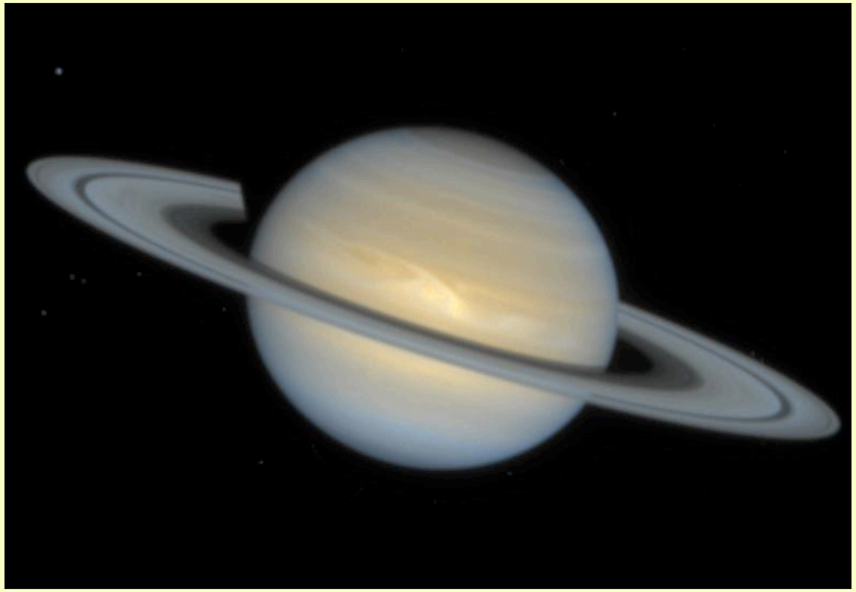
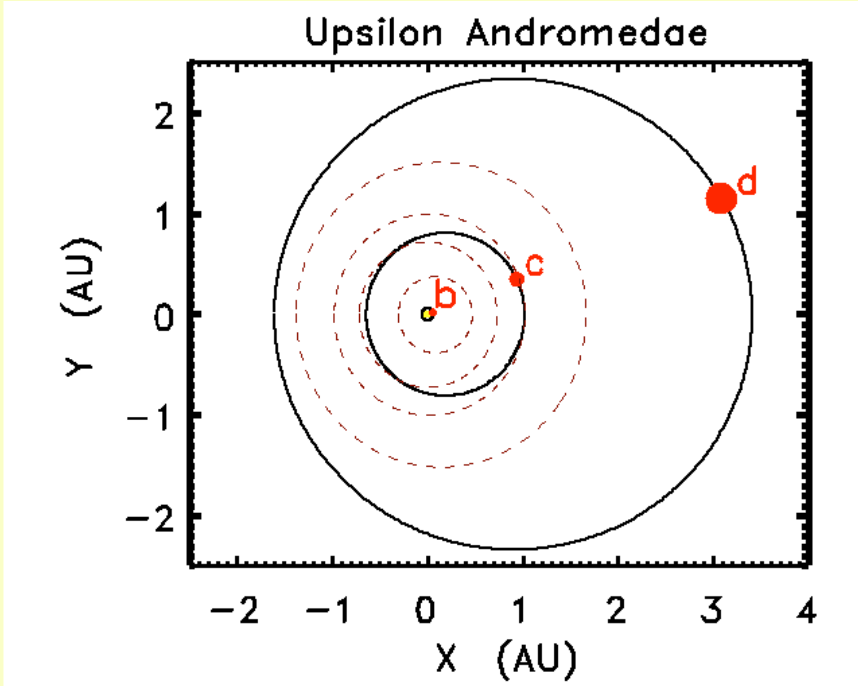
Helmholtz International Summer School on Dense Matter,
Dubna/Russia, Aug 21 – Sep 1, 2006

Warm dense matter in giant planets and exoplanets





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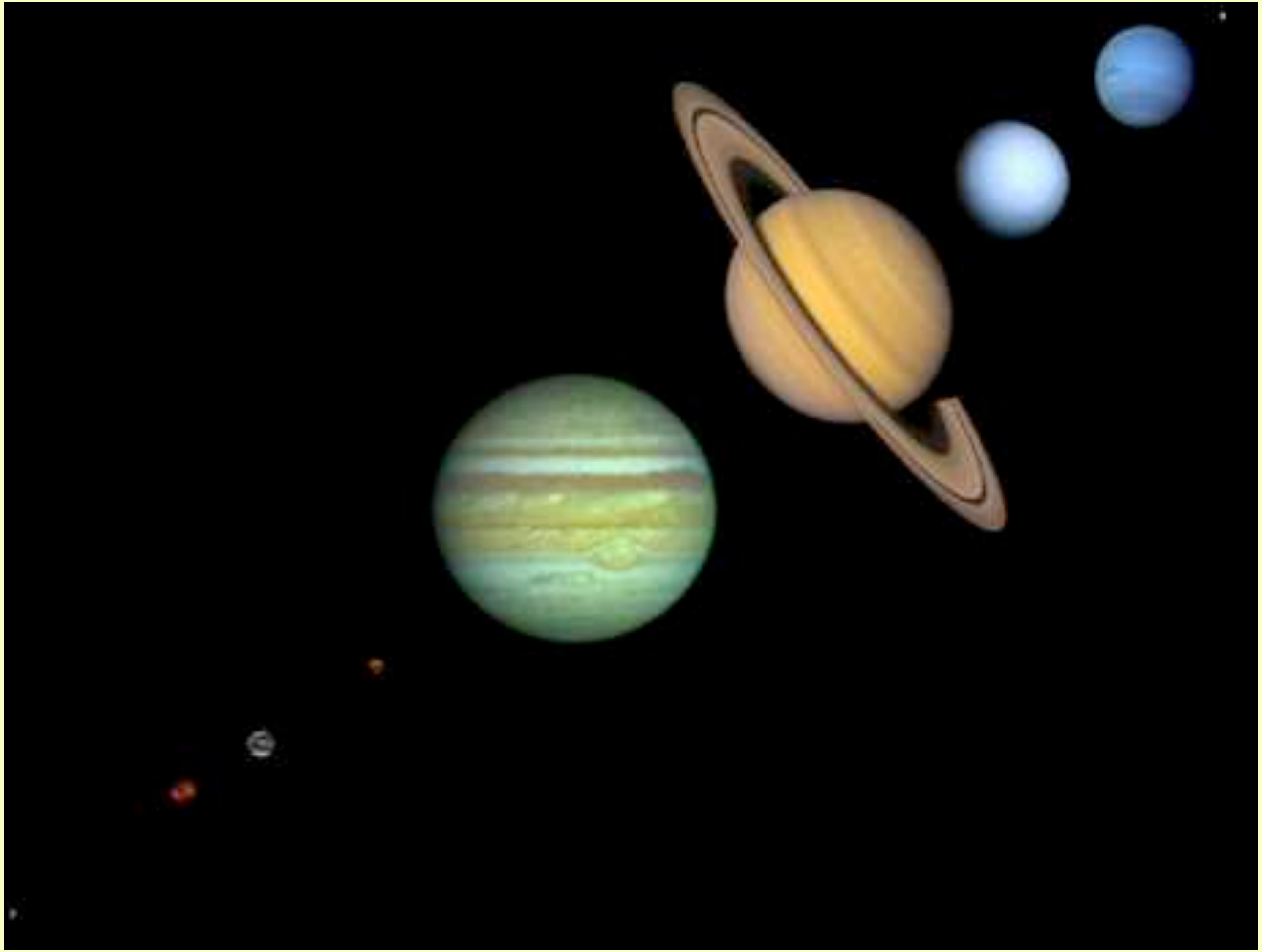
Contents

