

# The Spectral Analysis of the Quark-Gluon Plasma

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- Calculate an observable quantity in statistical QCD;
- measure that quantity in nuclear collision experiments;
- see if the two agree.

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**How?**

- Calculate an observable quantity in statistical QCD;
- measure that quantity in nuclear collision experiments;
- see if the two agree.

$\exists$  many interesting phenomenological model approaches to help our understanding.

**But** there is no free lunch!

## How hot is the Quark-Gluon Plasma?

What can we use as QGP Thermometer?

hadron abundances  $\Rightarrow$  hadronization stage of QGP

$\exists$  probe of earlier, hot QGP, “smoking gun”?

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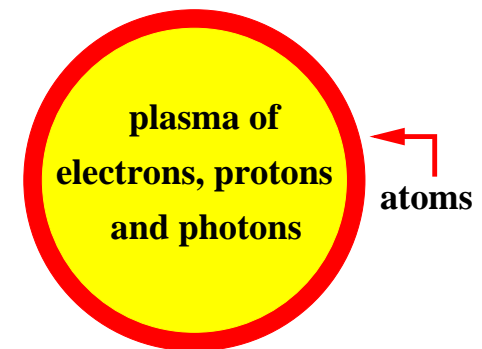
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$\exists$  probe of earlier, hot QGP, “smoking gun”?

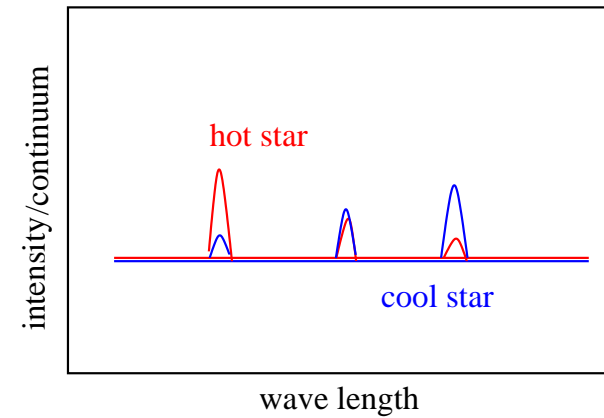
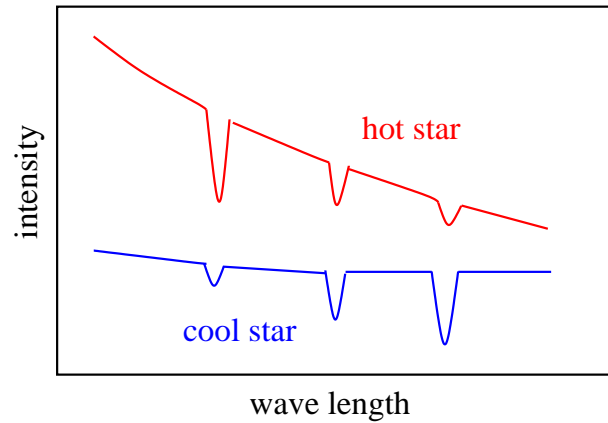
better look at a **shining star** ★  
than for a **smoking gun**...

How does one measure temperatures of stellar interiors?

photons from plasma core are emitted,  
absorbed by atoms in crust, lead to  
absorption lines in stellar spectra







- absorption lines indicate presence of atomic species
- absorption strength gives temperature of stellar interior

Conjecture: **Quarkonia** are the spectral lines of the QGP

Matsui & HS, 1986

$\exists$  no crust of QGP, but  $\exists$  early hard production of quarkonia

they're there when QGP appears, and its effect on different quarkonium states tells how hot the QGP is.

## Contents

1. Quarkonia are very **unusual** hadrons
2. Quarkonia **melt** in a hot QGP
3. Quarkonium production is **suppressed** in nuclear collisions
4. Quarkonia can be **created** at QGP hadronization

# 1. Quarkonia are very unusual hadrons

**heavy** quark ( $Q\bar{Q}$ ) bound states **stable** under strong decay

- **heavy**:  $m_c \simeq 1.2 - 1.4$  GeV,  $m_b \simeq 4.6 - 4.9$  GeV
- **stable**:  $M_{c\bar{c}} \leq 2M_D$  and  $M_{b\bar{b}} \leq 2M_B$

What is “**usual**”?

- light quark ( $q\bar{q}$ ) constituents
- hadronic size  $\Lambda_{\text{QCD}}^{-1} \simeq 1$  fm, independent of mass
- loosely bound,  $M_\rho - 2M_\pi \gg 0$ ,  $M_\phi - 2M_K \simeq 0$
- relative production abundances  $\sim$  energy independent, statistical: at large  $\sqrt{s}$ , rate  $R_{i/j} \sim$  phase space at  $T_c$
- $(dN_{\text{ch}}/dy) \sim \ln s$

Quarkonia: heavy quarks  $\Rightarrow$  non-relativistic potential theory

Jacobs et al. 1986

Schrödinger equation  $\left\{ 2m_c - \frac{1}{m_c} \nabla^2 + V(r) \right\} \Phi_i(r) = M_i \Phi_i(r)$

with confining (“Cornell”) potential  $V(r) = \sigma r - \frac{\alpha}{r}$

state	$J/\psi$	$\chi_c$	$\psi'$	$\Upsilon$	$\chi_b$	$\Upsilon'$	$\chi'_b$	$\Upsilon''$
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
$\Delta E$ [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
$\Delta M$ [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
radius [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

$(m_c = 1.25 \text{ GeV}, m_b = 4.65 \text{ GeV}, \sqrt{\sigma} = 0.445 \text{ GeV}, \alpha = \pi/12)$

excellent account of full quarkonium spectroscopy:

spin-averaged masses , binding energies, radii.

masses to better than 1 %...

