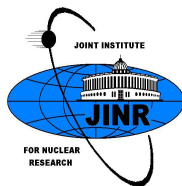


# Statistical Model for the QCD Phase Diagram.

J. Cleymans  
University of Cape Town, South Africa

Helmholtz International Summer School  
28 August - 8 September 2012  
JINR, Dubna



# Outline

Heavy Ion Collisions at the LHC

Thermal Model

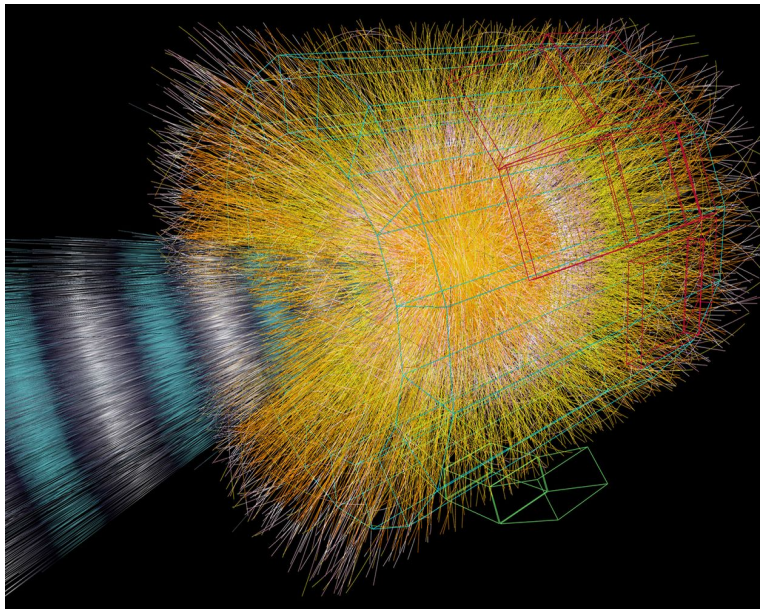
Hagedorn temperature

Surprise: No Dependence on the Size of the System.

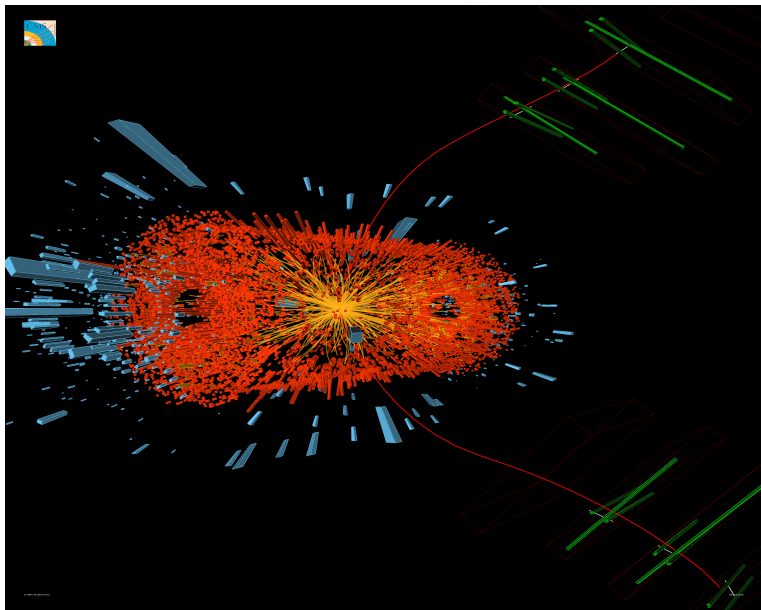
Heavy Ion Collisions at NICA/FAIR



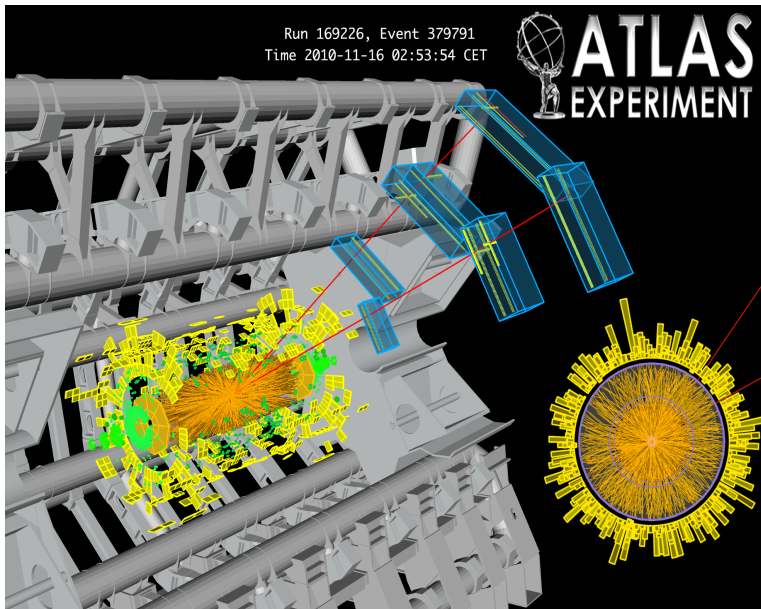
# Heavy Ion Collisions in ALICE



# Heavy Ion Collisions in CMS



# Heavy Ion Collisions in ATLAS



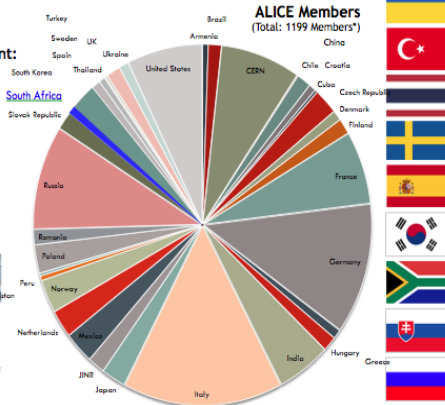
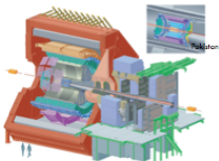
# South Africa

## The ALICE Collaboration

35 Countries - 124 Institutes - 158 MCHF capital cost

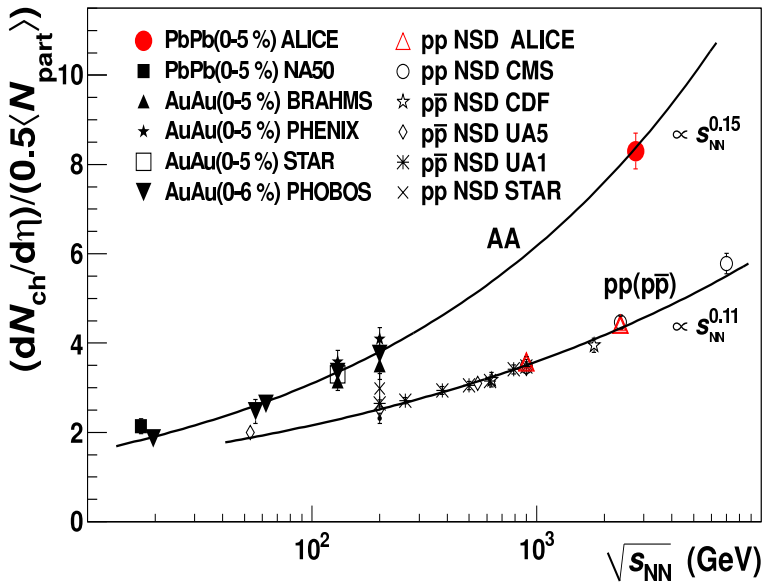
### History of the ALICE Experiment:

- 1990-1996 Design
- 1992-2002 R&D
- 2000-2010 Construction
- 2002-2007 Installation
- 2008 -> Commissioning
- 4 TP addenda along the way:
- 1996 Muon spectrometer
- 1999 TRD
- 2006 EMCAL
- 2010 DCAL

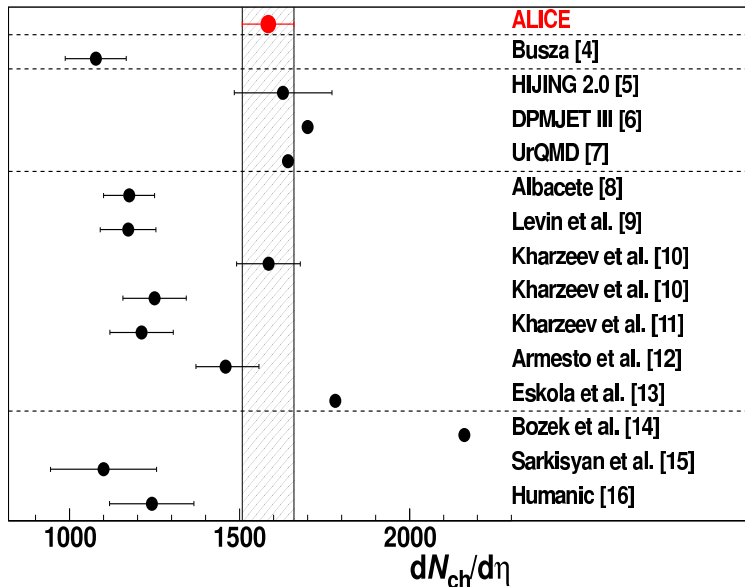


\*Alice Collaboration Data Base (ACDB) records, January 2012

# Particle Multiplicity in Heavy Ion Collisions

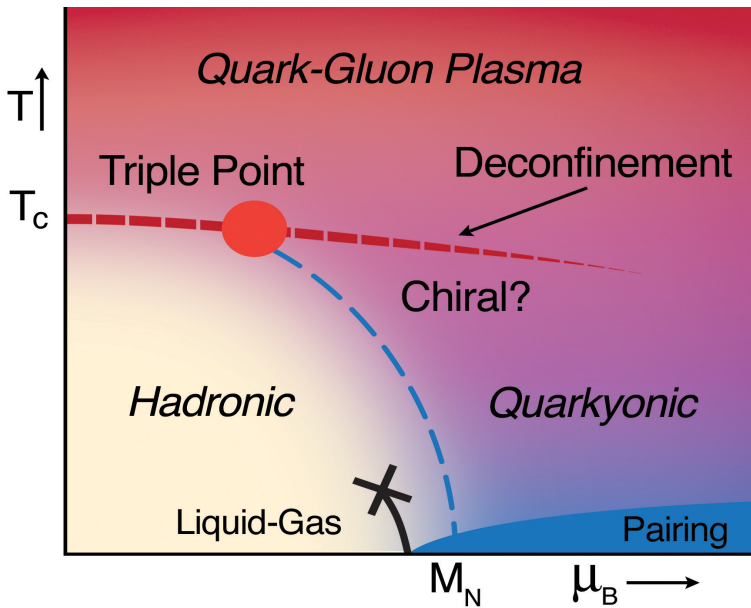


# Particle Multiplicity in Heavy Ion Collisions

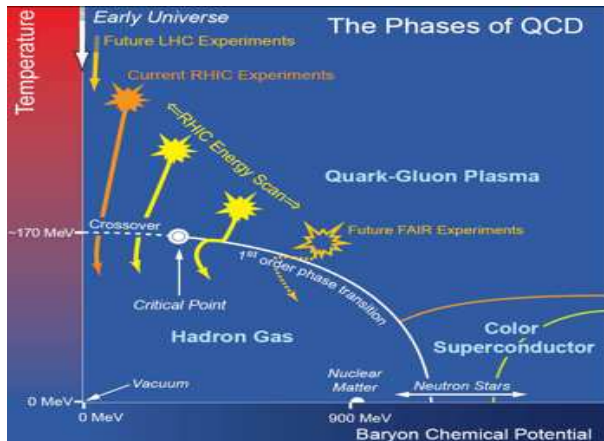




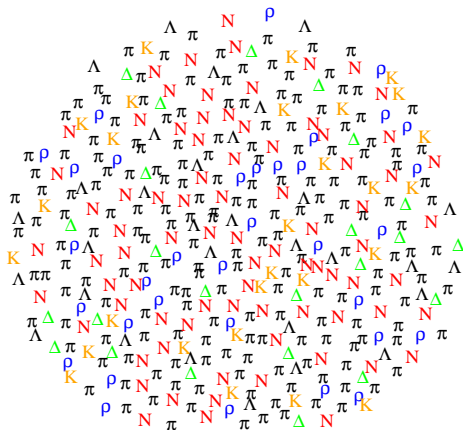
## Phase Diagram



# Phase Diagram



# Hadronic Gas before Chemical Freeze-Out



J.C. and H. Satz, Z. fuer Physik C57, 135, 1993.

# Thermal Equilibrium

In thermal equilibrium

$$Z = \text{Tr} e^{-\frac{H}{T} + \frac{\mu N}{T}}$$

$$\langle N \rangle = \frac{\text{Tr} N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\text{Tr} e^{-\frac{H}{T} + \frac{\mu N}{T}}}$$

$$\langle E \rangle = \frac{\text{Tr} E e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\text{Tr} e^{-\frac{H}{T} + \frac{\mu N}{T}}}$$



# Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion

After integration over  $m_T$

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

where  $N_i^0$  is the particle yield  
as calculated in a fireball **AT REST!**

**Effects of hydrodynamic flow cancel out in ratios.**



# Thermal Equilibrium

## Particle Number

$$\begin{aligned}
 \langle N \rangle &= \frac{\text{Tr} N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\text{Tr} e^{-\frac{H}{T} + \frac{\mu N}{T}}} \\
 &= \frac{T}{Z} \frac{\partial}{\partial \mu} \text{Tr} e^{-\frac{H}{T} + \frac{\mu N}{T}} \\
 &= T \frac{1}{Z} \frac{\partial Z}{\partial \mu} \\
 &= T \frac{\partial}{\partial \mu} \ln Z
 \end{aligned}$$



# Thermal Equilibrium

## Average Energy

$$\begin{aligned}
 \langle E \rangle &= \frac{\text{Tr } H e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\text{Tr } e^{\frac{-H}{T} + \frac{\mu N}{T}}} \\
 &= -\frac{1}{Z} \frac{\partial Z}{\partial \beta} + \mu \langle N \rangle \\
 &= T^2 \frac{\partial}{\partial T} \ln Z + \mu \langle N \rangle
 \end{aligned}$$



