Andreas Schmitt

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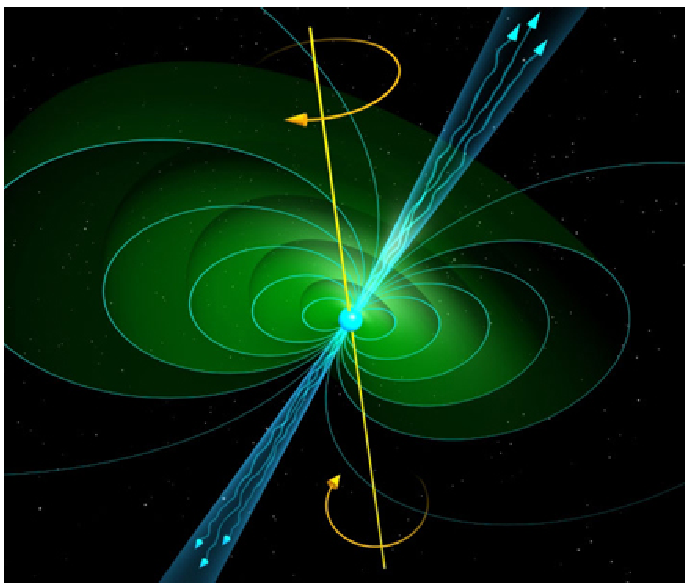
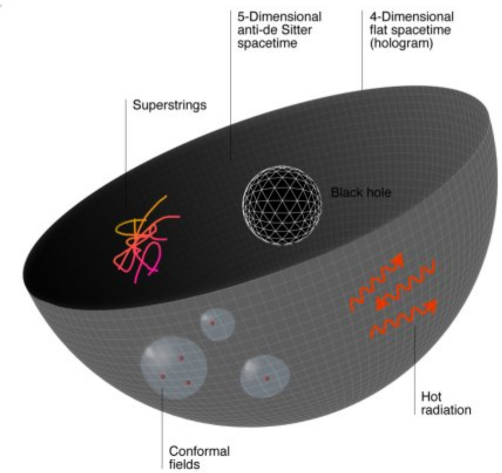
Technische Universität Wien
1040 Vienna, Austria

Dense matter in a magnetic field ...



... from a **field-theoretic** and



a **holographic** point of view

• Outline

1. Setting the stage: equilibrium phases of QCD
    - chiral symmetry breaking in QCD
    - QCD at nonzero T, μ, and magnetic field B
2. Effect of a magnetic field on chiral symmetry breaking
    - "*magnetic catalysis*" in the Nambu-Jona Lasinio (NJL) model
3. Brief introduction to AdS/CFT and the Sakai-Sugimoto model
    - the gauge/gravity duality and its application to QCD
    - the Sakai-Sugimoto model (and how chiral symmetry breaking is realized)
4. Holographic chiral symmetry breaking in a magnetic field
    - "*(inverse) magnetic catalysis*" in the Sakai-Sugimoto model
    - comparison to field-theoretical (NJL) results
5. (Homogeneous) holographic baryonic matter
    - baryons in the Sakai-Sugimoto model
    - large-N_c baryons vs. real-world baryons

• Outline

1. **Setting the stage: equilibrium phases of QCD**
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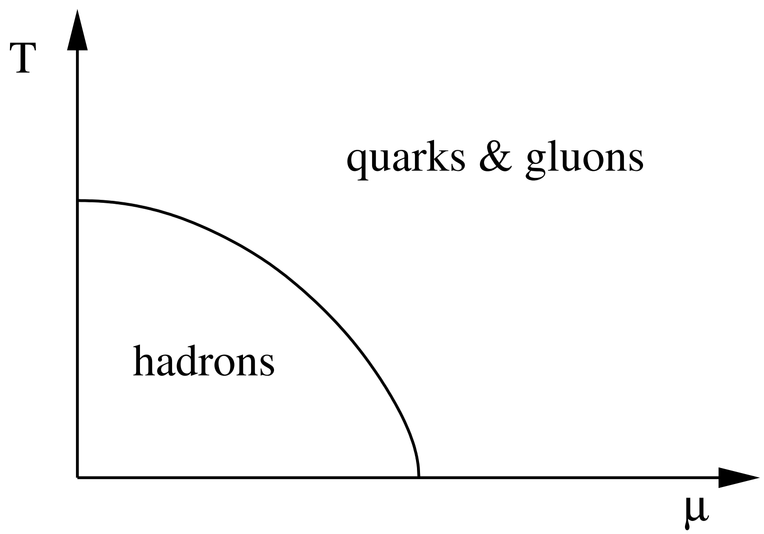
• QCD phase transitions at nonzero T and μ (page 1/2)

1. quarks & gluons at large T and/or μ are weakly coupled due to asymptotic freedom
   D.J. Gross, F. Wilczek, PRL 30, 1343 (1973); H.D. Politzer, *ibid.* 1346
2. at small T, μ we observe hadrons rather than quarks & gluons

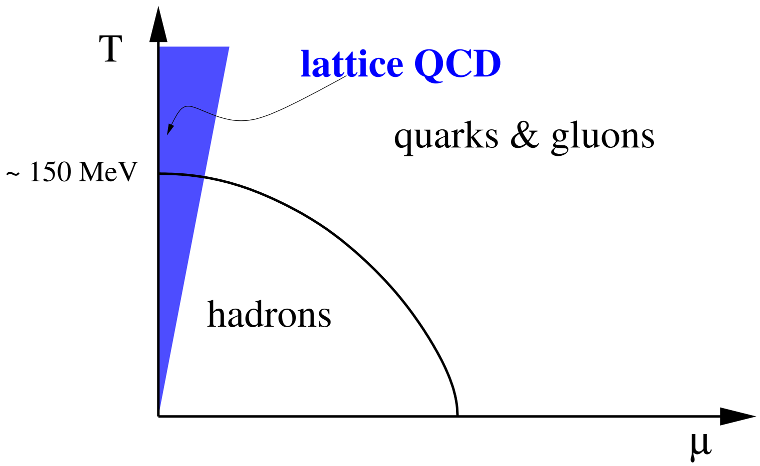
$\Rightarrow$ naive guess of the phase diagram:



- Nature of transition?
- Order parameter?
- How to observe it?
- How to compute it?

N. Cabibbo, G. Parisi, PLB 59, 67 (1975)

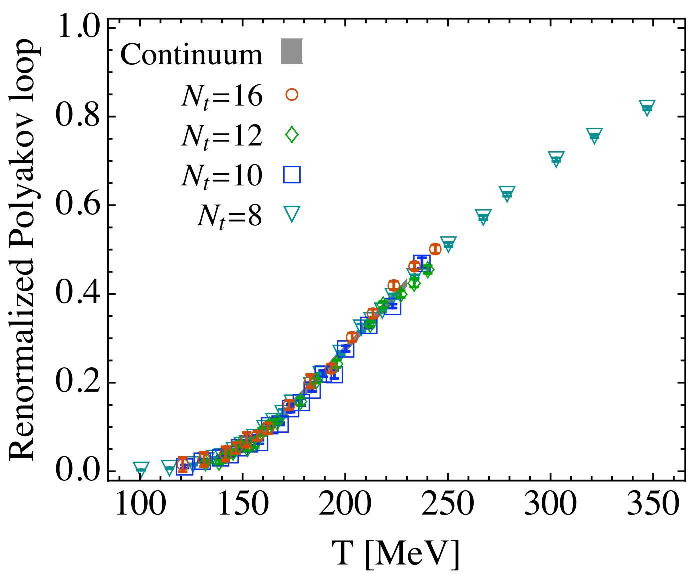
• QCD phase transitions at nonzero T and μ (page 2/2)



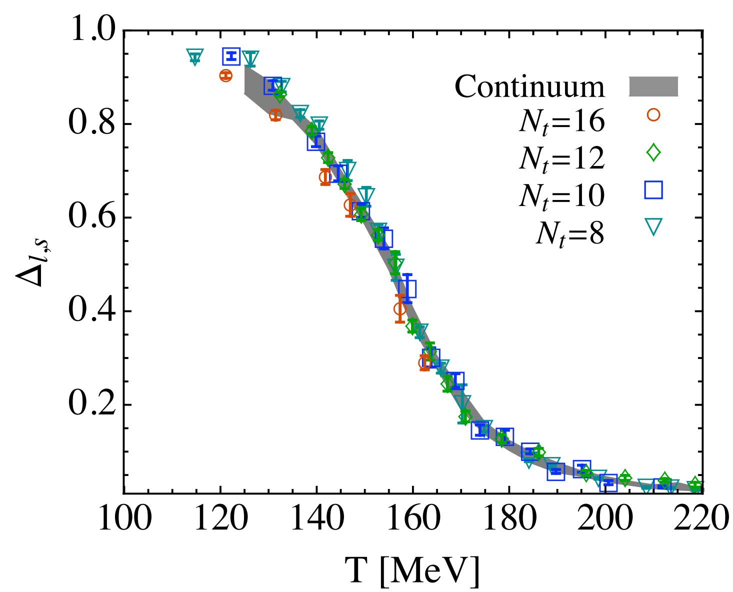
- zero chemical potential:

use lattice QCD
to compute transition

S. Borsanyi *et al.* JHEP 1009, 073 (2010)



deconfinement transition
(crossover)



chiral transition
(crossover)

• Chiral symmetry (breaking) in QCD (page 1/3)

QCD Lagrangian	**chiral fermions**
$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu}$ $= \bar{\psi}_R i\gamma^\mu D_\mu \psi_R + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L$ $-\bar{\psi}_R M \psi_L - \bar{\psi}_L M \psi_R - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu}$	$\psi_R \equiv P_R\psi\,, \quad \psi_L \equiv P_L\psi$ $P_R = \frac{1+\gamma^5}{2}\,, \quad P_L = \frac{1-\gamma^5}{2}$

$\Rightarrow M = 0$: $\mathcal{L}_{\mathrm{QCD}}$ invariant under $\psi_R \to \underbrace{e^{i\phi_R^a t_a}}_{\in U(N_f)_R}\psi_R\,,\ \psi_L \to \underbrace{e^{i\phi_L^a t_a}}_{\in U(N_f)_L}\psi_L$

$\Rightarrow$ *global* symmetry group

$$U(N_f)_R \times U(N_f)_L \cong \boxed{\underbrace{SU(N_f)_R \times SU(N_f)_L}_{\text{"chiral symmetry"}}} \times U(1)_B \times U(1)_A$$

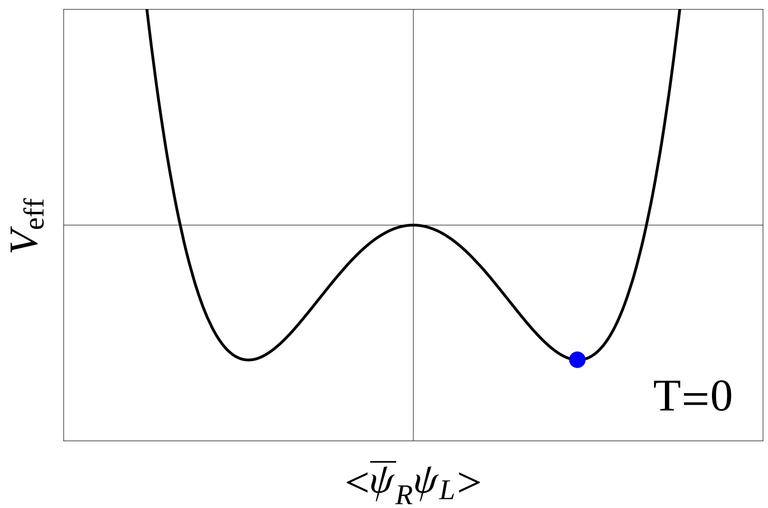
• Chiral symmetry (breaking) in QCD (page 2/3)

- quark mass(es) break chiral symmetry explicitly
- chiral condensate $\langle\bar{\psi}_R\psi_L\rangle$ breaks chiral symmetry spontaneously

$$\boxed{SU(N_f)_R \times SU(N_f)_L \longrightarrow SU(N_f)_{R+L}}$$

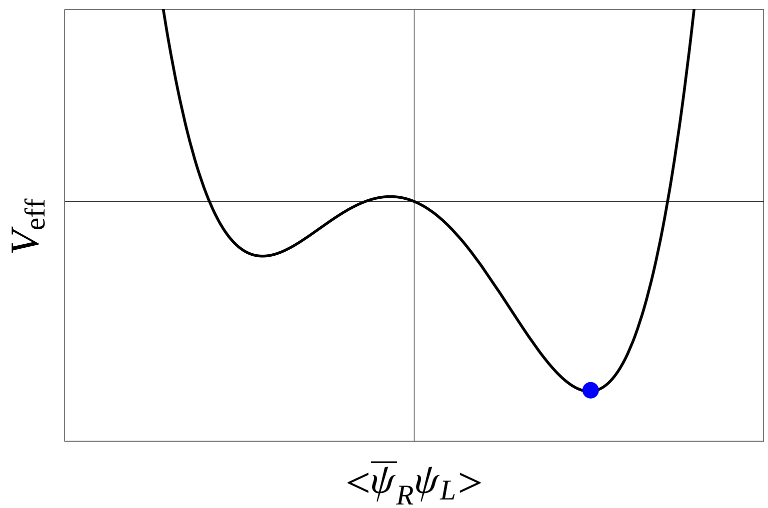
$M = 0$

$\langle\bar{\psi}_R\psi_L\rangle = 0$ for $T \geq T_c$



$M \neq 0$

$\langle\bar{\psi}_R\psi_L\rangle$ always nonzero



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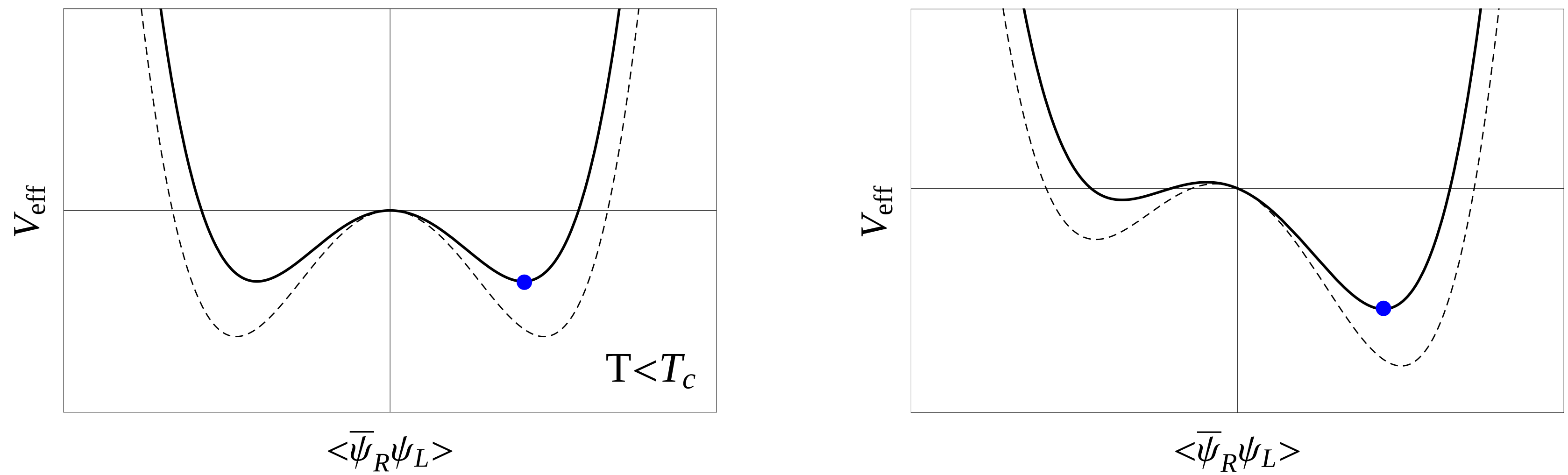

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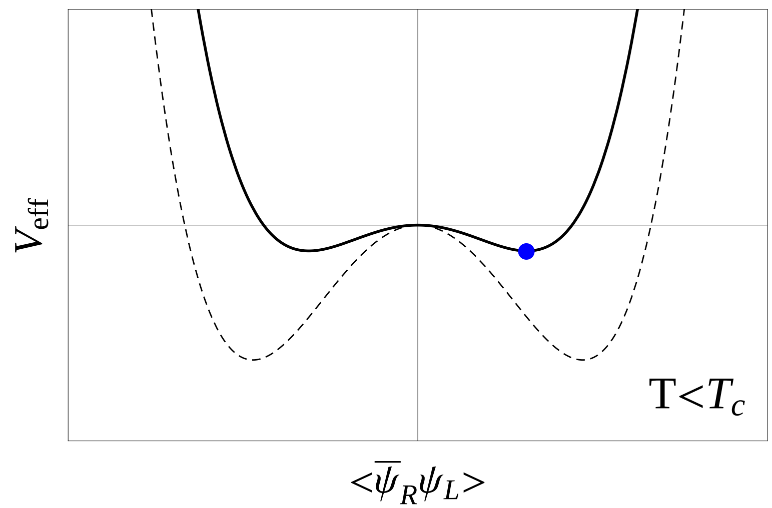

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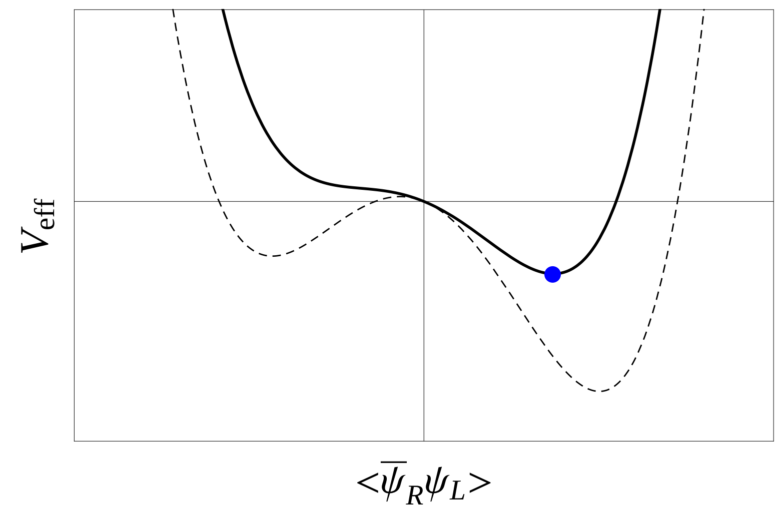

