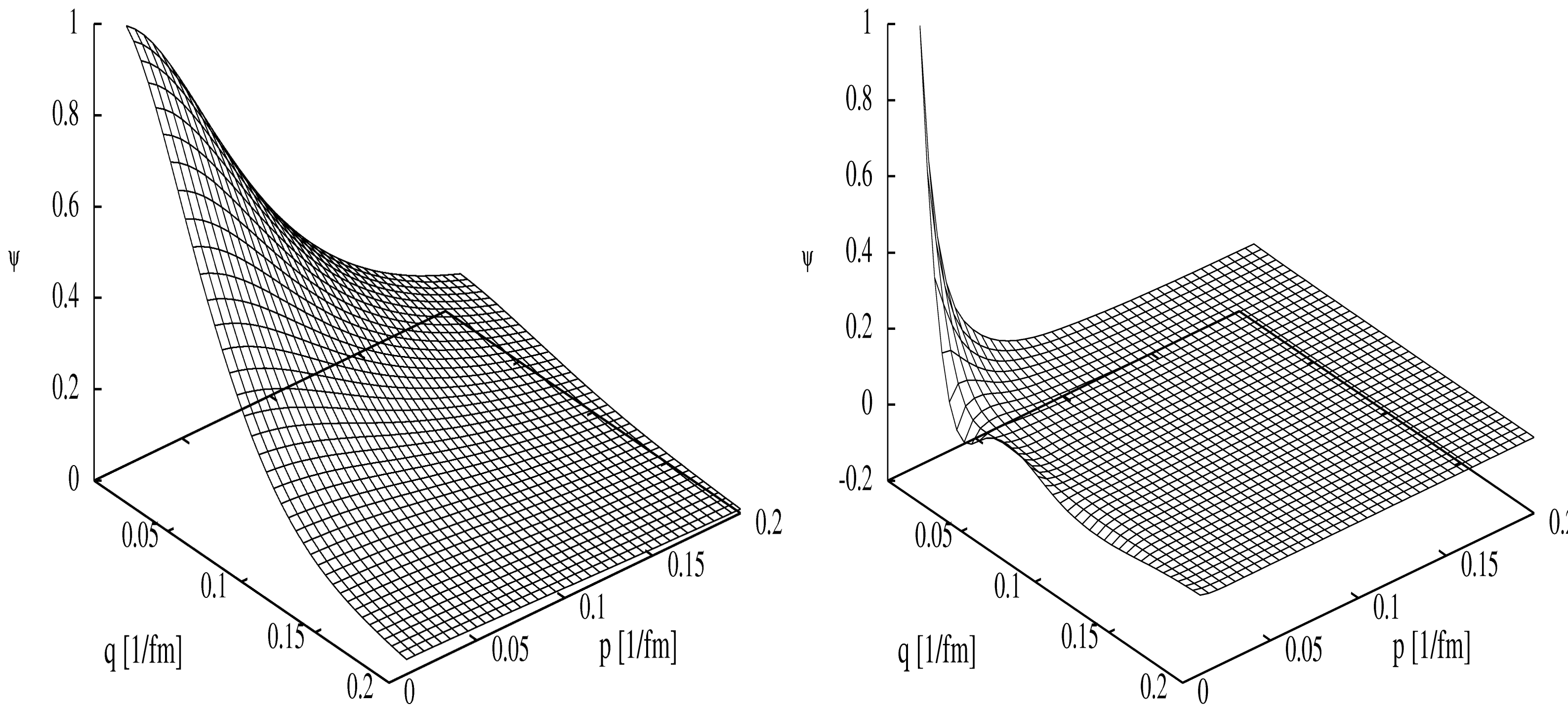
$$G_0^R(\vec{k}_1, \vec{k}_2, E) = \frac{Q_2(\vec{k}_1, \vec{k}_2)}{E - \epsilon(\vec{k}_1) - \epsilon(\vec{k}_2) + i\eta}, \quad (12)$$