Solar system: ~~(Nine)~~ Eight planets*



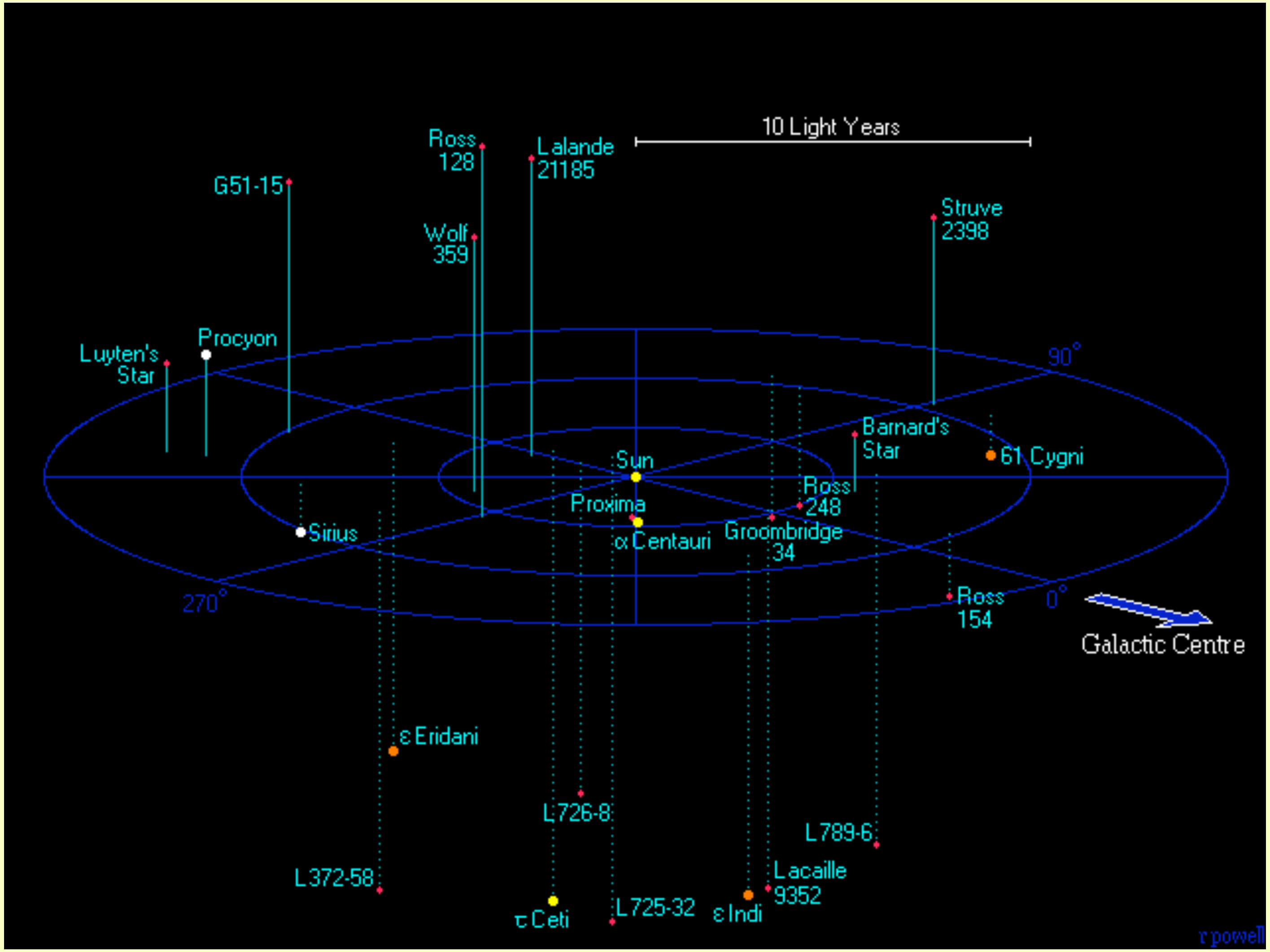
*IAU Meeting Prague 24.08.2006: Pluto is considered as „Dwarf Planet“

Planetary parameters

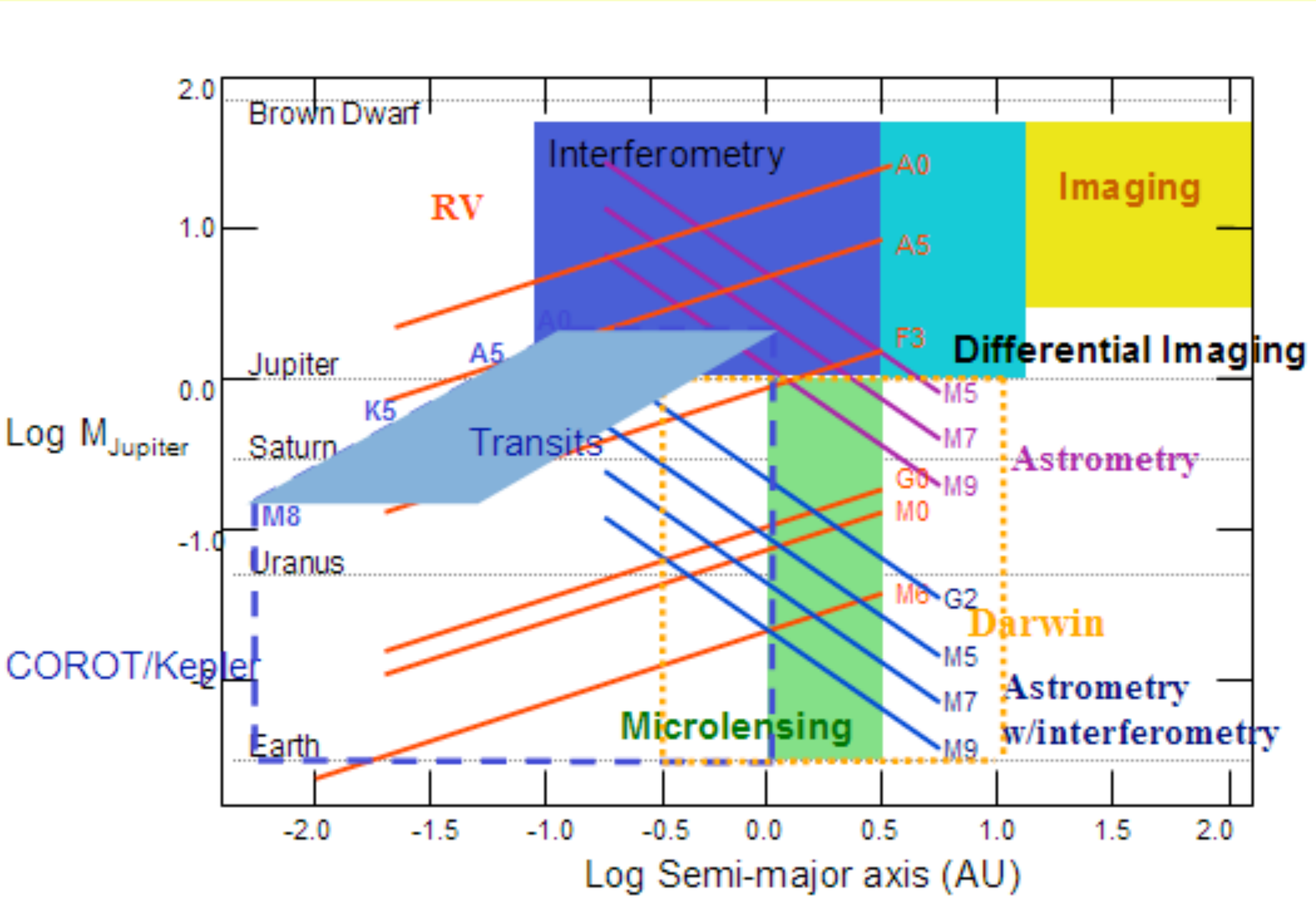
Object	Dist. (AE)	Mass (ME)	rho (g/cm^3)	G_{surf} (Earth)	T_{surf} (K)	Rot.- period	Orb.- period	Moons
Sun	-	333000	1.41	28	5800	25.4 d	-	-
Planets:								
Mercury	0.387	0.0553	5.43	0.378	440	59 d	88 d	-
Venus	0.724	0.8152	5.20	0.907	730	243 d	224.7d	-
Earth	1.000	1.0000	5.52	1.000	287	23.934 h	365 d	1
Mars	1.524	0.1075	3.93	0.377	218	24.623 h	687 d	2
Jupiter	5.203	317.88	1.33	2.364	120	9.925 h	11.856 a	61
Saturn	9.555	95.162	0.69	0.916	88	10.656 h	29.424 a	31
Uranus	19.204	14.535	1.32	0.889	59	17.24 h	83.75 a	22
Neptune	30.087	17.141	1.64	1.125	48	16.11 h	163.7 a	14
Dwarf Planets:								
Ceres	2.5-2.9	(Asteroid belt)					4.6 a	-
Pluto	39.505	0.0022	2.06	0.067	37	6.387 d	248.5 a	3
„Xena“	38-98	(Kuiper belt object 2003 UB_{313})					557 a	1

