Parton Recombination, Elliptic Flow and Event-by-Event Observables

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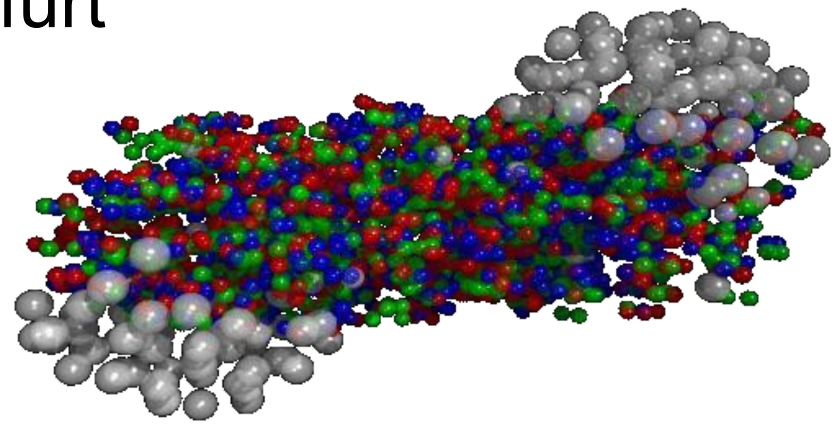


The Effect of Dynamical Parton Recombination on Event-by-Event Observables.
S. H., Stefan Scherer, Marcus Bleicher. e-Print: hep-ph/0702188

Thanks

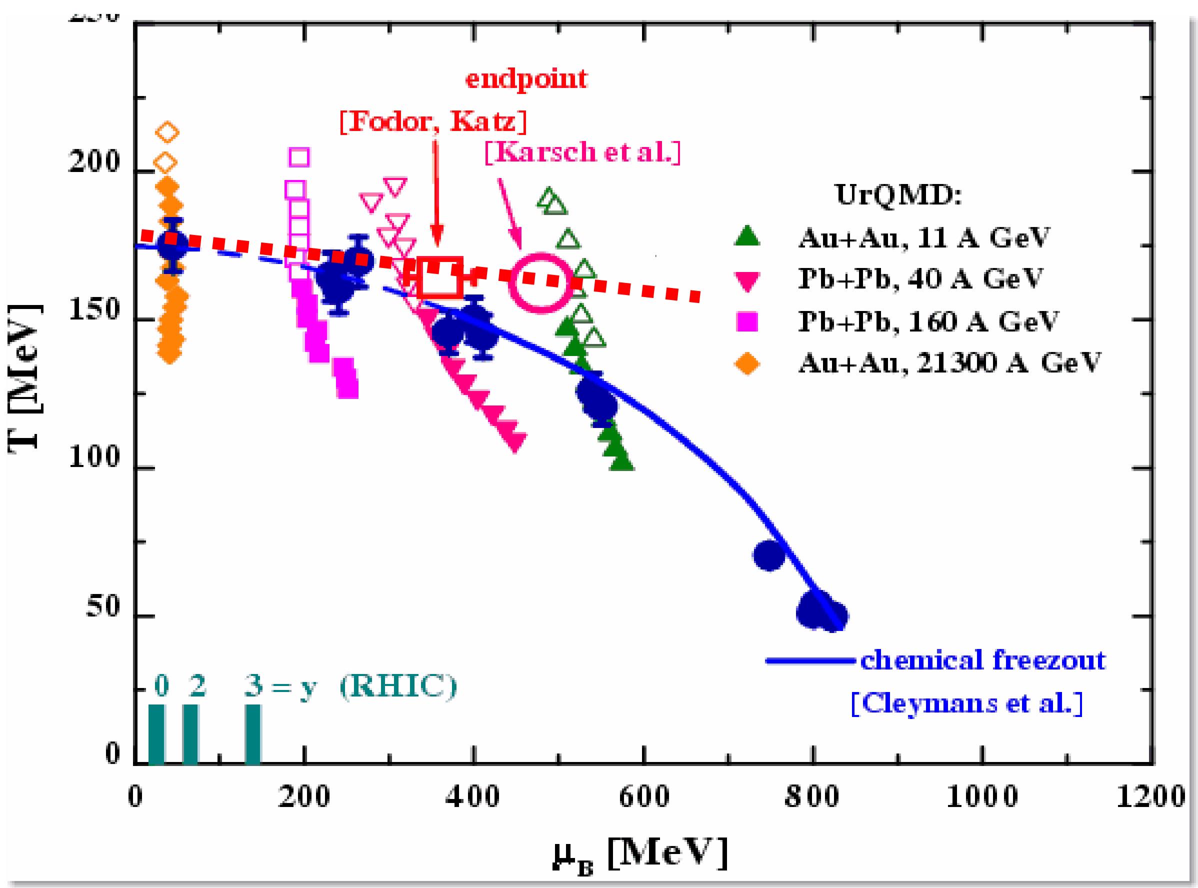
- Steffen Bass
- Rainer Fries
- Berndt Mueller
- Chiho Nonaka

for heavy use of their work in this presentation

Outline of the talk

- Introduction
- Elliptic Flow
- Static Parton Coalescence
- Scaling properties
- The Quark-Molecular Dynamics
- Charge fluctuations
- Charge transfer fluctuations
- Baryon-strangeness correlations
- Summary

Motivation



At **RHIC**:
look for signals of freely moving partons.
(D, C_{BS}, κ)

At **FAIR/SPS**:
look for the mixed phase and the onset of deconfinement
(ω, k/π, p/π)

E. Bratkovskaya, M.B. et al., PRC 2005

Elliptic flow

Ollitrault

Sorge

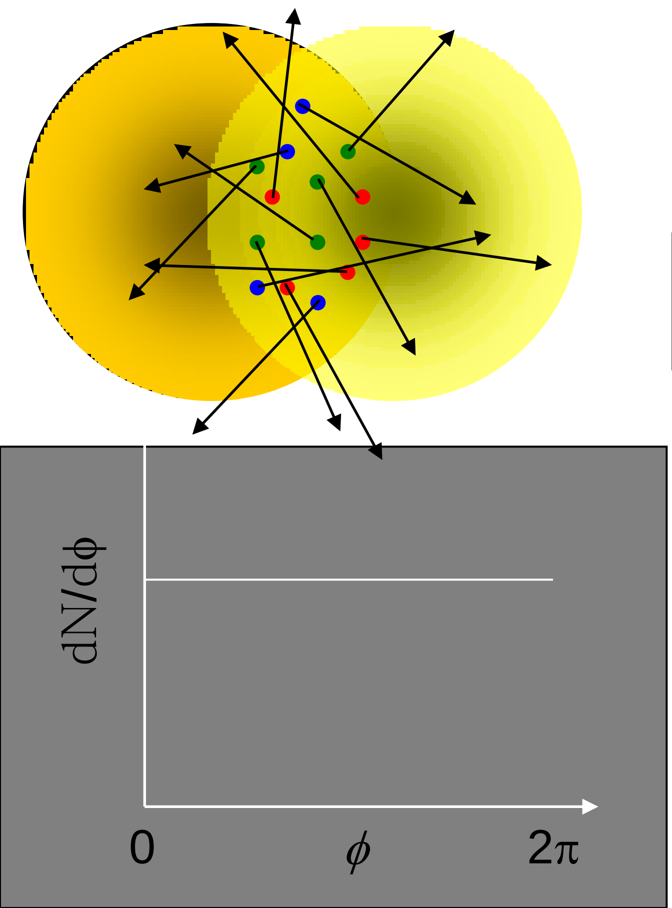
Elliptic Flow

Ollitrault ('92)

Response of the system to initial spatial anisotropy

No secondary interaction

$\lambda \to \infty$



Input

Spatial anisotropy ε

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Interaction among produced particles