NB:

recent work on field theoretical quarkonium studies,

NRQCD

Brambilla & Vairo 1999, Brambilla et al. 2000

⇒ quarkonia are unusual

– very small, mass-dependent size:

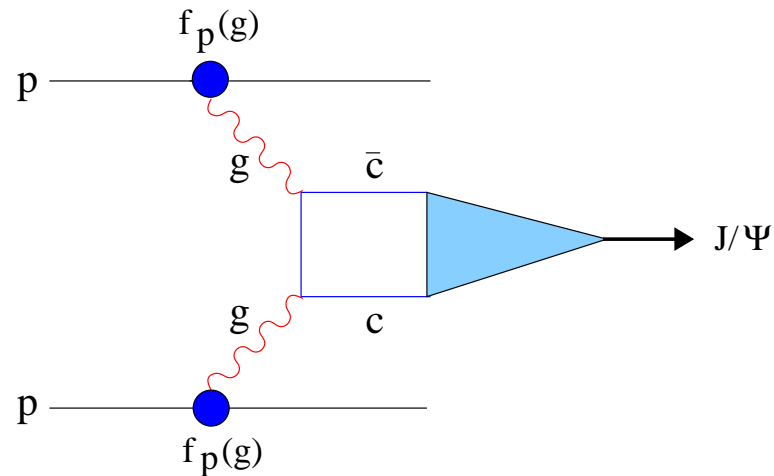
$$r_{J/\psi} \simeq 0.25 \text{ fm}, \quad r_{\Upsilon} \simeq 0.14 \text{ fm} \quad \ll \quad \Lambda_{\text{QCD}}^{-1} \simeq 1 \text{ fm}$$

– very tightly bound:

$$\begin{aligned} 2M_D - M_{J/\psi} &\simeq 0.64 \text{ GeV} \\ 2M_B - M_{\Upsilon} &\simeq 1.10 \text{ GeV} \end{aligned} \quad \gg \quad \Lambda_{\text{QCD}} \simeq 0.2 \text{ GeV}$$

## primary production via partonic interaction dynamics

Einhorn & Ellis 1975, Baier & Rückl 1983, Lansberg 2006



given parton distribution functions from DIS,  
 $c\bar{c}$  production is perturbatively calculable (cum grano salis)

$J/\psi$  binding is not, but it is independent of collision energy:

$$R[(J/\psi)/c\bar{c}] \sim |\phi_{J/\psi}(0)|^2 \neq f(s)$$

results for/from elementary collisions:

- $(dN_{c\bar{c}}/dy) \sim s^a$
- $(dN_{\text{ch}}/dy) \sim \ln s$
- $N_{c\bar{c}}/N_{\text{ch}}$  grows with collision energy      compare  $[N_{s\bar{s}}/N_{\text{ch}}]$

⇒ heavy flavor production is dynamical and not statistical

- $(dN_{J/\psi}/dy)/(dN_{c\bar{c}}/dy) \simeq 0.02$ ,      compare  $[N_{\rho}/N_{\text{ch}}]$   
factor 10 bigger than ratio of statistical weights at  $T_c$   
much more hidden charm than statistically predicted
- $(dN_{\psi'}/dy)/dN_{J/\psi}/dy) \simeq 0.2$ ,      compare  $[N_{\rho}/N_{\omega}]$   
factor five bigger than ratio of statistical weights at  $T_c$   
ratios of states  $\sim$  wave functions, not Boltzmann factors

⇒ quarkonium binding is dynamical and not statistical

Quarkonium production in elementary collisions: no medium  
What happens to quarkonia in hot strongly interacting media?

## 2. Quarkonia melt in a hot QGP

Matsui & HS 1986, Karsch et al. 1988

- QGP consists of deconfined color charges, hence  
 $\exists$  color screening for  $Q\bar{Q}$  state
- screening radius  $r_D(T)$  decreases with temperature  $T$
- if  $r_D(T)$  falls below binding radius  $r_i$  of  $Q\bar{Q}$  state  $i$ ,  
 $Q$  and  $\bar{Q}$  cannot bind, quarkonium  $i$  cannot exist
- quarkonium dissociation points  $T_i$ , from  $r_D(T_i) = r_i$ ,  
specify temperature of QGP

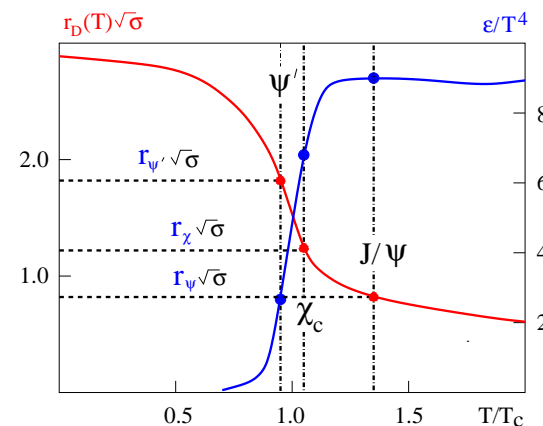


Color screening  $\Rightarrow$  binding **weaker** and of **shorter range**

when force range/screening radius become less than binding radius,  $Q$  and  $\bar{Q}$  cannot “see” each other

$\Rightarrow$  quarkonium dissociation points

determine temperature  $\Rightarrow$  energy density of medium

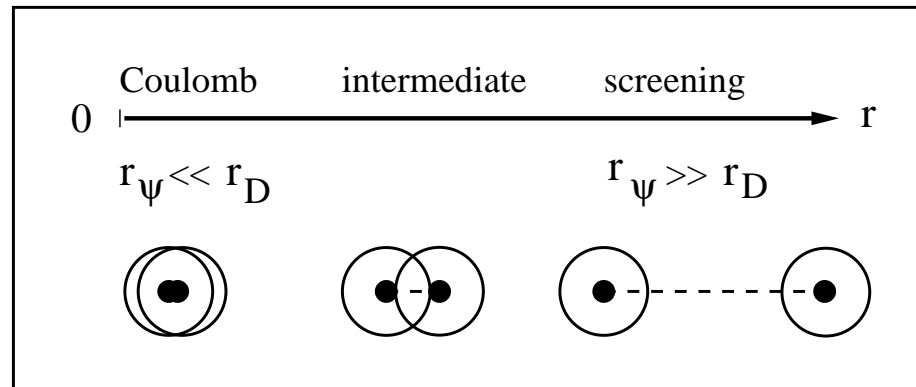


How to calculate quarkonium dissociation temperatures?

- determine heavy quark potential  $V(r, T)$  in finite temperature QCD, solve Schrödinger equation
- calculate in-medium quarkonium spectrum  $\sigma(\omega, T)$  directly in finite temperature lattice QCD

Consider static  $Q\bar{Q}$  pair in QGP above  $T_c$ , at separation  $r$

$\exists$  three interaction ranges,  
depending on  $Q\bar{Q}$   
separation distance



- $r_{J/\psi} \ll r_D(T)$ : quarkonium does not see medium
- $r_{J/\psi} \gg r_D(T)$ :  $Q$  does not see  $\bar{Q}$
- $r_{J/\psi} \sim r_D(T)$ : complex interactions

How to calculate  $Q\bar{Q}$  potential?