Temperature dependent binding energies of triton in nuclear matter

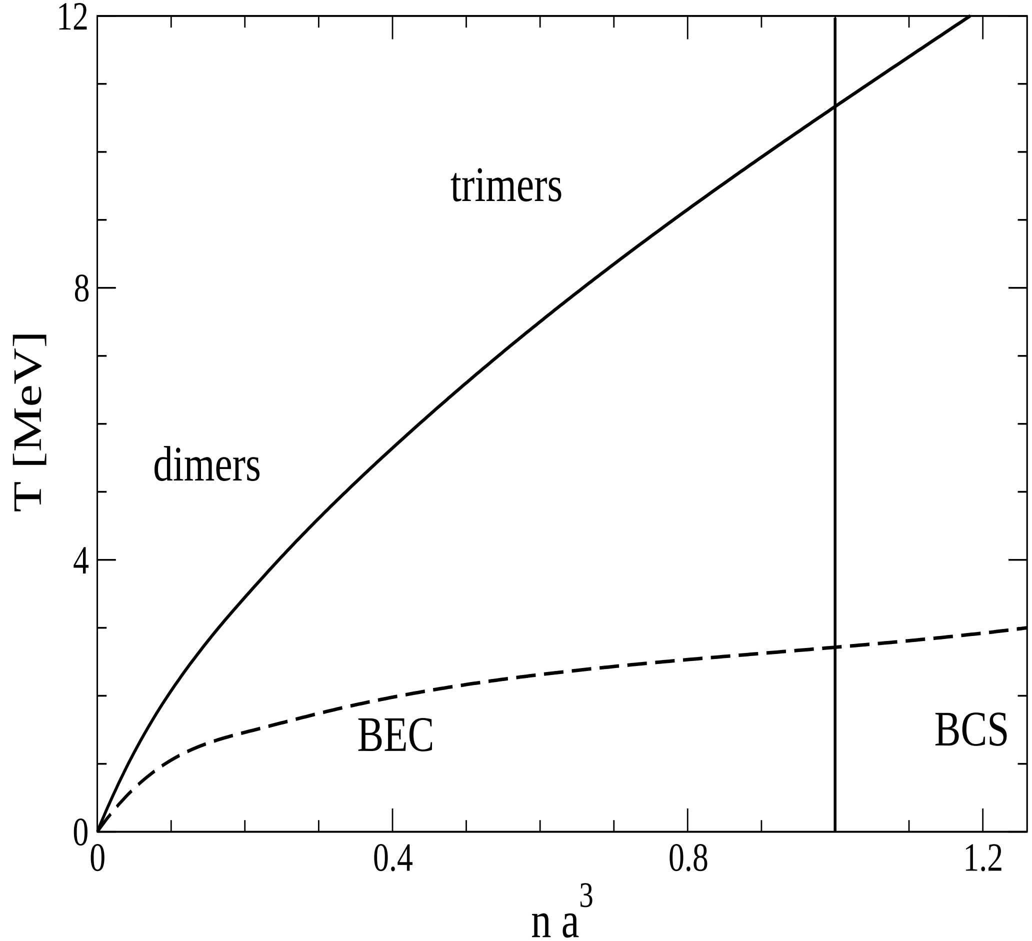


The ratio $\eta = E_{3B}(T)/E_{2B}(T)$ *is independent of temperature.*

The three-body wave-function



The phase diagram



Summary on three-body problem

- The background medium modifies the three-body w.f. and binding energy and leads to extinction of bound states at a critical temperature
- The ratio of the three-body to two-body bound states energies is found to be independent of temperature.
- Implications for the composition of dilute nuclear matter and physics of dilute multi-species atomic gases remains to be explored. Of particular interest are the Efimov states recently observed in gases.

Alpha condensation

- Phenomenology of alpha condensation:
  - The excited states of $4N$ nuclei are well described within the α particle model: elementary degrees of freedom are α's interacting via a α-α potential: ^{8}Be (unstable) ^{12}C (first stable α nucleus), ^{16}O, ^{40}Ca.
  - Recent work suggest that these systems are well described by single wave function (BEC in systems with a few particles ?).
  - This motivates the study of Bose-Einstein condensation in *infinite alpha matter* - start with $N \rightarrow \infty$ system and follow the crossover as N is reduced. (*see work by G. Röpke, P. Schuck + Japanese colleagues*).

