Color Superconducting Quark Matter

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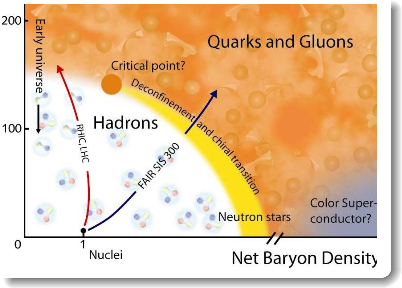
Dubna International Advanced School for Theoretical Physics
and HIC-for-FAIR School and Workshop
"Dense QCD Phases in Heavy Ion Collisions",

JINR Dubna (Russia), August 21 – September 4, 2010.

Motivation



- QCD phase diagram
- focus of this talk:
  large density, low temperature

→ color superconductivity ?

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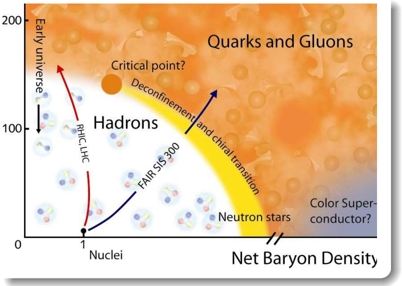

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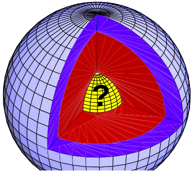
  - heavy-ion reactions: unlikely
  - compact stars:

    $\rho_{center} = 3 - 10\,\rho_0, \quad T \approx 0 \qquad (✔)$



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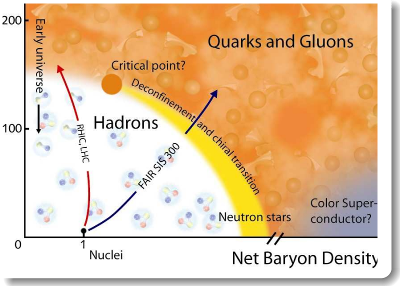

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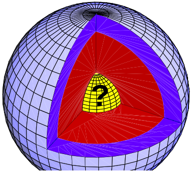
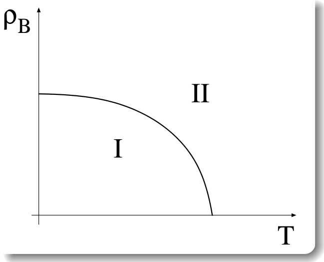


➜ QCD phase diagram under compact star conditions ?

QCD phase diagram (short history)

- early conjecture:



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I hadronic phase (confined)

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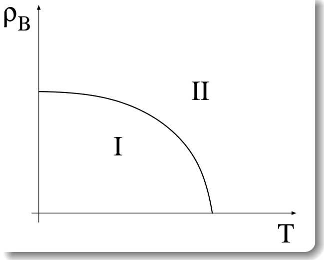

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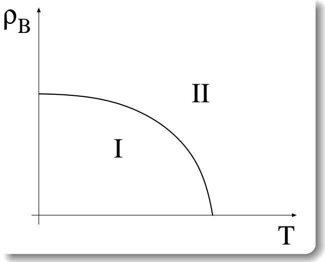

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II quark-gluon plasma (deconfined)

"The true phase diagram may actually be substantially more complex . . . "

- Collins & Perry, PRL (1975)

  "Also we might expect superfluidity or superconductivity."

Color superconducting phases

- early work: Barrois (1977); Frautschi (1978); Bailin & Love (1984)
- "rediscovery": Alford, Rajagopal, Wilczek, PLB (1998); Rapp, Schäfer, Shuryak, Velkovsky, PRL (1998).

$$\Delta \sim 100 \text{ MeV} \quad \rightarrow \quad T_c \sim 50 \text{ MeV}$$

- suggested phase diagrams (schematic)

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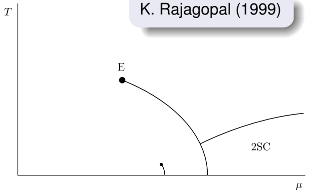

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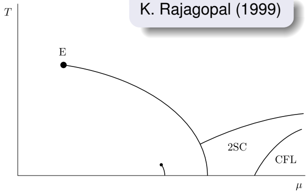

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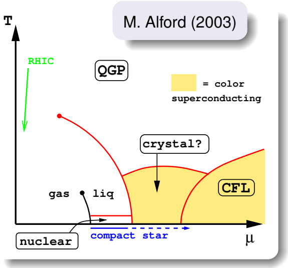

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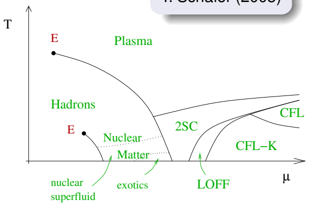

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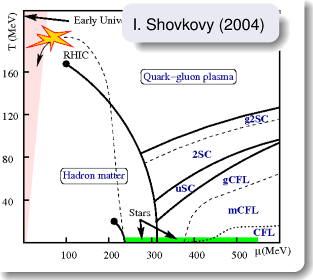

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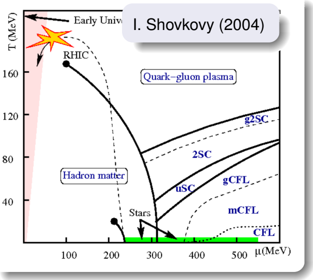

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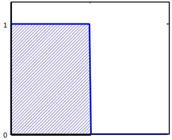
Outline

1. overview: phase diagrams
2. pairing patterns
3. gap equations
4. realistic quark masses, neutral matter
5. inhomogeneous phases
6. ...

Cooper instabilities

- ideal Fermi gas:
  - ➜ pair creation @ Fermi surface with no free energy

occupation #



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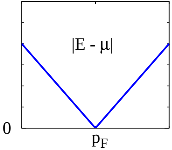

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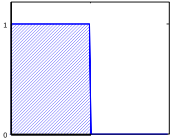

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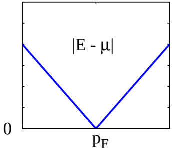

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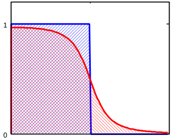

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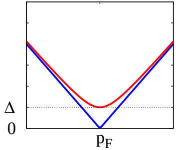

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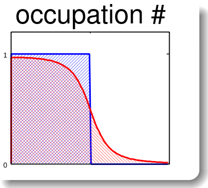

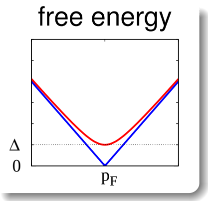

- QCD: attractive qq interaction ➜ diquark condensates

Field operators

- quark field operator: $q(x) = \begin{pmatrix} q_1(x) \\ \vdots \\ q_{4N_fN_c}(x) \end{pmatrix}$
  - 4 Dirac $\times$ N_f flavor $\times$ N_c color components
  - annihilates a quark or creates an antiquark

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- adjoint operator: $q^\dagger, \qquad \bar{q} = q^\dagger \gamma^0$
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Quark-antiquark condensates

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  - $\hat{\mathcal{O}}$ = operator in color, flavor, and Dirac space (including derivatives)
- examples:
  - "chiral condensate": $\langle \bar{q} q \rangle$
  - quark number density: $\langle \bar{q}\gamma^0 q \rangle = \langle q^\dagger q \rangle$
  - electric charge density:
    $\langle \bar{q}\,\hat{Q}\gamma^0\,q \rangle = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s, \qquad \hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}_f$
  - color charge densities

