Recent Developments of the Hadron Resonance Gas Model

K.A. Bugaev,

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Dubna, September, 2012

HRG: a Multi-component Model

Traditional HRG model: one hard-core radius R=0.25-0.3 fm

A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006)777

Two hard-core radii: R_pi =0.62 fm, R_other = 0.8 fm

G. D.Yen. M. Gorenstein, W. Greiner, S.N. Yang, PRC (1997)56

Or: R_mesons =0.25 fm, R_baryons = 0.3 fm

A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006) 777,
PLB (2009) 673

Overall description of data (mid-rapidity or 4π multiplicities or ratios is good (remember J. Cleymans talk)!

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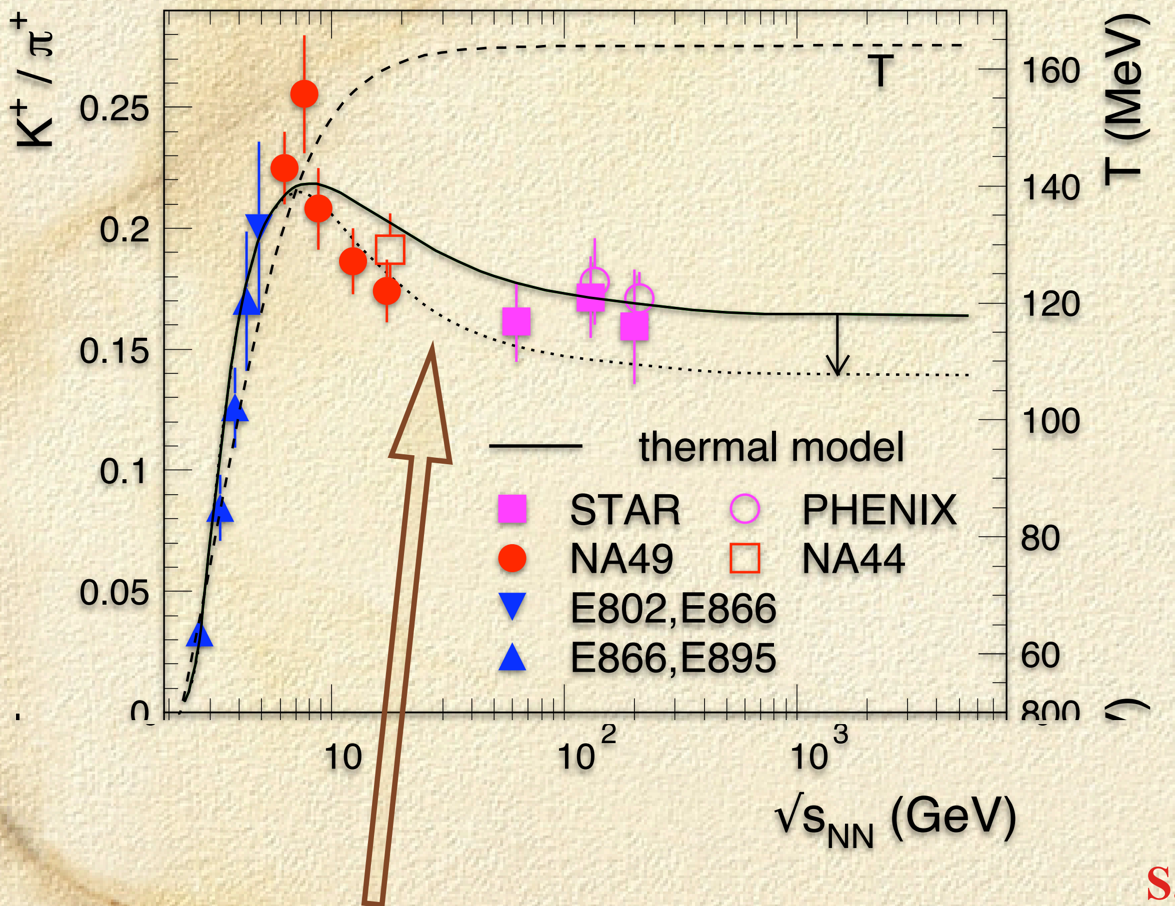
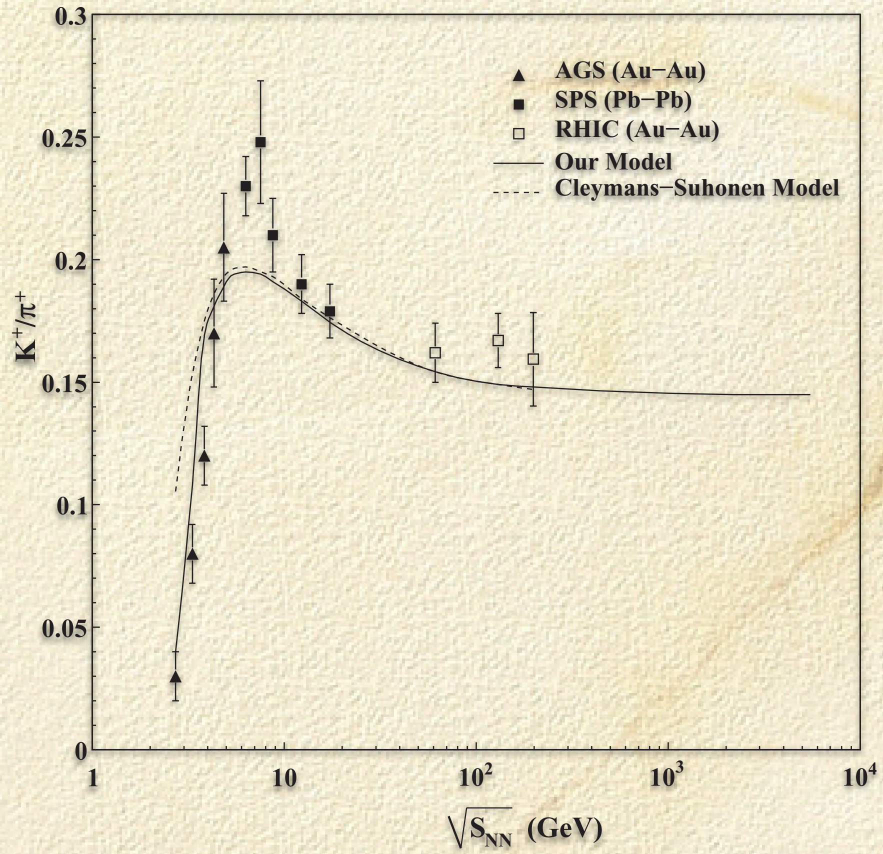
PLB (2009) 673

