Низкоразмерные малочастичные системы в физике ультрахолодных квантовых газов

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ЛТФ ОИЯИ, Дубна

Дубна, 6-7 февраля 2014

Outline

- Quantum gas, what is this? Why it is interesting?
- Low-dimensional quantum systems in confining traps
- Theoretical models (pseudopotential approach)
- Widths and shifts of magnetic Feshbach resonances in atomic traps
- Resonance mechanism of molecule formation in 1D trap with CM excitation
- Dipolar confinement-induced resonances
- Outlook

1997 Nobel prize in physics
S. Chu, C. Cohen Tannoudji, W. Phillips
Laser manipulation of atoms

2001 Nobel prize in physics
E. Cornell, W. Ketterle, C. Wieman
Bose-Einstein Condensation
In atomic gases

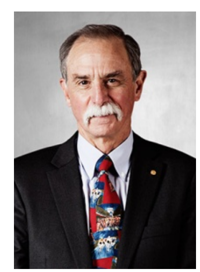


2005 Nobel prize in physics
J. Hall, T. Haensch, R. Glauber
Laser precision spectroscopy and
optical frequency comb



2012 Nobel prize in physics
S. Haroche, D.J. Wineland

Ground-breaking experimental methods
that enable measuring and manipulation of
individual quantum systems

Pre-history of laser cooling

- atoms and molecules at room temperatures - <V> ~ 300-1000 m/sec
- end of XIX century liquid N_2 (T=77 K) - <V> ~ 150 m/sec
- 1911 liquid He superconductivity (T=1-4 K) - <V> ~ 90 m/sec
- 1938 superfluidity ^{4}He
- 1972 superfluidity ^{3}He (T=1mK)

- ~1980 laser cooling ⟶ 10^{-6}-10^{-9}K

Основная мотивация для охлаждения нейтральных атомов и ионов

- уменьшить уширение и смещение спектральных линий за счёт теплового движения атомов в спектроскопии и в атомных часах

- исследовать новые явления:

  - бозе-эйнштейновская конденсация в атомных газах

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  - для ионов: квантовые скачки, квантовые логические затворы

Две идеи:

- “дипольная сила” – взаимодействие индуцированного дипольного момента атома с и градиента поля падающего света

  Г.А.Аскарьян ЖЭТФ (1962)

  В.С.Летохов Письма в ЖЭТФ (1968) (использовать для пленения атомов)

- использование эффекта Доплера для уменьшения тепловых скоростей атомов (лазерное охлаждение)

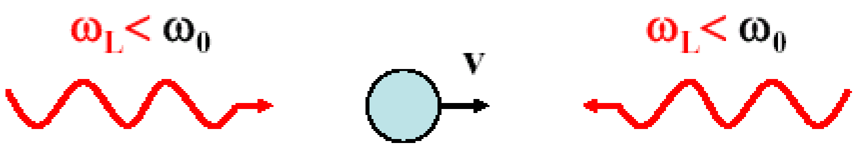
  T. Heansch & A. Schawlow Opt. Commun (1975)

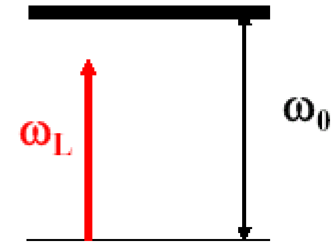
  D. Wineland & H. Dehmelt Bul. Amm. Phys. Soc. (1975)

Hänsch, Schawlow
Wineland, Dehmelt
1975

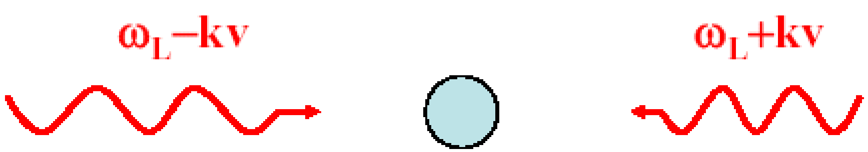
Doppler Cooling

Doppler effect





Laboratory frame



Atom frame

Absorption of the photon ω_L**+kv,** followed by a spontaneous emission equiprobable in all directions of the space.

$$\omega = \omega_L \left(1 + \frac{v}{c} \cos\theta\right)$$

Лазерное охлаждение атомных пучков (лазер с фиксированной частотой)

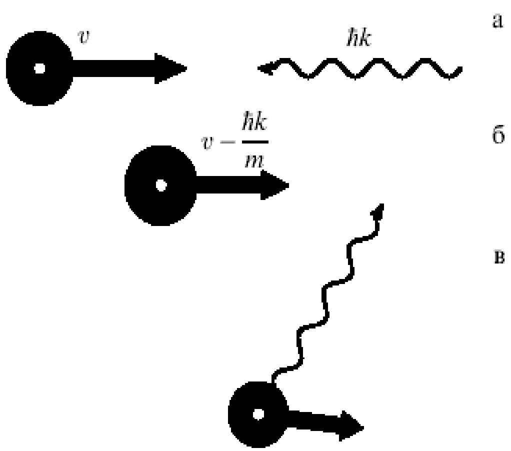


Рис. 1. Атом, движущийся со скоростью v, сталкивается с фотоном с импульсом $\hbar k$ (а); после поглощения фотона атом замедляется на $\hbar k/m$ (б); после переизлучения фотона в произвольном направлении атом в среднем движется медленнее, чем на рис. а (в).

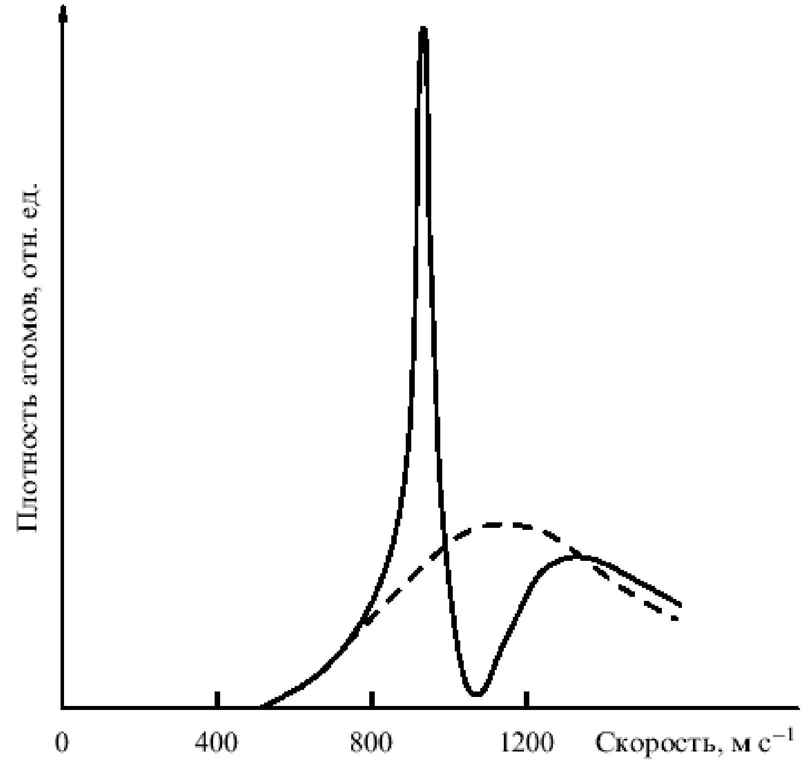


Рис. 3. Охлаждение атомного пучка лазером с фиксированной частотой. Штриховая кривая представляет распределение по скоростям до охлаждения, сплошная кривая — после охлаждения. Атомы из узкого диапазона скоростей перешли в несколько более узкий диапазон с центром на меньшей скорости.

Лазерное охлаждение атомных пучков (лазер с изменяющейся частотой)

V.Letokhov, V. Minogin & B. Pavlik Opt.Comm. (1976)

Лазерное чирпирование

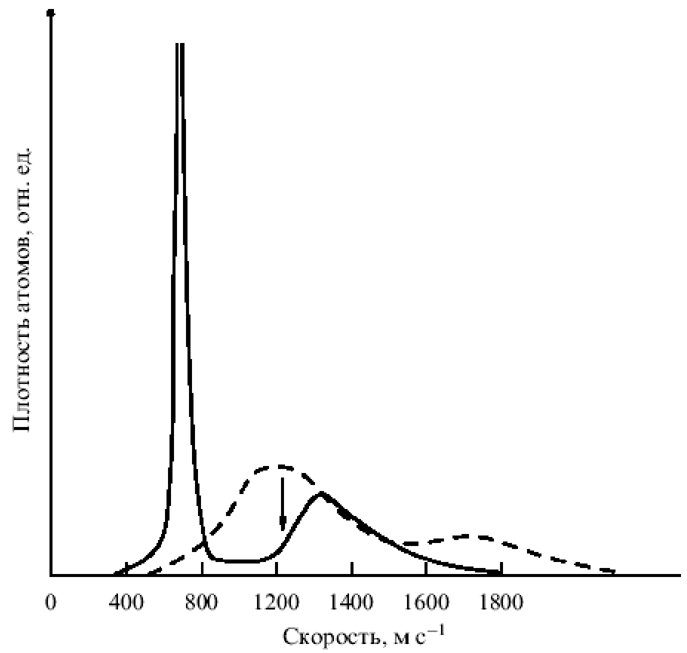
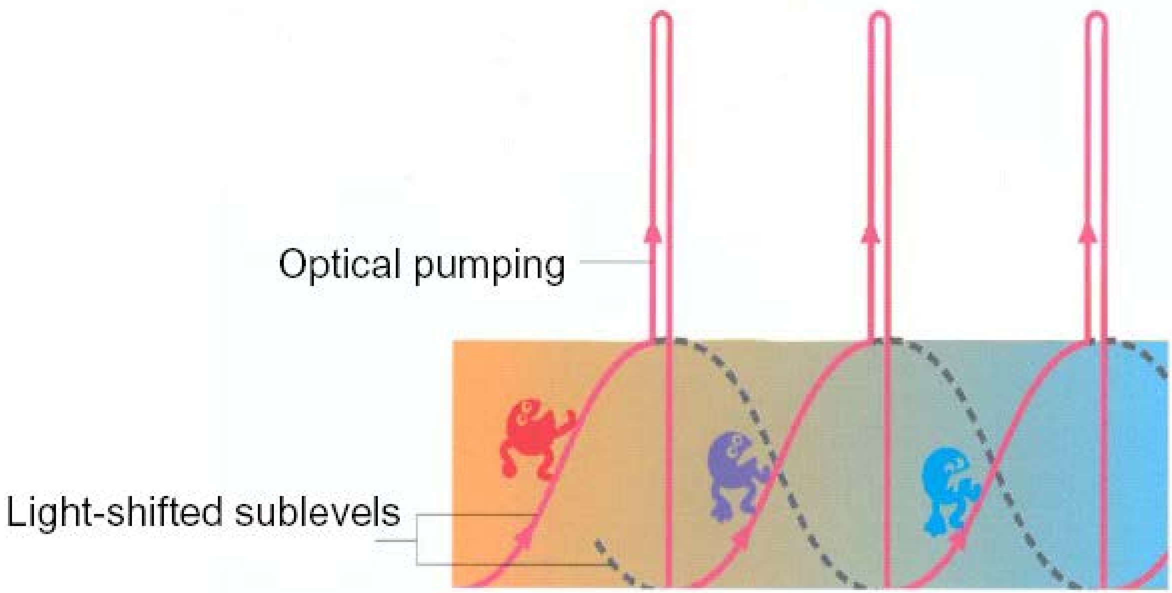


Рис. 5. Распределение по скоростям до (штриховая линия) и после (сплошная линия) зеемановского охлаждения. Стрелкой отмечена максимальная скорость, резонансная с замедляющим лазером (дополнительный максимум при 1700 м с$^{-1}$ дают атомы в состоянии F = 1, которые оптически накачиваются в состояние F = 2 в течение процесса охлаждения).

Sisyphus cooling

J. Dalibard, C. Cohen-Tannoudji



$$k_B T = U_0 / 4$$

Limit Temperature: about 10 times recoil energy

Trapping Atoms in Light Field - Optical Dipole Potentials

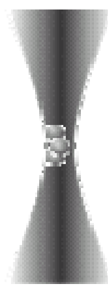
Energy of a dipole in an electric field: $U_{dip} = -\vec{d} \cdot \vec{E}$

An electric field induces a dipole moment: $\vec{d} = \alpha \vec{E}$

$$U_{dip} \propto -\alpha(\omega) I(\vec{r})$$

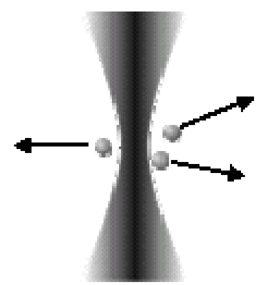
Red detuning:

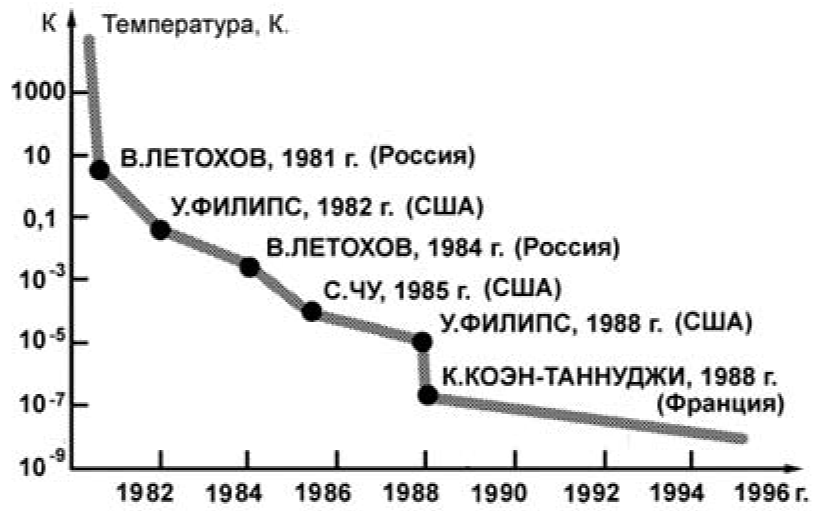
Atoms are trapped in the intensity maxima