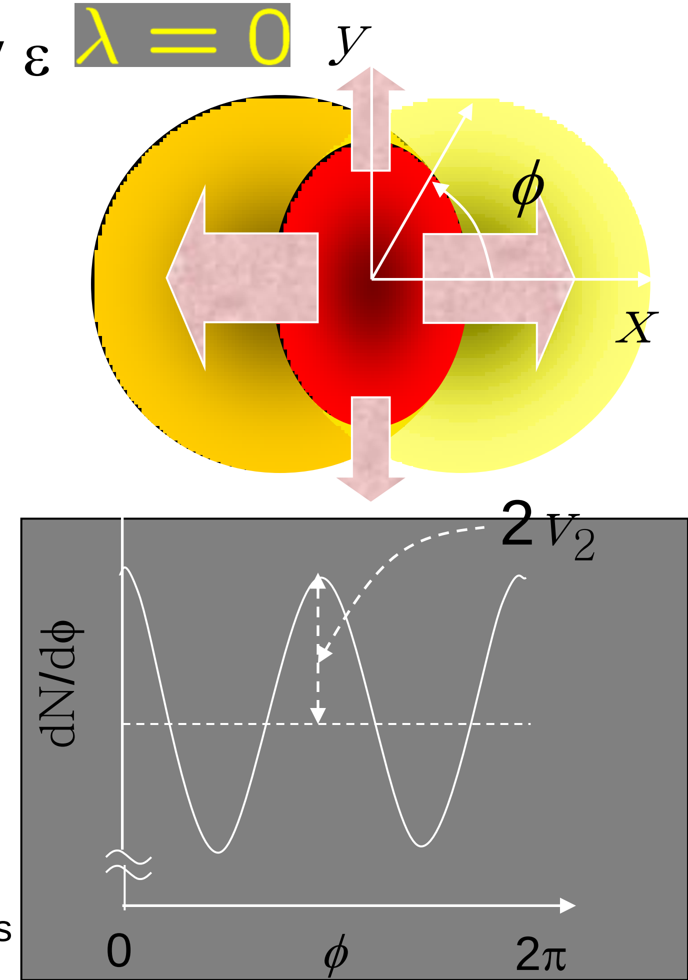
Output

Momentum anisotropy v_2

$$v_2 = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle$$

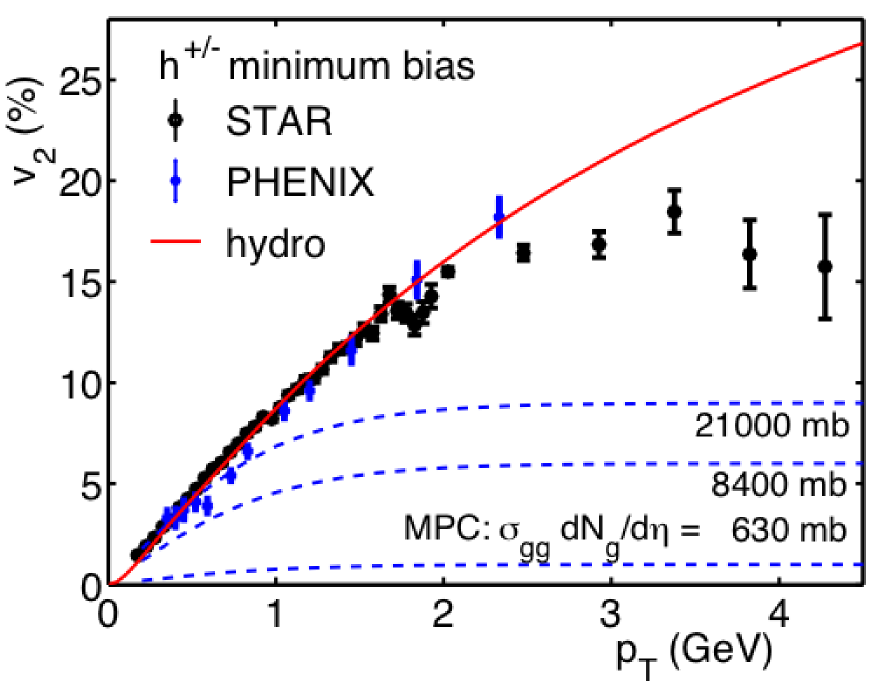
Hydrodynamic behavior

$\lambda = 0$

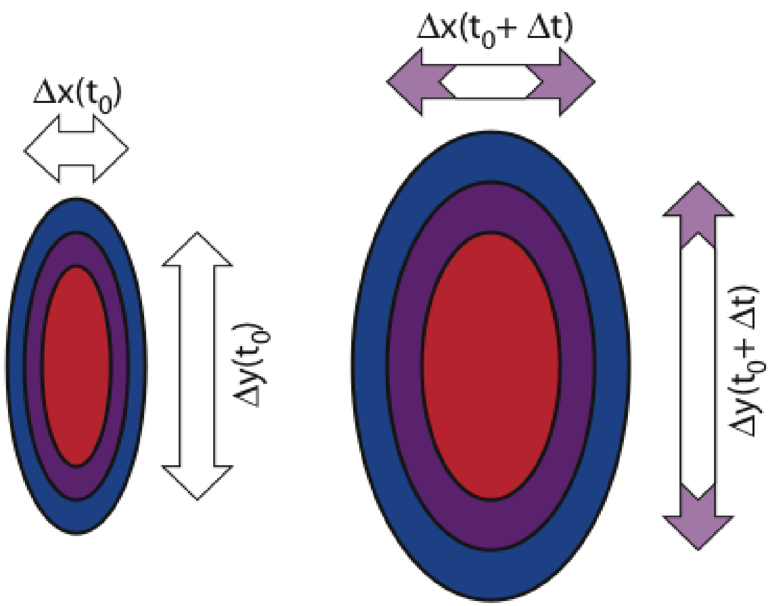


Momentum Anisotropy and Strong rescattering

$$v_2(p_T;b) = \langle\cos(2\varphi_p)\rangle = \frac{1}{\frac{dN}{dy p_T dp_T}} \int d\varphi_p \frac{dN}{dy p_T dp_T d\varphi_p} \cos 2\varphi_p$$



D. Molnar and M. Gyulassy, NPA 698 (2002) 379
PFK et al. PLB. 500 (2001) 232



PFK, J. Sollfrank and U.Heinz, PRC 62 (2000) 054909

* strong rescattering required

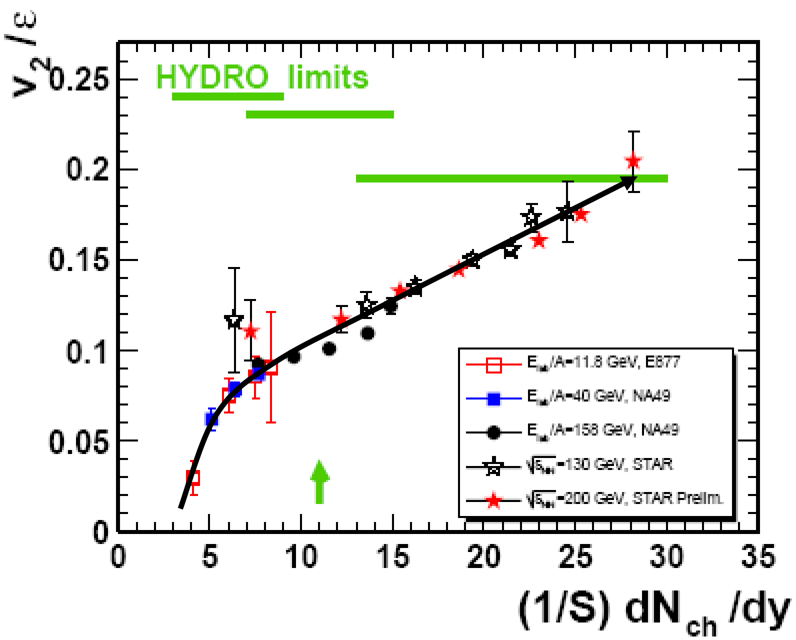
  the hydrodynamic limit appears to be exhausted

* transition of spatial to momentum anisotropy has to occur early

  can't allow a delay larger than 1 fm/c

Particle Density Dependence of Elliptic Flow

NA49('03)



(response)=(output)/(input)

Number density per unit transverse area

Kolb, Sollfrank, Heinz ('00)

- **<u>Dimension</u>**
  - 2D+boost inv.
- **<u>EoS</u>**
  - QGP + hadrons (chem. eq.)
- **<u>Decoupling</u>**
  - **Sudden freezeout**

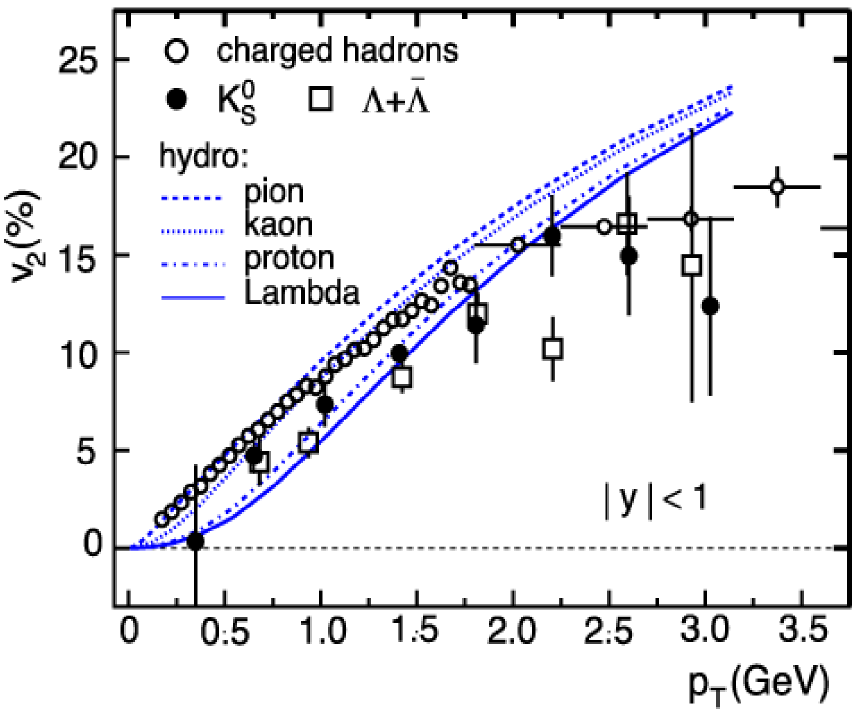
•Hydrodynamic response is const. $v_2/\varepsilon \sim 0.2$ @ RHIC
•Exp. data reach hydrodynamic limit at RHIC for the first time.

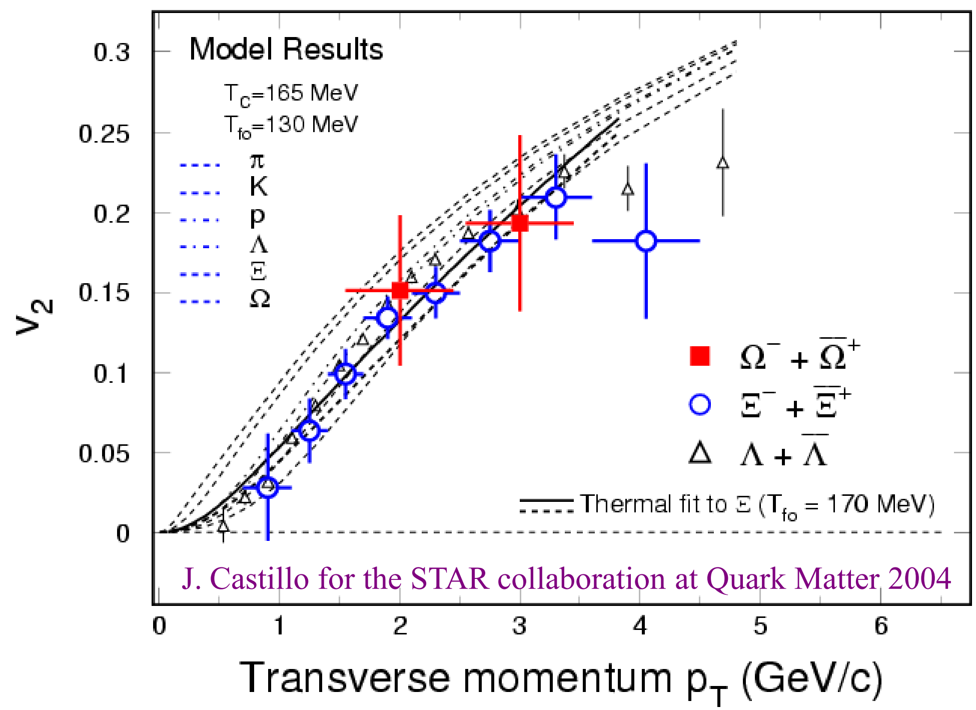
Dawn of the hydro age?