But there are problems with K+/pi+ and Λ/pi- ratios at SPS energies!!!

Strangeness Horn Description Puzzle

Too slow decrease after maximum!





Short dashed line: a desired result

A. Andronic, P.Braun-Munzinger, J. Stachel, PLB (2009) 673

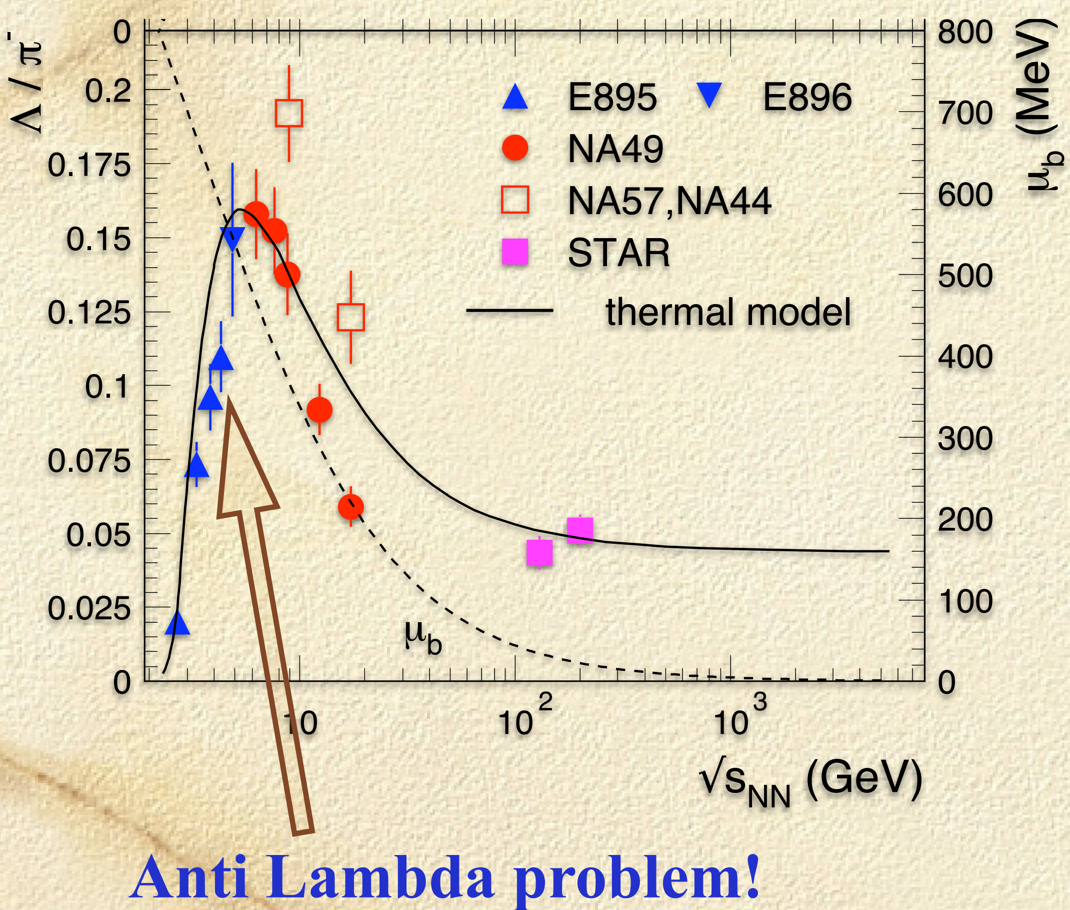
S. K. Tiwari, P. K. Srivastava, and C. P. Singh, PRC(2012) 85