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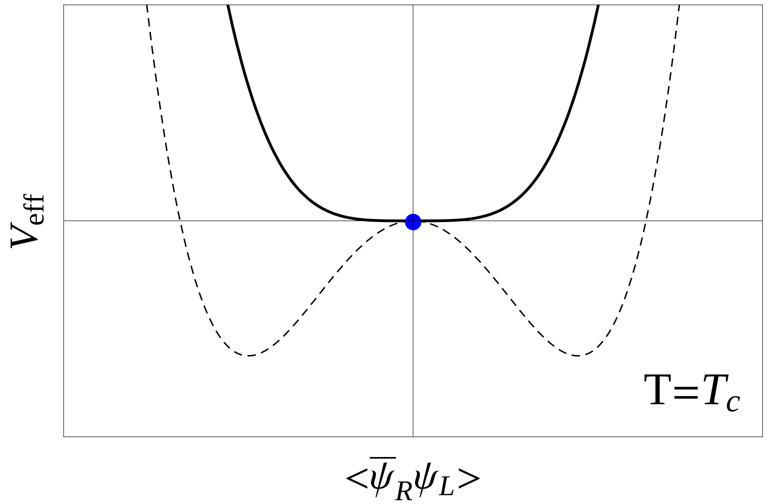

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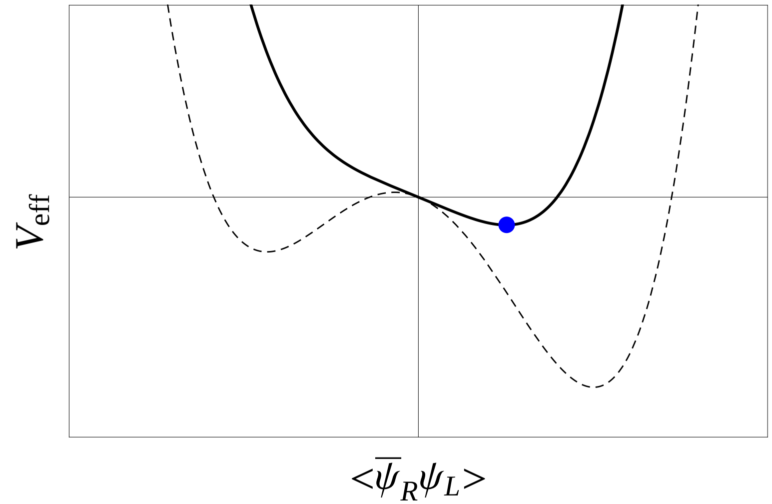

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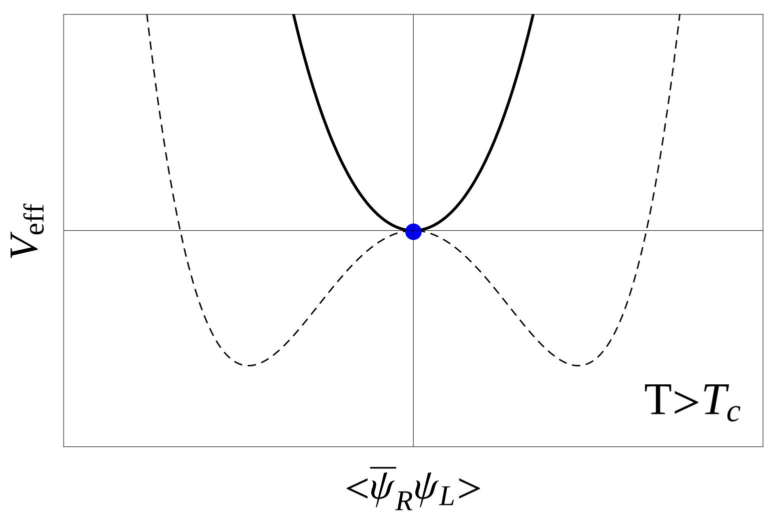

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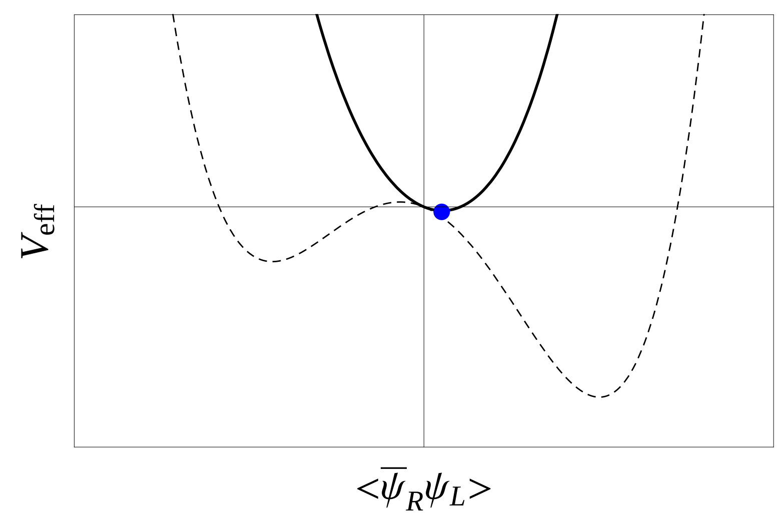
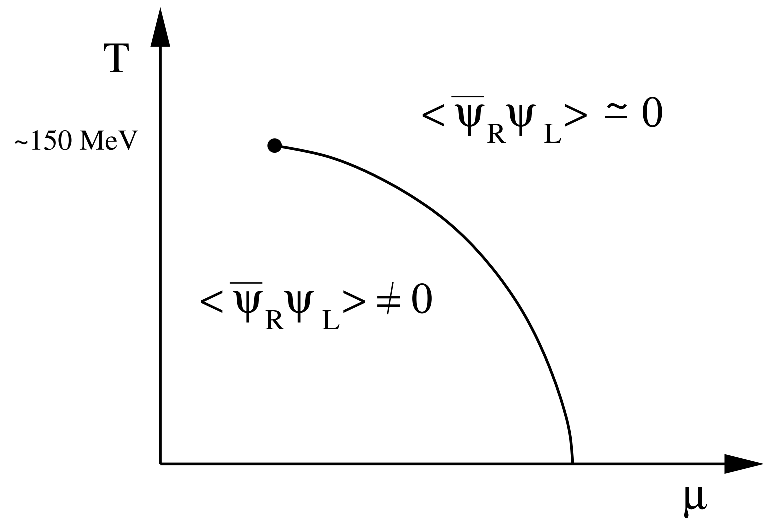


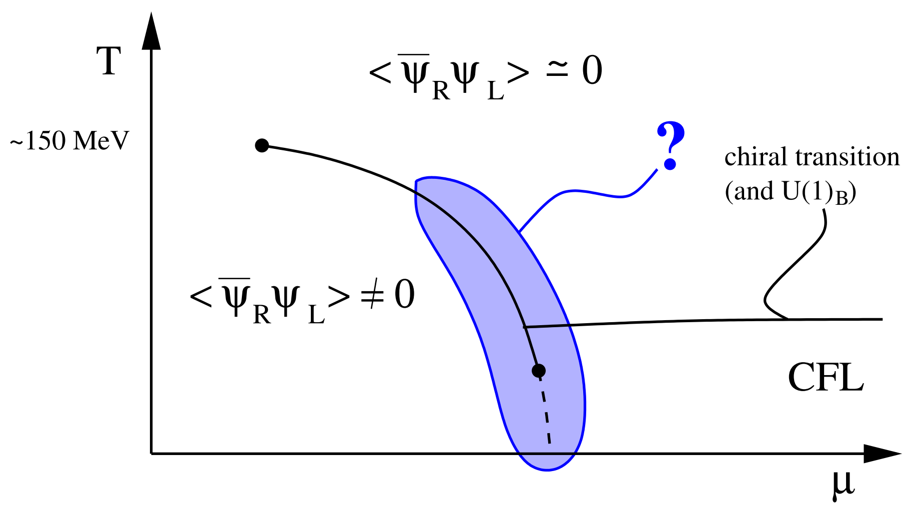
- nonzero quark masses in real world $\longrightarrow$ crossover at $\mu = 0$ (possibly 1st order transition at $\mu \neq 0$)

• Chiral symmetry (breaking) in QCD (page 3/3)

$\longrightarrow$ refined guess of phase diagram

- no first-principle calculation for intermediate μ



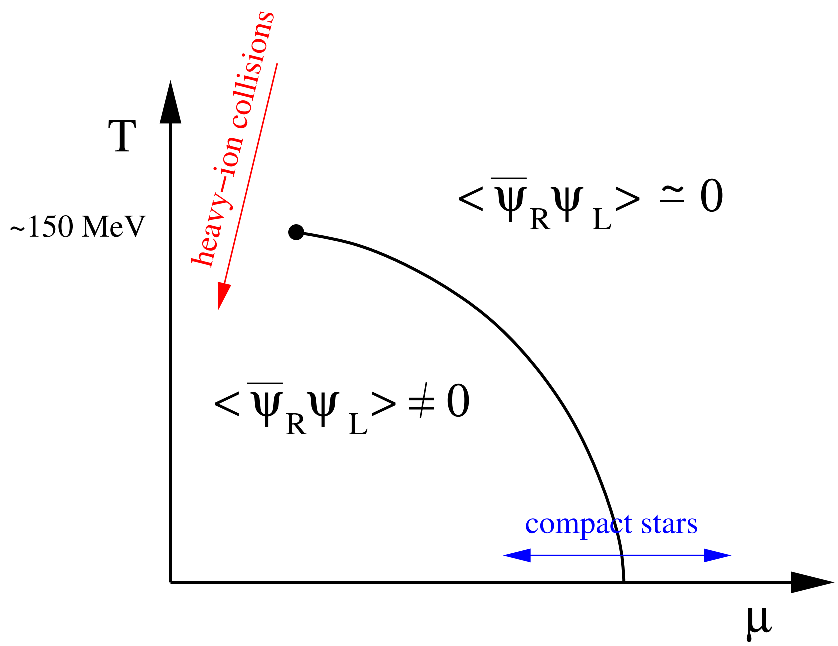


- chiral symmetry also broken spontaneously at asymptotically large μ by color-flavor locking (CFL) ($N_f = 3$)

$\longrightarrow$ CFL will be ignored for the remainder of the lecture

- **"Laboratories" for probing QCD phase transitions (page 1/3)**

- theoretically, "intermediate" regions very challenging:
  - energies too small to use perturbation theory (strong coupling!)
  - energies too large to use conventional nuclear physics
- how about experiments?

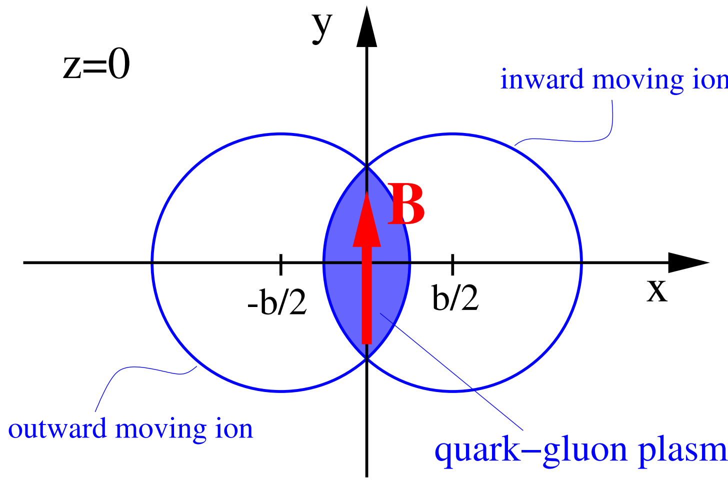


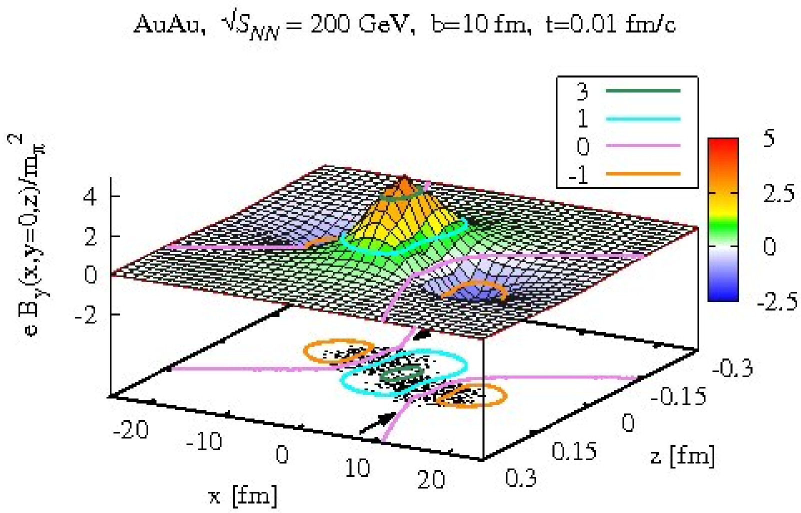
- Heavy-ion collisions: signatures of quark-gluon plasma? (large $T \gtrsim T_c$, small $\mu \ll T$)
- Compact stars: neutron stars or quark stars or hybrid stars? (large $\mu \sim 400\,\mathrm{MeV}$, small $T \ll \mu$)

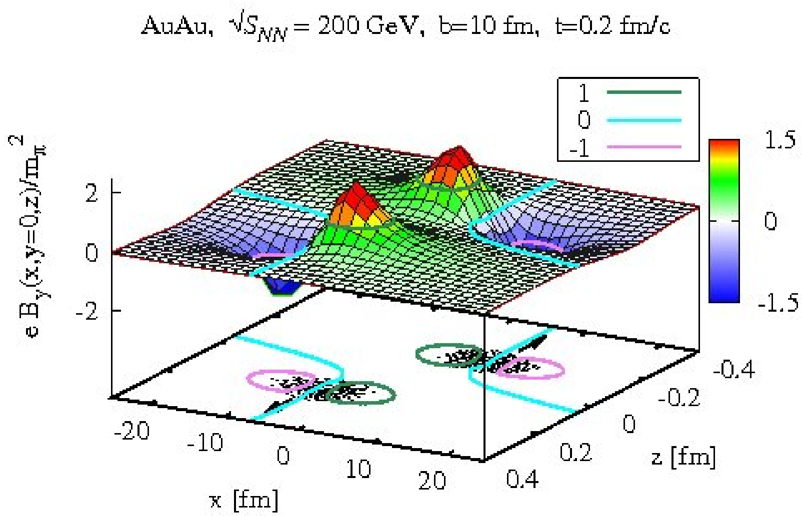
- **In both instances large magnetic fields are present!**

- **“Laboratories” for probing QCD phase transitions (page 2/3)**

(1) Non-central heavy-ion collisions:

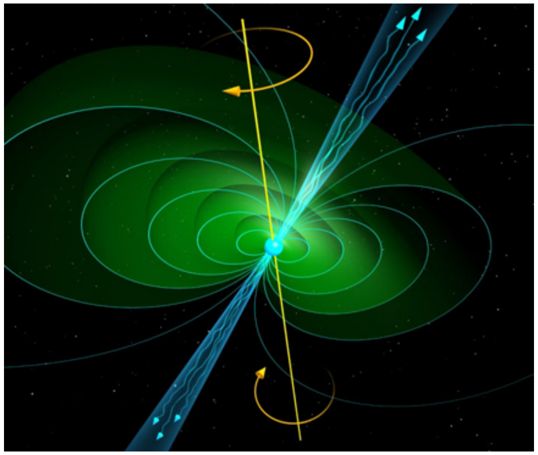




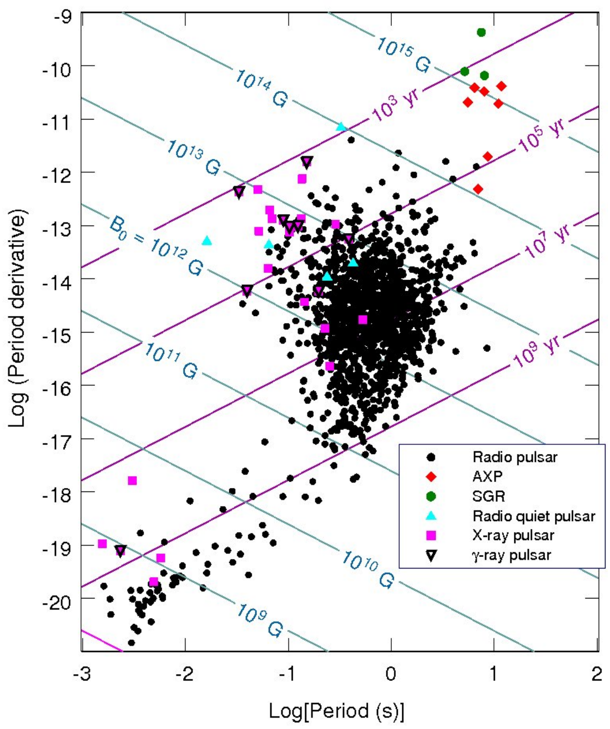


V. Voronyuk, *et al.* PRC 83, 054911 (2011)

- **“Laboratories” for probing QCD phase transitions (page 3/3)**



(2) Compact stars (“Magnetars”):



- magnetic fields from star's progenitor, strongly enhanced (flux conserved)
- surface magnetic field measured via

$$B \propto (P\dot{P})^{1/2}$$

(magn. dipole radiation)

A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

• QCD at nonzero T, μ, and B (page 1/3)

- heavy-ion collisions:

  temporarily $B \lesssim 10^{19}\,\mathrm{G}$

  Skokov, Illarionov, Toneev,
  Int. J. Mod. Phys. A 24, 5925 (2009)

  (*compare:*
  earth's magn. field: $B \simeq 0.6\,\mathrm{G}$
  LHC supercond. magnets: $B \simeq 10^5\,\mathrm{G}$)

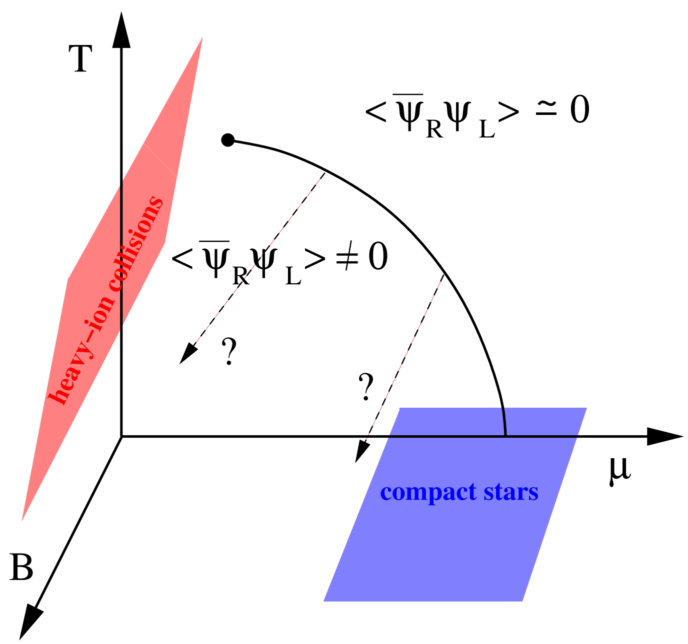
- magnetars:

  at surface $B \lesssim 10^{15}\,\mathrm{G}$

  Duncan, Thompson, Astrophys.J. 392, L9 (1992)

  larger in the interior,
  $B \sim 10^{18-20}\,\mathrm{G}$?