Systematic Mass Effects

Huovinen, PFK, Heinz, Ruuskanen, Voloshin, PLB 503 (2001) 58

$$v_2(p_T;b) = \langle\cos(2\varphi_p)\rangle = \frac{1}{\frac{dN}{dyp_T dp_T}} \int d\varphi_p \frac{dN}{dyp_T dp_T d\varphi_p} \cos 2\varphi_p$$





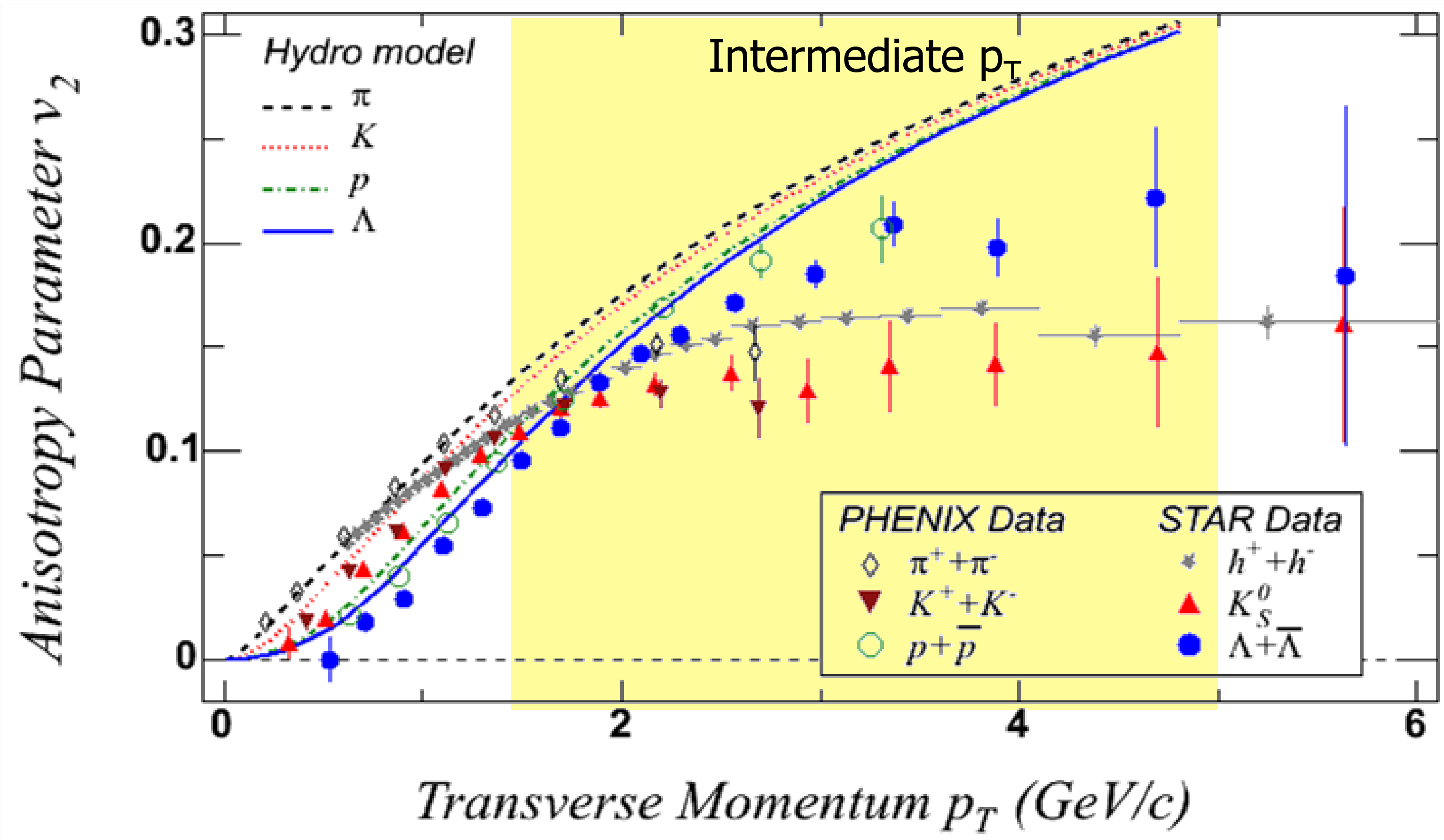
* elliptic flow shows a strong hydrodynamic mass effect

all quark flavors share a common flow field

v_2 vs. p_T

Large values indicate strong sensitivity to the system geometry for production at all measured p_T

v_2 at intermediate p_T is grouped by quark number

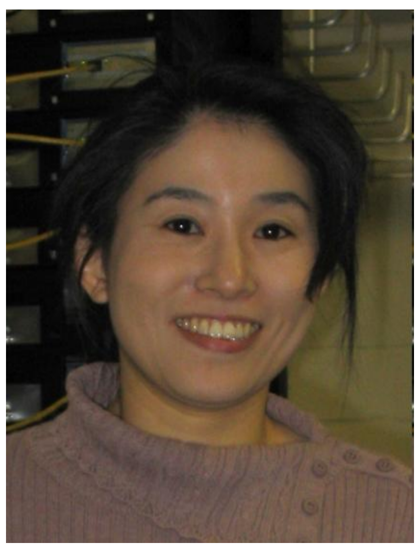


PRL **92** (2004) 052302; PRL **91** (2003) 182301

A possible solution to the puzzle:

- parton recombination

R.J. Fries, C. Nonaka, B. Mueller & S.A. Bass, PRL 90 202303 (2003)



Recombination+Fragmentation Model

basic assumptions:

- at low p_t, the quarks and antiquark spectrum is thermal and they recombine into hadrons locally "at an instant":

$$q\bar{q} \rightarrow M \qquad\qquad qqq \rightarrow B$$

  - ➢ features of the parton spectrum are shifted to higher p_t in the hadron spectrum

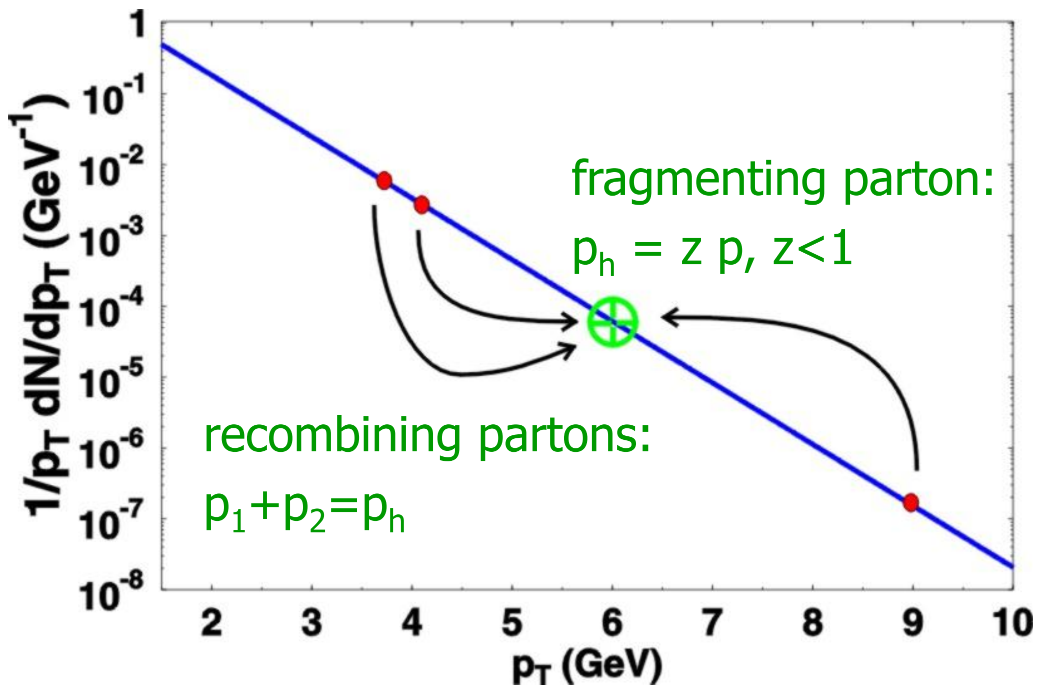
- at high p_t, the parton spectrum is given by a pQCD power law, partons suffer jet energy loss and hadrons are formed via fragmentation of quarks and gluons

Recombination: Pro’s & Con’s

Pro’s:

- for exponential parton spectrum, recombination is more effective than fragmentation
- baryons are shifted to higher p_t than mesons, for same quark distribution

➢ understand behavior of protons!



Con’s:

- recombination violates entropy conservation
- gluons at hadronization need to be converted

Recombination: new life for an old idea

Heavy-Ion Phenomenology:

- T. S. Biro, P. Levai & J. Zimanyi, Phys. Lett. B347, 6 (1995)

  ALCOR: a dynamical model for hadronization

➢ yields and ratios via counting of constituent quarks

- R.C. Hwa & C.B. Yang, Phys. Rev. C66, 025205 (2002)
- R. Fries, B. Mueller, C. Nonaka & S.A. Bass, Phys. Rev. Lett. 90
- V. Greco, C.M. Ko and P. Levai, Phys. Rev. Lett. 90
- R. Rapp & E.V. Shuryak, Phys. Rev. D67, 074036 (2003)

Anisotropic flow:

- S. Voloshin, QM2002, nucl-ex/020014
- Z.W. Lin & C.M. Ko, Phys. Rev. Lett 89, 202302 (2002)
- D. Molnar & S. Voloshin, nucl-th/0302014

Recombination: nonrelativistic formalism

- use thermal quark spectrum given by: w(p) = exp(-p/T)

- for a Gaussian meson wave function with momentum width Λ_M, the meson spectrum is obtained as:

$$\frac{dN_M}{d^3P} = C_M \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} w\left(\tfrac{1}{2}P - q\right) w\left(\tfrac{1}{2}P + q\right) \left|\hat{\varphi}_M(q)\right|^2$$

$$= C_M \frac{V}{(2\pi)^3} \left[w\left(\tfrac{1}{2}P\right)\right]^2 \left(1 - \frac{2\Lambda_M^2}{TP} + \cdots\right)$$

- similarly for baryons:

$$\frac{dN_B}{d^3P} = C_B \frac{V}{(2\pi)^3} \left[w\left(\tfrac{1}{3}P\right)\right]^3 \left(1 - O(\Lambda_B^2 / TP)\right)$$

Recombination: relativistic formalism

- choose a hypersurface Σ for hadronization
- use local light cone coordinates (hadron defining the + axis)
- $w_a(r,p)$: single particle Wigner function for quarks at hadronization
- Φ_M & Φ_B: light-cone wave-functions for the meson & baryon respectively
- x, x′ & (1-x): momentum fractions carried by the quarks
- integrating out transverse degrees of freedom yields:

$$E\frac{dN_M}{d^3P} = \int_\Sigma d\sigma \frac{P\cdot u}{(2\pi)^3} \sum_{\alpha,\beta} \int dx\, w_\alpha(R, xP^+)\, \bar{w}_\beta(R, (1-x)P^+) \left|\bar{\phi}_M(x)\right|^2$$

$$E\frac{dN_B}{d^3p} = \int_\Sigma d\sigma \frac{P\cdot u}{(2\pi)^3} \sum_{\alpha,\beta,\gamma} \int dx\, dx'\, w_\alpha(R, xP^+)\, w_\beta(R, x'P^+)\, w_\gamma(R, (1-x-x')P^+) \left|\bar{\phi}_B(x, x')\right|^2$$

Elliptic Flow: partons at low p_t

- azimuthal anisotropy of parton spectra is determined by elliptic flow:

$$\frac{d^2N}{p_t\, dp_t\, d\phi_p} = \frac{1}{2\pi}\left[\frac{dN}{p_t dp_t}\right]\left(1+2v_2\cos\left(2\phi_p\right)\right)$$ (Φ_p: azimuthal angle in p-space)