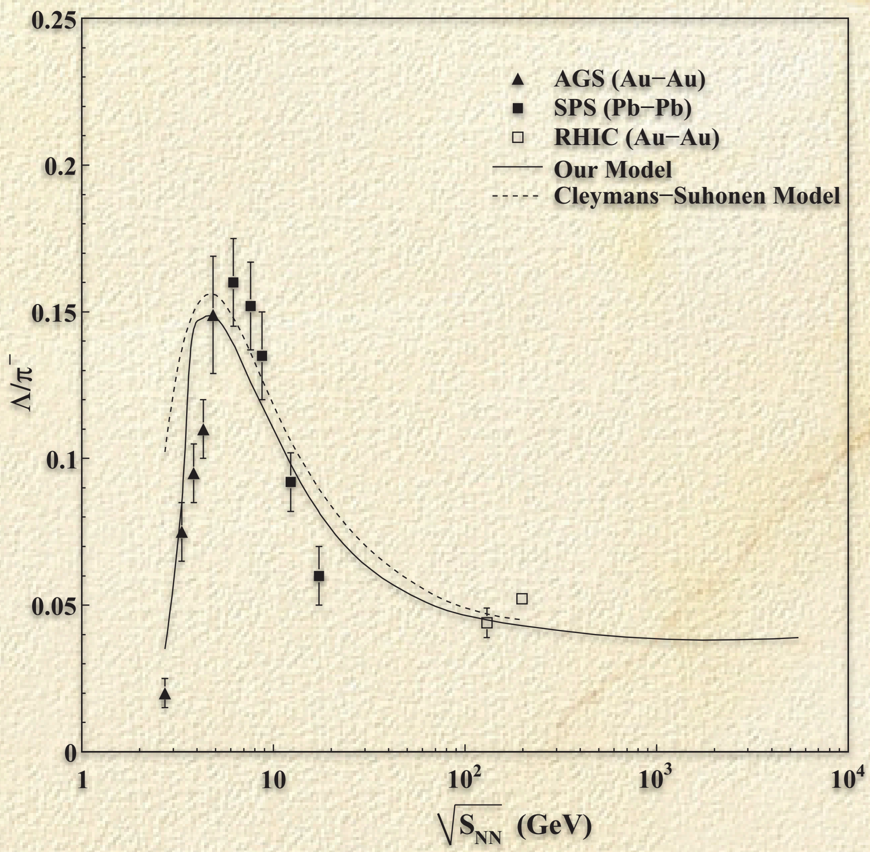
R_pi =0. fm, R_other = 0.8 fm

Very sophisticated excluded volume, but thermodynamically inconsistent model!

Further Problems at SPS Energies

Too steep increase before maximum and too slow decrease after it!





A. Andronic, P.Braun-Munzinger, J. Stachel, PLB (2009) 673

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Simple Solution to Horn Puzzle

Use four hard-core radii: R_pi, R_K are fitting parameters; R_mesons = 0.3 fm, R_baryons = 0.5 fm are fixed

G. Zeeb, K.A. Bugaev, P.T. Reuter and H. Stoecker, Ukr. J. Phys. 53, 279 (2008)

D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin, arXiv:1204.0103 [hep-ph].

p is pressure K-th charge density of i-th hadron sort is n_i^K ($K \in \{B, S, I3\}$)

$\mathcal{B}$ the second virial coefficients matrix $b_{ij} \equiv \frac{2\pi}{3}(R_i + R_j)^3$

$$p = T \sum_{i=1}^{N} \xi_i \,, \quad n_i^K = Q_i^K \xi_i \left[1 + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^{N} \xi_j} \right]^{-1} \,, \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \ldots \\ \xi_s \end{pmatrix} \,,$$

NO strangeness suppression is included!

the variables ξ_i are the solution of the following system:

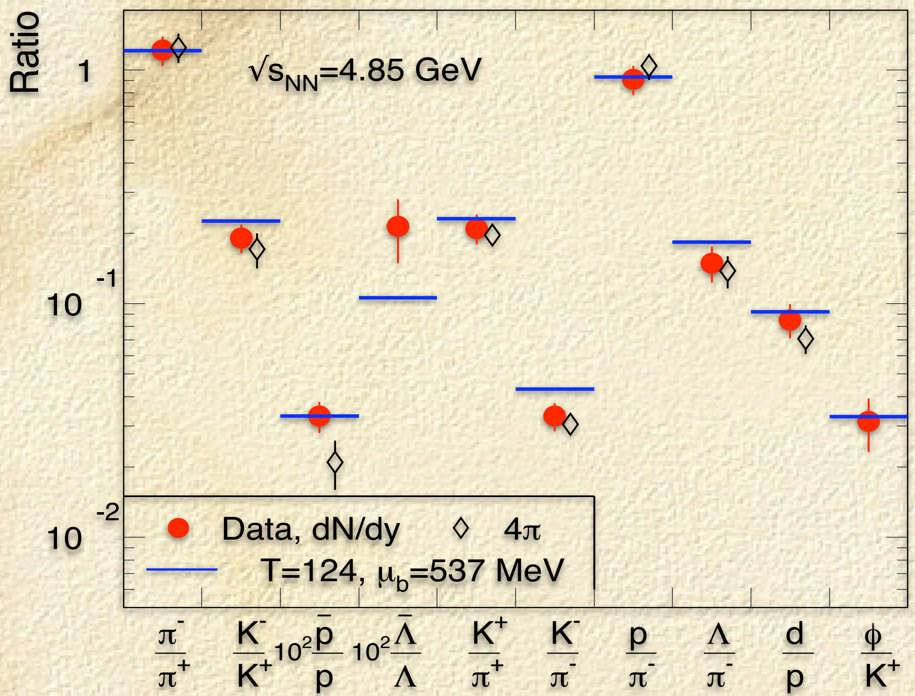
$$\xi_i = \phi_i(T) \exp\left(\frac{\mu_i}{T} - \sum_{j=1}^{N} 2\xi_j b_{ij} + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^{N} \xi_j} \right) \,, \quad \underbrace{\phi_i(T) = \frac{g_i}{(2\pi)^3} \int \exp\left(-\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3k}_{\text{THERMAL DENSITY}}$$