  Lai, Shapiro, Astrophys.J. 383, 745 (1991)
  E. J. Ferrer *et al.*, PRC 82, 065802 (2010)



effect on QCD phase transitions?

$$\Lambda^2_{\mathrm{QCD}} \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$$

$(1\,\mathrm{eV}^2 \simeq 51.189\,\mathrm{G})$

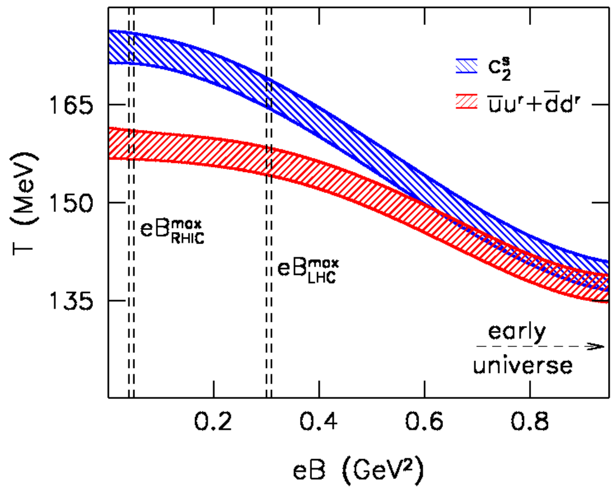
• QCD at nonzero T, μ, and B (page 2/3)

A (very incomplete) collection of recent "magnetic activities":

- QCD phase transitions in a magnetic field on the lattice

  M. D'Elia, S. Mukherjee, F. Sanfilippo, PRD 82, 051501 (2010)

  G.S. Bali, *et al.*, JHEP 1202, 044 (2012) (see plot)



- "splitting" of deconfinement and chiral symmetry breaking

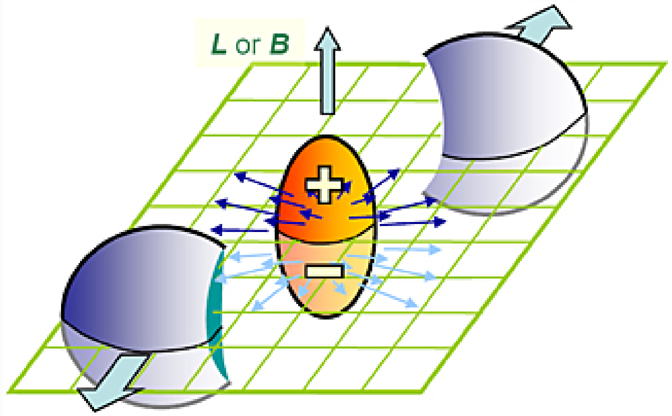
  R. Gatto, M. Ruggieri, PRD 83, 034016 (2011)

  A. J. Mizher, M. N. Chernodub, E. S. Fraga, PRD 82, 105016 (2010)

  holographically: F. Preis, A. Rebhan and A. Schmitt, JHEP 1103, 033 (2011)

• QCD at nonzero T, μ, and B (page 3/3)

- chiral magnetic effect
  Kharzeev, McLerran, Warringa, NPA 803, 227 (2008)
  holographically: H. -U. Yee, JHEP 0911, 085 (2009)
  Rebhan, Schmitt, Stricker, JHEP 1001, 026 (2010)
  A. Gynther, K. Landsteiner, F. Pena-Benitez
  and A. Rebhan, JHEP 1102, 110 (2011)



- ρ meson condensation through magnetic field
  M. N. Chernodub, PRD 82, 085011 (2010)
  holographically: N. Callebaut, D. Dudal, H. Verschelde, arXiv:1105.2217 [hep-th]

- anomalous hydrodynamics
  D. T. Son and P. Surowka, PRL 103, 191601 (2009)
  K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107, 021601 (2011)

$\longrightarrow$ D. Kharzeev, K. Landsteiner, A. Schmitt, H.-U. Yee (Eds.),
"Strongly interacting matter in magnetic fields", Lect. Notes Phys., to appear in late 2012

- **Summary part 1**

- QCD phase structure is very difficult to compute (especially at finite μ)
- both instances that probe QCD phase transitions involve huge magnetic fields
- also theoretically, nonzero B might help to understand QCD phases (B as another “knob” like N_c, μ_I etc.)

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2. **Effect of a magnetic field on chiral symmetry breaking**
3. Brief introduction to AdS/CFT and the Sakai-Sugimoto model
4. Holographic chiral symmetry breaking in a magnetic field
5. (Homogeneous) holographic baryonic matter

• Magnetic catalysis (page 1/5)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

• (massless) fermions in **Nambu-Jona-Lasinio (NJL)** model

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma^\mu\partial_\mu - \mu\gamma^0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]$$

Mean-field approximation:

$$\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle + \underbrace{(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle)}_{\text{small fluctuation}} \Rightarrow (\bar{\psi}\psi)^2 \simeq -\langle\bar{\psi}\psi\rangle^2 + 2\langle\bar{\psi}\psi\rangle\bar{\psi}\psi$$

$$\Rightarrow \quad \mathcal{L}_{\text{mean field}} = \bar{\psi}(i\gamma^\mu\partial_\mu - M - \mu\gamma^0)\psi - \frac{M^2}{4G}$$

$\Rightarrow$ chiral condensate induces "constituent quark mass"

$$\boxed{M = -2G\langle\bar{\psi}\psi\rangle}$$

• Magnetic catalysis (page 2/5)

- determine M from minimizing free energy

$$\frac{\partial\Omega}{\partial M} = 0 \qquad \Rightarrow \qquad \boxed{M = 2G\sum_e \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{M}{E_k}\tanh\frac{E_k - e\mu}{2T}}$$

"gap equation" $(B = 0)$

$$E_k = \sqrt{k^2 + M^2}$$

- gap equation at $T = \mu = 0$

$$1 - \frac{1}{g} = \frac{M^2}{\Lambda^2}\ln\frac{\Lambda}{M}$$

- Λ momentum cutoff
- $g \equiv G\Lambda^2/\pi^2$ dimensionless coupling

Zero magnetic field:

dynamical fermion mass

$$M \propto \langle\bar\psi\psi\rangle \neq 0$$

only for coupling $g > g_c = 1$

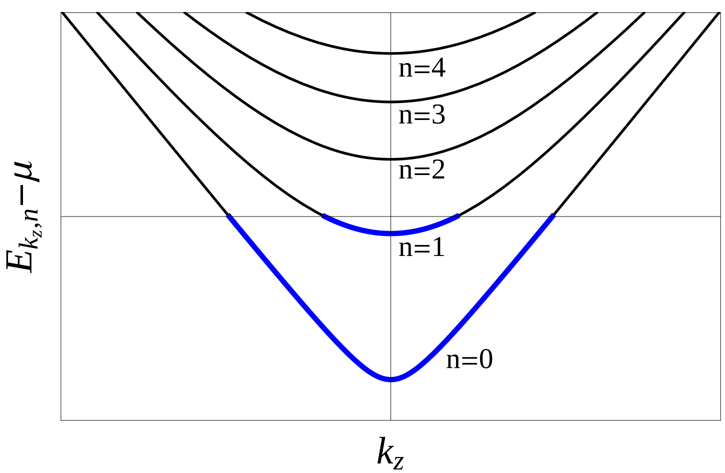
• Magnetic catalysis (page 3/5)

• include magnetic field $\vec{B} = (0, 0, B)$

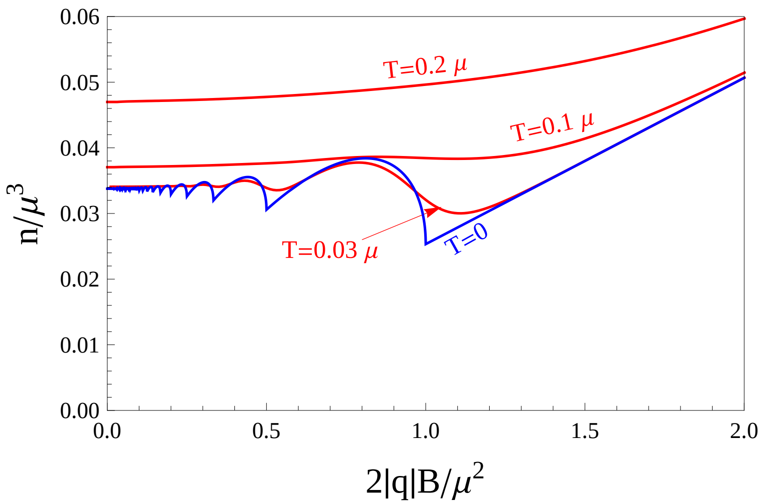
$$2\int \frac{d^3\mathbf{k}}{(2\pi)^3} \to \frac{|q|B}{2\pi}\sum_{n=0}^{\infty}(2-\delta_{n0})\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}$$

$$E_k \to E_{k_z,n} = \sqrt{k_z^2 + 2n|q|B + M^2}$$

• remember Landau levels n:



fermion excitations



density (massless fermions)

• Magnetic catalysis (page 4/5)

- gap equation with magnetic field ($\mu = T = 0$), $x \equiv \frac{M^2}{2|q|B}$

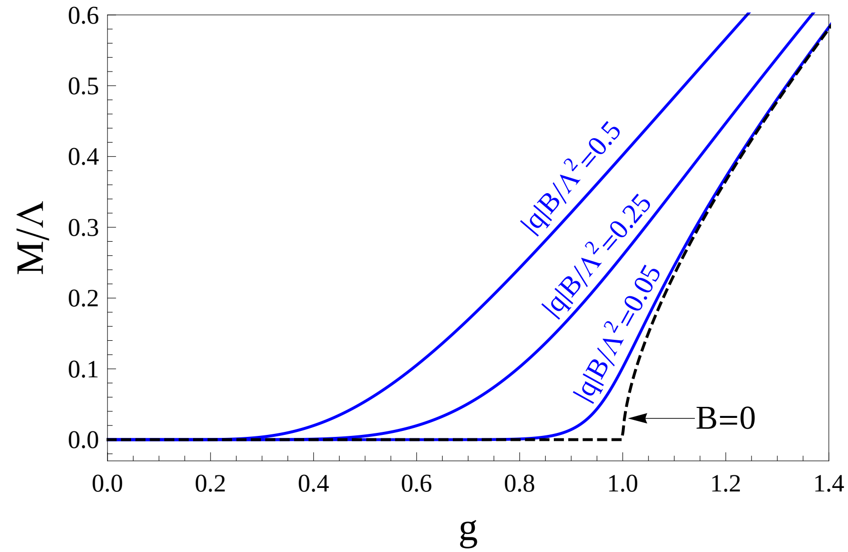
$$1 - \frac{1}{g} = \frac{M^2}{\Lambda^2} \ln \frac{\Lambda}{M} - \underbrace{\frac{|q|B}{\Lambda^2} \left[\left(\frac{1}{2} - x \right) \ln x + x - \frac{1}{2} \ln 2\pi + \ln \Gamma(x) \right]}_{\simeq \frac{|q|B}{\Lambda^2} \ln \frac{\sqrt{|q|B}}{M\sqrt{\pi}} \quad (M^2 \ll |q|B)} .$$

Nonzero magnetic field:

$M \neq 0$ for *arbitrarily small* g,

$$M \simeq \sqrt{\frac{|q|B}{\pi}} e^{-\Lambda^2/(|q|Bg)}$$

at weak coupling $g \ll 1$

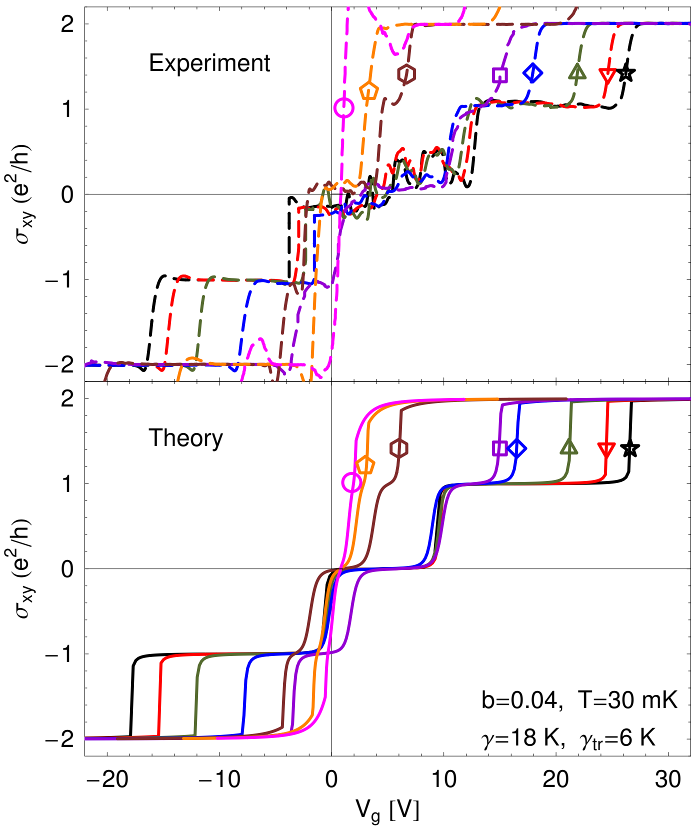


- **Magnetic catalysis (page 5/5)**

Analogy to BCS Cooper pairing:

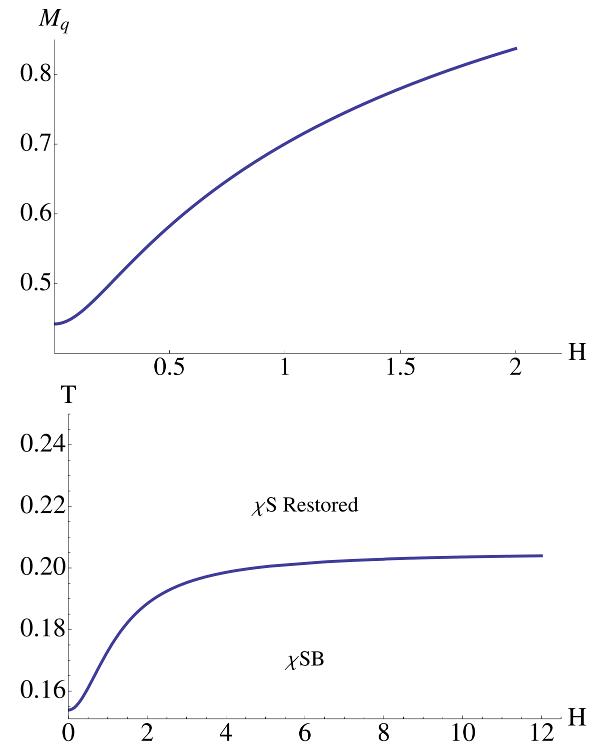
BCS superconductor	Magnetic catalysis
Cooper pair condensate $\langle \psi\psi \rangle$	chiral condensate $\langle \overline{\psi}\psi \rangle$
$\Delta \propto \mu\, e^{-\mathrm{const.}/G\nu_F}$ (ν_F: d.o.s. at $E = \mu$ Fermi surface)	$M \propto \sqrt{eB}\, e^{-\mathrm{const.}/G\nu_0}$ (ν_0: d.o.s. at $E = 0$ surface)
pairing dynamics effectively (1+1)-dimensional because of Fermi surface	effectively (1+1)-dimensional in lowest Landau level (LLL) because of magn. field
gap equation $\Delta = \frac{\mu^2 G}{2\pi^2} \int_0^\infty dk \frac{\Delta}{\sqrt{(k-\mu)^2 + \Delta^2}}$	gap equation (LLL) $M = \frac{\vert q\vert BG}{2\pi^2} \int_{-\infty}^\infty dk_z \frac{M}{\sqrt{k_z^2 + M^2}}$

- **Magnetic catalysis in the real world and in holography**



V.P.Gusynin *et al.*, PRB 74, 195429 (2006)

- **graphene**: appearance of additional plateaus in strong magnetic fields [$B = 9\,\mathrm{T}$ (pink), $B = 45\,\mathrm{T}$ (black)]



C.V.Johnson, A.Kundu, JHEP 0812, 053 (2008)

- **Sakai-Sugimoto**: magnetic field enhances dynamical mass M_q and critical temperature T_c

$\longrightarrow$ *see next part of this lecture*

- **Summary part 2**

Magnetic catalysis

=

magnetic field favors/enhances $\bar{\psi} - \psi$ pairing

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• The gauge/gravity duality: basic idea

J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

"pedestrian's guide": S. S. Gubser and A. Karch, Ann. Rev. Nucl. Part. Sci. 59, 145 (2009)

string theory (in higher dimensions)	$\Leftrightarrow$	gauge theory (on boundary)

original "AdS/CFT correspondence":

string theory on $AdS_5 \times S^5 \Leftrightarrow \mathcal{N} = 4\ SU(N_c)$ SYM theory on $\mathbb{R}^{3,1}$

$$\frac{(\text{curvature radius})^4}{(\text{string length})^4} = \frac{R^4}{\ell_s^4} = g^2_{\rm YM} N_c \equiv \lambda \quad \text{'t Hooft coupling}$$

$\ell_s \ll R$		$\lambda \gg 1$
supergravity limit (*easy!*)	$\Leftrightarrow$	strong coupling limit (*difficult!*)

• Applications of the gauge/gravity duality to QCD

- compare with $\mathcal{N} = 4$ SYM

  typically in the context of heavy-ion collisions

  see for instance the review

  Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann, arXiv:1101.0618 [hep-th]

