Past, Present and Future of the Statistical Bootstrap Model

Kyrill Alekseevich Bugaev

Bogolyubov ITP, Kiev, Ukraine

Dubna, September 2012

Outline

- A+A (HIC) experiments and their goals
- General remarks on QCD matter phase diagram
- QCD matter phase diagram from lattice QCD
- Statistical Bootstrap Model and limiting T for hadrons
- Concept of Hagedorn thermostat and nonequivalence of ensembles for H-spectrum
- Theoretical and practical importance
- Conclusions

Experiments on A+A Collisions

SIS100 (GSI)
C.M.S. energy/nucl 2 - 4 GeV
AGS (BNL) 4.9 GeV
SPS (CERN) 6.1 - 17.1 GeV
RHIC (BNL) 62, 130, 200 GeV

} Completed

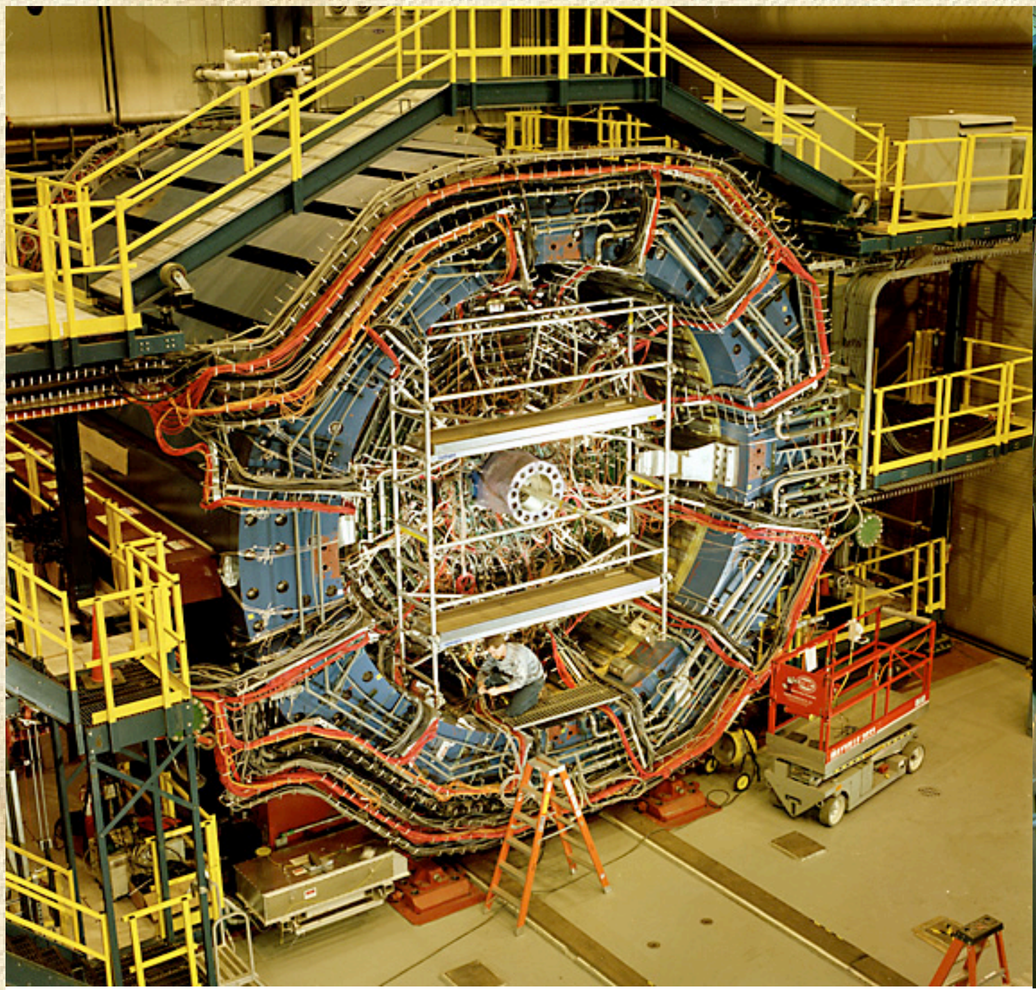
Ongoing HIC experiments
LHC (CERN) > 1 TeV (high energy)
RHIC (BNL) low energy
SPS (CERN) low energy

Future HIC experiments
NICA(JINR, Dubna)
SIS300 = FAIR (GSI)

RHIC Detectors

RHIC - Relativistic Heavy Ion Collider (Brookhaven, USA)

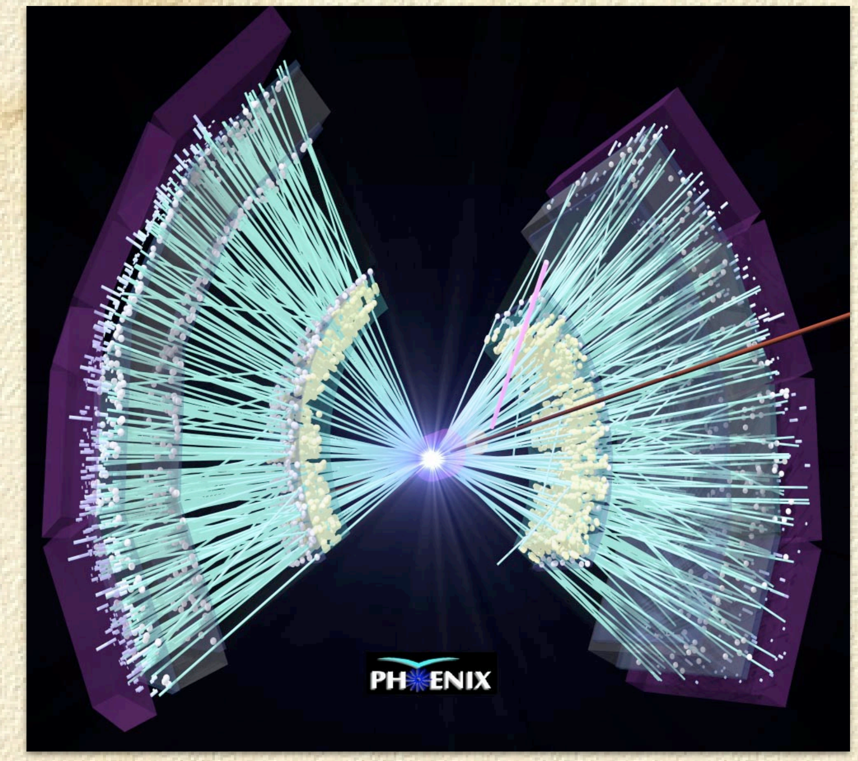
center of mass energy up to 200 GeV/nucleon

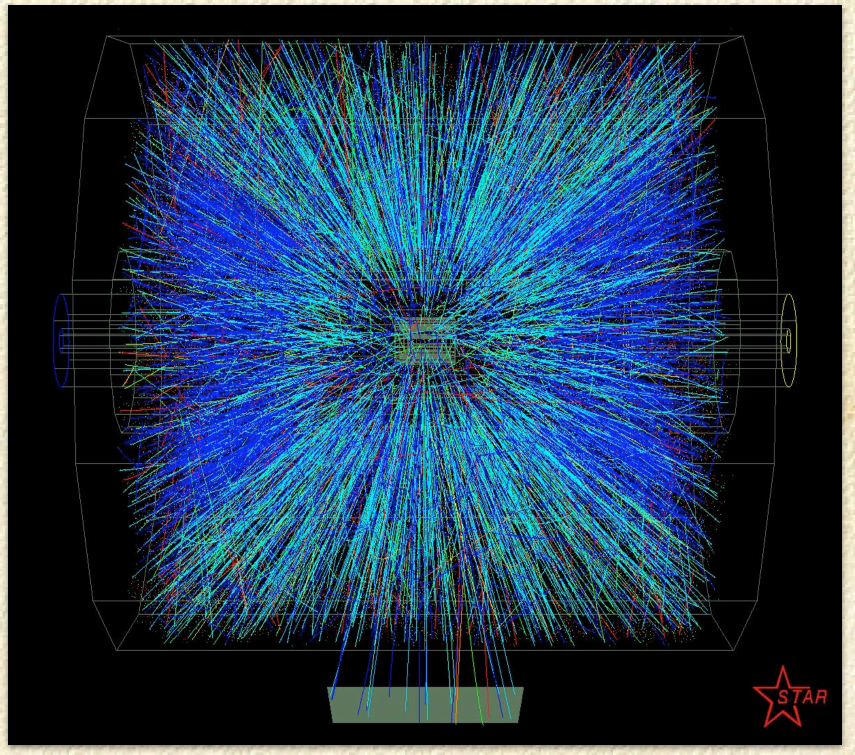


STAR

PHENIX

Single Collision at RHIC Energies



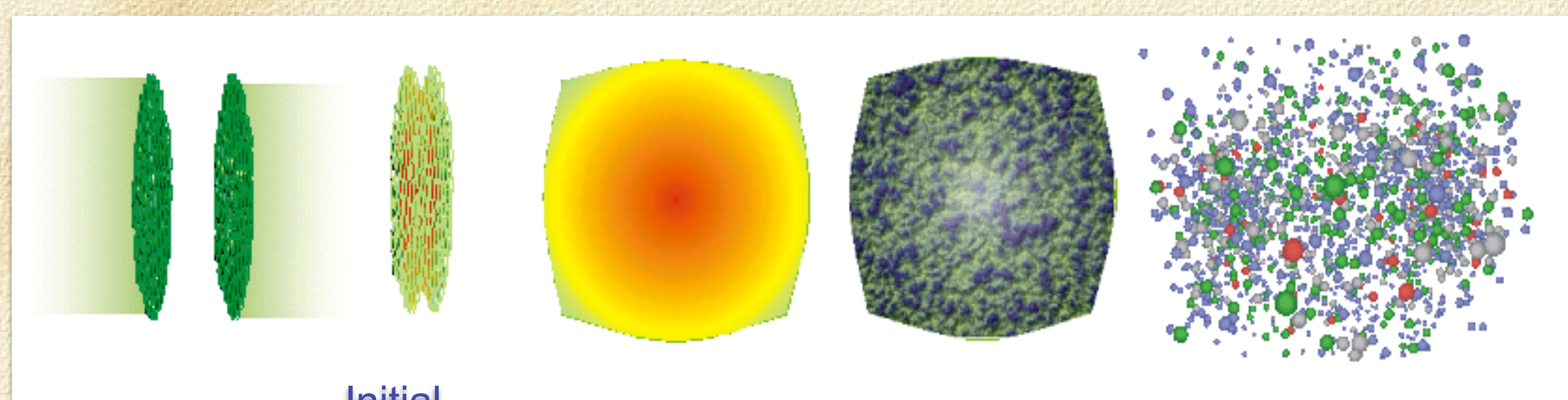


2000-10000 particles are registered and identified in each event!