Blue detuning:

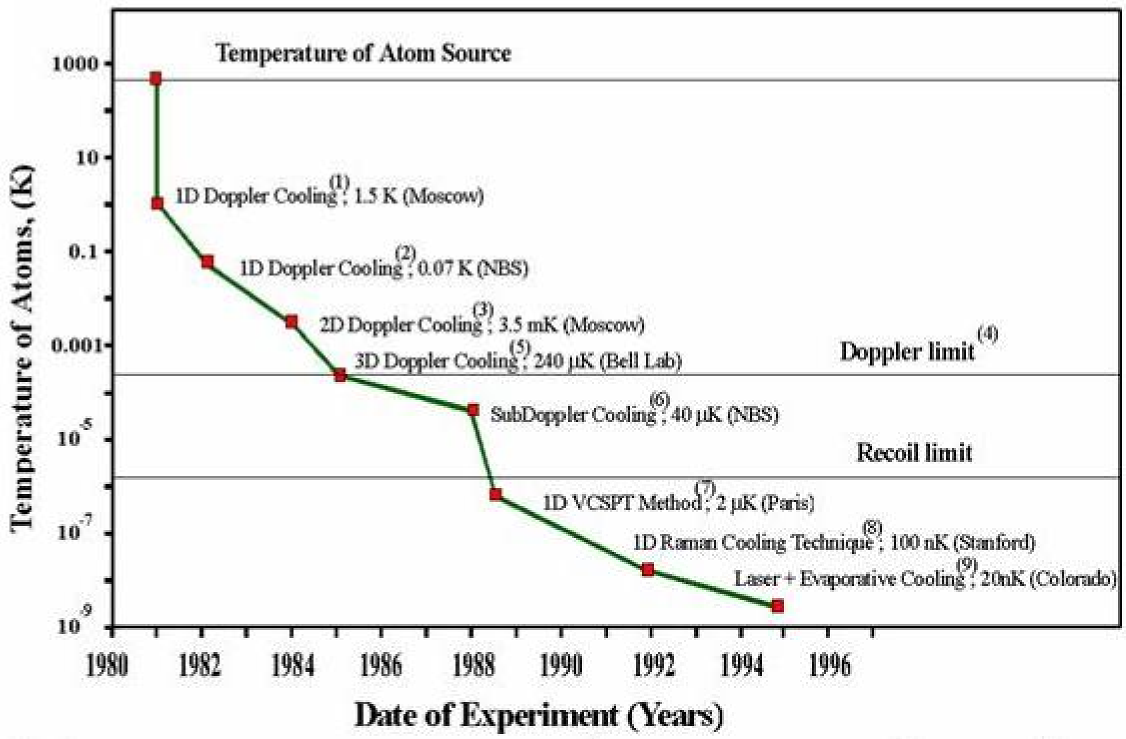
Atoms are repelled from the intensity maxima




К
Температура, К.
1000
10
0,1
10 -3
10 -5
10 -7
10 -9
В.ЛЕТОХОВ, 1981 г. (Россия)
У.ФИЛИПС, 1982 г. (США)
В.ЛЕТОХОВ, 1984 г. (Россия)
С.ЧУ, 1985 г. (США)
У.ФИЛИПС, 1988 г. (США)
К.КОЭН-ТАННУДЖИ, 1988 г.
(Франция)
1982
1984
1986
1988
1990
1992
1994
1996 г.

Creation of ultracold atoms and molecules (quantum gases)

Progress in production of ultracold atoms and molecules: laser cooling



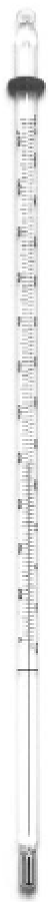
T.Heansch and A. Schawlow Opt.Comm. (1975)

V.S.Letokhov,V.G.Minogin andB.D.Pavlik, ZhTPH, 72, 1328 (1977)

The Nobel Prize in Physics 1997

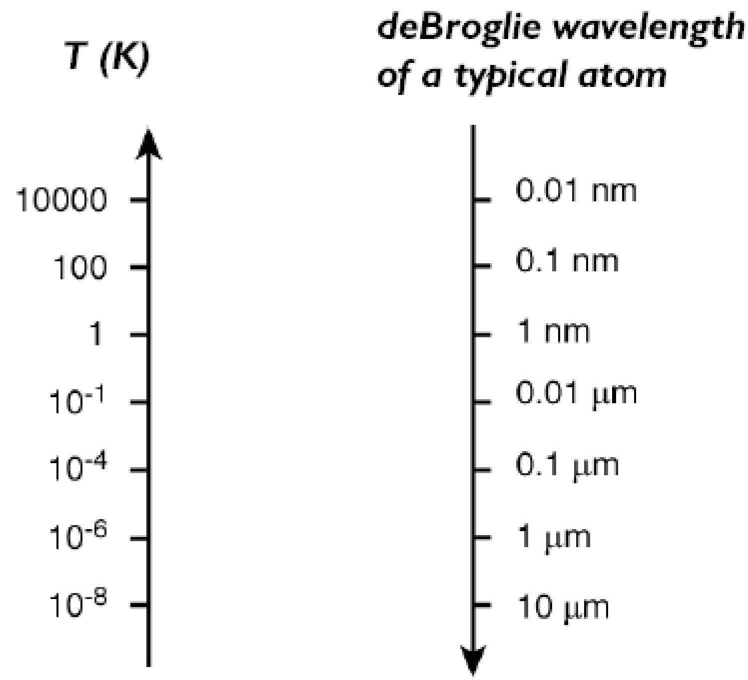
Steven Chu Claude Cohen-Tannoudji William D. Phillips

de Broglie Wavelength

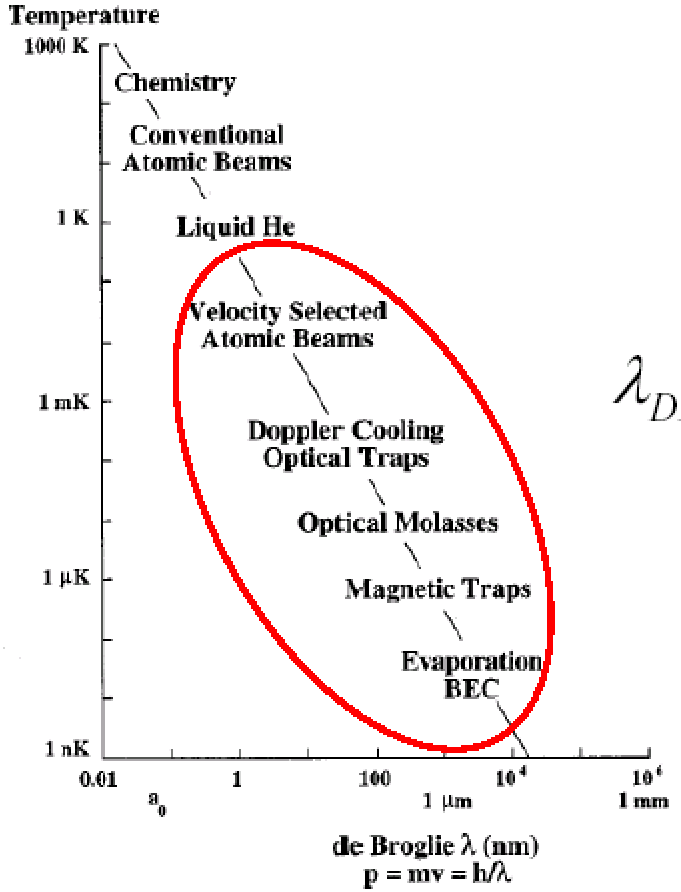


Thermal deBroglie wavelength

$$\lambda_T = \frac{h}{mV} = \frac{2\pi\hbar}{\sqrt{3mkT}}$$

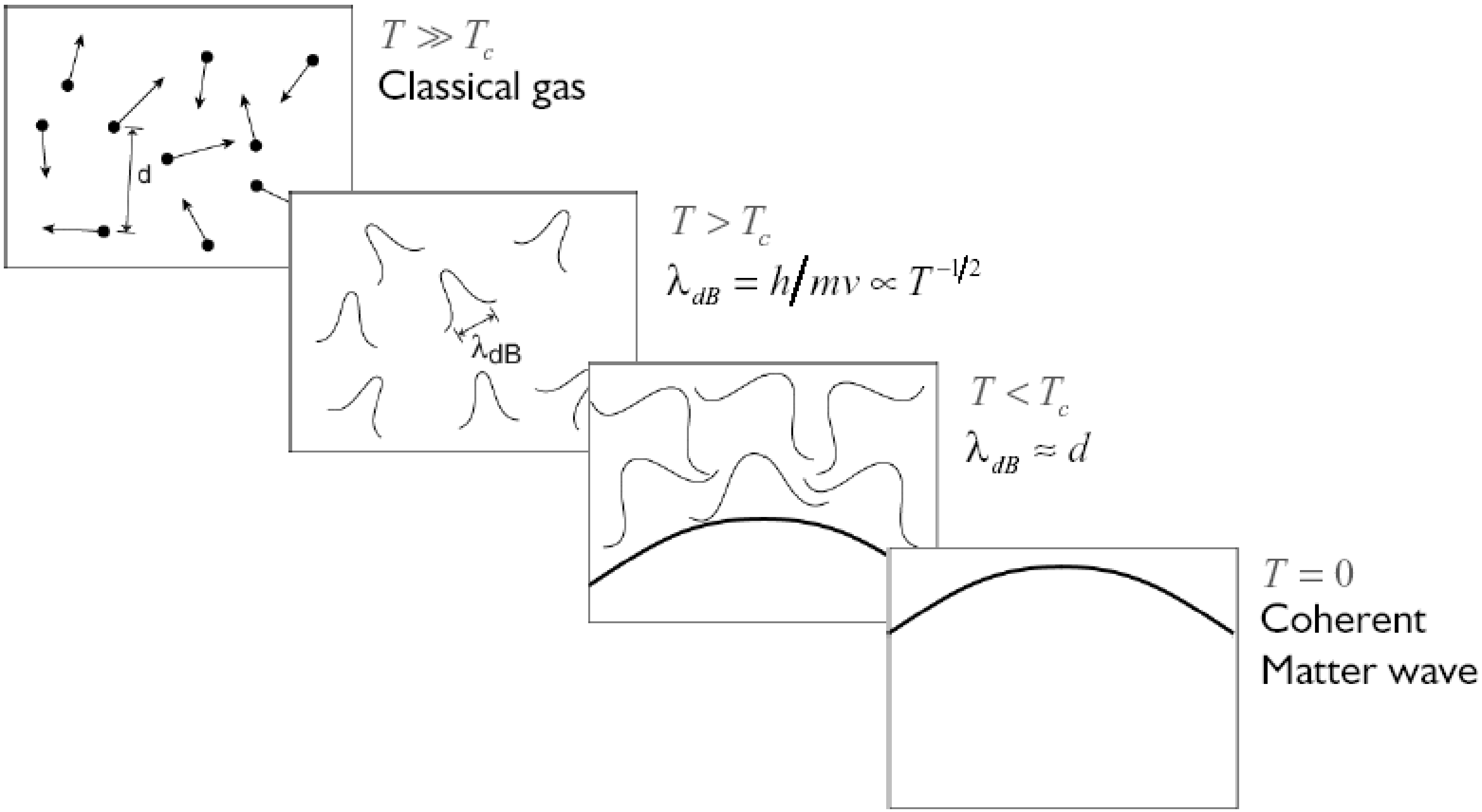


Temperature scale



$$\lambda_{DB} = \frac{h}{Mv}$$

From a Classical Gas to a Bose-Einstein Condensate

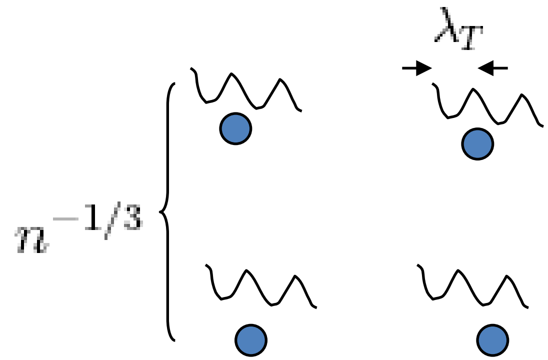


Dilute gas → **two distance scales:**

$$\lambda_T = \frac{h}{mV} = \frac{2\pi\hbar}{\sqrt{3mkT}}$$ **and** $$n^{-1/3}$$