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- Bogoliubov rotation:

$$|g.s.\rangle \;=\; \prod_{\vec{p},s,c,c'} \left[\cos\theta_s^b(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^b(\vec{p})}\, \sin\theta_s^b(\vec{p})\, b^\dagger(\vec{p},s,u,c)\, b^\dagger(-\vec{p},s,d,c')\right]$$
$$\left[\cos\theta_s^d(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^d(\vec{p})}\, \sin\theta_s^d(\vec{p})\, d^\dagger(\vec{p},s,u,c)\, d^\dagger(-\vec{p},s,d,c')\right]|0\rangle$$

  - quasiparticles $=$ superpositions of particles and holes ($+$ antiparticles) which annihilate $|g.s.\rangle$

Diquark condensates

- diquark condensates: $\langle q^T \hat{\mathcal{O}} q \rangle$
- Pauli principle: $q_i q_j = -q_j q_i$

$$\Rightarrow \; q^T \hat{\mathcal{O}} q \; = \; q_i \hat{\mathcal{O}}_{ij} q_j \; = \; -q_j \hat{\mathcal{O}}_{ij} q_i \; = \; -q_j \hat{\mathcal{O}}^T_{ji} q_i \; = \; -q^T \hat{\mathcal{O}}^T q$$

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➜ $\hat{\mathcal{O}}$ must be **totally antisymmetric**: $\hat{\mathcal{O}}^T = -\hat{\mathcal{O}}$

Operators in flavor and color space

- Pauli matrices (for two flavors):

$$\underbrace{\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\text{symmetric triplet}}, \quad \underbrace{\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\text{antisymm. singlet}}$$

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- Gell-Mann matrices (for three flavors or colors):

$$\underbrace{\mathbb{1}, \; \lambda_1, \; \lambda_3, \; \lambda_4, \; \lambda_6, \; \lambda_8,}_{\text{symmetric sextet}} \qquad \underbrace{\lambda_2, \; \lambda_5, \; \lambda_7}_{\text{antisymmetric antitriplet}}$$

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  - antitriplet: The vector $\langle q^T \begin{pmatrix} \lambda_7 \\ -\lambda_5 \\ \lambda_2 \end{pmatrix} q\rangle$ transforms like an antiquark $\bar{q} = \begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$ under $SU(3)_c$.

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- antisymmetric:
  - $C\gamma_5$ (scalar)
  - C (pseudoscalar)
  - $C\gamma^\mu\gamma_5$ (vector)
- symmetric:
  - $C\gamma^\mu$ (axial vector)
  - $C\sigma^{\mu\nu}$ (tensor)

Combined operators

	symmetric	antisymmetric
Dirac	$C\gamma^\mu$, $C\sigma^{\mu\nu}$ A T	C, $C\gamma_5$, $C\gamma_5\gamma^\mu$ P S V
$U(2)$	$\underbrace{\mathbb{1}, \tau_1, \tau_3}_{3}$	$\underbrace{\tau_2}_{1}$
$U(3)$	$\underbrace{\mathbb{1}, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8}_{6}$	$\underbrace{\lambda_2, \lambda_5, \lambda_7}_{\bar{3}}$

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→ many possibilities . . .

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$$\langle q^T C\gamma_5 \tau_2 \lambda_2 q \rangle \sim \underbrace{(\uparrow\downarrow - \downarrow\uparrow)}_{\text{spin}} \otimes \underbrace{(ud - du)}_{\text{flavor}} \otimes \underbrace{(r\,g - g\,r)}_{\text{color}}$$

Symmetry properties: color

- only red and green quarks are paired:

$$\langle q^T \lambda_2 q \rangle \sim \langle rg - gr \rangle \hat{=} \langle \bar{b} \rangle$$ "antiblue"

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- more general ansatz?

$$\langle q^T (\alpha\lambda_2 + \beta\lambda_5 + \gamma\lambda_7) q \rangle = \alpha\langle \bar{b} \rangle - \beta\langle \bar{g} \rangle + \gamma\langle \bar{r} \rangle \equiv \begin{pmatrix} \gamma \\ -\beta \\ \alpha \end{pmatrix}$$

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  - can always be rotated into the "antiblue" direction by a global color transformation $q \rightarrow \exp(i\theta_a \cdot \frac{\lambda_a}{2})\, q$

Symmetry properties: color

- only red and green quarks are paired:

$$\langle q^T \lambda_2 q \rangle \sim \langle rg - gr \rangle \hat{=} \langle \bar{b} \rangle$$ "antiblue"

→ $SU(3)_c$ "spontaneously" broken to $SU(2)_c$

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  - $SU(2)_A$: $\quad q \;\rightarrow\; e^{i\theta_a\frac{\tau_a}{2}\gamma_5}q \quad\Rightarrow\quad \delta \;\rightarrow\; \delta$ conserved

Three-flavor color superconductors

- scalar color-antitriplet condensates:
  - $s_{AA'} \;=\; \langle q^T\, C\gamma_5\, \tau_A\, \lambda_{A'}\, q\rangle$
  - notation:
    - τ_A = antisymmetric flavor generator
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  ➜ diagonalization by combined color and flavor rotations:

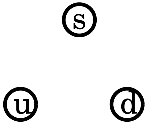
$$s \;\rightarrow\; s' \;=\; V\,s\,U \;=\; \begin{pmatrix} s_{22} & 0 & 0 \\ 0 & s_{55} & 0 \\ 0 & 0 & s_{77} \end{pmatrix}, \qquad U \in SU(3)_c,\;\; V \in SU(3)_f$$

Pairing patterns

- eight possible phases:

normal quark matter (NQ)

$s_{22} = s_{55} = s_{77} = 0$



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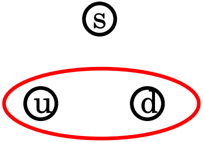
$s_{22} \neq 0, \quad s_{55} = s_{77} = 0$

$+$ two more phases of this kind



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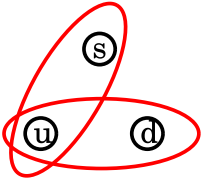

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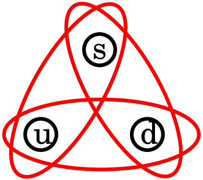

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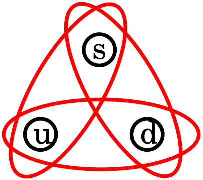
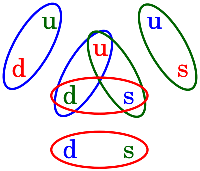


- CFL pairing pattern (more explicitly):

$$(\uparrow\downarrow - \downarrow\uparrow) \otimes \Big(\Delta_2\,(ud - du) \otimes (r\,g - g\,r) + \Delta_5\,(ds - sd) \otimes (g\,b - b\,g) + \Delta_7\,(su - us) \otimes (b\,r - r\,b) \Big)$$



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Theoretical approach

- weak-coupling QCD → D. Rischke, Prog. Part. Nucl. Phys. (2004)
  - $\mu \gg \Lambda_{QCD} \;\Rightarrow\; \alpha_s(\mu) \ll 1$ ➜ systematic expansion
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  - *some highlights later* ...
- NJL-type models
  - schematic quark models with point interactions (like BCS!)
  - not predictive
  - suited for explorative studies with competing condensates

Two-flavor model

- Lagrangian with interaction in the desired channel:

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + H \sum_{A=2,5,7} (\bar{q}\, i\gamma_5\tau_2\lambda_A\, C\bar{q}^T)(q^T C\, i\gamma_5\tau_2\lambda_A\, q)$$

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$$\Rightarrow \mathcal{L}_{int} = 4H \sum_{A=2,5,7} \bar{\Psi} \begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix} \Psi\, \bar{\Psi} \begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix} \Psi$$

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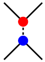
➜ vertices:



$$= \; 4Hi\, \Gamma_A^{\uparrow} \otimes \Gamma_A^{\downarrow} \;=\; 4Hi \begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix}$$

Nambu-Gorkov propagator

- kinetic term + chemical potential:

$$\mathcal{L}_{kin} + \mu\, q^{\dagger} q \;=\; \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q$$

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$$
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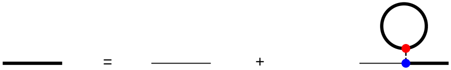
➜ inverse bare propagator in momentum space:

$$S_0^{-1}(p) = \begin{pmatrix} p\!\!\!/ + \mu\gamma^0 & 0 \\ 0 & p\!\!\!/ - \mu\gamma^0 \end{pmatrix}$$

Mean-field propagator

- dressed propagator (Hartree approximation):



$$iS(p) = iS_0(p) + iS_0(p)\,(-i\Sigma)\,iS(p)$$

$$\Leftrightarrow\; S^{-1}(p) = S_0^{-1}(p) - \Sigma$$

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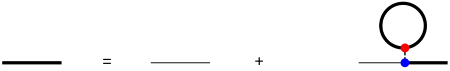

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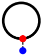
- self-energy:

$$-i\Sigma \; = $$



$$= \; 4iH\sum_A \Big\{ \; \Gamma_A^{\uparrow} \int \frac{d^4k}{(2\pi)^4}\,(-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\downarrow}\, iS(k)] \\ + \; \Gamma_A^{\downarrow} \int \frac{d^4k}{(2\pi)^4}\,(-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\uparrow}\, iS(k)] \Big\}$$

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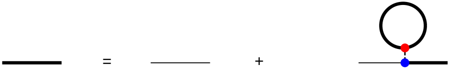

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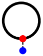
$$-i\Sigma \; = $$  $$= \; 4iH \sum_A \Big\{ \; \Gamma_A^{\uparrow} \int \frac{d^4k}{(2\pi)^4} \, (-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\downarrow}\, iS(k)] + \Gamma_A^{\downarrow} \int \frac{d^4k}{(2\pi)^4} \, (-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\uparrow}\, iS(k)] \Big\}$$

→ selfconsistency problem!

Gap equation

- selfconsistency problem: $S^{-1} = S_0^{-1} - \Sigma[S]$

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$$\Sigma = \begin{pmatrix} 0 & -\Delta\,\gamma_5\tau_2\lambda_2 \\ \Delta^*\,\gamma_5\tau_2\lambda_2 & 0 \end{pmatrix} \Rightarrow S^{-1} = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}$$

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- result:

$$\Delta = 16H\,\Delta\,i\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k_0^2-\omega_-^2}+\frac{1}{k_0^2-\omega_+^2}\right)$$ "gap equation"

quasiparticle dispersion laws: $\omega_\mp = \sqrt{(|\vec{k}| \mp \mu)^2 + |\Delta|^2}$

Propagator

- dressed propagator: $S = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}^{-1}$
  - dimension: $2 \times 4 \times N_f \times N_c$

    ➜ 48×48 matrix for $N_f = 2$, $N_c = 3$
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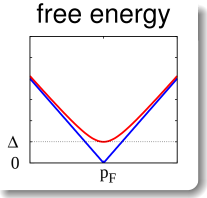
- diagonalization:

$$S(p^0, \vec{p}) = U(\vec{p}) \begin{pmatrix} \frac{1}{p^0 - \lambda_1(\vec{p})} & & \\ & \ddots & \\ & & \frac{1}{p^0 - \lambda_{48}(\vec{p})} \end{pmatrix} U^\dagger(\vec{p})\,\gamma^0$$

  - $U(\vec{p})$ = unitary matrix, does not depend on p^0 !

Dispersion relations



- 48 eigenvalues

  = 24 quasiparticle dispersion relations:

  - $\omega_-(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |\Delta|^2}$ (8-fold) red and green quarks
  - $\omega_+(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |\Delta|^2}$ (8-fold) " antiquarks
  - $\epsilon_-(\vec{p}) = \big|\, |\vec{p}| - \mu \,\big|$ (4-fold) blue quarks
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- $+$ 24 quasiholes: $-\omega_\mp(\vec{p}),\ -\epsilon_\mp(\vec{p})$

Dispersion relations (CFL)

- 72 eigenvalues

  $=$ 36 quasiparticle dispersion relations:

$\omega_{8,-}(\vec{p}) = \sqrt{(\lvert\vec{p}\rvert - \mu)^2 + \lvert\Delta\rvert^2}$	(16-fold)	quark octet $\times$ spin
$\omega_{8,+}(\vec{p}) = \sqrt{(\lvert\vec{p}\rvert + \mu)^2 + \lvert\Delta\rvert^2}$	(16-fold)	antiquark "
$\omega_{1,-}(\vec{p}) = \sqrt{(\lvert\vec{p}\rvert - \mu)^2 + \lvert 2\Delta\rvert^2}$	(2-fold)	quark singlet $\times$ spin
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- $+$ 36 quasiholes: $-\omega_{8,\mp}(\vec{p}),\ -\omega_{1,\mp}(\vec{p})$

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➜ nontrivial solutions always exist for $H > 0$!

Mean-field Lagrangian

- back to our Lagrangian:

$$\hat{\mathcal{L}} = \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q + H \sum_{A=2,5,7} (\bar{q}\, i\gamma_5\tau_2\lambda_A\, C\bar{q}^T)(q^T C\, i\gamma_5\tau_2\lambda_A\, q)$$

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$$\mathcal{L}_{int} = \frac{1}{2} \sum_A \left\{ (q^T C \gamma_5 \tau_2 \lambda_A q)^\dagger \varphi_A + h.c. - \frac{1}{2H} \varphi_A^\dagger \varphi_A \right\}$$

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- result, using Nambu-Gorkov spinors:

$$\mathcal{L}_{MF} = \bar{\Psi} \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & -i\overleftarrow{\partial\!\!\!/} - \mu\gamma^0 \end{pmatrix} \Psi - \frac{|\Delta|^2}{4H}$$

Thermodynamic potential

- (grand canonical) thermodynamic potential:

$$\Omega(T,\mu) \;=\; -\frac{T}{V}\ln\mathcal{Z} \;=\; -\frac{T}{V}\ln\mathbf{Tr}\,\exp\left(-\frac{1}{T}\int d^3x\,(\mathcal{H}-\mu q^\dagger q)\right)$$

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- general result:

$$\Omega(T,\mu) \;=\; -T\sum_n\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\,\ln\left(\frac{1}{T}\,S^{-1}(i\omega_n,\vec{k})\right) \;+\; \mathcal{V}$$

  - $\ln A = \ln\big((1-(1-A)\big) = \sum_{n=1}^{\infty}\frac{1}{n}(1-A)^n$
  - useful formula: $\mathrm{Tr}\,\ln A = \ln \mathrm{Det}\,A$