# Thermal Equilibrium

$$\begin{aligned}
 N_i &= g_i V \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E}{T}\right) e^{\frac{\mu_i}{T}} \\
 &= g_i V \frac{4\pi}{(2\pi)^3} \int p^2 dp \exp\left(-\frac{\sqrt{p^2 + m_i^2}}{T}\right) e^{\frac{\mu_i}{T}} \\
 &= g_i V \frac{4\pi}{(2\pi)^3} T^3 \int x^2 dx \exp\left(-\sqrt{x^2 + m_i^2/T^2}\right) e^{\frac{\mu_i}{T}} \\
 &= g_i V \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
 \end{aligned}$$





# Thermal Equilibrium

$$n_i = g_i \frac{1}{2\pi^2} T m_i^2 K_2 \left( \frac{m_i}{T} \right) e^{\frac{\mu_i}{T}}$$

$$\epsilon_i = g_i \frac{1}{2\pi^2} T m_i^3 \left[ K_1 \left( \frac{m_i}{T} \right) + 3 \frac{T}{m} K_2 \left( \frac{m_i}{T} \right) \right] e^{\frac{\mu_i}{T}}$$

$$s_i = g_i \frac{1}{2\pi^2} m_i^3 \left[ K_1 \left( \frac{m_i}{T} \right) + \frac{4T}{m} K_2 \left( \frac{m_i}{T} \right) - \frac{\mu_i}{m} K_2 \left( \frac{m_i}{T} \right) \right] e^{\frac{\mu_i}{T}}$$

$$P_i = g_i \frac{1}{2\pi^2} T^2 m_i^2 K_2 \left( \frac{m_i}{T} \right) e^{\frac{\mu_i}{T}}$$



# Chemical Equilibrium

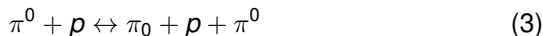
In equilibrium

$$E_1 + E_2 + \dots = E_3 + E_4 + E_5 + \dots \quad (1)$$

for the chemical potentials

$$\mu_1 + \mu_2 + \dots = \mu_3 + \mu_4 + \mu_5 + \dots \quad (2)$$

As an example



leads to

$$\mu_{\pi^0} + \mu_p = \mu_{\pi^0} + \mu_p + \mu_{\pi^0} \quad (4)$$

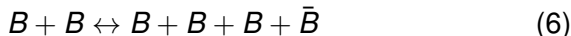
which leads to

$$\mu_{\pi^0} = 0 \quad (5)$$



## Chemical Equilibrium

In equilibrium



$$dE = -pdV + TdS + \mu_B dN_B + \mu_{\bar{B}} dN_{\bar{B}}$$

Due to baryon number conservation one has

$$N_B - N_{\bar{B}} = \text{constant}$$

and

$$dN_B = dN_{\bar{B}}$$

The energy is a minimum for

$$dE = (\mu_B + \mu_{\bar{B}})dN_B = 0 \quad (7)$$

$$\mu_B = -\mu_{\bar{B}} \quad (8)$$



# Chemical Equilibrium

In equilibrium

$$N_B = g V \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E}{T} + \frac{\mu_B}{T}\right)$$

$$N_{\bar{B}} = g V \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E}{T} - \frac{\mu_B}{T}\right)$$

$$N_B = N_{\bar{B}} \rightarrow \mu_B = 0$$

$$N_B \geq N_{\bar{B}} \rightarrow \mu_B \geq 0$$

$$N_B \leq N_{\bar{B}} \rightarrow \mu_B \leq 0$$



	Chemical Equilibrium	No Chem. Equil.
$\pi$	$\exp\left[-\frac{E_\pi}{T}\right]$	$\exp\left[-\frac{E_\pi}{T} + \frac{\mu_\pi}{T}\right]$
$N$	$\exp\left[-\frac{E_N}{T} + \frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T} + \frac{\mu_N}{T}\right]$
$\bar{N}$	$\exp\left[-\frac{E_N}{T} - \frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T} + \frac{\mu_{\bar{N}}}{T}\right]$
$\Lambda$	$\exp\left[-\frac{E_\Lambda}{T} + \frac{\mu_B}{T} - \frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T} + \frac{\mu_\Lambda}{T}\right]$
$\bar{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T} - \frac{\mu_B}{T} + \frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T} + \frac{\mu_{\bar{\Lambda}}}{T}\right]$
$K$	$\exp\left[-\frac{E_K}{T} + \frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T} + \frac{\mu_K}{T}\right]$
$\bar{K}$	$\exp\left[-\frac{E_K}{T} - \frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T} + \frac{\mu_{\bar{K}}}{T}\right]$



The number of particles of type  $i$  is determined by:

$$N_i = V g_i \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

For bosons:

$$N_i = V g_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) - 1}$$

For fermions:

$$N_i = V g_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) + 1}$$



# Chemical Equilibrium

Only conserved quantum numbers matter for chemical equilibrium: In equilibrium

$$\mu_i = B_i \mu_B + Q_i \mu_Q + S_i \mu_S + C_i \mu_C + \dots \quad (9)$$



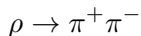
$g_i$	$m_i$	stat	$S_i$	$B_i$	$Q_i$	Particle $i$
1	0.140	-1	0	0	1.	$\pi^+$
1	0.135	-1	0	0	0.	$\pi^0$
1	0.140	-1	0	0	-1.	$\pi^-$
1	0.547	-1	0	0	0.	$\eta$
3	0.770	-1	0	0	1.	$\rho^+$
3	0.770	-1	0	0	0.	$\rho^0$
3	0.770	-1	0	0	-1.	$\rho^-$
3	0.782	-1	0	0	0.	$\omega$
1	0.958	-1	0	0	0.	$\eta'$
1	0.980	-1	0	0	0.	$f_0$
1	0.982	-1	0	0	1.	$a_0^+$
1	0.982	-1	0	0	0.	$a_0^0$
1	0.982	-1	0	0	-1.	$a_0^-$
3	1.019	-1	0	0	0.	$\phi$
3	1.170	-1	0	0	0.	
3	1.230	-1	0	0	1.	
3	1.230	-1	0	0	0.	
3	1.230	-1	0	0	-1.	
3	1.229	-1	0	0	1.	
3	1.229	-1	0	0	0.	
3	1.229	-1	0	0	-1.	
5	1.275	-1	0	0	0.	
3	1.282	-1	0	0	0.	
1	1.297	-1	0	0	0.	
1	1.300	-1	0	0	1.	
1	1.300	-1	0	0	0.	





# The Role of Resonances

## Example: $\rho$ 's



Final, observed, number of  $\pi^+$  is given by

$$N_{\pi^+} = N_{\pi^+}(\text{thermal}) + N_{\pi^+}(\text{resonance decays})$$

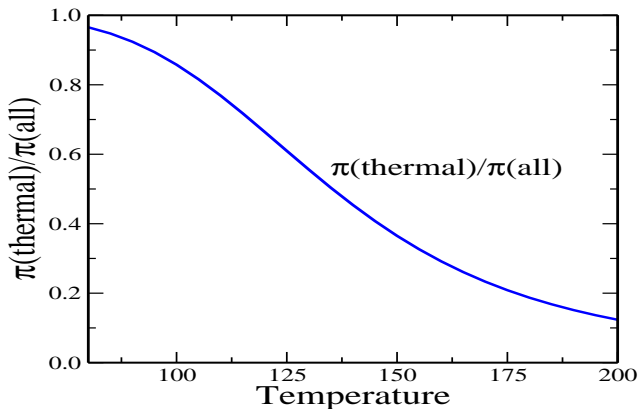
depending on the temperature, over 80% of observed pions are due to resonance decays