- Heavy Quark Studies in Finite Temperature QCD

Hamiltonian  $\mathcal{H}_Q$  for QGP with color singlet  $Q\bar{Q}$  pair:

$$F_Q(r, T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_Q/T\}$$

Hamiltonian  $\mathcal{H}_0$  for QGP without  $Q\bar{Q}$  pair:

$$F_0(T) = -T \ln \int d\Gamma \exp\{-\mathcal{H}_0/T\}$$

study free energy difference  $F(r, T) = F_Q(r, T) - F_0(T)$

internal energy difference  $U(r, T)$  & entropy difference  $S(r, T)$

$$U(r, T) = -T^2 \left( \frac{\partial [F(r, T)/T]}{\partial T} \right) = F(r, T) + TS(r, T)$$

relation to potential?  $V = U$  or  $V = F$  or mixture?

- weakly interacting plasma (QED, perturbative QCD)

Laine et al. 2007, Beraudo et al. 2008, Brambilla et al. 2008, Escobedo & Soto 2008, Burnier et al. 2009

real-time propagator of  $Q\bar{Q}$  pair in medium

$$V_w(r, T) = -\alpha \left[ \mu(T) - \frac{1}{r} e^{-\mu(T)r} \right]$$

with  $\mu(T) = 1/r_D(T) \sim \alpha T$

imaginary-time propagator of  $Q\bar{Q}$  pair in medium

$$F_w(r, T) = -\alpha \left[ \mu(T) - \frac{1}{r} e^{-\mu(T)r} \right]$$

in perturbative limit, potential (real part) is free energy

entropy  $TS_w(r, T) = -\alpha \mu(T) \left[ 1 - e^{-\mu(T)r} \right]$

internal energy  $U_w(r, T) = -\alpha \left[ \mu(T) - \frac{1}{r} \right] e^{-\mu(T)r}$

large distance limit (screening regime)

$$F_w(\infty, T) = -TS_w(\infty, T) = -\alpha\mu; \quad U_w(\infty, T) = 0$$

( $\alpha\mu/2$  is “mass” of polarization cloud)

short distance limit (Coulomb regime)

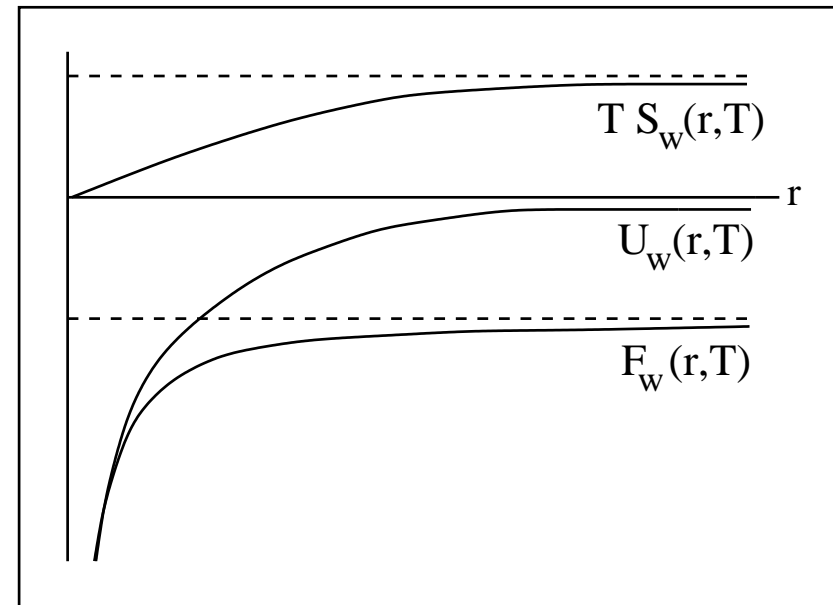
$$F_w(r, T) = U_w(r, T) = -\frac{\alpha}{r}$$

$$TS_w(r, T) \rightarrow 0$$

melting process:

work done to separate  $Q\bar{Q}$   
is converted into entropy

overall energy balance = 0



so far: perturbative limit  $\sim$  weakly interacting plasma  
 (Debye-Hückel theory, slightly non-ideal gas)

QCD: very high  $T \gg \Lambda_{\text{QCD}}$  and/or very small  $r \ll \Lambda_{\text{QCD}}^{-1}$

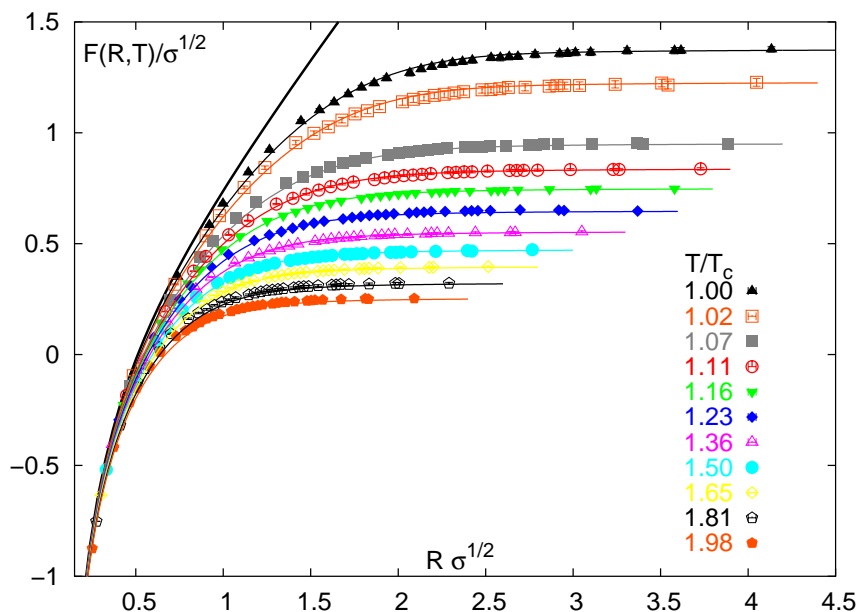
- strongly interacting QGP ( $T_c \leq T \leq 3 T_c$ )