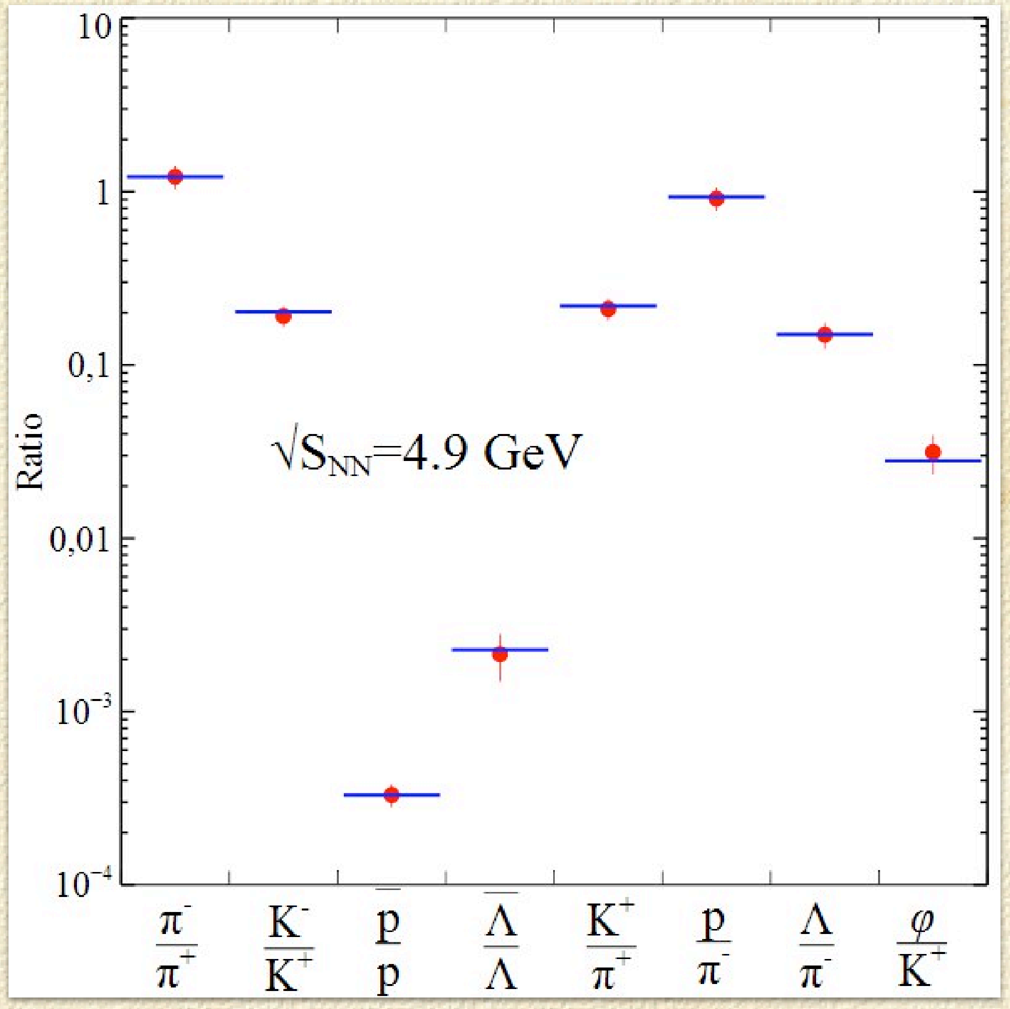
Chemical potential of i-th hadron sort: $\mu_i \equiv Q_i^B \mu_B + Q_i^S \mu_S + Q_i^{I3} \mu_{I3}$

Q_i^K are charges, m_i is mass and g_i is degeneracy of the i-th hadron sort

Results for Ratios (AGS)

There is NO anti Lambda problem here and all ratios are well described!





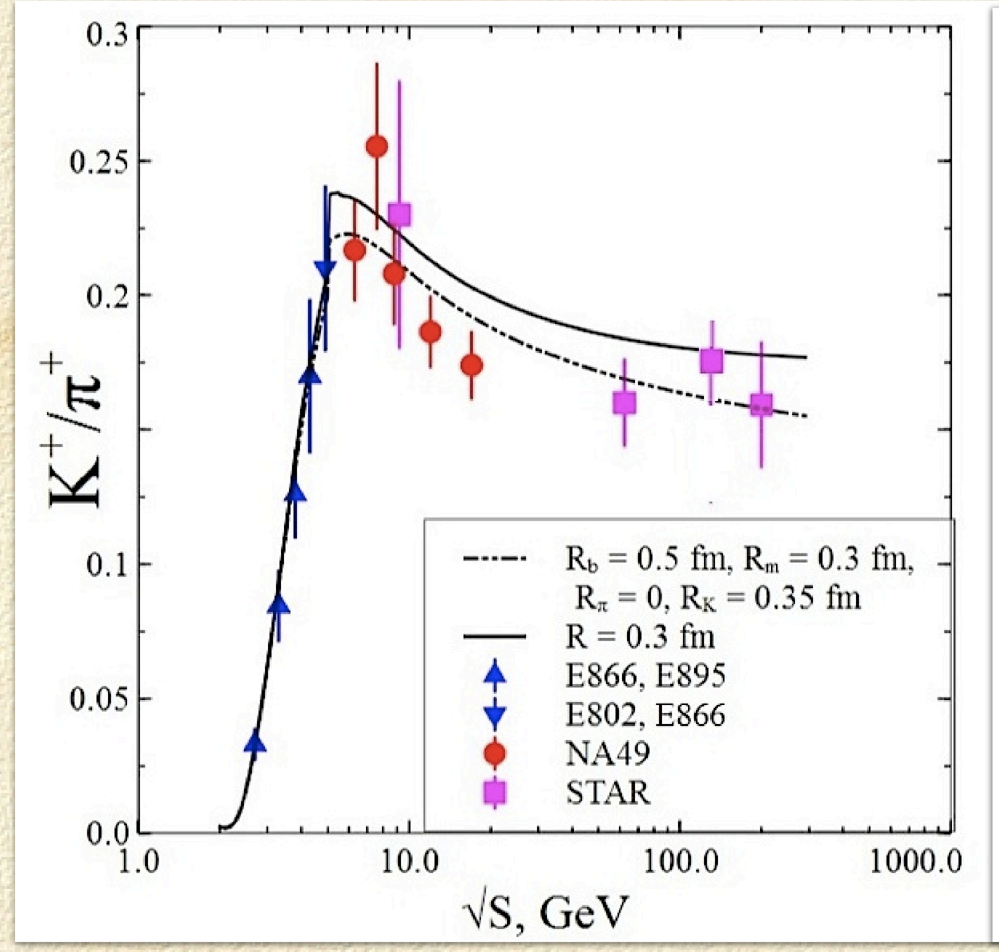
There is an anti Lambda problem! Also K-/K+ and K/pi and Lambda/pi- are not well described!