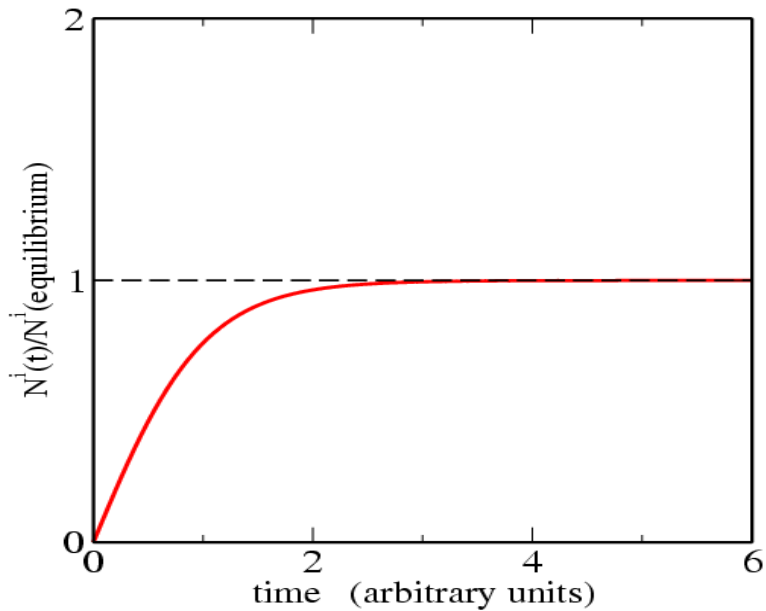
$g_i$	$m_i$	stat	$S_i$	$B_i$	$Q_i$	BR $\rightarrow \pi^+$	Particle $i$
1	0.140	-1	0	0	1.	1.000	$\pi^+$
1	0.135	-1	0	0	0.	0.000	$\pi^0$
1	0.140	-1	0	0	-1.	0.000	$\pi^-$
1	0.547	-1	0	0	0.	0.285	$\eta$
3	0.770	-1	0	0	1.	1.000	$\rho^+$
3	0.770	-1	0	0	0.	1.000	$\rho^0$
3	0.770	-1	0	0	-1.	0.000	$\rho^-$
3	0.782	-1	0	0	0.	0.910	$\omega$
1	0.958	-1	0	0	0.	0.965	$\eta'$
1	0.980	-1	0	0	0.	0.521	$f_0$
1	0.982	-1	0	0	1.	1.285	$a_0^+$
1	0.982	-1	0	0	0.	0.285	$a_0^0$
1	0.982	-1	0	0	-1.	0.285	$a_0^-$
3	1.019	-1	0	0	0.	0.155	$\phi$
3	1.170	-1	0	0	0.	1.000	$h_1$
3	1.230	-1	0	0	1.	1.500	
3	1.230	-1	0	0	0.	0.50	
3	1.230	-1	0	0	-1.	0.50	
3	1.229	-1	0	0	1.	1.91	
3	1.229	-1	0	0	0.	0.91	
3	1.229	-1	0	0	-1.	0.91	
5	1.275	-1	0	0	0.	0.69	
3	1.282	-1	0	0	0.	1.00	
1	1.297	-1	0	0	0.	1.11	
1	1.300	-1	0	0	1.	2.00	
1	1.300	-1	0	0	0.	1.50	



# Importance of Resonances.



## Strangeness saturation?



## Strangeness saturation?

$$N_i = \boxed{\gamma_s^{|S|}} V g_i \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

with

$\gamma_s < 1$  strangeness under-saturation

$\gamma_s = 1$  strangeness in chemical equilibrium

$\gamma_s > 1$  strangeness over-saturation



## SPS data.

	Measurement
Pb–Pb 158A GeV	
$(\pi^+ + \pi^-)/2.$	$600 \pm 30$
$K^+$	$95 \pm 10$
$K^-$	$50 \pm 5$
$K_S^0$	$60 \pm 12$
$p$	$140 \pm 12$
$\bar{p}$	$10 \pm 1.7$
$\phi$	$7.6 \pm 1.1$
$\Xi^-$	$4.42 \pm 0.31$
$\Xi^-$	$0.74 \pm 0.04$
$\bar{\Lambda}/\Lambda$	$0.2 \pm 0.04$



## SPS data.

SPS: Chemical Freeze-Out Parameters:

$$T = 156.0 \pm 2.4 \text{ MeV}$$

$$\mu_B = 239 \pm 12 \text{ MeV}$$

$$\gamma_s = 0.862 \pm 0.036$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich  
Physical Review C64 (2001) 024901.



# AGS data.

	Measurement
<b>Au–Au 11.6A GeV</b>	
<b>Participants</b>	$363 \pm 10$
$K^+$	$23.7 \pm 2.9$
$K^-$	$3.76 \pm 0.47$
$\pi^+$	$133.7 \pm 9.9$
$\Lambda$	$20.34 \pm 2.74$
$p/\pi^+$	$1.234 \pm 0.126$
$\bar{p}$	$>0.0185 \pm 0.0018$





## AGS data.

AGS: Chemical Freeze-Out Parameters:

$$T = 130.6 \pm 5.5 \text{ MeV}$$

$$\mu_B = 594 \pm 26 \text{ MeV}$$

$$\gamma_s = 0.883 \pm 0.124$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich  
Physical Review C64 (2001) 024901.



# SIS data.

	Measurement
<b>Au–Au 1.7A GeV</b>	
$\pi^+/\text{p}$	$0.052 \pm 0.013$
$\text{K}^+/\pi^+$	$0.003 \pm 0.00075$
$\pi^-/\pi^+$	$2.05 \pm 0.51$
$\eta/\pi^0$	$0.018 \pm 0.007$



# SIS data.

SIS: Chemical Freeze-Out Parameters:

$$T = 49.7 \pm 1.1 \text{ MeV}$$

$$\mu_B = 818 \pm 15 \text{ MeV}$$

$$\gamma_s = 1 \text{ (fixed)}$$

J. C., H. Oeschler and K. Redlich)  
Physical Review C59, (1999) 1663.



## RHIC data.

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu, Phys. Rev. C71, 0409071 (2005)

