Surface Tension of Quark Gluon Bags and Physical Mechanism of the (Tri)critical Endpoint Generation

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Dubna, September, 2012

Outline

- A bit of history of the Statistical Approach to RHIC
- A few words on statistical models
- About the VdWaals equation of state
- The induced surface tension in hadron resonance gas
- Confinement model and the role of surface tension of QGP bags
- Determination of surface tension from lattice QCD
- Exactly solvable model of Quark Gluon Bags with Surface Tension
- Conclusions

Statistical Approach: Gas of Bags

* 1965 Hagedorn suggested an exponentially growing mass spectrum for heavy hadrons. The model led to the idea of limiting temperature for hadrons.

* 1974 MIT Bag model is proposed. It treats hadrons as QG bags.
  A.Chodos et. al., Phys. Rev. D 9, (1974) 3471.

* 1975 Cabbibo and Parisi conjectured that limiting temperature evidences for the new physics above T_H. The relevant d.o.f. are quarks and gluons.

QCD era begins!

* 1981 Kapusta showed that MIT Bags have the Hagedorn mass spectrum. The Gas of Bags model is suggested. It unifies the three previous ideas. Hence, heavy hadrons = QGP bags. PRD 23 (1981) 2444.

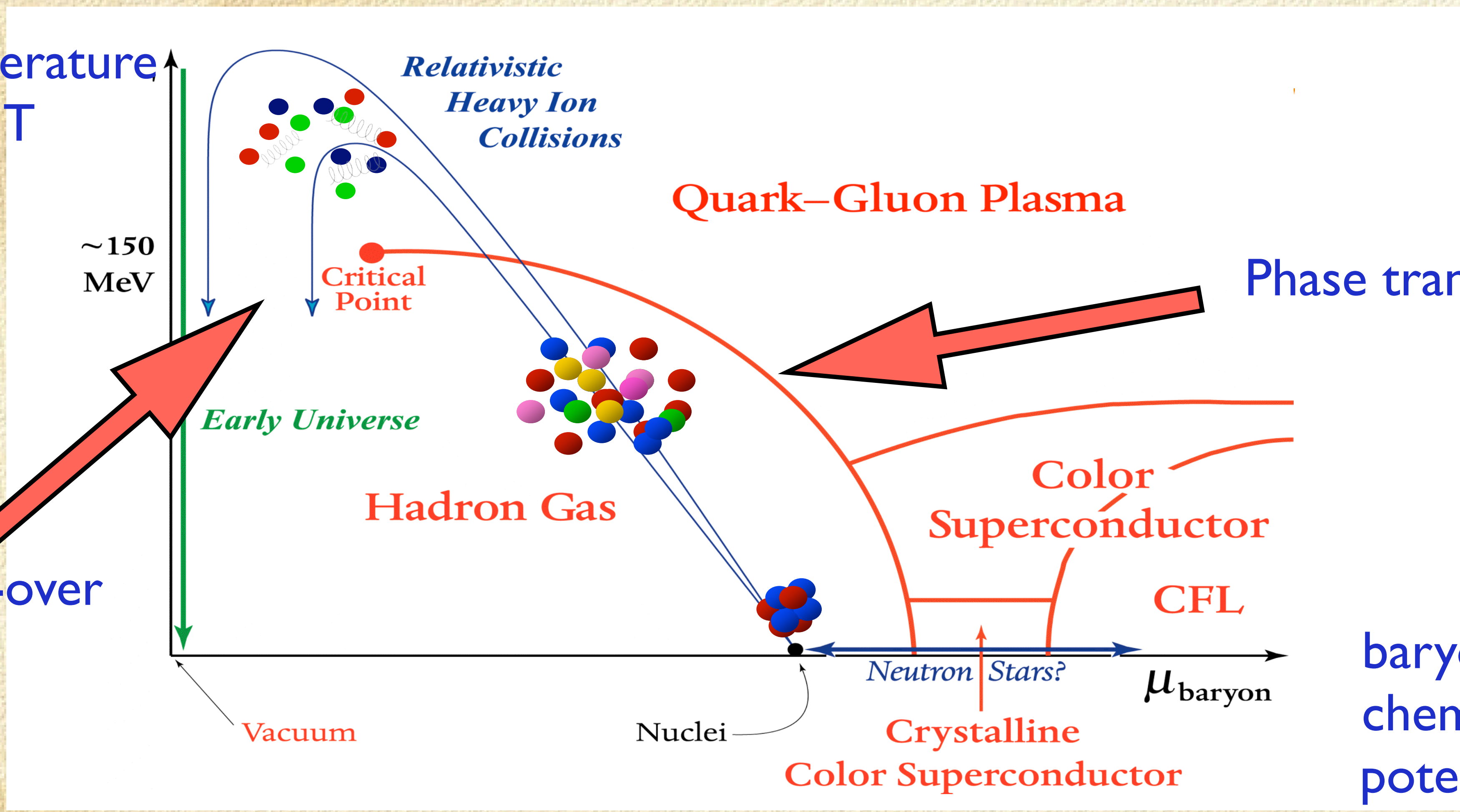
* 1981 An exact analytical solution of the Gas of Bags Model (GBM) is found. The conditions for 1-st, 2-nd order deconfinement PT are discussed.
  M.I.Gorenstein, V.K. Petrov and G.M. Zinovjev, Phys. Lett. B 106 (1981) 327.

* 2004 Shift of the paradigm: from noninteracting quarks and gluons inside QGP to strongly interacting QGP = sQGP (liquid-like phase)

sQGP era begins!

QCD EoS is unknown beyond CEP

Temperature T



Phase transition

Cross-over

baryonic chemical potential

QGP is a dense phase, i.e. it is liquid-like!

But in contrast to our everyday experience (boiling water) QGP appears at higher temperatures!

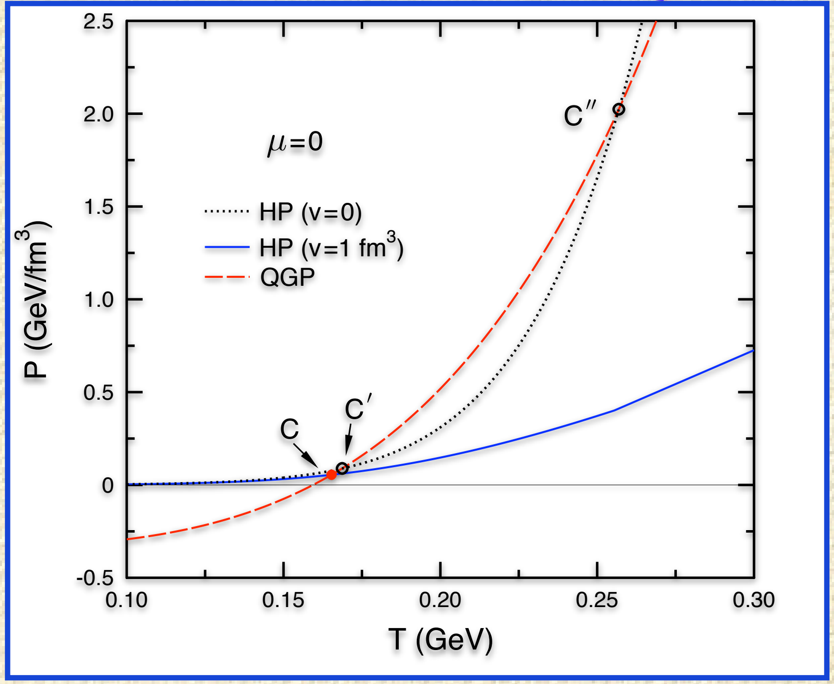
Strategy to build up sQGP EoS

- Extend an exactly solvable model with PT (Gas of Bags Model) to describe QGP liquid
- Use universality properties of liquid-gas EoS and study QCD phase diagram
- Generalize exact solutions to finite systems and define finite volume analogs of phases
- Formulate PT signals for finite systems

What do we need to include into QGP EoS

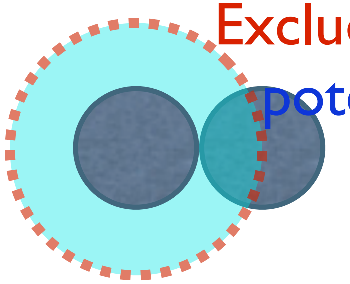
Short range repulsion - otherwise no QGP exists at high T!

Ideal hadron gas has higher pressure and energy density than QGP!

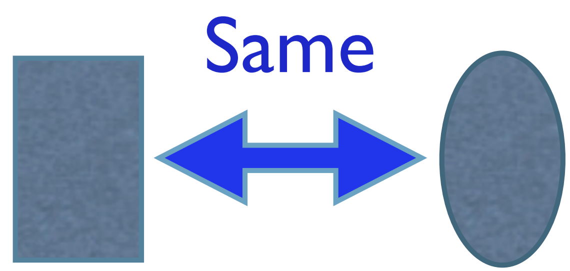


M.I. Gorenstein, G.M. Zinovjev, V.K. Petrov and V.P. Shelest, Teor. Mat. Phys. 52 (1982) 346.

Interaction: Hard core repulsion a la VDW



Excluded Volume (per particle) of hard core potential of radius R is 4 eigen volumes!



Like in SBM

Attraction: is accounted by many sorts of clusters and by their chemical equilibrium.

Eigen volume approximation means that bags are deformable!
Is good for high densities!

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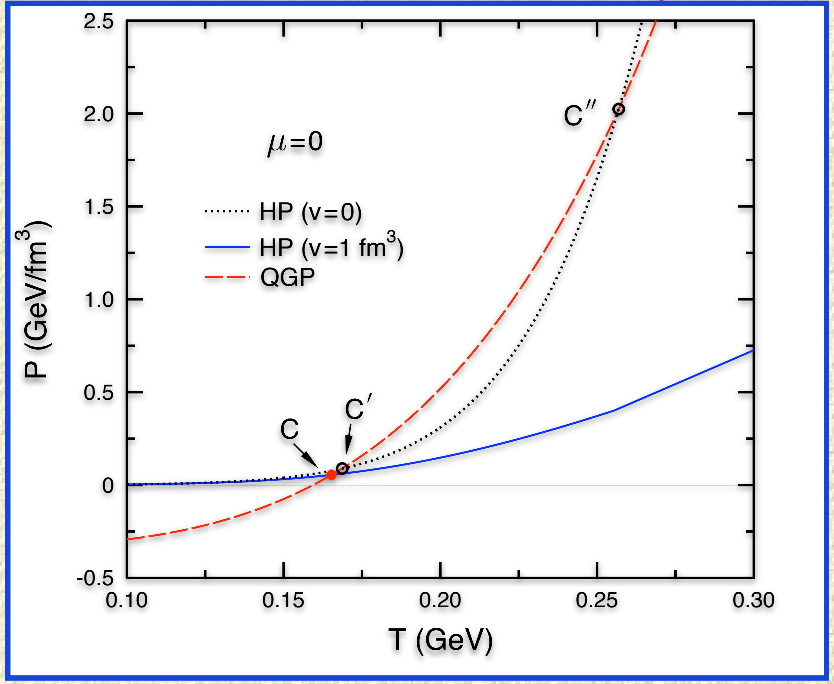


Surface tension of QGP bags since they are similar to liquid droplets!

Use the fact that real gases consist of droplets of all possible sizes!

Model the color confinement!

Basics of the VdWaals EOS

$$P = \underbrace{\frac{NT}{V - Nb}}_{repulsion} - \underbrace{\frac{N^2 a}{V^2}}_{attraction} \quad \text{or} \quad \left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = NT$$

This VdWaals equation cannot be derived rigorously. It is a postulate.

VdWaals EOS is nonstatistical (=classical), but it is simple and it is a first example of the critical point model!