Quest for extrasolar planets

Scan the neighborhood of the sun



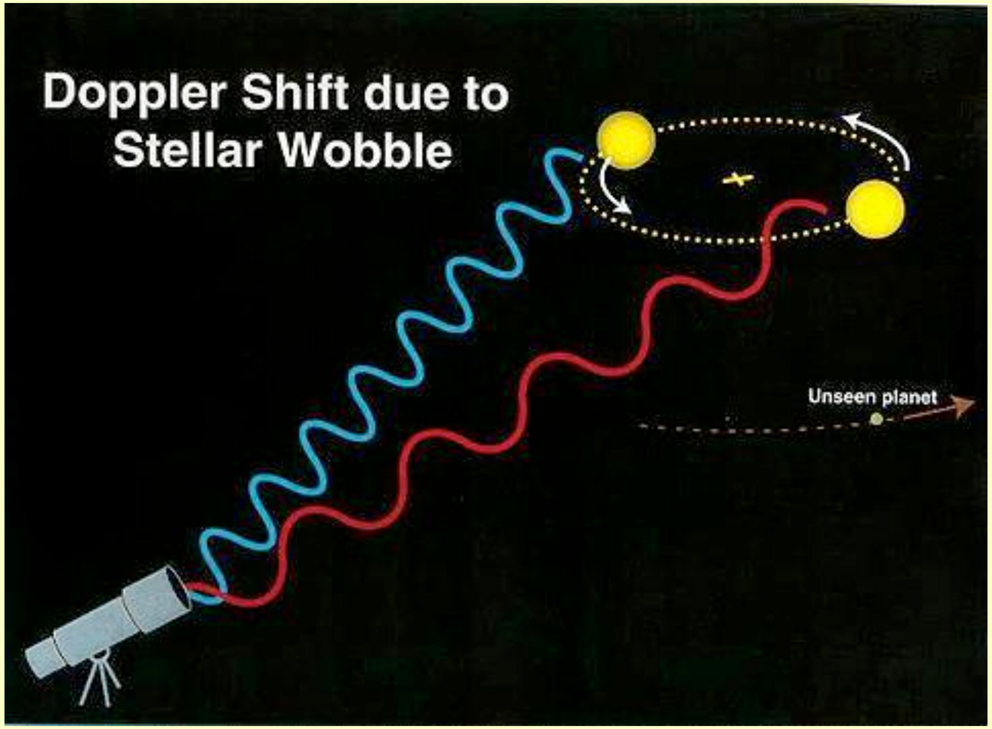
Extrasolar planets: Detection methods



Radial velocity method

Measurement of the periodic Doppler shift of the stellar spectral lines

- Successful method: 180 detections so far
- Several planetary systems with 2 and 3 planets
- Method restricted to close-in planets with short orbital distances
- Method restricted to main sequence stars of spectral type F7-M5



Signal detection limit :

$$v_r = c\frac{d\lambda}{\lambda} \approx 2 \text{ m/s}$$

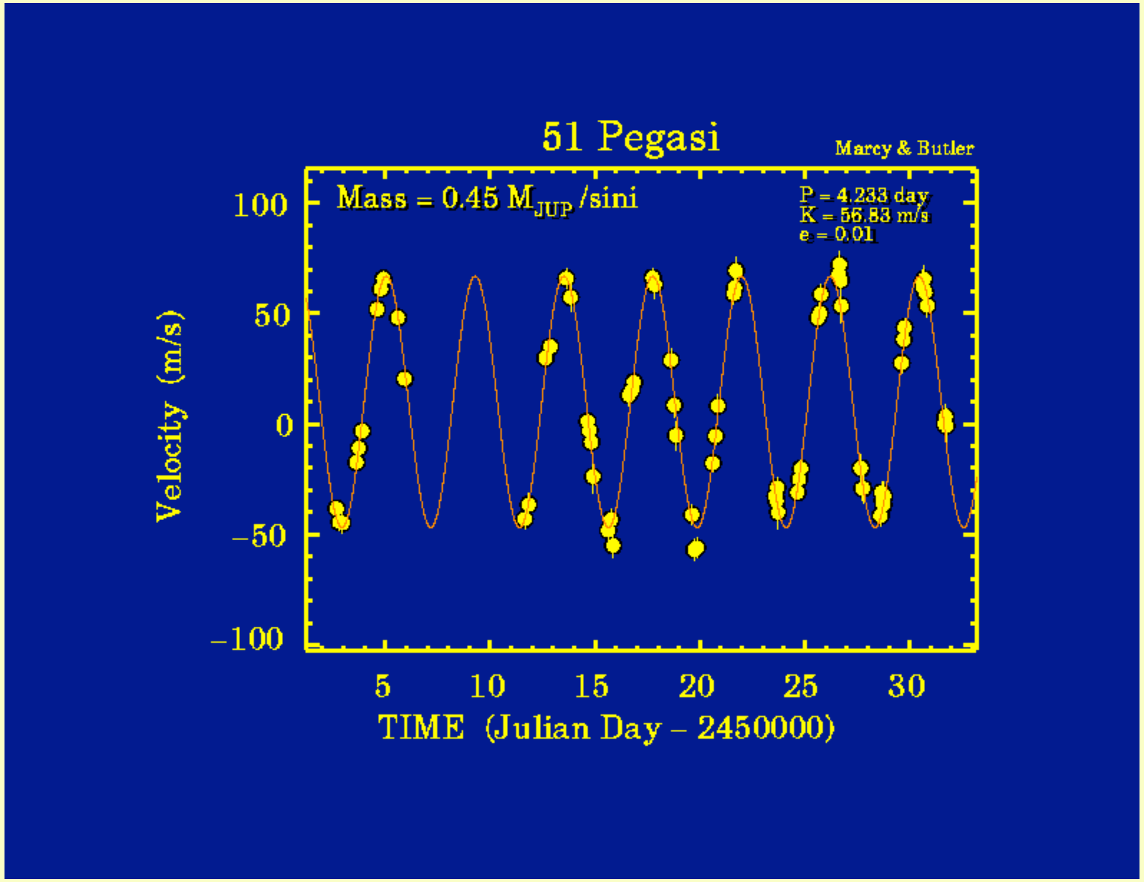
Earth: $v_r \approx 0.1 \text{ m/s}$

Jupiter: $v_r \approx 13 \text{ m/s}$

Saturn: $v_r \approx 3 \text{ m/s}$

First exoplanet detected

- Star: 51 Pegasi $(M_* = 1.06 M_\odot\ ,\ d = 45\,\mathrm{ly})$
- Mass: $M_P \sin i = 0.45 M_J$
- Period: $T = 4.233\,\mathrm{d}$
- Semi-major axis: $a_P = 0.051\,\mathrm{AE}$



M. Mayer and D. Queloz, Nature **378**, 355 (1995)

Radial velocity method and orbital parameters

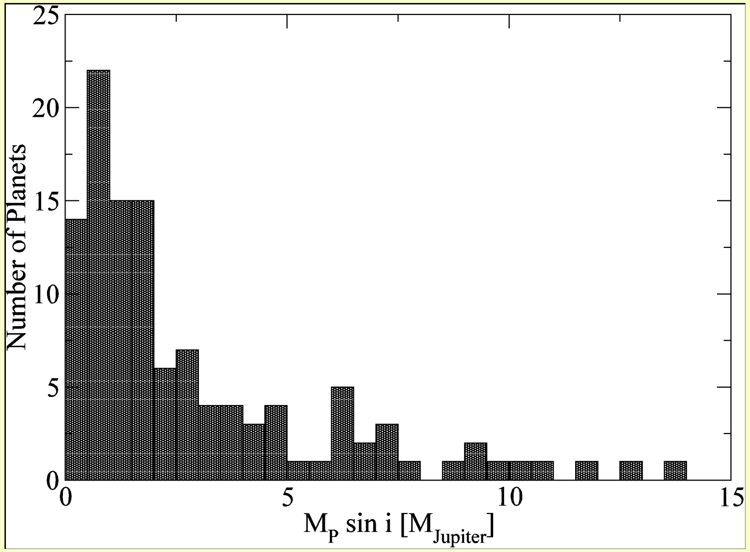
Measured parameters

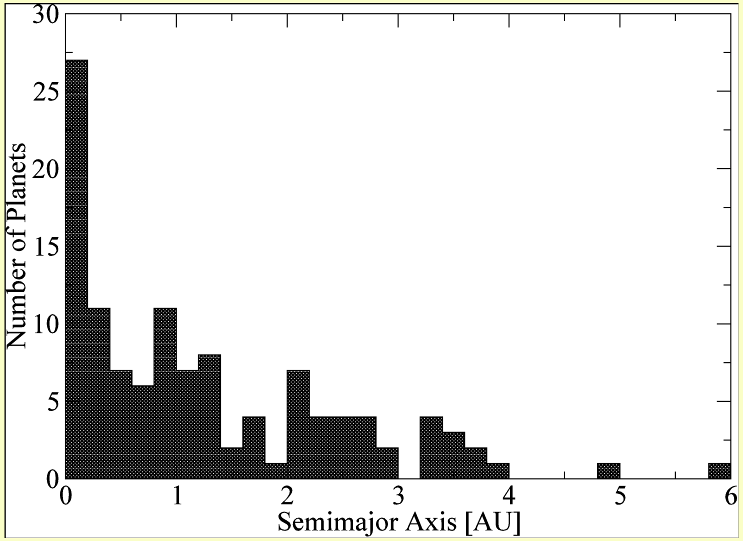
Radial velocity $v_r = \frac{v_*}{\sin i} = c\frac{d\lambda}{\lambda}$