Ratio	Experiment	Central	Mid-Central	Peripheral
$\pi_{(2)}^- / \pi_{(2)}^+$	BRAHMS	0.990±0.100		
	PHENIX	0.960±0.177	0.920±0.170	0.933±0.172
	PHOBOS	1.000±0.022		
	STAR	1.000±0.073	1.000±0.073	1.000 ± 0.073
$K_{(2)}^+ / K_{(2)}^-$	PHENIX	1.152±0.240	1.292±0.268	1.322±0.284
	PHOBOS	1.099±0.111		
	STAR	1.109±0.022	1.105±0.036	1.120±0.040
$\bar{p}_{(1)} / p_{(1)}$	PHENIX	0.680±0.149	0.671±0.142	0.717±0.157
$\bar{p}_{(2)} / p_{(2)}$	BRAHMS	0.650±0.092		
	PHOBOS	0.600±0.072		
	STAR	0.714±0.050	0.724±0.050	0.764±0.053
$\bar{\Lambda}_{(1)} / \Lambda_{(1)}$	PHENIX	0.750±0.180	0.798±0.197	0.795±0.197
$\bar{\Lambda}_{(2)} / \Lambda_{(2)}$	STAR	0.719±0.090	0.739±0.092	0.744±0.100
$\Xi_{(2)}^+ / \Xi_{(2)}^-$	STAR	0.840±0.053	0.822±0.114	0.815±0.096
$\bar{\Omega}^+ / \Omega^-$	STAR	1.062±0.410		
$K_{(2)}^- / \pi_{(2)}^-$	PHENIX	0.151±0.030	0.134±0.027	0.116±0.023
	STAR	0.151±0.022	0.147±0.022	0.130±0.019
$K_S^0 / \pi_{(2)}^-$	STAR	0.134±0.022	0.131±0.022	0.108±0.018
$\bar{p}_{(1)} / \pi_{(2)}^-$	PHENIX	0.049±0.010	0.047±0.010	0.045±0.009
$\bar{p}_{(2)} / \pi_{(2)}^-$	STAR	0.069±0.019	0.067±0.019	0.067±0.019
$\Lambda_{(1)} / \pi_{(2)}^-$	STAR	0.043±0.008	0.043±0.008	0.039±0.007
$\Lambda_{(2)} / \pi_{(2)}^-$	PHENIX	0.072±0.017	0.068±0.016	0.074±0.017
$< K^{*0} > / \pi_{(2)}^-$	STAR	0.039±0.011		
$\phi / \pi_{(2)}^-$	STAR	0.022±0.003	0.021±0.004	0.022±0.004
$\Xi_{(2)}^- / \pi_{(2)}^-$	STAR	0.0093±0.0012	0.0072±0.0011	0.0060±0.0008



## RHIC data.

RHIC: Chemical Freeze-Out Parameters:

$$T = 169 \pm 4.2 \text{ MeV}$$

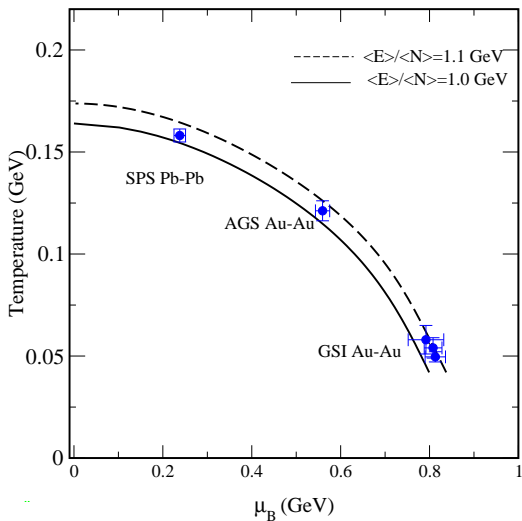
$$\mu_B = 39.6 \pm 6 \text{ MeV}$$

$$\gamma_s = 0.9 \pm 0.1$$

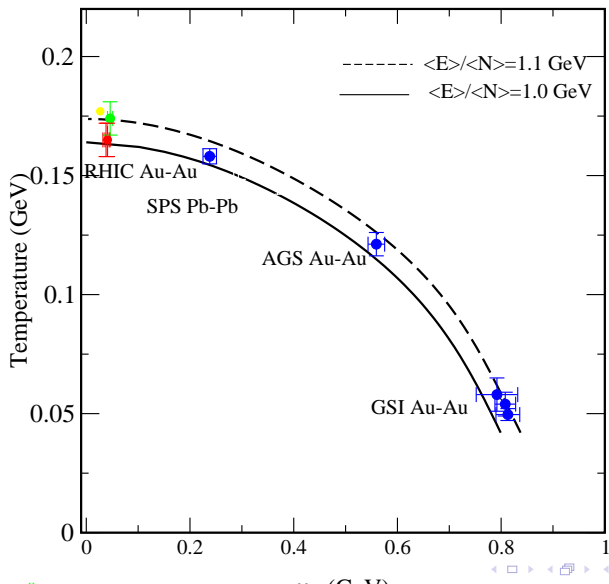
J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu  
Phys. Rev. C71, 0409071 (2005)