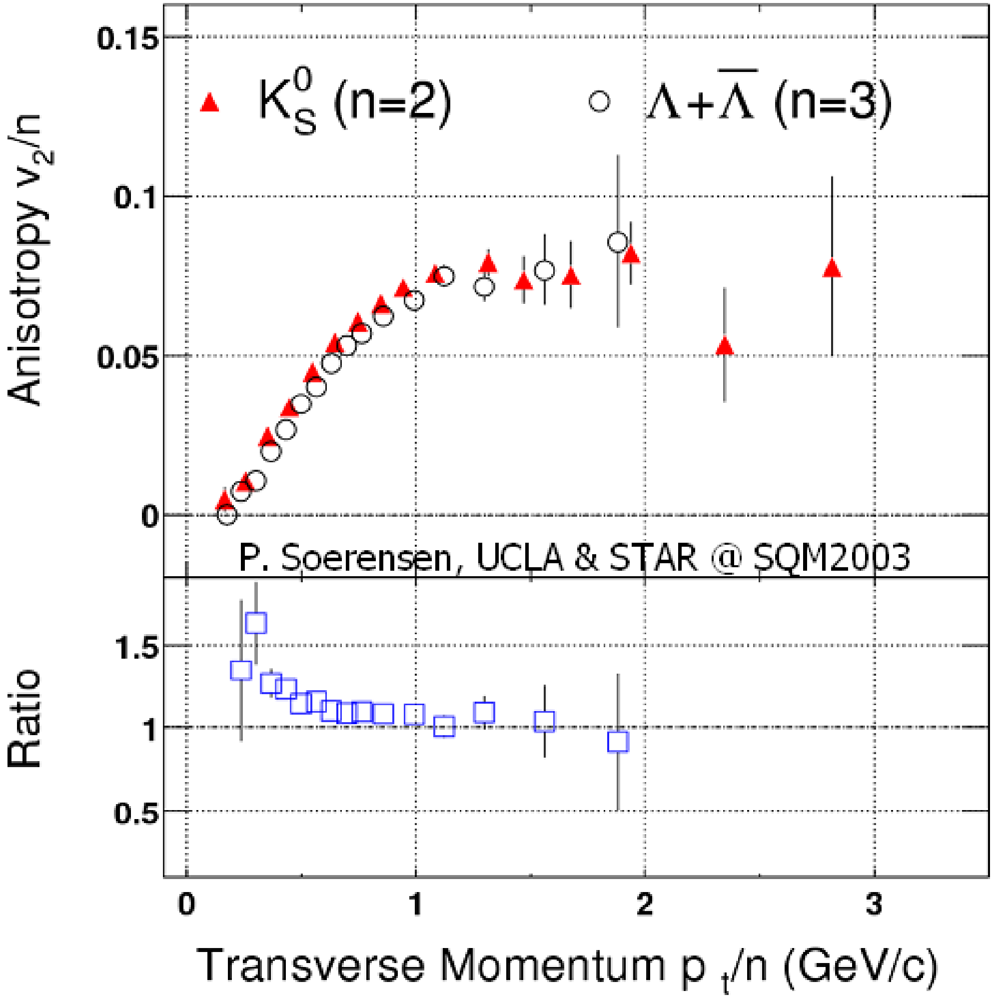
- with Blastwave parametrization for parton spectra:

$$v_2\left(p_t\right)=\left\langle\cos\left(2\phi_p\right)\right\rangle=\frac{\int_0^{2\pi} d\phi_s \cos\left(2\phi_s\right) I_2\left(\frac{p_t \sinh\left(\rho\left(\phi_s\right)\right)}{T}\right) K_1\left(\frac{m_t \cosh\left(\rho\left(\phi_s\right)\right)}{T}\right)}{\int_0^{2\pi} d\phi_s\, I_0\left(\frac{p_t \sinh\left(\rho\left(\phi_s\right)\right)}{T}\right) K_1\left(\frac{m_t \cosh\left(\rho\left(\phi_s\right)\right)}{T}\right)}$$

- azimuthal anisotropy is parameterized in coordinate space and is damped as a function of p_t:

$$\rho\left(\phi_s\right)=\frac{1}{2}\ln\left(\frac{1+\beta_t}{1-\beta_t}\right)\left(1+\alpha_p\left(p_t\right)\cos\left(2\phi_s\right)\right) \quad \text{and} \quad \alpha_p\left(p_t\right)=-\alpha_0\frac{1}{1+\left(p_t/p_0\right)^2}$$

Parton Number Scaling of v_2



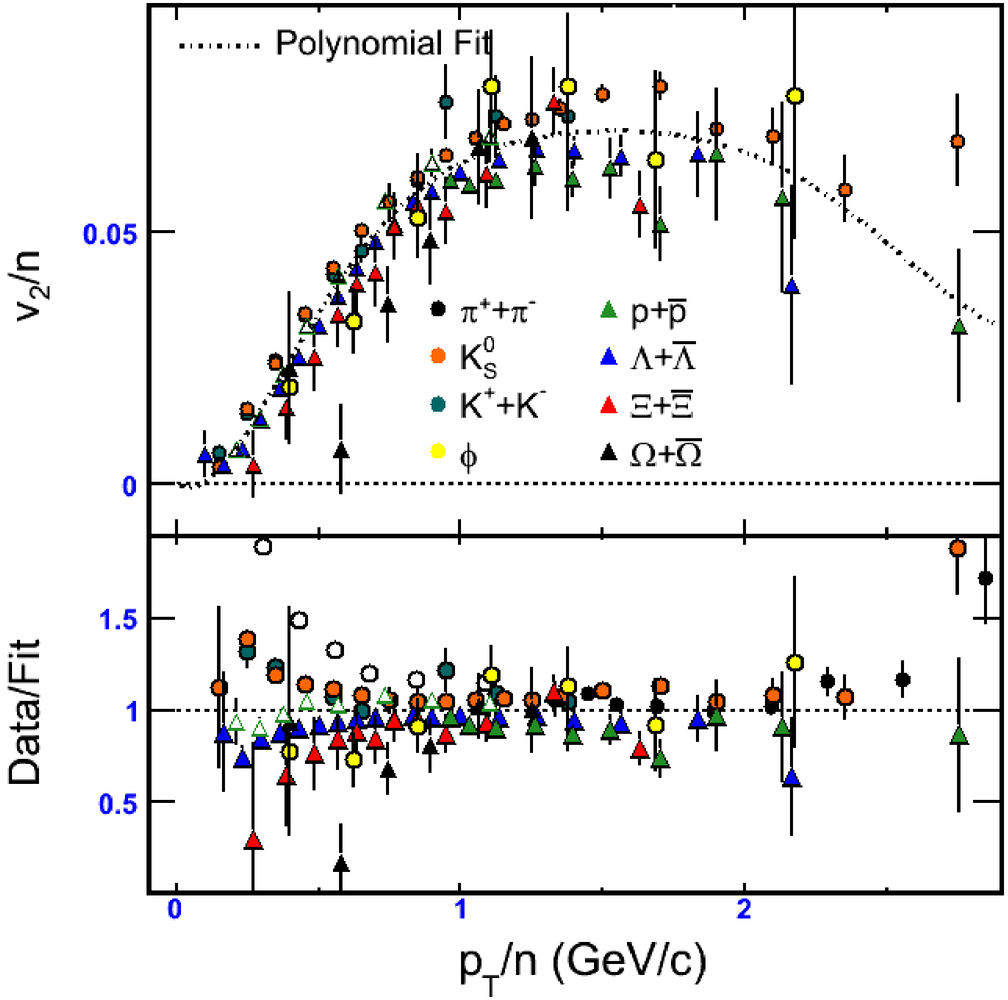
- in leading order of v_2, recombination predicts:

$$v_2^M(p_t) = 2v_2^p\left(\frac{p_t}{2}\right)$$

$$v_2^B(p_t) = 3v_2^p\left(\frac{p_t}{3}\right)$$

- smoking gun for recombination
- measurement of partonic v_2 !

Do multi-strange hadrons flow?



P. Sorensen. SOM 2006

- Data indicates approximate 3:2 scaling with constituent quarks
- Baryons are generally below mesons
- Decrease of v_2 at high p_T

Is this rough scaling a signal for recombination?

The tool: qMD

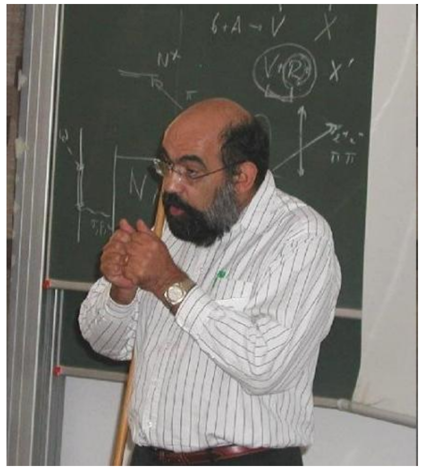
qMD : Quark Molecular Dynamics
(a toy model for hadronization)

- out-of-equilibrium transport model, (Vlasov equation)
- provides a hadronization prescription
- essentially realizes a dynamical quark recombination approach

Hofmann, Bleicher, Scherer, Neise, Stoecker, Greiner. Phys.Lett.B478:161-171,2000.

Parton/Hadron Transport

Stoecker



Aichelin

Ko

Cassing

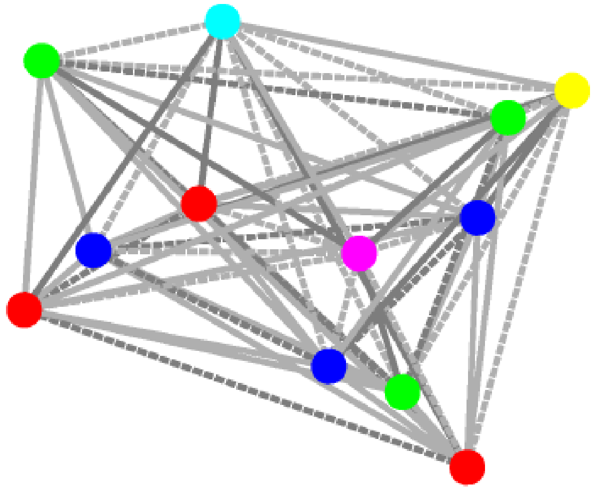
Fluctuations are THE tool!?