Period T

Known parameters

Stellar mass M_*





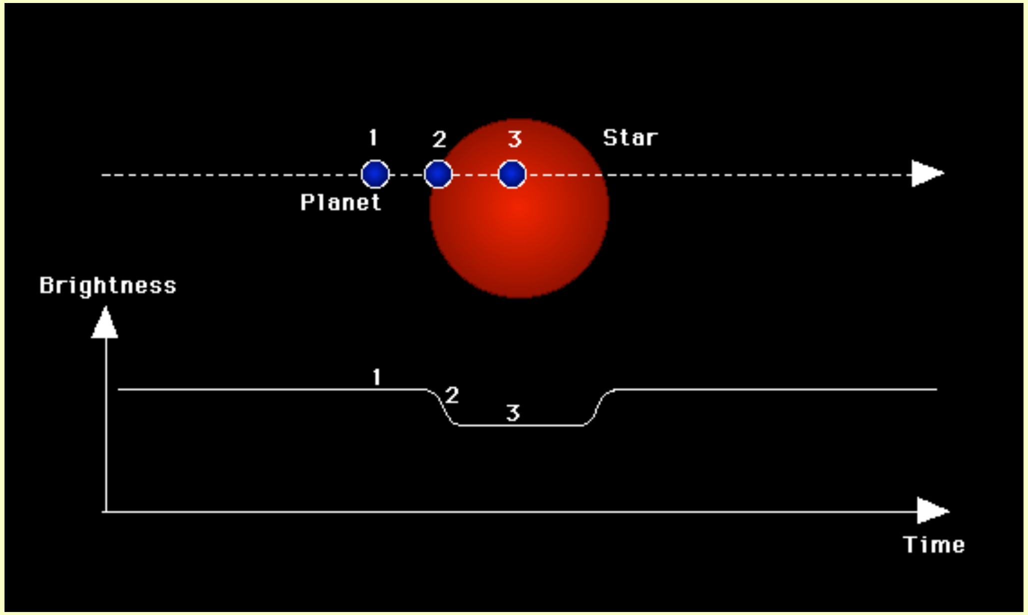
Derivation of M_P, a_P by means of Keplerian laws

3rd Keplerian law : $\frac{a_P^3}{T^2} = \frac{G(M_* + M_P)}{4\pi^2}$

Common center of mass : $M_P a_P = M_* a_*$, $2\pi a_* = T v_*$

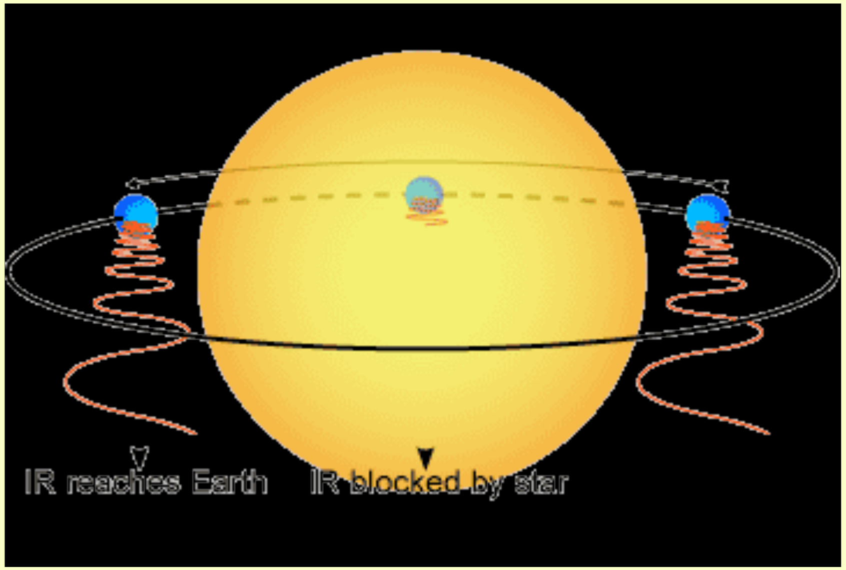
$$M_P = M_* \frac{T v_*}{2\pi} \frac{1}{a_P} \Rightarrow M_P \sin i = v_r \left(\frac{M_*^2 T}{2\pi G}\right)^{1/3}$$

Transits: Relative photometry



Transits

measurement of the decreasing stellar luminosity during occultation by transiting planet



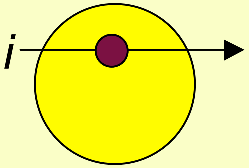
“Secondary Transits“

measurement of the decreasing planetary infrared-intensity during occultation by the star

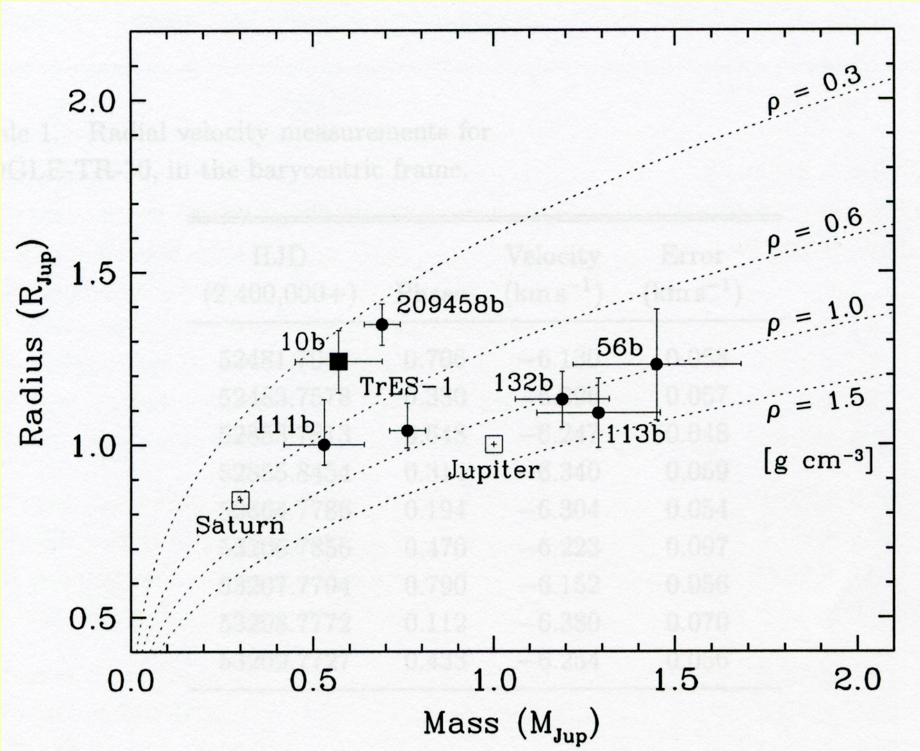
Transits: Observational parameters

Direct measurements: period *P*, transit duration *T*, change of intensity *dI /I*

Parameters derived: radius R_P, semi-major axis a_P, inclination *i*



$P \rightarrow a_P$: $\gamma M_* P^2 = a_P^3$, $dI/I = R_P^2 / R_*^2$, $T \rightarrow i$:

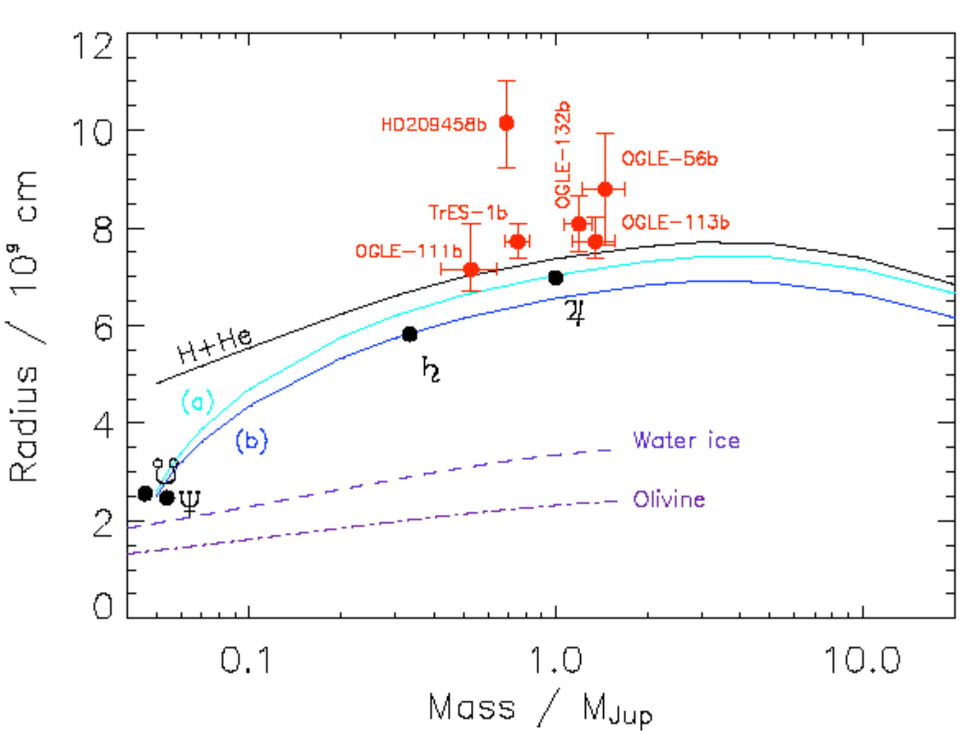


Combination with radial velocity results:

mass-radius relationship!