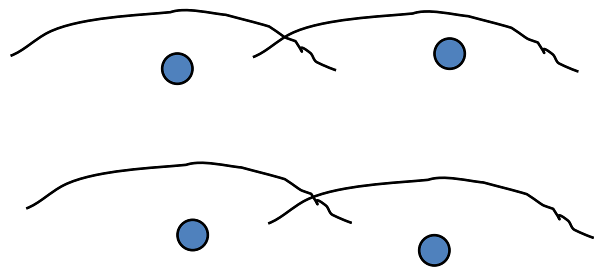
Boltsmann gas

$n\lambda_T^3 \ll 1$ →



Quantum statistics

$n\lambda_T^3 > 1$ →



bosons → $n\lambda_T^3 \geq 2.61$ → **BEC**

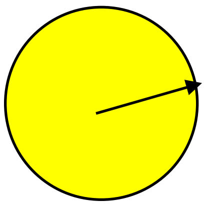
= macroscopic occupation of a single quantum state

$$T \leq T_c = 3.31\frac{\hbar^2}{m}n^{2/3}$$

BEC objects

$$n\lambda_T^3 \geq 2.61$$

- **Neutron stars**



$n \sim 10^{39} cm^{-3}$

$T \sim 10^6 - 10^7 K$

$R \sim 10$ km

- **Superfluid** ^4He

$n \sim 10^{22} cm^{-3}$

$T \leq 1K$

- **Exitones in semiconductors**

$n \sim 10^{18} cm^{-3}$

$T \leq 1K$

- **Dilute atomic gases**

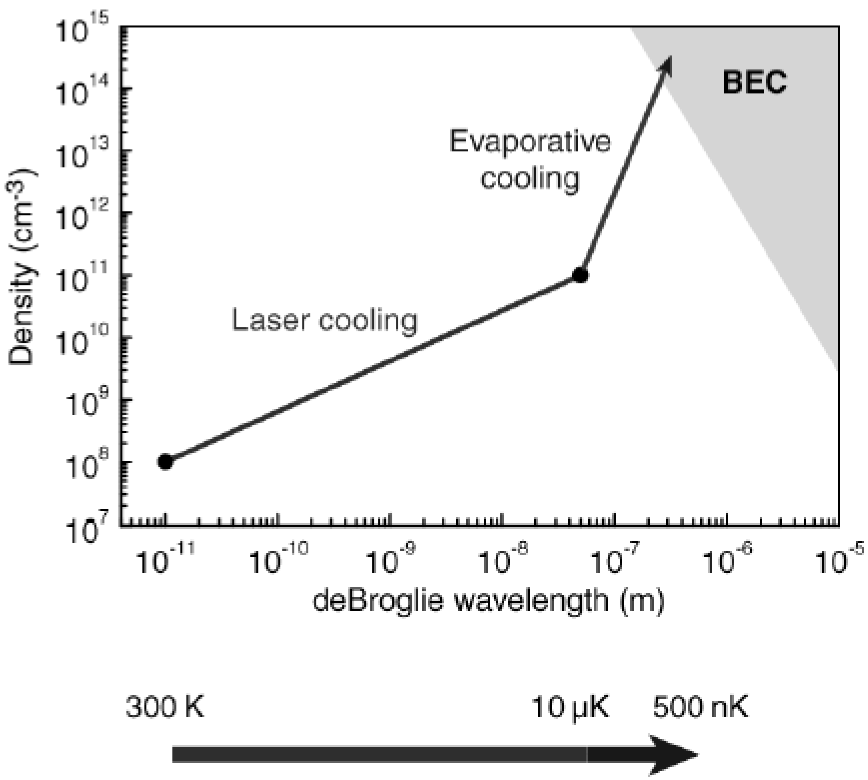
$n \sim 10^{12} - 10^{15} cm^{-3}$

$T \sim 3nK \rightarrow \mu K$

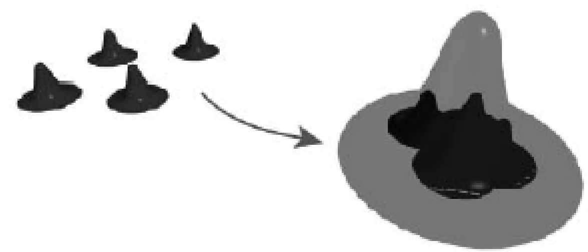
$N \sim 10^4 - 10^7$

Collective behavior of extremely dilute systems
Generation of coherent matter waves

Reaching Bose-Einstein Condensation

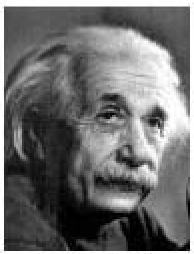


$$n \cdot \lambda^3 \approx 1$$



Experimenting with the coldest objects in the whole universe

Predicted 1924...

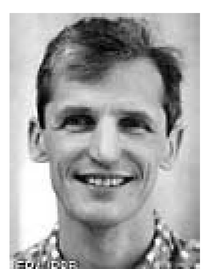


A. Einstein

S. Bose

- *1924 Bose sends Einstein his work on the statistics of photons. Einstein translates this works.*
- *1924 Only eight days later Einstein has developed his „Quantum theory of the single atomic ideal gas“ .*
- *1925 Einstein continuies his work on the ideal gas with Bose statistics, and for the first time notes the phenomena of Bose-Einstein condensation.*

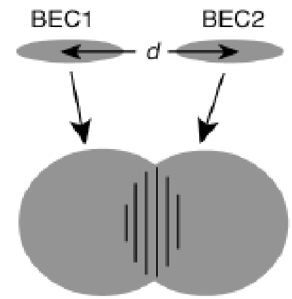
- *1995 Bose-Einstein condensation in a dilute gas of ^{87}Rb atoms is achieved by Eric Cornell and Carl Wieman (JILA) and a few months later with ^{23}Na by Wolfgang Ketterle (MIT)*



Nobel prize 2001 !

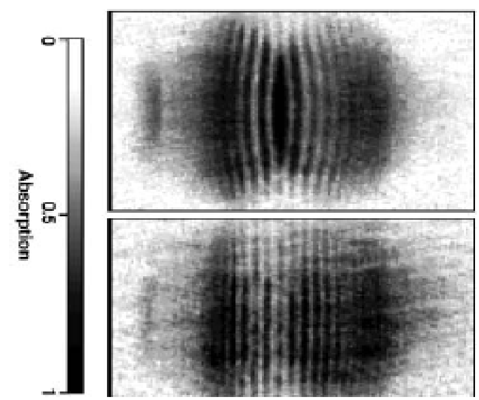
Interference of two Bose-Einstein Condensates

Trapped BEC's



BEC's after an expansion time t

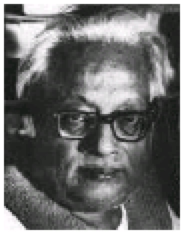
$$\lambda = \frac{h}{m\Delta v} = \frac{ht}{md}$$

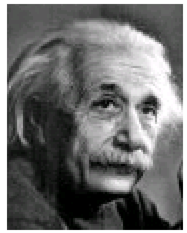


M. R. Andrews *et. al.*
Science 275, ff. 637, 1997

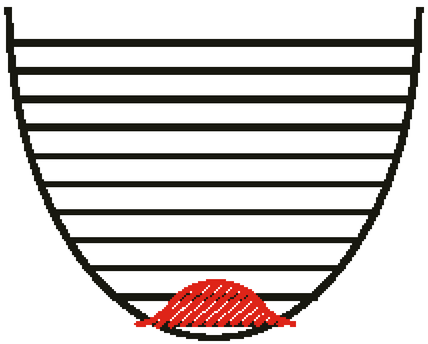
Quantum statistics in harmonic traps

- Bose-Einstein statistics (1924)





Bose-Einstein condensate



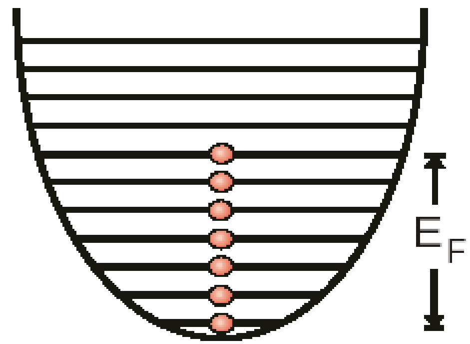
Bose enhancement

$$T_C = \frac{\hbar\omega}{k_B}(0.83\ N)^{1/3}$$

Dilute gases: 1995, JILA, MIT

- Fermi-Dirac statistics (1926)

Fermi sea



Pauli Exclusion

$$T \ll T_F = \frac{\hbar\omega}{k_B}(6\ N)^{1/3}$$

Dilute gases: 1999, JILA

Classical and quantum rotation

Rotating classical gas

rigid body rotation $\vec{v} = \vec{\Omega} \times \vec{r} \longrightarrow \vec{\nabla} \times \vec{v} = 2\vec{\Omega}$

Rotating quantum gas

macroscopic wave function: $\psi(\vec{r}) = \sqrt{\rho(\vec{r})}\, e^{i\phi(\vec{r})}$

At a place where $\rho(\vec{r}) \neq 0$, the velocity field a zero curl: $\vec{v} = \frac{h}{m}\vec{\nabla}\phi$

The only way to generate a non-trivial rotation field

is to nucleate quantum vortices

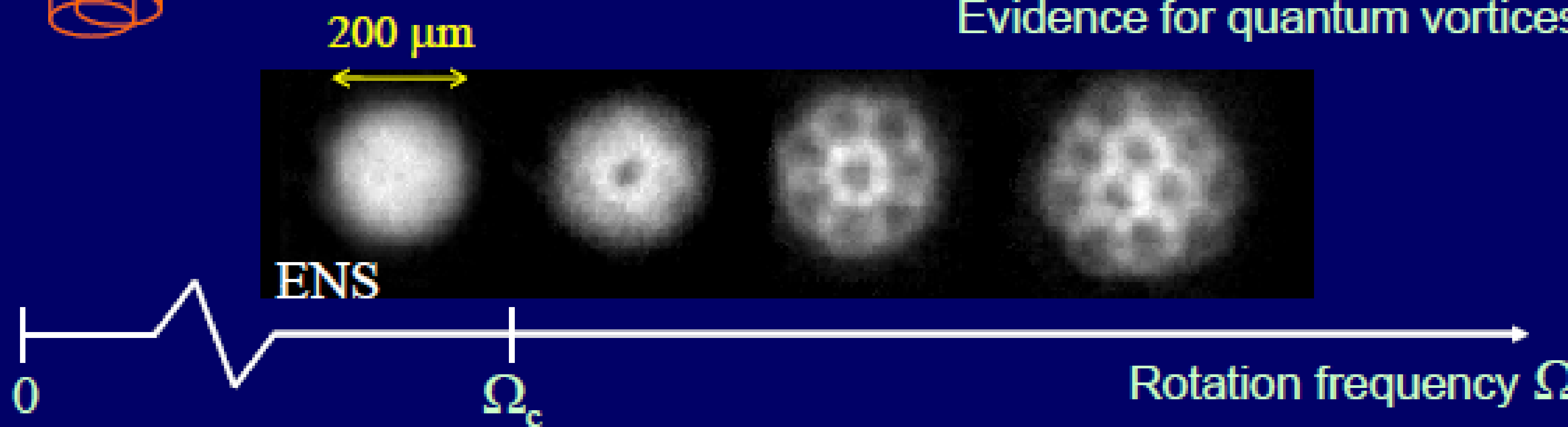
Feynman, Onsager

Rotating condensates

We stir our condensate with a moving laser beam and take an image after ballistic expansion

K. Madison, F. Chevy, V. Bretin, P. Rosenbuch

Evidence for quantum vortices



Clear proof of superfluidity

Towards quantum Hall effect?

Наблюдение решетки вихрей в бозе-эйнштейновском конденсате

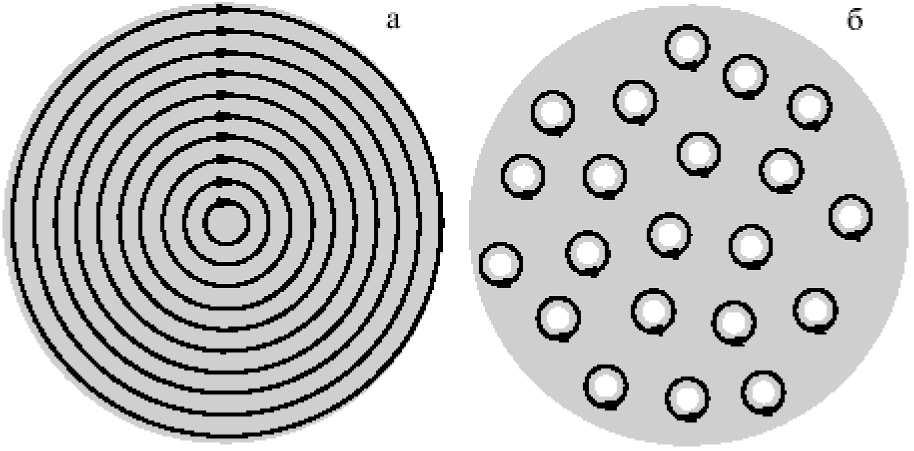


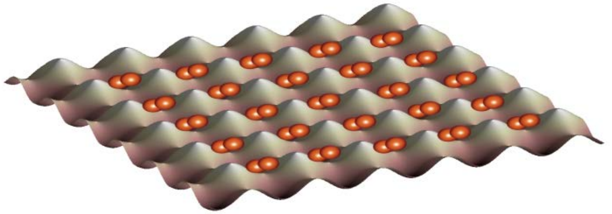
Рис. 19. Сравнение поля скоростей при вращении обычной и сверхтекучей жидкостей. Обычная жидкость вращается как твердое тело (а), а в сверхтекучей возникает решетка квантовых вихрей (б).

Quantum simulation with fully controlled few-body systems

control over: quantum states, particle number, interaction

- attractive interactions ⟶ BCS-like pairing in finite systems
- repulsive int.+splitting of trap ⟶ entangled pairs of atoms (quantum information processing)
- + periodic potential ⟶ quantum many-body physics (systems with low entropy to explore such as quantum magnetism)
- ...