Thermodynamic potential

- result after Matsubara summation:

$$
\begin{aligned}
\Omega(T,\mu) \;=\; -\int \frac{d^3p}{(2\pi)^3} \Big\{ \;& 8\left(\frac{\omega_-}{2} + T\,\ln(1+e^{-\omega_-/T}) \right. \\
& \left. +\frac{\omega_+}{2} + T\,\ln(1+e^{-\omega_+/T}) \right) \\
& +4\left(\frac{\epsilon_-}{2} + T\,\ln(1+e^{-\epsilon_-/T}) \right. \\
& \left. +\frac{\epsilon_+}{2} + T\,\ln(1+e^{-\epsilon_+/T}) \right) \Big\} \\
& + \frac{|\Delta|^2}{4H}
\end{aligned}
$$

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$$\rightarrow \Delta = 4H\,T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\left[\,iS\,\Gamma_2^{\downarrow}\,\right]$$

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- integrand strongly peaked at $|\vec{p}| = \mu$ $\rightarrow$ $\int p^2\,dp \approx \mu^2\int dp$
- Taylor expansion of the remaining integral in Δ

$\rightarrow$ -$\delta\Omega \propto \mu^2\Delta^2$

Thermodynamic quantities

- standard thermodynamic relations:
  - pressure: $p = -\Omega$
  - density: $n = -\frac{\partial\Omega}{\partial\mu}$
  - entropy density: $s = -\frac{\partial\Omega}{\partial T}$
  - energy density: $\varepsilon = -p + Ts + \mu n$

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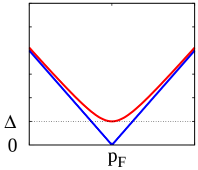

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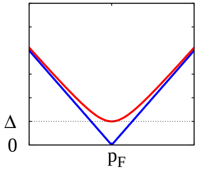

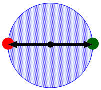

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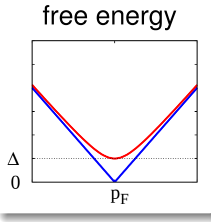

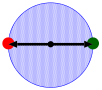

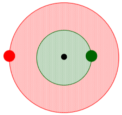

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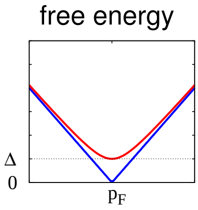

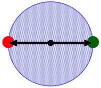

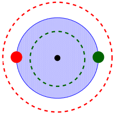

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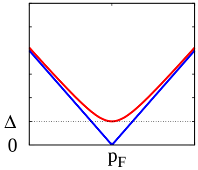

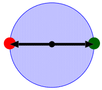

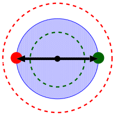

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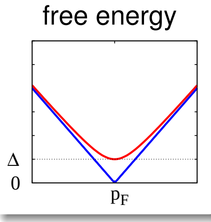

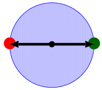

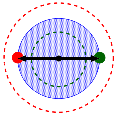

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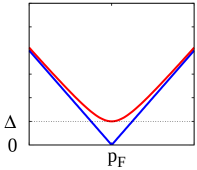

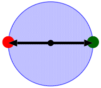

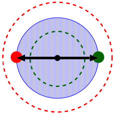

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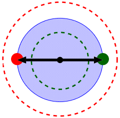

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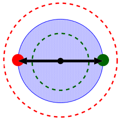

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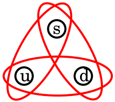

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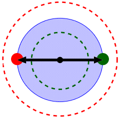

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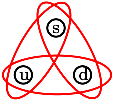
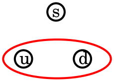


- case 2: *moderate densities*

$$M_s \gtrsim \mu \gg M_{u,d} \;\rightarrow\; p_F^s \ll p_F^d \approx p_F^u \;\rightarrow\; \text{2SC}$$



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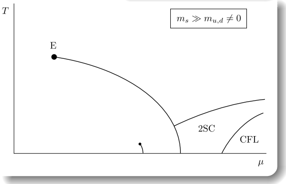
K. Rajagopal (1999)



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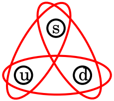

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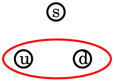

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- four conserved charges; densities: $n_r,\ n_g,\ n_b,\ n_Q$

$$\Leftrightarrow\ n = n_r + n_g + n_b, \quad n_3 = n_r - n_g, \quad n_8 = \frac{1}{\sqrt{3}}(n_r + n_g - 2n_b), \quad n_Q$$

➜ four independent chemical potentials: $\mu,\ \mu_3,\ \mu_8,\ \mu_Q$

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but one should always check . . .

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- constraints in compact stars:
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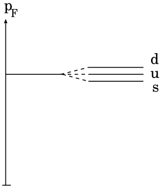

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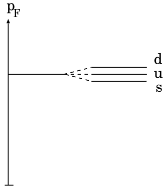

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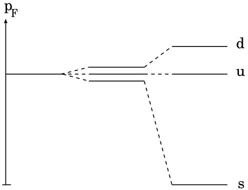

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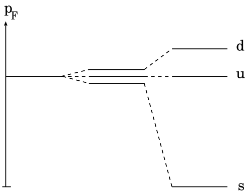

- strong coupling: 2SC possible !

3-flavor NJL model

- Lagrangian: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$
  - free part: $\mathcal{L}_0 = \bar{q}(i\not{\partial} - \hat{m})q \,, \qquad \hat{m} = diag_f(m_u, m_d, m_s)$
  - quark-quark interaction:

$$\mathcal{L}_{qq} = H \sum_{A,A'} (\bar{q}\, i\gamma_5 \tau_A \lambda_{A'}\, C\bar{q}^T)(q^T C\, i\gamma_5 \tau_A \lambda_{A'}\, q)$$

  - standard $SU(3)$ quark-antiquark interaction:

$$\begin{aligned}\mathcal{L}_{\bar{q}q} &= G \sum_{a=0}^{8} \left\{ (\bar{q}\tau_a q)^2 + (\bar{q} i\gamma_5 \tau_a q)^2 \right\} \\ &\quad - K \left\{ \det_f \left(\bar{q}(1+\gamma_5)q \right) + \det_f \left(\bar{q}(1-\gamma_5)q \right) \right\}\end{aligned}$$

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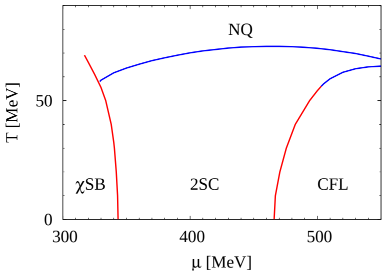
- mean-field approximation:
  - qq-condensates: $\langle ud \rangle$, $\langle us \rangle$, $\langle ds \rangle$ $\leftrightarrow$ *diquark gaps*
  - $\bar{q}q$-condensates: $\langle \bar{u}u \rangle$, $\langle \bar{d}d \rangle$, $\langle \bar{s}s \rangle$ $\leftrightarrow$ *dynamical masses*

Model calculations

- NJL model *without* imposing neutrality



M.B., M. Oertel, NPA (2002); M. Oertel, M.B., hep-ph/0202098.

also: Gastineau, Nebauer, Aichelin, PRC(2002).