RHIC Stages

Probe QGP – a new form of matter predicted by Quantum Chromodynamics (QCD)

1 fm $\simeq 10^{-15}$ m   1 fm/c $\simeq 3.3\ 10^{-23}$ s



	CGC	Initial Singularity	Glasma	sQGP	Hadron Gas
		quantum fluctuations	local thermalization	strongly interacting QGP	expansion and decay of resonances
proper time:		$\tau \simeq$ 0-0.1 fm/c	$\tau \simeq$ 0.1-1. fm/c	$\tau \simeq$ 1.-10. fm/c	$\tau >$ 10. fm/c

CGC - color glass condensate (coherent high density gluons)

The Complexity of RHICs

During A+A collision the nuclear matter (in general) has several transformations: … → Two nuclei (cold nuclear matter)

→ Evolution of excited NON-equilibrated q, g plasma

→ EXPANSION of the equilibrated q, g plasma

→ Transformation into hadrons (HADRONIZATION), partial or complete (cross-over or PT), DURING expansion

→ Kinetic freeze-out (spectra of secondaries do not change)

→ Detection → Analysis → Acceptance of measurement → Publication
→ Comparison with theoretical model → …

Each stage (after arrow) requires a working MODEL!

The worst is that some stages HAPPEN simultaneously!

Goals of HIC experiments

- To learn QCD matter Equation of State = QCD matter phase diagram
- Understanding such fundamental phenomena as:
  color confinement, nature of deconfinement, nature of chiral symmetry restoration
- Understanding the Early Universe history, the properties of neutron, quark, strange e.c.t. stars + exotica (strangelets, dibaryons)

Since QCD is not solved, we have to use lattice QCD, other theoretical and phenomenological models

sQGP is the most perfect fluid!

Anti de Sitter Conformal Field Theory (AdS/CFT) is holographically dual to QCD = string model

G. Policastro, D. T. Son and A. O. Starinets, JHEP **0209**, 043 (2002) [arXiv:hep-th/0205052].

P. K. Kovtun and A. O. Starinets, Phys. Rev. D **72**, 086009 (2005) [arXiv:hep-th/0506184].

D. Teaney, Phys. Rev. D **74**, 045025 (2006) [arXiv:hep-ph/0602044].

P. Kovtun and A. Starinets, Phys. Rev. Lett. **96**, 131601 (2006) [arXiv:hep-th/0602059].

P. K. Kovtun and A. O. Starinets, Phys. Rev. D **72**, 086009 (2005) [arXiv:hep-th/0506184].

AdS/CFT Predicted:

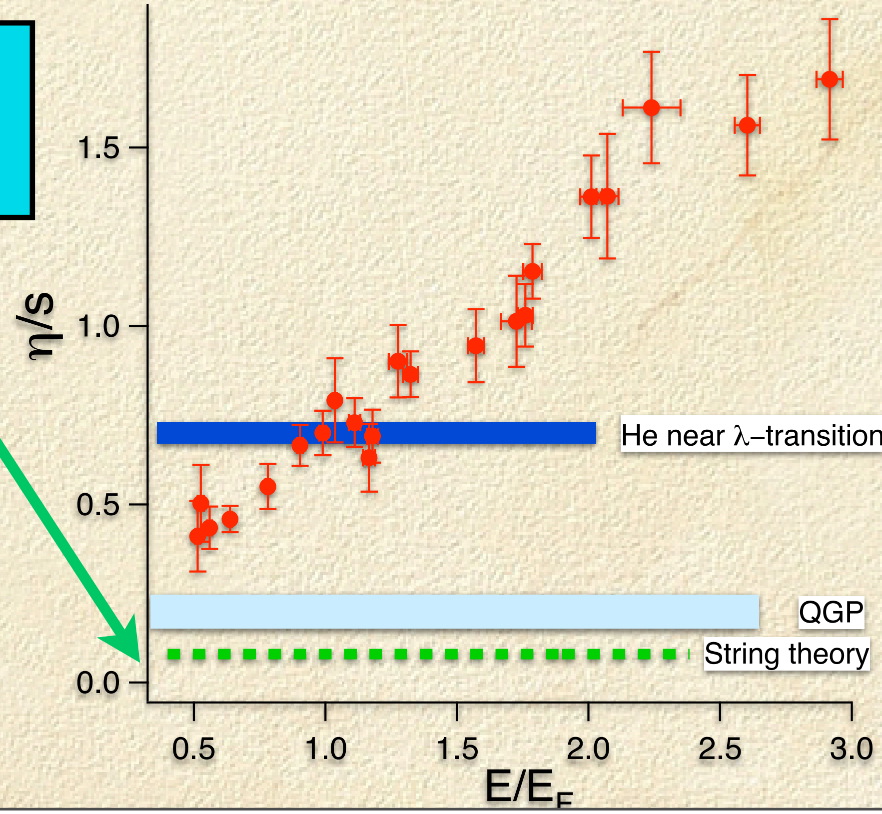
$$\min \frac{\text{shear viscosity}}{\text{entropy density}} = \frac{1}{4\pi}$$

Shear viscosity=means momentum transfer between fluid layers via a unit area

η <=> <p> <free path> <particle density>

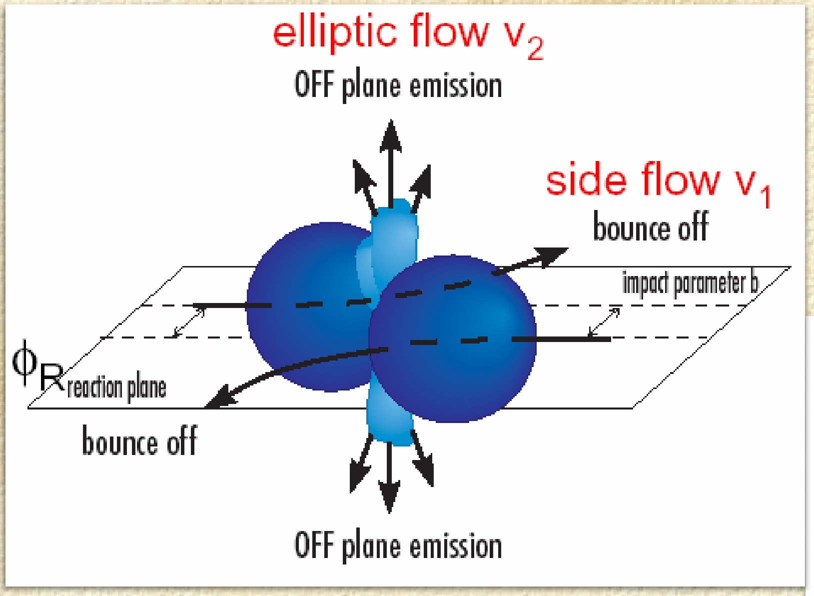
s <=> <number of states/particle> <particle density>

Red dots: cold atoms in traps



Viscosity "measurement"

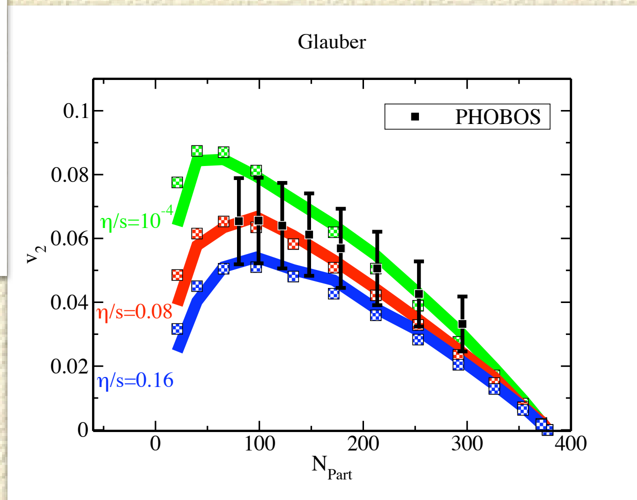
Non-central collision:

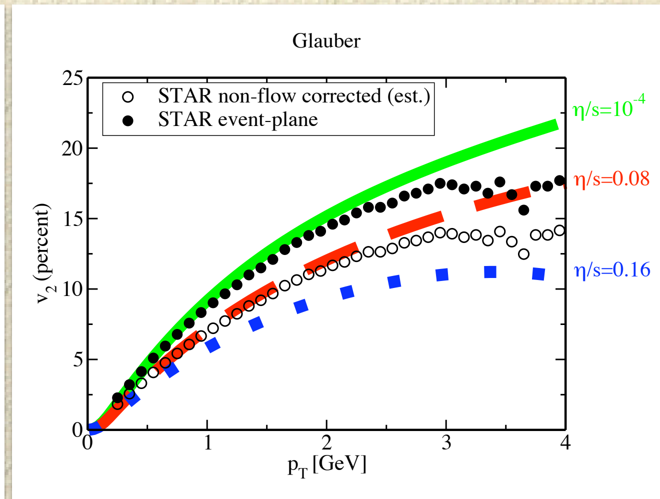


Azimuthal distributions with respect to reaction plane

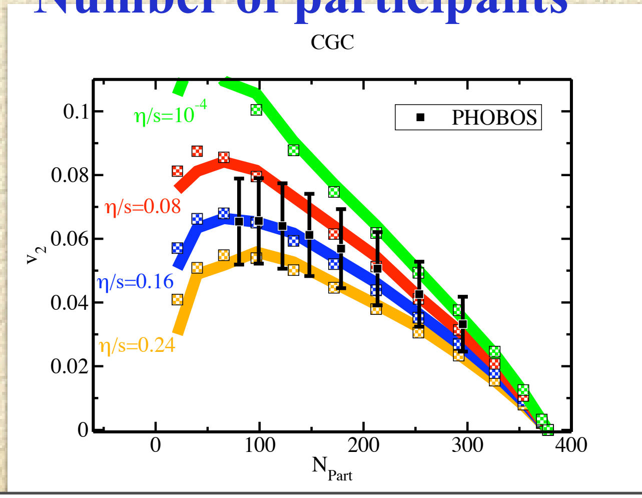
$$\varphi' := \varphi - \Phi_R$$

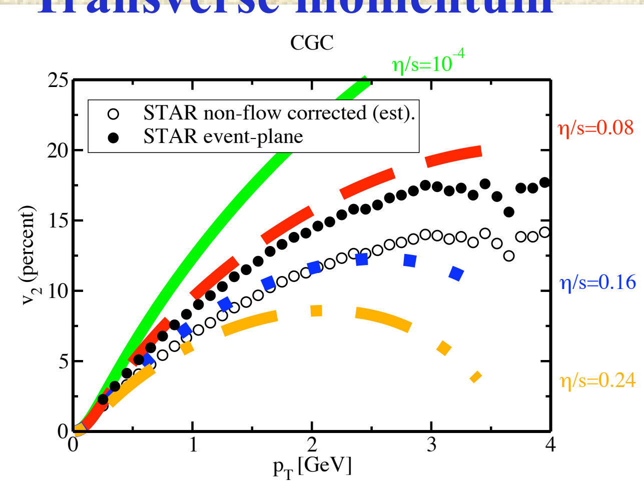
$$\frac{d^3N}{p_t dp_t dy d\varphi'} \propto (1 + 2v_1 \cos(\varphi') + 2v_2 \cos(2\varphi') + \ldots)$$





Number of participants **Transverse momentum**





Elliptic flow coefficient stores information about early stage of collision!

Comparison of hydro simulations with experimental data (RHIC)

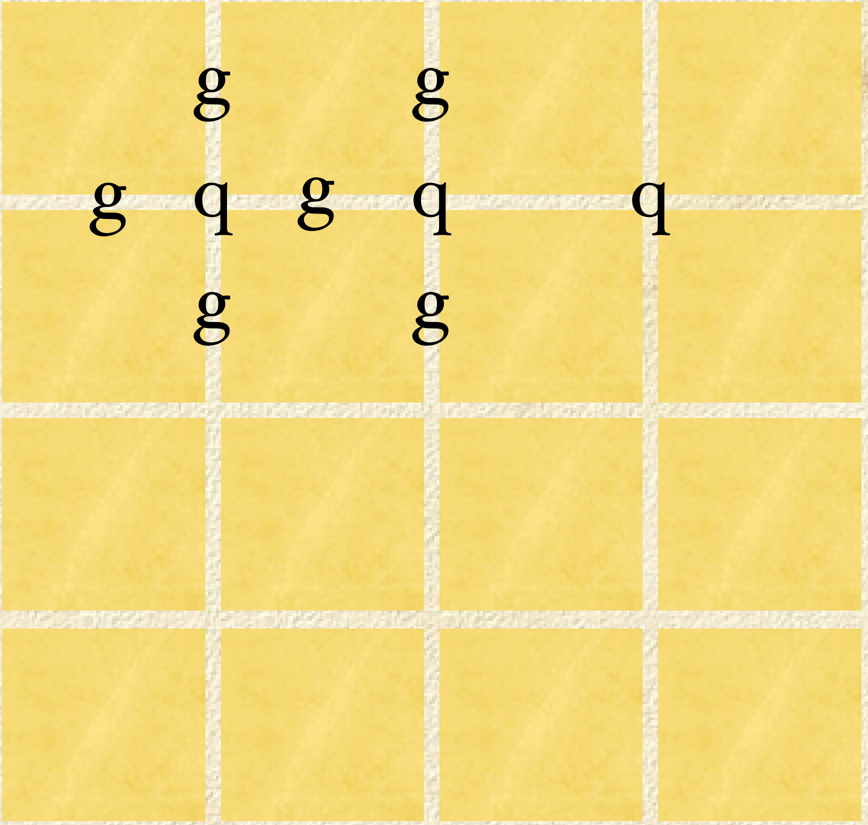
Lattice QCD

70-th & 80-th

K. Wilson, J. Kogut, M. Creutz and others

suggested to discretize space-time continuum and to consider q & g fields on lattice: quarks (q) are located in sites and gluons (g) are existing on links, connecting the sites.

Then field integrals can be approximated by integrals of large, but finite dimension and can be calculated NUMERICALLY, using Monte Carlo method!



See Prof. M. Ilgenfritz lecture

Lattice QCD and QCD inspired models with Zero Quark Masses

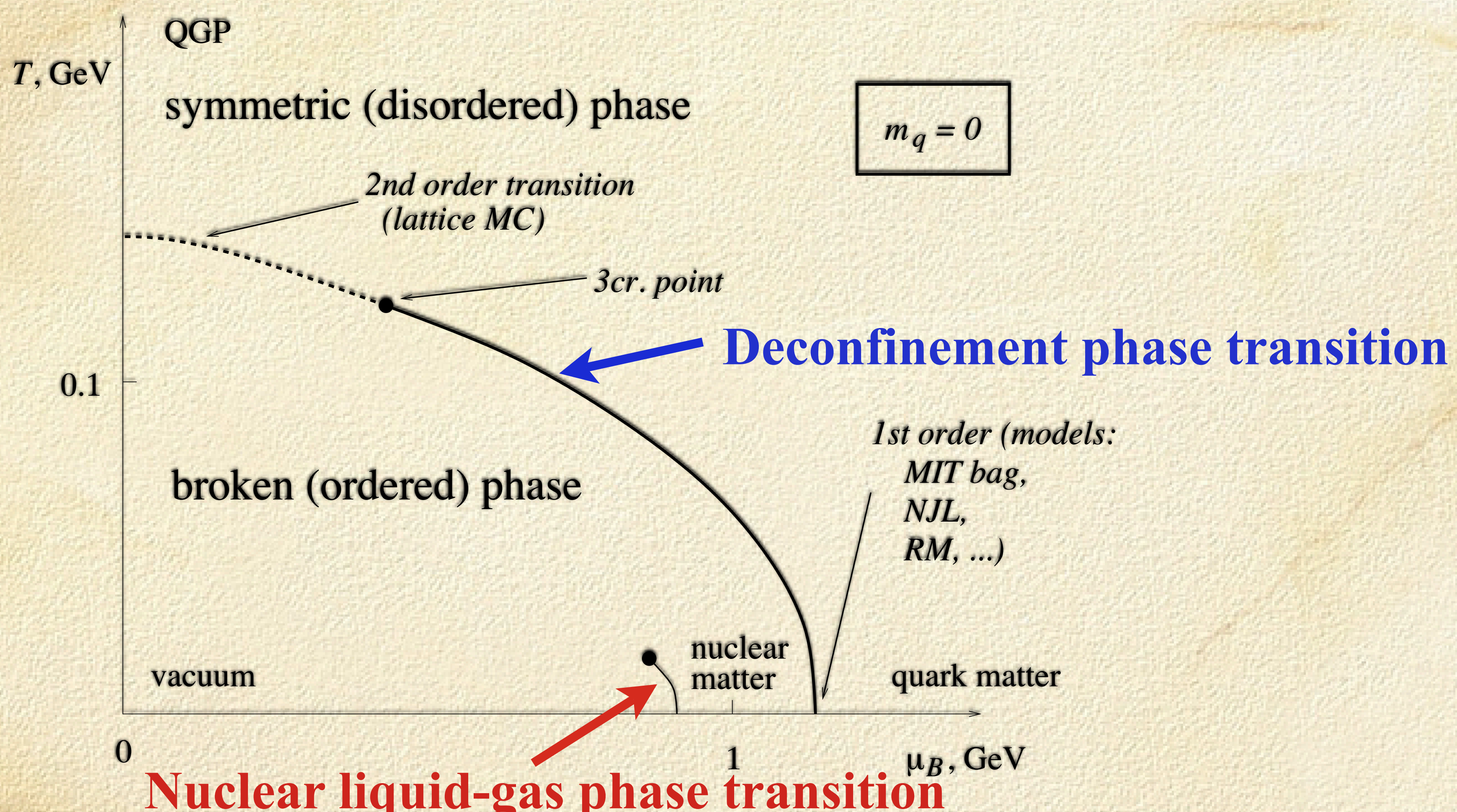
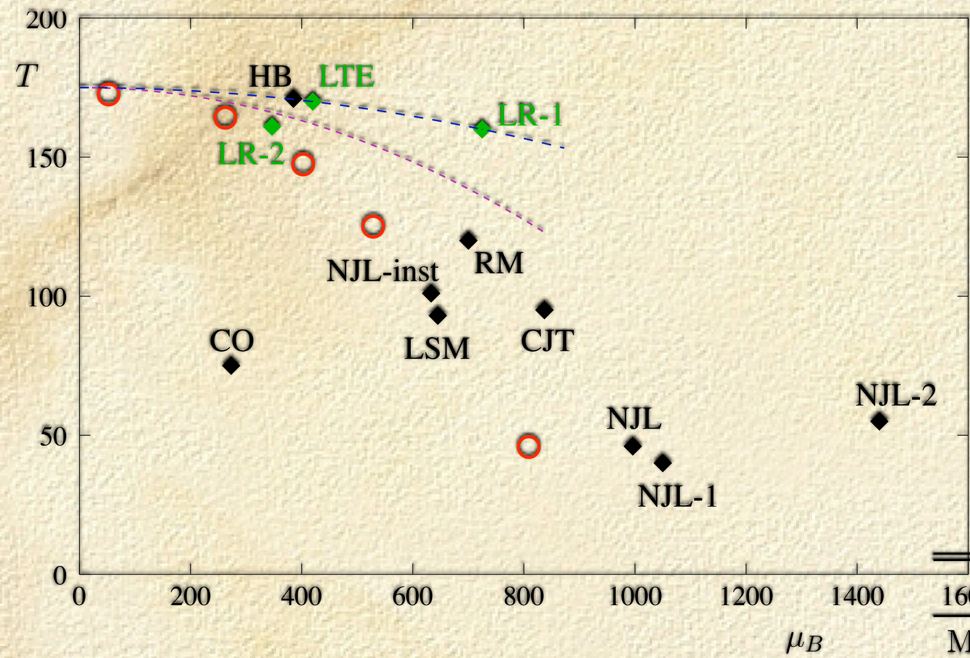


Fig. 2. Phase diagram of QCD with two *massless* quarks. The chiral symmetry order parameter *qualitatively* distinguishes two phases: $\langle\bar{\psi}\psi\rangle \neq 0$ in the broken phase and $\langle\bar{\psi}\psi\rangle = 0$ in the symmetric phase.