- Fluctuations might provide information on
  - deconfinement/confinement
  - correlation length
  - thermalization
  - nature of the QGP
  - critical point
- Is it that easy?
  - finite time and volume
  - non-equilibrium
  - hadronization

Quark Molecular Dynamics

Hamiltonian of the model :

$$H = \sum_{i=1}^{N} \sqrt{\mathbf{p}_i^2 + m_i^2} + \frac{1}{2} \sum_{i \neq j} C_{ij} V(|\mathbf{r}_i - \mathbf{r}_j|)$$



- Potential :

  linear potential $V(r) = \kappa r$
- Color factor C_{ij} :

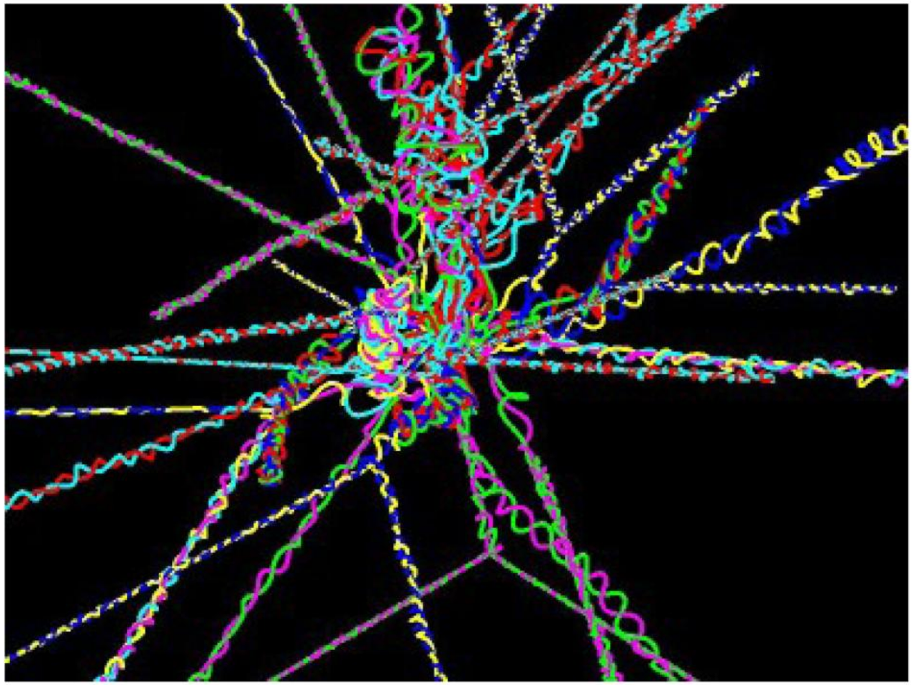
  can be attractive or repulsive depending on the color of the quarks
- Quarks :

  classical point-particles with light masses $m_{u,d} = 5$ MeV, $m_s = 150$ MeV

Trajectories

qMD features :

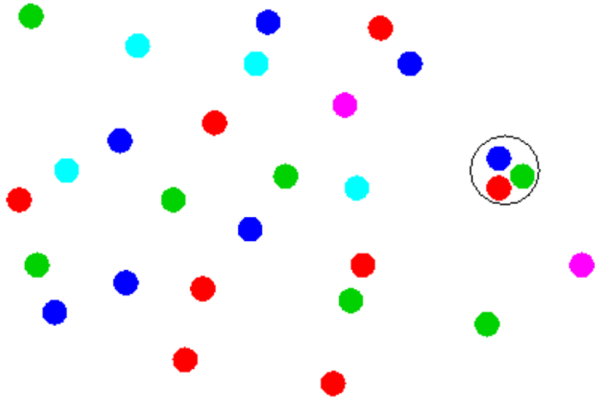
- mesons
- baryons
- confinement
- recombination
- out-of-equilibrium



M. Hofmann Ph.D. thesis

Hadronization procedure

- color neutral clusters
- separation in space
- small remaining interaction



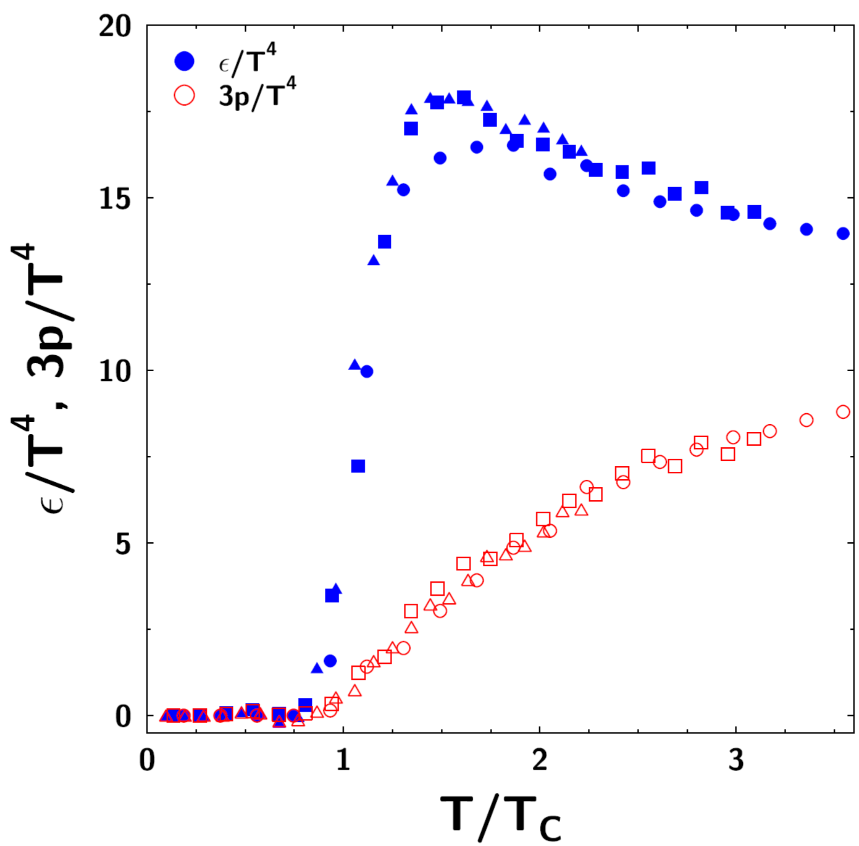
- force on quark i

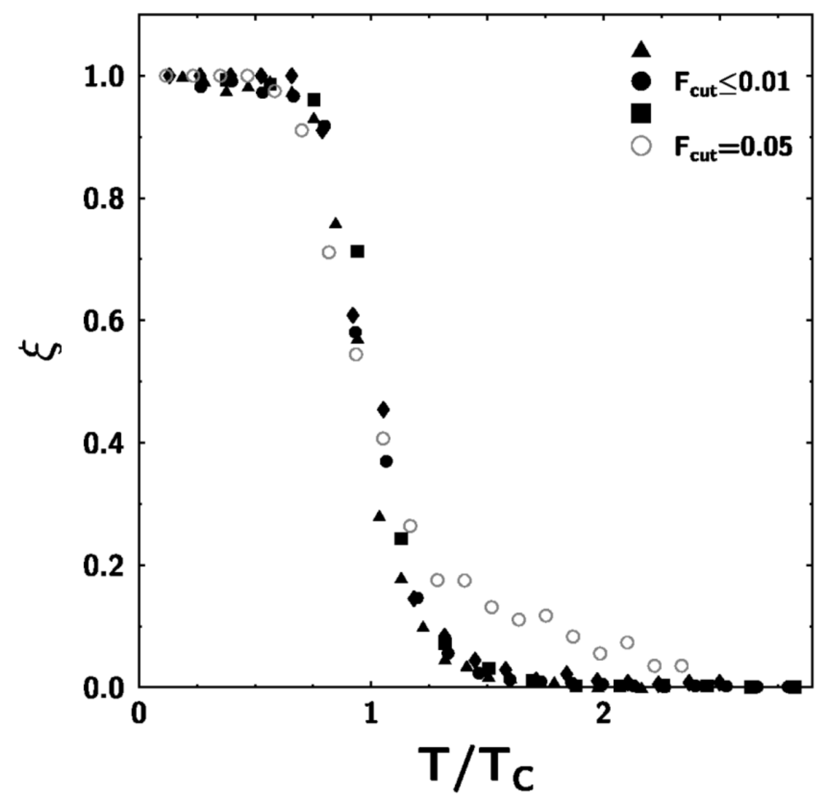
$$\vec{F}_i = \sum_j \vec{F}_{ij} = \sum_j C_{ij} \nabla_j V(|\vec{r}_i - \vec{r}_j|)$$

- Remaining interaction on a cluster

$$|\vec{F}_{cluster}| = |\frac{1}{N_{cluster}} \sum_{i \in cluster} \vec{F}_i| < \kappa_{min} = F_{cut}\ \kappa$$

Some properties: equilibrium

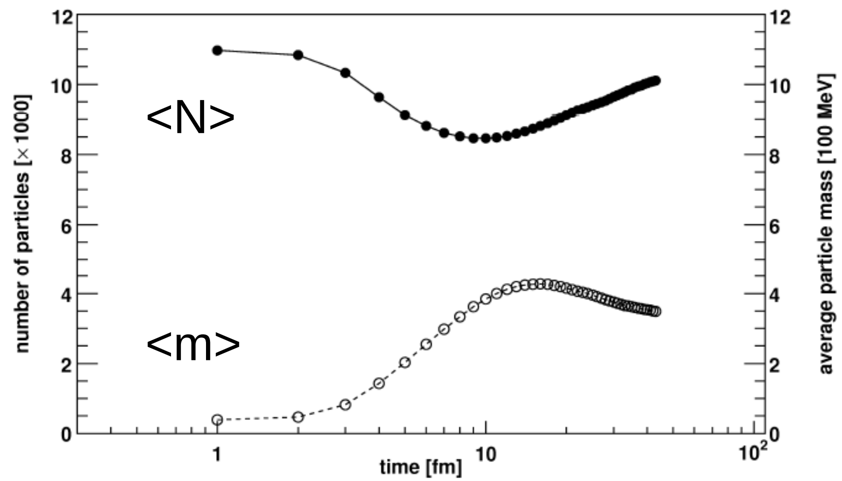




$$\xi = N_{hadrons} / N_{all\ \ particles}$$

- For ‘real’ physics use UrQMD initial state
- dissolve strings into ‘free’ quarks
- evolve system with qMD

Entropy consideration

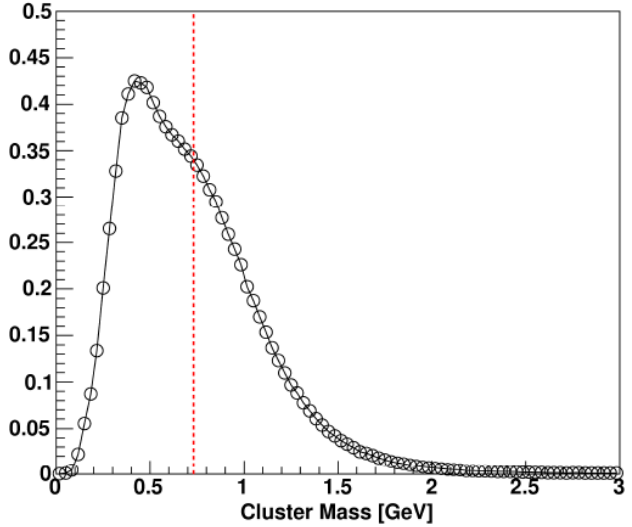


Do we violate the 2^{nd} law of thermodynamics ?

- Entropy can be estimated by measuring the number of particles.
- As many particles at the begining and at the end of the calculation
- Heavy clusters will decay into numerous particles

The decay of resonance increases entropy

Entropy and recombination



Do we violate the 2^{nd} law of thermodynamics ?