Konacki et al. (2003)

(Extrasolar) giant planets: Modelling the interiors



H+He: 25% Helium, without core

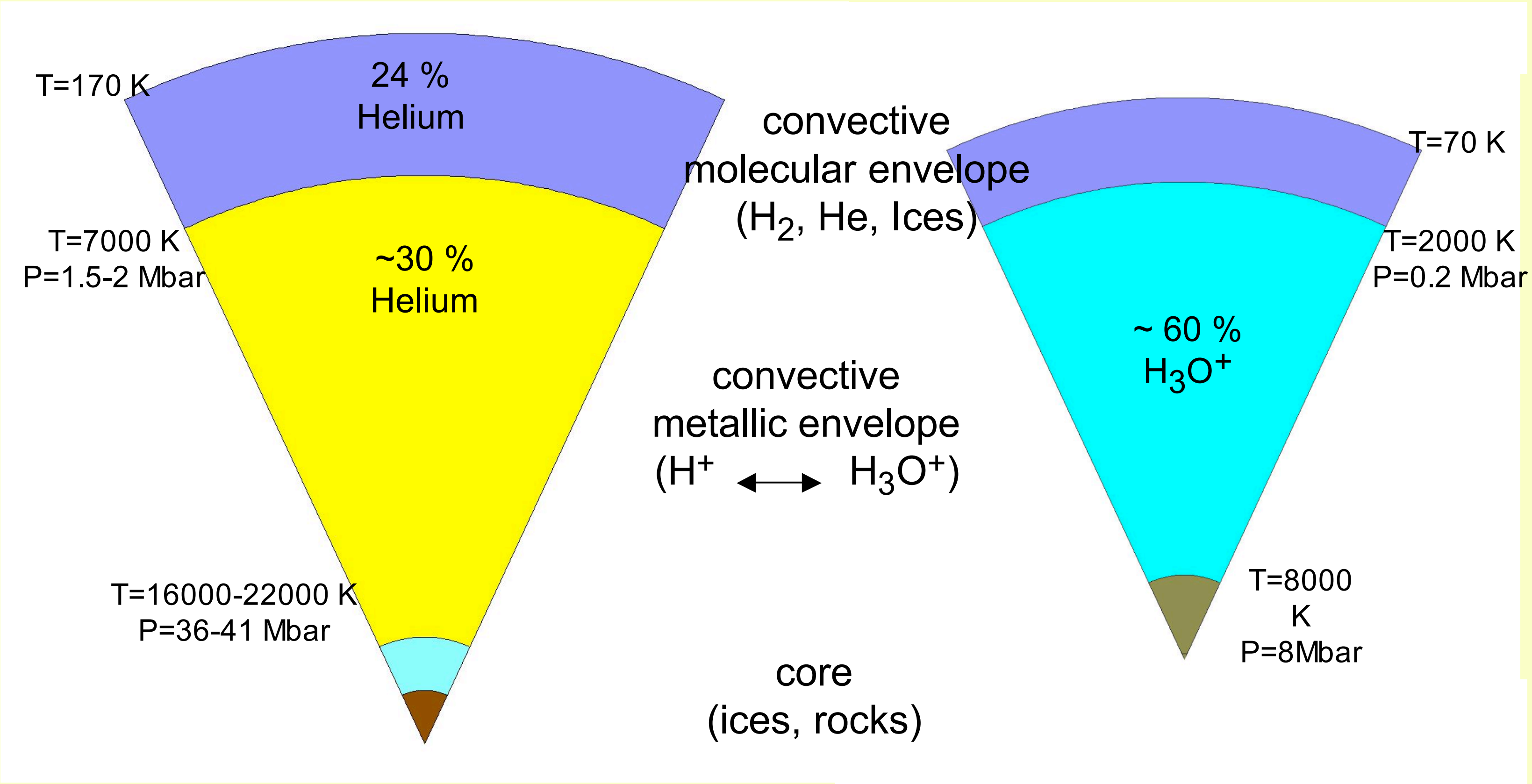
a) 30% helium, core mass=15 M_E

b) 36% helium, core mass=15 M_E

Planet	$M \sin i [M_{jup}]$	$R [R_{jup}]$	a [AU]	P [d]
HD 209458b	0.69	1.42	0.0462	3.5
OGLE-56	1.45	1,08	0.0225	3.0
OGLE-113	0.765	1.25	0.0228	1.2
OGLE-132	1.19	1.00	0.0307	4.0
OGLE-111	0.53	1,08	0.0470	1.4
TrES-1	0.75	1.13	0.0393	1.7

→ **Hot Jupiters !**

Solar giant planets: Schematic 3-layer model



Jupiter (gas giant)

Neptune (ice giant)

Observables and free parameters

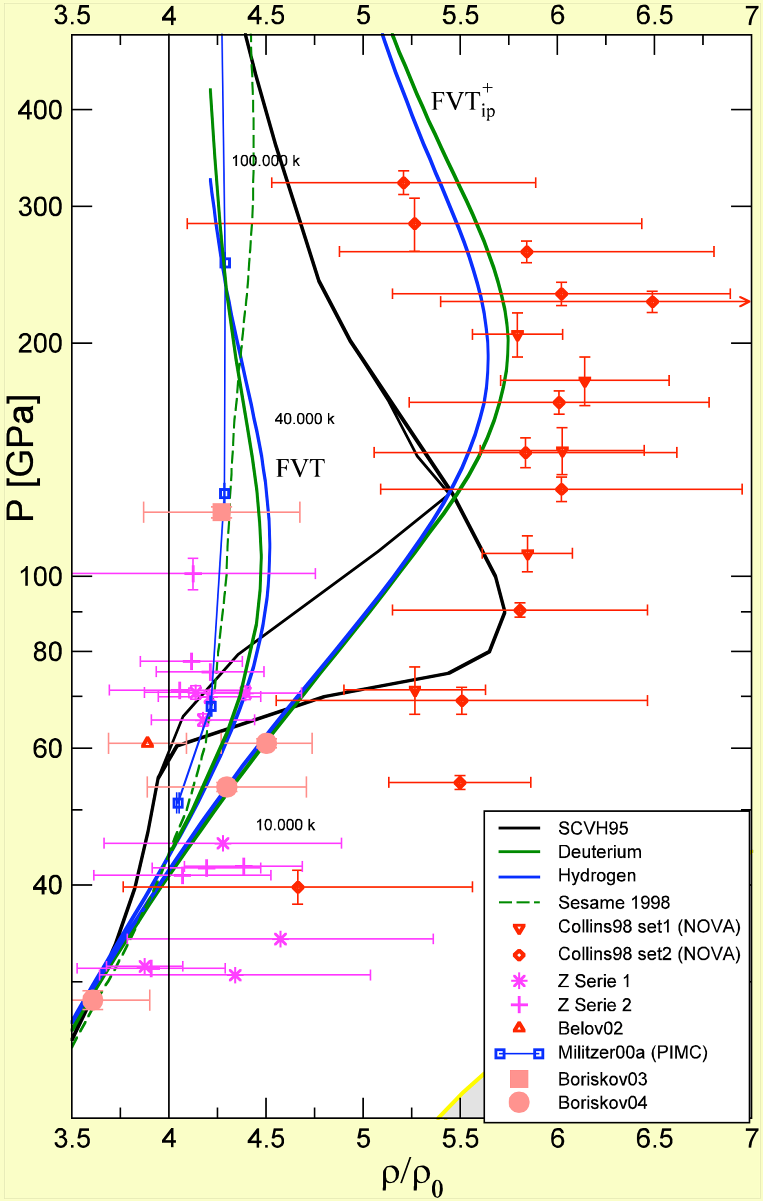
Constraints	Jupiter	Saturn
M [M_{EARTH}]	318	95
R [R_{EARTH}]	11.2	9.4
T (1 bar) [K]	165	135
Period [h]	9.9	10.7
X_{He} (1 bar)	23,8 %	6 %
X_{He} (average)	(27,5 %)	(27,5 %)
gravitational moments J2, J4, J6		

free parameters
core mass
ice: rock ratio (core)
X_{Metals} (molecular envelope)
X_{Metals} (metallic envelope)
transition pressure

Most important input parameter:

EOS of H (He) !