$\Rightarrow$  very different behavior  
 (lattice results,  $N_f = 2$ )

separate strong part

$$F(r, T) = F_w(r, T) + F_s(r, T)$$

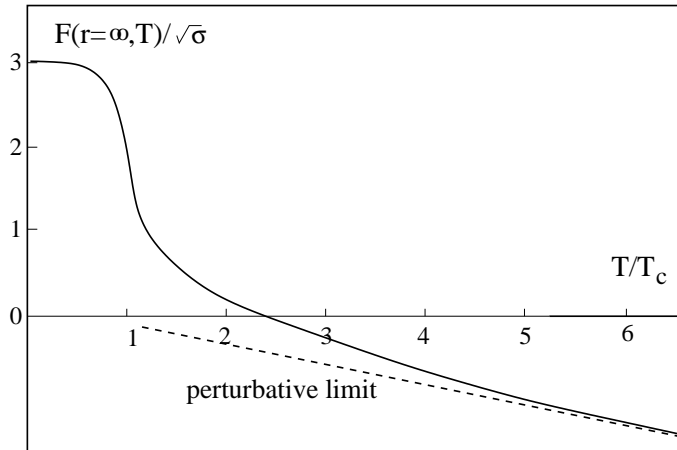
Kaczmarek & Zantow 2005



$T_c \leq T \lesssim 3 T_c$ : strong deviations from perturbative limit

large distance limit

to parametrize lattice results  
use 1-d Schwinger string form:



$$F_s(r, T) = \sigma r \left[ \frac{1 - e^{-\mu(T)r}}{\mu(T)r} \right] = \frac{\sigma}{\mu(T)} \left[ 1 - e^{-\mu(T)r} \right]$$

large distance limit  $F_s(\infty, T) = \sigma / \mu(T)$

in contrast to  $F_w(\infty, T) = -\alpha \mu(T)$

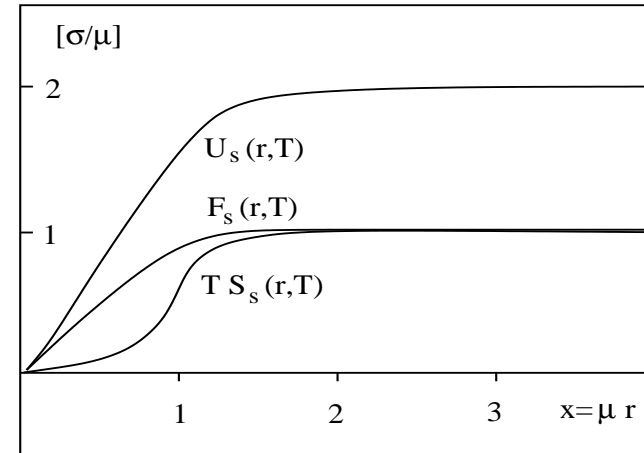
near  $T_c$ ,  $F_s \gg F_w$ :  $Q\bar{Q}$  in strongly interacting QGP?

two modifications:

- with  $\mu(T) \sim T$ , now obtain

$$T S_s(r, T) = \frac{\sigma}{\mu} [1 - (1 + \mu r) e^{-\mu r}]$$

$$U_s(r, T) = \frac{\sigma}{\mu} [2 - (2 + \mu r) e^{-\mu r}]$$



need one  $\sigma/\mu$  to separate  $Q$  and  $\bar{Q}$ , and another  $\sigma/\mu$   
to form polarization clouds (entropy change)

Who pays for what?

$V(r, T) = U(r, T)$  — the heavy quark pair pays all

$V(r, T) = F(r, T)$  — the medium pays the entropy change

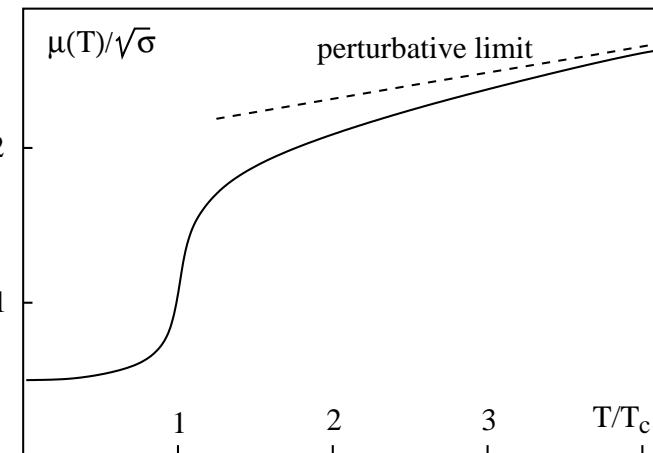
$V(r, T) = xF(r, T) + (1 - x)U(r, T)$

— medium and pair split the entropy cost



the more the pair pays, the tighter is its binding...with obvious consequences on dissociation temperatures

- in the critical region  $\mu(T) \not\propto T$ ,  
 much stronger variation  
 potential model calculations  
 must use  
 parametrization of lattice data



indicative results  
 for  $T_{\text{diss}}/T_c$

state	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$
$V(r, T) = U(r, T)$	2.1	1.2	1.1
$V(r, T) = F(r, T)$	1.2	1.0	1.0

Digal et al. 2001; Shuryak & Zahed 2004; Wong 2004/5; Alberico et al. 2005;  
 Digal et al. 2005; Mocsy & Petreczky 2005/6

- Lattice Studies of Quarkonium Spectrum

Calculate correlation function  $G_i(\tau, T)$  for mesonic channel  $i$  determined by quarkonium spectrum  $\sigma_i(\omega, T)$

$$G_i(\tau, T) = \int d\omega \sigma_i(\omega, T) K(\omega, \tau, T)$$

relates imaginary time  $\tau$  and  $c\bar{c}$  energy  $\omega$  through kernel

$$K(\omega, \tau, T) = \frac{\cosh[\omega(\tau - (1/2T))]}{\sinh(\omega/2T)}$$

invert  $G_i(\tau, T)$  to get quarkonium spectra  $\sigma_i(\omega, T)$

### Basic Problem

correlator given at discrete number  $N_\tau/2$  of lattice points with limited precision; presently best  $N_\tau = 96$  ( $0.75 T_c$ ),  $48$  ( $1.5 T_c$ )

want spectra  $\sigma_i(\omega, T)$  at  $\sim 1000$  points in  $\omega$

- brute force solution: calculate correlators for  $N_\tau = 2000$  then inversion is well-defined – project for FAR distant future
- in the meantime: invert  $G(\tau, T)$  by MEM to get  $\sigma(\omega, T)$

Maximum Entropy Method (MEM) here: [Asakawa and Hatsuda 2004](#)

what is the most likely solution for given data, given errors and some basic information?