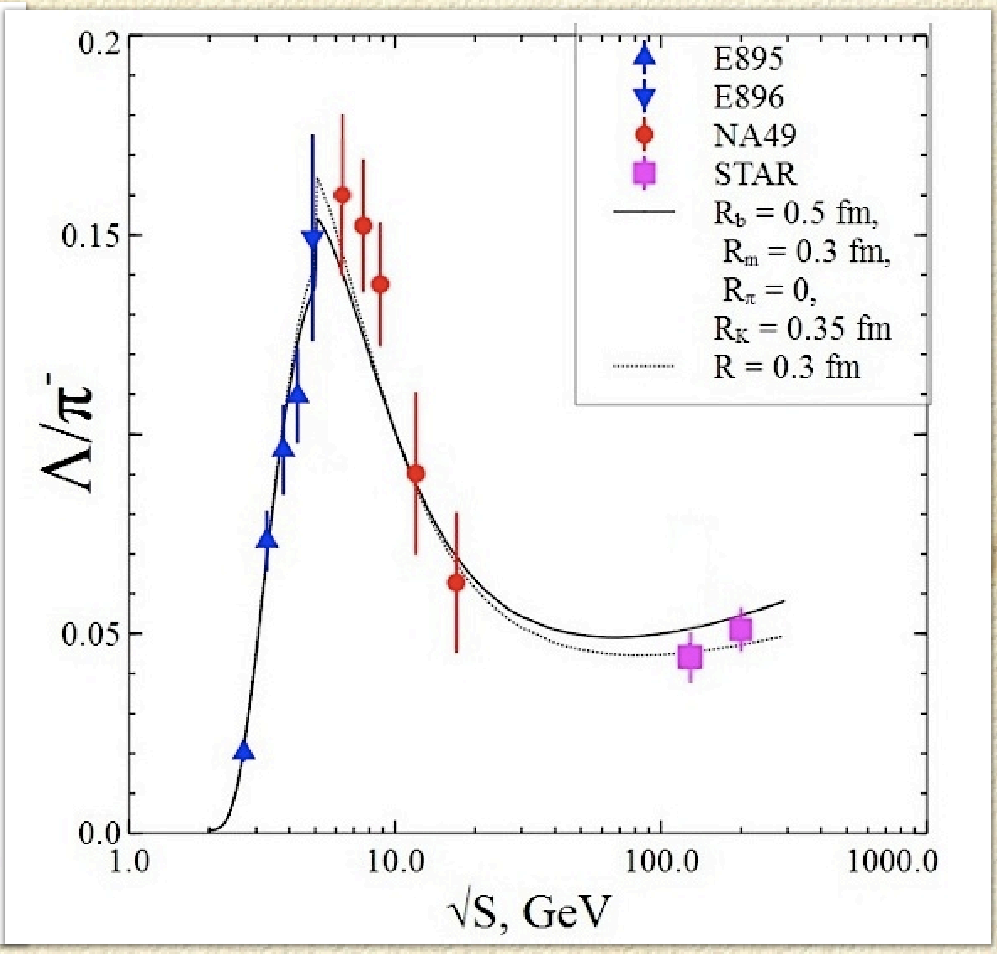
$T \simeq 131$ MeV, $\mu_B \simeq 539$ MeV, $\mu_{I3} \simeq -16$ MeV

A. Andronic, P.Braun-Munzinger, J. Stachel, NPA (2006)777

K.A.B., D.R. Oliinychenko, A.S. Sorin, G.M. Zinovjev, arXiv:1208.5968 [hep-ph].

Description of Horns at SPS





Best global fit of all ratios gives R_pi=0. fm, R_K =0.35 fm,

χ^2/dof =1.018 for fixed: R_baryons =0.5 fm, R_mesons = 0.3 fm

Note that Lambda and other hyperons can be described better!

Wide Resonances in Thermal Media

Wide resonances are VERY important in a thermal model. For instance, description of pions cannot be achieved without

σ meson: $m_\sigma = 484 \pm 24$ MeV, width $\Gamma_\sigma = 510 \pm 20$ MeV

R. Garcia-Martin, J. R. Pelaez and F. J. Yndurain, PRD (2007) 76

$$n_X^{tot} = n_X^{thermal} + n_X^{decay} = n_X^{th} + \sum_Y n_Y^{th} Br(Y \to X)$$

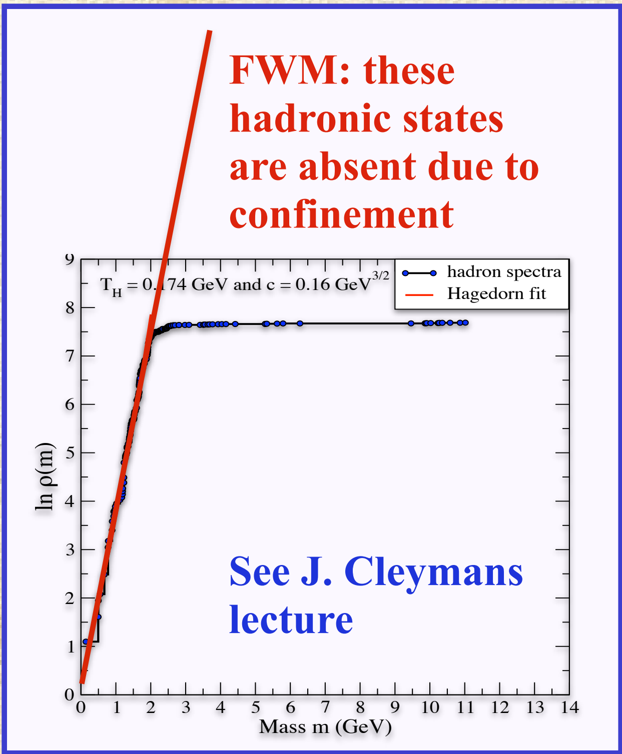
$Br(Y \to X)$ is decay branching of Y-th hadron into hadron X