Alpha condensation

- Astrophysical motivation:
  - Supernova matter at densities $\rho \sim 10^{12}$ g cm^{-3} and temperatures $T \leq 10$ MeV contains $15 - 20\%$ α particles
  - α effect and its impact on the r-process (*McLaughlin, Fuller and Wilson, 1996*).
  - triple-alpha fusion in accreting neutrons stars $\alpha + \alpha + \alpha \rightarrow {}^{12}C+Q$ *(Langanke et al 1992)).*

From Hamiltonian to effective action

- *Consider gas of α particles interacting with 2 and 3-body forces*

$$\begin{aligned} H &= \int d^3x \Bigg[\frac{\hbar^2}{2m} \nabla \psi^\dagger(\mathbf{x}) \nabla \psi(\mathbf{x}) - \mu \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}) \\ &\qquad + \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}) U(\mathbf{x}) \Bigg], \end{aligned}$$

- *Interactions:*

$$\begin{aligned} U(\mathbf{x}) &= \int d\mathbf{x}' V_2(\mathbf{x}', \mathbf{x}) \psi^\dagger(\mathbf{x}') \psi(\mathbf{x}') \\ &+ \int d\mathbf{x}' \int d\mathbf{x}'' V_3(\mathbf{x}, \mathbf{x}', \mathbf{x}'') \psi^\dagger(\mathbf{x}'') \psi(\mathbf{x}') \psi^\dagger(\mathbf{x}'') \psi(\mathbf{x}'), \end{aligned}$$

Coupling constants

Assume contact form of interaction (need lattice regularization):

$$U(\mathbf{x}) = g_2\psi^\dagger(\mathbf{x})\psi(\mathbf{x}) + g_3\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x}),$$

$g_2 = 4\pi\hbar^2 a_{\mathrm{sc}}^{(2)}/m$ is related to the scattering length:

$$a_{\mathrm{sc}}^{(2)} = \frac{m}{4\pi\hbar^2}\int d\mathbf{x}\; V_2(\mathbf{x}).$$

and assume that

$$V_3(\mathbf{x},\mathbf{x}',\mathbf{x}'') = \tilde{V}_2^{(1)}(\mathbf{x},\mathbf{x}') + \tilde{V}_2^{(2)}(\mathbf{x},\mathbf{x}'') + \tilde{V}_2^{(3)}(\mathbf{x}',\mathbf{x}'').$$

Effective action

- Expand the fields ψ and $\psi^\dagger$ in Matsubara sums and keep near T_c the zeroth order term (Baym, Blaizot, Zinn Justin '99).

$$\psi(\mathbf{x},\omega_\nu) \;=\; \psi_0(\mathbf{x}) + \sum_{\nu=-\infty,\ \nu\neq 0}^{\infty} e^{i\omega_\nu\tau}\psi(\mathbf{x},\tau), \qquad (13)$$

- New fields: $\psi = \eta(\phi_1 + i\phi_2)$, where $\phi_{1,2}$ -real, $\eta = \sqrt{m/\hbar^2\beta}$.

Partition function

- Continuum action describes a *classical $O(2)$ symmetric scalar ϕ^6 field theory in 3 dimensions* $\phi^2 = \phi_1^2 + \phi_2^2$:

$$\mathcal{S}(\phi) = \int d^3x \left\{ \frac{1}{2} \sum_{\nu} \left[\partial_\nu \phi\right]^2 + \frac{r}{2}\phi^2 - \frac{u}{4!}\left[\phi^2\right]^2 + \frac{w}{6!}\left[\phi^2\right]^3 \right\}.$$