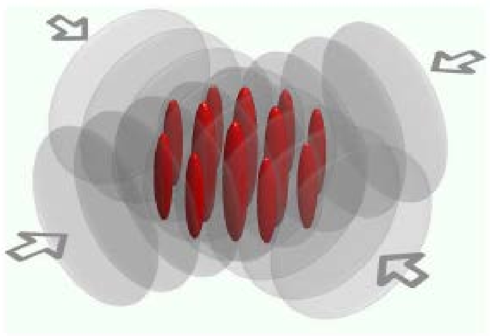
Bose-Hubbard Physics



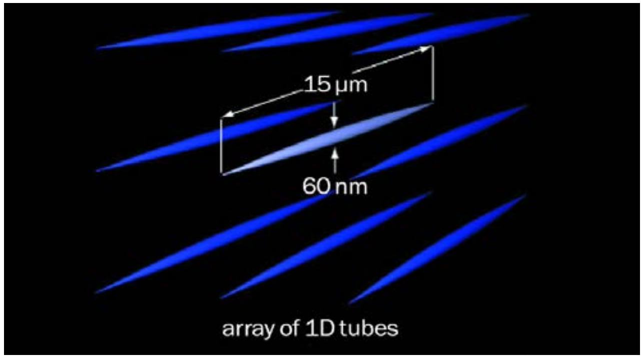
Investigation of quantum gases confined in traps

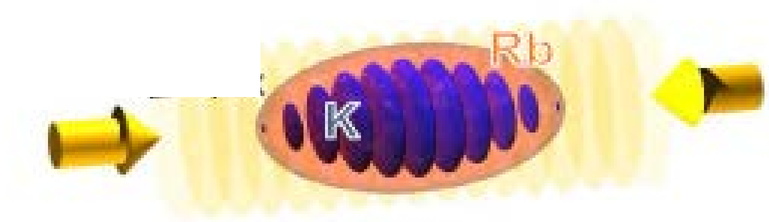
System:

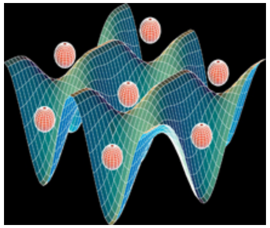
- horizontal tubes
- transversal trap frequency 2π 14000 Hz



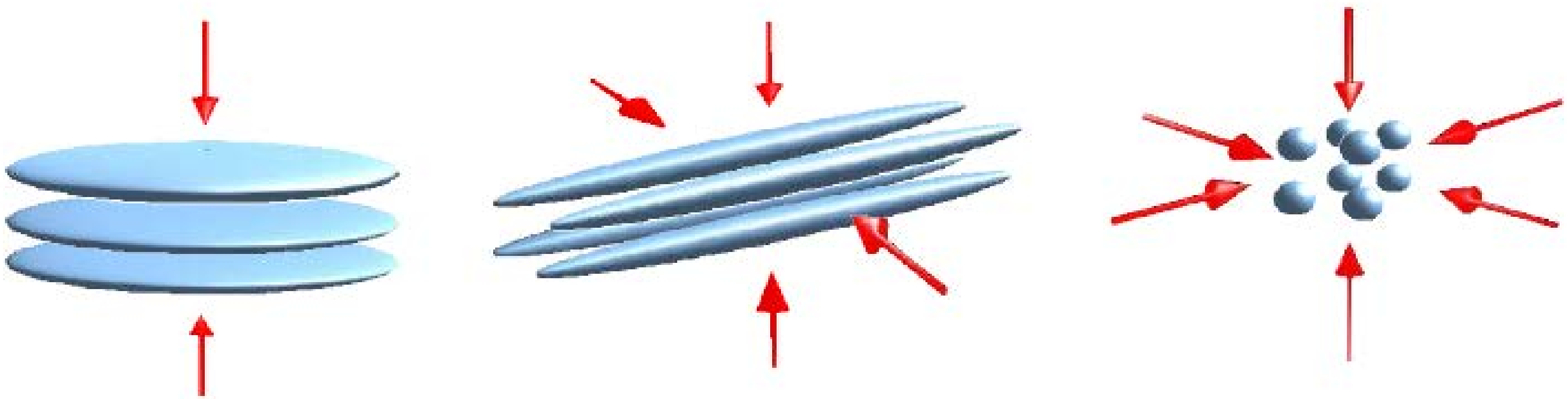
Experimental setup. The lattice potential is created by two retro-reflected laser beams confining the atoms to an array of 1D tubes with equipotential surfaces shown in red.



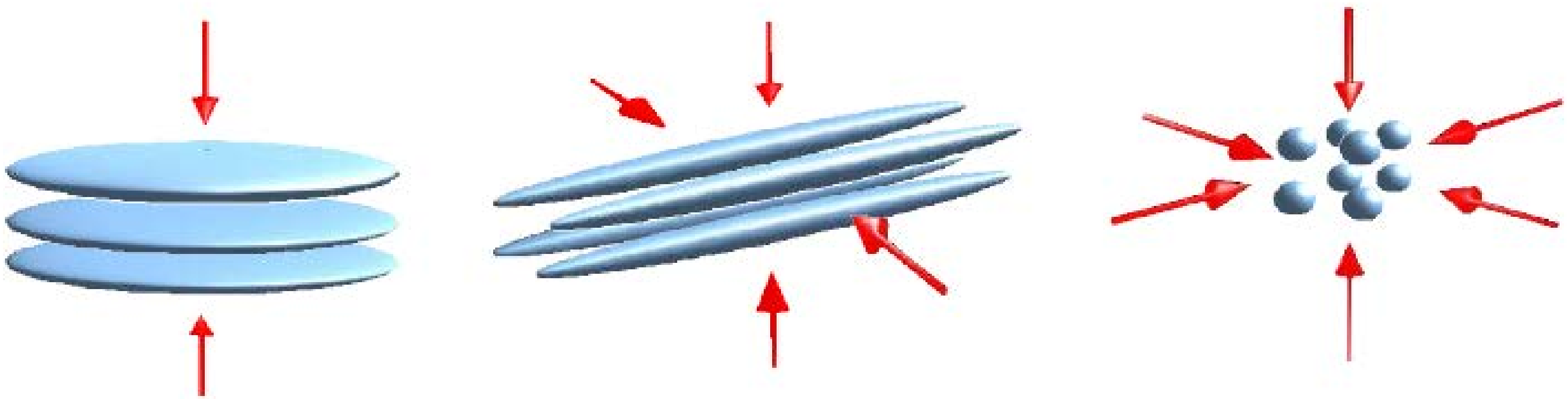




ultracold atoms in optical traps, atomic chips, quantum wires



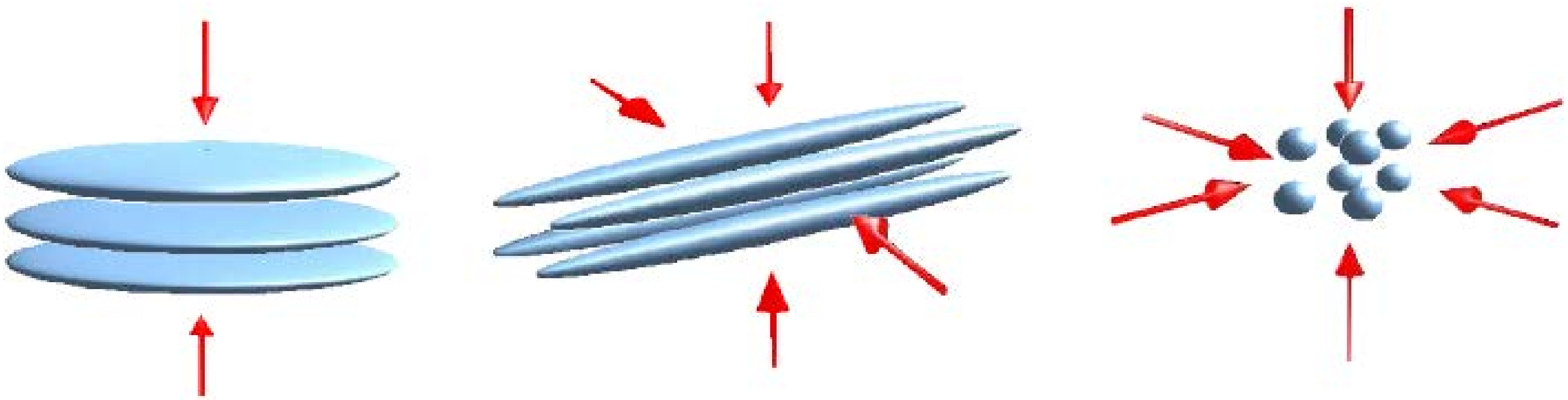
Lattices formed by applying orthogonal standing waves in one, two, and three directions.



Lattices formed by applying orthogonal standing waves in one, two, and three directions.

An electric field induces a dipole moment: $\vec{d} = \alpha \vec{E}$

Energy of a dipole in an electric field: $U_{dip} = -\dot{d} \cdot \vec{E}$



Lattices formed by applying orthogonal standing waves in one, two, and three directions.

An electric field induces a dipole moment: $\vec{d} = \alpha \vec{E}$

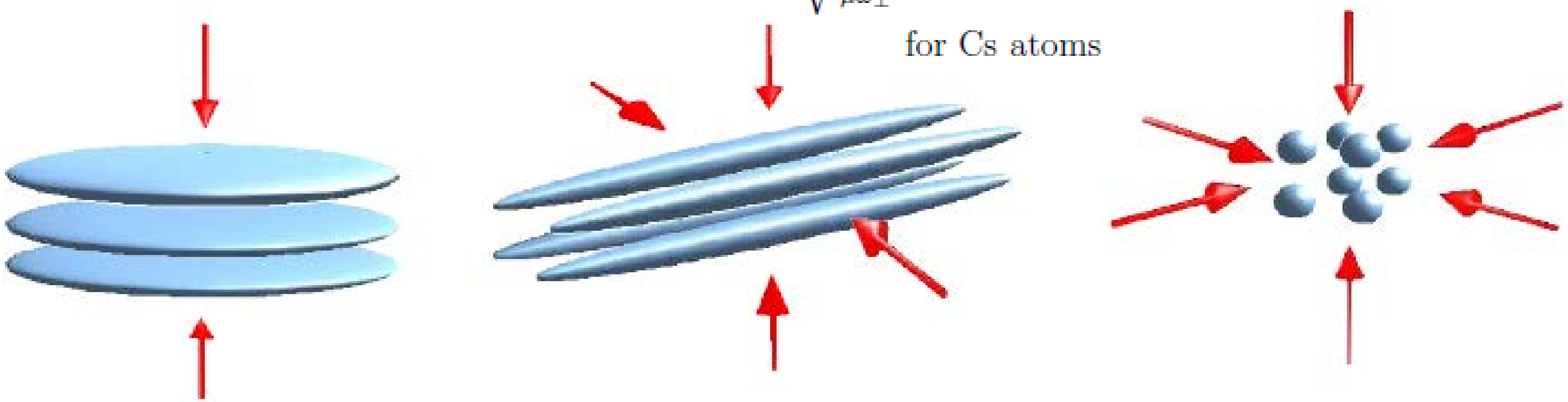
Energy of a dipole in an electric field: $U_{dip} = -\vec{d} \cdot \vec{E}$

$$U_{dip} \propto -\alpha(\omega) I(\vec{r})$$

$$\omega_\perp \sim 14\text{kHz}$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}} \sim 70 - 100nm$$

for Cs atoms



Lattices formed by applying orthogonal standing waves in one, two, and three directions.

An electric field induces a dipole moment: $\vec{d} = \alpha \vec{E}$

Energy of a dipole in an electric field: $U_{dip} = -\vec{d} \cdot \vec{E}$

$$U_{dip} \propto -\alpha(\omega)\, I(\vec{r})$$

motivation in brief

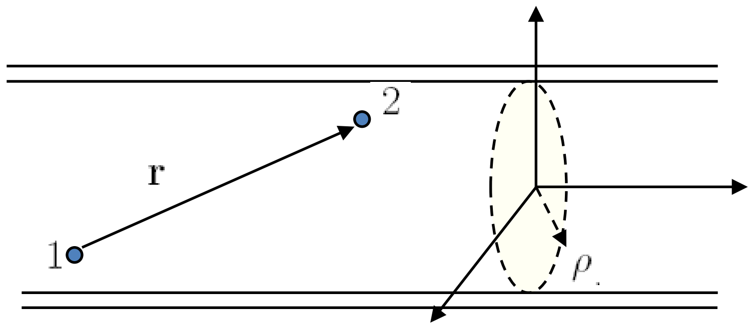
theoretical aspects

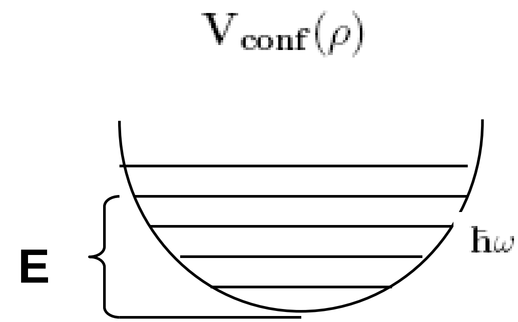
motivation in brief

theoretical aspects

3D free-space scattering theory is no longer valid and development of low-dimensional theory including influence of the trap is needed

- **What happens if atoms scatter in confined geometry (quasi-1D) ?**





- **What happens in collision of two distinguishable atoms in harmonic trap, or identical atoms in anharmonic trap ?**

$$H'(\boldsymbol{\rho_R}, \boldsymbol{r}) = -\frac{1}{2M}\left(\frac{\partial^2}{\partial \rho_R^2} + \frac{1}{4\rho_R^2}\right) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial \phi_R^2} + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + \underline{\mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos(\phi - \phi_R)} + V(r)$$

$$\mathbf{5D} \quad \rightarrow \quad \{r, \theta, \phi, \rho_R, \phi_R\}$$

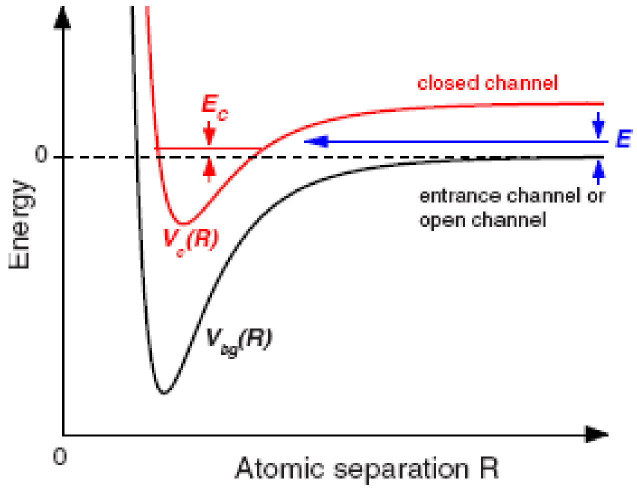
non-separable two-body problem

motivation in brief

experimental aspects

Feshbach resonances

phenomenon occurs when particles in an entrance channel resonantly couple to bound state supported by the closed channel potential



H.Feshbach, Ann.Phys.5(1958);19(1961)

(nuclear reactions)

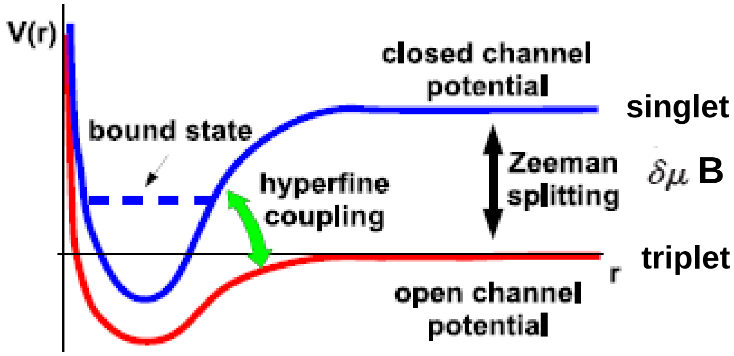
U.Fano, Phys.Rev.124(1961)