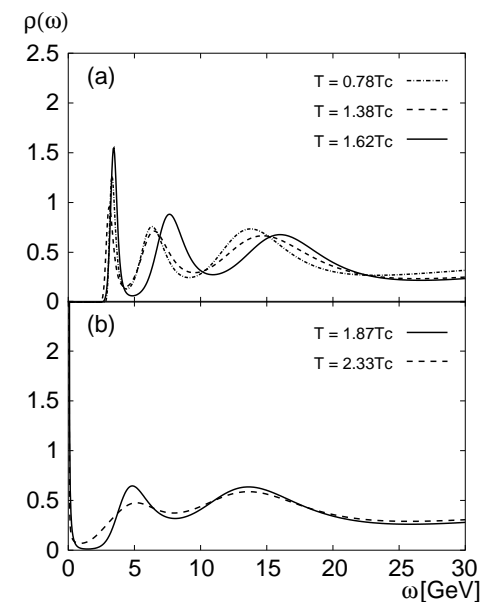
charmonia quenched:

- Umeda et al. 2001
- Asakawa & Hatsuda 2004
- Datta et al. 2004
- Iida et al. 2005
- Jakovac et al. 2005

charmonia unquenched:

- Aarts et al. 2005, 2007

first results  $\Rightarrow$



- MEM requires input reference (“default”) function for  $\sigma$ ;  
form of and dependence on default function?

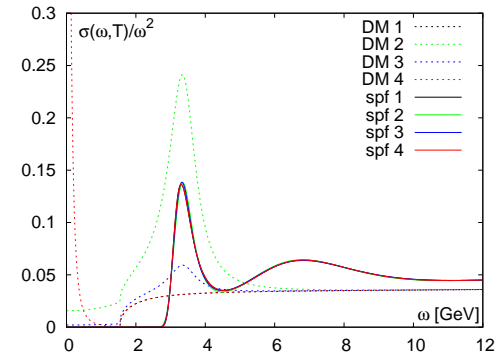
Preliminary work: [Heng-Tong Ding](#), [O. Kaczmarek](#), [F. Karsch](#), [HS](#)

choose two extreme cases as DF

- DF1:  $\sigma(\omega, T = 0)$ , quarkonium spectrum in vacuum
- DF2:  $\sigma_{\text{free}}(\omega, T)$ , spectrum for free  $Q\bar{Q}$  pair at  $T$
- what does MEM specify for  $\sigma(\omega, T)$  from correlators at  $T$ ?
- consider calculations in quenched QCD for PS channel
  - at  $T = 0.75 T_c$  for  $N_x = 128$ ,  $N_\tau = 96$ , 132 configs.
  - at  $T = 1.50 T_c$  for  $N_x = 128$ ,  $N_\tau = 48$ , 471 configs.
  - at  $T = 2.25 T_c$  for  $N_x = 128$ ,  $N_\tau = 36$ , xxx configs.
  - at  $T = 3.00 T_c$  for  $N_x = 128$ ,  $N_\tau = 24$ , xxx configs.

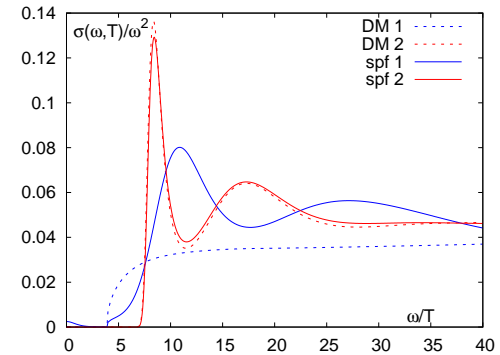
information sufficient  
 for unique MEM results;  
 spatial lattice size  
 insufficient for resonance width

$$T = 0.75 T_c$$



information insufficient  
 for unique MEM results;  
 spatial lattice size  
 insufficient for resonance width

$$T = 1.50 T_c$$



- better statistics, larger  $N_\tau$  should resolve MEM results
- larger  $N_x$  should (eventually) resolve resonance width

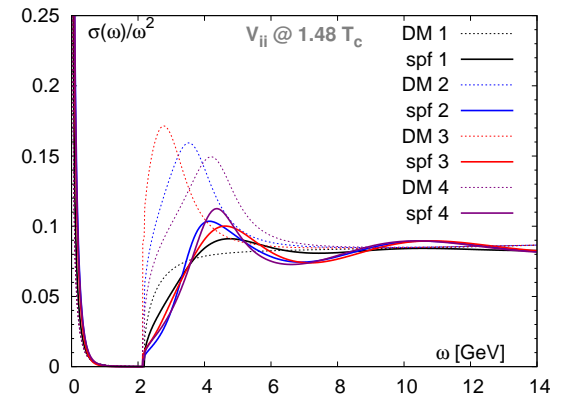
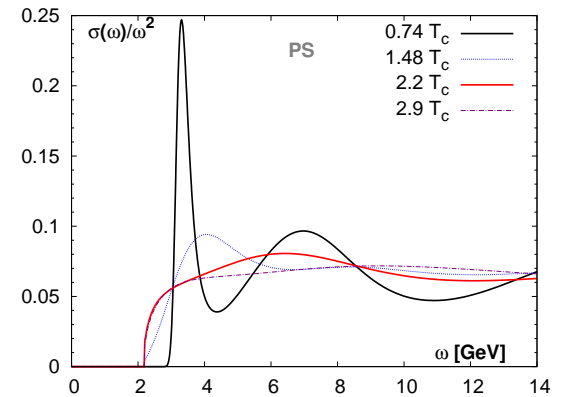
Some further results:

increasing  $T$  shifts “peak”  
to higher mass:  
 $\Rightarrow$  thermal peak of  
unbound heavy quarks

change of default peak  
not accepted by MEM

Tentative summary so far:

- $J/\psi$  survives up to  $T \simeq 1.5 - 2.0 T_c$
- $\chi$  and  $\psi'$  dissociated at or slightly above  $T_c$



But there are **further questions**:

- Schrödinger equation provides dissociation temperature as point where  $J/\psi$  radius diverges, binding energy vanishes;  
 $R \simeq 5$  fm,  $\Delta E \simeq 10$  MeV in medium of  $T \simeq 250$  MeV?
- Lattice calculations provide quarkonium spectrum with given resonance width, position;  
how wide can it get, how far can it shift and still be  $J/\psi$ ?