Basics of the VdWaals EOS

$$P = \frac{N\,T}{\underbrace{V - N\,b}_{repulsion}} - \underbrace{\frac{N^2 a}{V^2}}_{attraction} \quad \text{or} \quad \left(P + \frac{N^2 a}{V^2}\right)(V - N\,b) = NT$$

Consider the reduced form of the one component VdWaals EOS:

$$\left(p_R + \frac{3}{v_R^2}\right)\left(v_R - \frac{1}{3}\right) = \frac{8}{3} T_R \quad \text{with}$$

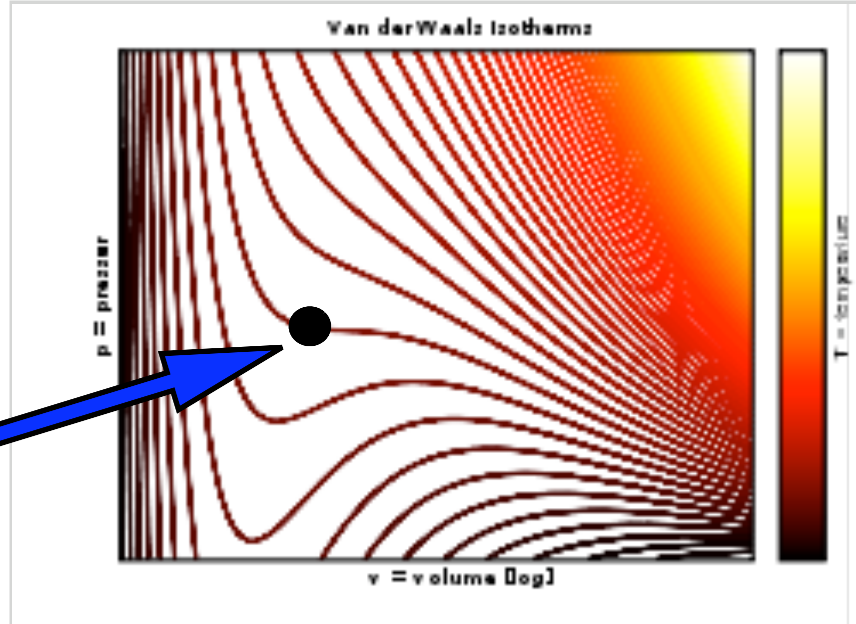
$$T_R = \frac{T}{T_c}\,, \quad p_R = \frac{p}{p_c}\,, \quad v_R = \frac{\rho_c}{\rho}$$

Critical endpoint

$$T_c = \frac{8\,a}{27\,b}\,, \quad p_c = \frac{a}{27\,b^2}\,, \quad \rho_c = \frac{1}{3\,b}$$

Critical parameters follow from

$$\left(\frac{\partial p_R}{\partial v_R}\right)_{T_c} = 0, \quad \left(\frac{\partial^2 p_R}{\partial v_R^2}\right)_{T_c} = 0$$



The van der Waals isotherms: the model correctly predicts a mostly incompressible liquid phase, but the oscillations in the phase transition zone do not fit experimental data.

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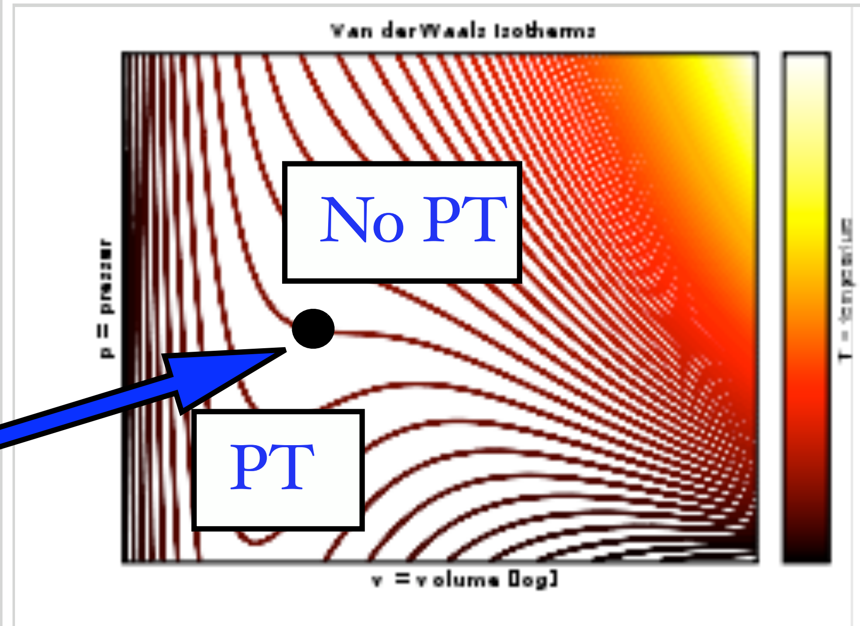

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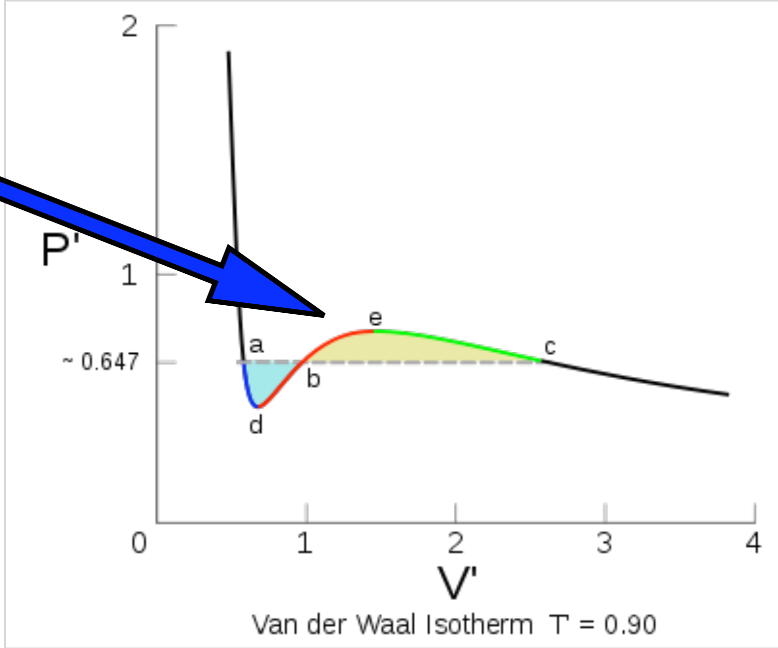
Law of Corresponding States

Although VdWaals EOS behavior contradicts the 2-nd Van Hove axiom of statistical mechanics it was important to formulate the law of corresponding states!



Maxwell's rule eliminates the oscillating behavior of the isotherm in the phase transition zone by defining it as a certain isobar in that zone.

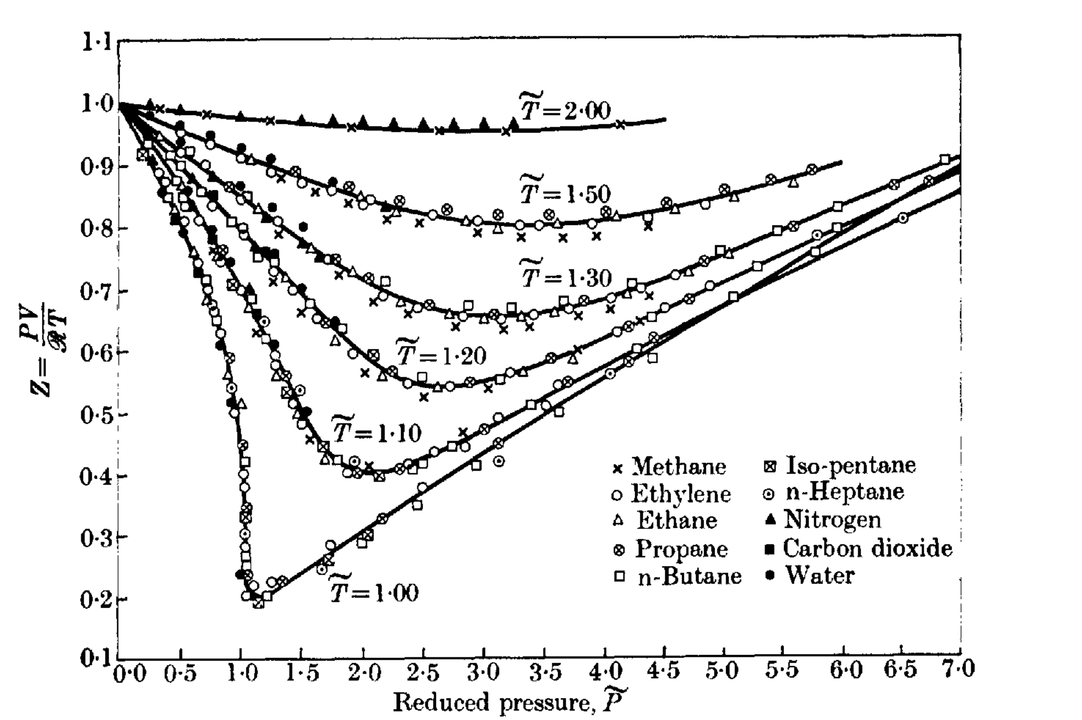


FIG. 5.3. Dependence of the compressibility ratio $Z \equiv PV/\mathscr{R}T$ upon reduced pressure $\widetilde{P}$ for different reduced temperatures $\widetilde{T}$. The fact that the data for a wide variety of fluids fall on identical curves supports the law of corresponding states. After Su (1946).

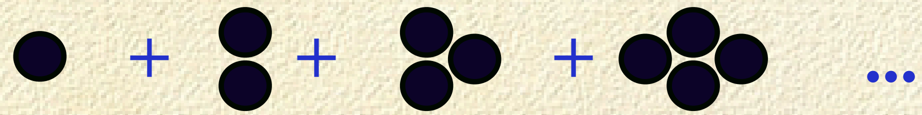
Law of corresponding states:

There exist universal functions (Z=PV/(RT) or similar ones) that show a universal behavior on reduced quantities for all substances within the same universality class!

Why the Van der Waals EoS is Wrong?

Experiments and exactly solvable models of liquid states show that

The real gases consist of droplets of all possible sizes!

Real gas = 

Only this fact explains the reason of how the liquid appears from gas!

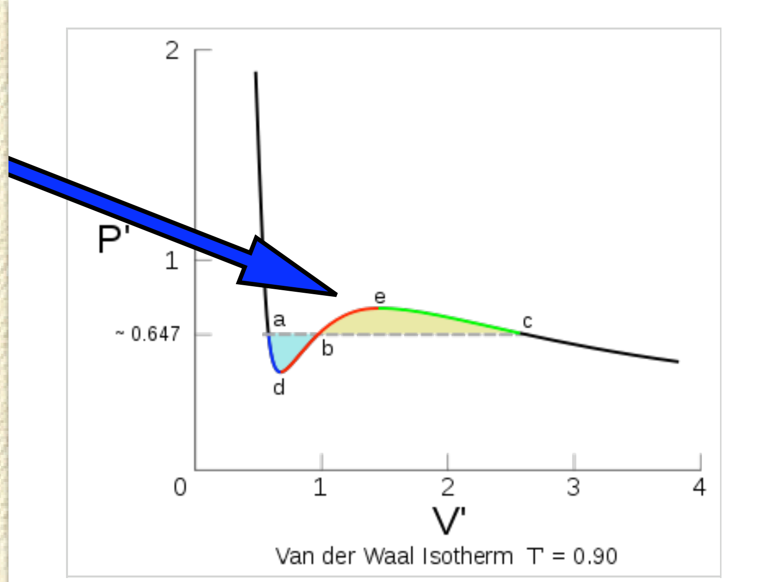
Model	References	Comment
Fisher Droplet Model (FDM)- Condensation of gases	M. Fisher, Physica 3 (1967); J.B. Elliott et al, nucl-ex/0608022 (2006)	Describes the gas only! NO liquid phase!
Statistical Multifragmentation Model (SMM) [without Coulomb interaction]- Liquid-Gas PT in nuclear matter	J. P. Bondorf et al, Phys. Rep. 257(1995); K.A.B., Phys. Part. Nucl. 38 (2007);	Elaborate model, but liquid phase has limiting density! $\Rightarrow$problems at high pressure!

Despite all problems these models describe the (tri)critical endpoint very well, since they account for vanishing surface tension at endpoint!

AntiRandrup & Co

2. The mechanically unstable part of VdW isotherms does not correspond to a HOMOGENEOUS matter!

In physics (and in stat.mechanics) the matter is DISCRETE! Recall systems of hadrons, of nuclei, of electrons e.t.c. Homogeneity is always a question of scale.



Maxwell's rule eliminates the oscillating behavior of the isotherm in the phase transition zone by defining it as a certain isobar in that zone.

3. J. Randrup & Co convert this homogeneous matter into droplets with some tricks.

But the elaborate statistical models of phase transitions are dealing not with the molecules, but with droplets of all possible sizes. These are relevant dof!

4. In hydrodynamics the evolution of supercooled droplets is well known from works of L. van Hove, M. Gyulassy, H. Bartz, L. Csernai e.t.c.

Therefore, it is unclear why J. Randrup & Co need an approximated (linearized) hydro, if we have deflagration and detonation!

More to AntiRandrup & Co

5. Suppose we have to accurately model the QCD (tri)critical endpoint obtained by lQCD. Then we cannot use the VdWaals EOS because it has different exponents than QCD (tri)critical endpoint.