- Entropy can be estimated by measuring the number of particles
- Without decay, the number of particles decreases at hadronization
- Entropy depends also on the mass of the particle
- for $m/T > 3$:

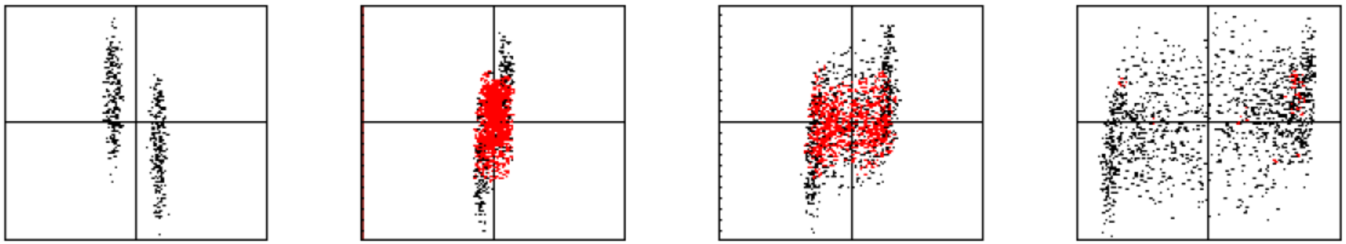
$$S/N = 3.5 + m/T$$

At the transition, $S_{QGP} < S_{HG}$

$$\begin{aligned} S_{QGP} &= 2\ N_{hadrons}\ 3.5 &= 7\ N_{hadrons} \\ S_{HG} &= N_{hadrons}(3.5 + 750/150) &= 8.5\ N_{hadrons} \end{aligned}$$

Recombination can be compatible with entropy conservation

The idea behind conserved charge fluctuations



Electric charge for example ($Q = Q_+ - Q_-$) :

Hadronic degrees of freedom :

$$\begin{aligned} i &= (\pi^+, \pi^-) \\ Q_i &= \pm 1 \end{aligned}$$

Partonic degrees of freedom :

$$\begin{aligned} i &= (u, \bar{u}, d, \bar{d}) \\ Q_i &= \pm(\tfrac{1}{3}, \tfrac{2}{3}) \end{aligned}$$

$$\langle \delta Q^2 \rangle = \langle (\sum_i Q_i \delta N_i)^2 \rangle$$

Build quantities sensitive to the fractional charges of the partons

Fluctuations and suceptibilities

$$Z = \sum_i \exp[-\beta(E_i - \mu_Q Q_i - \mu_B B_i - \mu_S S_i)]$$

$$(X, Y) = (Q, B, S)$$

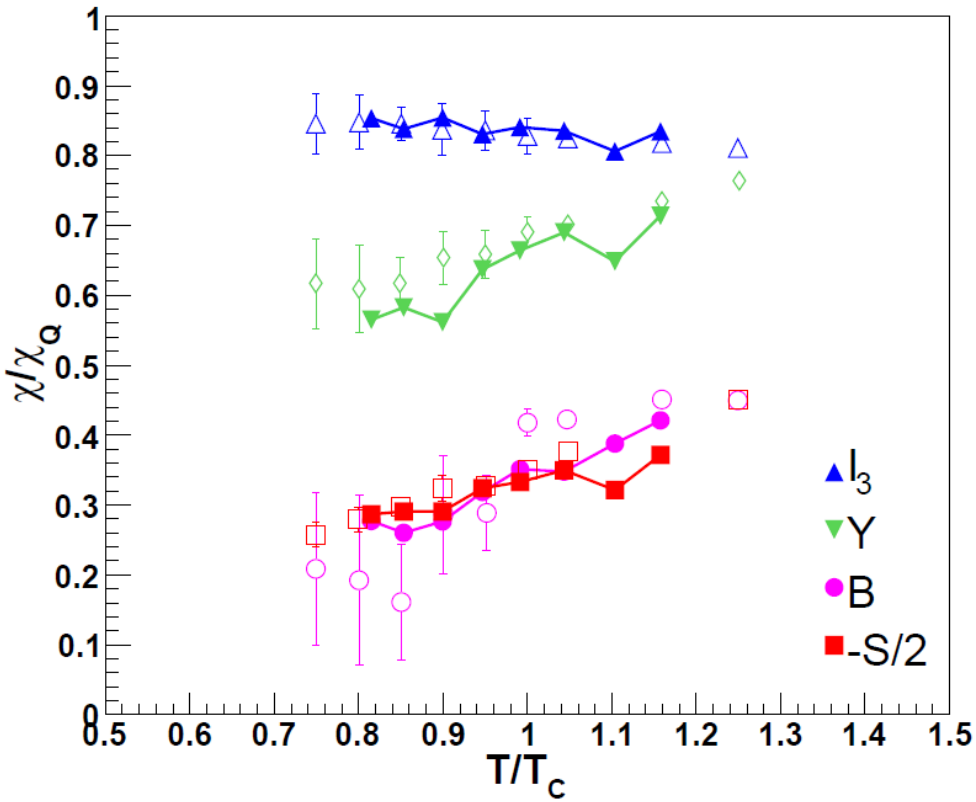
variances and correlations

$$\begin{aligned} \langle(\delta X)^2\rangle &= T^2 \frac{\partial^2}{\partial \mu_X^2} \log(Z) &= -T \frac{\partial^2}{\partial \mu_X^2} F \\ \langle(\delta X)(\delta Y)\rangle &= T^2 \frac{\partial^2}{\partial \mu_X \mu_Y} \log(Z) &= -T \frac{\partial^2}{\partial \mu_X^2 \mu_Y^2} F \end{aligned}$$

susceptibilities

$$\begin{aligned} \langle \delta X^2 \rangle &= -\frac{1}{V} \frac{\partial^2}{\partial \mu_X^2} F &= V\ T\ \chi_X \\ \langle \delta X \delta Y \rangle &= -\frac{1}{V} \frac{\partial^2}{\partial \mu_X \partial \mu_Y} F &= V\ T\ \chi_{XY} \end{aligned}$$

Comparison to lQCD (I)

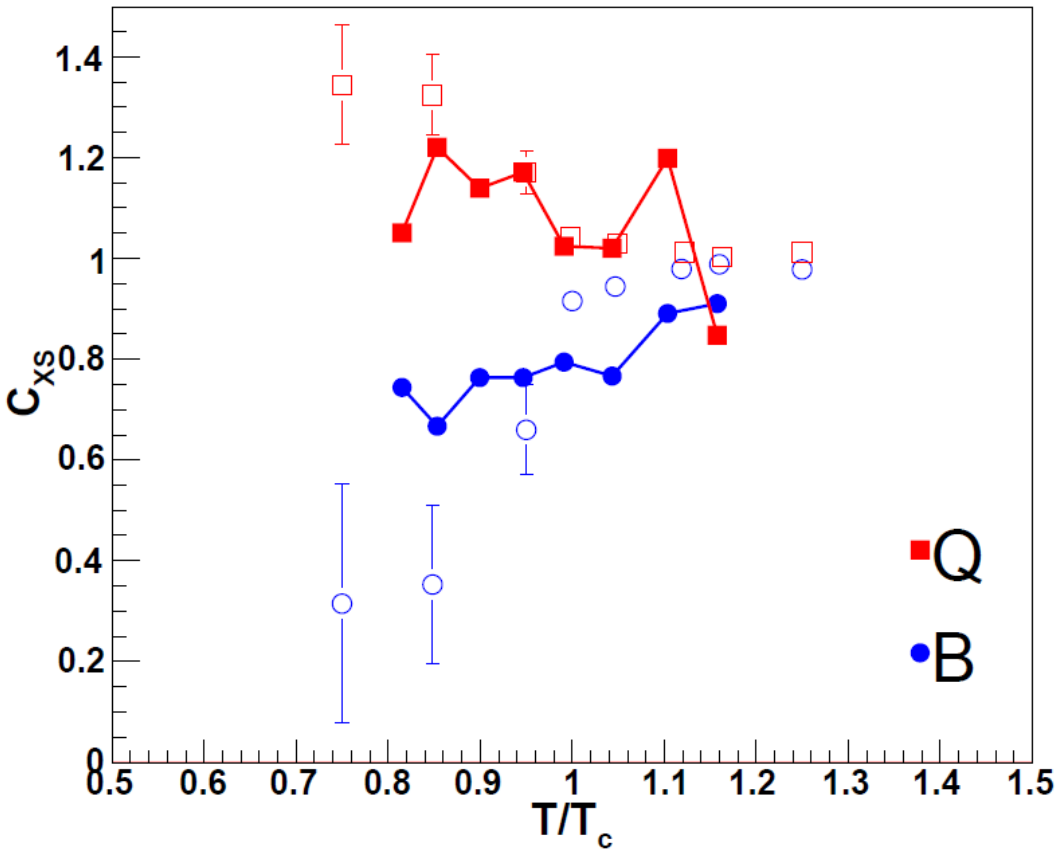


$$\frac{\chi_{XQ}}{\chi_Q} = \frac{\langle XQ\rangle - \langle X\rangle\langle Q\rangle}{\langle Q^2\rangle - \langle Q\rangle^2}$$

Open symbols : lattice data from Gavai, Gupta. Phys.Rev.D73:014004,2006

Full symbols with lines are the result of qMD calculations

Comparison to lQCD (II)



$$C_{BS} = -3\frac{\chi_{BS}}{\chi_S}$$

$$C_{QS} = 3\frac{\chi_{QS}}{\chi_S}$$

Open symbols : lattice data from Gavai, Gupta. Phys.Rev.D73:014004,2006

Full symbols with lines are the result of qMD calculations

- Hadron gas seems to be pretty similar to QGP…

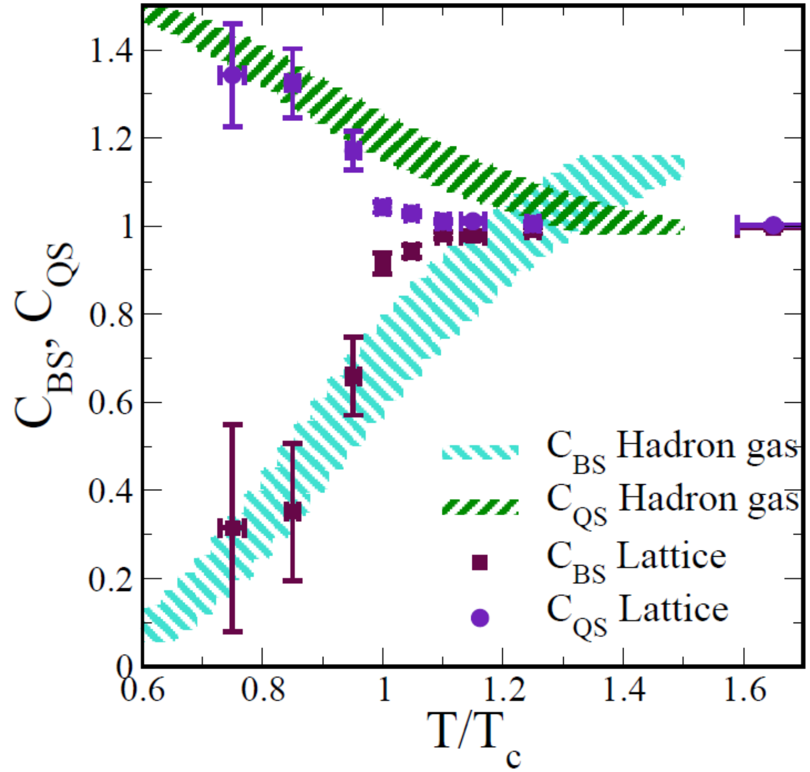


FIG. 3: (Color online) A comparison of the C_{BS} and C_{QS} calculated in a truncated hadron resonance gas at $\mu_B = \mu_S = \mu_Q = 0$MeV compared to lattice calculations at $\mu = 0$ from Ref. [22]. The two hazed bands for C_{BS} and C_{QS} for the hadron gas plots reflect the uncertainty in the actual value of the phase transition temperature T_c., which is assumed to lie in the range $T_c = 170 \pm 10$MeV.