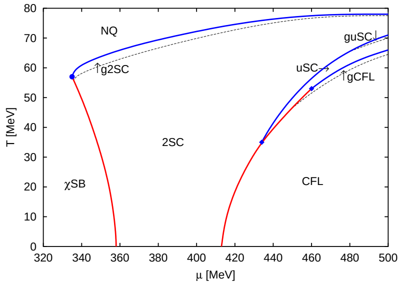
- quark phases at T=0:

$$(\chi SB \to)\ \textit{2SC} \to \textit{CFL}$$

Model calculations

- NJL model *with* neutrality constraints

  (assuming homogeneous phases)
- “strong diquark coupling”: $H = G$



- quark phases at T=0:
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Rüster, Werth, M.B., Shovkovy, Rischke, PRD (2005)

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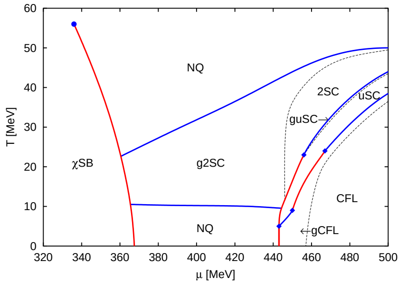
- "intermediate diquark coupling": $H = 0.75\,G$



- quark phases at T=0:
  - "strong coupling":
    2SC → *CFL*
  - "intermediate coupling":
    normal → *gCFL* → *CFL*
    - no 2SC!
    - gapless phases

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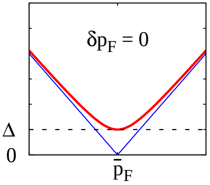

Gapless color superconductors

- unequal Fermi momenta:
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  - splitting of the quasiparticle modes:
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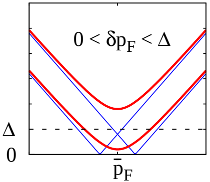
free energy



Gapless color superconductors

- unequal Fermi momenta:
  $p_F^{a,b} = \bar{p}_F \pm \delta p_F$
  - splitting of the quasiparticle modes:
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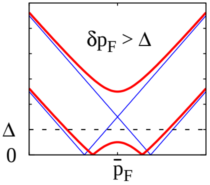

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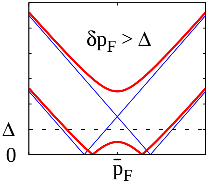

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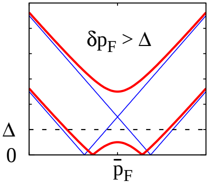

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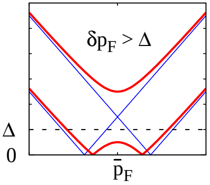

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- problem: imaginary Meissner masses ➜ instability !

Inhomogeneous phases

- alternative to BCS pairing:
  pairs with nonzero total momentum

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- FF (Fulde, Ferrell, 1964):
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    $\langle q(\vec{x})q(\vec{x})\rangle \sim \Delta\, e^{2i\vec{q}\cdot\vec{x}}$ for fixed $\vec{q}$
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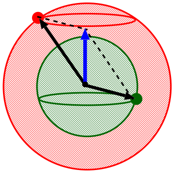

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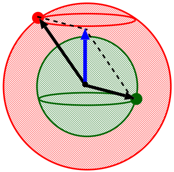

- LO (Larkin, Ovchinnikov, 1964):
  - multiple plane waves (e.g., $\cos(2\vec{q}\cdot\vec{x})$)

Model

- Model Lagrangian with NJL-type qq interaction:

$$\mathcal{L} = \bar{q}\,(i\partial\!\!\!/ + \hat{\mu}\gamma^0)\,q + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = H \sum_{A,A'=2,5,7} (\bar{q}\, i\gamma_5\tau_A\lambda_{A'}\, q_c)(\bar{q}_c\, i\gamma_5\tau_A\lambda_{A'}\, q)\,, \qquad q_c = C\bar{q}^T$$

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- mean-field approximation:

$$\langle\,\varphi_{AA'}(x)\rangle = \Delta_A(x)\,\delta_{AA'}\,, \qquad \langle\,\varphi^\dagger_{AA'}(x)\rangle = \Delta^*_A(x)\,\delta_{AA'}$$

  - $\Delta_A(x)$ *classical* fields
  - retain space-time dependence!

Mean-field model

- Lagrangian:

$$\mathcal{L}_{MF}(x) = \bar{\Psi}(x)\, S^{-1}(x)\, \Psi(x) \; - \; \frac{1}{4H} \sum_A |\Delta_A(x)|^2$$

  - Nambu-Gor'kov bispinors: $\Psi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} q(x) \\ q_c(x) \end{pmatrix}$
  - inverse dressed propagator:

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- Thermodynamic potential:

$$\Omega_{MF}(T,\mu) = -\frac{1}{2}\frac{T}{V}\mathrm{Tr}\ln\frac{S^{-1}}{T} + \frac{T}{V}\sum_A \int_{[0,\frac{1}{T}]\otimes V} d^4x\, \frac{|\Delta_A(x)|^2}{4H}$$

  - $\mathrm{Tr}\ln S^{-1}$ nontrivial because of x-dependent gap functions!

Crystalline ansatz

- gap functions: time-independent, periodic in space
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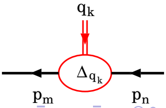

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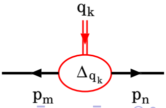

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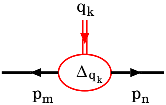
  - diagonal in energy ➜ Matsubara sum as usual



Hamiltonian

- The inverse propagator can be put into the form

$$S^{-1}_{p_m,p_n} = \gamma^0 \left(i\omega_{p_n} - \mathcal{H}_{\vec{p}_m,\vec{p}_n} \right) \delta_{\omega_{p_m},\omega_{p_n}}$$

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$$\Omega_{MF} = -\frac{1}{4V} \sum_{\lambda} \left[E_\lambda + 2T \ln \left(1 + 2e^{-E_\lambda/T} \right) \right] + \sum_A \sum_{q_k} \frac{|\Delta_{A,q_k}|^2}{4H}$$

  - E_λ: eigenvalues of $\mathcal{H}$

Thermodynamic potential

- remaining problem: diagonalize $\mathcal{H}$

$$\mathcal{H}_{\vec{p}_m,\vec{p}_n} = \begin{pmatrix} (\gamma^0 \vec{p\!\!\!/}_n - \hat{\mu})\,\delta_{\vec{p}_m,\vec{p}_n} & -\sum_{\vec{q}_k} \hat{\Delta}_{q_k} \gamma^0 \gamma_5\, \delta_{\vec{q}_k,\vec{p}_m-\vec{p}_n} \\ \sum_{\vec{q}_k} \hat{\Delta}^*_{q_k} \gamma_0 \gamma_5\, \delta_{\vec{q}_k,\vec{p}_n-\vec{p}_m} & (\gamma^0 \vec{p\!\!\!/}_n + \hat{\mu})\,\delta_{\vec{p}_m,\vec{p}_n} \end{pmatrix}$$

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➜ We finally obtain:

$$\Omega_{MF} = -\frac{1}{4}\int_{B.Z.} \frac{d^3k}{(2\pi)^3} \sum_\lambda \left[E_\lambda(\vec{k}) + 2T\,\ln\left(1 + 2e^{-E_\lambda(\vec{k})/T}\right)\right] + \sum_A \sum_{q_k} \frac{|\Delta_{A,q_k}|^2}{4H}$$

  - $E_\lambda(\vec{k})$: eigenvalues of $\mathcal{H}(\vec{k})$.