VdWaals exponents: $\alpha' = 0, \quad \beta = \frac{1}{2}, \quad \gamma' = 1, \quad \delta = 3$

Recall A. Ivanytskyi talk on exponents

	2d Ising model	Simple liquids	3d Ising model
α'	0	0.09-0.11	0.1096 ± 0.0005
β	$\frac{1}{8}$	0.32-0.35	0.3265 ± 0.0001
γ'	$\frac{7}{4}$	1.2-1.3	1.2373 ± 0.0002
δ	15	4.2-4.8	4.7893 ± 0.0008

Relevant to QCD

	O(3)	O(4)
α'	-0.115(9)	-0.19(6)
β	0.3645(25)	0.38(1)
γ'	1.386(4)	1.44(4)
δ	4.802(37)	4.82(5)

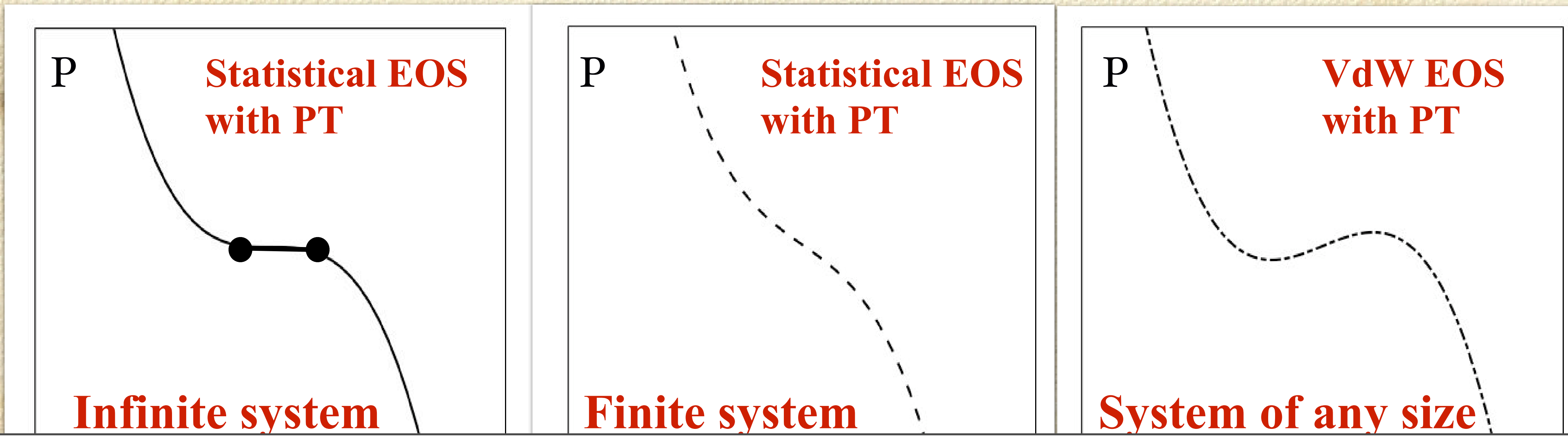
From my experience the choice of critical indices defines very strong restrictions on the statistical model parameters! If you use the wrong exponents, then you cannot properly describe thermodynamics data!

Even More to AntiRandrup & Co

6. The reason of why VdWaals EOS has a wrong mechanism of the critical endpoint generation is in the absence of surface tension

7. What should we do in finite systems we are dealing with?
In finite systems the phase transitions (in strict statistical sense) do not exist. True statistical models do show such a behavior.

However, VdWaals EOS has a phase transition even at vanishing volume of the system! Therefore, I do not understand such a logic: according to J. Randrup & Co one has to use VdWaals EOS in a finite system and, thus, to generate a phase transition which does not exist in it. What for?



Source of Induced Surface Tension

Pressure of N-sorts particles with hard core radii R_k up to 2-nd virial coefficient

$$p(T,\mu) \approx T \sum_{k=1}^{N} \phi_k e^{\frac{\mu_k}{T}} (1 - \sum_{n=1}^{N} a_{kn}\, \phi_n\, e^{\frac{\mu_n}{T}}), \quad \phi_n(T) \quad \text{is thermal particle density}$$

a_{kn} **is the 2-nd virial coefficient between hard core radii R_k and R_n**

$$a_{kn} = \frac{2}{3\pi}(R_k + R_n)^3 = \frac{2}{3\pi}(R_k^3 + 3R_k^2 R_n + 3R_k R_n^2 + R_n^3)$$

Usual VdWaals approximation: the pressure is extrapolated to high density as

$$p = \sum_{k=1}^{N} p_k \approx T \sum_{k=1}^{N} \phi_k e^{\frac{\mu_k}{T}} \left(1 - \sum_{n=1}^{N} a_{kn} \frac{p_n}{T}\right) \approx T \sum_{k=1}^{N} \phi_k \exp\left[\frac{\mu_k}{T} - \sum_{n=1}^{N} a_{kn} \frac{p_n}{T}\right]$$

But it is not unique procedure! Substituting a_{nk} and regrouping terms we have

$$p \approx T \sum_{k=1}^{N} \phi_k e^{\frac{\mu_k}{T}} \left[1 - \frac{4}{3}\pi R_k^3 \cdot \sum_{n=1}^{N} \phi_n e^{\frac{\mu_n}{T}} - 4\pi R_k^2 \cdot \sum_{n=1}^{N} R_n \phi_n e^{\frac{\mu_n}{T}}\right]$$

$$= T \sum_{k=1}^{N} \phi_k e^{\frac{\mu_k}{T}} \left[1 - \frac{4}{3}\pi R_k^3 \cdot \frac{p}{T} - 4\pi R_k^2 \cdot W_1\right]$$

$$\simeq \boxed{T \sum_{k=1}^{N} \phi_k e^{\frac{\mu_k}{T}} \exp\left[-\underbrace{\frac{4}{3}\pi R_k^3 \cdot \frac{p}{T}}_{volume\ part} - \underbrace{4\pi R_k^2 \cdot W_1}_{surface\ part}\right]}, \quad \text{with} \quad W_1(T,\mu) = \sum_{k=1}^{N} R_k \phi_k e^{\frac{\mu_k}{T}}$$

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density

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pressure/T

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Hadronic Surface Tension

V. Sagun, A. Ivanytskyi, K.A.B., I.N. Mishustin in preparation

I. Hard core repulsion => the energy part of surface free energy

II. The attraction => the entropy part of surface free energy

D. Oliinychenko, K.A.B., A.S. Sorin, arXive:1204.0103 hep-ph

Hadron Resonance Gas with surface tension

Surface free energy like in Fisher droplet model:

$$F_{surf} = \sigma_0 \left(1 - \frac{T}{T_0}\right) 4\pi R_k^2, \quad k \in \{\text{Baryons, Mesons}\}$$

Collision energies set, $\sqrt{S_{NN}}$	χ^2/NDF with surface tension	σ_0, MeV fm^{-2}	T_0, MeV
2.7 - 7.6	$25.8043/31 = 0.832$	$\mathbf{0.91 \cdot 10^{-2}}$	61
2.7 - 200	$103.036/80 = 1.288$	$\mathbf{-1.37 \cdot 10^{-2}}$	57
2.7 - 62.4 (no 130 and 200)	$85.268/63 = 1.3534$	$\mathbf{-3.21 \cdot 10^{-2}}$	62
12, 17, 62.4, 130, 200	$62.1454/35 = 1.776$	0.654	147

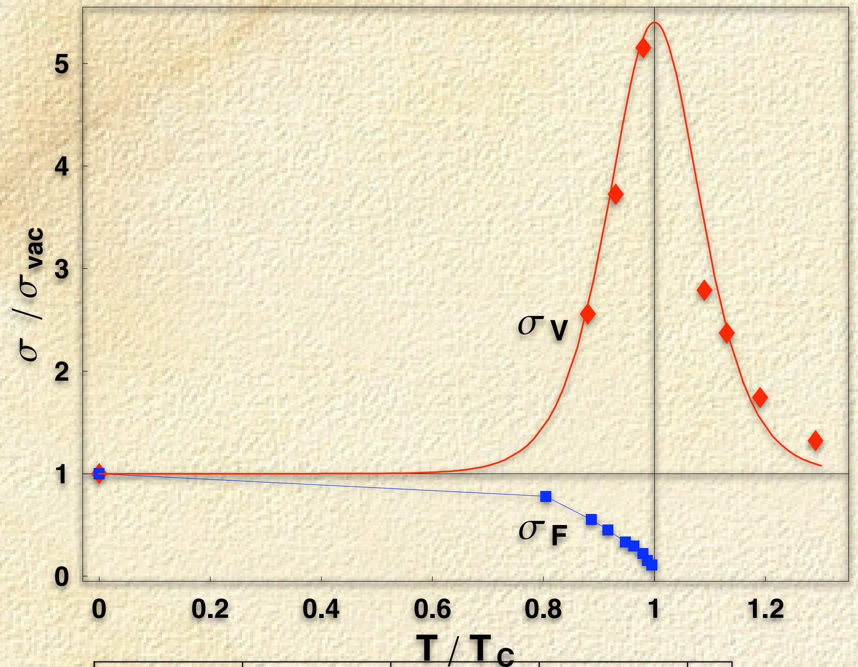
Table 1: **Results of the global fit, including the extracted surface tension parameters.**

Can We Find the Surface Tension of QG bags?

Confinement by Color String within sQGP

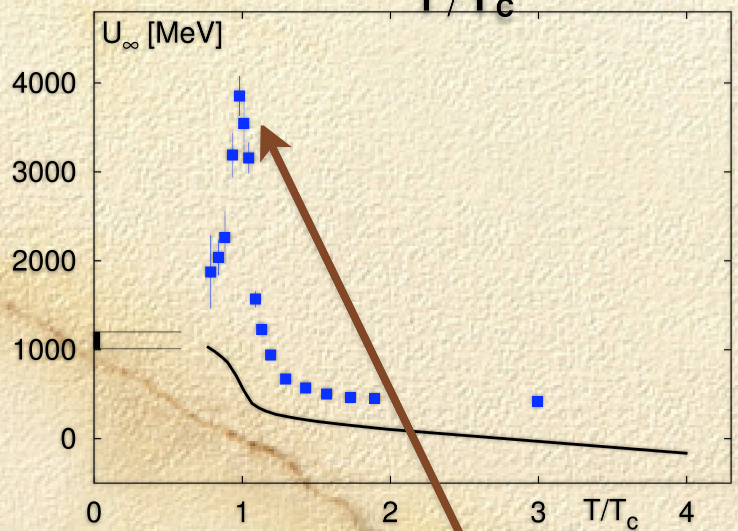
Internal energy U, entropy S

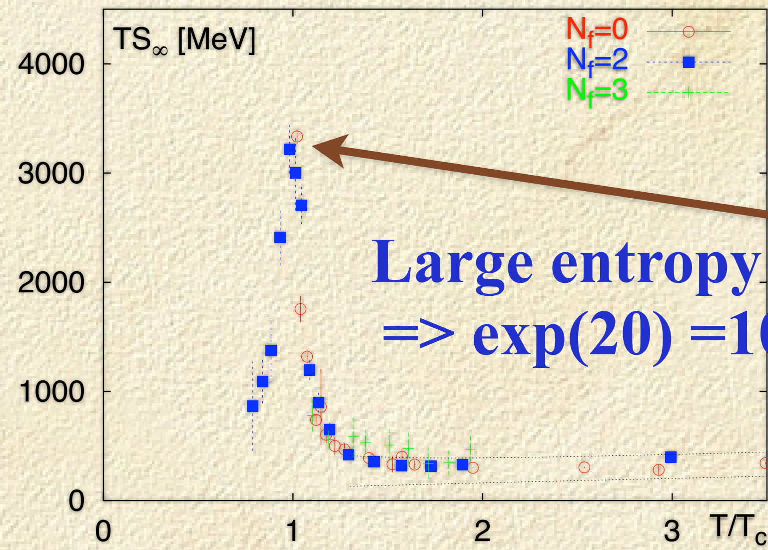
$$U(T,r) = F - TdF/dT = F + TS$$



String tension for internal energy (V)

String tension for free energy (F) —> 0





Large entropy S = 20!
=> exp(20) =108 !?**

Very strong interaction! => No color charge separation!