(atomic collisions)

the resonances occur when the scattering energy is varied

Magnetic Feshbach resonances

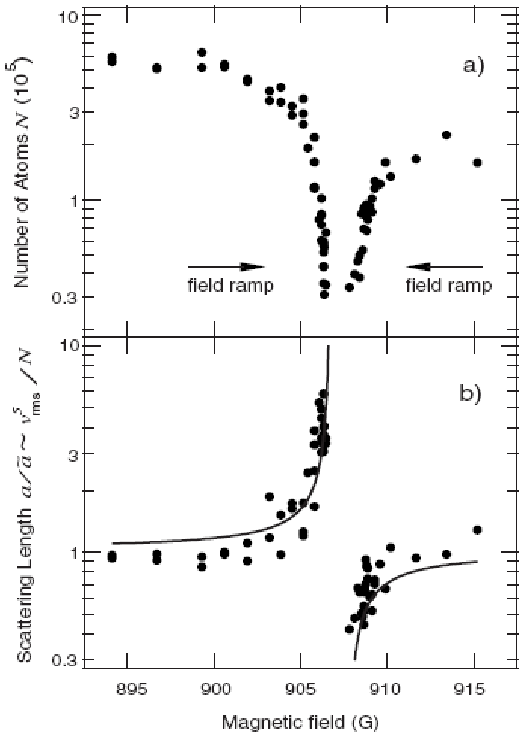
in ultracold gases scattering takes place in the zero-energy limit and the resonances occur when an external field B tunes bound state near threshold

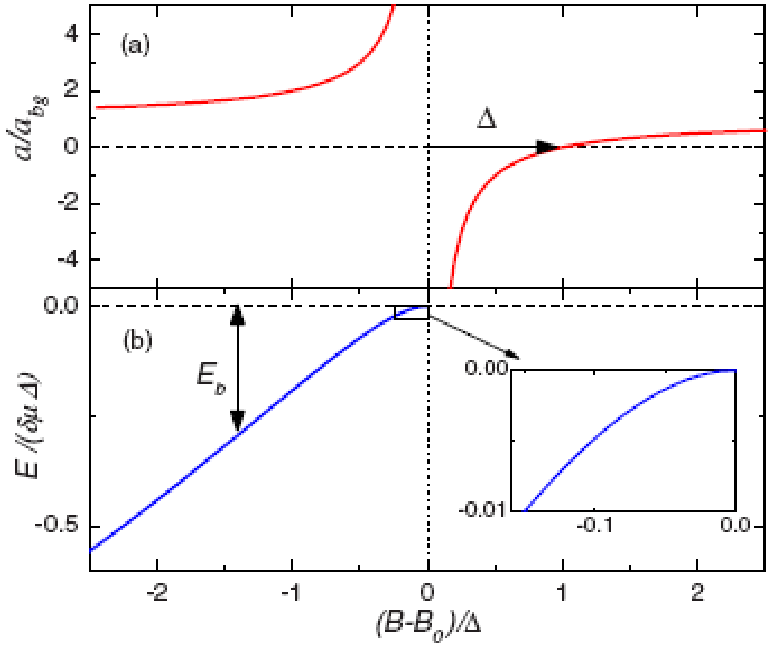


Simplified sketch of two collision potentials for atomic scattering in a magnetic field. The closed channel potential is for a predominantly singlet total electronic spin state, and the open channel is predominantly triplet.

Magnetic Feshbach resonances

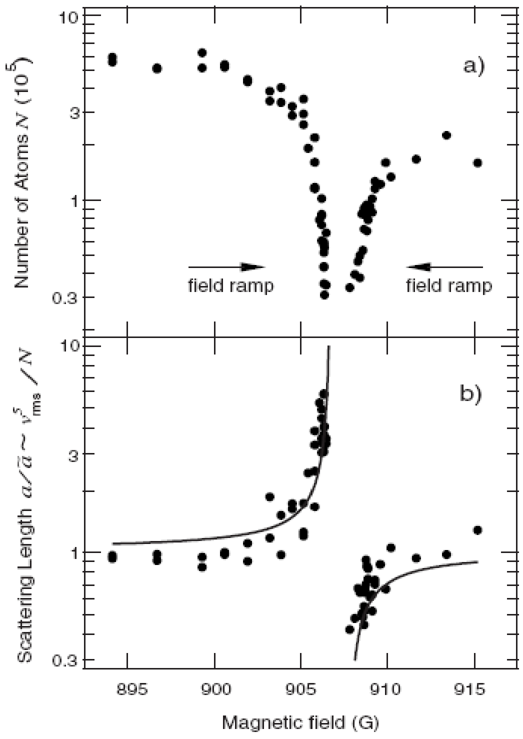
S.Innouye et. al. Nature 392 (1998): Observation of FR in BEC

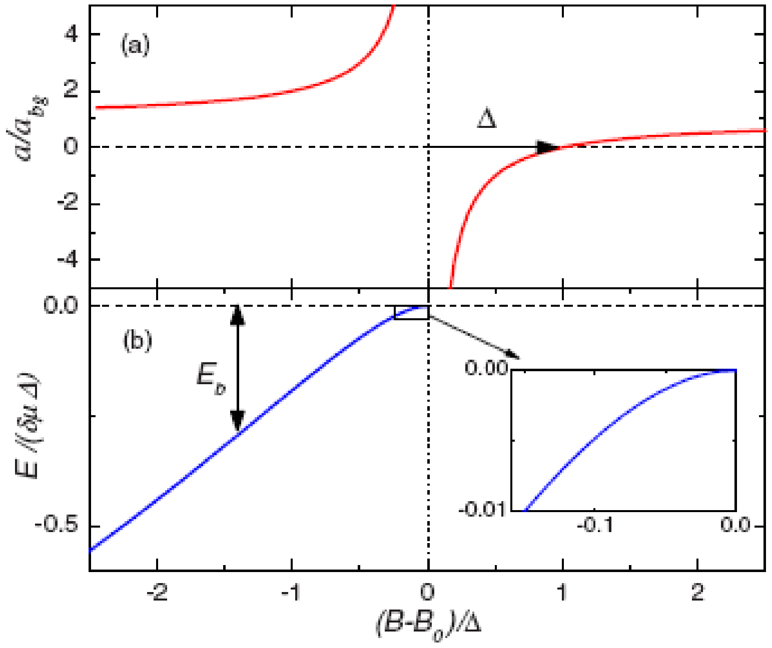




Magnetic Feshbach resonances

S.Innouye et. al. Nature 392 (1998): Observation of FR in BEC

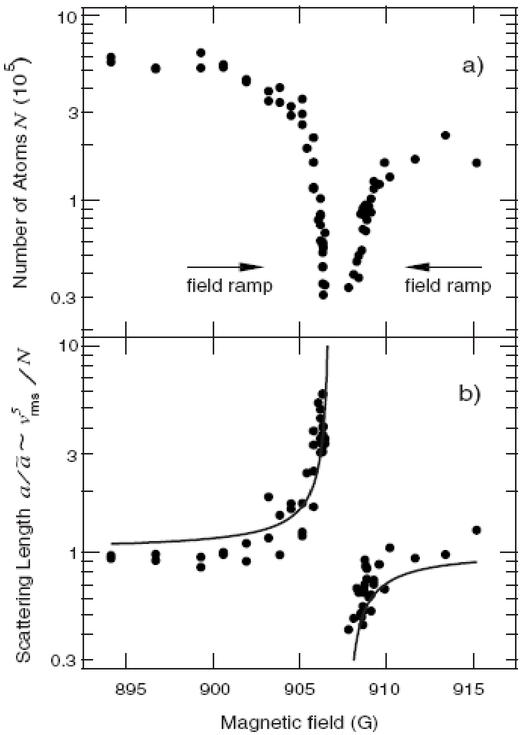


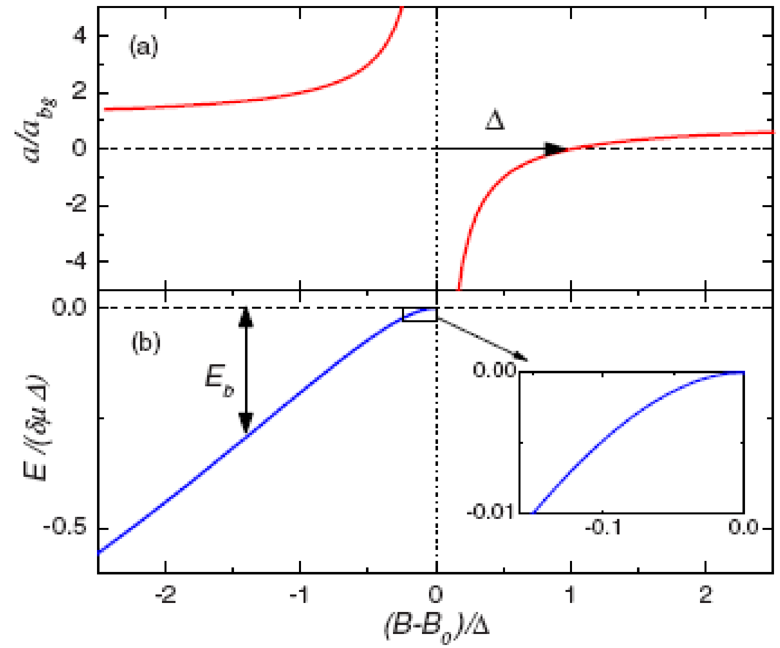


possibility to tune the interaction from strong attraction to strong repulsion

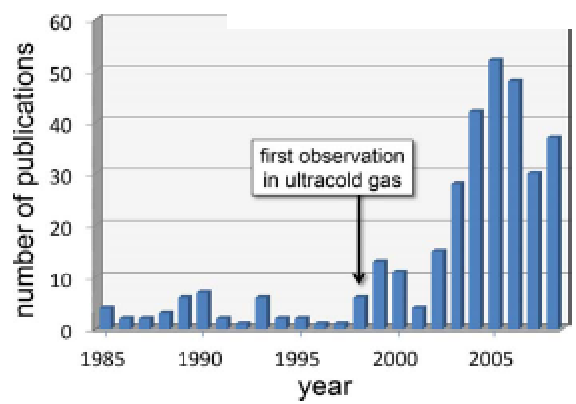
Magnetic Feshbach resonances

S.Innouye et. al. Nature 392 (1998): Observation of FR in BEC





possibility to tune the interaction from strong attraction to strong repulsion

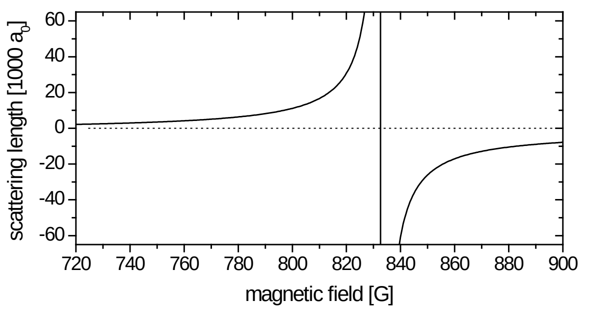


Number of publications per year (from 1985 to 2008) with Feshbach resonances appearing in the title. Data from ISI Web of Science.

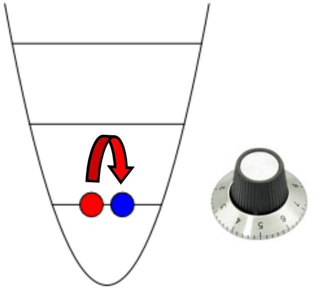
Experiments with deterministically prepared quantum systems

- **control interparticle interaction**

2 interacting particles in a 1D potential



magnetic Feshbach resonance



Experiments with deterministically prepared quantum systems

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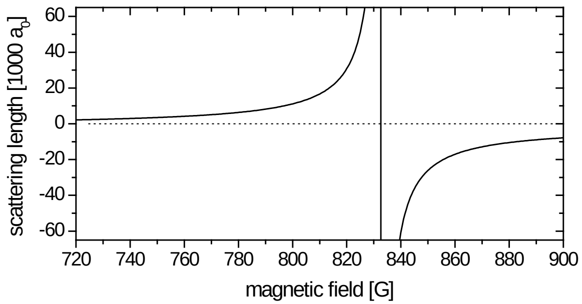
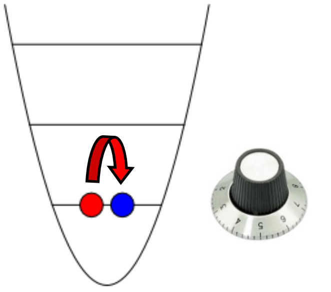
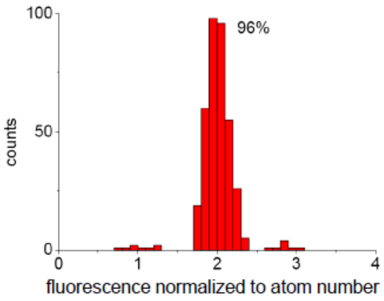


magnetic Feshbach resonance



- **control over quantum states and particle number with long lifetime**



Experiments with deterministically prepared quantum systems

- **control interparticle interaction**

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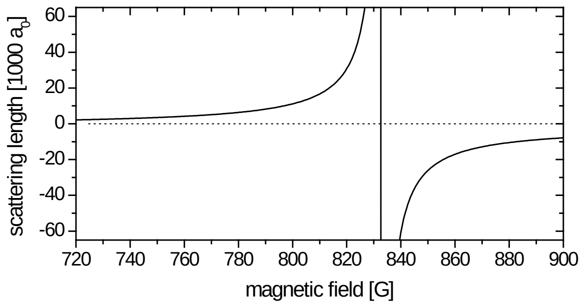
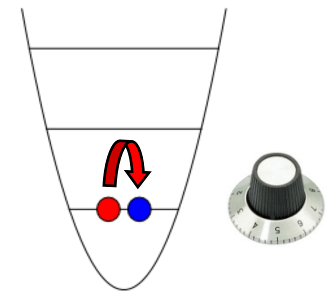


magnetic Feshbach resonance



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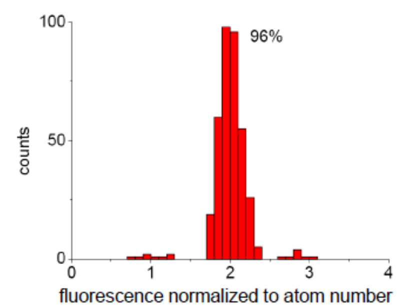