Lattice QCD and QCD inspired models with Non-Zero Quark Masses



There are technical difficulties to extend LQCD to nonzero baryonic chemical potentials.
=> Phenomenological models are unavoidable!

Green dots are lattice QCD results.
Red circles are chemical freeze-out points

Source	(T, μ_B), MeV	Comments	Label
MIT Bag/QGP	none	*only 1st order*	—
Asakawa,Yazaki '89	(40, 1050)	NJL, CASE I	NJL/I
ibidem	(55, 1440)	NJL, CASE II	NJL/II
Barducci, *et al.* '89-94	$(75, 273)_{TCP}$	composite operator	CO
Berges, Rajagopal '98	$(101, 633)_{TCP}$	instanton NJL	NJL/inst
Halasz, *et al.* '98	$(120, 700)_{TCP}$	random matrix	RM
Scavenius, *et al.* '01	(93,645)	linear σ-model	LSM
ibidem	(46,996)	NJL	NJL
Fodor, Katz '01	(160, 725)	lattice reweighting I	LR-1
Hatta, Ikeda, '02	(95, 837)	effective potential (CJT)	CJT
Antoniou, Kapoyannis '02	(171, 385)	hadronic bootstrap	HB
Ejiri, *et al.* '03	(–,420)	lattice Taylor expansion	LTE
Fodor, Katz '04	(162, 360)	lattice reweighting II	LR-2

Confinement by Color String before sQGP

Confinement = absence of free color charges

Consider confining string between static q & anti q of length L and radius R<<L

color anticolor

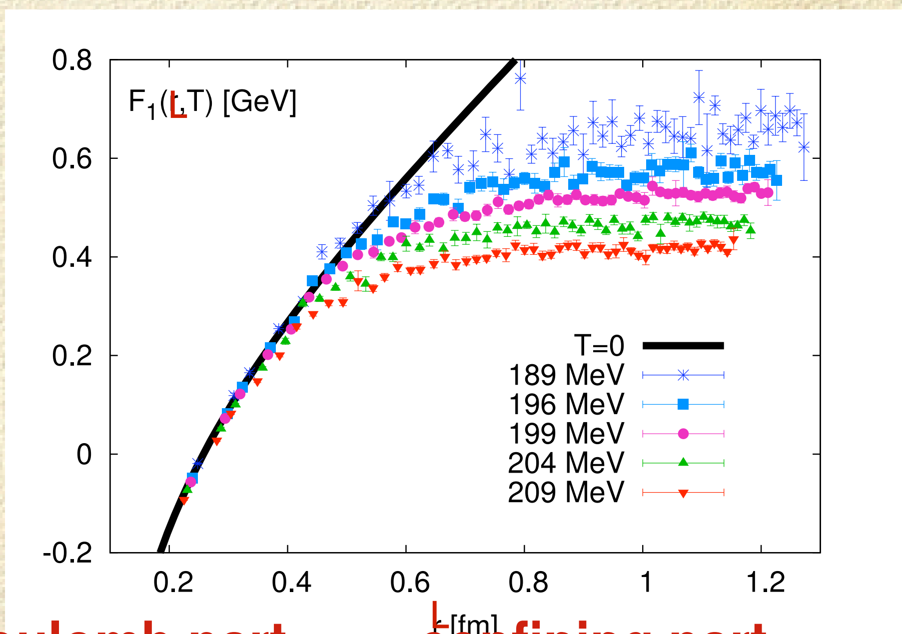
Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Confinement means infinite free energy for infinite L

Deconfinement means that string tension vanishes

Can be rigorously found by Lattice QCD

At T=0 the string tension = 12 tons!

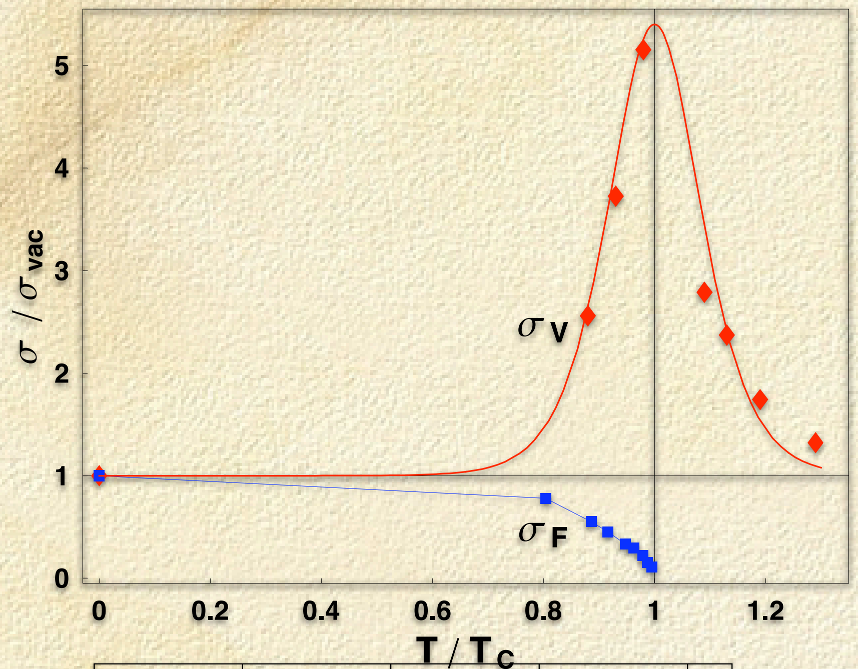


Coulomb part confining part

Confinement by Color String within sQGP

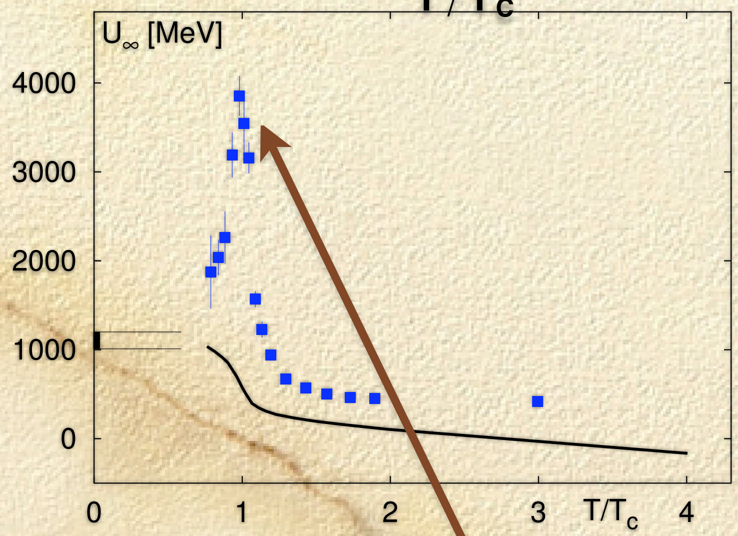
Internal energy U, entropy S

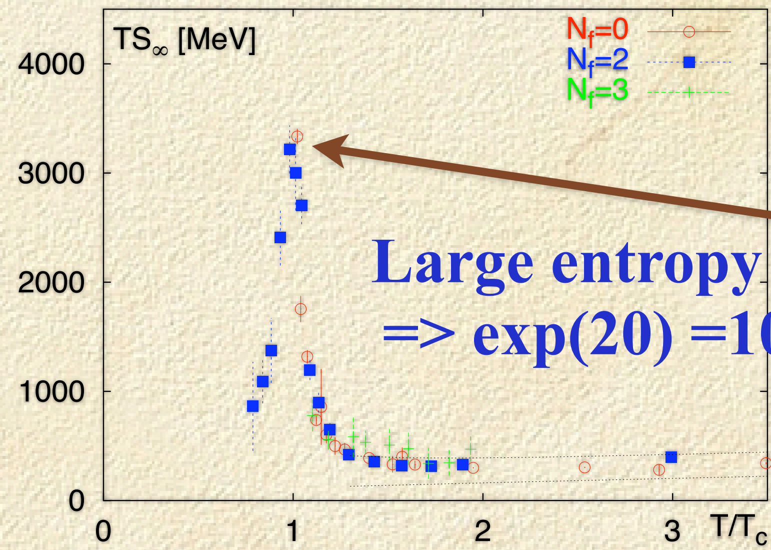
$$U(T,r) = F - TdF/dT = F + TS$$



String tension for internal energy (V)

String tension for free energy (F) —> 0





Large entropy S = 20!
=> exp(20) =108 !?**

Very strong interaction! => No color charge separation!

sQGP is a strongly interacting liquid !?

Plasma Parameter $\Gamma = \frac{\text{Interaction energy}}{\text{kinetic energy}} = U/T$

Depending on magnitude of this parameter Γ classical plasmas have the following regimes:

i. a weakly coupled or gas regime, for $\Gamma < 1$;
ii. a liquid regime for $\Gamma \approx 1 - 10$; **QGP range!**
iii. a glassy liquid regime for $\Gamma \approx 10 - 100$;
iv. a solid regime for $\Gamma > 300$.