Confining String = Color Tube

Consider confining string between static q & anti q of length L and radius R<<L

outer pressure Ptot

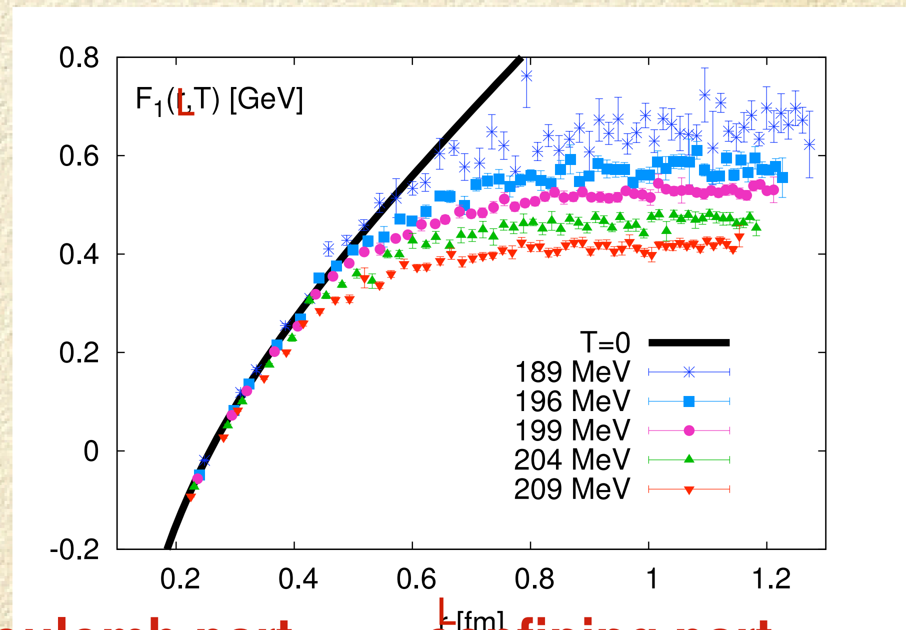
color anticolor

Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Confinement means infinite free energy for infinite L

Deconfinement means that string tension vanishes

Can be rigorously found by Lattice QCD



Coulomb part confining part

String Tension vs Surface Tension

K.A.B., G.M. Zinovjev, Nucl. Phys. A848 (2010)

Consider now this tube as the cylindrical bag of length L and radius R<<L

Neglect effects of color sources and get cylinder FREE ENERGY as:

$$F_{cyl}(T,L,R) \equiv -\underbrace{p_v(T)\pi R^2 L}_{thermal} + \underbrace{\sigma_{surf}(T) 2\pi R L}_{surface} + \underbrace{T\tau \ln \frac{V}{V_0}}_{small}$$

Equating the cylinder FREE ENERGY to string free energy $F_{str} = \sigma_{str} L$

$$\boxed{\sigma_{str}(T) = \sigma_{surf}(T)\, 2\pi R \; - \; p_v(T)\pi R^2} + \cancel{\frac{T\tau}{L} \ln\left[\frac{\pi R^2 L}{V_0}\right]}$$

We got a new possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T=0) = -(0.25)^4$ GeV^4, $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2$ GeV^2 $\Rightarrow$

$$\sigma_{surf}(T=0) = (0.2229 \text{ GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183 \text{ GeV})^3} \approx 157.4 \text{ MeV fm}^{-2}.$$

Surface Tension at Cross-over

For vanishing σ_{str} one has $\quad \sigma_{str}^{LQCD} \approx \frac{\ln(L/L_0)}{R^2} C$

This is due to increase of surface fluctuations $\quad \Rightarrow \quad$ in general

$$\sigma_{str}(T)\, R^k \to \omega_k > 0 \quad \text{for} \quad k > 0$$

Parametrize $\sigma_{str} = \sigma_{str}^0\, t^\nu, \quad$ where $\quad t \equiv \frac{T_{tr}(\mu) - T}{T_{tr}(\mu)} \to +0$

and find total pressure and total entropy density

for $\mu = 0$ (baryonic chemical potential)

$$p_{tot} = p_v(T) - \frac{\sigma_{surf}(T)}{R} \equiv \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{\pi R^2} \to \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \left[\sigma_{surf} - \frac{\omega_k}{\pi}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{k+1}{k}}\right]$$

$$s_{tot} = \left(\frac{\partial\, p_{tot}}{\partial\, T}\right)_\mu \to \underbrace{\frac{1}{k\, \sigma_{str}} \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\, \sigma_{str}}{\partial\, T}\, \sigma_{surf}}_{dominant \quad since \quad \sigma_{str} \to\ 0} + \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\ \sigma_{surf}}{\partial\ T} - \frac{k+2}{\pi\, k}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{2}{k}} \frac{\partial\ \sigma_{str}}{\partial\ T}$$

For finite σ_{surf} and $\frac{\partial\ \sigma_{str}}{\partial\ T} < 0$ $\quad \Rightarrow \quad \sigma_{surf} < 0 \quad$ since $\quad s_{tot} > 0$

Comparison with LQCD

$\Rightarrow$ Assume: we can apply our results to LQCD data with $L \gg R$

For $\sigma_{str} \to 0 \quad \Rightarrow \quad R \to \frac{2\,\sigma_{surf}}{p_v}$ and lattice entropy is

$$\frac{S_{lat}}{L} = -\frac{1}{L}\frac{\partial F_{lat}}{\partial T} \to -\frac{s_{tot}\,k\,\sigma_{str} R}{\sigma_{surf}} = -\frac{s_{tot}\,k\,\omega_k}{\sigma_{surf} R^{k-1}} \to t^{\nu-1}$$

$\Rightarrow$ again $\sigma_{surf} < 0$

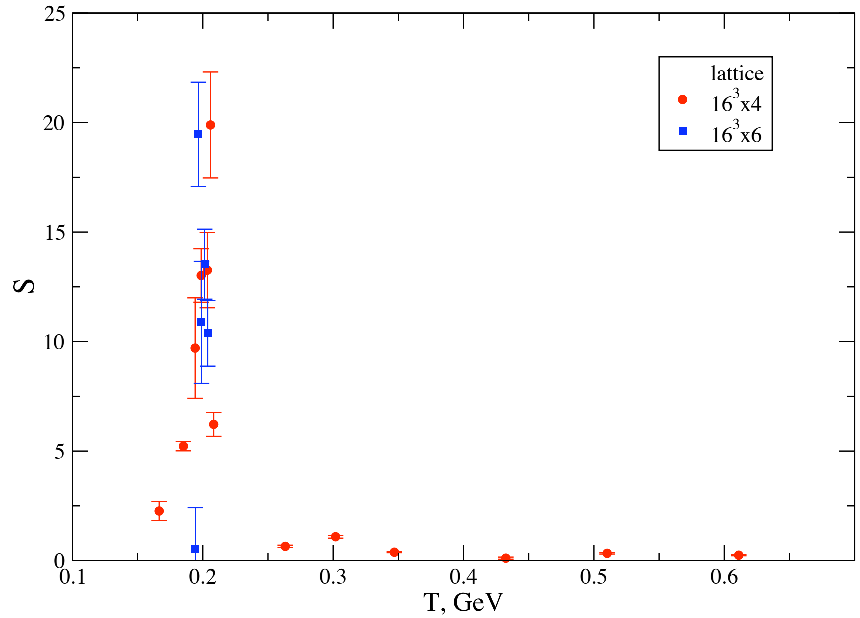
$\Rightarrow$ S_{lat} diverges for $\nu < 1$ and $R \to \infty$

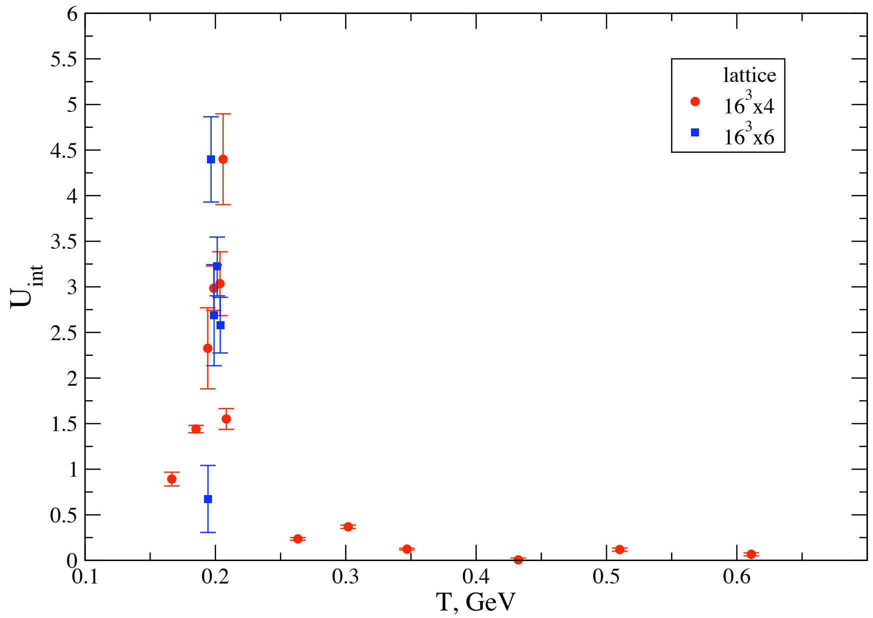
$\Rightarrow$ S_{lat} has a sharp inclease for $\nu < 1$ and $R \to R_{lat} < \infty$

Can we verify this result with LQCD data?

Mysterious Maximum

Entropy and Internal Energy





Similarly, consider the fall down of S_{lat} due to strong s_{tot} decrease

This explains ‘a mysterious maximum in S_{lat}’ (E. Shuryak)

Why Does the String Entropy Diverge at the Cross-over ?

$$\frac{S_{str}}{L} = \frac{\sigma_{str}^0 \, \nu}{T_{tr}} \, t^{\nu-1} \to \frac{\nu}{T_{tr}} \left[\frac{\sigma_{str}^0}{\omega_k^{1-\nu}} \right]^{\frac{1}{\nu}} \boxed{R^{\frac{k(1-\nu)}{\nu}}}$$

String entropy diverges for $\nu < 1$ and $t \to +0$.

$\boxed{R \text{ power } \frac{k(1-\nu)}{\nu}}$ is FRACTAL for any $\nu \neq \frac{k}{k+n}$ where $n = 1, 2, 3, ...$

In LQCD the fractal structures are well known.

In this model the fractals appear at $t \to +0$ as surface deformations due to zero total pressure inside the color tube $\Rightarrow$ $\boxed{\text{at NO ENERGY costs!}}$

=> At the cross-over temperature there exist FRACTALS!

Surface Tension Summary

I. We got a possibility to determine QGP bag surface tension directly from LQCD!

$$\sigma_{surf}(T=0) = (0.2229\ \mathrm{GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183\ \mathrm{GeV})^3} \approx 157.4\ \mathrm{MeV\ fm}^{-2}.$$

II. At T= 0 the bag surface tension is rather large!

III. At the cross-over temperature there exist FRACTALS!

$$\frac{S_{str}}{L} = \frac{\sigma^0_{str}\,\nu}{T_{tr}}\, t^{\nu-1} \longrightarrow \frac{\nu}{T_{tr}} \left[\frac{\sigma^0_{str}}{\omega_k^{1-\nu}}\right]^{\frac{1}{\nu}} R^{\frac{k(1-\nu)}{\nu}}$$

String entropy diverges for $\nu < 1$ and $t \longrightarrow +0$.

R power $\frac{k(1-\nu)}{\nu}$ is FRACTAL for any $\nu \neq \frac{k}{k+n}$ where $n = 1, 2, 3, \ldots$

VI. Like in ordinary liquids: zero surface tension defines T of (tri)critical point!

$$T_{cep} = T_\sigma = 152.9 \pm 4.5\ \mathrm{MeV}$$

K.A.B. et al, arXiv:1101.4549

V. At the cross-over temperature the bag surface tension must be negative!

Surface Tension Summary

Remarkable fact: chemical FO data for rHIC gives T_σ = 147 ± 7 MeV

which is almost the same as color tube model predictions!

$T_{cep} = T_\sigma = 152.9 \pm 4.5$ MeV **K.A.B. et al, Phys. Atom. Nucl. 75 (2012)**

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Is there any problem with negative surface tension coefficient?

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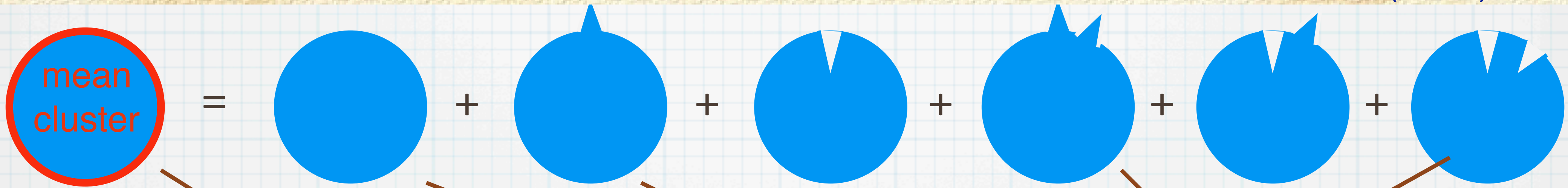
Surface Free Energy: F = E -TS

To find surface F one has to count for ALL surface deformations together with energy costs

Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A.B. et al, PRE 72 (2005)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy}\times\left\{1 + \left(\underbrace{w_H N_H}_{1\ Hill}+\underbrace{w_D N_D}_{1\ Dale}\right)\exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \quad \geq \quad F \quad \geq \quad \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}\,, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A.B. & Elliott, UJP 52 (2007)

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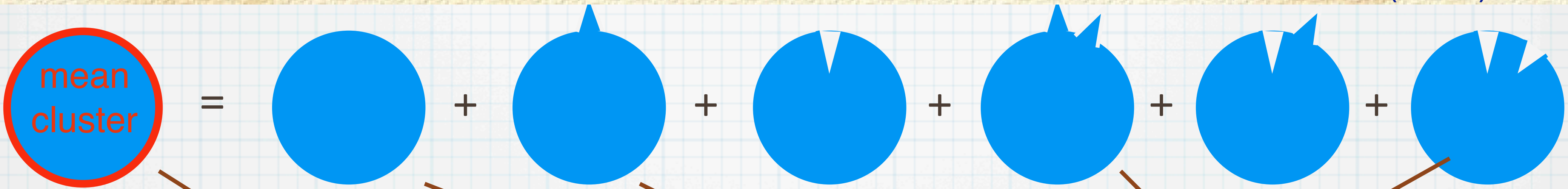

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K.A.B. & Elliott, UJP 52 (2007)

Thus, there is NOTHING wrong, if surface F < 0 above critical T!