⇓

quantum simulation with fully controlled few-body systems

G.Zürn et. al. Phys. Rev. Lett. 108, 075303 (2012)

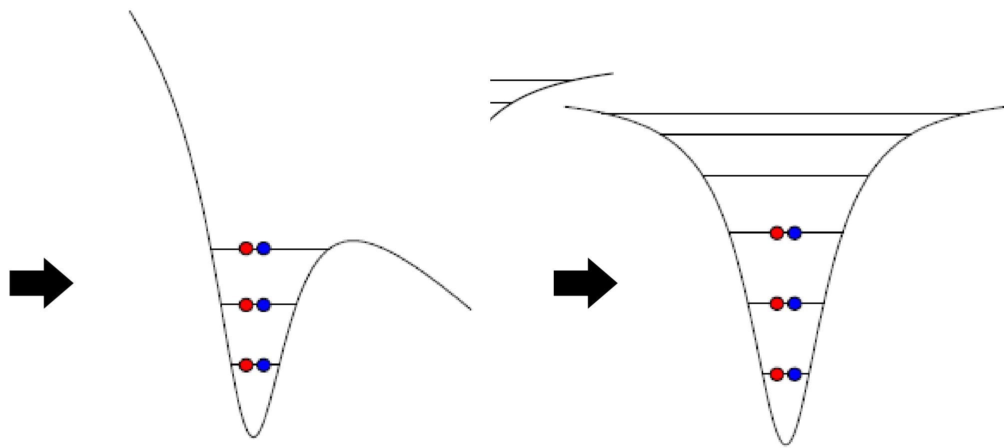
Preparation

- 2-component mixture in reservoir T=250nK
- superimpose microtrap

  scattering → thermalisation

  expected degeneracy: $T/T_F = 0.1$

- switch off reservoir



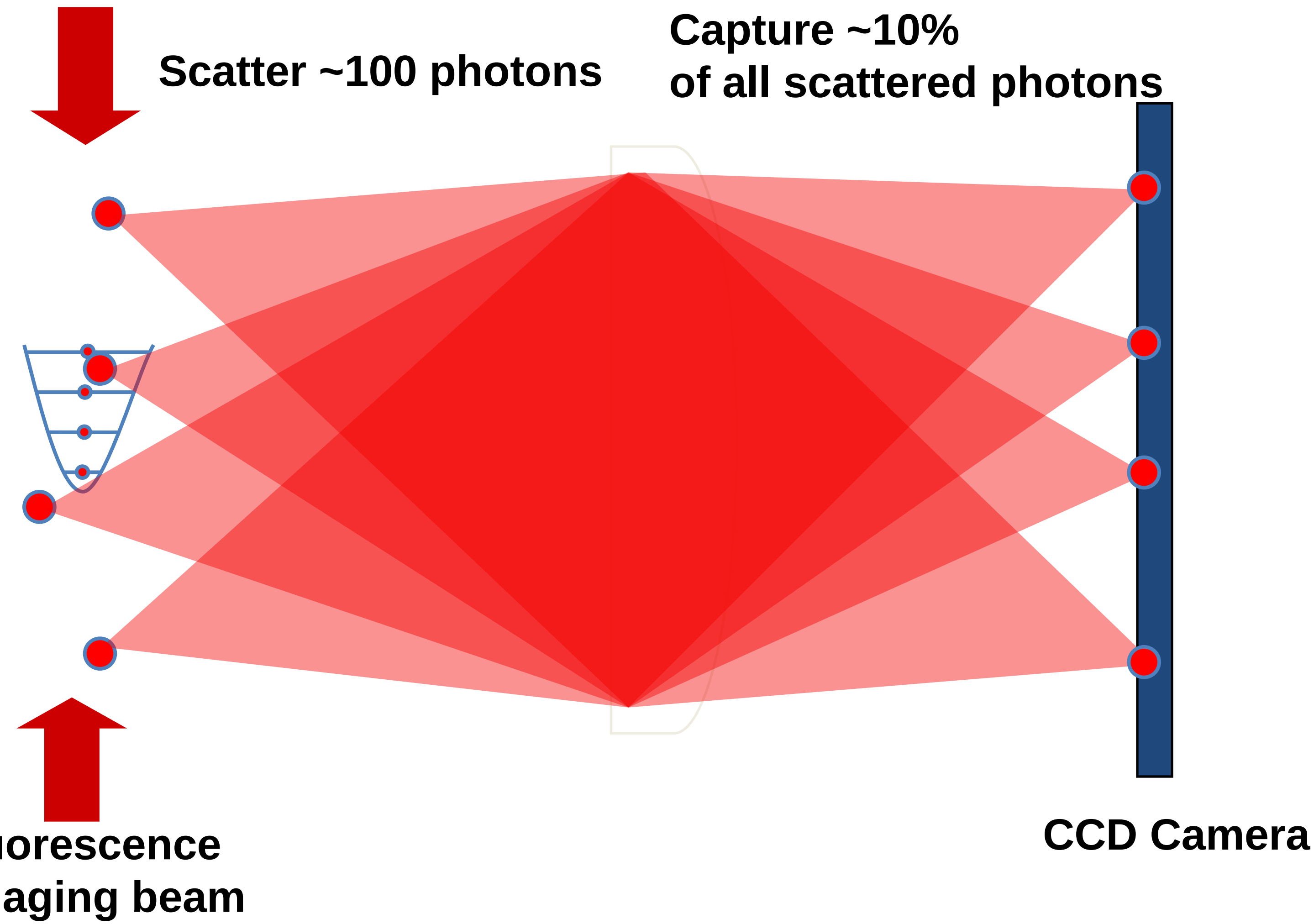
count the atoms

+ magnetic field gradient in axial direction

S.Serwane et.al. Science 332 (2011)

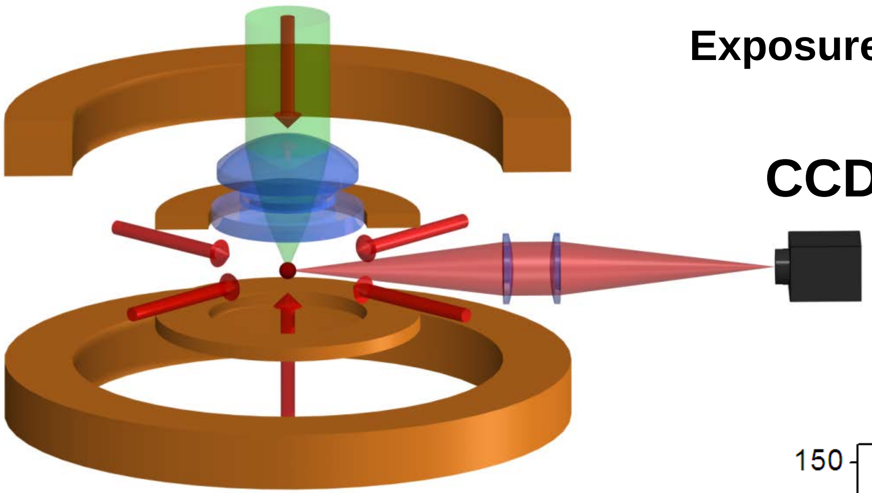
Detection:

Modern CCD Cameras allow detection of single photons:



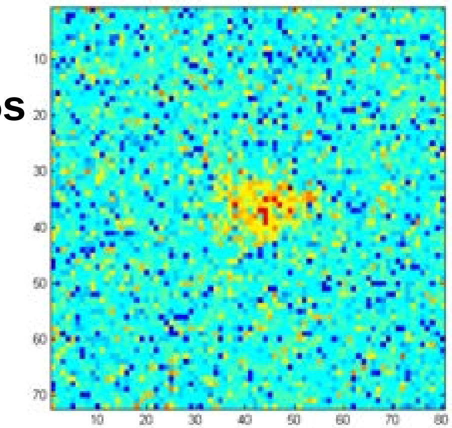
S.Serwane et.al. Science 332 (2011)

Single atom detection

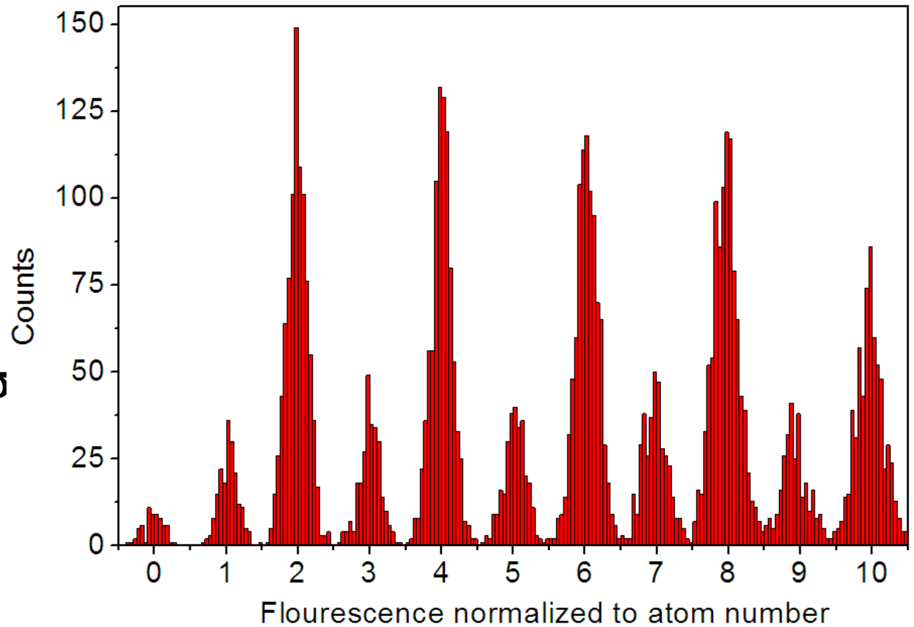


1/e-lifetime: 250s

Exposure time 0.5s



one atom in a Magneto Optical Trap

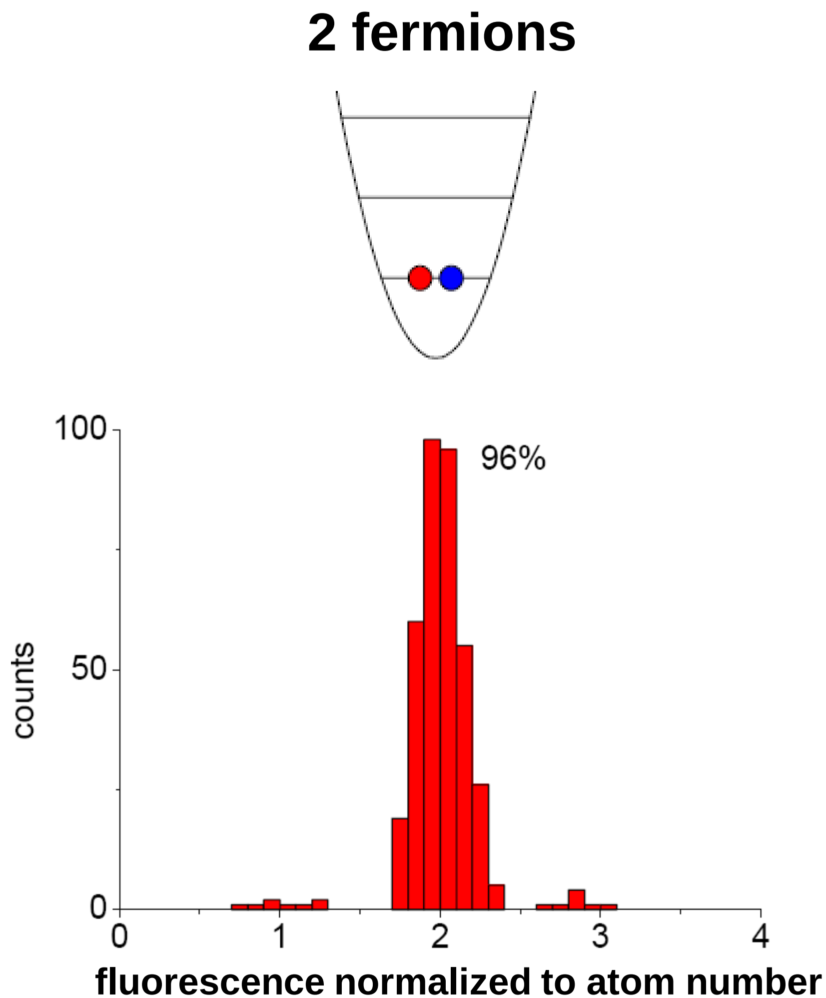


distance between 2 neighbouring atom peaks: ~ 6σ

1-10 atoms can be distiguished with high fidelity > 99%

S.Serwane et.al. Science 332 (2011)