(A. Majumder et al, Phys. Rev.C 74 (2006) 054901)

Charge ratio fluctuations

Jeon, Koch. Phys.Rev.Lett.85:2076-2079,2000.

Bleicher, Jeon, Koch. Phys.Rev.C62:061902,2000.

The Measure :

$$D = \langle N_{ch} \rangle \langle \delta R^2 \rangle$$

$$\begin{aligned} Q &= N_+ - N_- \\ N_{ch} &= N_+ + N_- \\ R &= N_+ / N_- \end{aligned}$$

$$D \approx 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{ch} \rangle}$$

Corrections

$$\tilde{D}(\Delta y) = D(\Delta y) / (C_\mu \; C_y)$$

$$\begin{aligned} C_\mu &= \left(\frac{\langle N_+ \rangle_{\Delta y}}{\langle N_- \rangle_{\Delta y}} \right)^2 \\ C_y &= 1 - \frac{\langle N_{ch} \rangle_{\Delta y}}{\langle N_{ch} \rangle_{total}} \end{aligned}$$

Expectation values :

- $\tilde{D} = 1$ in a QGP
- $\tilde{D} = 4$ in an uncorrelated Pion Gas
- $\tilde{D} = 2.8$ in a Resonance Gas

$$D \approx 4\frac{\langle \delta Q^2\rangle}{\langle N_{ch}\rangle}$$

$$\begin{aligned}
\delta Q &= \delta N_{\pi+} - \delta N_{\pi-} \\
(\delta Q)^2 &= \delta N^2_{\pi+} + \delta N^2_{\pi-} + correlations \\
\langle(\delta Q)^2\rangle &= \langle \delta N^2_{\pi+}\rangle + \langle \delta N^2_{\pi-}\rangle \\
\langle(\delta Q^2)\rangle &= N_{\pi^+} + N_{\pi^-} = N_{ch}
\end{aligned}$$

Pion gas, D ~ 4

Assumptions :

$$\begin{aligned}
correlations &= 0 \\
\langle \delta N^2_{\pi+}\rangle &= \langle N_{\pi+}\rangle \\
\langle \delta N^2_{\pi-}\rangle &= \langle N_{\pi-}\rangle
\end{aligned}$$

$$\begin{aligned}
\delta Q &= Q_u(\delta N_u - \delta N_{\bar{u}}) + Q_d(\delta N_d - \delta N_{\bar{d}}) \\
\delta Q^2 &= Q_u^2(\delta N_u^2 + \delta N_{\bar{u}}^2) + Q_d^2(\delta N_d^2 + \delta N_{\bar{d}}^2) \\
&\quad + correlations \\
\langle \delta Q^2\rangle &= Q_u^2\langle N_{u+\bar{u}}\rangle + Q_d^2\langle N_{d+\bar{d}}\rangle
\end{aligned}$$

$$D = 4\frac{Q_u^2\frac{\langle N_{ch}\rangle}{2} + Q_d^2\frac{\langle N_{ch}\rangle}{2}}{\langle N_{ch}\rangle}.$$

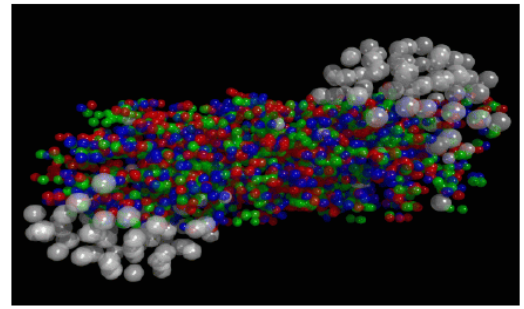
$$D = 4\frac{(1/3)^2\frac{1}{2} + (2/3)^2\frac{1}{2}}{1}.$$

Quark gas, D ~ 1

Assumptions :

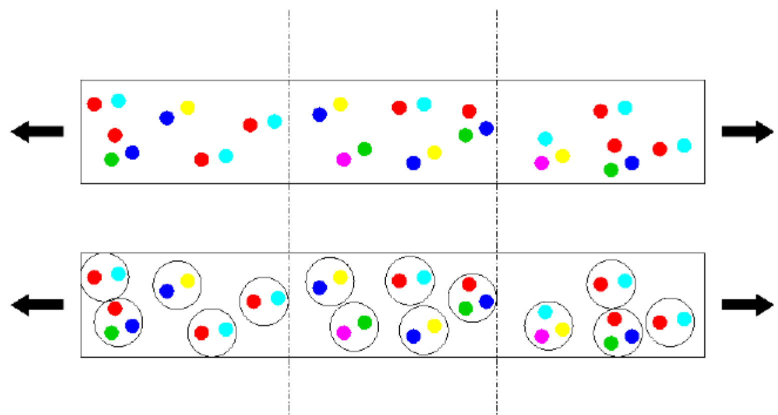
$$\begin{aligned}
correlations &= 0 \\
\langle \delta N_u^2\rangle &= \langle N_u\rangle \\
\langle \delta N_d^2\rangle &= \langle N_d\rangle \\
\langle N_{u+\bar{u}}\rangle &= \langle N_{ch}\rangle/2 \\
\langle N_{d+\bar{d}}\rangle &= \langle N_{ch}\rangle/2 \\
\langle N_{ch}\rangle &= \langle N_{q+\bar{q}}\rangle
\end{aligned}$$

Can one observe the fluctuations in the initial state?



Longitudinal flow

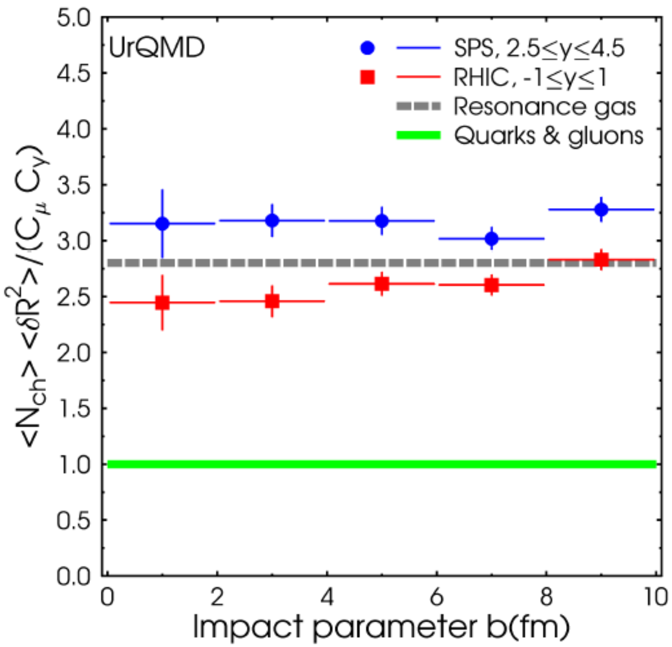
- Initial fluctuations should be frozen in a given rapidity window
- Different studies show that rescattering is not sufficient to dampen the signal
- $\Delta y_{kick} \ll \Delta y_{accept} \ll \Delta y_{total}$
- A key point is the influence of hadronization

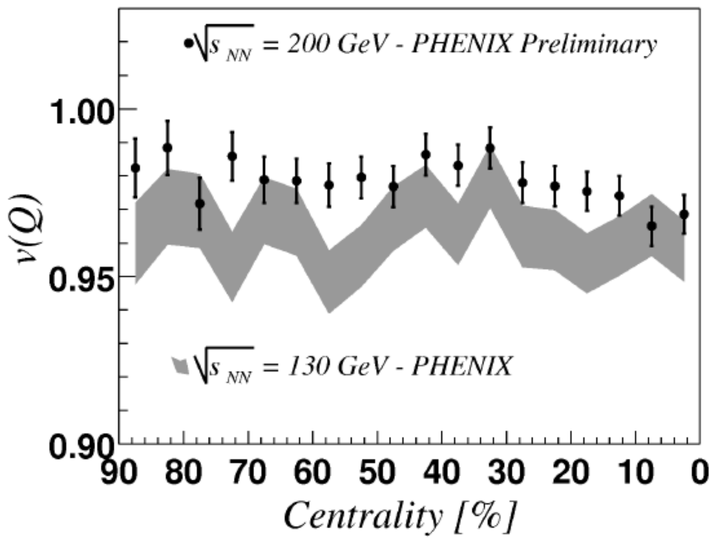


See e.g. Shuryak et al, Phys.Rev.C63:064903,2001

Experimental results

Bleicher, Jeon, Koch. Phys.Rev.C62:061902,2000

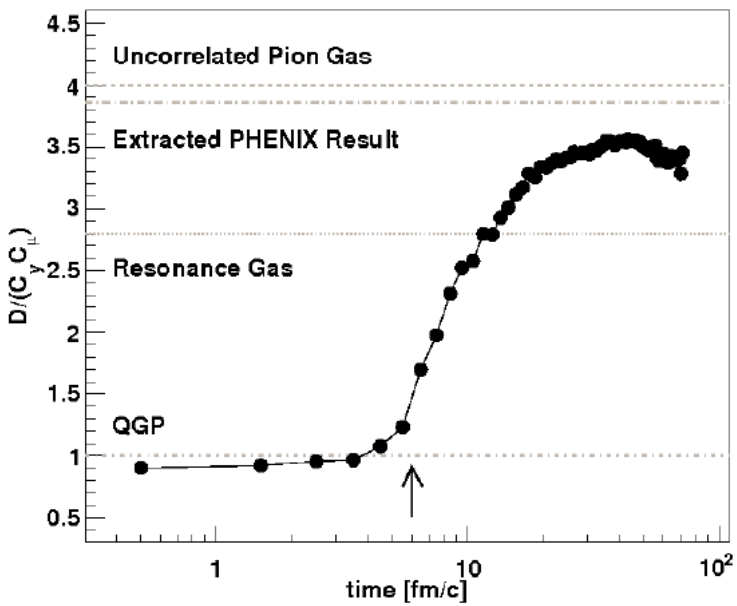


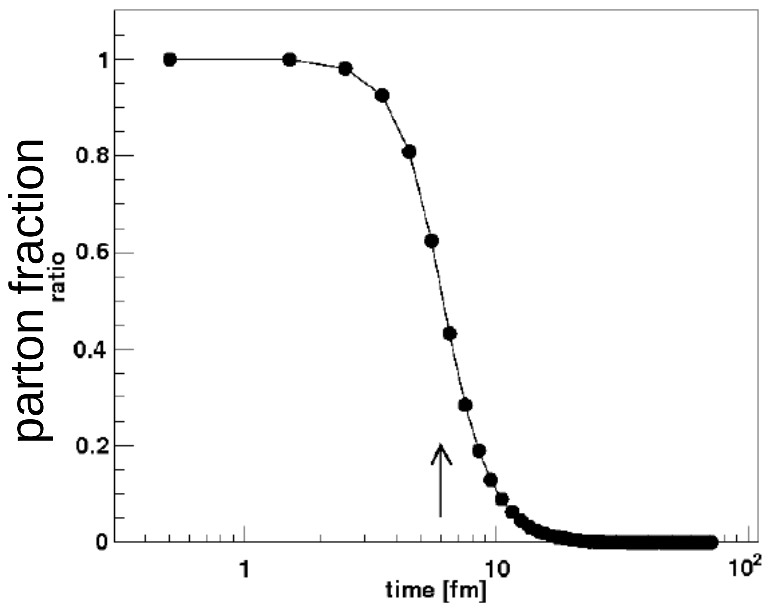


$$\tilde{D} = \frac{\langle N_{ch} \rangle \langle \delta R^2 \rangle}{C_\mu \;\, C_y} \approx 4 \;\nu(Q)$$