Hydrogen EOS used for interior models



Hugoniot curves:

Sesame tables (Kerley 1972): limit for a „stiff“ EOS, agrees with PIMC

FVT (Rostock): applicable for P < 0.5 Mbar including pressure dissociation, agrees with experiments and QMD results

FVT$^+_{ip}$ (Rostock): includes plasma contributions → more compressible, reproduces NOVA data („other limit“)

Saumon, Chabrier, Van Horn (SCVH): commonly used for Jupiter and Saturn, two versions with/without PPT, yields also a higher compression ratio

Modelling solar giant planets: Basic equations

mass conservation:

$$dm = 4\pi r^2 \rho(r)\, dr$$

hydrostatic equation of motion:

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{dU}{dr}\ , \qquad U = V + Q$$

gravitational potential:

$$V(\vec{r}) = -G\int_{V_0} d^3r'\, \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

expansion into Legendre polynomials:

$$V(r,\theta) = -\frac{GM}{r(\theta)}\left(1 - \sum_{i=1}^{\infty}\left(\frac{R_{eq}}{r(\theta)}\right)^{2i} J_{2i} P_{2i}\left(\cos\theta\right)\right)$$

gravitational moments:

$$J_{2i} = -\frac{1}{MR_{eq}^{2i}}\int d^3r'\, \rho(r'(\theta'))\, r'^{2i}\, P_{2i}(\cos\theta')$$

Multipole expansion of the gravitational potential

$$\boxed{V(\vec{r}) = -G\int_{V_0} d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}}$$ gravitational potential of a mass distribution $\rho(r)$

multipole expansion:

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \begin{cases} \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^{n} P_n(\cos\psi) & : r > r' \\ \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^{-n-1} P_n(\cos\psi) & : r < r' \end{cases} \quad , \psi = \angle(\vec{r},\vec{r}\,')$$

$$V(\vec{r}) = -G\int d\Omega \int_0^r dr'\rho(r')r'^2 \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^{n} P_n(\cos\psi) \quad \text{: external field}$$

$$-G\int d\Omega \int_r^R dr'\rho(r')r'^2 \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^{-n-1} P_n(\cos\psi) \quad \text{: internal field}$$

- axial symmetry: $P_n(\cos\psi) = P_n(\cos\theta)\cdot P_n(\cos\theta') + \text{terms}(\varphi,\varphi')$ (struck through)
- north-south symmetry: $P_n(\theta) = P_n(-\theta) \rightarrow n = 2k$ even
- spherical symmetry: no terms containing $P_n\big|_{n>0}$, $P_0 = 1$

Potential of spherical symmetric planets

$$V(r) = -\frac{Gm(r)}{r} \quad - \quad 4\pi G\int_r^R dr'\rho(r')r'$$

external internal field

derivative of the gravitational potential:

$$\frac{dV}{dr} = \frac{Gm(r)}{r^2} - \frac{G}{r}\underbrace{\frac{dm}{dr}}_{4\pi r^2\rho(r)} - 4\pi G\underbrace{\frac{d}{dr}\int_r^R dr'\rho(r')r'}_{-\rho(r)r} = \frac{Gm(r)}{r^2}$$

centrifugal potential: $Q(r) = \frac{1}{2}\omega^2 r^2 \sin^2\theta = \frac{1}{2}\omega^2 r^2 \frac{2}{3}\left(1 - P_2(\cos\theta)\right)^{sphere} = \frac{1}{3}\omega^2 r^2$

derivative: $\frac{dQ}{dr} = \frac{2}{3}\omega^2 r$

Potential of nearly spherical planets

Apply perturbation theory for non-relativistic compact objects

$$V(r,t) = -\frac{G}{r}\sum_{n=0}^{\infty}\left(\underbrace{r^{-2n}\int_{r'<r} d^3r'\rho(r')r'^{2n}\,P_{2n}(t')}_{I^{e}_{2n}(r)} \;+\; \underbrace{r^{2n+2}\int_{r<r'} d^3r'\rho(r')r'^{-(2n+2)}\,P_{2n}(t')}_{I^{i}_{2n}(r)} \right) P_{2n}(t)$$

$(t = \cos\theta)$

Aim:

- calculation of the density integrals $I^{e}_{2n}(r)\ ,\ I^{i}_{2n}(r)$
- $U(r,t) \rightarrow U(l)$ to solve the equations of motion

Method: Theory of figures by Zharkov & Trubitsyn (1978)

equipotential surfaces $r(\theta) = l(1+\sum_{n=0}^{A} s_{2n}(l)P_{2n}(t))$, $s_{2n}(l)$: figure functions

scaling : $\frac{4}{3}\pi l^3 = \int_V d^3r(l,\theta)$

approximation schema $A = \begin{cases} 1: J_2 \\ 2: J_2, J_4 \\ 3: J_2, J_4, J_6 \end{cases}$

Aim : $U(l)=\sum_{n=0}^{\infty} U_{2n}(l)P_{2n}(t)$ with $U_{2n}=0$ for $n>0$

Density integrals : $I_{2n}^{e}(r)\ ,\ I_{2n}^{i}(r)$

- $r^m = l^m\left(1+\sum_{n=0}^{A} s_{2n}(l)P_{2n}(t)\right)^m$ = ...binomial expansion
- Products of Legendre polynomials $P_m \cdot P_n = \sum_{i=0}^{n+m} q_i P_i$
- Example : $I_0^e(l) = \frac{4}{3}\pi\bar{\rho}l_1^3\frac{m(l)}{M} \rightarrow U(l)=\frac{-Gm(l)}{l}+\dots$

$$I_0^i(l)=\frac{4}{3}\pi\int_l^{l_1} dl'\rho(l')\frac{d}{dl'}\left[l'^2\left(\frac{3}{2}-\frac{3}{10}s_2^2(l')-\frac{2}{35}s_2^3(l')\right)\right]$$ (3rd order)

U(l) formally determined!

$$U(l)=-G\left(\left(1+\frac{2}{5}s_2^2\right)\frac{I_0^e(l)}{l}-\frac{3}{5}s_2\frac{I_2^e(l)}{l^3}+I_0^i(l)+\frac{2}{5}s_2 l^2 I_2^i(l)\right)+\omega^2 l^2\left(\frac{1}{3}-\frac{10}{21}s_2\right)$$ (2nd order)

Figure functions s_{2n}: Iterative solution

$$\boxed{U_{2n} = 0 \text{ for } n > 0}$$

$$\text{e.g.}\quad 0 = U_4(l) = -s_4 + \frac{18}{35} s_2^2 I_0^e(l) - \frac{54}{35} s_2 I_2^e(l) + I_4^e + \frac{36}{35} s_2 I_2^i(l) + I_4^i(l) - \omega^2 \frac{12}{35} s_2(l)$$

(2nd order)