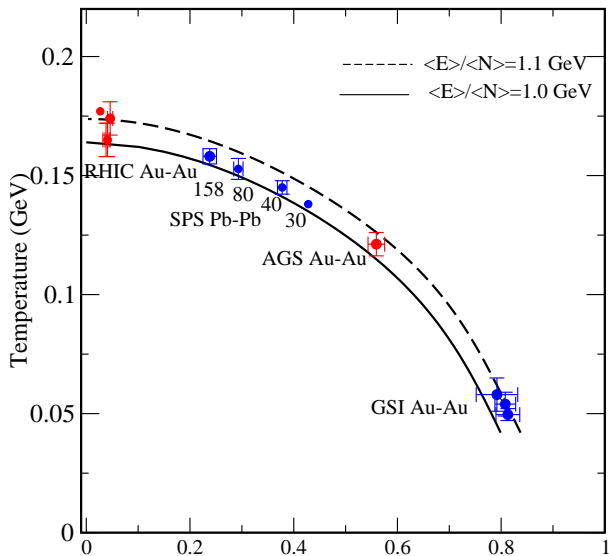
## E/N in 1999



## E/N in 2000

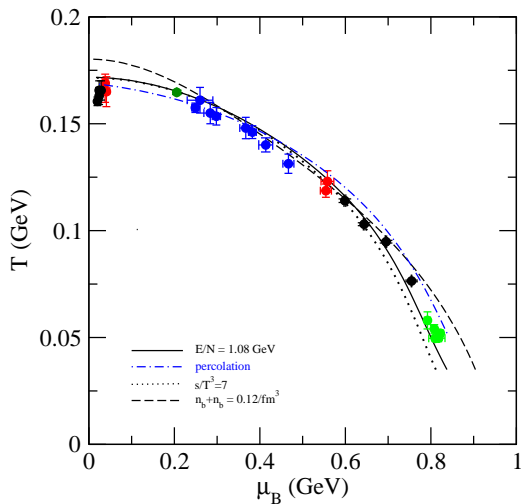


## E/N in 2005

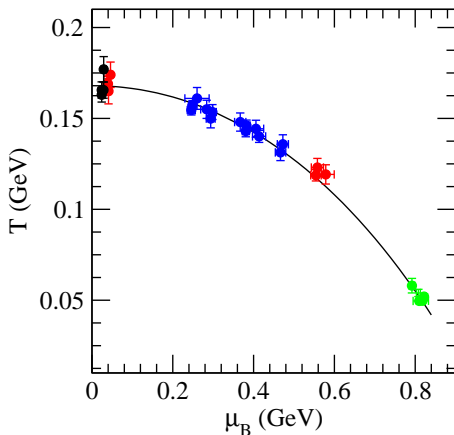




# Chemical Freeze-Out: Status in 2005



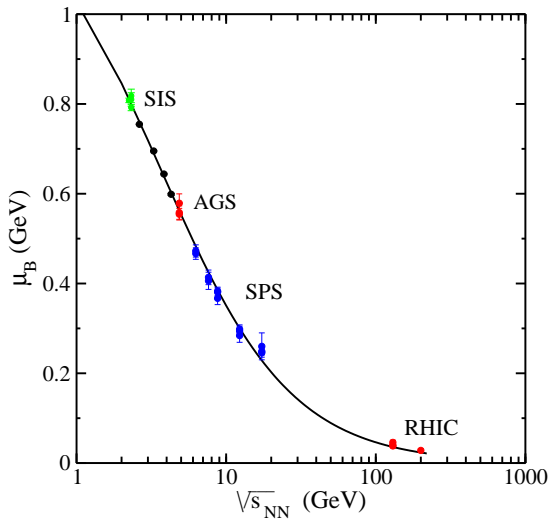
# Chemical Freeze-Out: Status in 2005



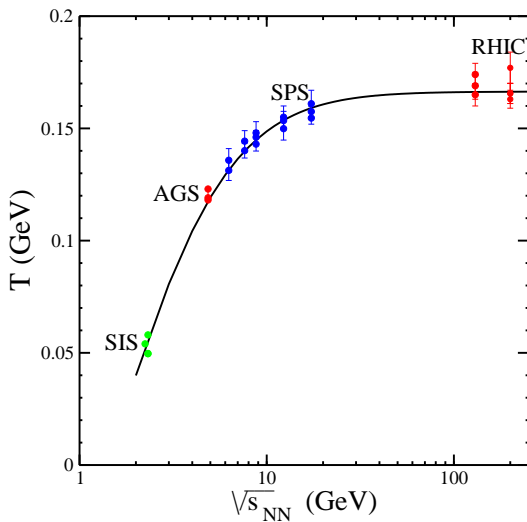
$$T = 0.166 - 0.139 \mu_B^2 - 0.053 \mu_B^4$$

J.C., H. Oeschler, K. Redlich, S. Wheaton,  
PR C73, 034905 (2006)



$\mu_B$  as a function of  $\sqrt{s_{NN}}$ 

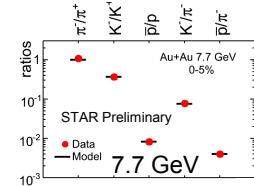
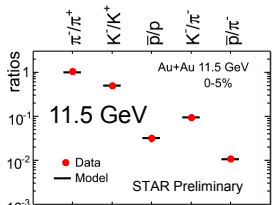
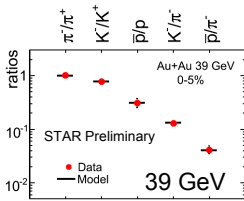
# $T$ as a function of $\sqrt{s_{NN}}$



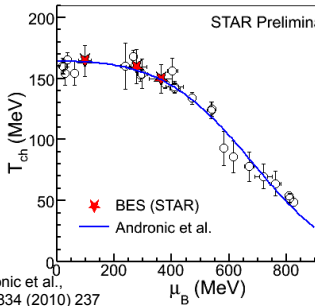
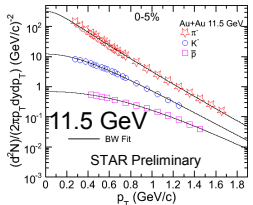
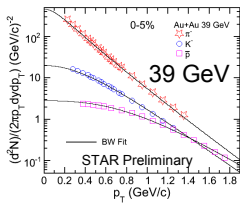


# Freeze-out Conditions

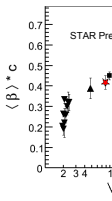
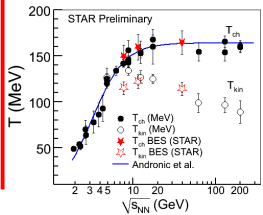
QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.



## Chemical freeze-out: Particle ratios

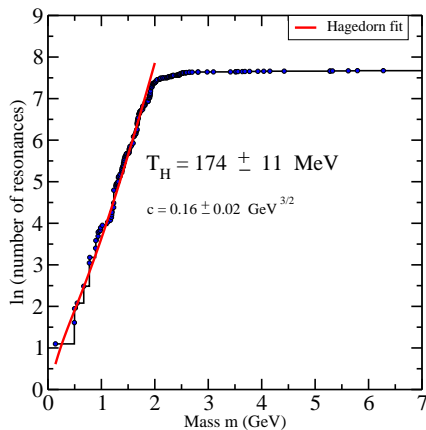


QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.



## Kinetic freeze-out : Momentum distributions

# THE HAGEDORN TEMPERATURE.



Keep on adding the number of hadronic resonances.

J.C. and Dawit Worku, Mod. Phys. Lett. A26 (2011) 1197; arXiv:  
1103.1463

# HADRONS DO NOT EXIST ABOVE THE HAGEDORN TEMPERATURE.

Thermodynamic quantities like particle density, energy density, pressure ,... all involve a summation over hadron species:

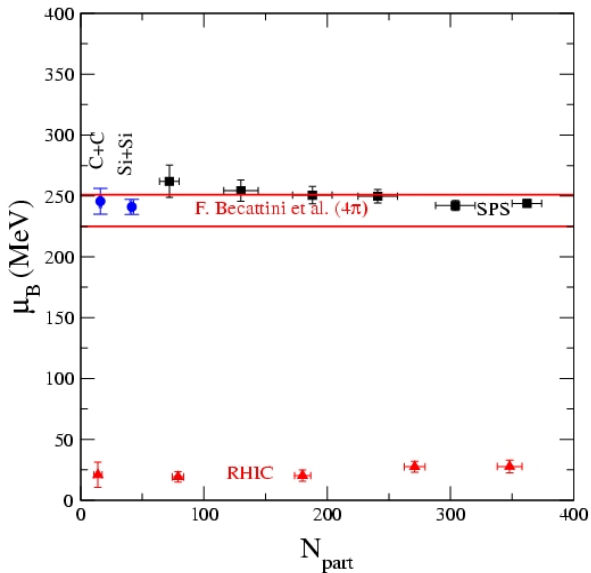
$$\sum_i \exp -\frac{E_i}{T}$$

and the sum becomes (too) large due to the number of resonances.