Large width of QG bags is absolutely necessary to explain the deficit in the number of observed heavy resonances compared to Hagedorn mass spectrum and to model a color confinement phenomenon. See the finite width model of QG bags

K.A.B., V.K. Petrov, G.M. Zinovjev, Europhys. Lett. (2009) 85; PRC (2009) 79

Unfortunately, there is NO THEORY how to do this rigorously for an ensemble of about 100 different hadronic (mesonic & baryonic) species...

Resonances in Thermal Media



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Resonance Contribution in Thermal Media

Therefore, at chemical freeze-out we substitute

D. Hahn and H. Stoecker, Nucl. Phys. A (1986) 452
K.G. Denisenko and St. Mrowczynski, PRC (1987) 35

$$\int \exp\left(-\frac{\sqrt{k^2+m_j^2}}{T}\right) d^3k \rightarrow \frac{\int\limits_{M_j^{Th}}^{\infty} dx\, \rho_j(x) \int \exp\left(-\frac{\sqrt{k^2+x^2}}{T}\right) d^3k}{\int\limits_{M_j^{Th}}^{\infty} dx\, \rho_j(x)},$$

M_j^{Th} is a threshold of dominant decay channel

Mass distribution of j-th resonance is

$$\rho_j(x) = \begin{cases} \frac{1}{(x-m_j)^2+\Gamma_j^2/4}, & \text{for hadrons with } m_j \leq 2.5 \text{ GeV} \\ \exp\left[-\frac{(m_j-m)^2}{2\,\sigma_j^2}\right] & \text{for hadrons \& QG bags with } m_j > 2.5 \text{ GeV} \end{cases}$$

used in all advanced thermal models

used in finite width model of QG bags

Gaussian width is $\sigma_j = \Gamma_j/Q \simeq \Gamma_j/2.355$

K.A.B., V.K. Petrov, G.M. Zinovjev,
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This approximation is usually criticized, but

1. within approaches that employ a few hadronic dof
2. at least at low T the deviation from elaborate formulae is about experimental uncertainty for the Delta 1232 peak

W. Weinhold, B. Friman, W. Noerenberg, PLB (1998) 433

However, since more elaborate approximations lead to huge complications without qualitative and quantitative improvements, the thermal models use this one at chemical freeze-out.

Sigma Meson at Low T

In nonrelativistic approximation $M_k^{Th} \gg T$

$$\Longrightarrow \int \frac{d^3p}{(2\pi)^3} \exp\left[-\frac{\sqrt{p^2+m^2}}{T}\right] = \left[\frac{m\,T}{2\,\pi}\right]^{\frac{3}{2}} \exp\left[-\frac{m}{T}\right]$$

For convenience let's use the Gaussian attenuation:

$$\underbrace{\Theta\left(m - M_k^{Th}\right) \exp\left[-\frac{(m_k - m)^2}{2\,\sigma_k^2}\right]}_{grows\ fast\ near\ threshold} \overbrace{\exp\left[-\frac{m}{T}\right]}^{decreases\ fast} \underbrace{\left[\frac{m\,T}{2\,\pi}\right]^{\frac{3}{2}}}_{\simeq\ const}$$

After making a full square in exponential $\Longrightarrow$

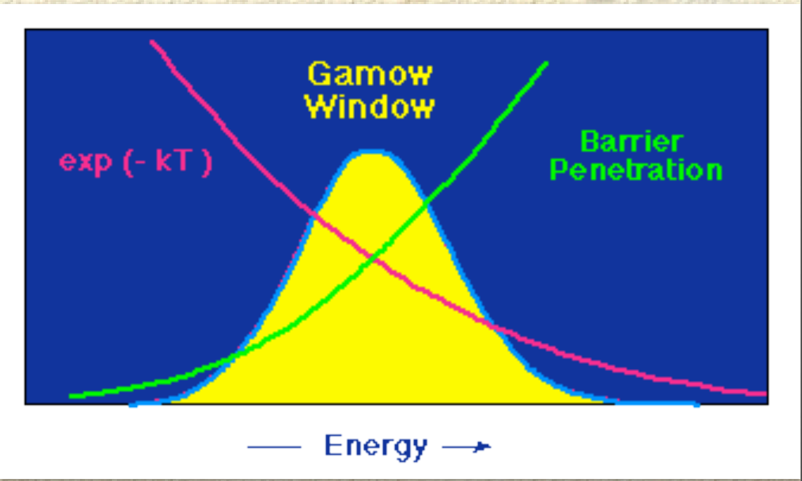
$$\underbrace{\Theta\left(m - M_k^{Th}\right) \exp\left[-\frac{(m_k - \frac{\sigma_k^2}{T} - m)^2}{2\,\sigma_k^2}\right]}_{peak\ is\ shifted\ to\ threshold} \overbrace{\exp\left[\frac{\sigma_k^2}{2\,T}\right]}^{leads\ to\ enhancement} \exp\left[-\frac{m_k}{T}\right] \underbrace{\left[\frac{m\,T}{2\,\pi}\right]^{\frac{3}{2}}}_{\simeq\ const}$$