High fidelity preparation



F. Serwane et al., Science 332, 336-338 (2011)

theoretical models for describing
low-dimensional few-body dynamics
in confined geometry of traps
are required

low energy collisions can be described by one parameter

$$\mathrm{k} \to 0$$

$$f(k) = \frac{\tan \delta(k)}{k} \to k^{2l}$$

$$f(k) = \frac{\tan \delta(k)}{k} \to -a \quad \text{s-wave scattering length}$$

low energy collisions can be described by one parameter

$$\mathbf{k} \rightarrow 0$$

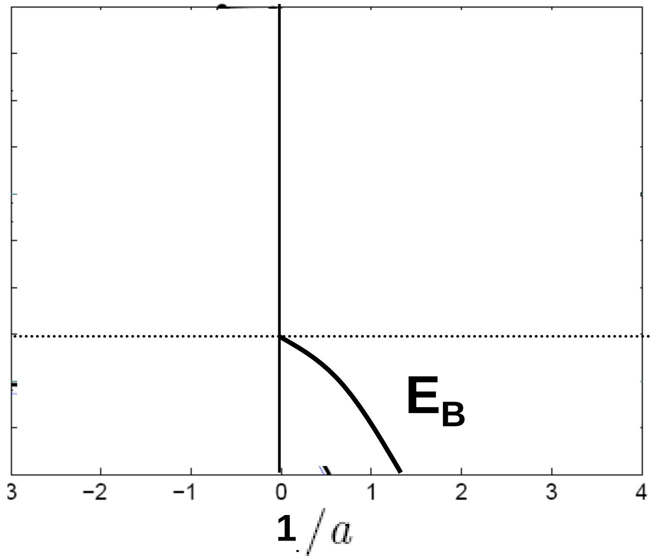
$$f(k) = \frac{\tan\delta(k)}{k} \rightarrow k^{2l}$$

$$f(k) = \frac{\tan\delta(k)}{k} \rightarrow -a \quad \text{s-wave scattering length}$$

$$\mathbf{V}(\mathbf{r}) = \frac{2\pi\hbar^2 a}{\mu}\delta^3(r) \quad \text{pseudopotential}$$

Pseudopotential approximation in 3D free space

$$\hat{V}(r) = \frac{2\pi\hbar^2 a}{\mu}\delta^3(r)$$

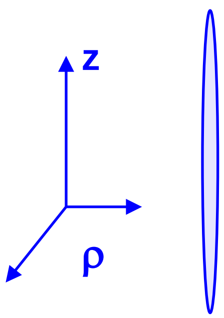


at a >0 a bound state E_B appears in $\hat{V}(r)$

$$E_B = -\frac{\hbar^2}{2\mu a^2}$$

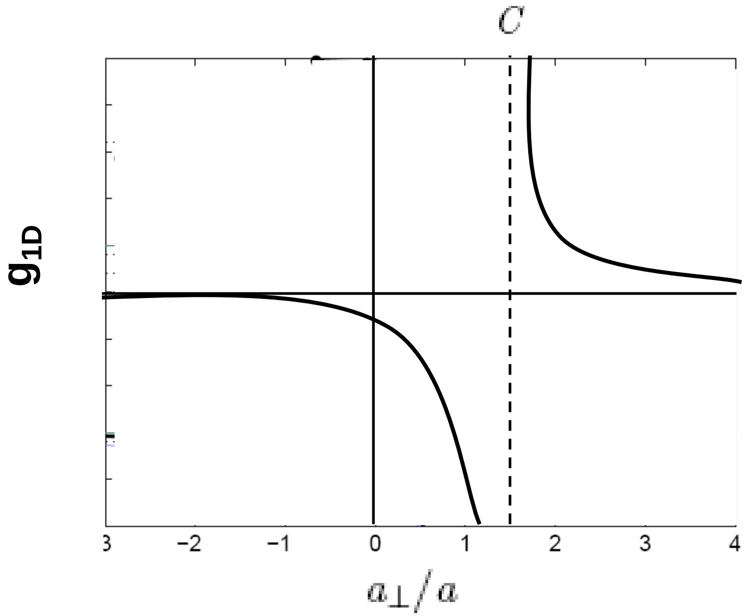
Pseudopotential approximation in quasi-1D

("zero-range" potentials Yu.N.Demcov & V.N.Ostrovskii)



$$H_{1D} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + g_{1D}\delta(z)$$

$$g_{1D} = \frac{2\hbar^2 a}{\mu a_\perp^2}\frac{1}{(1 - Ca/a_\perp)}$$



M.Olshanii,Phys.Rev.Lett. 81(1998)938

$$\hat{H} = \hat{H}_z + \hat{H}_\perp + \hat{V}$$

$$\hat{H}_z = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2}; \quad \hat{V}(\mathbf{r}) = \frac{2\pi\hbar^2 a}{\mu}\delta^3(\mathbf{r})$$

$$\hat{H}_\perp = -\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}\right] + \frac{\mu}{2}\omega_\perp^2\rho^2$$

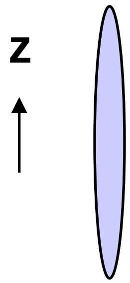
$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$$a_\perp / a = C \qquad \textbf{CIR}$$

$$C = 1.46\ldots$$

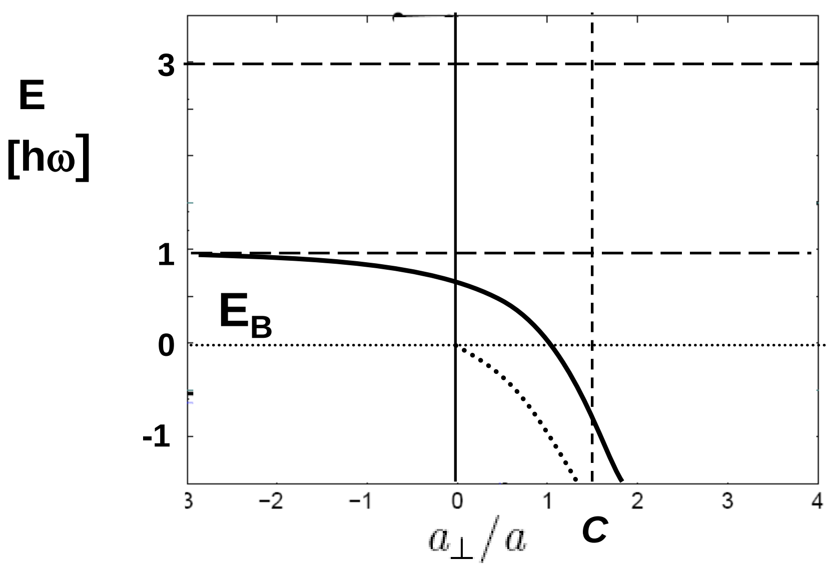
Pseudopotential approximation in quasi-1D

M.Olshanii,Phys.Rev.Lett. 81(1998)938



quasi-1D harmonic trap

$$H_{1D} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + g_{1D}\delta(z)$$

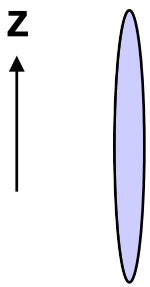


bound state E_B is shifted by the trap to $\hbar\omega$

bound state E_B exists for any a in quasi-1D trap !!!

Pseudopotential approximation in quasi-1D

M.Olshanii,Phys.Rev.Lett. 81(1998)938



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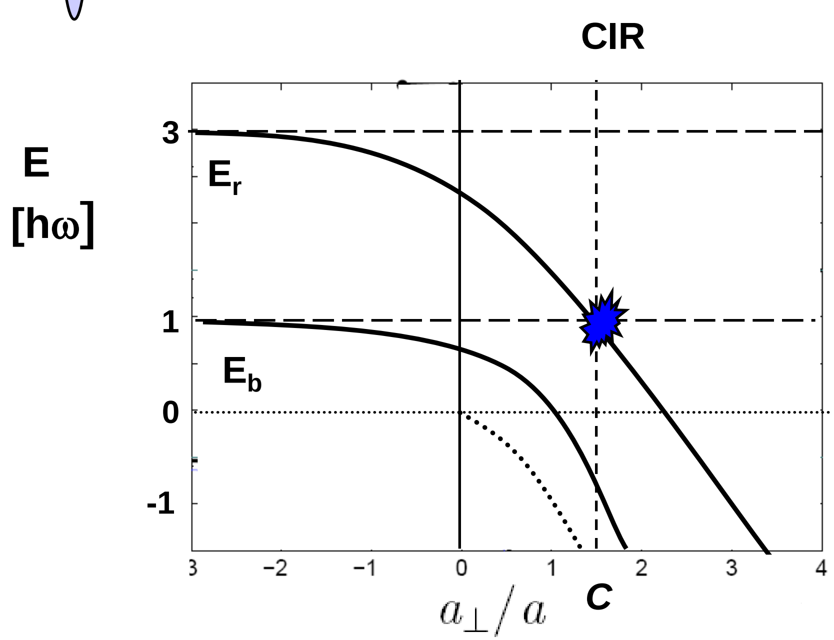
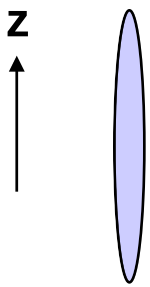


E_r resonance state in closed channel (first excited state in trap)

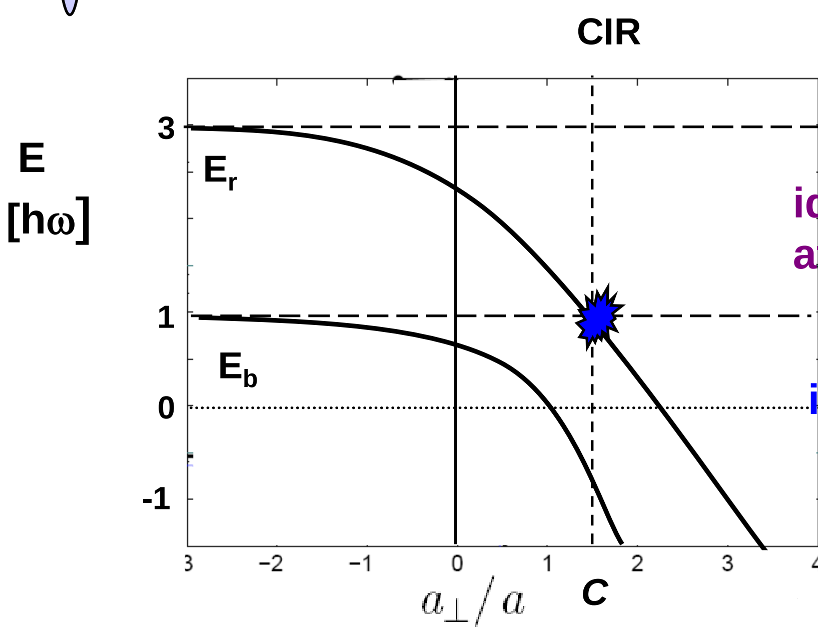
crosses threshold at poir$a_\perp / a = C$

Pseudopotential approximation in quasi-1D (gas of impenetrable bosons at $a_\perp / a = C$)

$$g_{1\mathrm{D}} = -\frac{\hbar^2}{\mu a_{1\mathrm{D}}} = \frac{2\hbar^2 a}{\mu a_\perp^2}\frac{1}{(1 - Ca/a_\perp)} \underset{k\to 0}{=} \frac{\hbar^2 k\ Re\{f_0^+\}}{\mu\ Im\{f_0^+\}}$$



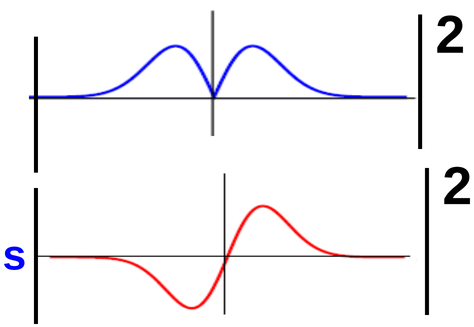
$$\text{CIR:}\quad g_{1D} \to \pm\infty \quad a_{1D} \to 0 \qquad f_0^+ = -\frac{1}{1+ika_{1D}} \to -1$$



$$T = |\ 1 + f_0^+\ |^2 \to 0 \quad !!$$

identical bosons at $g_{1\mathrm{D}} \to \pm\infty$

identical fermions



impenetrable bosons (Tonks&Girardeau 1960)

M.D. Girardeau, PRA 82, 011607(R) (2010)

4D quant. two-body problem in 1D harmonic trap

$$i\frac{\partial}{\partial t}\psi(\rho_R,\mathbf{r},t) = H(\rho_R,\mathbf{r})\psi(\rho_R,\mathbf{r},t)$$

4D : transformed Hamiltonian

$$H(\rho_R,\mathbf{r}) = H_{CM}(\rho_R) + H_{rel}(\mathbf{r}) + W(\rho_R,\mathbf{r}) \ .$$

$$H_{CM} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{\rho_R^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{4\rho_R^2}\right) + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$H_{rel} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + V(r)$$

2D if $\omega_1=\omega_2$ $\quad W(\rho_R,\mathbf{r}) = \mu(\omega_1^2 - \omega_2^2) r\rho_R \sin\theta\cos\phi$ ← **: CM coupling**

$$\psi(\rho_R,\mathbf{r},t=0) = Nr\sqrt{\rho_R}\exp\left\{-\frac{\rho_1^2}{2a_1^2} - \frac{\rho_2^2}{2a_2^2} - \frac{(z-z_0)^2}{2a_z^2} + ik_0 z\right\}$$

Wave-packet propagation method

V.S.Melezhik Phys.Lett:A230, 203(1997)

V.S.Melezhik and D.Baye Phys.Rev.C59, 3232(1999)

V.S.Melezhik, J.I.Kim and P.Schmelcher Phys.Rev. A76, 053611(2007)

Quantum two-body problem in 1D harmonic trap

- 2D: non-separable quantum two-body problem ($m_a=m_b$)
- 4D: center-of-mass non-separability ($m_a \neq m_b$)
- Multichannel scattering in 1D trap: transverse excitation/deexcitation
- 3D scattering problem: anisotropic interatomic interaction
- 5D: anharmonisity → center-of-mass non-separability ($m_a=m_b$)
- …

Multichannel atomic scattering and confinement-induced resonances in waveguides

S.Saeidian,V.Melezhik, P.Schmelcher, Phys.Rev. A77, 042721 (2008)

transverse excitations and resonant atomic processes in waveguides were analyzed for the first time