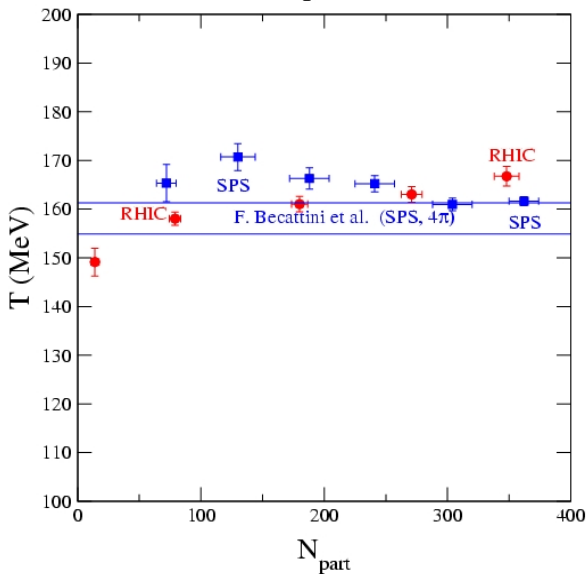


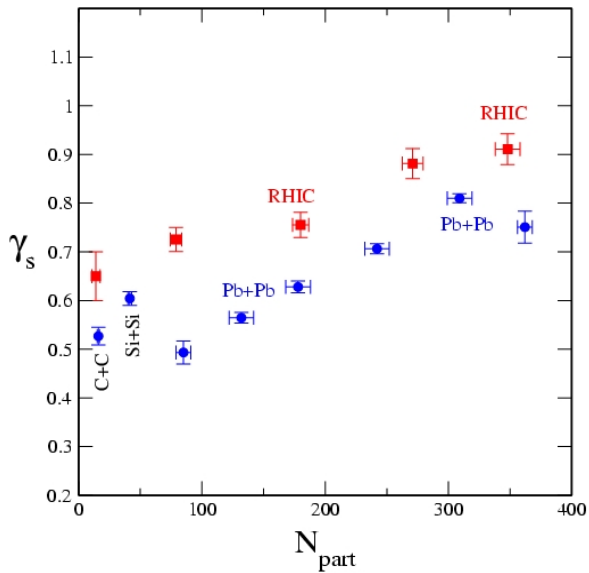


# Centrality Dependence of the Baryon Chemical Potential



# Centrality Dependence of the Chemical Freeze-out Temperature





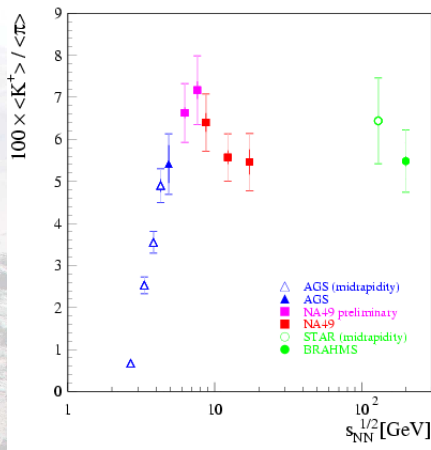
The NA49 Collaboration has performed a series of measurements of Pb-Pb collisions at 20, 30, 40, 80 and 158 AGeV beam energies . When these results are combined with measurements at lower beam energies from the AGS they reveal an unusually sharp variation with beam energy in the  $\Lambda / \langle \pi \rangle$ , with  $\langle \pi \rangle \equiv 3/2(\pi^+ + \pi^-)$ , and  $K^+ / \pi^+$  ratios.

Such a strong variation with energy does not occur in pp collisions and therefore indicates a major difference in heavy-ion collisions. This transition has been referred as the “horn”.



# The Elephant in the Room

Friese  
Dinkelaker  
Blume  
Speltz



Difficult to avoid, Hard to Model

→ But no unambiguous corroborating evidence

## Strangeness in Heavy Ion Collisions

vs

## Strangeness in pp - collisions

Use the Wroblewski factor

$$\lambda_s = \frac{2 \langle s\bar{s} \rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle}$$

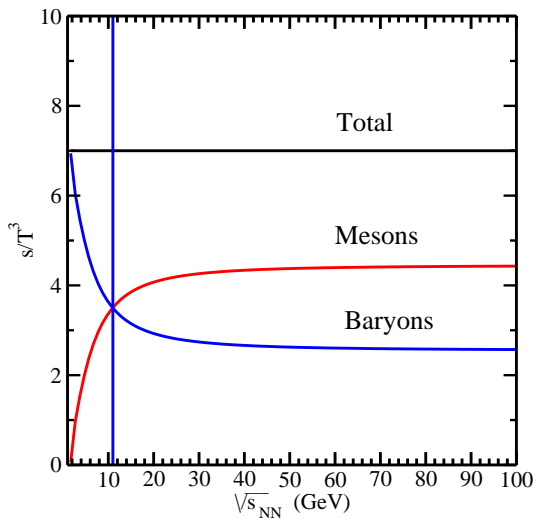
This is determined by the number of **newly** created quark – anti-quark pairs and **before** strong decays, i.e. before  $\rho$ 's and  $\Delta$ 's decay.

Limiting values :

$\lambda_s = 1$  all quark pairs are equally abundant, SU(3) symmetry.

$\lambda_s = 0$  no strange quark pairs.



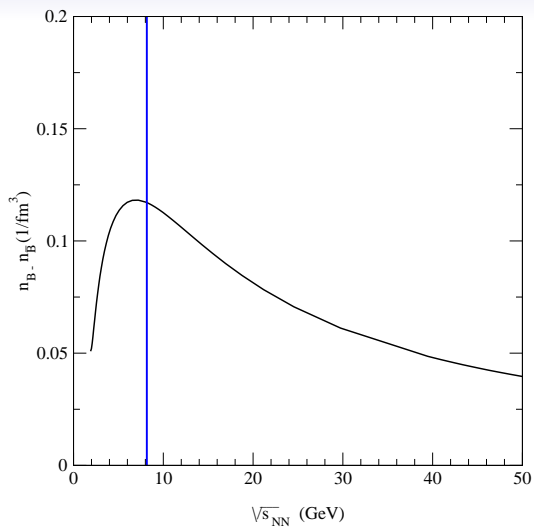


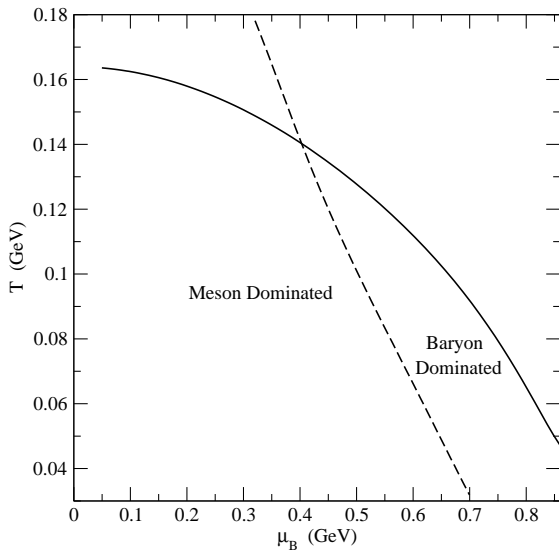
# J.C., H. Oeschler, K. Redlich, S. Wheaton, Phys. Lett. B615 (2005) 50-54

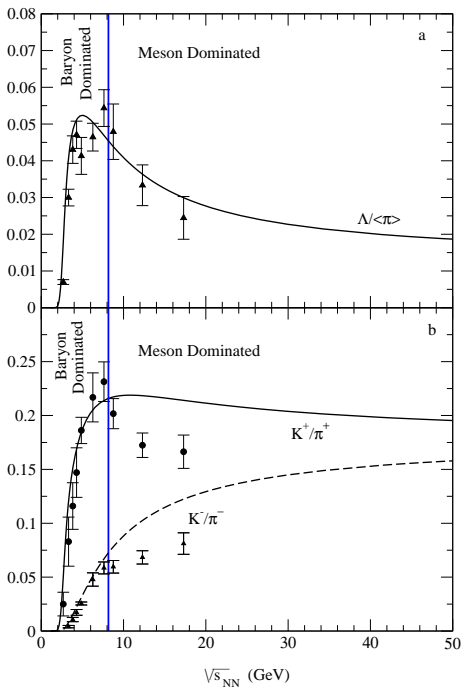
In the statistical model a rapid change is expected as the hadronic gas undergoes a transition from a baryon-dominated to a meson-dominated gas. The transition occurs at a temperature  $T = 151$  MeV and baryon chemical potential  $\mu_B = 327$  MeV corresponding to an incident energy of  $\sqrt{s_{NN}} = 11$  GeV.