The shifted peak goes UNDER threshold, if

$$M_k^{Th} \geq \tilde{m}_k \equiv m_k - \frac{\sigma}{T}$$

For σ meson $\Longrightarrow$

$$M_\sigma^{Th} = 2\,m_\pi \simeq 280 \text{ MeV}$$



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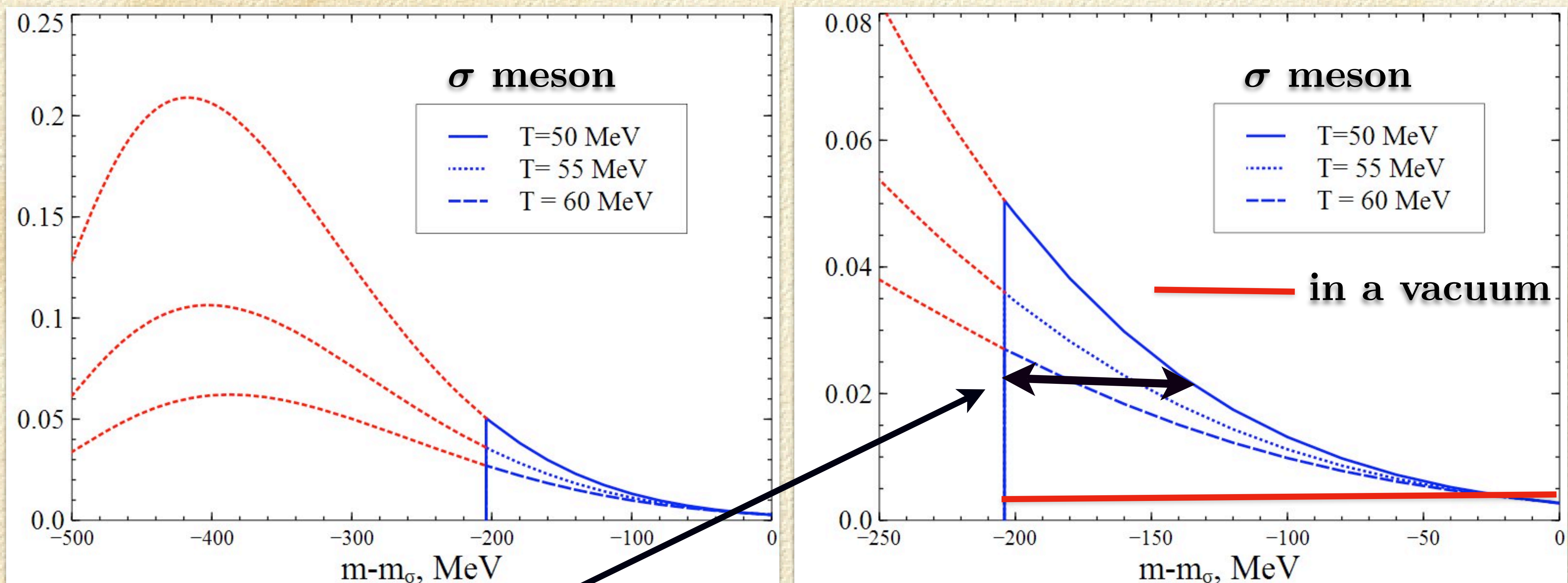
The shifted peak goes UNDER threshold, if

$$M_k^{Th} \geq \tilde{m}_k \equiv m_k - \frac{\sigma_k^2}{T} \quad \Leftrightarrow \quad \text{for} \quad \boxed{T < T_k^+ \equiv \frac{\sigma_k^2}{m_k - M_k^{Th}}}$$

For σ meson $\Longrightarrow$

$$M_\sigma^{Th} = 2\,m_\pi \simeq 280 \text{ MeV} \quad \text{and} \quad T_\sigma^+ \simeq 92 \text{ MeV}$$

Wide Resonance Sharpening and Enhancement Near Threshold



σ meson effective width decreased in 7 times!

$\Gamma_\sigma^{eff}(T = 50\,\mathrm{MeV}) \simeq 62.5$ MeV, $\quad \Gamma_\sigma^{eff}(T = 55\,\mathrm{MeV}) \simeq 71.5$ MeV,

$\Gamma_\sigma^{eff}(T = 60\,\mathrm{MeV}) \simeq 82.5$ MeV

Is well described by the formula

$$\Gamma_k^N(T) \simeq \frac{\ln(2)}{\frac{1}{T} - \frac{1}{T_k^+} - \frac{3}{2\,M_k^{Th}}}$$