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- consider only 2SC-like pairing
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- remaining diagonalization problem:

$$(\mathcal{H}_{\Delta,\delta\mu})_{\vec{p}_m,\vec{p}_n} = \begin{pmatrix} (p_m - \bar{\mu} - \delta\mu)\,\delta_{\vec{p}_m,\vec{p}_n} & \Delta_{p_m - p_n} \\ \Delta^*_{p_n - p_m} & -(p_m - \bar{\mu} + \delta\mu)\,\delta_{\vec{p}_m,\vec{p}_n} \end{pmatrix}$$

Regularization

- unregularized expression for Ω_{MF} divergent
  - ➜ needs to be regularized
- 3-momentum cutoff ?
- inhom. phases: $\mathcal{H}$ depends on two momenta!
  - cut off both of them, e.g., $|\vec{p}_m|, |\vec{p}_n| \leq \Lambda$?
  - ➜ strong regularization artifacts:
    - large $|\vec{q}|$ suppressed, e.g., $|\vec{q}| < 2\Lambda$
    - violates "model independent" low-energy results

➜ Pauli-Villars-like scheme:

$$F(E_\lambda) \to \sum_{j=0}^{2} F(E_{\lambda,j}) \,, \qquad E_{\lambda,j}(\vec{k}) = \sqrt{E_\lambda^2(\vec{k}) + j\Lambda^2}$$

Numerical investigation

- crystal structure:
  - general problem (too) difficult
  - ➜ consider one-dimensional modulations (in 3+1 D):

$$\Delta(z) = \sum_k \Delta_k \, e^{2ikqz}$$

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- external parameters: $T, \bar{\mu}, \delta\mu$
  - here: $T = 0, \ \bar{\mu} = 400$ MeV ➜ only $\delta\mu$ is varied

step 1: minimize Ω at fixed q

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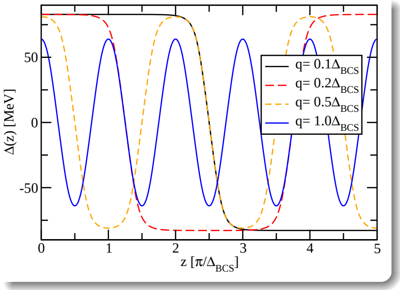
- example: $\delta\mu = 0.7\ \Delta_{BCS}$



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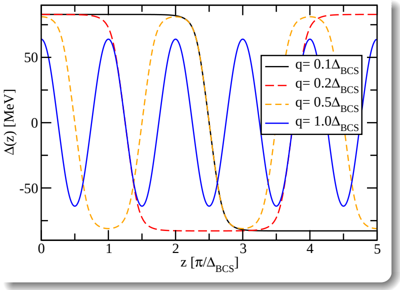

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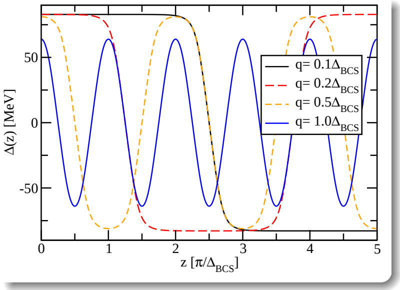

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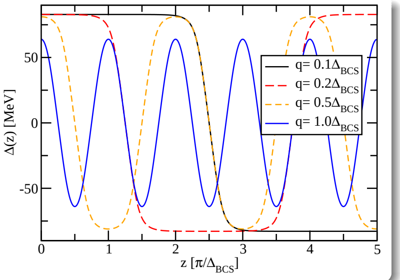

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- parametrization of the gap function:

$$\Delta(z) = A\,\mathrm{sn}\big(\kappa(z - z_0); \nu\big)$$

(works extremely well)

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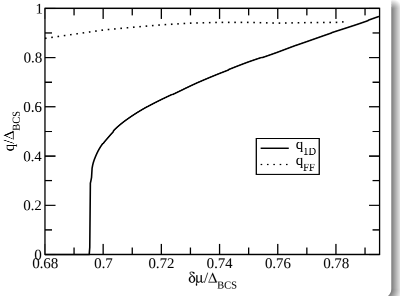

- inhom. - BCS: $q \to 0$ ➜ 2nd order (FF - BCS: 1st order)!

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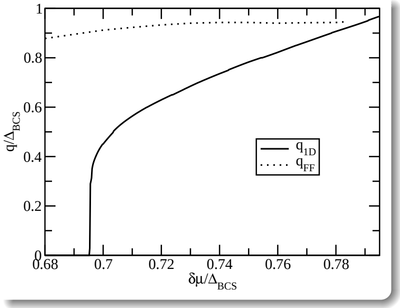
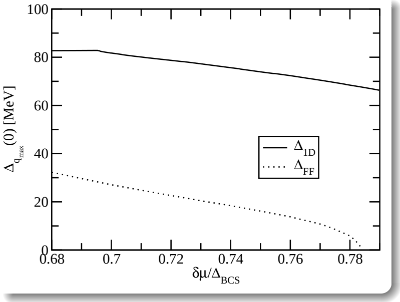


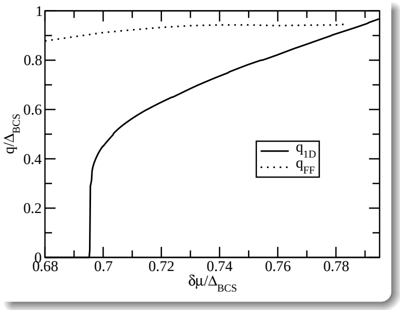
- amplitude:



- inhom. - BCS: $q \to 0$ ➜ 2nd order (FF - BCS: 1st order)!
- inhom. - normal: 1st order (FF - normal: $\Delta \to 0$ ➜ 2nd order)!

step 2: minimize Ω w.r.t. q

- preferred q:



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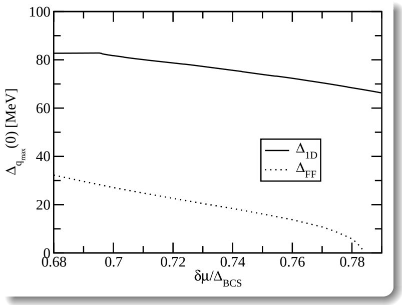

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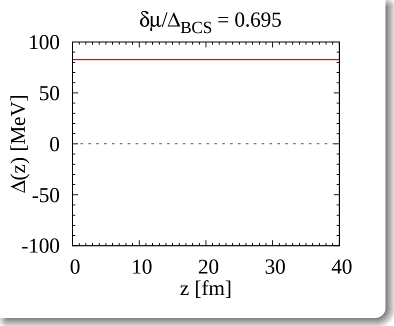
- inhom. - normal: 1st order (FF - normal: $\Delta \to 0$ ➜ 2nd order)!
- $\Delta_{inhom.} \gg \Delta_{FF}$

Gap functions

- putting everything together:



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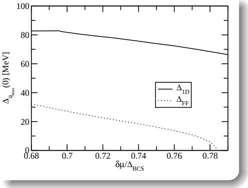

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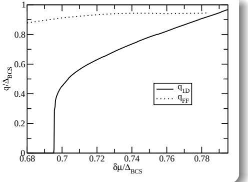

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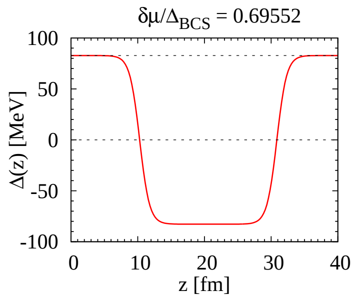

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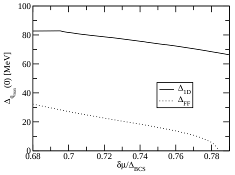