  – viscosity G. Policastro, D. T. Son, A. O. Starinets, PRL 87, 081601 (2001)

  – jet quenching H. Liu, K. Rajagopal, U. A. Wiedemann, PRL 97, 182301 (2006)

  – expanding plasma R. A. Janik, R. B. Peschanski, PRD 73, 045013 (2006)

- towards a gravity dual of QCD

  – add flavor to AdS/CFT A. Karch, E. Katz, JHEP 0206, 043 (2002)

  – "bottom-up" approach Erlich, Katz, Son, Stephanov, PRL 95, 261602 (2005)

  – Sakai-Sugimoto model ("top-down")
  T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

• Example: shear viscosity η

- what is $\frac{\eta}{s}$ for the quark-gluon plasma (QGP)?
  - weak coupling: $\frac{\eta}{s}(\lambda \to 0) = \dfrac{A}{\lambda^2 \ln(B/\sqrt{\lambda})}$ parametrically large

    P. B. Arnold, G. D. Moore and L. G. Yaffe, JHEP 0011, 001 (2000)
  - lattice QCD: transport properties very difficult to compute

    see however: H. B. Meyer, PRD 76, 101701 (2007)
  - experiment: infer value with hydro simulation $\frac{\eta}{s} \simeq 0.08 - 0.2$

    M. Luzum and P. Romatschke, PRC 78, 034915 (2008)

strong coupling via AdS/CFT:

$$\boxed{\frac{\eta}{s}(\lambda \to \infty) = \frac{1}{4\pi} \simeq 0.08}$$

- only AdS/CFT comes close to QGP
- transport properties discriminate between weak and strong coupling

  $\Rightarrow$ QGP is strongly coupled

G. Policastro, D. T. Son, A. O. Starinets, PRL 87, 081601 (2001)

• The Sakai-Sugimoto model in two steps

1. Background geometry with D4-branes

   E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)

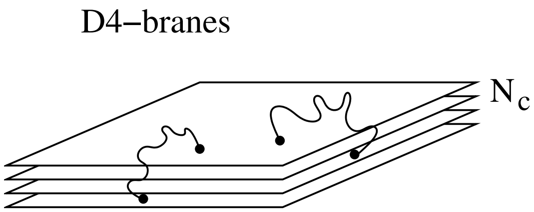
   M. Kruczenski, D. Mateos, R. C. Myers, D. J. Winters, JHEP 0405, 041 (2004)

2. Add flavor D8-branes

   T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

• Sakai-Sugimoto model: background geometry (p. 1/3)

N_c D4-branes compactified on circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$



- 4-4 strings $\longrightarrow$ adjoint scalars & fermions, gauge fields
- periodic $x_4 \longrightarrow$ break SUSY by giving mass $\sim M_{\rm KK}$ to scalars & fermions

$\Rightarrow SU(N_c)$ gauge theory

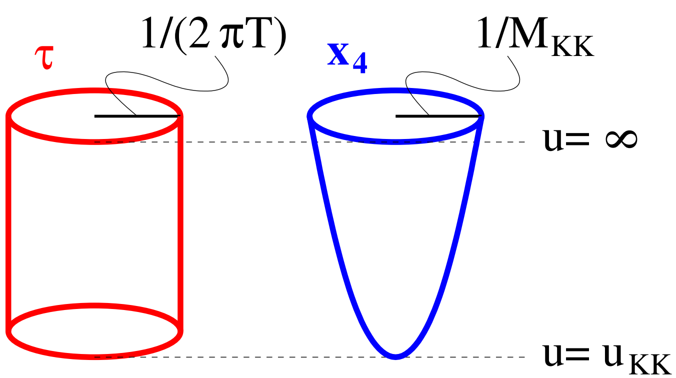
$$\lambda = \frac{g_5^2 N_c}{2\pi/M_{\rm KK}}$$

	$\lambda \ll 1$	$\lambda \gg 1$
dual to large-N_c QCD (at energies $\ll M_{\rm KK}$)	$\surd$ $\Lambda_{\rm QCD} \ll M_{\rm KK}$	**x** $\Lambda_{\rm QCD} \sim M_{\rm KK}$
gravity approximation	**x**	$\surd$

• Background geometry (page 2/3): two solutions

Confined phase

$$ds^2_{\rm conf} = \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + \tilde{f}(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right]$$

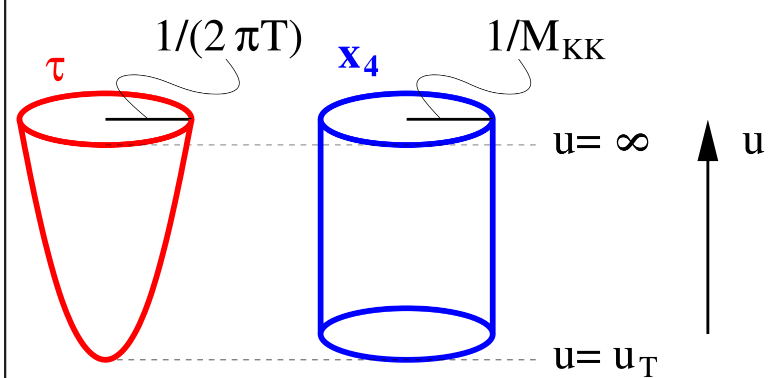


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u^3_{\rm KK}}{u^3}$$

Wick rotated regular geometry

Deconfined phase

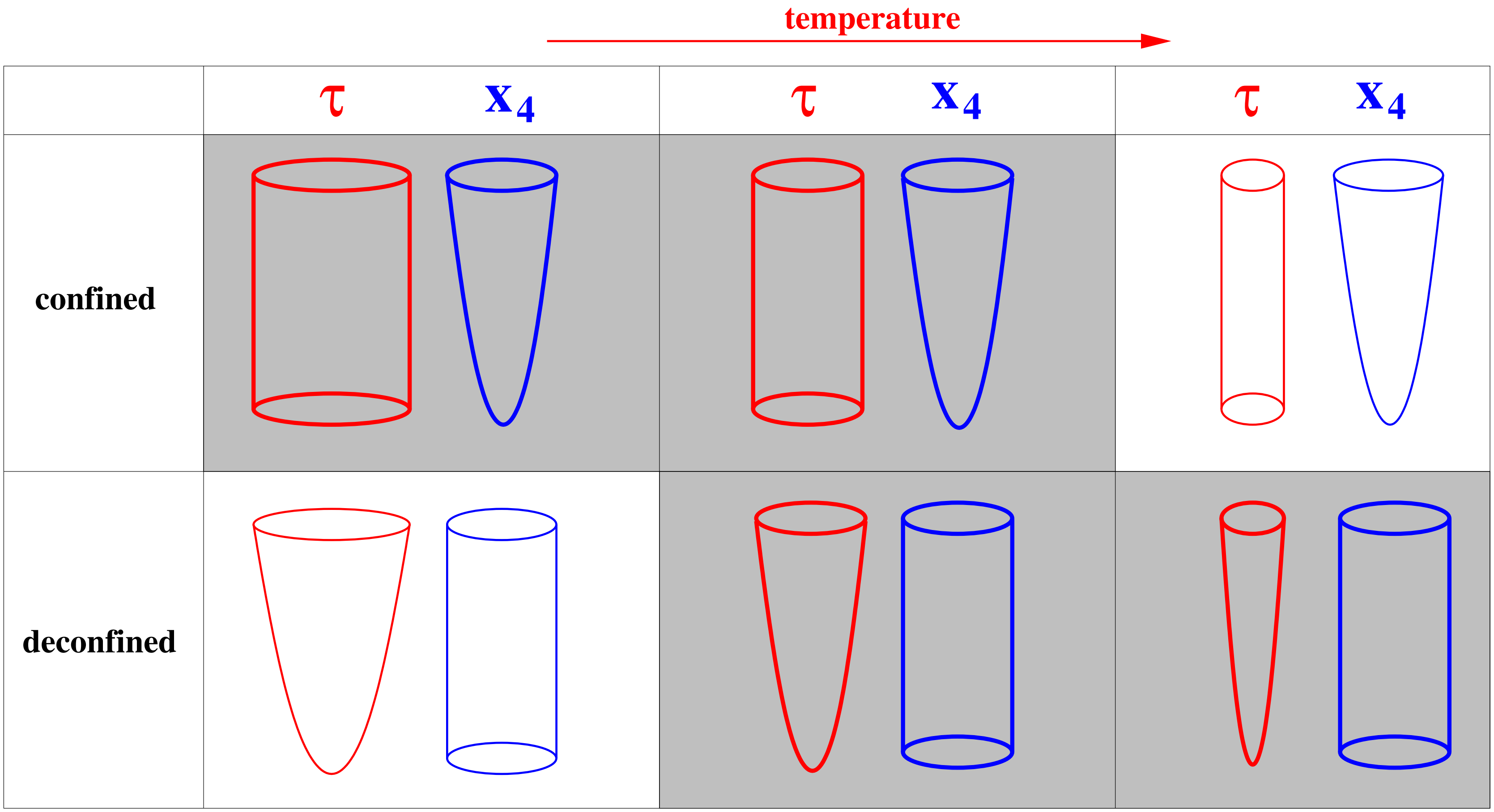
$$ds^2_{\rm deconf} = \left(\frac{u}{R}\right)^{3/2}\left[f(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u^3_T}{u^3}$$

Wick rotated black brane

- **Background geometry (page 3/3): deconfinement phase transition**



$$\boxed{T_c = \frac{M_{\rm KK}}{2\pi}}$$

fit $M_{\rm KK} = 949\,\mathrm{MeV}$ to reproduce ρ mass

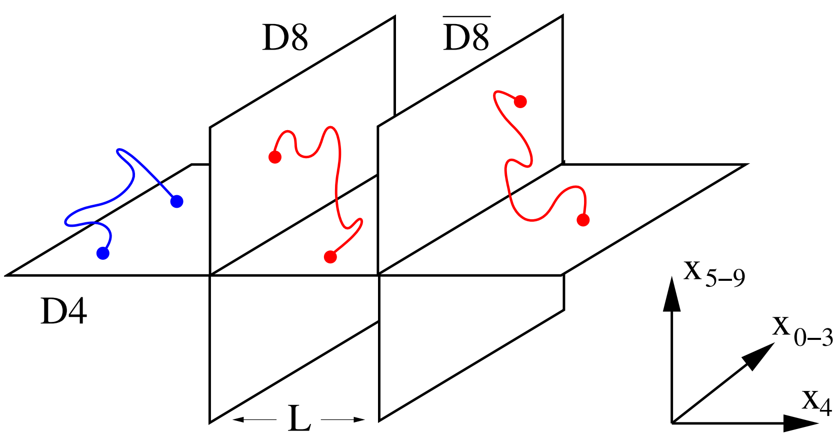
$\Rightarrow T_c \simeq 150\,\mathrm{MeV}$

• Add flavor (page 1/2)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

- add N_f D8- and $\overline{\text{D8}}$-branes, separated in x_4

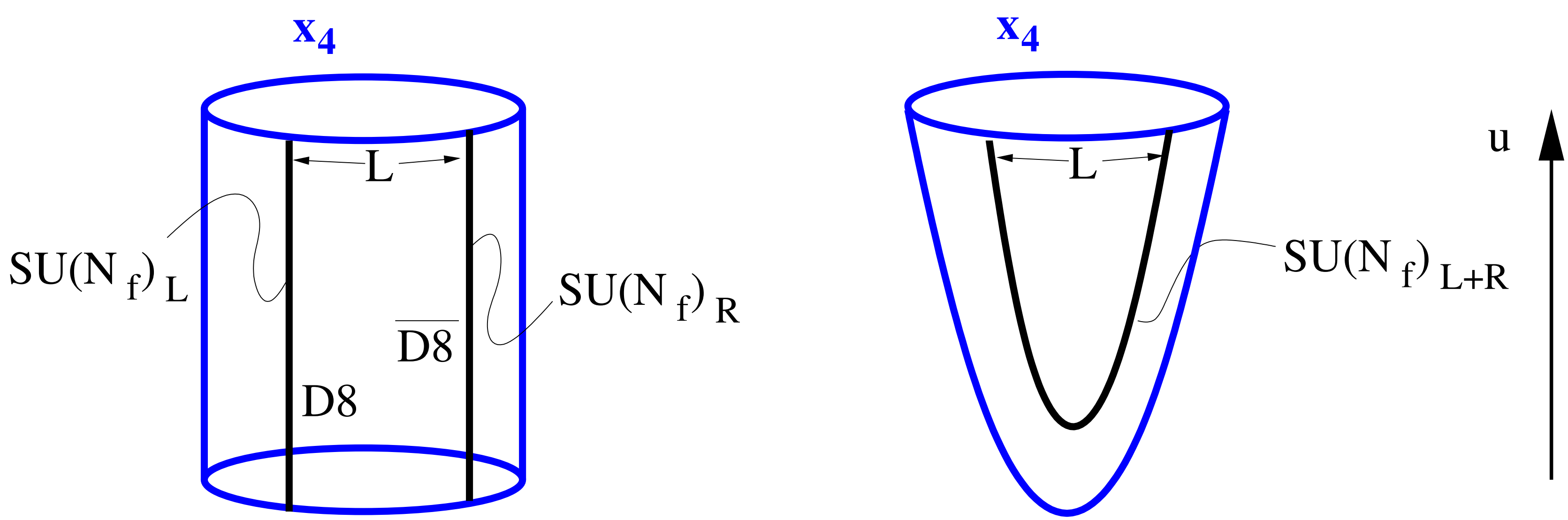
	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\text{D8}}$	x	x	x	x		x	x	x	x	x



- 4-8, 4-$\bar{8}$ strings
  $\longrightarrow$ fundamental, massless chiral fermions
  under $U(N_f)_L \times U(N_f)_R$
  $\Rightarrow$ quarks & gluons

• Add flavor (page 2/2): Chiral symmetry breaking

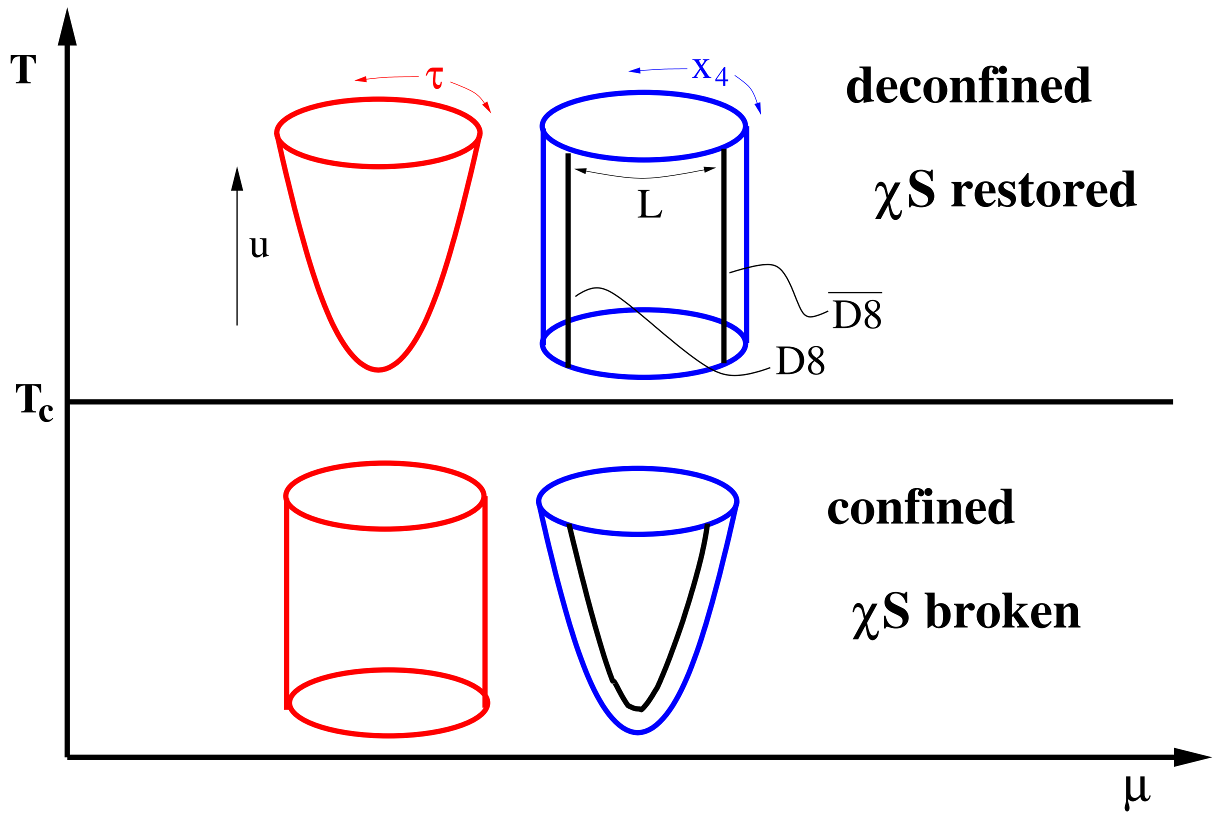
- background geometry unchanged if $N_f \ll N_c$ ("probe branes")
  $\longrightarrow$ "quenched" approximation
- gauge symmetry on the branes $\longrightarrow$ global symmetry at $u = \infty$



- chiral symmetry breaking

$$\boxed{SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}}$$

• Chiral transition in the Sakai-Sugimoto model (p. 1/3)

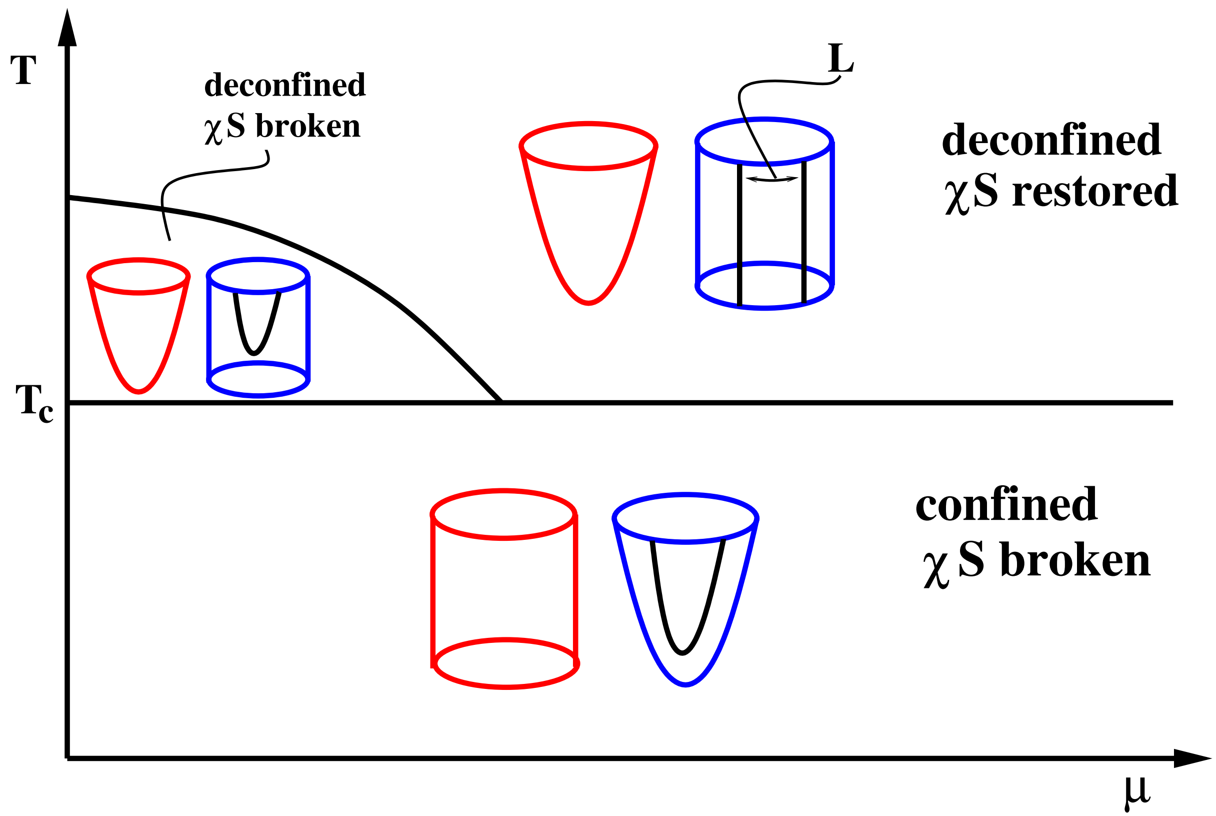


- not unlike expectation from large-N_c QCD
- in probe brane approximation: **chiral transition** unaffected by quantities on flavor branes (μ, B, ...)