Possible way out: melting region is quite narrow in  $T$ ?

$\exists$  observable consequences for nuclear collision experiments?

### 3. Quarkonium production is **suppressed** in nuclear collisions

...but for a variety of reasons

- nuclear modification (“shadowing”) of parton distribution functions
- parton energy loss in cold nuclear matter
- pre-resonance dissociation (“absorption”) in cold nuclear matter
- dissociation by screening (“melting”) and/or collisions in hot QGP

assume both initial & final state cold nuclear matter effects are taken into account correctly;

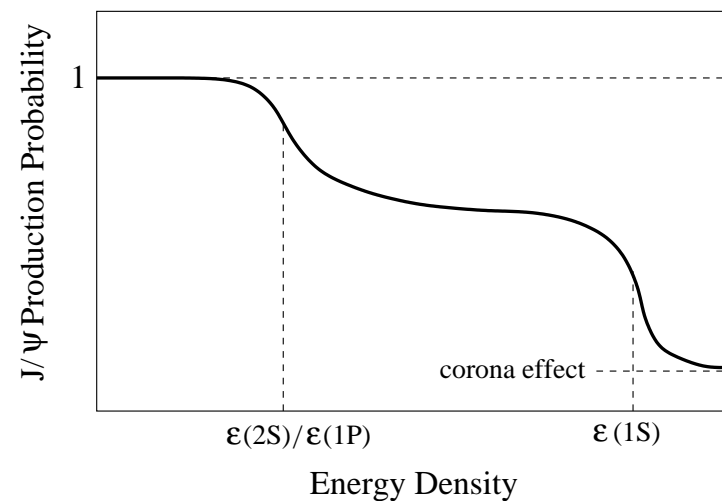
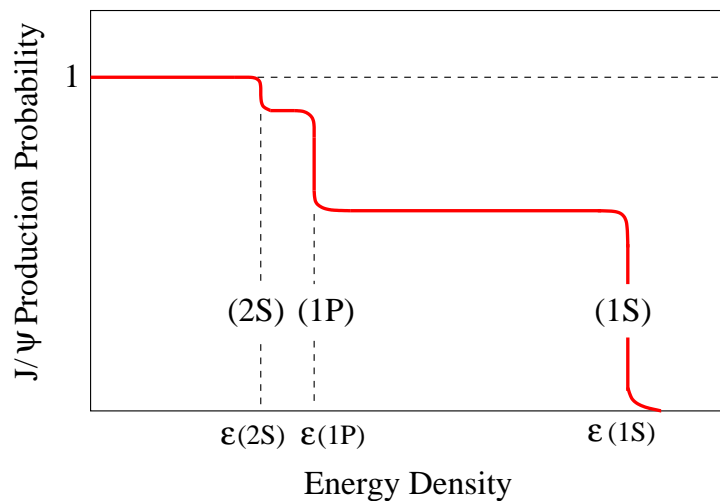
SPS & RHIC:  $\exists$  remaining 50 %  $\pm$ ? “anomalous” suppression

If due to melting in hot QGP  $\Rightarrow$  sequential  $J/\psi$  suppression

Karsch & HS 1991; Gupta & HS 1992; Karsch, Kharzeev & HS 2006



- measured  $J/\psi$ 's are about 60% direct 1S, 30%  $\chi_c$  decay, 10%  $\psi'$  decay
- narrow excited states  $\rightarrow$  decay outside medium; medium affects excited states
- $J/\psi$  survival rate shows sequential reduction: first due to  $\psi'$  and  $\chi_c$  melting, then later direct  $J/\psi$  dissociation
- experimental smearing of steps; corona effect



IF charmonium/bottomonium thresholds are measurable:

- experimental test of quantitative statistical QCD results

⇒ no charmonium production at the LHC?

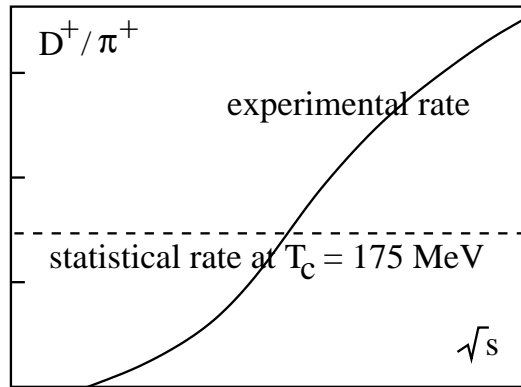
– corona effect

– significant  $B$  production → charmonium production via feed-down from  $B$  decay; check through  $pp$  studies. **And:**

#### 4. Quarkonia can be **created** at QGP hadronization

Braun-Munzinger & Stachel 2001, Thews et al. 2001, Grandchamp & Rapp 2002  
Andronic et al. 2003, Zhuang et al. 2006

- $c\bar{c}$  production is dynamical “hard process”:  
at high energy, produced medium contains more than the  
“statistical” number of charm quarks



- assume
  - charm quark abundance constant in evolution to  $T_c$
  - charm quarks form part of equilibrium QGP at  $T_c$
  - equilibrium QGP at  $T_c$  hadronizes statistically
  - charmonium production via statistical  $c\bar{c}$  fusion
- “secondary” charmonium production by fusion of  $c$  and  $\bar{c}$  produced in different primary collisions
- insignificant at “low” energy, since very few charm quarks; could be dominant production mechanism at high energy

- simplified illustration...assume at “LHC” per event

100  $c\bar{c}$  pairs

1000  $q\bar{q}$  pairs

non-statistical fraction; statistical  $\sim 10^{-3}$  for  $T_c = 175$  MeV

primary rates:

1  $J/\psi$ , 99  $D$ , 99  $\bar{D}$ , 901 light hadrons  $\Rightarrow R_{AA} \simeq 1$

rates for statistical combination of given quark abundances:

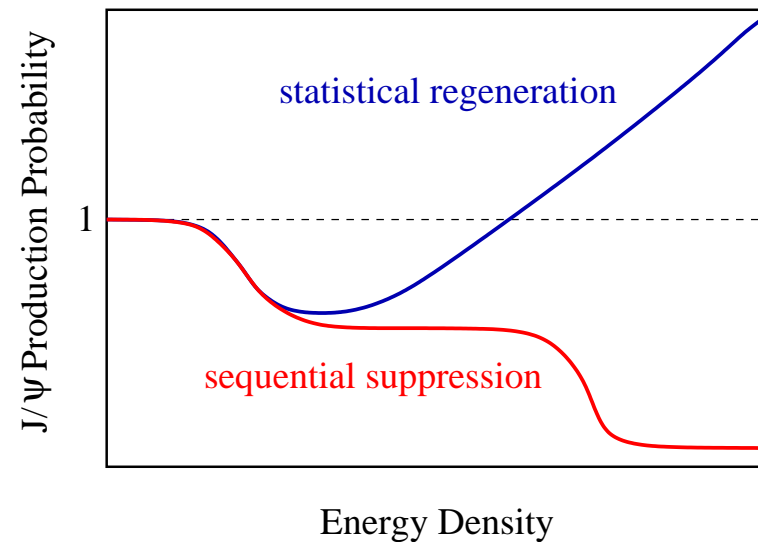
10  $J/\psi$ , 90  $D$ , 90  $\bar{D}$ , 910 light hadrons  $\Rightarrow R_{AA} \simeq 10$

$\Rightarrow J/\psi$  production strongly enhanced re scaled  $pp$  rate

$$\Rightarrow \frac{J/\psi}{D} \simeq 0.1 \text{ instead of } 0.01 \text{ in } pp$$

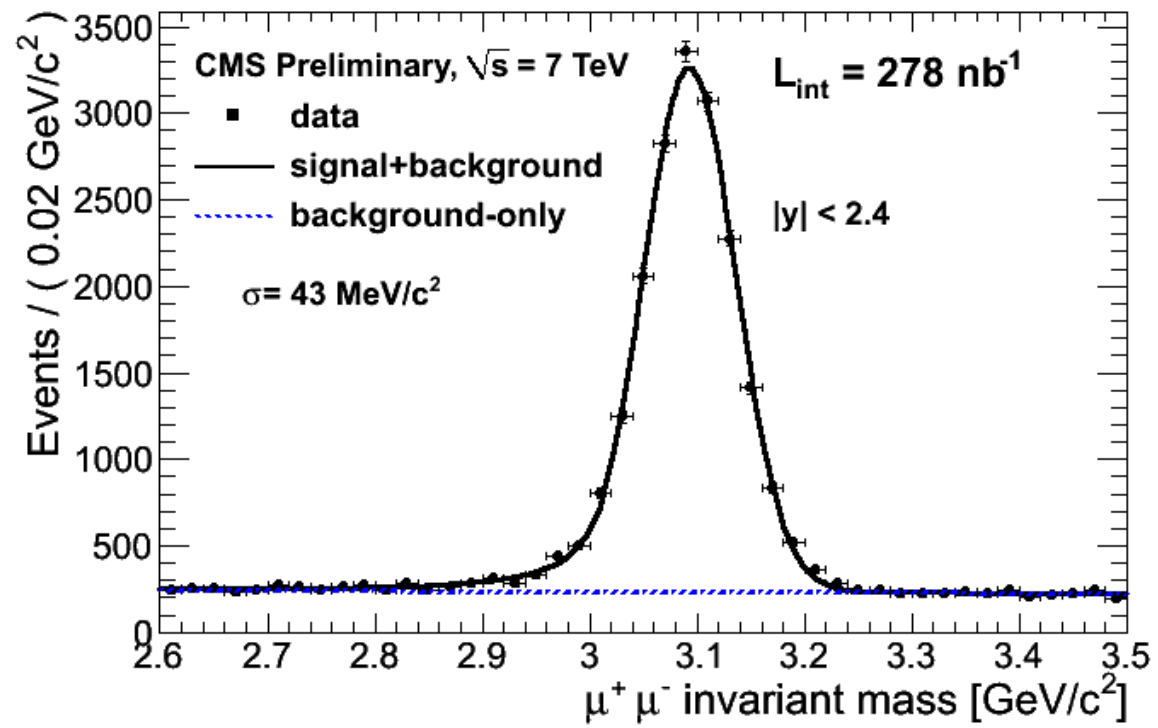
ratio of hidden/open charm strongly enhanced re  $pp$  ratio

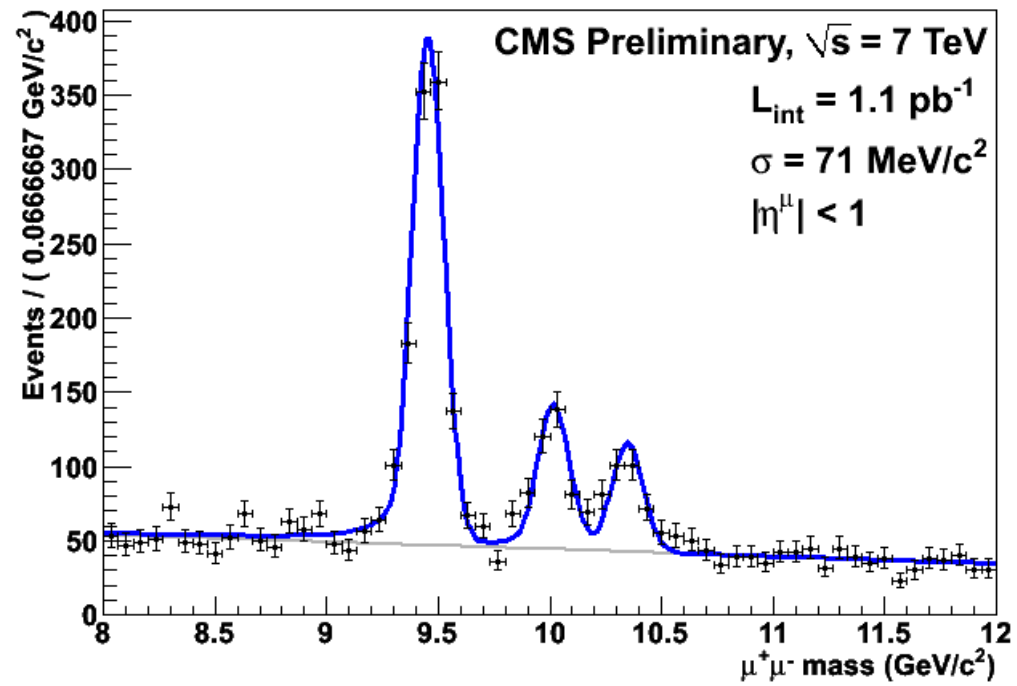
two readily distinguishable  
predictions for  
anomalous  $J/\psi$  production