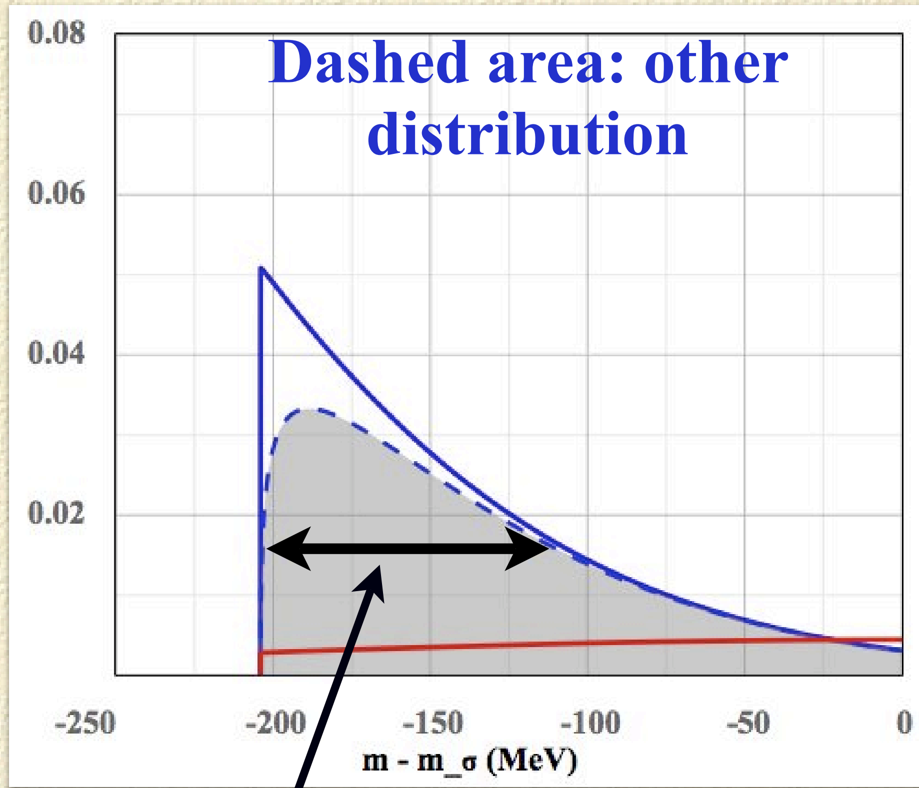
A Few Remarks

1. Results are shown for extreme case, but formula

$$\Gamma_k^N(T) \simeq \frac{K}{\frac{1}{T} - \frac{1}{T_k^+} - \frac{3}{2\, M_k^{Th}}}$$

with $K \in [0.7;\ 1.5]$

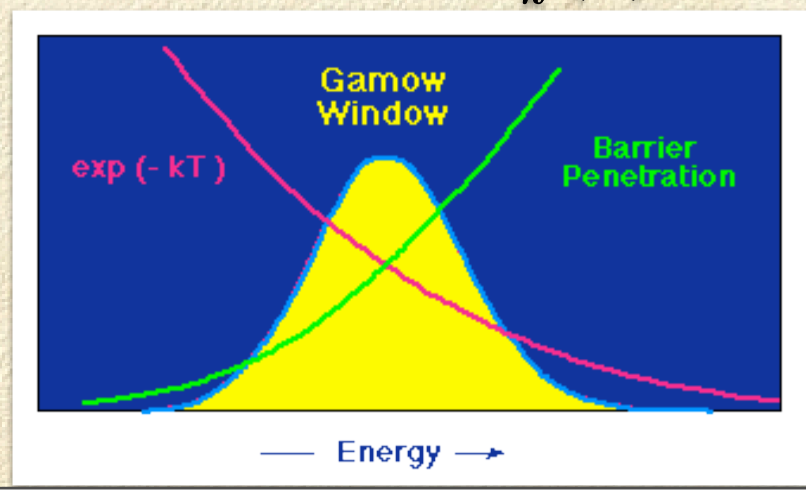
works well for other parameterizations!



Enhancement is 0.864 of blue line

Effective width is $\Gamma_k^N(T) \simeq 90$ MeV

2. Source of resonance distribution has many similarities with famous Gamov window for thermonuclear reactions of charged particles!



The main difference is due threshold

Finite Width Model of QG Bags

Finite width model of QG bags predicts that

K.A.B., V.K. Petrov, G.M. Zinovjev, Europhys. Lett. (2009) 85

The bag width is $\Gamma_B \simeq \Gamma_0(T) \left[\frac{M_B}{M_0}\right]^{\frac{1}{2}}$

with $M_0 \simeq 2.5$ GeV and

$$\Gamma_0(T) \simeq \begin{cases} 400 \text{ MeV}\,, & \text{if } T = 0 \text{ MeV}\,, \\ 800 \text{ MeV}\,, & \text{if } T \simeq 90 \text{ MeV}\,, \\ 1400 \text{ MeV}\,, & \text{if } T \simeq 170 \text{ MeV} \end{cases}$$

Finite width model was successfully verified on a variety of lattice QCD thermodynamics data

K.A.B., V.K. Petrov, G.M. Zinovjev, PRC (2009) 79

For T=0 this width relation perfectly describes the imaginary part of leading Regge trajectories for $\rho_{J^{--}}, \omega_{J^{--}}, a_{J^{++}}$ and $f_{J^{++}}$ **mesons with the spins J < 7**

K.A.B., E,G. Nikonov, A.S. Sorin, G.M. Zinovjev, JHEP02 (2011) 059

Application to Quark Gluon Bags

$$\Gamma_B^N(T) \simeq \left. \frac{K}{\frac{1}{T} - \frac{1}{T_k^+}} \right|_{T \ll T_B^+} \to T\,K\,, \quad \text{where} \quad K \in [0.7;\ 1.5]$$

Thus, the lower T, the sharper is bag!

However, the finite width model predicts a huge suppression for T < 80 MeV => one can hope that for T = 90-120 MeV the QG bag width can be between 60 and 160 MeV!

=> Perhaps the QG bags can be observed as sharp resonances with mass about 2.5 GeV which are absent in Particle Data Group

=> Such a hypothesis can be verified at NICA energies!

Further Estimates

For a decay into L pions (L = 1–5) one finds

$$T_B^{+} \simeq \frac{[\Gamma_0(T)]^2}{Q^2\, M_0 \left(1 - \frac{M_B^{Th}}{M_B}\right)} \simeq \left(1 + \frac{M_B^{Th}}{L\, m_\pi}\right) \cdot \begin{cases} 11.5\ \text{MeV}\,, & \text{if } T = 0\ \text{MeV}\,, \\ 46.2\ \text{MeV}\,, & \text{if } T \simeq 90\ \text{MeV}\,, \\ 141\ \text{MeV}\,, & \text{if } T \simeq 170\ \text{MeV} \end{cases}$$

Due to Boltzmann factor it is better to keep threshold mass to 2.5 GeV

=> For T = 90 MeV the necessary condition for T_B^{+} is satisfied for L < 6

Thanks for your attention!