Compatible with the hadronic expectation

Recombination and fluctuation





Recombination kills the fluctuations

- $\tilde{D} = 1$ in the quark matter phase
- $\tilde{D}$ is compatible with the experiment result in the late stage
- Hadronization and the increase of $\tilde{D}$ occur at the same time

See also…

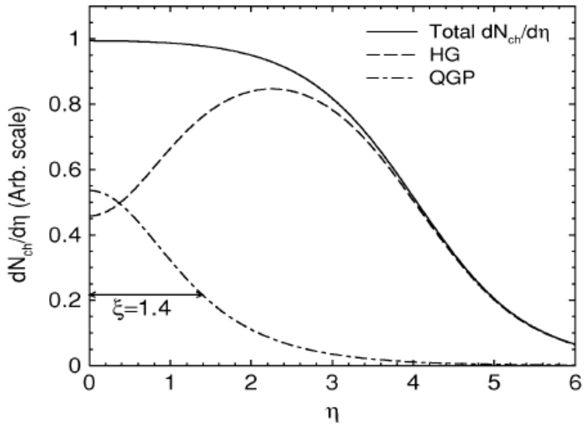
- Bialas: Recombination blur ratio fluctuations (Phys.Lett.B532:249-251,2002)
- Nonaka: Recombination blurs ratio fluctuations (Phys.Rev.C71:051901,2005)
- Ma: Hadronization blurs ratio fluctuations (SQM 2007)
- Present work:

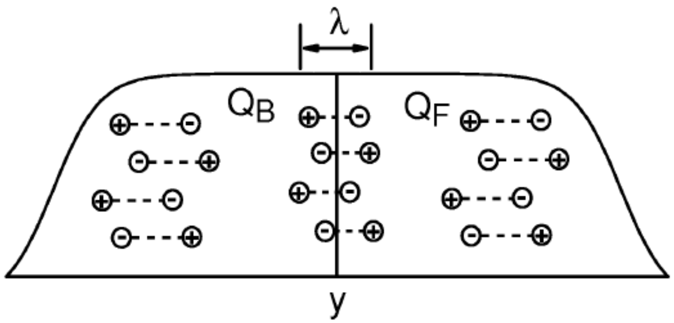
The Effect of Dynamical Parton Recombination on Event-by-Event Observables.
S. H., Stefan Scherer, Marcus Bleicher. e-Print: hep-ph/0702188

Charge transfer fluctuations

Shi, Jeon. Phys.Rev.C72:034904,2005

Jeon, Shi, Bleicher. Phys.Rev.C73:014905,2006



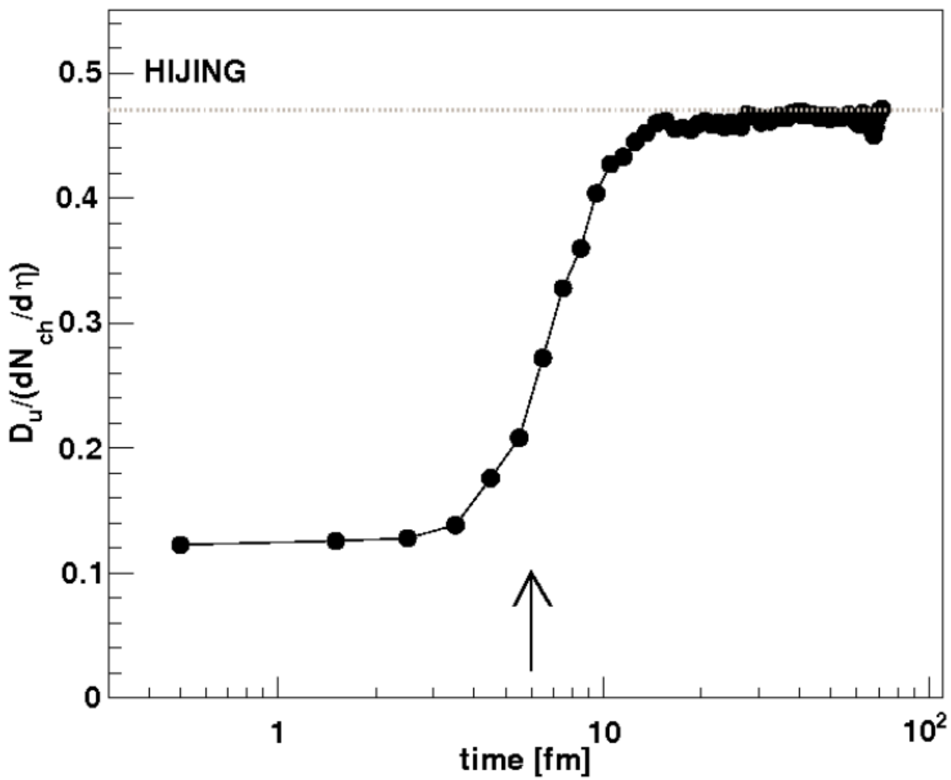


Idea

- $D_u(\eta) = \langle u(\eta)^2 \rangle - \langle u(\eta) \rangle^2$
- $u(\eta) = [Q_F(\eta) - Q_B(\eta)]/2$
- $\kappa(y) = \frac{D_u(y)}{dN_{ch}/dy}$
- κ is proportional to the charge correlation length

qMD results on kappa

calculate $\frac{D_u(y)}{dN_{ch}/dy}$ at midrapidity where the signal should be the strongest



Baryon-Strangeness Correlations

Koch, Majumder, Randrup. Phys.Rev.Lett.95:182301,2005.
S. H., Stoecker, Bleicher. Phys.Rev.C73:021901,2006.

In a QGP, strangeness is always carried together with baryon number
In a Hadron Gas, Strangeness can be carried without baryon number

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} \approx -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

expectation values :

- $C_{BS} = 1$ in a QGP
- $C_{BS} = 0.66$ in a HG ($T = 170$ MeV, $\mu = 0$)

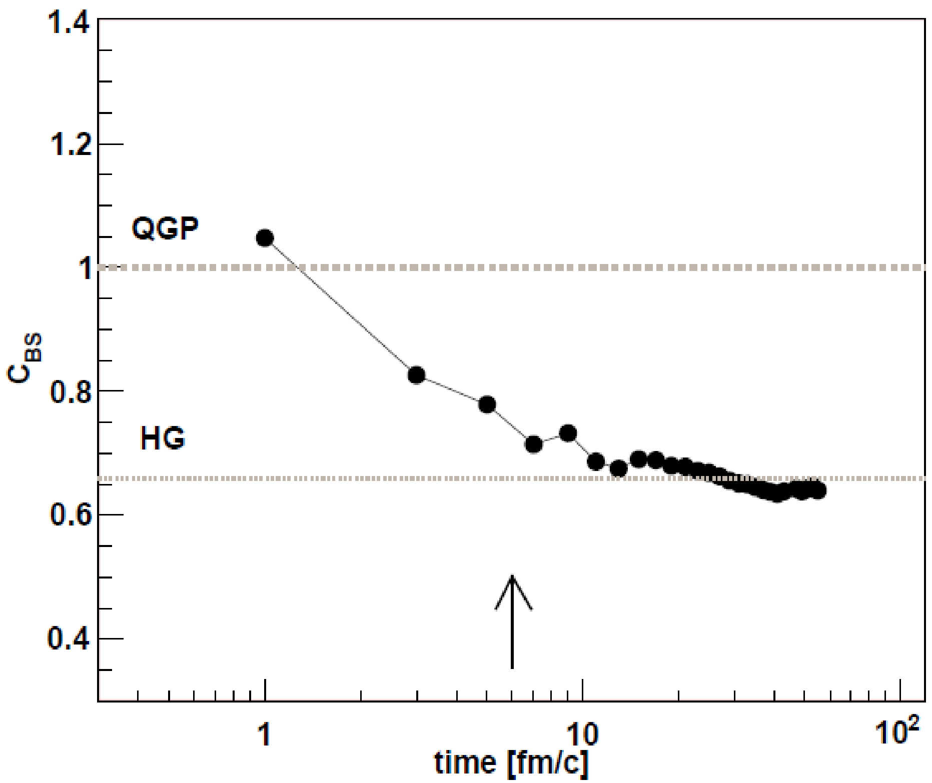
related quantities :

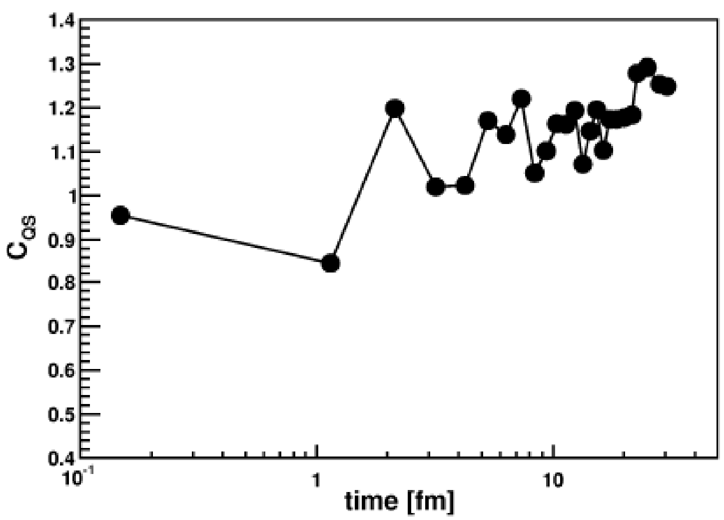
some particles are difficult to measure

- $C_{QS} = \frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} \approx \frac{3 - C_{BS}}{2}$
- $C_{MS} \approx C_{BS}$ with $M = B + 2I_3$
- take into account only strange charged particles

Time evolution

The signal vanishes with hadronization for all these quantities





Conclusions

- qMD performs dynamical recombination to describe the hadronization of quarks into hadrons
- charge fluctuations were measured in different experiment and yield the hadron-gas expectation
- C_{BS} was measured on the lattice and yield the expected QGP result
- recombination kills all "smokin' gun" signals related to the fluctuations and correlations of conserved charges
- conversely, the agreement with experimental results can be seen as another evidence for recombination