- **Chiral transition in the Sakai-Sugimoto model (p. 2/3)**
  - less “rigid” behavior for smaller L
  - deconfined, chirally broken phase for $L < 0.3\,\pi/M_{\mathrm{KK}}$

    O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)

    N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)

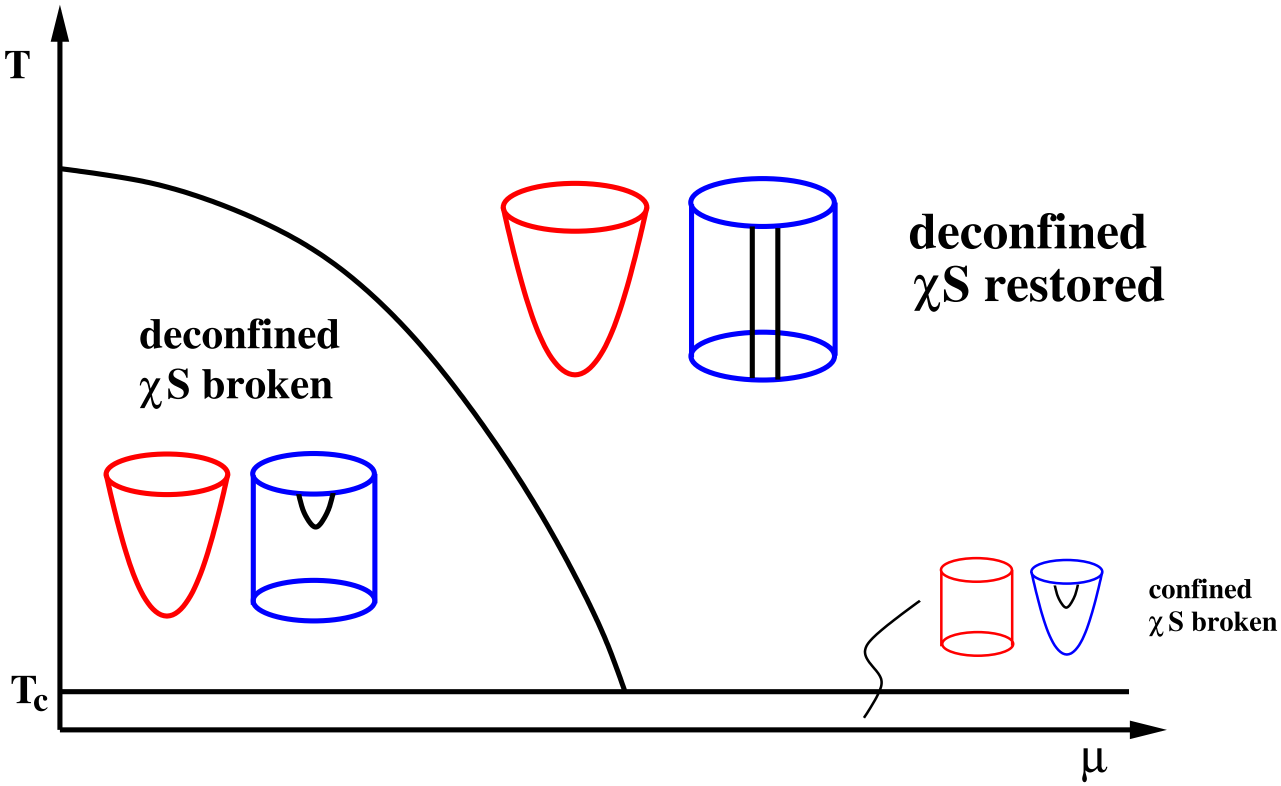


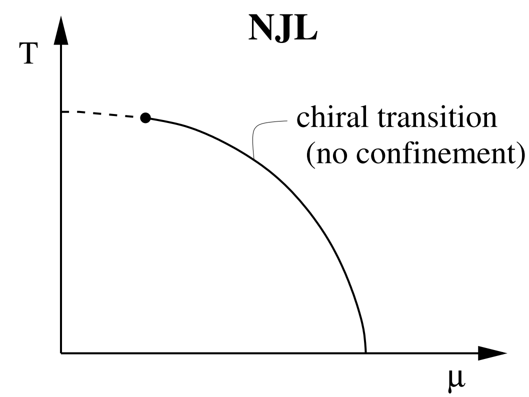
• Chiral transition in the Sakai-Sugimoto model (p. 3/3)

- $L \ll \pi/M_{\rm KK}$ corresponds to (non-local) NJL model

  E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

  J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)





- "decompactified" limit → gluon dynamics decouple
- this limit is considered in the following calculation ...

• Summary part 3

- the gauge/gravity duality provides a tool for strongly coupled physics
- in AdS/CFT, we study the correct limit (strong coupling) but the wrong theory ($\mathcal{N} = 4$ SYM)
- the Sakai-Sugimoto model comes closer to QCD
  - it has confinement and chiral symmetry breaking
  - it still is, at best, dual to large-N_c QCD

- **Outline**

1. Setting the stage: equilibrium phases of QCD
2. Effect of a magnetic field on chiral symmetry breaking
3. Brief introduction to AdS/CFT and the Sakai-Sugimoto model
4. **Holographic chiral symmetry breaking in a magnetic field**
5. (Homogeneous) holographic baryonic matter

• Sketch of the holographic calculation (page 1/3)

- D8-brane action

$$S = \underbrace{T_8 V_4 \int d^4x \int dU\, e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' F)}}_{\text{Dirac-Born-Infeld (DBI)}} + \underbrace{\frac{N_c}{24\pi^2} \int d^4x \int A_\mu F_{u\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}}_{\text{Chern-Simons (CS)}},$$

- deconfined geometry, $N_f = 1$

$$S = \mathcal{N} \int du \sqrt{u^5 + b^2 u^2} \sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2} + \frac{3\mathcal{N}}{2} b \int du\, (a_3 a_0' - a_0 a_3')$$

(dimensionless quantities, $a_\mu = \frac{2\pi\alpha'}{R} A_\mu$, $b = 2\pi\alpha' B$)

- chemical potential $\mu = a_0(\infty)$
- magnetic field in 3-direction $b = F_{12}(\infty)$
- $a_3(u)$ induced $\rightarrow$ anisotropic condensate $a_3(\infty) = \nabla\pi^0$

• Sketch of the holographic calculation (page 2/3)

- equations of motion:

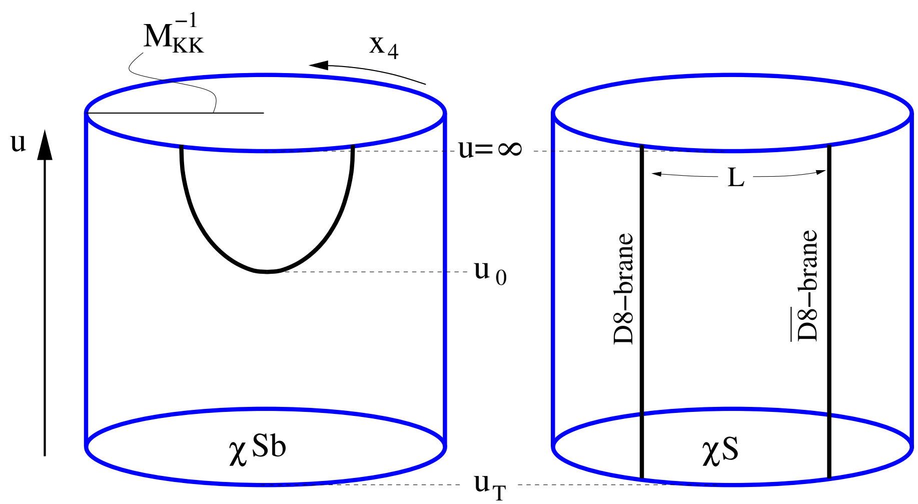
$$\partial_u\left(\frac{a_0'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_3'$$

$$\partial_u\left(\frac{f\,a_3'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_0'$$

$$\partial_u\left(\frac{u^3f\,x_4'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 0$$

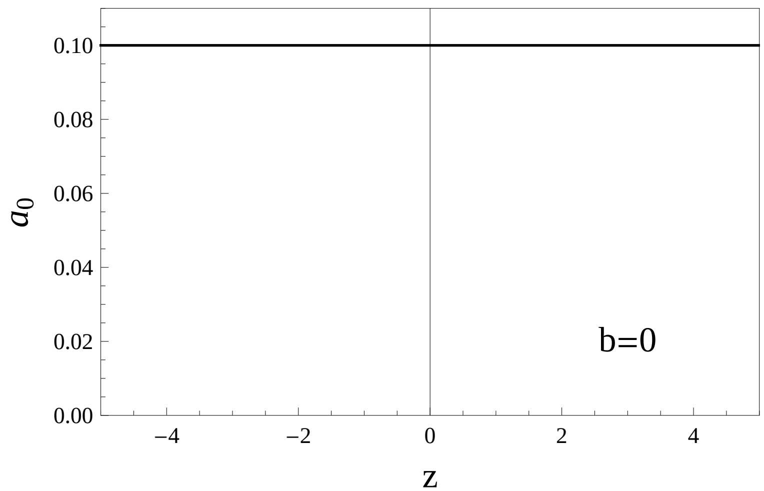
- to be solved for $a_0(u)$, $a_3(u)$, $x_4(u)$

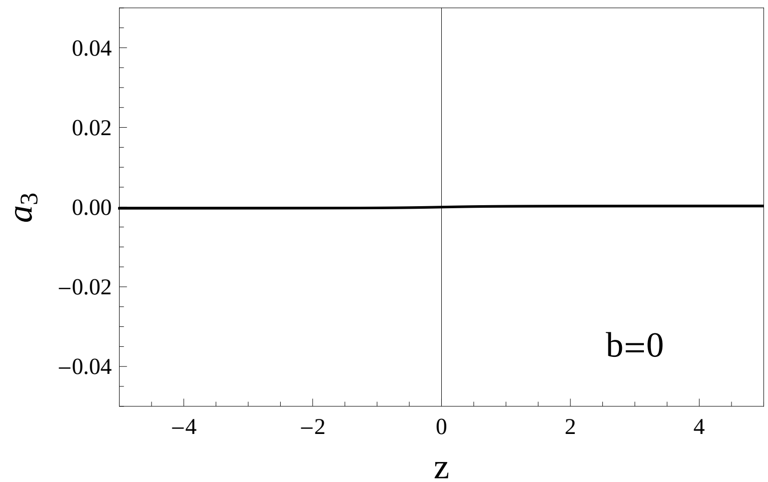
$$x_4(u) = \begin{cases} \text{const.} & \chi\text{S} \\ \text{nontrivial} & \chi\text{Sb} \end{cases}$$

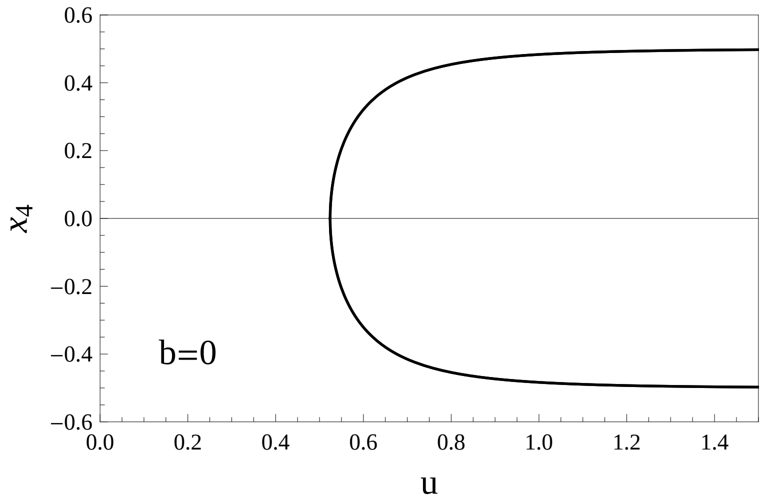


• Sketch of the holographic calculation (page 3/3)

- solutions of EoM; e.g., chirally broken phase, $u = (u_0^3 + u_0 z^2)^{1/3}$







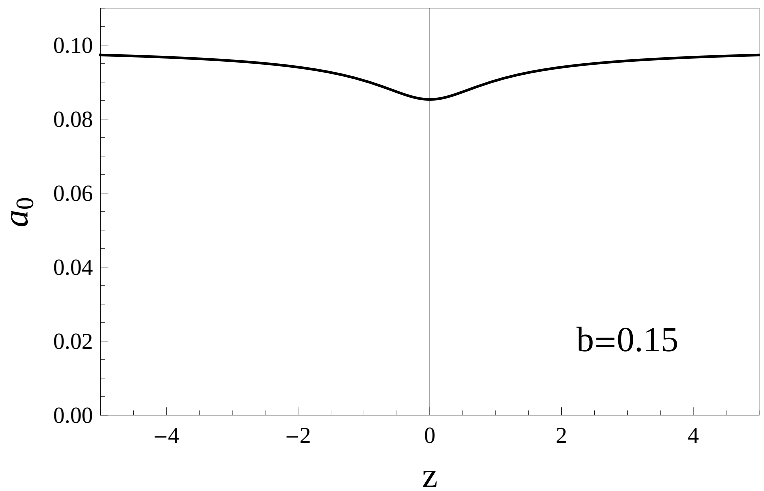
$\rightarrow$ insert solutions back into

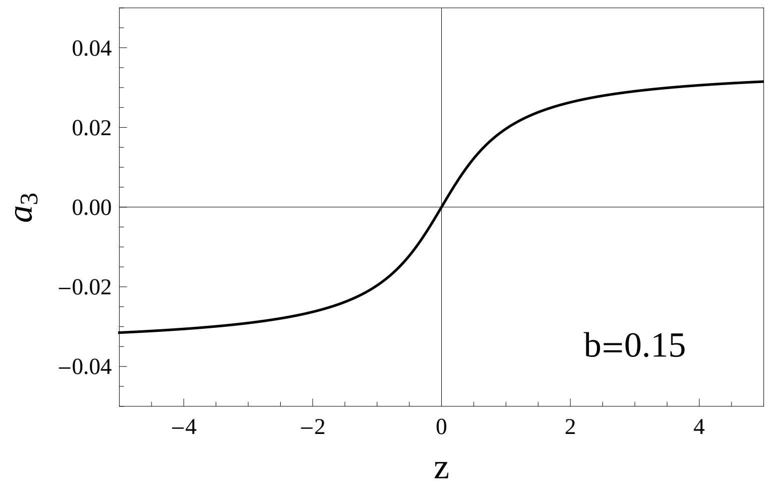
$$\Omega = \frac{T}{V} S_{\text{on-shell}}$$

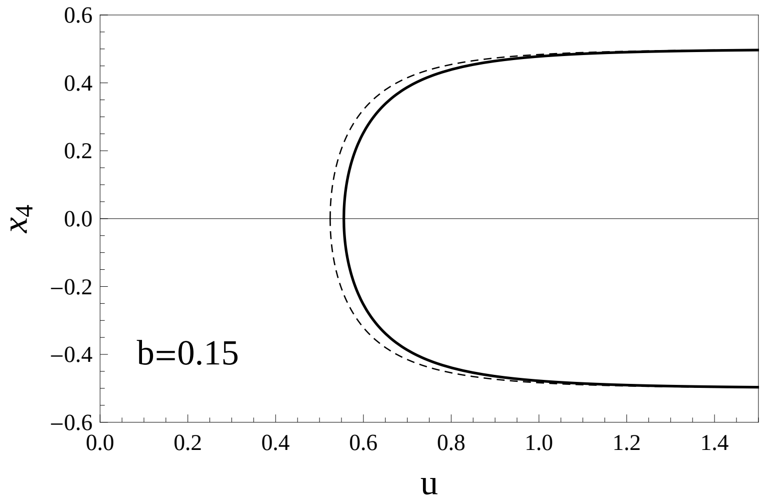
to compute
chiral phase transition

• Sketch of the holographic calculation (page 3/3)

• solutions of EoM; e.g., chirally broken phase, $u = (u_0^3 + u_0 z^2)^{1/3}$







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• Sketch of the holographic calculation (page 3/3)