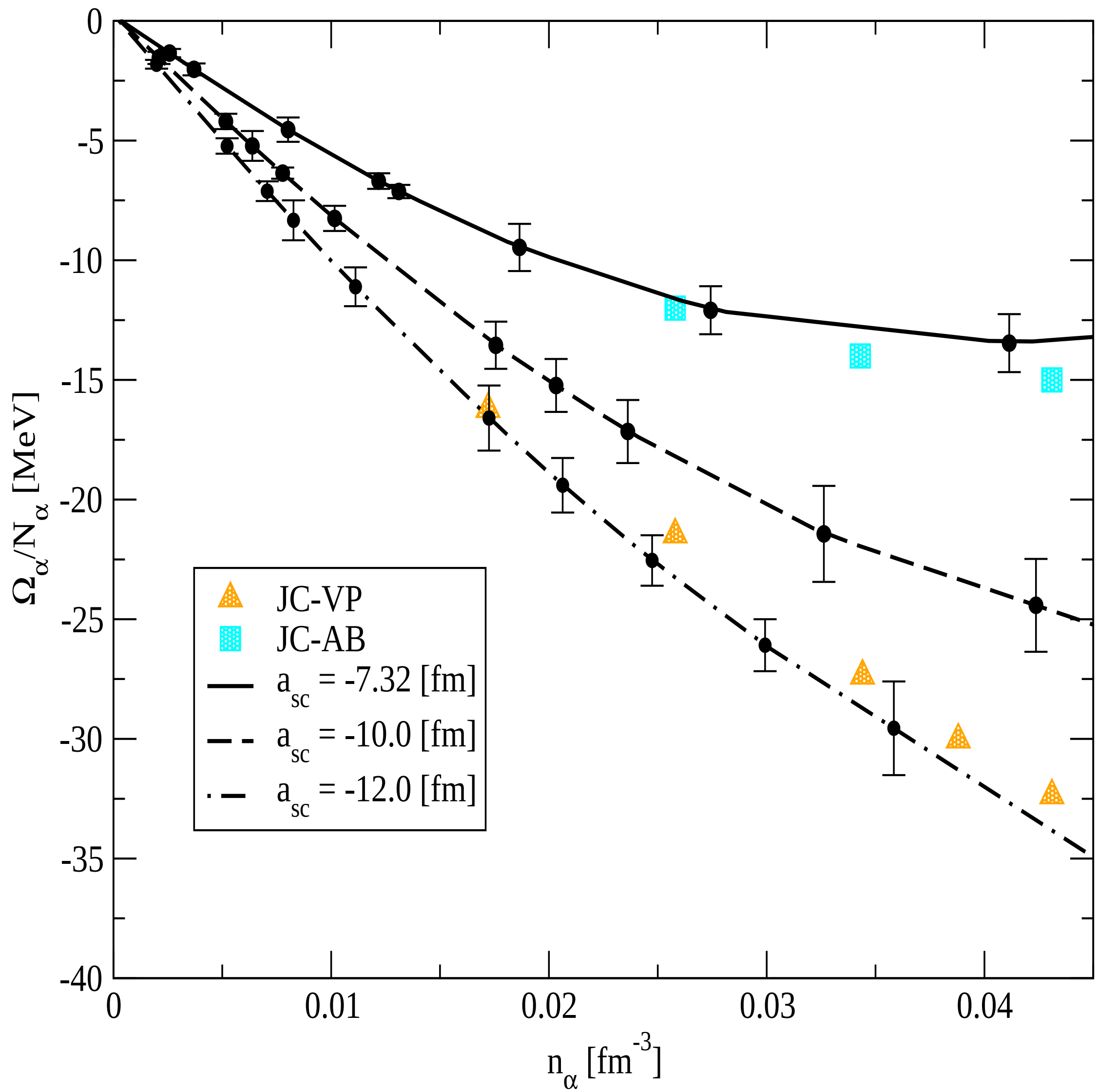
with parameters: $r = -2\beta\mu\eta^2$, $u = 4!\beta g_2 \eta^4$, $w = 6!\beta g_3 \eta^6$.