This means only that entropy dominates!

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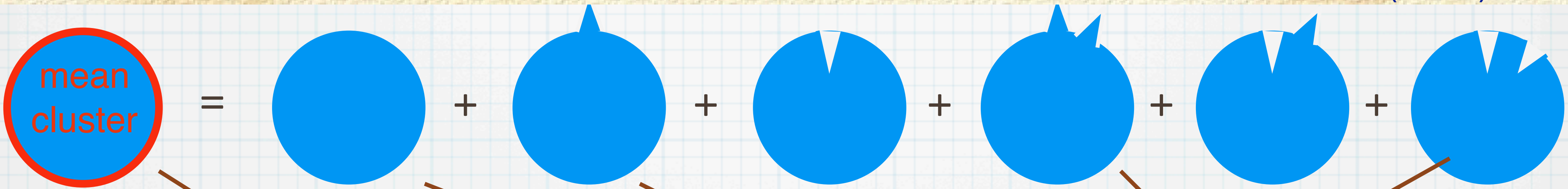

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Story is not over yet! The surface tension is even more important!

Thus, there is NOTHING wrong, if surface F < 0 above critical T!

This means only that entropy dominates!

What About Ordinary Liquids?

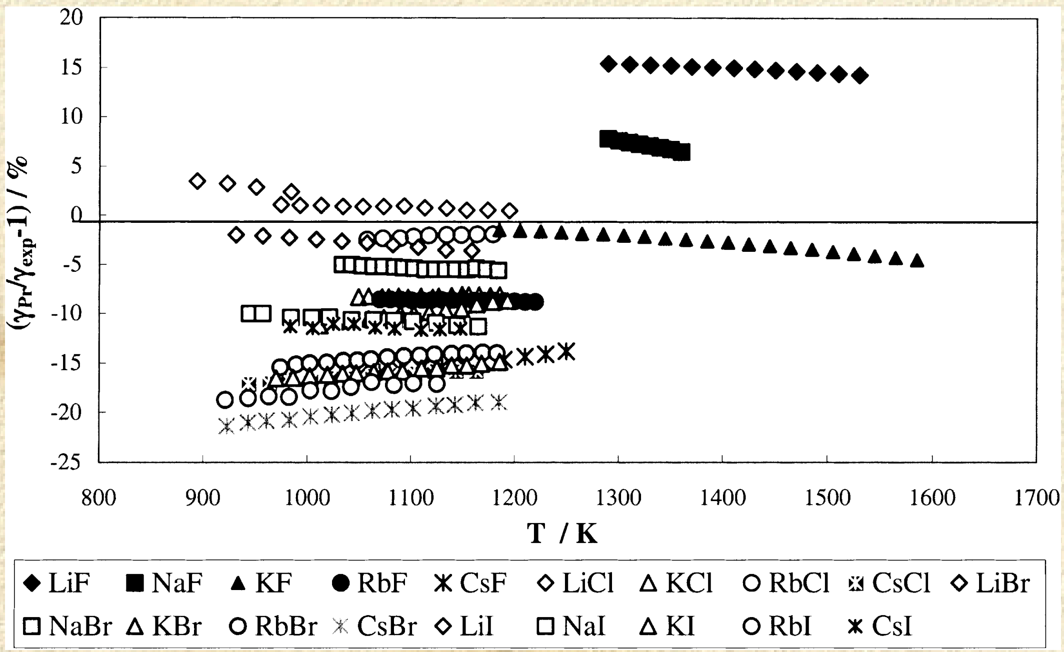
РОССИЙСКАЯ АКАДЕМИЯ НАУК
НАУЧНЫЙ СОВЕТ ПО ФИЗИКЕ НИЗКОТЕМПЕРАТУРНОЙ ПЛАЗМЫ
/Секция термодинамических, оптических и переносных свойств/
ИССЛЕДОВАТЕЛЬСКИЙ ЦЕНТР "ФАИР - РОССИЯ"
ОБЪЕДИНЕННЫЙ ИНСТИТУТ ВЫСОКИХ ТЕМПЕРАТУР РАН
МОСКОВСКИЙ ФИЗИКО-ТЕХНИЧЕСКИЙ ИНСТИТУТ

Физика вещества с высокой концентрацией энергии

Научно-координационная Сессия "Исследования неидеальной плазмы"

1 - 2 декабря 2010 г., ПРЕЗИДИУМ РАН, Ленинский пр-т 32а, Москва

N. Galamba et al. / Fluid Phase Equilibria 183–184 (2001) 239–245



Surface tension deviations calculated as a function of temperature for all the molten alkali halides studied.

1. Present day models for surface tension are not precise to make some certain conclusions.

2. So far, the specialists in liquids overlooked that negative values of the surface tension coefficient can exist.

3. The existence of negative surface tension coefficients does not contradict to any known fact!

The Van der Waals Repulsion

The Grand canonical partition (GCP) of n hadronic bags
with the hard-core repulsion of the Van der Waals type ($\mu_B = 0$)

$$\mathbf{Z}(V,T) = \sum_{\{N_k\}} \left[\prod_{k=1}^{n} \frac{\left[(V - v_1 N_1 - ... - v_n N_n)\, \phi_k(T) \right]^{N_k}}{N_k!} \right] \boxed{\theta\left(V - v_1 N_1 - ... - v_n N_n \right) \,,}$$

the particle density of bags of mass m_k and eigen volume v_k and degeneracy g_k

$$\phi_k(T) \equiv g_k\ \phi_k(T) \equiv \frac{g_k}{2\pi^2} \int\limits_0^\infty p^2 dp\ e^{-\frac{(p^2\ +\ m_k^2)^{1/2}}{T}} = g_k \frac{m_k^2 T}{2\pi^2}\ K_2\left(\frac{m_k}{T}\right)$$

Using the standard Laplace transformation with respect to volume V,
one gets the **isobaric partition with the simple pole:**

$$\hat{Z}(s,T) \equiv \int\limits_0^\infty dV \exp(-sV)\ Z(V,T) = \frac{1}{[s - F(s,T)]}$$

describes hard core repulsion in GC ensemble

with $\quad F(s,T) \equiv \sum_{j=1}^{n} \boxed{\exp\left(-v_j s\right)} g_j \phi(T, m_j)\ .$

- The Θ function is VERY important because ensures that bags do not overlap!

Basic Ingredients of QGBST Model

If the number of bag kinds is infinite, there may appear an essential singularity of the Isobaric Partition. This is used in GBM and QGBST to generate PT. This can be seen as follows (also for non-zero μ):

For $V \to \infty$ the whole analysis is reduced to the analysis of the Singularities of IP!

After Inverse Laplace transform GCP becomes

$$Z(V,T,\mu) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} ds\ Z(s,T,\mu)\ e^{s\,V} =$$

$$\sum_{s_i^*} Res\left(\ Z(s_i^*,T,\mu)\ \ e^{s_i^*\,V}\right) \longrightarrow e^{V\,\max\,(s_i^*)}$$

Comparing with

$$Z(V,T,\mu) \longrightarrow e^{\frac{p\,V}{T}} \Longrightarrow p(T,\,\mu) = T\,\max\,(s_i^*)\ ,$$

Equation for Singularities:

$$s^*(T)\ =\ F(s^*,T)$$

where $\sigma > \max Re(s_i^*)$ - **the most right singularity.**

- PT happens, if two singularities coincide.

Mass-Volume Spectrum of QGBST Model

Assume: there exist the **discrete mass-volume spectrum** $F_H(s,T)$ of hadrons lighter than M_0 and the **continuous volume spectrum** $F_Q(s,T)$

discrete part **continuous part**

$$F(s,T) \equiv F_H(s,T) + F_Q(s,T) =$$

Hagedorn spectrum

$$\sum_{j=1}^{n} g_j e^{-v_j s} \phi(T, m_j) + u(T) \int\limits_{V_0}^{\infty} dv \, \frac{\exp\left[\left(s_Q(T) - s\right) v - \sigma(T)\, v^{\varkappa}\right]}{v^{\tau}}$$

hadron resonance gas **QGbags**

- K.A.B., PRC 76 (2007)

Term F_H has no s-singularities at any T and generates a simple pole only!

The bag spectrum $F_Q(s,T)$ is chosen to give $\boxed{\text{an essential singularity } s_Q(T) \equiv \frac{p_Q(T)}{T}.}$

$s_Q(T)$ defines QGP pressure $p_Q(T)$ at zero baryonic density (MIT Bag Model).

The (reduced) $\boxed{\textbf{surface tension coefficient}}$ $\sigma(T) = \frac{\sigma_o}{T} \cdot \left[\frac{T_{cep} - T}{T_{cep}}\right]^{2k+1}$ $(k = 0, 1, 2, ...)$.

$\sigma_o = Const > 0$, but can be a smooth function of T (and μ_B).

- Note, here T_{cep} = Const, but later it will be μ_B dependent!

The Role of Surface Tension. I

Case **I:** $\sigma(T) > 0$ is very similar to GBM with $\tau > 2$.

$s_Q(T) < 0$ at low T $\Rightarrow$ the simple pole $s^* = s_H(T)$ is the rightmost singularity.

At very high T the QGP pressure dominates $\Rightarrow$ $s^* = s_Q(T)$ is the rightmost singularity.

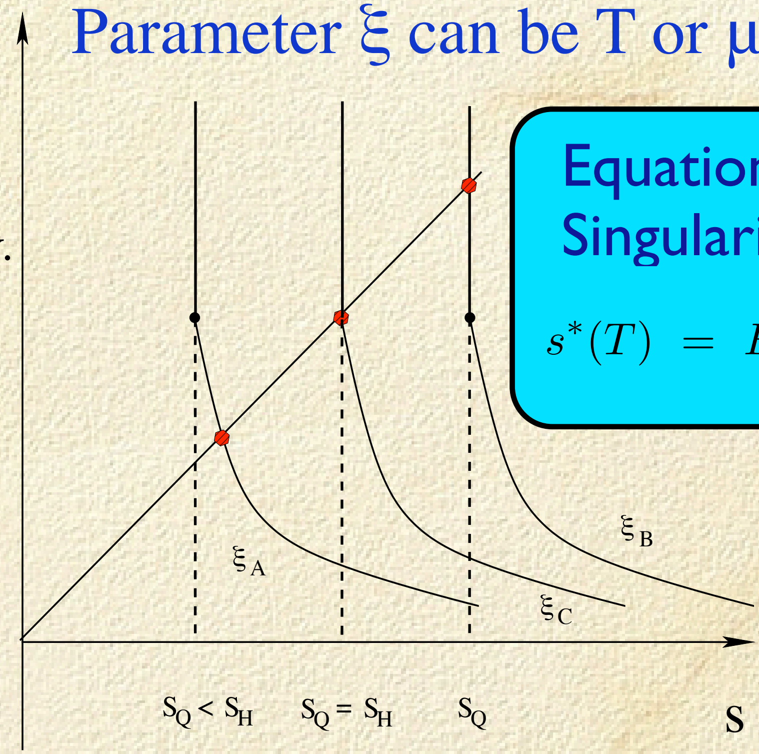
PT occurs, when the singularities coincide:

$$s_H(T_c) \equiv \frac{p_H(T_c)}{T_c} = s_Q(T_c) \equiv \frac{p_Q(T_c)}{T_c} \quad \text{or } \Delta = 0$$

which is just **Gibbs criterion.**

PT order follows from T-derivatives of $s_H(T)$.

Parameter ξ can be T or μB



Equation for Singularities:

$$s^*(T) \;=\; F(s^*, T)$$

$$s'_H \;=\; \frac{G \;+\; u\,\mathcal{K}_{\tau-1}(\Delta, -\sigma)\cdot s'_Q}{1 \;+\; u\,\mathcal{K}_{\tau-1}(\Delta, -\sigma)}\,, \quad \text{where } G \equiv F'_H + \frac{u'}{u} F_Q + \frac{(T_{cep}-2kT)\sigma(T)}{(T_{cep}-T)\,T}\, u\,\mathcal{K}_{\tau-\varkappa}(\Delta, -\sigma)\,,$$

$$\Delta \equiv s_H - s_Q \text{ and } \mathcal{K}_{\tau-a}(\Delta, -\sigma) \equiv \int_{V_o}^{\infty} dv\, \frac{\exp\left[-\Delta v - \sigma(T) v^{\varkappa}\right]}{v^{\tau-a}}\,,$$

Since for $\sigma(T) > 0$ all integrals are finite $\Rightarrow$ $s'_Q(T_c) \neq s'_H(T_c)$, there must exists $\mathbf{1}^{st}$ **order PT**.