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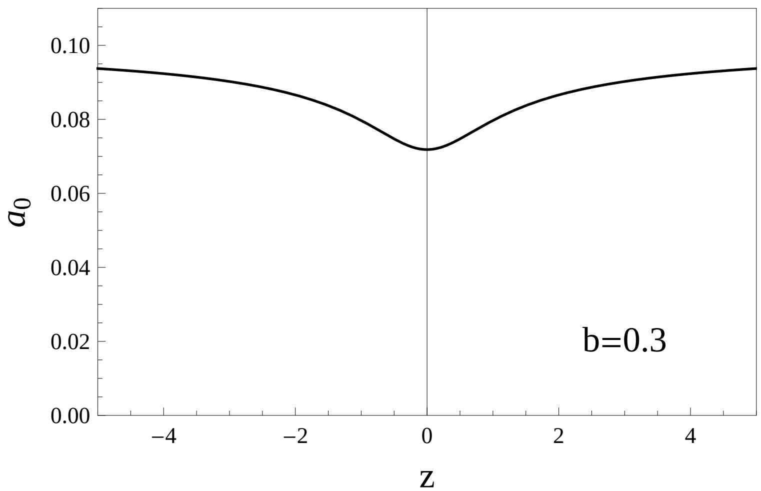
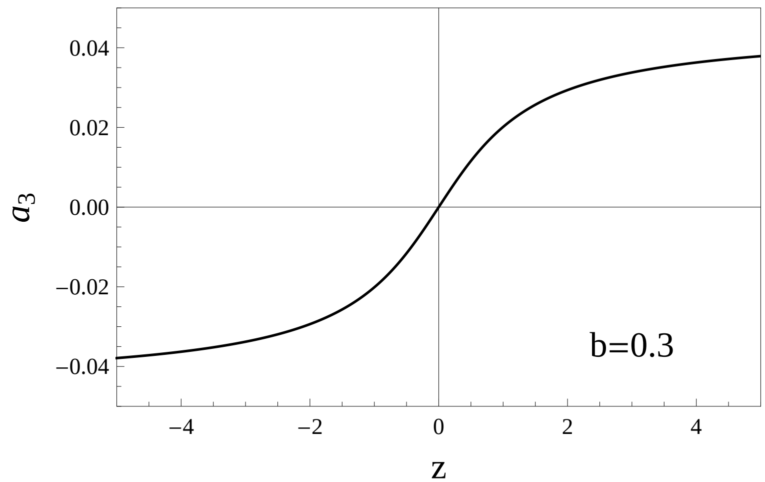
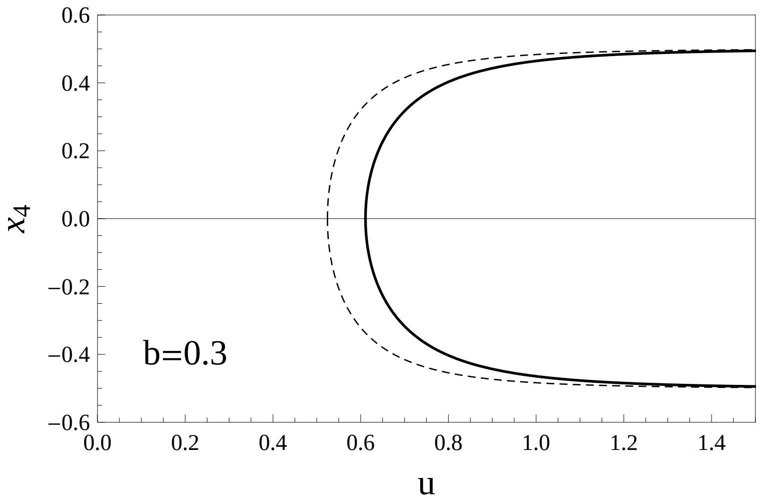






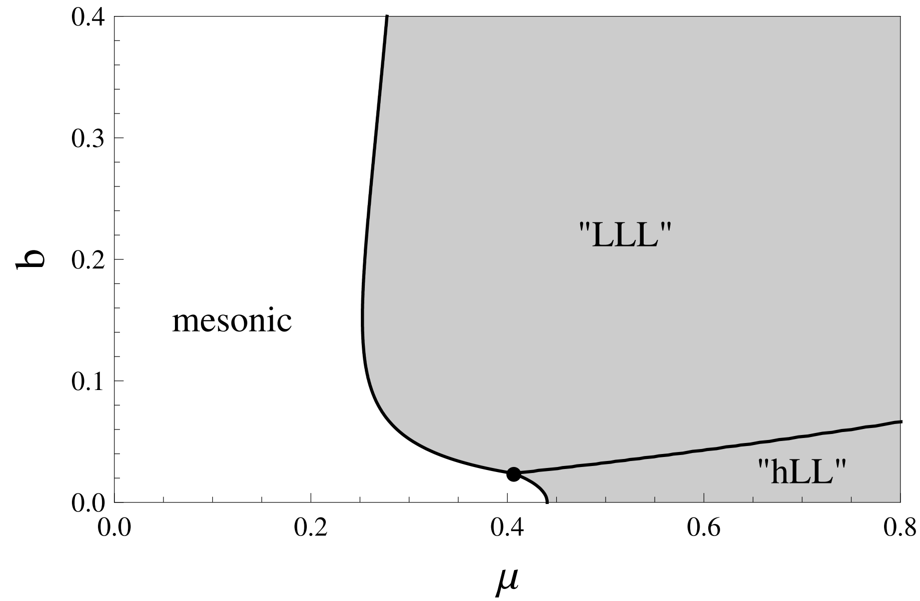
$\rightarrow$ insert solutions back into

$$\Omega = \frac{T}{V} S_{\mathrm{on-shell}}$$

to compute

chiral phase transition

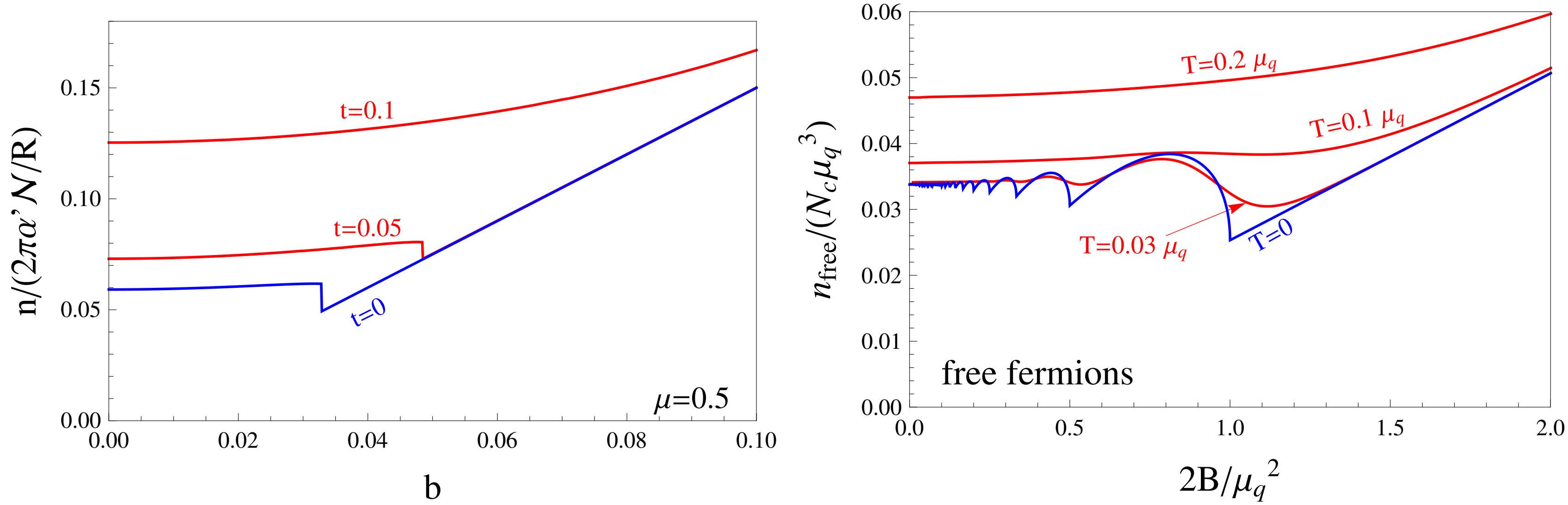
- **$T = 0$ phase diagram**



- Two main observations:
  - apparent Landau level transition
    G. Lifschytz, M. Lippert, PRD 80, 066007 (2009)
  - non-monotonic behavior of critical μ
    (doesn't magnetic catalysis suggest monotonic increase?)

• "LLL" in the Sakai-Sugimoto model

- compare density with free fermion system:



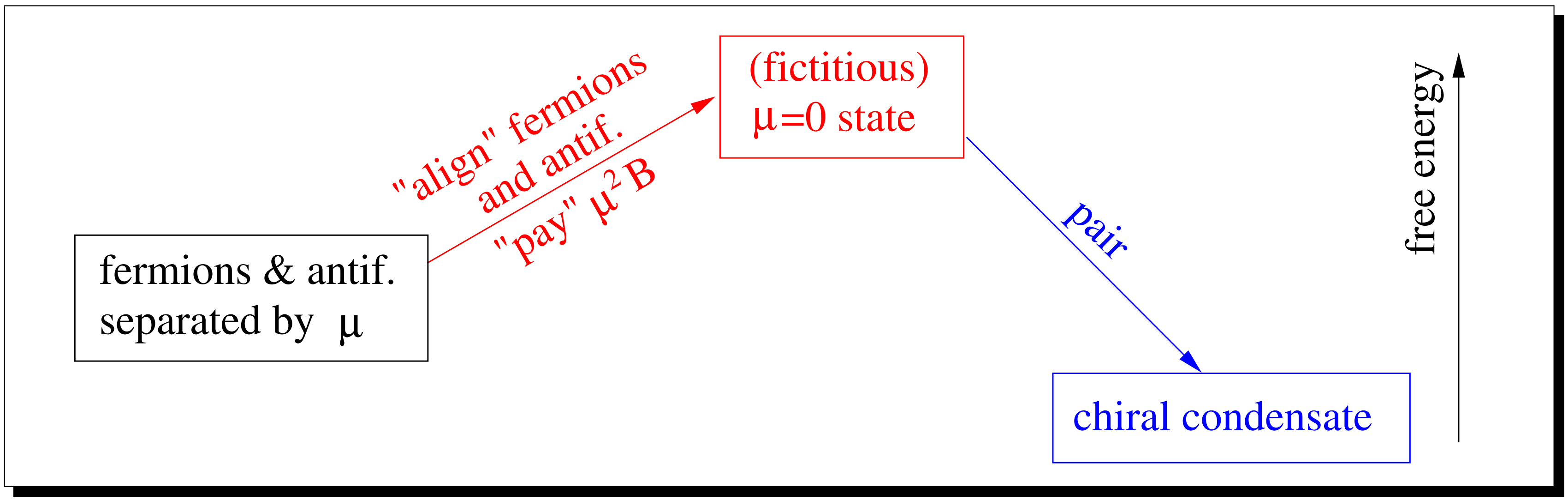
- no higher LL oscillations (expected due to strong coupling)
- linear behavior of n for large B exactly like for free fermions in LLL (all model parameters drop out!)

$$\boxed{n = \frac{\mu B}{2\pi^2}}$$

• Inverse magnetic catalysis (page 1/2)

Why does B *restore* chiral symmetry for certain μ?
("Inverse Magnetic Catalysis")

- chiral condensation (isotropic) at nonzero μ:



(analogous to Cooper pairing with mismatched Fermi surfaces)

- μ induces free energy *cost* for pairing; this cost depends on B!
- free energy *gain* from $\bar{\psi} - \psi$ pairing increases with B (magnetic catalysis)

• Inverse magnetic catalysis (page 2/2)

- this shows that inverse catalysis *can* happen
- whether it *does* happen, depends on details (and on coupling strength!)

NJL (weak coupling):

E. V. Gorbar *et al.*, PRC 80, 032801 (2009)

$$\Delta\Omega \propto B[\mu^2 - M(B)^2/2]$$

just like Clogston limit $\delta\mu = \frac{\Delta}{\sqrt{2}}$ in superconductivity

A. Clogston, PRL 9, 266 (1962)

B. Chandrasekhar, APL 1, 7 (1962)

→ **no inverse catalysis**

Sakai-Sugimoto:

large B:

$$\Delta\Omega \propto B[\mu^2 - 0.12\, M(B)^2]$$

→ **no inverse catalysis**

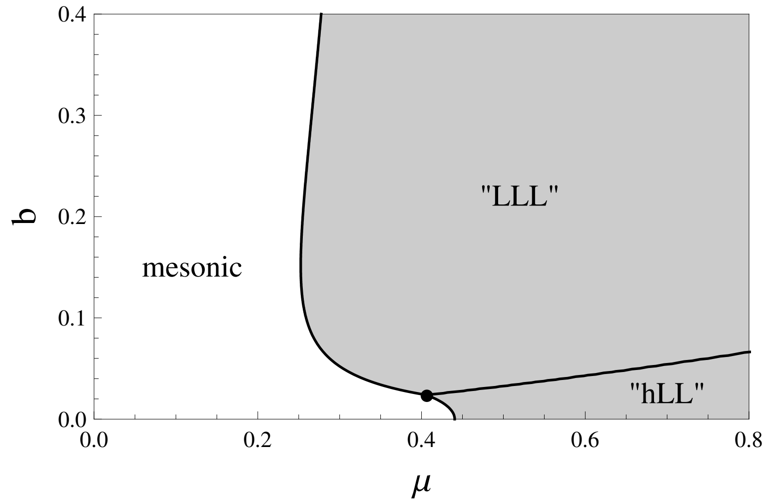
small B:

$$\Delta\Omega \propto \mu^2 B - \text{const} \times M(B)^{7/2}$$

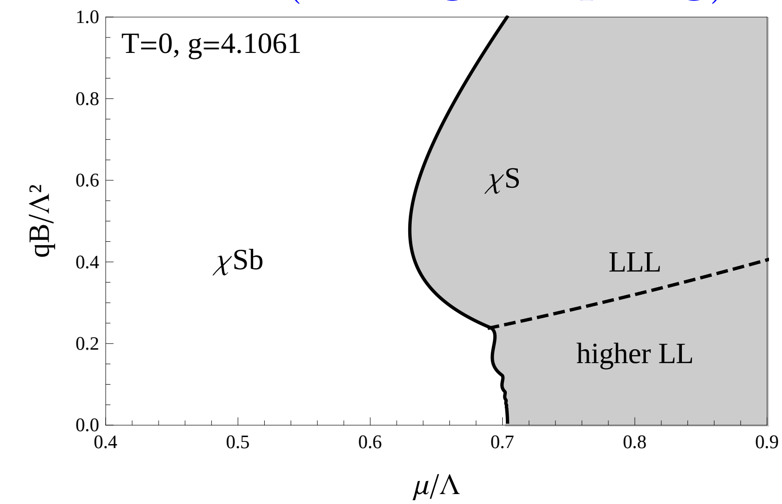
→ **inverse catalysis possible**

• Comparison with NJL calculation ($T = 0$)

Sakai-Sugimoto



NJL (strong coupling)



- NJL at sufficiently large g:
  - inverse magnetic catalysis just like in Sakai-Sugimoto!
- inverse magnetic catalysis in NJL and related models:

D. Ebert, K. G. Klimenko, M. A. Vdovichenko and A. S. Vshivtsev, PRD 61, 025005 (2000)

T. Inagaki, D. Kimura and T. Murata, Prog. Theor. Phys. 111, 371 (2004)

B. Chatterjee, H. Mishra and A. Mishra, PRD 84, 014016 (2011)

S. S. Avancini, D. P. Menezes, M. B. Pinto and C. Providencia, PRD 85, 091901 (2012)

J. O. Andersen and A. Tranberg, JHEP 1208, 002 (2012)

• Physical units

• original version of Sakai-Sugimoto model ($L = \frac{\pi}{M_{\rm KK}}$):

choose $M_{\rm KK} \simeq 949\,\text{MeV}$ and $\kappa \equiv \frac{\lambda N_c}{216\pi^3} \simeq 0.007$ to fit m_ρ and f_π

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 114, 1083 (2005)

($\rightarrow$ deconfinement temperature $T_c \simeq 150\,\text{MeV}$)

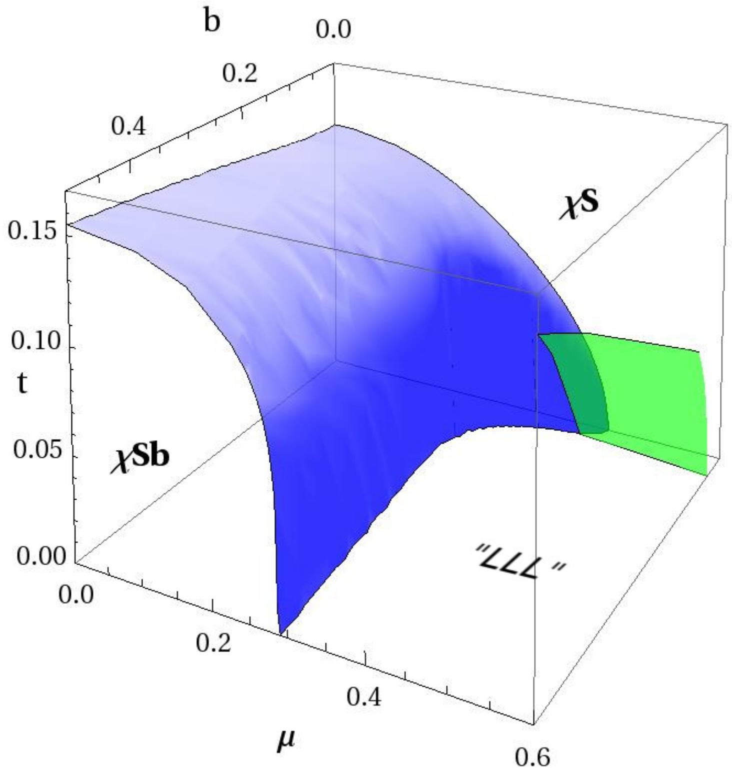
• here (non-asymptotic separation $L \ll \frac{\pi}{M_{\rm KK}}$):

$$\mu_q = \frac{R^3}{2\pi\alpha'}\frac{\mu\ell^2}{L^2}\,, \qquad T = \frac{t\ell}{L}\,, \qquad B = \frac{R^3}{2\pi\alpha'}\frac{b\ell^3}{L^3} \qquad (\ell = \frac{L}{R})$$