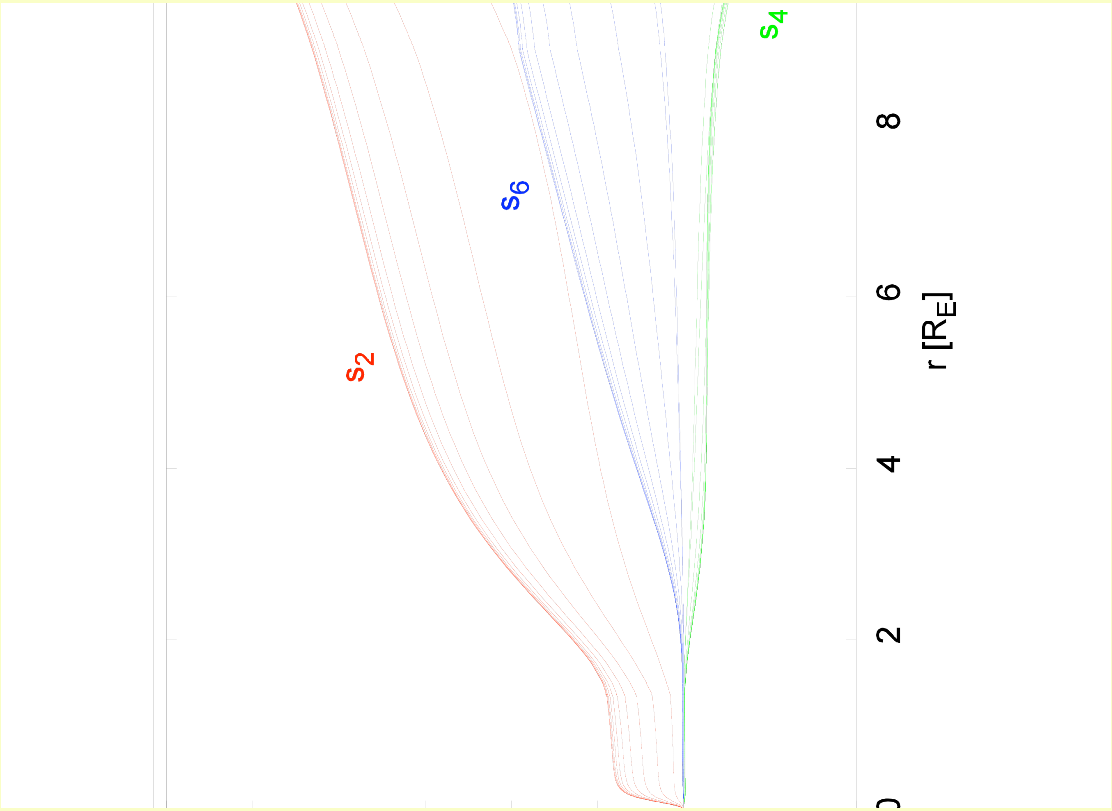


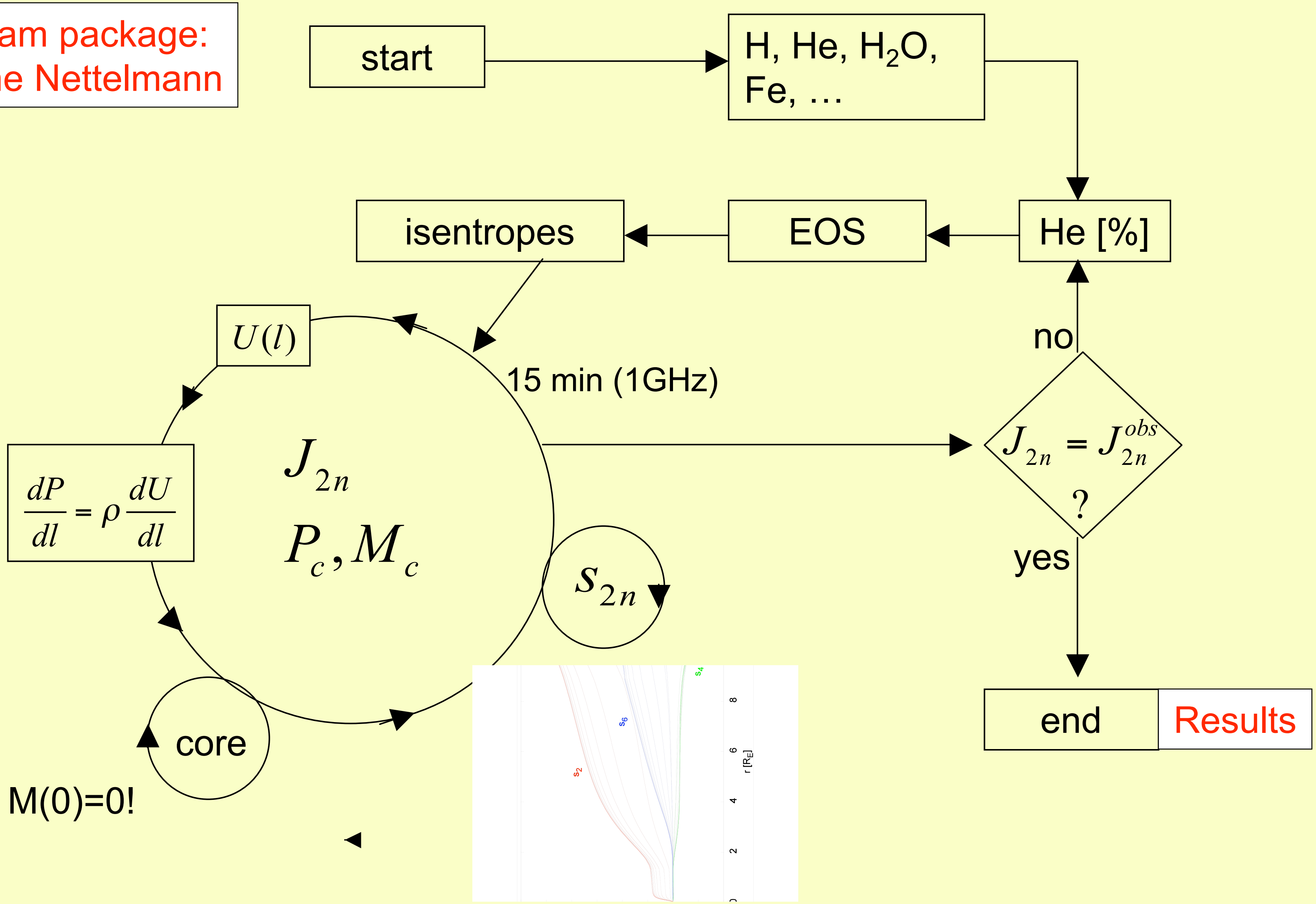
Figure functions for Jupiter

Gravitational moments:

$$J_{2n}(l) = \left(\frac{l}{MR_{eq}}\right)^{2n+2} I_{2n}^e(1)$$

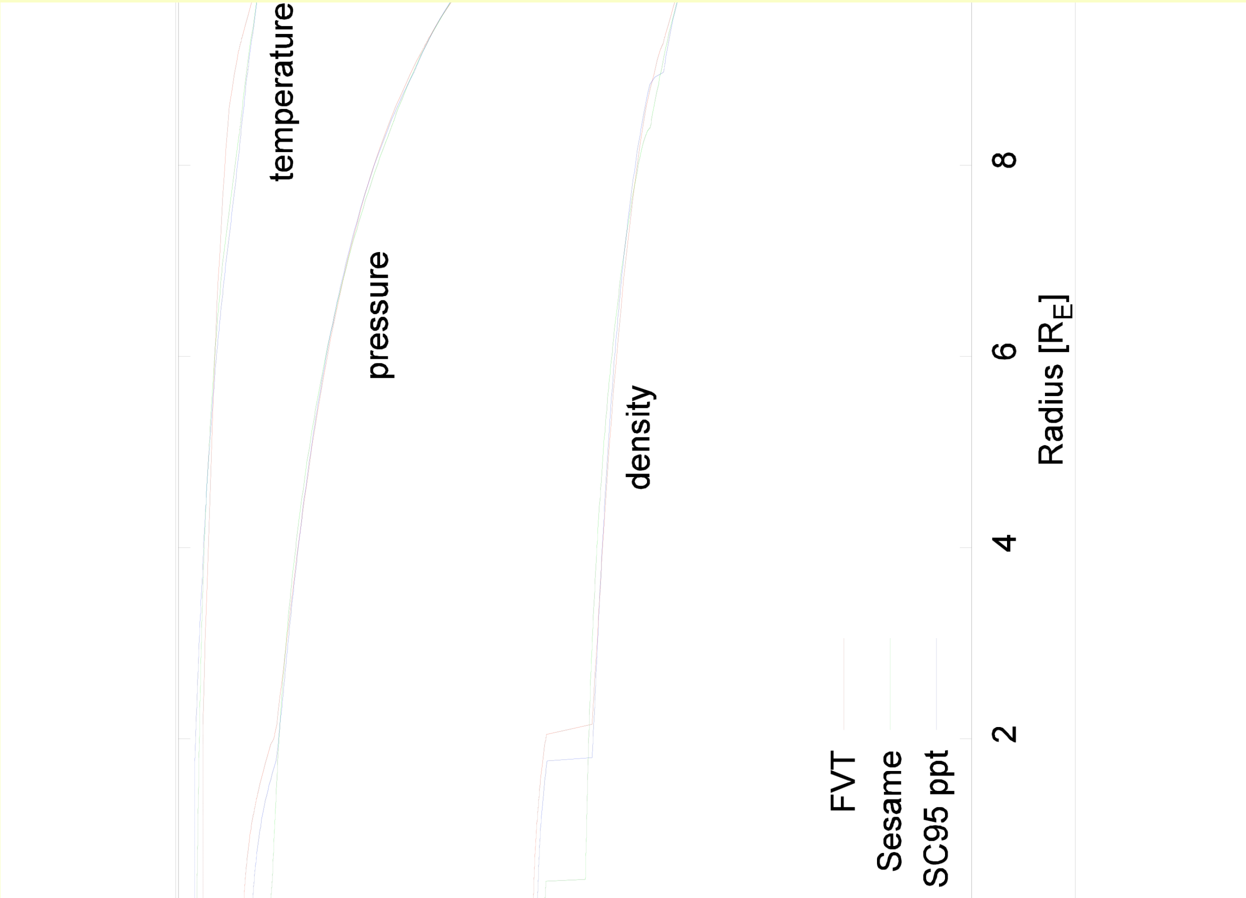
Program package for modelling giant planets

Program package:
Nadine Nettelmann



Results Jupiter: P-, T-, ρ - profiles

Very similar although different EOS are used!