E. Shuryak, Prog. Part. Nucl. Phys. (2009) 62

sQGP is a strongly interacting liquid !?

Plasma Parameter $\Gamma = \frac{\text{Interaction energy}}{\text{kinetic energy}} = U/T$

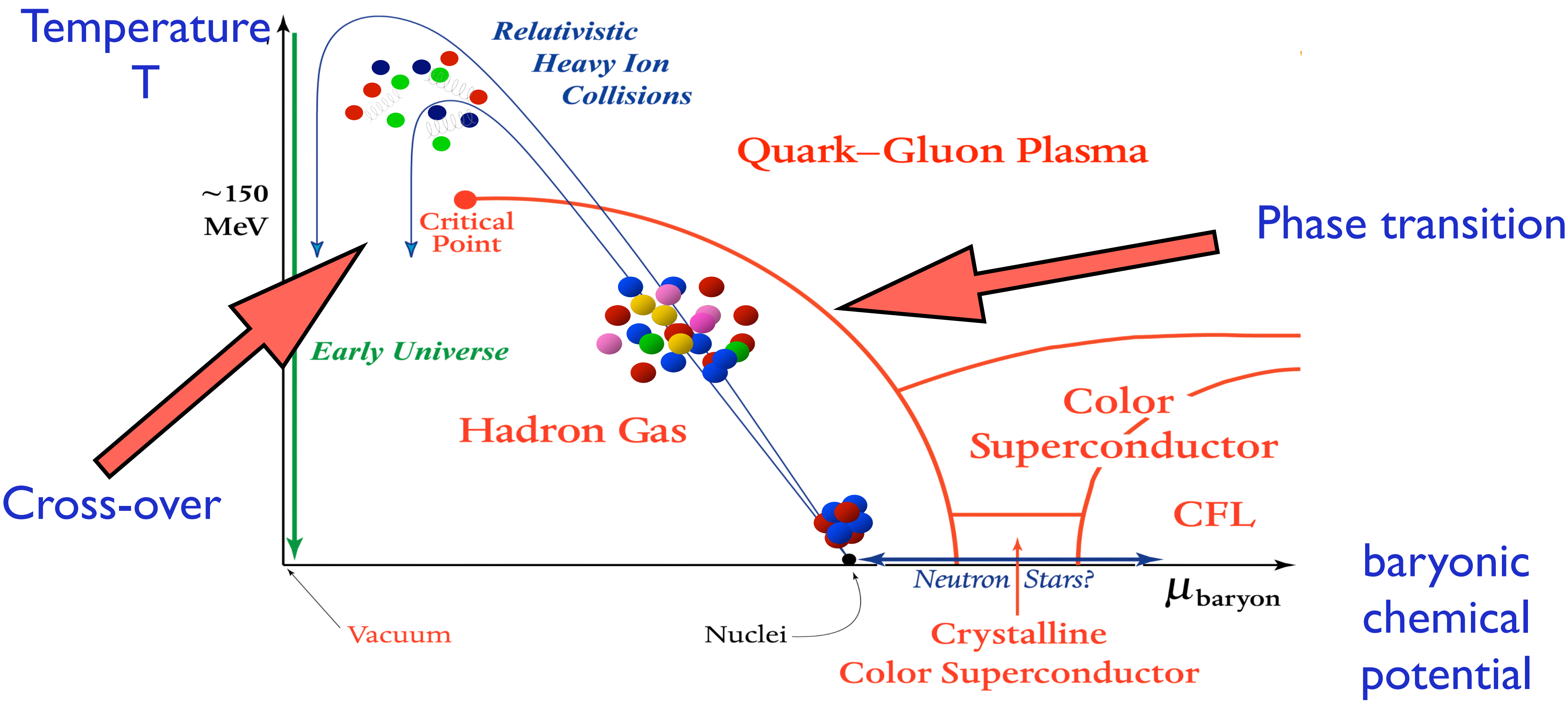
Depending on magnitude of this parameter Γ classical plasmas have the following regimes:

i. a weakly coupled or gas regime, for $\Gamma < 1$;
ii. a liquid regime for $\Gamma \approx 1 - 10$; **QGP range!**
iii. a glassy liquid regime for $\Gamma \approx 10 - 100$;
iv. a solid regime for $\Gamma > 300$.

DeConfinement = absence of free color charges too!
=> sQGP = clusters of q anti-q, qqq and so on states!
=> sQGP is liquid like phase!

E. Shuryak, Prog. Part. Nucl. Phys. (2009) 62

QCD EoS is unknown beyond CEP



QGP is a dense phase, i.e. it is liquid-like!

But in contrast to our everyday experience (boiling water)
QGP appears at higher temperatures!