The Role of Surface Tension. II

Case **II:** $T = T_{cep} \Rightarrow \sigma(T) = 0$ is simply equivalent to GBM.

At $s = s_Q(T_{cep})$ there exists PT for $\tau > 1$. The PT order depends on τ:

$$s \;=\; F_H(s,T) \;+\; F_Q(s,T) \quad \text{with} \quad F_Q(s,T) \equiv u(T) \int_{V_0}^{\infty} dv \, \frac{1}{v^{\tau}} < \infty \,, \quad \text{if} \quad \tau > 1$$

$$\mathcal{K}_{\tau-1}(0,0) \equiv \int_{V_o}^{\infty} dv \, \frac{1}{v^{\tau-1}} \to \infty \,, \quad \text{if} \quad \tau < 2 \qquad \Rightarrow \qquad s'_H \;=\; \frac{G \;+\; u\,\mathcal{K}_{\tau-1}(\Delta,-\sigma)\cdot s'_Q}{1 \;+\; u\,\mathcal{K}_{\tau-1}(\Delta,-\sigma)} \,,$$

For $\tau > 2 \Rightarrow \; s'_H(T_{cep}) \neq s'_Q(T_{cep})$, i.e. PT is 1^{st} order.

For $\tau \leq 2 \Rightarrow \; s'_H(T_{cep}) = s'_Q(T_{cep})$, i.e. PT is 2^{nd} or higher order.

Can be shown from second derivative that 2^{nd} order PT exists for $\frac{3}{2} < \tau \leq 2$.

In general for $(n+1)/n \leq \tau < n/(n-1)$ $(n = 3, 4, 5, ...)$ there is a n^{th} order phase transition

$$\begin{aligned} s_H(T_c) \;&=\; s_Q(T_c) \,, \quad s'_H(T_c) \;=\; s'_Q(T_c) \,, \; ... \\ s_H^{(n-1)}(T_c) \;&=\; s_Q^{(n-1)}(T_c) \,, \quad s_H^{(n)}(T_c) \;\neq\; s_Q^{(n)}(T_c) \,, \end{aligned}$$

with $s_H^{(n)}(T_c) = \infty$ for $(n+1)/n < \tau < n/(n-1)$ and

with a finite value of $s_H^{(n)}(T_c)$ for $\tau = (n+1)/n$.

The Role of Surface Tension. III

Case **III**: $\sigma(T) < 0$ is principally different from GBM

and **provides the cross-over existence.**

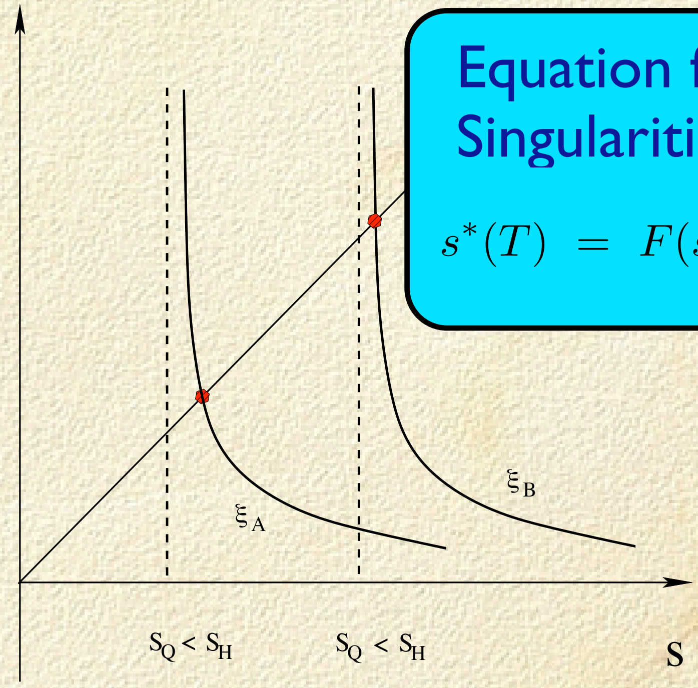
$\mathcal{K}_\tau(0, -\sigma)$ diverges irrespective to τ value!

$\mathcal{K}_\tau(s - s_Q(T) > 0, -\sigma)$ is finite and decreasing function of s

$\Rightarrow$ **simple pole** is rightmost singularity as long as $\sigma(T) < 0$

$s_Q(T)$ can be rightmost singularity at $s_Q(T) \to \infty$
($\equiv T \to \infty$) only!

Parameter ξ can be T or μ_B



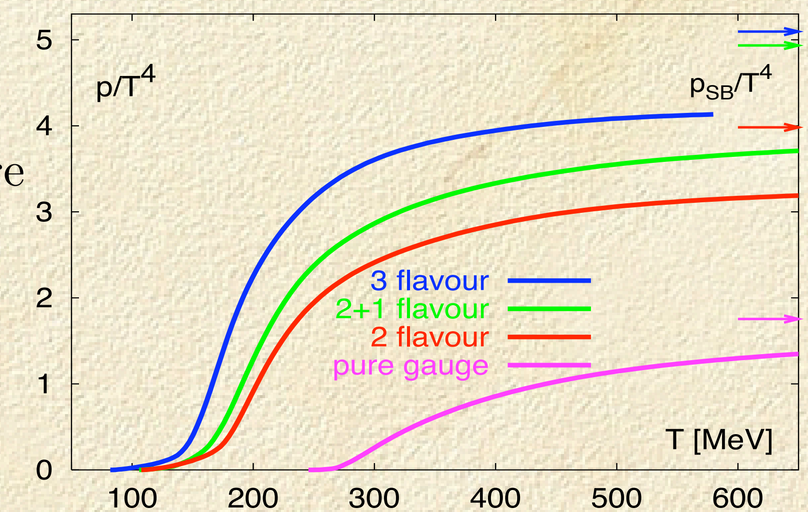
Equation for Singularities:

$$s^*(T) \ = \ F(s^*, T)$$

Compare this with Lattice QCD data and
$N = 2$ SUSY YM (Seiberg-Witten theory):

In Lattice QCD the **Stefan-Boltzmann limit** for pressure and energy density of free $q, \bar{q}, g$ **has not been seen yet** above PT!

$N = 2$ SUSY YM (Seiberg-Witten theory) predicts such a behavior for finite T!



QGBST model can easily handle such a behavior due to **cross-over!**

Non-zero Baryonic Densities

Inclusion of baryonic charge does not change the two types of singularities:

Equation for Singularities:

$$s^*(T) \;=\; F(s^*, T)$$

μ_B is baryonic chemical potential, b_j is charge of j-th hadron;
$u(T, \mu_B)$ can be derived from some spectrum $\rho(m, v, b)$

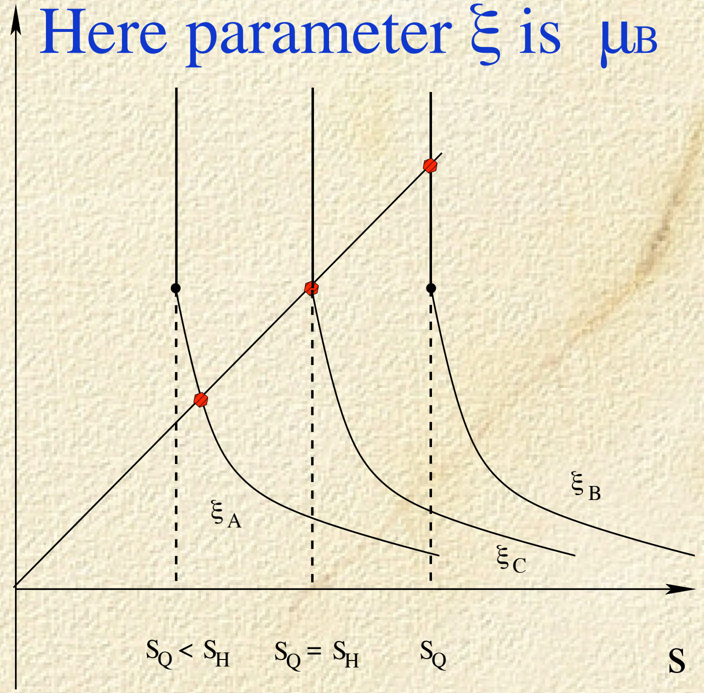
$$F_H(s, T, \mu_B) = \sum_{j=1}^{n} g_j e^{\frac{b_j \mu_B}{T} - v_j s} \phi(T, m_j)\,,$$

$$F_Q(s, T, \mu_B) = u(T, \mu_B) \int\limits_{V_0}^{\infty} dv\ \frac{\exp[(s_Q(T,\mu_B) - s)v - \sigma(T) v^{\varkappa}]}{v^{\tau}}\,.$$

QGP pressure $p_Q = T s_Q(T, \mu_B)$ can be chosen in several ways.
For definiteness we use the MIT Bag model pressure

$$p_Q = \frac{\pi^2}{90} T^4 \left[\frac{95}{2} + \frac{10}{\pi^2} \left(\frac{\mu_B}{T} \right)^2 + \frac{5}{9\pi^4} \left(\frac{\mu_B}{T} \right)^4 \right] - B$$

Here parameter ξ is μ_B



$u(T, \mu_B), B$ should obey the **sufficient conditions** for a PT existence:

$$F(s_Q(T, \mu_B = 0) + 0,\ T,\ \mu_B = 0) > s_Q(T, \mu_B = 0)\,,$$

$$F(s_Q(T, \mu_B) + 0,\ T,\ \mu_B) < s_Q(T, \mu_B)\,,\ \text{for all}\ \ \mu_B > \mu_A.$$

Phase Diagrams for TriCEP

Assume: the sufficient conditions are satisfied. $\Rightarrow$

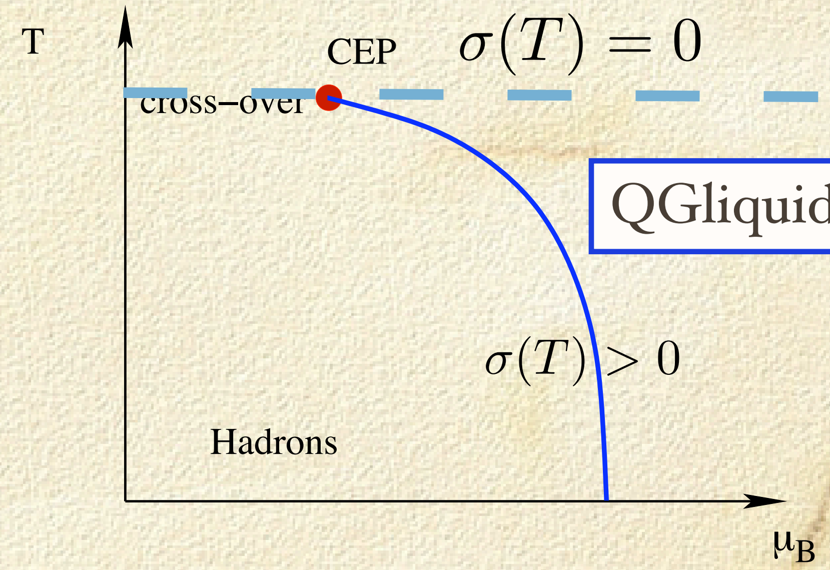
Equality of two singularities gives the Gibbs criterion:

$$s_H(T, \mu_B^c(T)) = s_Q(T, \mu_B^c(T))$$

$\Rightarrow \quad \mu_B = \mu_B^c(T)$ phase equilibrium line.

The shape of $\rho_B - T$ diagram depends on τ value.