- Compute the *partition function* on 3d spatial lattice

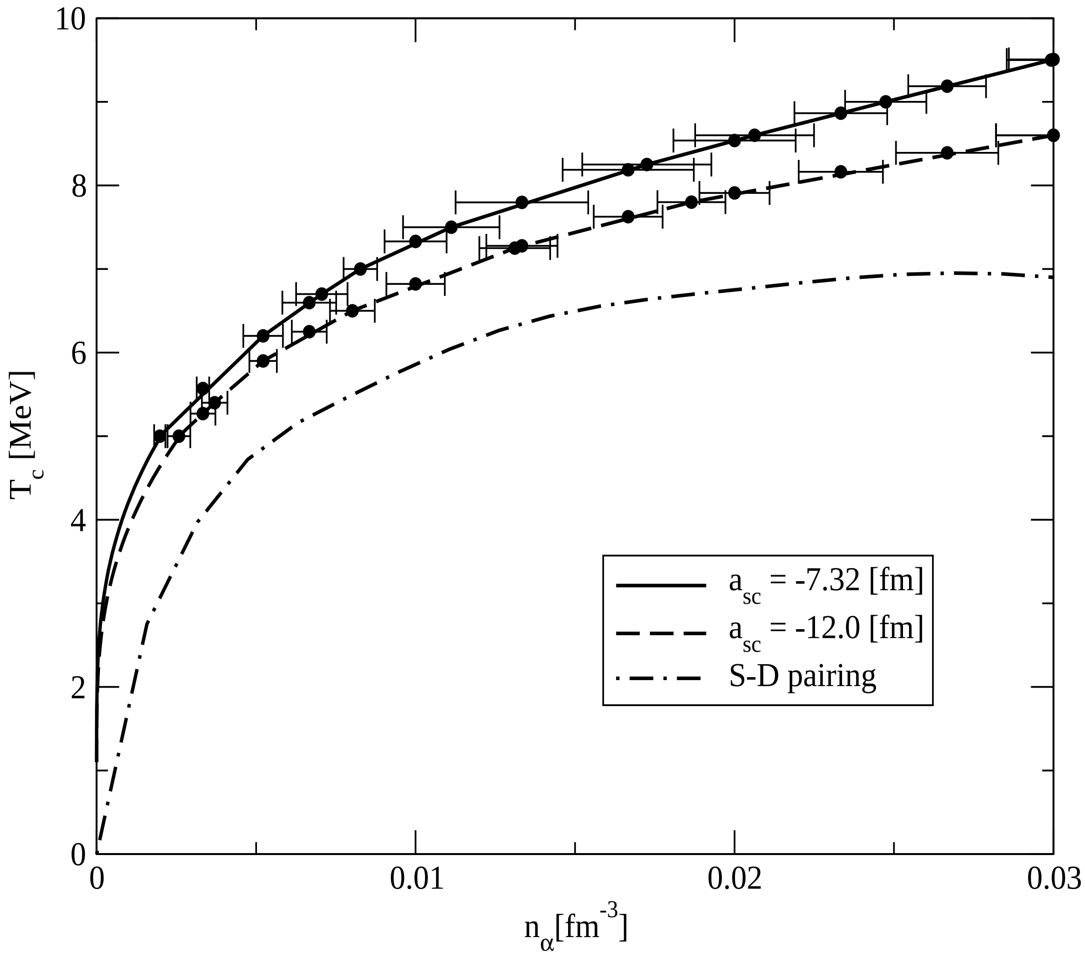
$$\mathcal{Z} = \int [d\phi(\mathbf{x})] \exp\left[-\mathcal{S}(\phi)\right],$$

Evaluated on a cubic lattice with Monte-Carlo methods *AS, H. Müther, P. Schuck, Nucl. Phys. A766 (2006) 97*.

EOS of α matter



Critical temperature for BEC



Summary on α condensation

- Alpha matter is simulated on a lattice after near T_c after re-casting the theory as $O(2)$ symmetric ϕ^6 theory with negative quartic and positive sextic interactions (differs from conventional ϕ^4 theory!).
- α condensation dominates the quasi-deuteron condensation at low densities.
- details of the EOS depend on the α-α potential (in particular in the dilute system on the scattering length).