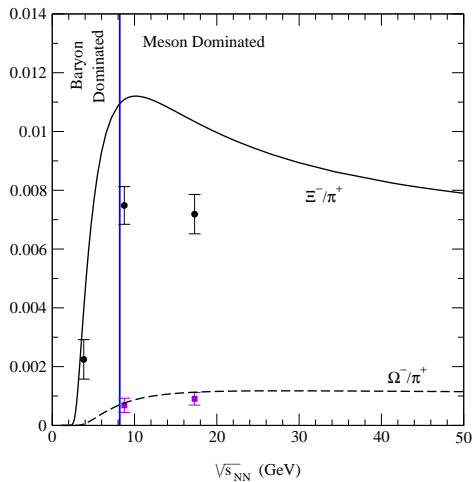












## Maxima in Particle Ratios predicted by the Thermal Model.

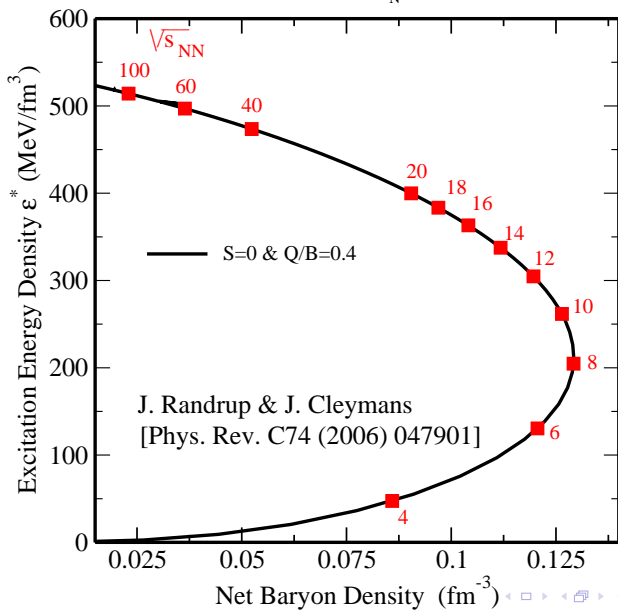
Ratio	Maximum at $\sqrt{s_{NN}}$ (GeV)	Maximum Value
$\Lambda / \langle \pi \rangle$	5.1	0.052
$\Xi^- / \pi^+$	10.2	0.011
$K^+ / \pi^+$	10.8	0.22
$\Omega^- / \pi^+$	27	0.0012

J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

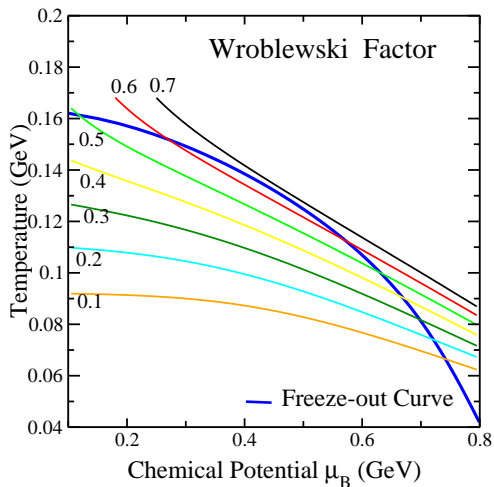


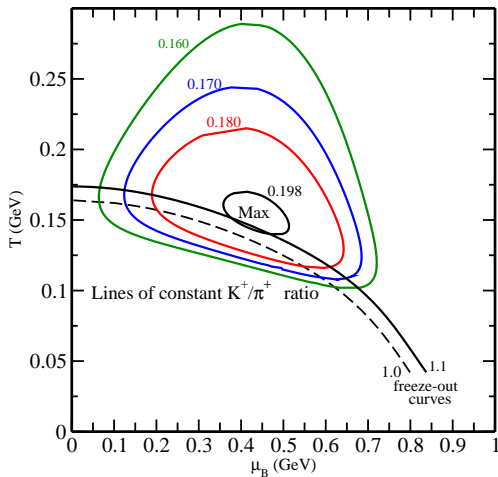
## Hadronic Freeze-Out

$$\varepsilon_* = \varepsilon - m_N \rho$$

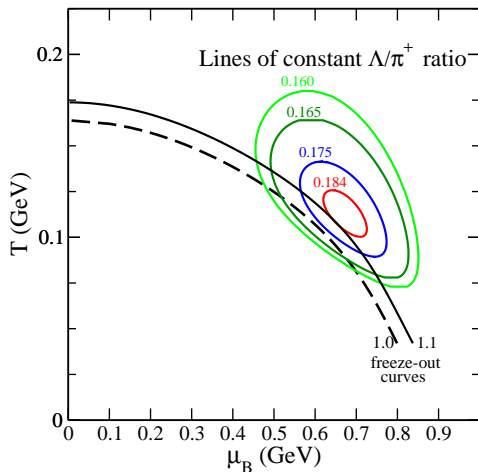


# Maxima in particle ratios : $K^+/\pi^+$



Maxima in particle ratios :  $K^+/\pi^+$ 



Maxima in particle ratios :  $K^+/\pi^+$ 

GOOD NEWS FOR NICA.



## Main goals of the project

1a) Heavy ion colliding beams  $^{197}\text{Au}^{79+} \times ^{197}\text{Au}^{79+}$  at

$$\sqrt{s_{\text{NN}}} = 4 \div 11 \text{ GeV} \quad (1 \div 4.5 \text{ GeV/u ion kinetic energy})$$

at

$$L_{\text{average}} = 10^{27} \text{ cm}^{-2} \cdot \text{s}^{-1} \quad (\text{at } \sqrt{s_{\text{NN}}} = 9 \text{ GeV})$$

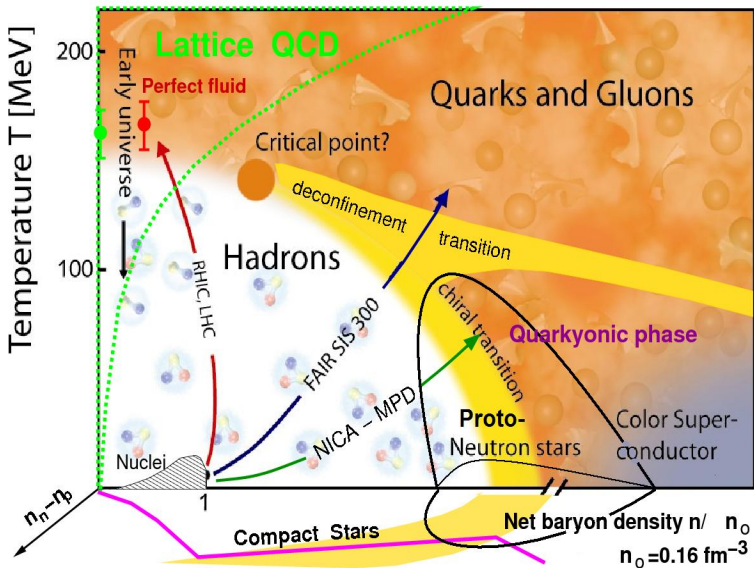
1b) Light-Heavy ion colliding beams of the same energy range and luminosity

2) Polarized beams of protons and deuterons:

$$p\uparrow p\uparrow \quad \sqrt{s_{\text{NN}}} = 12 \div 25 \text{ GeV} \quad (5 \div 12.6 \text{ GeV kinetic energy})$$

$$d\uparrow d\uparrow \quad \sqrt{s_{\text{NN}}} = 4 \div 13.8 \text{ GeV} \quad (2 \div 5.9 \text{ GeV/u ion kinetic energy})$$





GOOD LUCK NICA.