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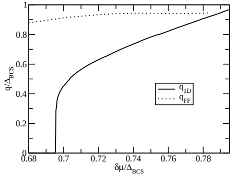

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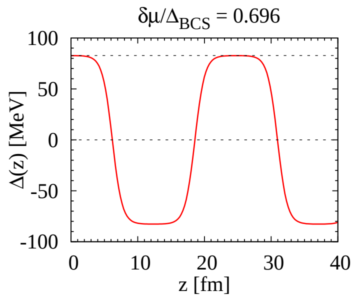

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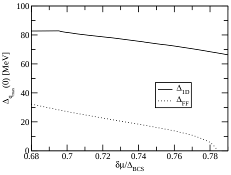

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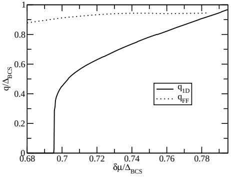

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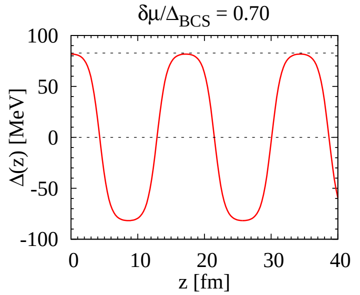

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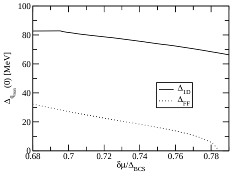

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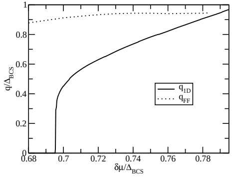

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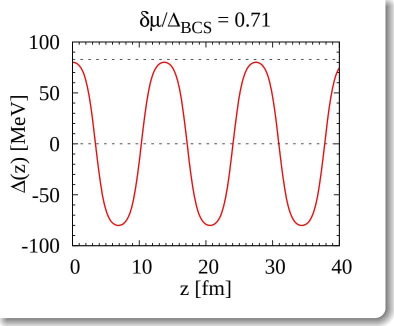

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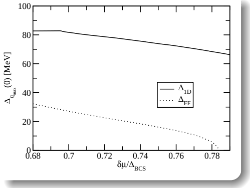

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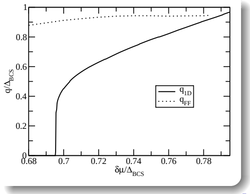

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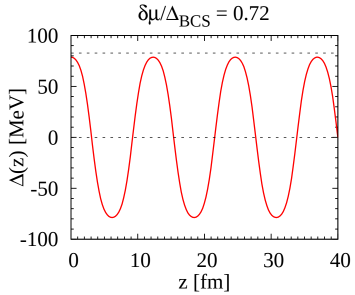

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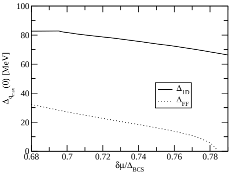

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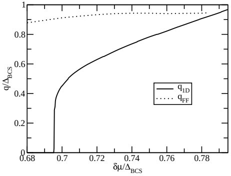

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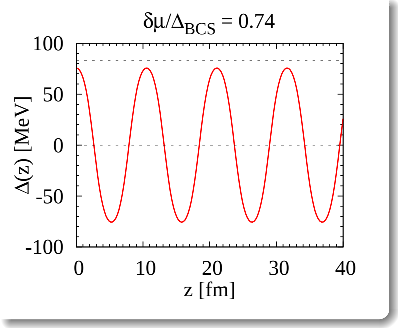

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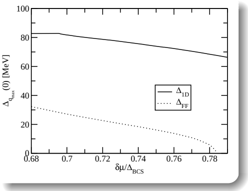

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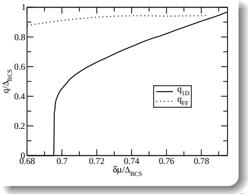

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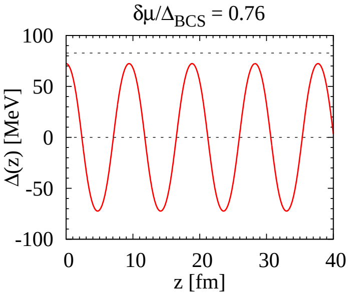

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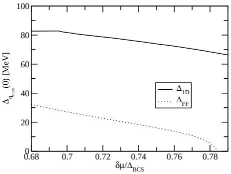

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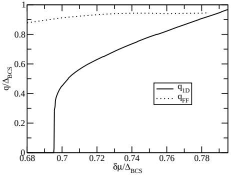

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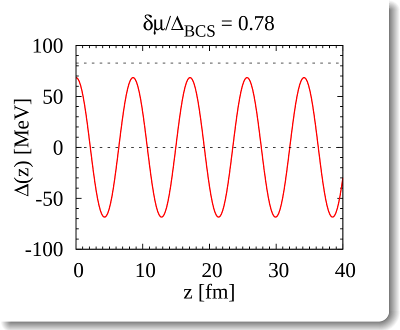

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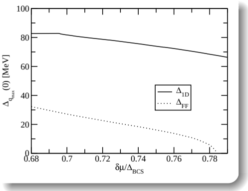

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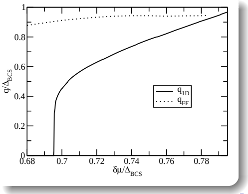

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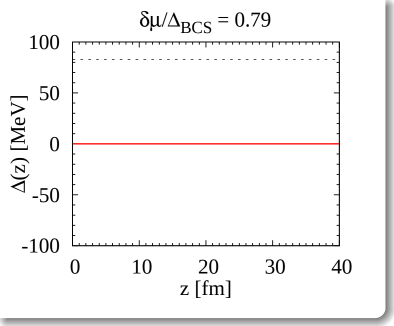

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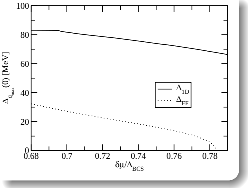

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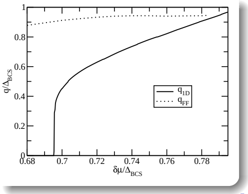
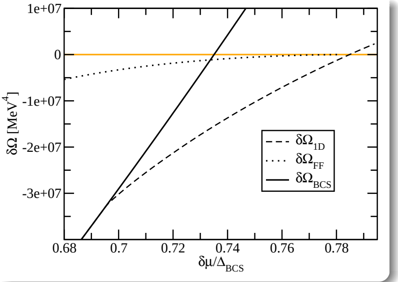


General one-dimensional solutions

- free-energy gain:



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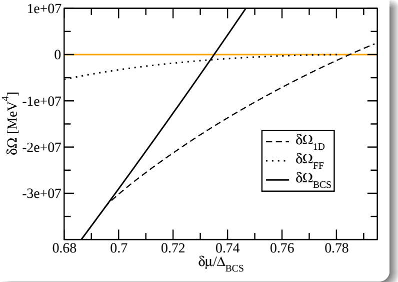

- LO window

  $\sim 2 \times$ FF window

Quasiparticle spectra

- anisotropic dispersion relations: $E_\lambda(\vec{k}) = E_\lambda(\vec{k}_\perp, k_z)$