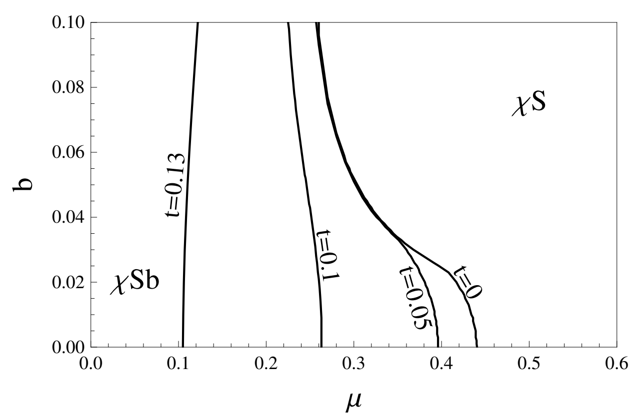
$$\Rightarrow B \simeq 5.1 \times 10^{19}\,\text{G}\left(\frac{\mu_{q,c}}{400\,\text{MeV}}\right)\left(\frac{T_c}{150\,\text{MeV}}\right) b\ell^3$$

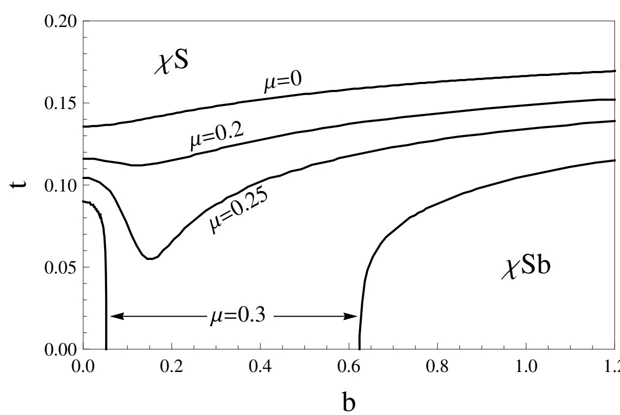
inverse magnetic catalysis reduces critical μ_q
from $\sim 400\,\text{MeV}$ ($B = 0$) to $\sim 230\,\text{MeV}$ ($B \simeq 1.0 \times 10^{19}\,\text{G}$)

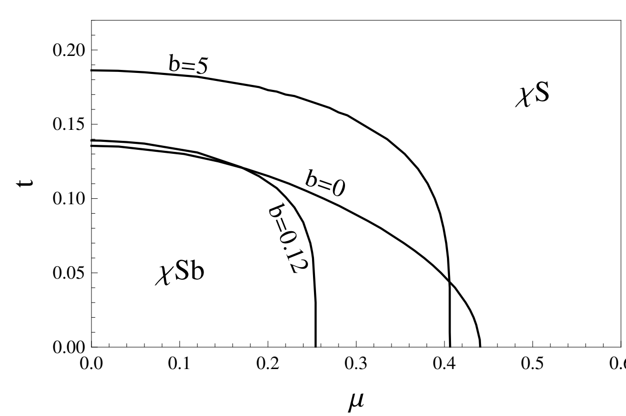
• Phase structure at nonzero temperature



blue: chiral phase transition
green: "LLL" transition



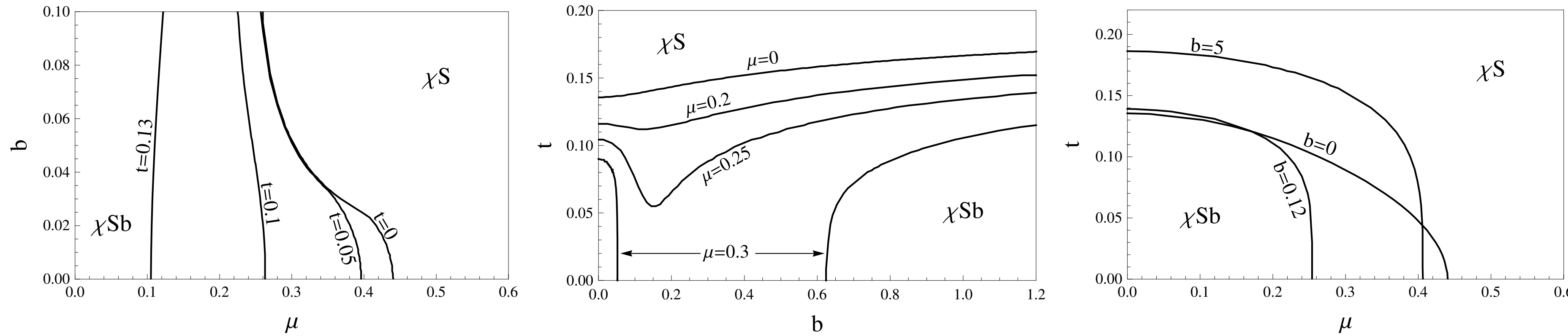




• Comparison with NJL calculation ($T \neq 0$)

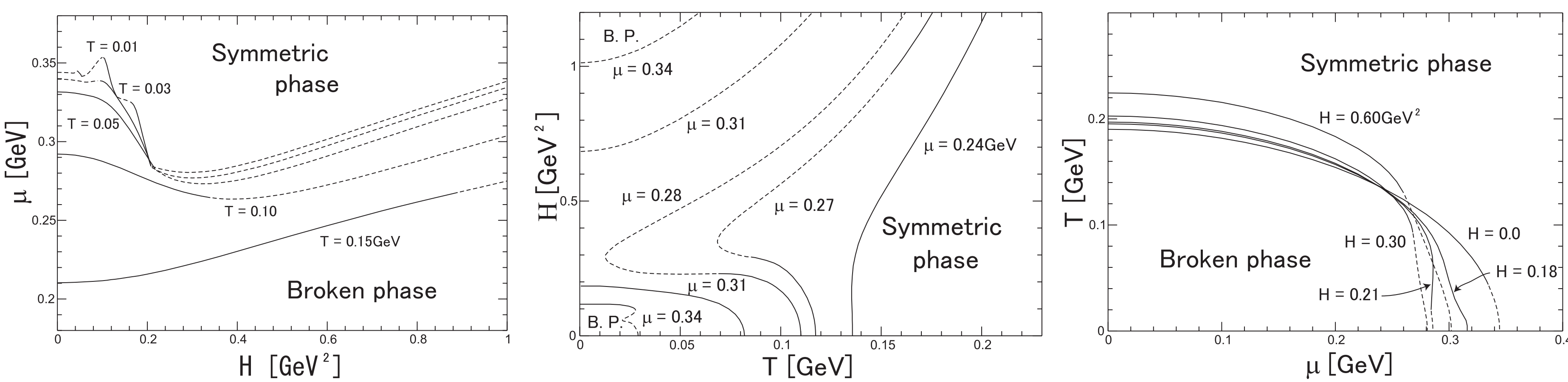
Sakai-Sugimoto:

F. Preis, A. Rebhan and A. Schmitt, JHEP 1103, 033 (2011)



NJL:

T. Inagaki, D. Kimura, T. Murata, Prog. Theor. Phys. 111, 371-386 (2004)



- **NJL vs. Sakai-Sugimoto (for small L)**

	NJL	**Sakai-Sugimoto (small L)**
MC	$\surd$	$\surd$
IMC at finite μ	$\surd$	$\surd$
chiral trans. $(m = 0)$	1st & 2nd	1st
$m \neq 0$	easy	difficult
LL oscillations	$\surd$	–
LLL	$\surd$	$\surd$ (indirect)
baryons	difficult	$\surd$ (large N_c)

• Summary part 4

- physics:
  - for dense matter, B has an unexpected effect on the chiral phase transition $\longrightarrow$ inverse magnetic catalysis
  - for compact star physics: if there is any effect of B on the phase transition, then it favors quark matter

- theory:
  - the Sakai-Sugimoto model interpolates between large-N_c QCD and an NJL-like model (asymptotic separation L being the interpolation parameter)

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1. Setting the stage: equilibrium phases of QCD
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5. **(Homogeneous) holographic baryonic matter**

• Homogeneous baryonic matter in Sakai-Sugimoto

- baryons in AdS/CFT: wrapped D-branes with N_c strings

  E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)

- baryons in Sakai-Sugimoto:
  - D4-branes wrapped on S^4
  - equivalently: instantons on D8-branes ($\longrightarrow$ skyrmions)

  T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

  H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)

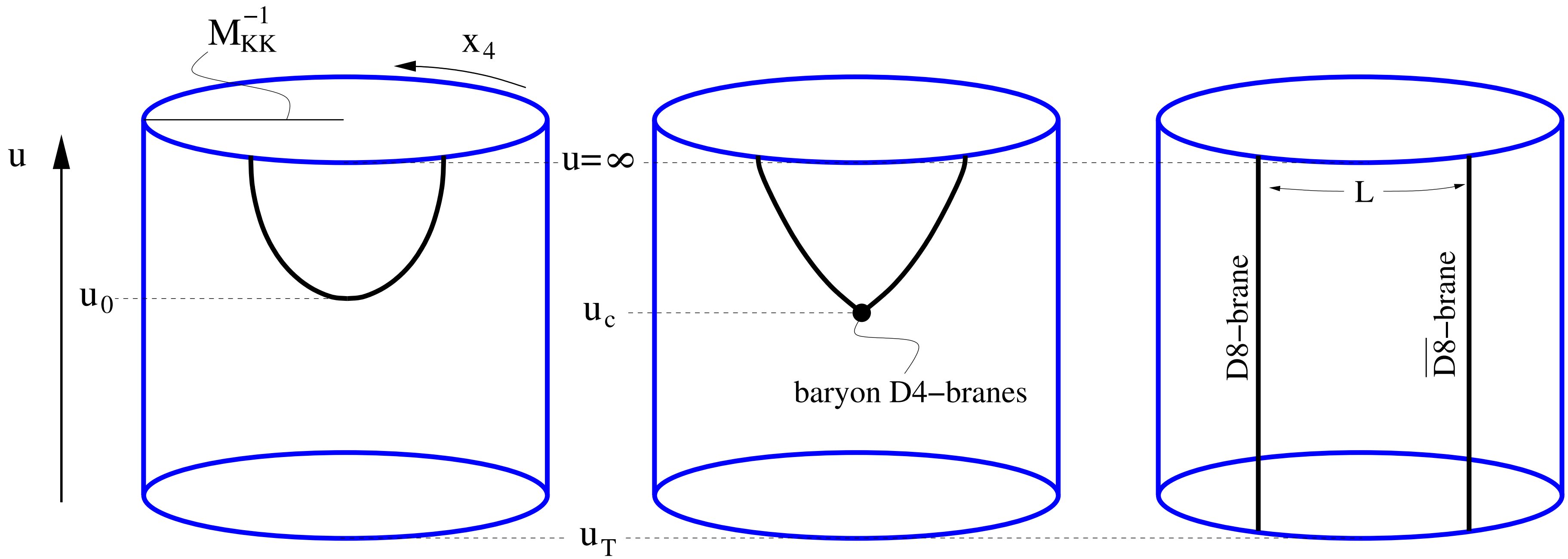
- pointlike approximation for $N_f = 1$:

  O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)

$$S = S_{\text{from above}} + \underbrace{N_4 T_4 \int d\Omega_4 d\tau\, e^{-\Phi}\sqrt{\det g}}_{\propto\, n_4 N_c M_q} + \underbrace{\frac{N_c}{8\pi^2}\int_{\mathbb{R}^4\times\mathcal{U}} A_0\, \mathrm{Tr}\, F^2}_{\propto\, n_4 \int A_0(u)\delta(u-u_c)}$$

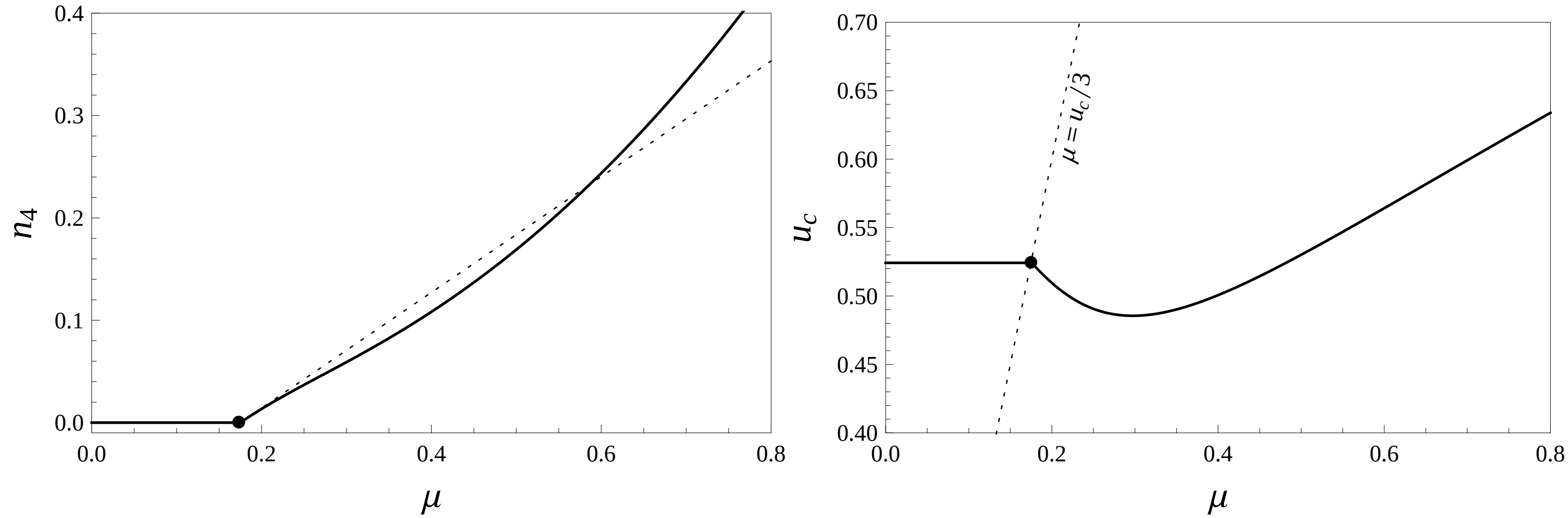
(n_4 baryon density, M_q constituent quark mass, u_c location of D4-branes)

- **Compare free energy of three phases**



mesonic	**baryonic**	**quark matter**
χS broken	χS broken	χS restored
$n_B \sim b\,\nabla\pi^0$	$n_B \sim n_4 + b\,\nabla\pi^0$	$n_B \sim N_c n_q$
$M_q \sim u_0$	$M_q \sim \dfrac{u_c}{3}$	$M_q = 0$

• Onset of baryons ($B = T = 0$)



- second-order transition at $\mu_q = M_q$
- linear behavior of baryon density close to onset

$$n_B(b=0) = \frac{2M_q^2}{0.17\,\lambda\,\frac{\pi/M_{\rm KK}}{L}}(\mu_q - M_q) + \ldots$$

- compare to ϕ^4 model: $n = \dfrac{2m^2}{\lambda}(\mu - m) + \ldots$

$\Rightarrow$ bosonic behavior of our large-N_c baryons

• Baryon onset in the real world (page 1/2)

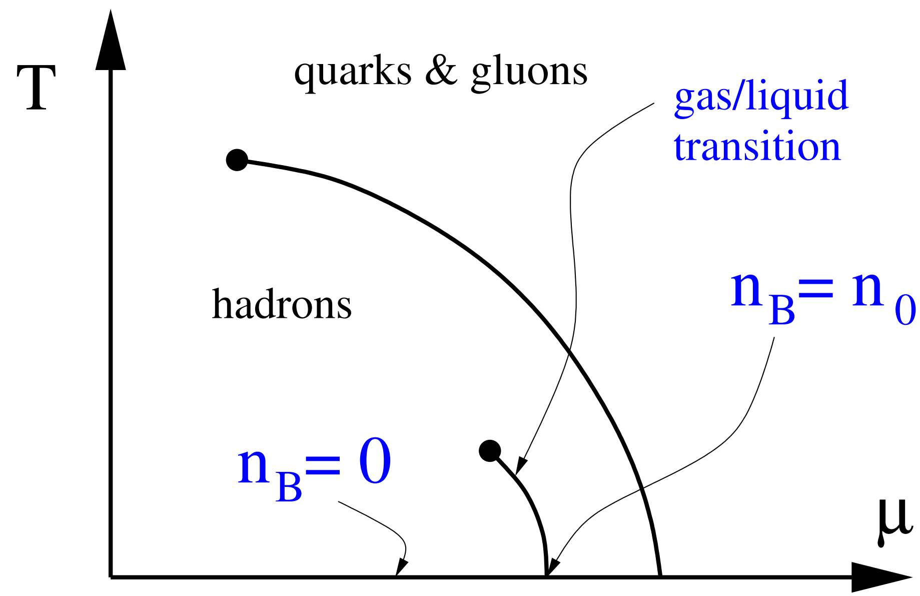
- nuclear matter onset at

$$\mu_B = M_B - E_{\rm bind}$$

  is first order!