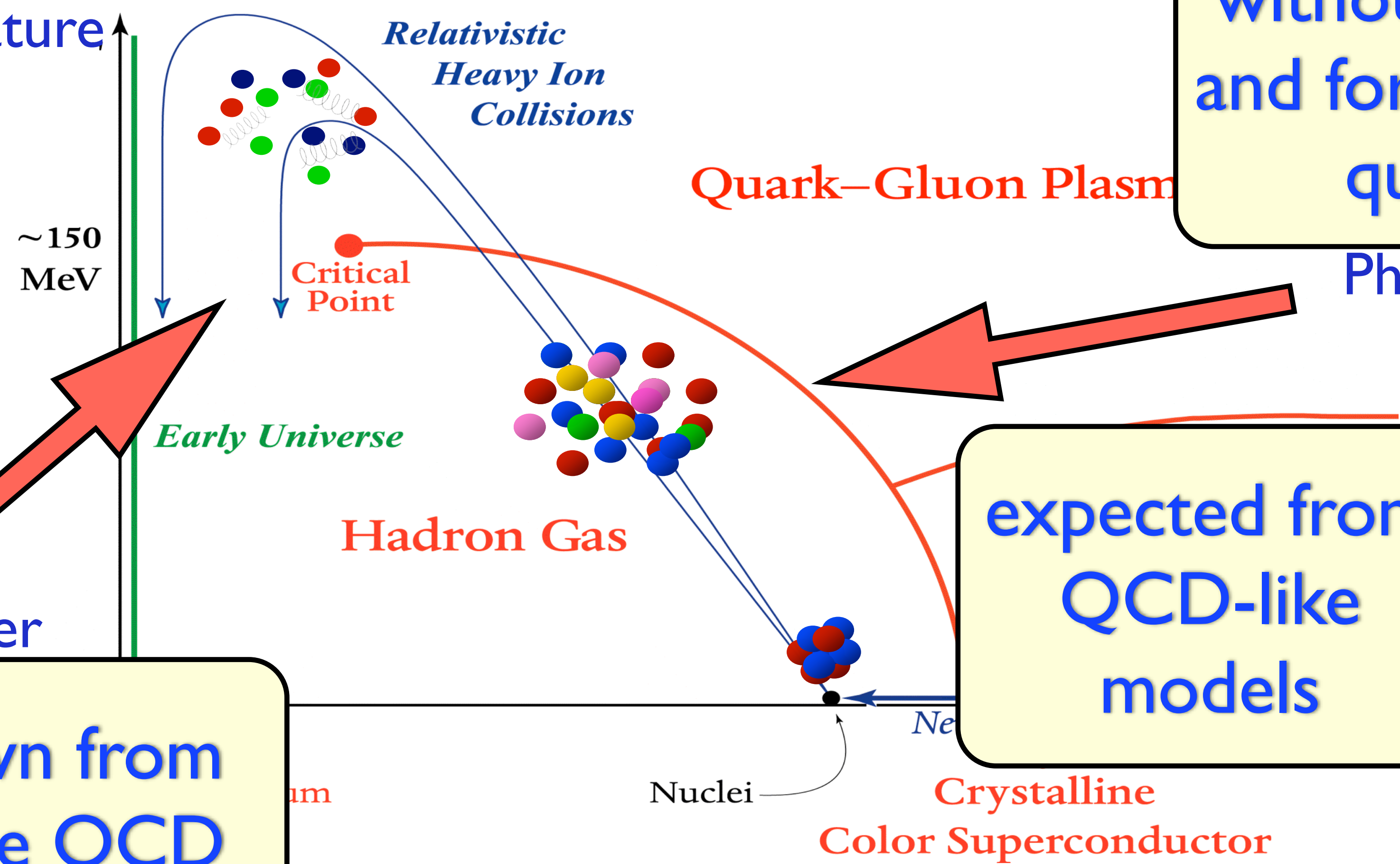
QCD EoS is unknown beyo

known from lattice QCD without quarks and for massless quarks

Temperature T

Phase transition

Cross-over



expected from QCD-like models

baryonic chemical potential

known from lattice QCD for realistic quark m

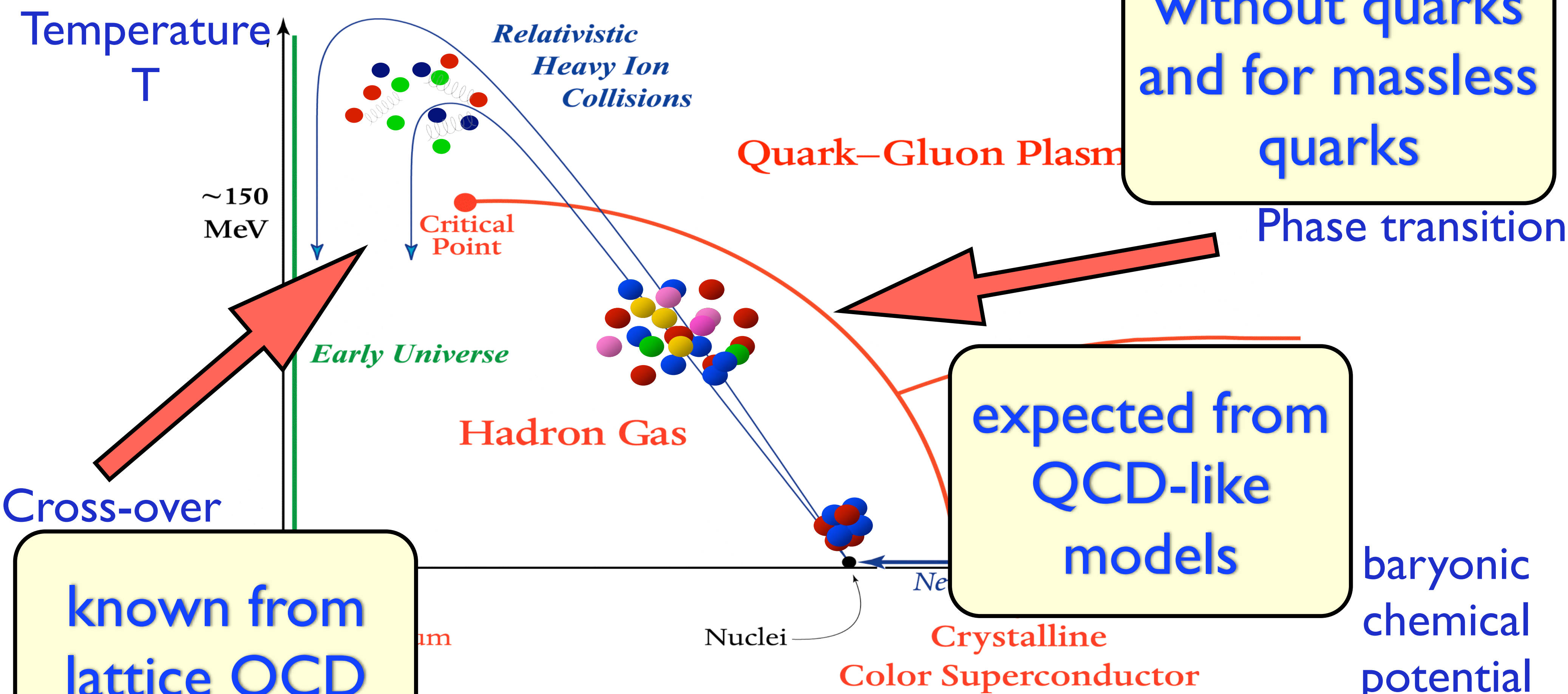
ase, i.e. it is liquid-like!

our everyday experience (boiling water)

QGP appears at higher temperatures!

QCD EoS is unknown beyo

known from lattice QCD without quarks and for massless quarks



expected from QCD-like models

known from lattice QCD for realistic quark m

ase, i.e. it is li

our everyday

QGP appears at high

What are other arguments for phase transition in QCD?

If sQGP is a liquid, then

- Can we find some general arguments that transition to sQGP is, indeed, a PT?
- What is the order of this PT?
- How to describe the strongly interacting liquid EoS?

Statistical Bootstrap Model

The first evidence for $\rho(E) = C\ e^{\alpha E}$ density of states was found **numerically** in 1958 having 15 particles only!

G. Fast, R. Hagedorn and L. W. Jones, Nuovo Cimento **27** (1963) 856;
G. Fast and R. Hagedorn, Nuovo Cimento **27** (1963) 208

Theory (**prediction**): $E^2\ \frac{d\sigma_{el}}{d\omega}|_{90} \approx A\ E\ e^{-3.17E}$ (1)

... And only in 1964 it was the first experimental evidence in favor of that. J. Orear, Phys. Lett. **13** (1964) 190

For large angle $p + p \to \pi + d$ at 2.4 GeV $\leq E \leq$ 6.8 GeV

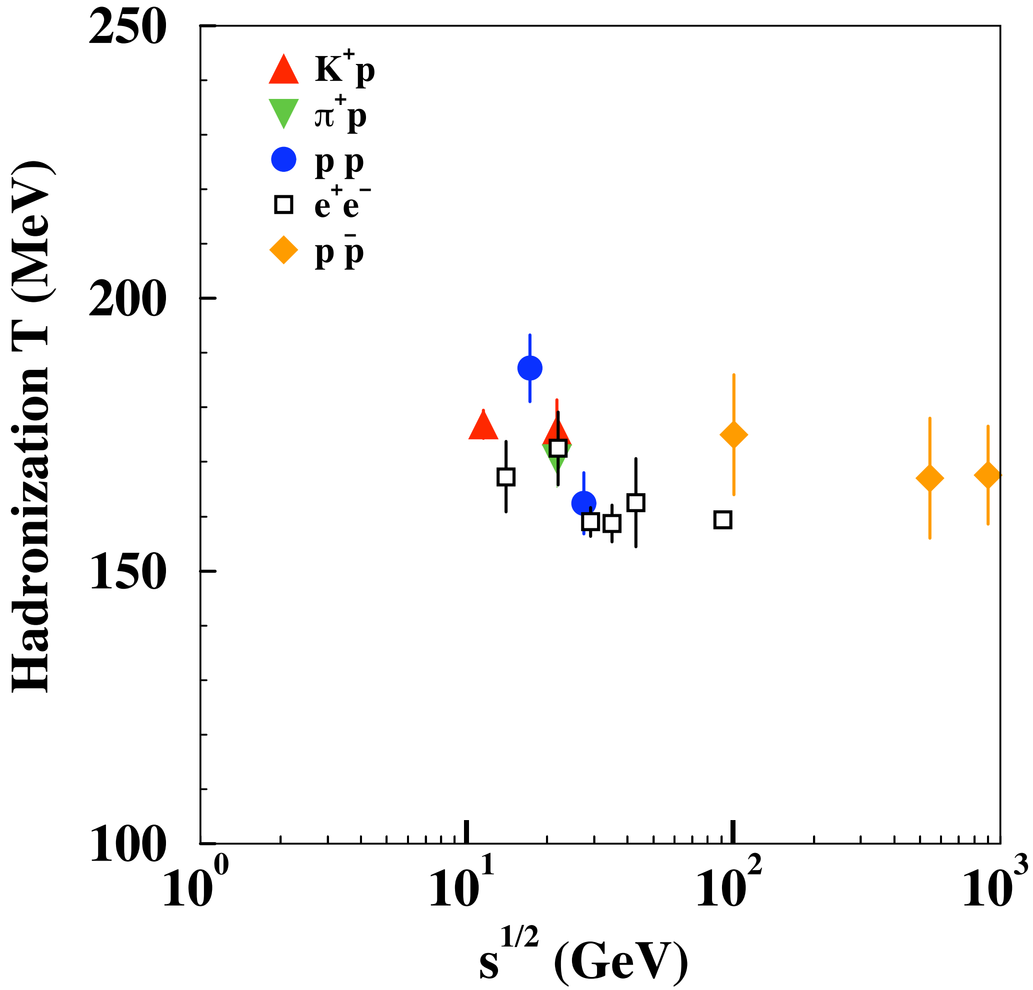
Consequence: For entropy $S = \alpha\ E^n\ \Rightarrow\ T = 1/(n\ \alpha\ E^{n-1})$

Then $T = Const$ leads to $n = 1\ \Rightarrow\ \rho(E) = C\ e^S = C\ e^{\alpha E}$
i.e. **exponentially growing spectrum!**
R. Hagedorn, Suppl. Nuovo Cimento **3** (1965) 147

Hadronization in Elementary Particle Collisions

- Stat. Hadronization Model: T = 175+/-15 MeV
  F.Becattini, A.Ferroni, Acta. Phys. Polon. B 35 (2004)

There are no quarks and gluons in this model! Only known hadrons!!!



SBM is still important since it is able to explain how "the particles are born in equilibrium"

Statistical Partitions

Canonical partition function of N classical (Boltzmann) particles is

N = fixed

$$\mathbf{Z_N(V,T)} = \frac{\mathbf{1}}{\mathbf{N!}} \int \prod_{\mathbf{i=1}}^{\mathbf{N}} \left[\frac{\mathbf{g\ d^3r_i d^3k_i}}{(\mathbf{2}\pi)^\mathbf{3}} \exp\left(-\frac{\mathbf{E_i}}{\mathbf{T}}\right)\right] \exp\left(-\frac{\mathbf{U}}{\mathbf{T}}\right) \quad \text{with} \quad \mathbf{E_i} = (\mathbf{m^2} + \mathbf{k_i^2})^{\mathbf{1/2}}$$

Interaction is given by the sum over of momentum dependent pair potentials:

$$\mathbf{U} \;=\; \sum_{\mathbf{1 \le i < j \le N}} \mathbf{u_{ij}} \quad \text{with} \quad \mathbf{u_{ij}} \equiv \mathbf{u}(\mathbf{r_i}, \mathbf{k_i};\ \mathbf{r_j}, \mathbf{k_j})$$

g is degeneracy factor

the Grand CP function:

$$\mathcal{Z}(\mathbf{V}, \mathbf{T}, \mu) \equiv \sum_{\mathbf{N=0}}^{\infty} \exp\left(\frac{\mu \mathbf{N}}{\mathbf{T}}\right) \mathbf{Z_N(V,T)}$$

Conserves mean number of particles (charges)

where $\mathbf{z} \equiv \exp(\mu/\mathbf{T})$ is fugacity

Statistical Bootstrap Partition

Consider Boltzmann n-particle Micro Canonical Partition

Ideal gas
U = 0

$$\sigma_n(E,V,m) = \frac{1}{n!}\left[\frac{V}{(2\pi)^3}\right]^n \int \delta\left(E - \sum_{i=1}^{n} E_i\right) \prod_{i=1}^{n} (4\pi p_i^2 \; dp_i) \qquad (2)$$

Its Laplace transform is the n-particle Canonical partition

$$Z_n(T,V,m) = \frac{1}{n!}\left[\frac{V}{(2\pi)^3}\right]^n \left[4\pi \int e^{-\frac{\sqrt{p^2+m^2}}{T}} p^2 \; dp\right]^n \qquad (3)$$

Summing up over all n = 0, 1, 2,..., one finds

$$Z(T,V,m) = \sum_{n=0}^{\infty} Z_n(T,V,m) = \sum_{n=0}^{\infty} \frac{Z_1(T,V,m)^n}{n!} =$$

$$\exp\left[\frac{VT}{2\pi^2} m^2 K_2\left(\frac{m}{T}\right)\right] \approx \exp\left[\left(\frac{mT}{2\pi}\right)^{3/2} V \exp\left(-\frac{m}{T}\right)\right]\Bigg|_{m>>T} \qquad (4)$$

Statistical Bootstrap Equation

For **a mixture** of two gases with particles of masses m_1 and m_2

$$Z(T,V,m_1,m_2) = Z(T,V,m_1)\cdot Z(T,V,m_2)$$

$\Rightarrow$ for spectrum $\rho(m)$ one obtains

$$Z_\rho(T,V) = \exp\left[\frac{VT}{2\pi^2}\int_0^\infty m^2 K_2\left(\frac{m}{T}\right)\rho(m)\ dm\right] \quad (5)$$

Where to get the spectrum $\rho(m)$ from?