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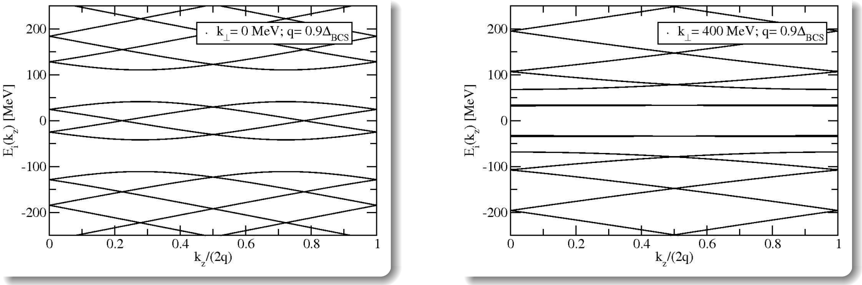
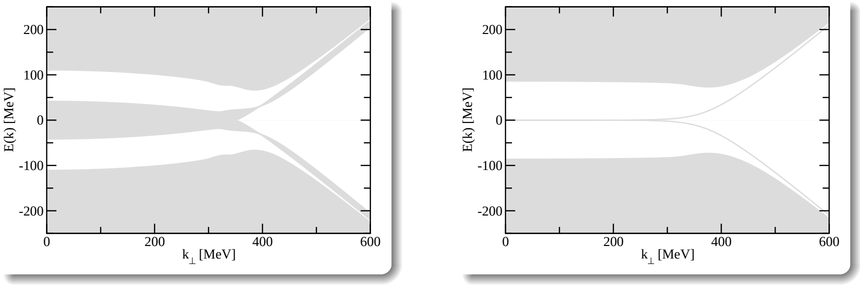
- anisotropic dispersion relations: $E_\lambda(\vec{k}) = E_\lambda(\vec{k}_\perp, k_z)$
- typical examples at fixed $k_\perp$:



Band structure

- superposition of the eigenvalue spectra of all k_z:

sinusoidal ($q = 0.9\ \Delta_{BCS}$) — soliton lattice ($q = 0.2\ \Delta_{BCS}$)



- “almost gapped” regions between low- and high-lying modes
- low-lying modes related to solitons: $q \to 0$ ➜ $E \to 0$

Dyson-Schwinger approach

- QCD Dyson-Schwinger equation for the quark propagator:

Nickel, Alkofer, Wambach, PRD (2006)

  - vertex function from DSE studies in vacuum
  - gluon propagator from DSE + particle-hole corrections (Debye screening and Landau damping)
  - include quark pairing

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- features:
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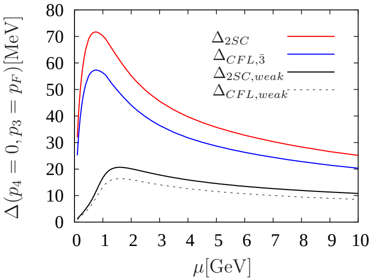
  - no free parameters
  - truncation
  - rather involved calculations . . .

DSE highlights: pairing gaps

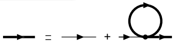


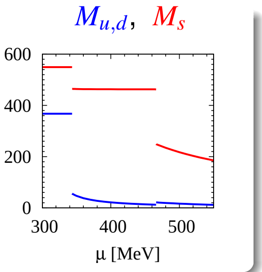
moderate densities:

3 times larger than extrapolated weak coupling results

Nickel, Alkofer, Wambach, PRD (2006)

DSE highlights: role of the strange quark mass

- NJL: M.B., Oertel, NPA (2002)
  - $M_s \sim G \langle \bar{s}s \rangle$ 
    - ➜ large in the 2SC phase
    - ➜ stabilizes 2SC



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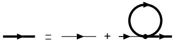
  - $M_s \sim G \langle \bar{s}s \rangle$ 
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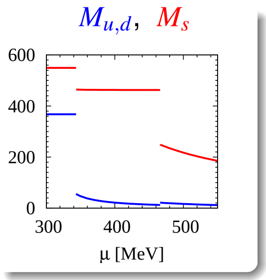

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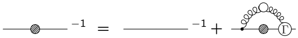
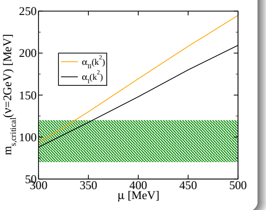


  - gluons screened by light quarks
    - ➜ M_s small
    - ➜ CFL favored much earlier



CFL + Goldstone phases

- CFL: chiral symmetry broken ➜ Goldstone bosons
  - "π", "K", "η" (by quantum numbers), but mainly diquarks
  - EFT prediction: very light, $m \sim \mathcal{O}(10\ \mathrm{MeV})$

    (Son & Stephanov, PRD 2000)

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- stress imposed by M_s ➜ K^0 condensation

  T. Schäfer, PRL (2000); Bedaque & Schäfer, NPA (2002).

  - heuristic argument: $p_F^s = \sqrt{\mu^2 - M_s^2} \simeq \mu - \frac{M_s^2}{2\mu}$

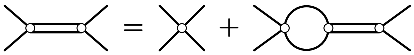
    ➜ effective strangeness chemical potential: $\mu_s \simeq \frac{M_s^2}{2\mu}$

    ➜ K^0 condensation if $\mu_s > m_{K^0}$

Goldstone bosons in the CFL phase

- explicit construction in NJL:

  Kleinhaus, M.B., Nickel, Oertel, PRD (2007)



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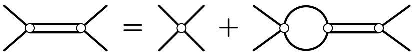
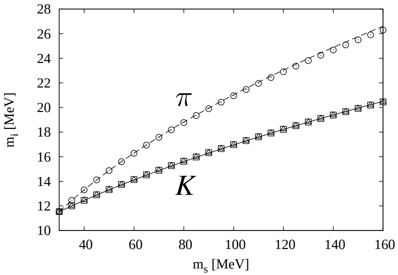


- meson masses



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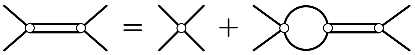

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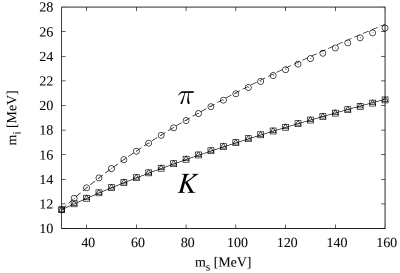
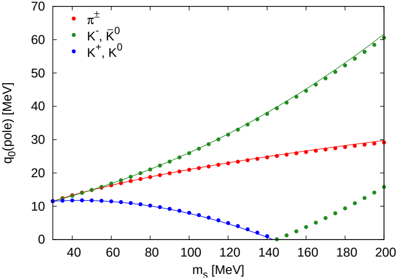


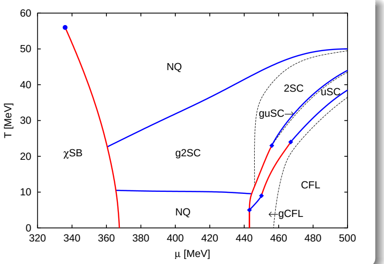
- meson poles $(m_i - \mu_i)$

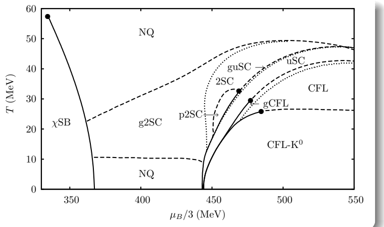


CFL+Goldstone phases

- phase diagram:
  include pseudoscalar diquark condensates
- result (for $H = 0.75G$):

H. Warringa, hep-ph/0606063





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