- nuclear ground state density

$$n_0 \simeq 0.15\,\mathrm{fm}^{-3}$$



- see for instance Walecka model:

$$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi + g_\sigma\bar\psi\sigma\psi - g_\omega\bar\psi\gamma^\mu\omega_\mu\psi + \mathcal{L}_{\sigma,\omega}$$

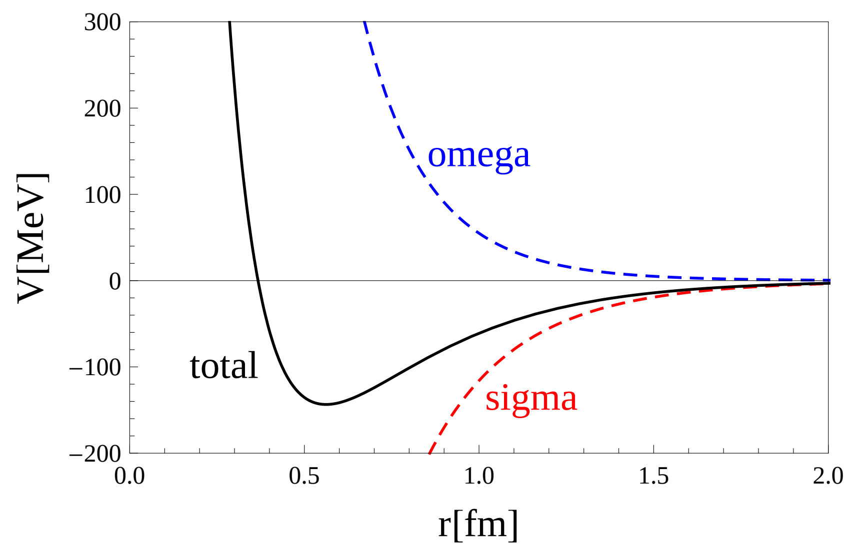
- attractive and repulsive interaction through sigma and omega exchange

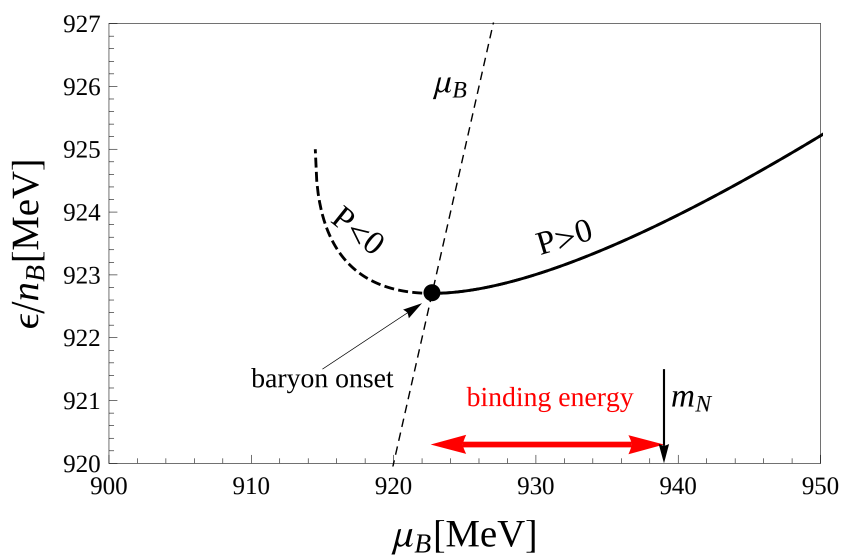
• Baryon onset in the real world (page 2/2)

- classical potential

$$V(r) = \frac{g_\omega^2}{4\pi}\frac{e^{-m_\omega r}}{r} - \frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma r}}{r}$$

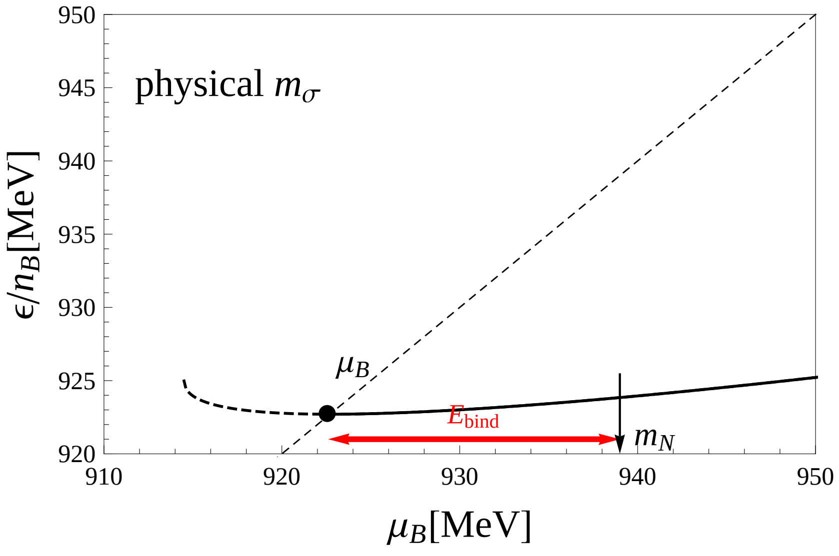
- nucleons "want" to sit $\sim 0.5\,\mathrm{fm}$ apart





- binding energy $E_{\mathrm{bind}} \simeq 16\,\mathrm{MeV}$
- nuclear matter is stable at $P = 0$
- onset with μ_B is first order

• Why is holographic onset second order?



- σ = quark-antiquark $\bar q\, q$

$$\Rightarrow \quad m_\sigma \propto N_c^0$$

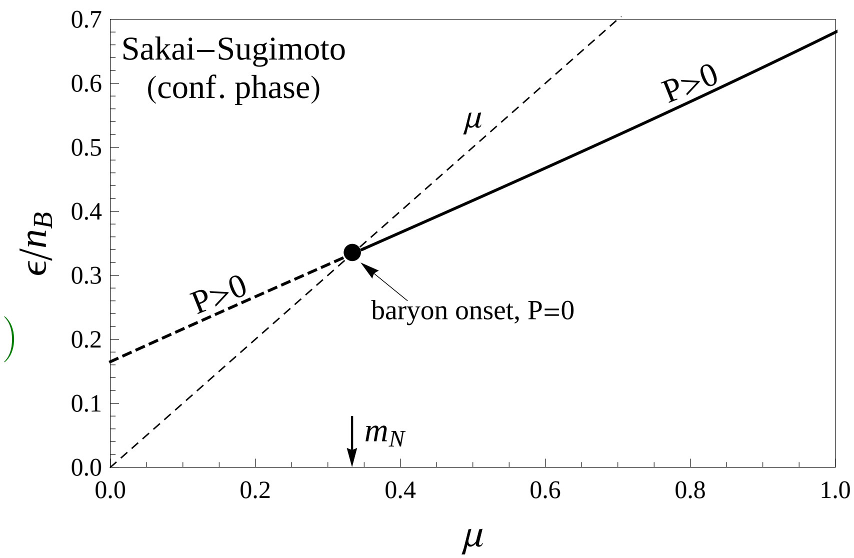
- σ = tetraquark $\underbrace{\bar q\,\bar q}_{N_c-1}\ \underbrace{q\,q}_{N_c-1}$

$$\Rightarrow \quad m_\sigma \propto 2(N_c-1) \sim N_c$$

- no $E_{\rm bind}$ in Sakai-Sugimoto: large-N_c effect due to heavy σ?

V. Kaplunovsky, J. Sonnenschein, JHEP 1105 (2011)

L. Bonanno, F. Giacosa, NPA 859, 49-62 (2011)



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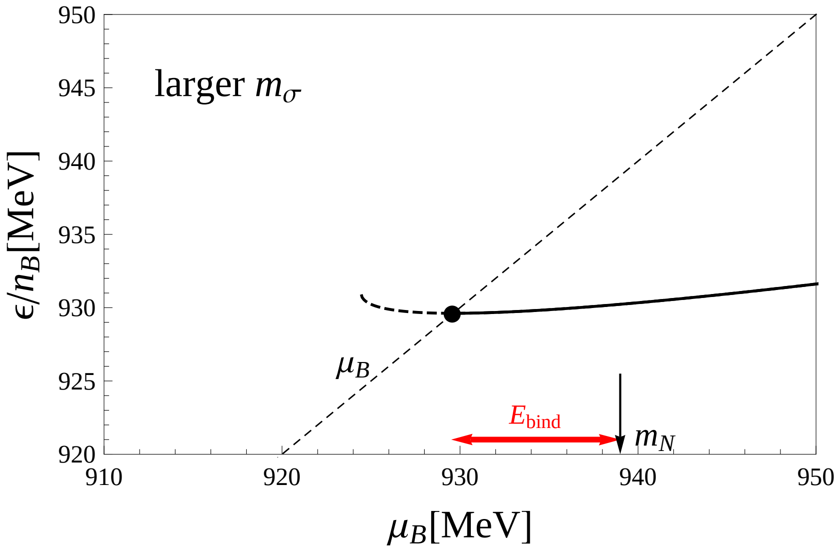

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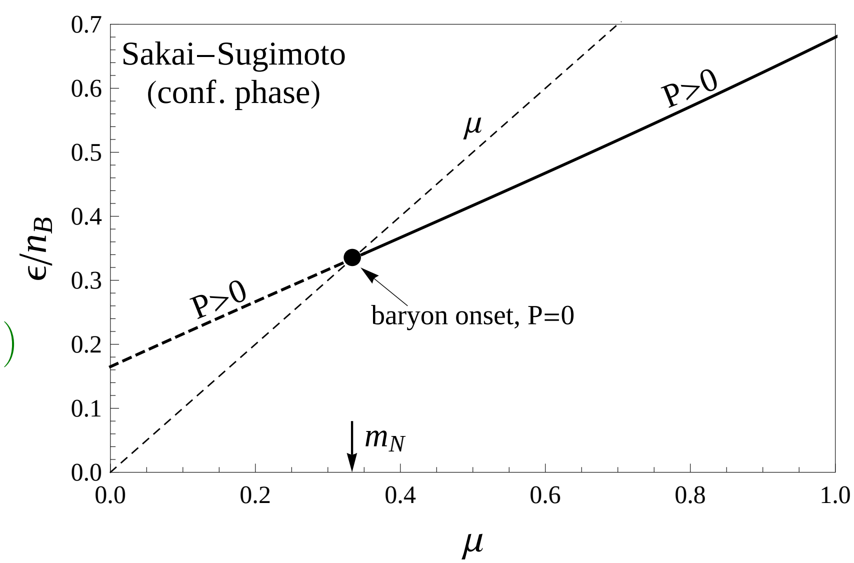

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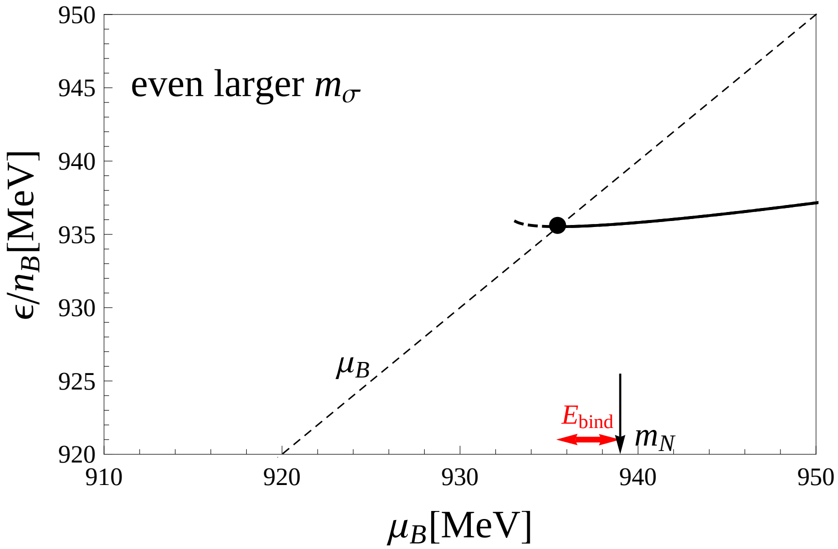

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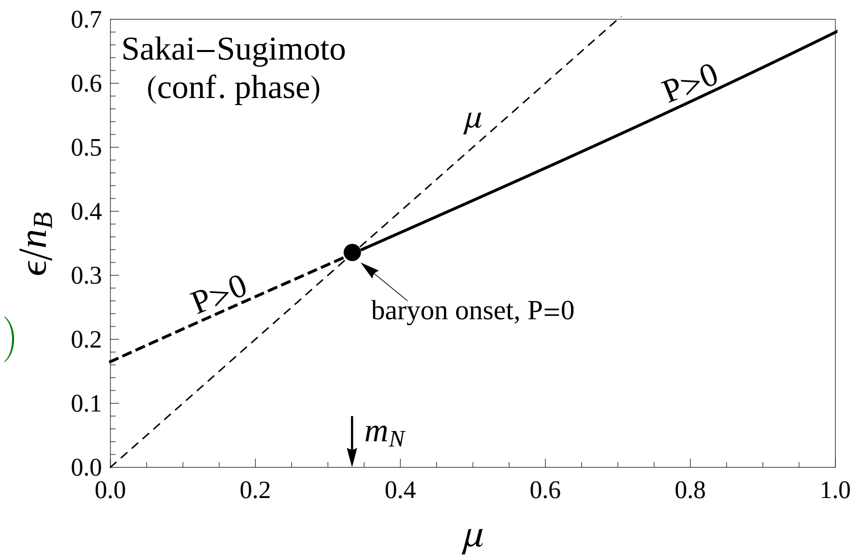
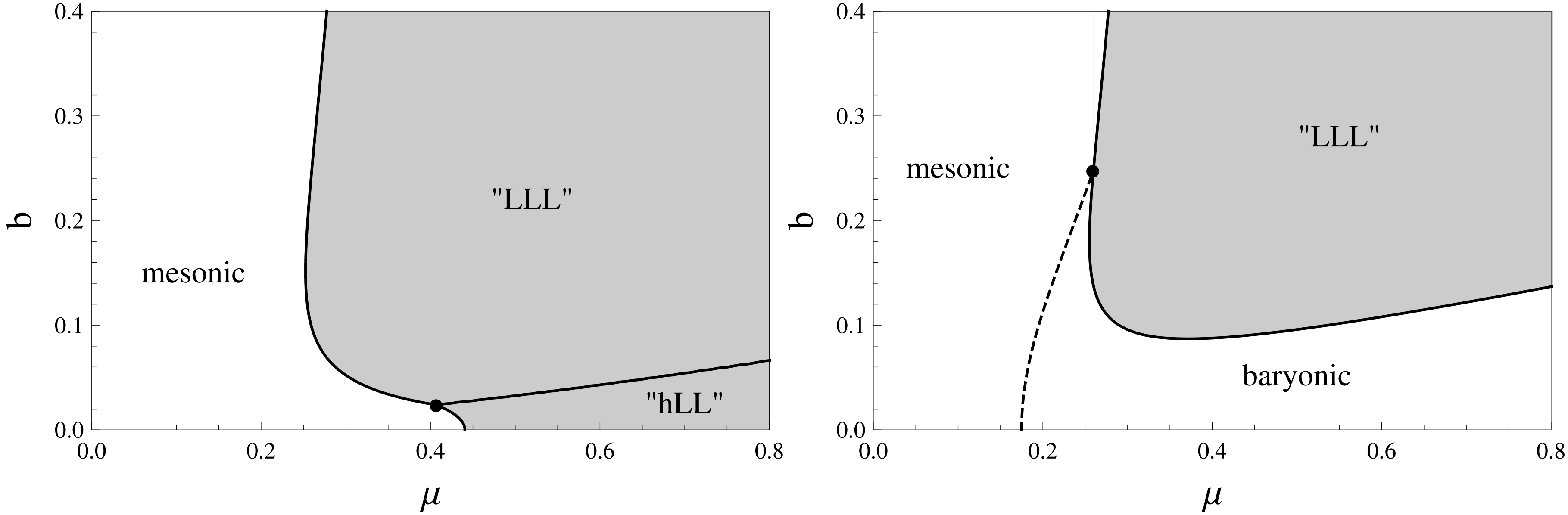


• Effect of baryons on $T = 0$ phase diagram

ignoring baryonic matter including baryonic matter



- small b: baryonic matter prevents the system from restoring chiral symmetry
- baryon onset line intersects chiral phase transition line $\longrightarrow$ large b: mesonic matter superseded by quark matter
- with baryonic matter, IMC plays an even more prominent role in the phase diagram

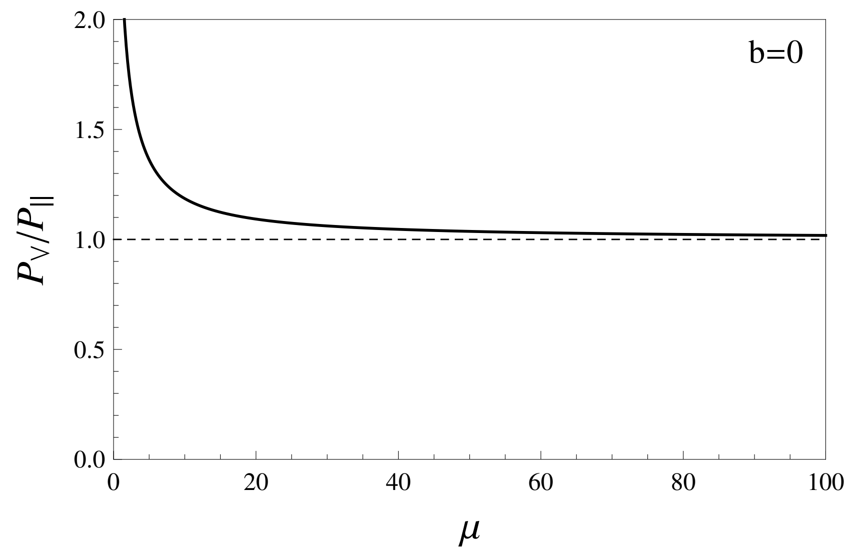
• Asymptotic baryonic matter

- For $\mu \to \infty$ baryonic and quark matter become indistinguishable:

$$P_{\vee}(b=0) \; = \; p\,\mu^{7/2} + \mathcal{O}(\mu^{5/2})$$

$$P_{||}(b=0) \; = \; p\,\mu^{7/2}$$

(where $p \equiv \frac{2}{7}\mathcal{N}\left[\frac{\Gamma\left(\frac{3}{10}\right)\Gamma\left(\frac{6}{5}\right)}{\sqrt{\pi}}\right]^{-5/2}$)



- is absence of chiral transition artifact of pointlike baryons?
  $\longrightarrow$ overlap of baryons shifted to $\mu \to \infty$
- should redo analysis with finite-size baryons
  (here: instantons, $N_f > 1$)

- **Summary part 5**

- large-N_c baryons are very different
  from real-world ($N_c = 3$) baryons
- this makes holographic description of baryonic matter
  unrealistic (if $N_c \to \infty$ limit is kept) or
  very challenging (if finite N_c calculation is attempted)
- with holographic baryonic matter
  the phase diagram changes dramatically

- **Conclusions: what can we learn from holography?**
  (in the given context of equilibrium phases of QCD)

- "Minimalistic" point of view:
  - consider Sakai-Sugimoto as just another model like NJL, PNJL, sigma model, ...
  - try to squeeze out model-independent physics
    (here: observe IMC, find physical picture which suggests model indep.)

- More "ambitious" point of view:
  - with AdS/CFT we have a "microscopic", reliable description of strongly coupled systems!
  - however, all systems considered so far are unrealistic
    (e.g., Sakai-Sugimoto dual to QCD at best for large-N_c and in inacc. limit)
  - try to learn about strongly coupled systems as such
    (absence of quasiparticles, viscosity bound, ...)
  - work hard to find gravity dual of QCD