S. Frautschi suggested the Bootstrap Equation of the form
S. Frautschi, Phys. Rev. **D3** (1971) 2821

$$\rho(m) = \delta(m-m_0) + \sum_{n=2}^{\infty}\frac{1}{n!}\int \delta\left(m - \sum_{i=1}^{n} m_i\right)\prod_{i=1}^{n}(\rho(m_i)\ dm_i) \quad (6)$$

$\Rightarrow$The fireball of mass m is either "input particle" with mass m_0,

or it is composed of any number of fireballs of any masses such that $\sum m_i = m$

Solution of Statistical Bootstrap Equation

Solution of SBE follows by the Laplace transform $e^{-m/T}$.
J. Yellin, Nucl. Phys. **B52** (1973) 583

With notations $z = \exp\left[-\frac{m_0}{T}\right]$; $\quad G(z) = \int_{m_0} \exp\left[-\frac{m}{T}\right] \rho(m) dm$

The SBE becomes $\quad z = 2G - \exp[G] + 1$

physical z ≥ 0

For $G \to 0 \quad \Rightarrow \quad z \approx G$, but for $G \to \infty \quad \Rightarrow \quad z \approx -\infty$

One can readily check that z(G) has a maximum!

$$\frac{dz}{dG} = 0 \;\Rightarrow\; z_{max} = z_0 = ln4 - 1 \approx 0.3863...; \quad G(z_0) = \ln 2$$

- Solution: $\rho(m) \approx m^{-3} \exp\left[\frac{m}{T_H}\right]$ for $m \to \infty$

- But this means that there exists a limiting temperature!?

$$T \;\leq\; T_H \;=\; -\frac{m_0}{\ln z_0} \approx \frac{m_0}{0.95} \approx \frac{m_\pi}{0.95} \approx 145 \text{ MeV}$$

Limiting T at fixed volume

As $T \to T_H - 0^+$ it follows $E \to \infty$

Grand canonical: fix volume V_{des} and T close to T_H

$$\frac{E}{V_{des}} \approx \int_{m_0}^{\infty} dm\ m \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[\frac{m}{T_H} - \frac{m}{T}\right] m^{-3}$$

Peculiar thing is that in the r.h.s. of mass integral

infinitely heavy states contribute! Where do they come from?

- Cabibbo and Parisi, Phys. Lett. **B59** (1975) 67, suggested that the limiting temperature T_H means a phase transition to quarks and gluons. **And PT is of 2-nd order!?**

Can we really prove this from SBE?

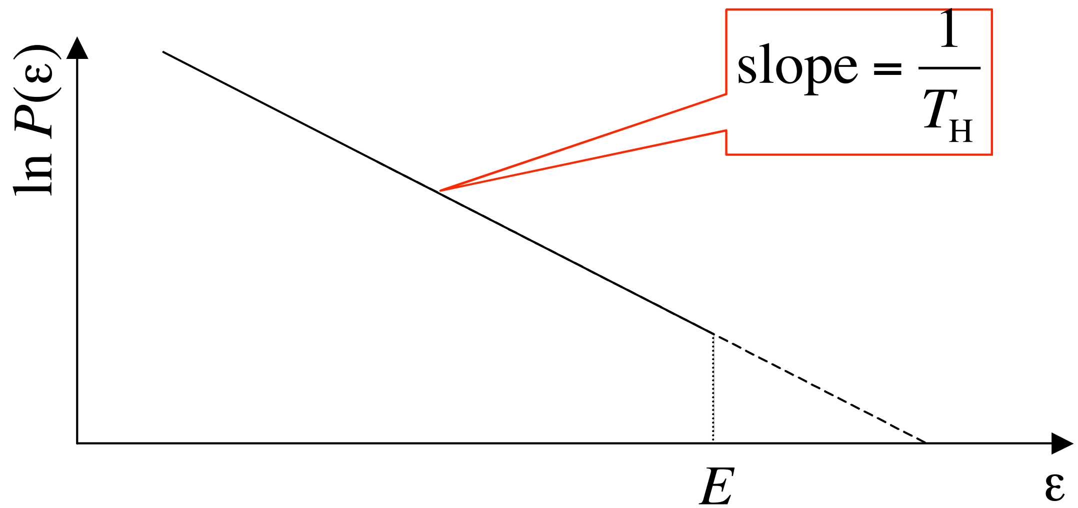
Microcanonical Ensemble
Example #1: 1-d Harmonic Oscillator

- For 1-d Harmonic Oscillator with energy ε in contact with Hagedorn resonance (just exponential spectrum for simplicity). Total energy is E. K.A.B.et al, Europhys. Lett. 76 (2006) 402

- The microcanonical probability of state & is:

$$P(\varepsilon) = \rho(E-\varepsilon) = \exp\left(\frac{E-\varepsilon}{T_H}\right) = \exp\left(\frac{E}{T_H}\right)\exp\left(-\frac{\varepsilon}{T_H}\right)$$

Exponent is Grand canonical! With fixed T!



Average value of & is

$$\bar{\varepsilon} = T_H\left(1 - \frac{E/T_H}{\exp(E/T_H)-1}\right)$$

For $E \to \infty$ **:** $\bar{\varepsilon} \to T_H$

Example #2: An Ideal Vapor coupled to Hagedorn resonance

- Consider microcanonical partition of N particles of mass m and kin. energy ε. The total level density is

$$P(E,\varepsilon) = \rho_{\mathrm{H}}(E-\varepsilon)\rho_{\mathrm{iv}}(\varepsilon) = \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{\mathrm{m}\varepsilon}{2\pi}\right)^{\frac{3}{2}N} \exp\left(\frac{E-\mathrm{m}N-\varepsilon}{\mathrm{T_H}}\right)$$

Exponent is Grand canonical! With fixed T!

The most probable energy partition is

$$\frac{\partial \ln P}{\partial \varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_{\mathrm{H}}} = 0 \Rightarrow \frac{\varepsilon}{N} = \frac{3}{2}T_{\mathrm{H}}$$

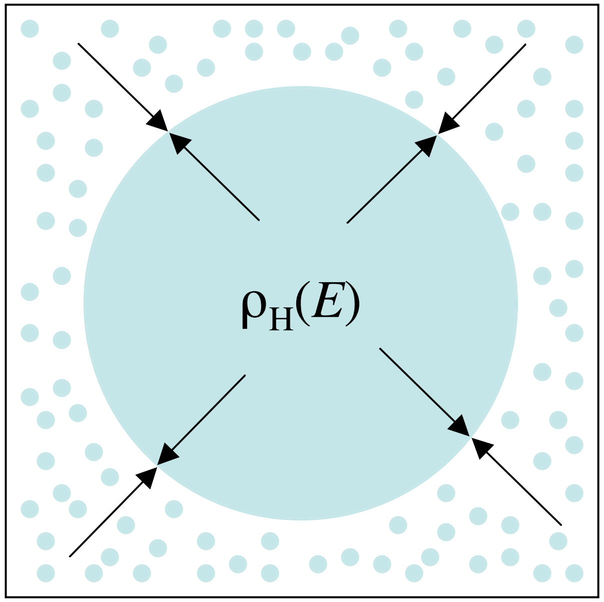
Homework No1: derive this result!

- T_H is the sole temperature characterizing the system:
- **A Hagedorn-like system is a perfect thermostat!**

Example #3: An Ideal Particle Reservoir

L.G. Moretto, K.A.B. et al, nucl-th/0601010

- If, in addition, particles are generated by the Hagedorn resonance, their concentration is volume independent!



ideal vapor ρ_{iv}
- particle mass = m
- volume = V
- particle number = N
- energy = ε

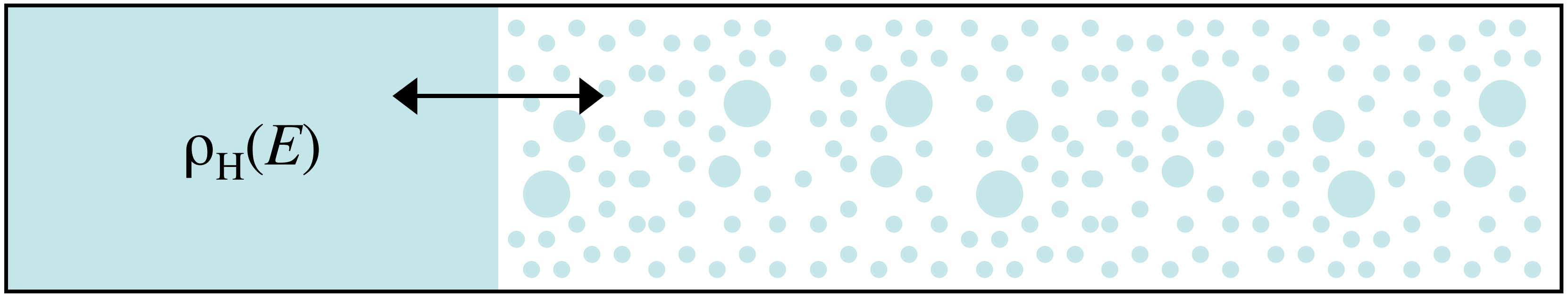
$$\left.\frac{\partial \ln P}{\partial N}\right|_V = -\frac{m}{T_H} + \ln\left[\frac{V}{N}\left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}}\right] = 0 \Rightarrow \boxed{\frac{N}{V} = \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right)}$$

Homework No2: derive this result!