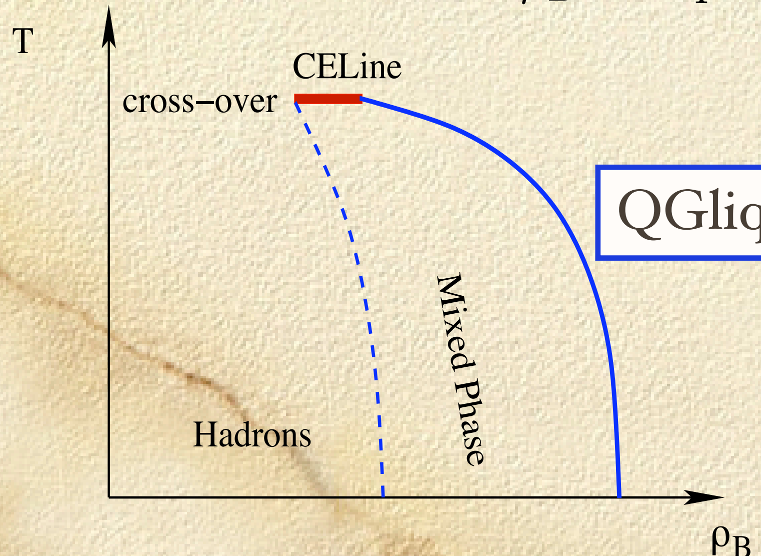
As we showed for $\sigma(T) > 0$ **there is 1st order PT**



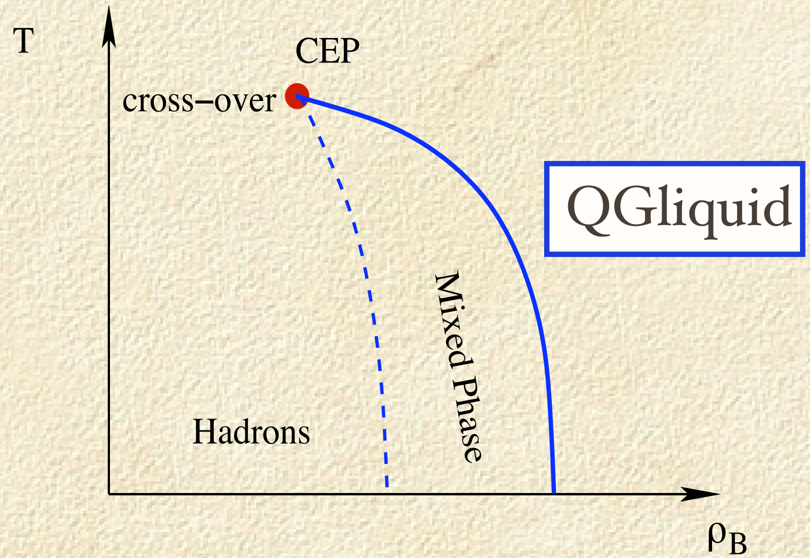
At $T = T_{cep} \Rightarrow \sigma(T) = 0$ and PT order depends on τ:

For $\tau > 2$ it is 1^{st} order PT

$\Rightarrow$ there is **a Critical Line** in $\rho_B - T$ plane!



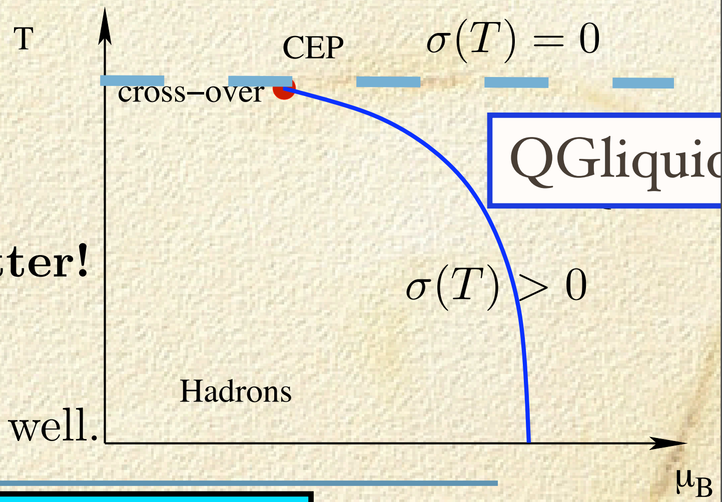
For $2 \geq \tau > \frac{3}{2}$ it is 2^{nd} order PT



Surface Tension Induced Phase Transition

The continuity of the solution: at the region $\mu_B < \mu_B^c(T_{cep})$ is easy to show.

For $T \leq T_{cep}$ **simple pole** is ALWAYS the rightmost singularity
$\Rightarrow$ the change of F_Q behavior does not matter.



For $T > T_{cep}$ **simple pole** is the ONLY singularity! $\Rightarrow$

I.e. since $\Delta \equiv s - s_Q(T, \mu_B) > 0$ and **sign of** $\sigma(T)$ **does not matter!**

All T and μ_B derivatives of F_Q exist and are finite $\Rightarrow$
all T and μ_B derivatives of pressure are finite as well.

For $\mu_B \geq \mu_B^c(T_{cep})$ and $T > T_{cep}$ **there exists PT of 2^{nd} or higher order.**

Consider the limit $T \to T_{cep} + 0$ or $\gamma^2 \equiv -\sigma(T) > 0$

K.A.B., PRC (2007) 76

There are two possibilities: either $\nu \equiv \gamma^2 \Delta^{-\varkappa} \to Const$ or $\nu \equiv \gamma^2 \Delta^{-\varkappa} \to 0$
(otherwise solution s^* does not exist)

Assuming that $\Delta = A\,\gamma^\alpha + O(\gamma^{\alpha+1})\,, \quad \Rightarrow \quad \frac{\partial \Delta}{\partial T} = \frac{\partial \gamma}{\partial T}\left[A\,\alpha\,\gamma^{\alpha-1} + O(\gamma^\alpha)\right] \sim \frac{(2\,k+1)\,A\,\alpha\,\gamma^\alpha}{2\,(T - T_{cep})}$,

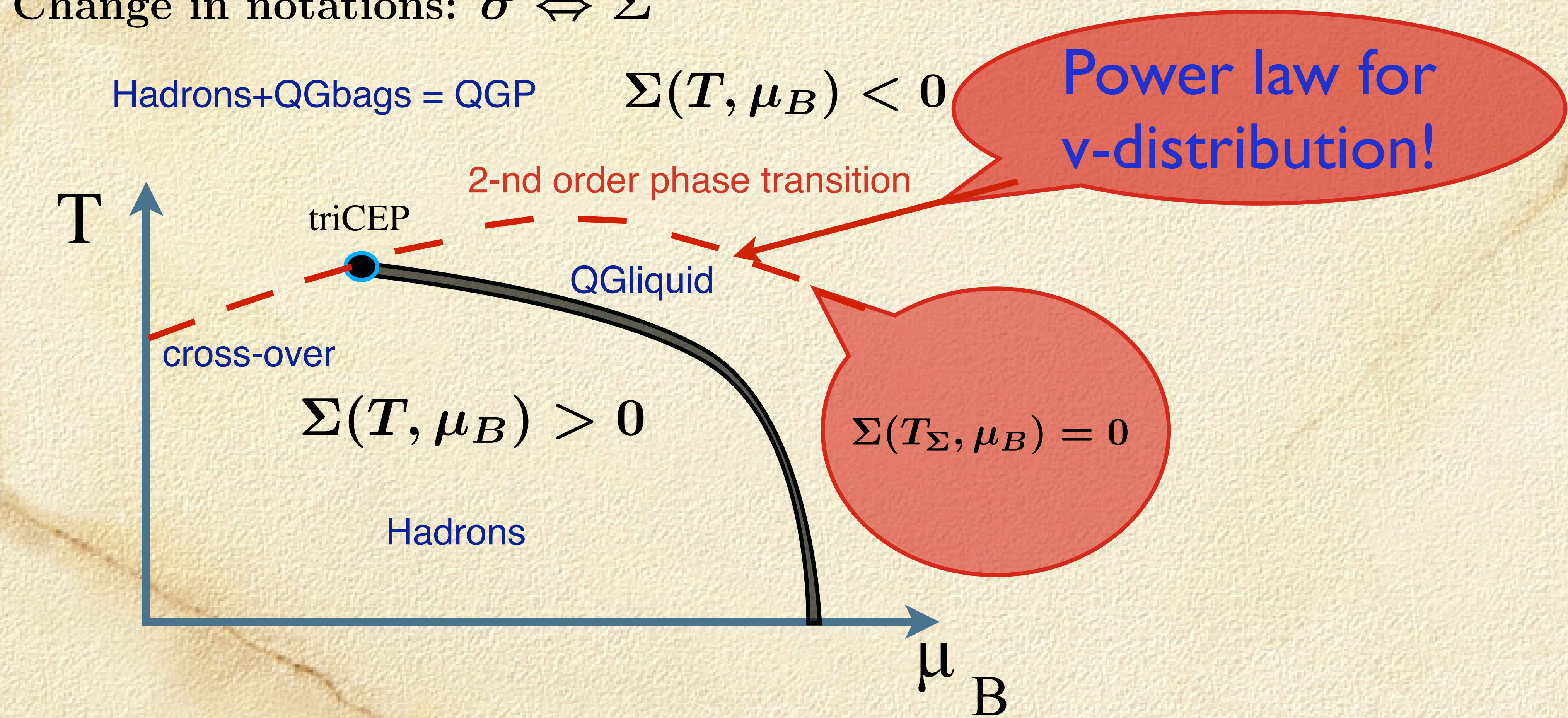
And comparing with
$$\frac{\partial \Delta}{\partial T} = \frac{G_2 + u\,\mathcal{K}_{\tau-\varkappa}(\Delta,\gamma^2)\,2\,\gamma\gamma'}{1 + u\,\mathcal{K}_{\tau-1}(\Delta,\gamma^2)} \approx \frac{\Delta^{2-\tau} G_2}{u\,\mathcal{K}_{\tau-1}(1,\nu)} + \frac{2\,\gamma\gamma'\Delta^{1-\varkappa}\left[\nu\,\varkappa\,\mathcal{K}_{\tau-2\varkappa}(1,\nu) - \mathcal{K}_{\tau-1-\varkappa}(1,\nu)\right]}{(\tau - 1 - \varkappa)\,\mathcal{K}_{\tau-1}(1,\nu)}$$

Results for TriCEP

Our group has calculated the critical indices for this case and found that the phase diagram must look like shown below

A. Ivanytskyi, NPA(2012) 880

Change in notations: $\boldsymbol{\sigma} \Leftrightarrow \boldsymbol{\Sigma}$



What about the critical endpoint?

CEP Generation

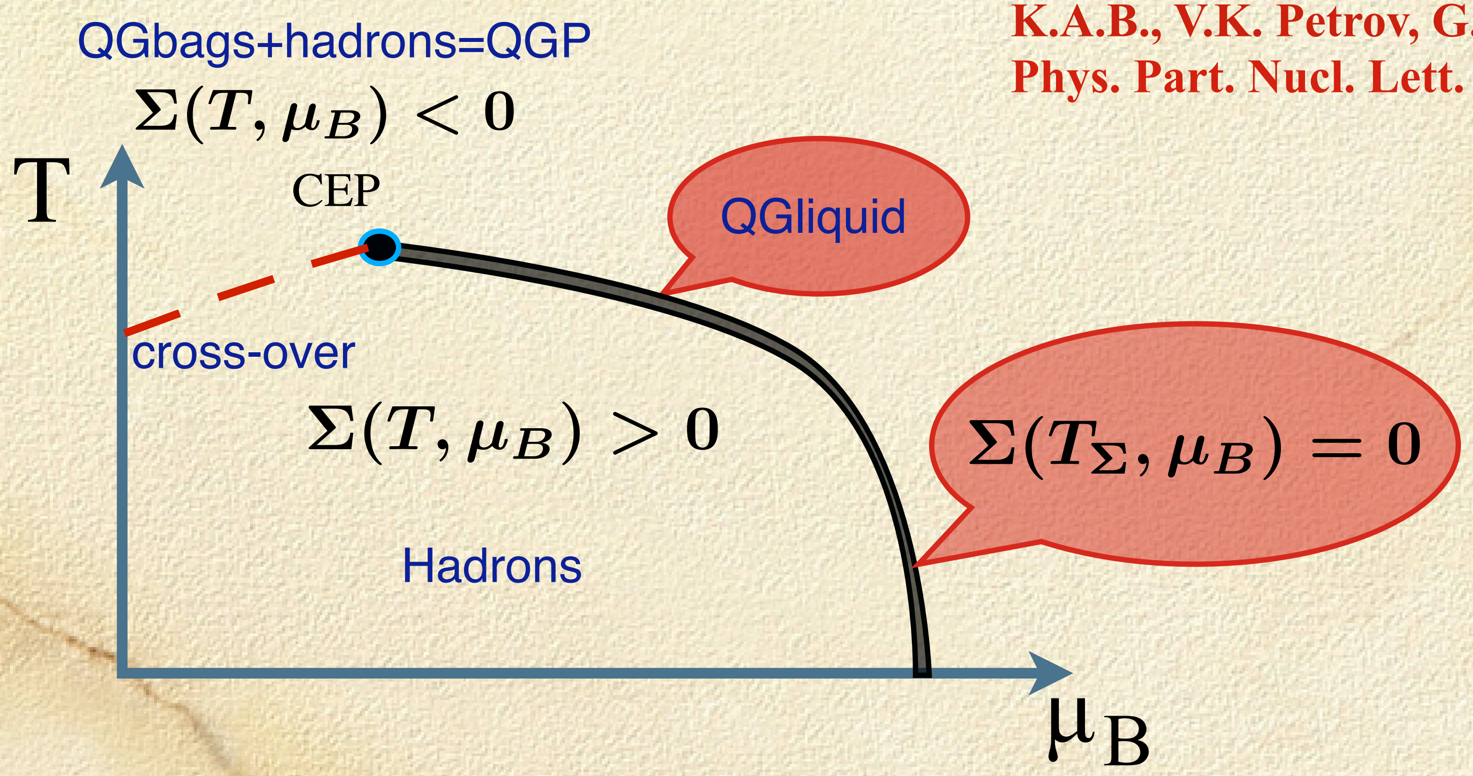
Main idea:

to match the curves of deconfinement PT and **$\Sigma = 0$!**

Prediction:

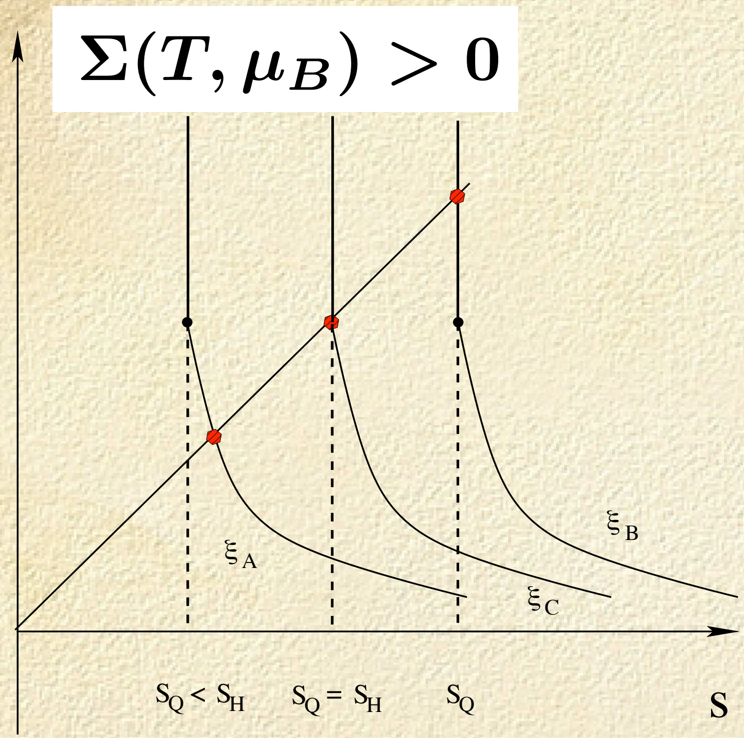
the power law in V-distribution of bags will be not just at CEP as one would expect, but in the mixed phase with **$\Sigma = 0$!**

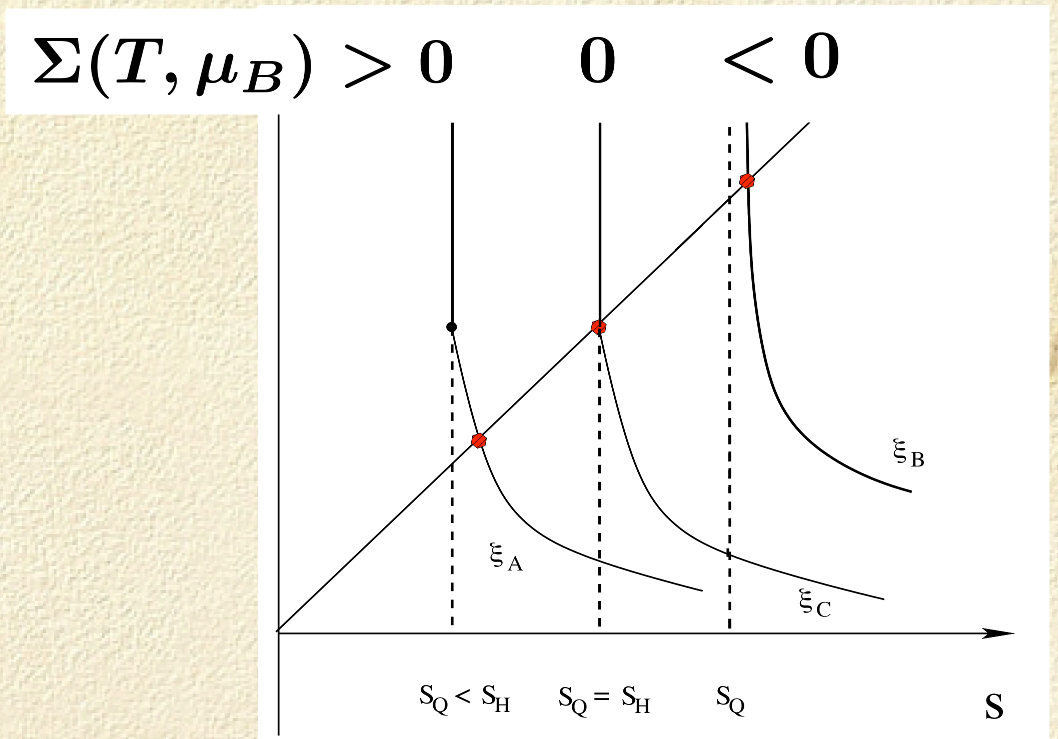
K.A.B., V.K. Petrov, G.M. Zinovjev, Phys. Part. Nucl. Lett. (2012) 9



Structure of singularities for CEP

Singularities of the IP and corresponding graphical solution of Eq. $s* = F(s*, T, \mu_B)$.





Case of triCEP PRC 76 (2007)

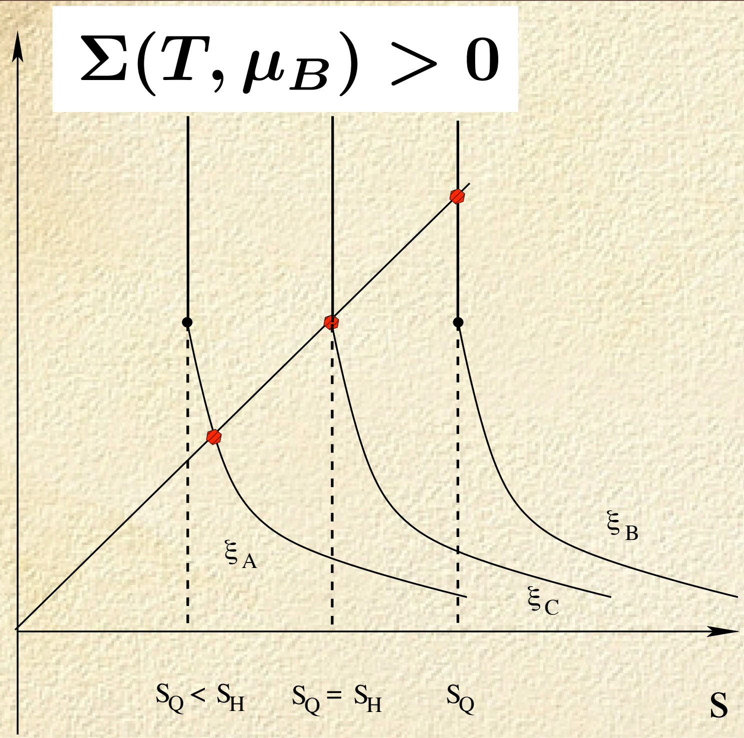
Case of CEP arXiv:0904.4420

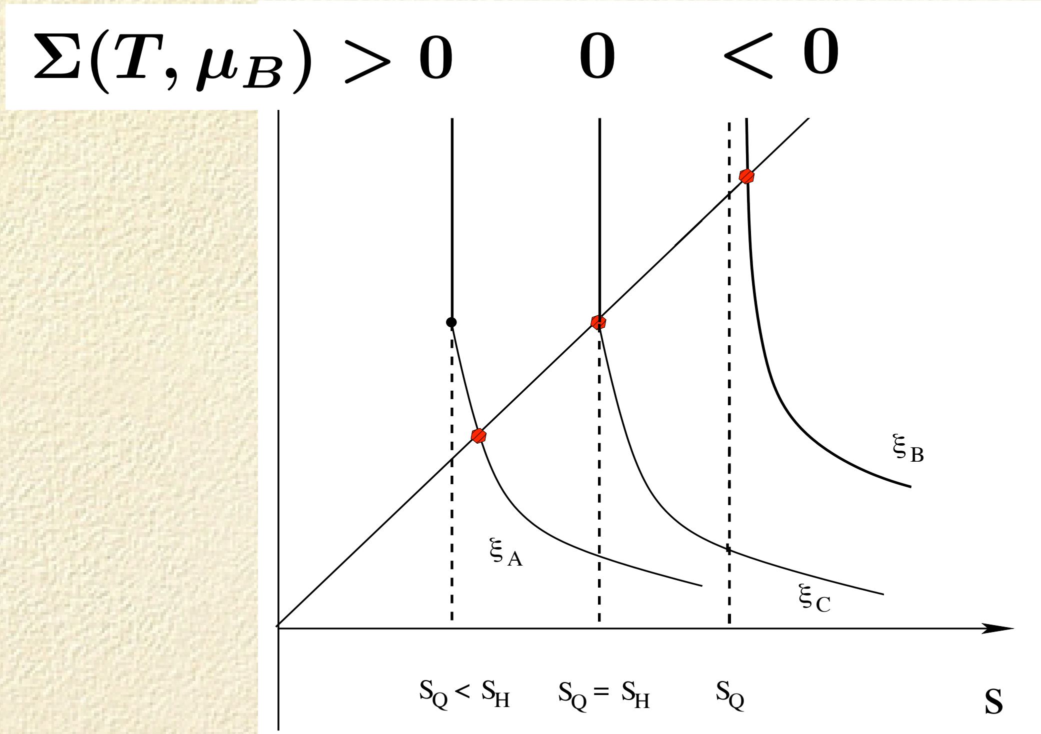
Parameter ξ can be either T or μ_B.

For example, if ξ is T, then $\xi_A < T_c$, $\xi_c = T_c$ and $\xi_B > T_c$.

Structure of singularities for CEP

- Thus, for the CEP case the rightmost singularity below and above PT line is a SIMPLE POLE!





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Parameter ξ can be either T or μ_B.

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Sufficient conditions for CEP existence

Let's denote $T^{\pm} \equiv T_{\Sigma}(\mu_B) \pm 0$ and same for p and ρ

Density of pure phases: $\rho^{\pm} = \frac{\partial p^{\pm}}{\partial \mu_B}$, where $p^{\pm} = T\,F(\frac{p^{\pm}}{T}, T, \mu_B)$.

$$\rho^{\pm} = T^{\pm} \frac{\frac{\partial F_H}{\partial \mu} + \frac{\partial u}{\partial \mu} I_{\tau}(\Delta^{\pm}, \Sigma^{\pm}) + \frac{\partial s_Q}{\partial \mu} u I_{\tau-1}(\Delta^{\pm}, \Sigma^{\pm}) - \frac{\partial \Sigma^{\pm}}{\partial \mu} u I_{\tau-\kappa}(\Delta^{\pm}, \Sigma^{\pm})}{1 + u I_{\tau-1}(\Delta^{\pm}, \Sigma^{\pm}) - \frac{\partial F_H}{\partial s}},$$

where $\Delta^{\pm} = s^{\pm} - s_Q(T^{\pm}, \mu_B)$ and $I_{\tau-\omega}(\Delta^{\pm}, \Sigma^{\pm}) = \int\limits_{V_0}^{\infty} \frac{dv}{v^{\tau-\omega}} e^{-\Delta^{\pm} v - \Sigma v^{\kappa}}$

Using $\Delta^{\pm}|_{T=T_c} = 0$ and $\Sigma^{\pm}|_{T=T_c} = 0 \quad \Rightarrow$

$$\Delta\rho = \left[\left(\frac{\partial \Sigma^{-}}{\partial \mu} - \frac{\partial \Sigma^{+}}{\partial \mu} \right) \frac{u I_{\tau-\kappa}(0,0)}{1 + u I_{\tau-1}(0,0) - \frac{\partial F_H}{\partial s}} \right]_{T=T_c}$$

does not vanish!

Condition for 1st order deconfinement PT existence:
finiteness of integrals $I_{\tau-\kappa}(0,0)$ and $I_{\tau-1}(0,0)$ and then $\tau > 2$.

Sufficient conditions for CEP existence

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* Thus, for the CEP case the 1-st order deconfinement PT is a SURFACE TENSION induced PT!

Using $\Delta^{\pm}|_{T=T_c} = 0$ and $\Sigma^{\pm}|_{T=T_c} = 0$ $\Rightarrow$

$$\Delta\rho = \left[\left(\frac{\partial \Sigma^-}{\partial \mu} - \frac{\partial \Sigma^+}{\partial \mu} \right) \frac{u I_{\tau-\kappa}(0,0)}{1 + u I_{\tau-1}(0,0) - \frac{\partial F_H}{\partial s}} \right]_{T=T_c}$$

does not vanish!

Condition for 1^{st} order deconfinement PT existence:
finiteness of integrals $I_{\tau-\kappa}(0,0)$ and $I_{\tau-1}(0,0)$ and then $\tau > 2$.

Conclusions

The relation between the string tension and the surface tension of QGP bags is found! It allows us to determine the surface tension of QGP bags directly from Lattice QCD.

The surface tension of QGP bags at T = 0 is large and at the cross-over T ~ 170 MeV the surface tension is negative!

At the cross-over T ~ 170 MeV there exist fractals => fractal surfaces!

On an example of exactly solvable models it is shown that the surface tension of QGP bags plays an important role in generation of the (tri)critical endpoint!

Thanks for your attention!