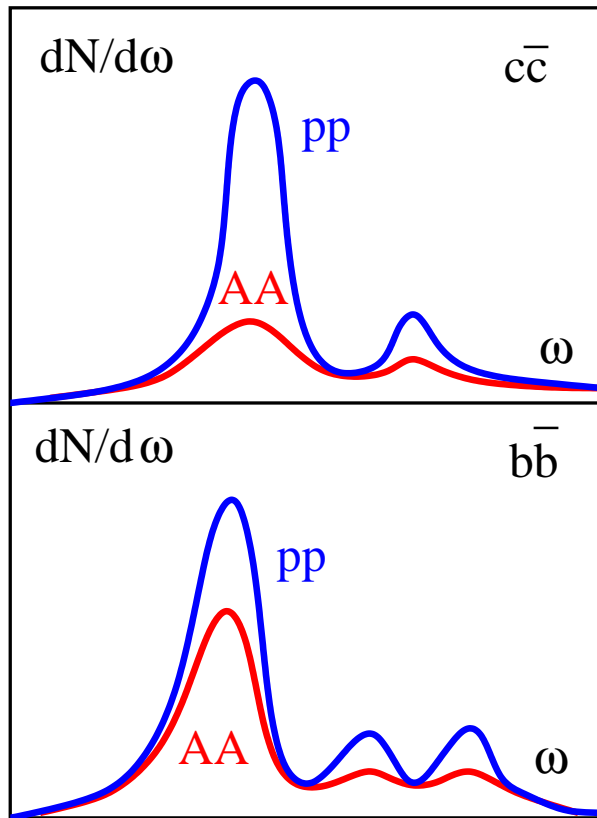


dynamical vs. statistical momentum spectra [Mangano & Thews 2003](#)

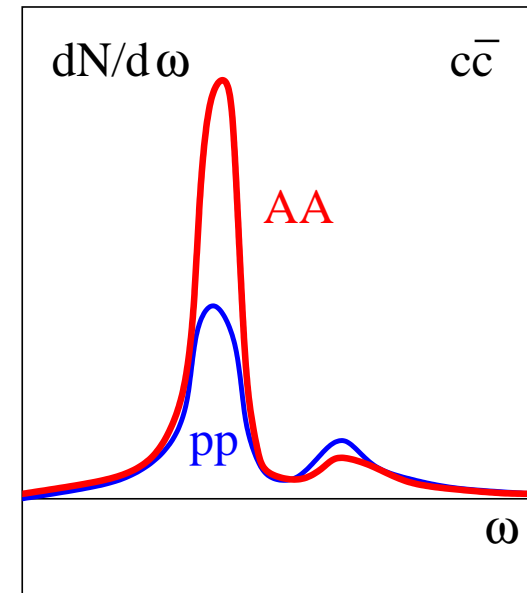
NB: assumption of statistical quarkonium binding...







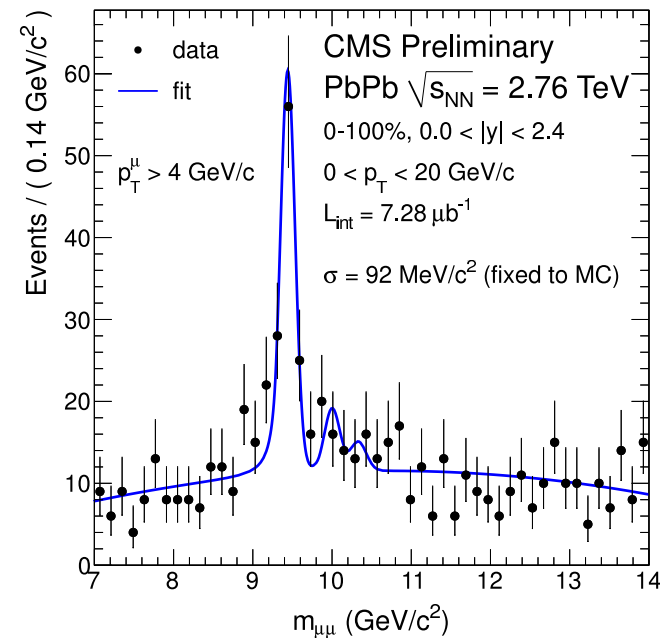
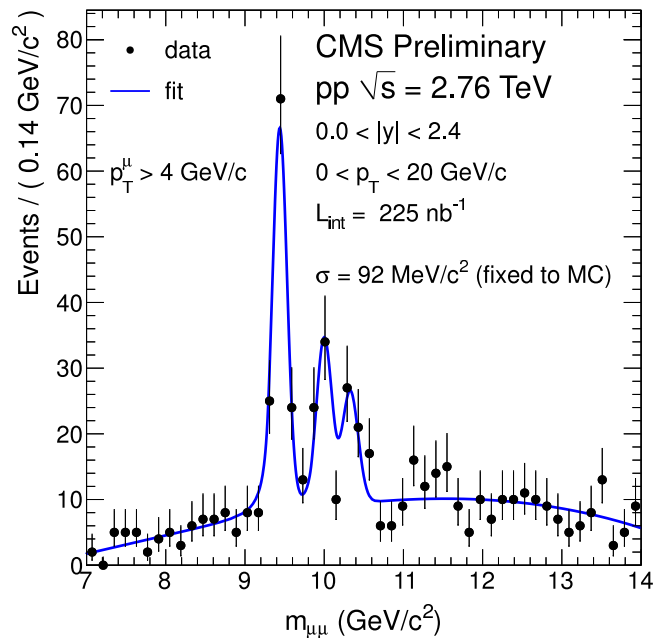
sequential suppression  
by color screening:  
only (possible) survivor is  $\Upsilon$



statistical regeneration:  
more  $J/\psi$  than in  
scaled  $pp$



If  $\exists$  statistical regeneration of  $J/\psi$ ,  
then use sequential suppression in bottomonium production  
as tool to compare heavy ion data to QCD calculations



work is in progress, data is great – stay tuned!

## Conclusions

Given reference measurements of open charm/bottom production,  
experimental quarkonium studies at the LHC can ask  
conceptual [model-independent] questions and provide  
conceptual [model-independent] answers to these.  
Quantitative details require specific theory/model input.