Remarkable result because it mean saturation between gas of particles and Hagedorn thermostat!

Important Finding!

- Volume independent concentration of vapor means:
- for increasing volume of system gas particles will be evaporated from Hagedorn resonance (till it vanishes);
- by decreasing volume we will absorb gas particles to Hagedorn resonance! Compare to ordinary water and its vapor!
- Literally, it is a liquid (Hagedorn resonance) in equilibrium with its vapor at Const. temperature!
- **=> This is mixed phase of the first order PT!**



Why In Previous Works There Was an Upper Temperature?

- Because they used canonical and grand canonical ensembles which are NOT equivalent to MCE in this case!

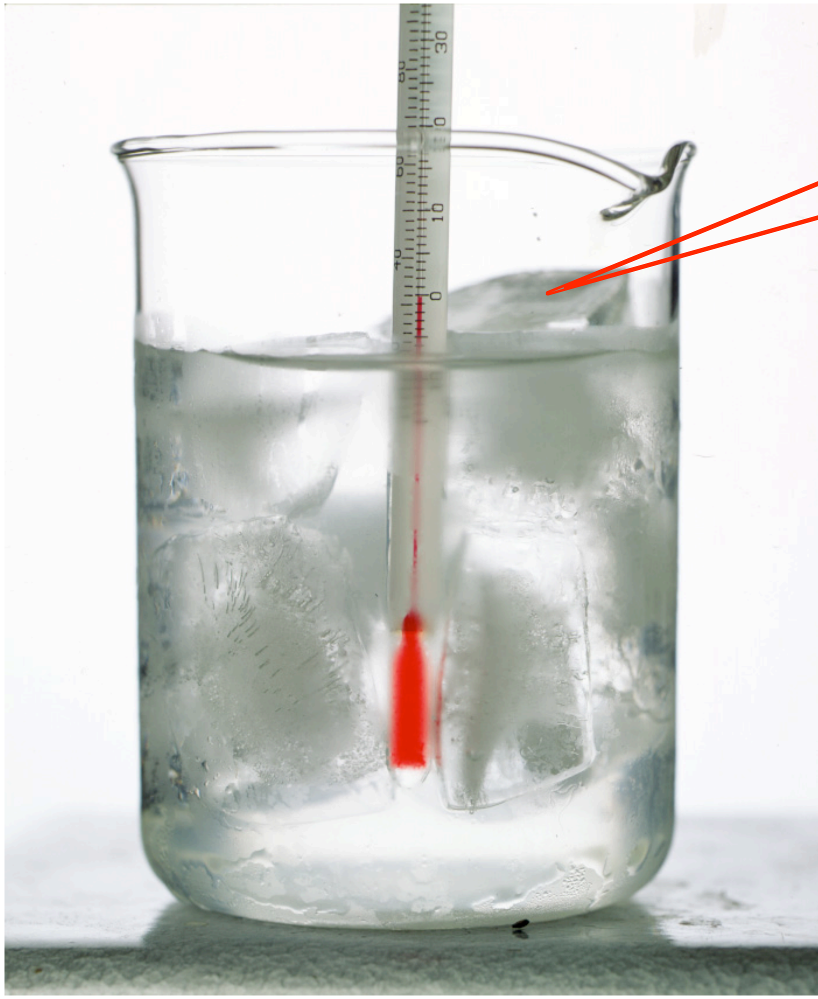
- Since the Hagedorn resonance is a perfect thermostat, the transform to (grand)canonical ensemble with other T does not make ANY SENSE!

$$Z_{Can} \equiv \int d\,E \;\; \rho_0 \;\; e^{\frac{E}{T_H} - \frac{E}{T}} = \rho_0 \;\; \frac{T_H\,T}{T_H - T}$$

it exists for $T < T_H$, but we know that two thermostats of different temperatures CANNOT BE IN EQUILIBRIUM!

Example with Explicit Thermostat:

- Export/import of heat does not change T!



$T = T_0 = 273\text{K}$
or
$0 \le T \le 273\text{K}$?

$$S = S_0 + \frac{\Delta Q}{T_0} = S_0 + \frac{E}{T_0} \Rightarrow \rho(E) = e^S = e^{S_0 + \frac{E}{T_0}}$$

- First take heat dQ=E from system with temperature T:
- Then give it to thermostat
- Is T_0 just a parameter?

$$Z(T) = \int dE \rho(E) e^{-E/T} = \frac{T_0 T}{T_0 - T} e^{S_0}$$

According to this logic, thermostat can have ANY T <T_0 !

Conclusions for Hagedorn thermostat

- Exponential mass spectrum is a very special object.
- It imparts the Hagedorn temperature to particles in contact with it = perfect thermostat!
- It is also a perfect particle reservoir!
- Grand canonical treatment should be used with great care! Microcanonical one is the right one.
- This is 1-st order phase transition in a finite system. No liberation of color d.o.f. is necessary for that!
- These simple findings took about 40 years (!) since before 2005 no one studied a PT in microcanonical ensemble at finite volumes

This is why "the particles are born in equilibrium"

The Refined Analysis Shows:

- The inverse slope that Hagedorn resonances are imparting is a kinetic temperature. K.A.B. et al, hep-ph/0504011
- The presence of the mass cut-off of the Hagedorn spectrum DOES NOT ALTER our conclusions: Hagedorn resonances are PERFECT THERMOSTATS and PARTICLE RESERVOIRS!
- Power prefactor in the Hagedorn spectrum changes the imparting temperature on 10-15%, and, perhaps, can lead to some experimental signals.

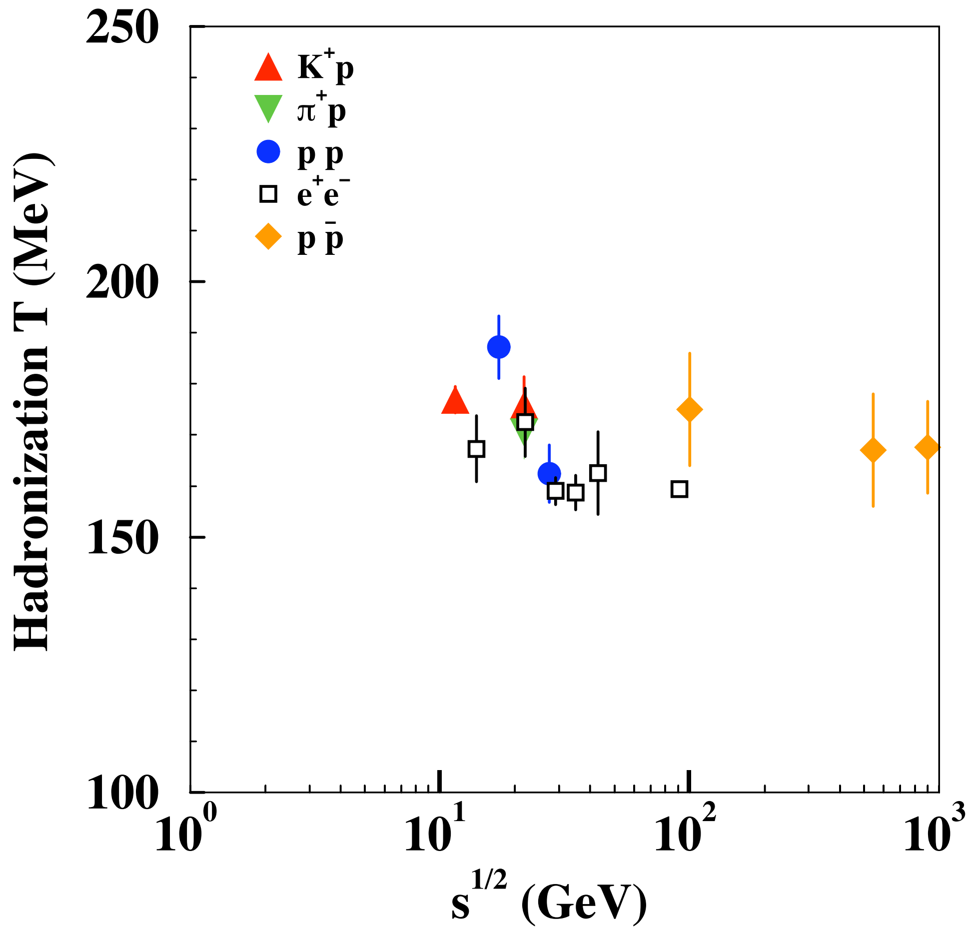
The Refined Analysis Shows:

- The inverse slope that Hagedorn resonances are imparting is a kinetic temperature. K.A.B. et al, hep-ph/0504011
- The presence of the mass cut-off of the Hagedorn spectrum DOES NOT ALTER our conclusions: **Hagedorn resonances are PERFECT THERMOSTATS and PARTICLE RESERVOIRS!**
- Power prefactor in the Hagedorn spectrum changes the imparting temperature on 10-15%, and, perhaps, can lead to some experimental signals.

Why is it important?

Hadronization in Elementary Particle Collisions

- Stat. Hadronization Model: T = 175+/-15 MeV
  F.Becattini, A.Ferroni, Acta. Phys. Polon. B 35 (2004)



These results **justify** the Statistical Hadronization Model and **explain** why hadronization T and inverse slopes in el. particle collisions are about 170 MeV.

NB: Hagedorn Spectrum Follows from

Stat.Bootstrap Model,
S.Frautschi, 1971

Hadrons are built from hadrons

Veneziano Model,
K.Huang,S.Weinberg,
1970

Used in string models

M.I.T. Bag Model,
J.Kapusta, 1981

Hadrons are quark-gluon bags

Large Nc limit of 3+1
QCD
T. Cohen, 2009

The real problem with H-spectrum is that experimentally it is not seen where it supposed to be seen!

To make SBE more realistic

- we have to understand why at low baryonic densities the 1-st order PT degenerates into a cross-over.
- we have to study the mechanism of the (tri)critical endpoint generation and the role of surface tension in it.
- we have to account for finite size of hadrons which might be not small.
- we have to account for finite life time of hadrons.

Thanks for your attention!