$P_c \approx 36-41$ Mbar

$T_c \approx 18-22000$ K

$\rho_c \approx 4$ g/cm^3

$r_{\mathrm{met}} \approx 0.85\ R_{\mathrm{Jup}}$

Core mass & heavy element abundance

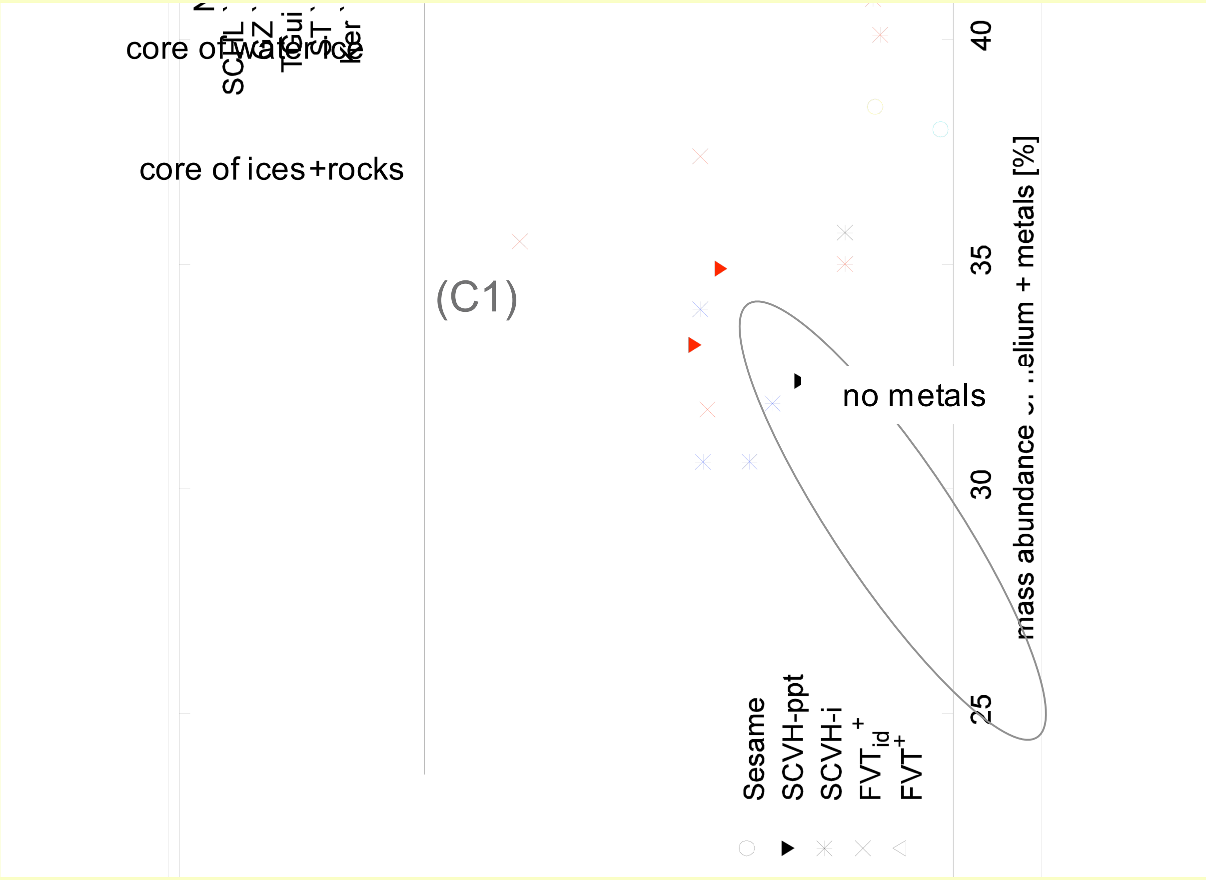
constraints from solar system evolution theory:

$\overline{X}_{He} = 27.5\%$ (C1)

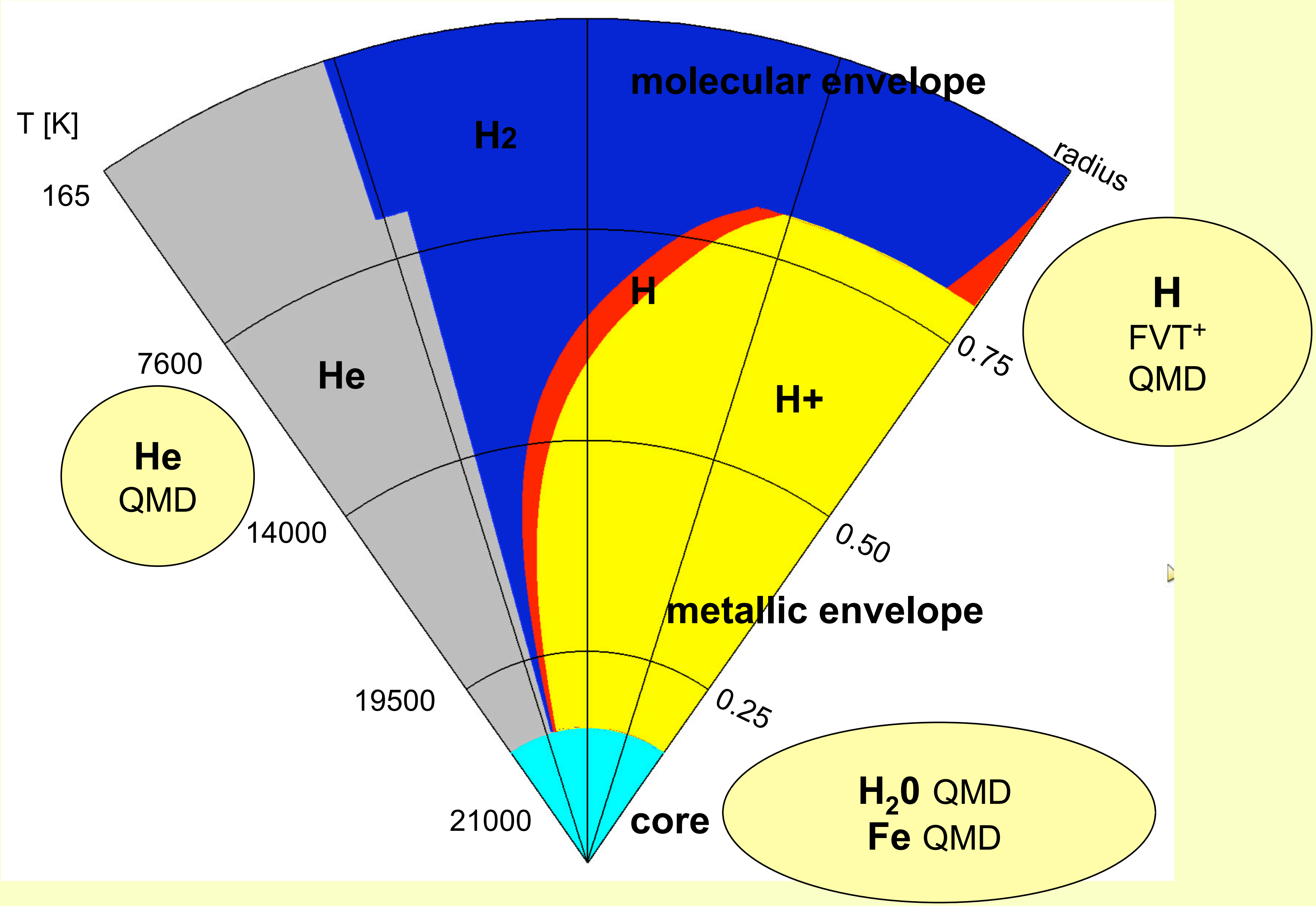
X_{He} (molecular layer) $= 24\%$

Scenario gravitational instability: $M_{core} < 10$ M_E

Scenario cluster formation: $10 < M_{core} < 20$ M_E



Internal composition of Jupiter



H-He EOS of Saumon, Chabrier, Van Horn, ApJS **99**, 713 (1995); H_2O EOS from Sesame tables (1972)

Summary

- Modelling giant planets is an important task of astrophysics →
  Structure and evolution of the solar system and of the universe
- Accurate models for giant planets in the solar system allow to check EOS data (H, He, H_20 …), especially in the WDM region
  → phase diagram at high pressures
  → plasma phase transition and nonmetal-to-metal transition
  → miscibility of helium in hydrogen, He droplet formation
- Exoplanets: New field of research
  → irradiation of nearby stars, opacity and circulation models
  → detection of Earth-like planets
  → new ground- and space-based instruments