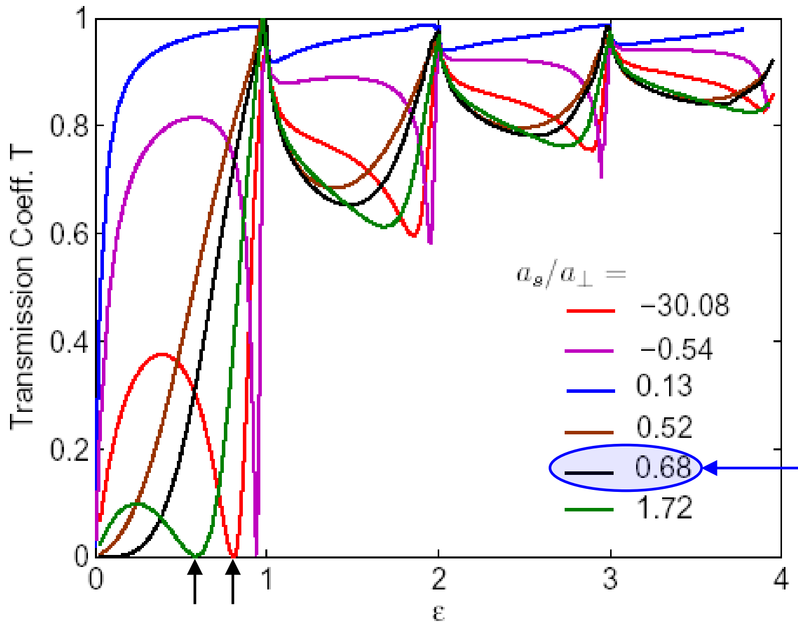
a_s - atom-atom scattering length

ω laser frequency defines:

$a_\perp = \sqrt{\frac{\hbar}{\mu\omega}}$ - transverse trap width

$\varepsilon_n = \hbar\omega(2n+1)$ - transverse excitations

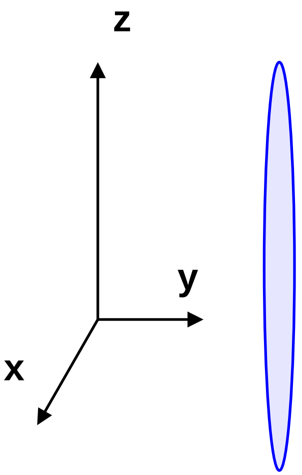
confinement-induced resonances (CIRs) were analyzed for non-zero colliding energy



zero-energy CIR (at E→hω)

reduced interatomic colliding energy $\varepsilon = \frac{E}{2\hbar\omega} - \frac{1}{2}$

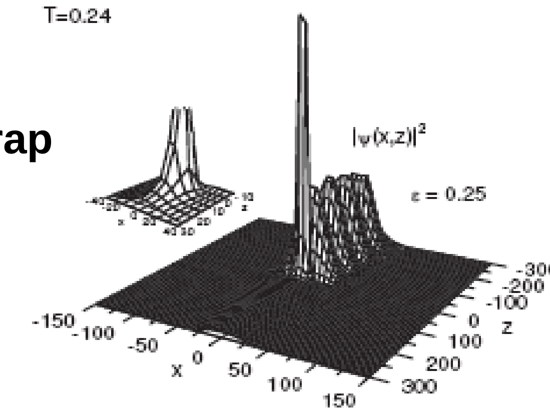
Interpretation of CIRs



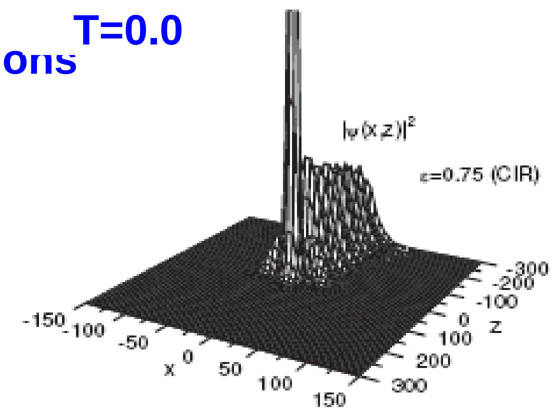
atom-atom collision in quasi-1D harmonic trap

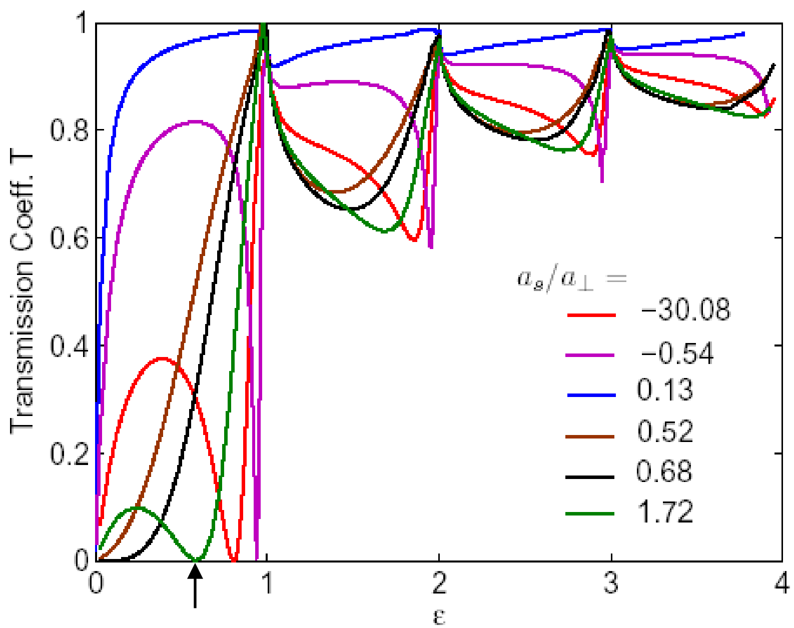
Ψ(x,z) – calculated scattering wave function

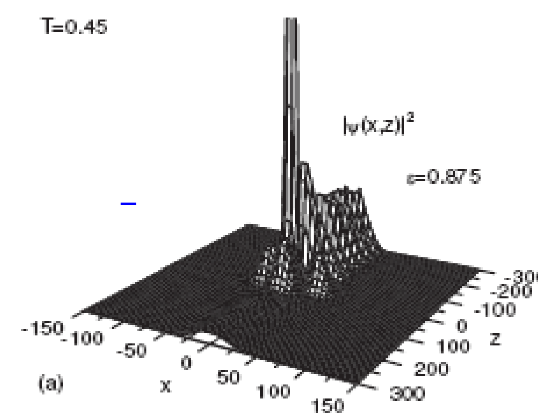
total reflection at the point of CIR:
identical bosons behave as identical fermions



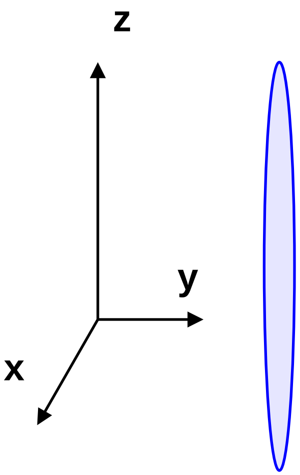
T=0.0







Interpretation of CIRs



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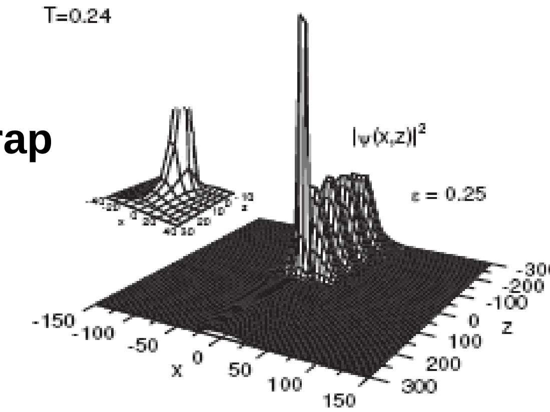
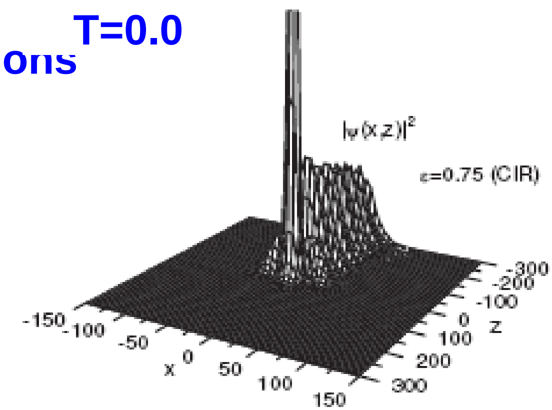
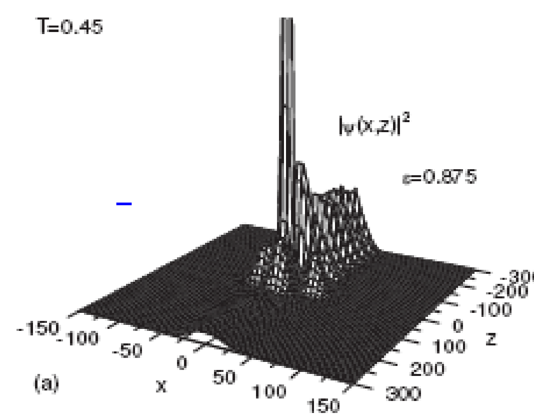
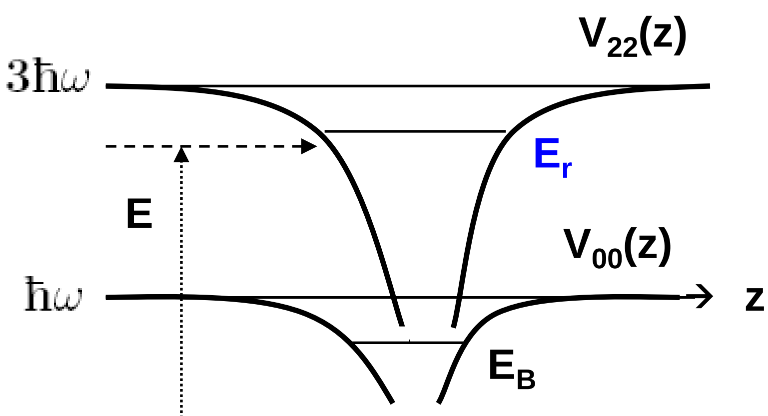


T=0.0





CIR is Feshbach-like resonance



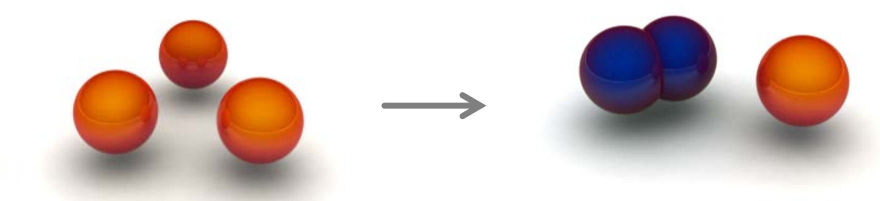
Experimental observation of CIR

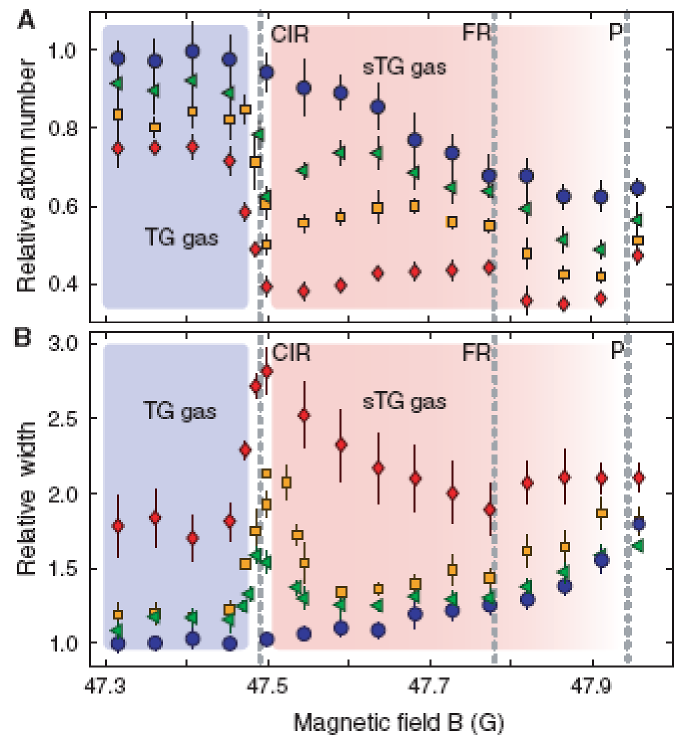
T. Kinoshita, T. Wenger, D. S. Weiss, Science 305, 1125 (2004).
B. Paredes *et al.*, Nature 429, 277 (2004).

strongly-correlated Tonks-Girardeau gas

E. Haller *et al.*, Science 325, 1224 (2009)

Detection of the CIR by an increase of three-body loss



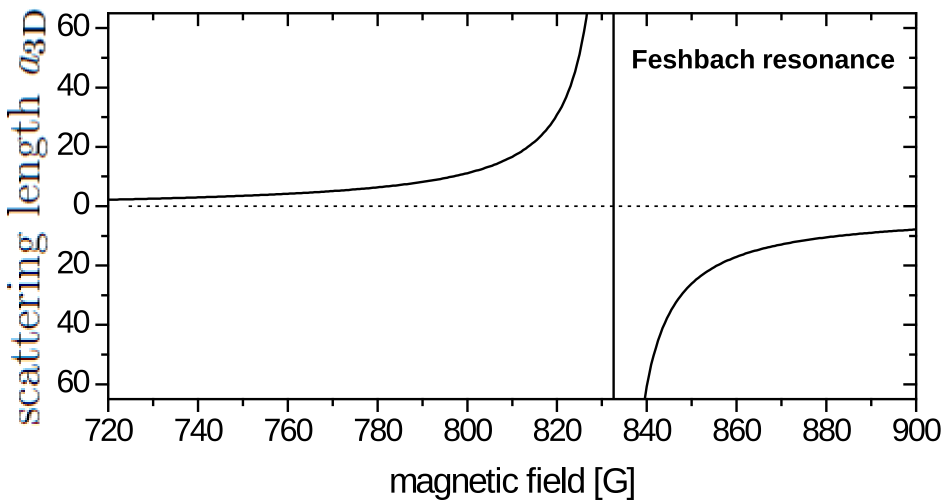


in agreement with predicted CIR position

$$\frac{a_{3D}}{a_{\perp}} \approx 1$$

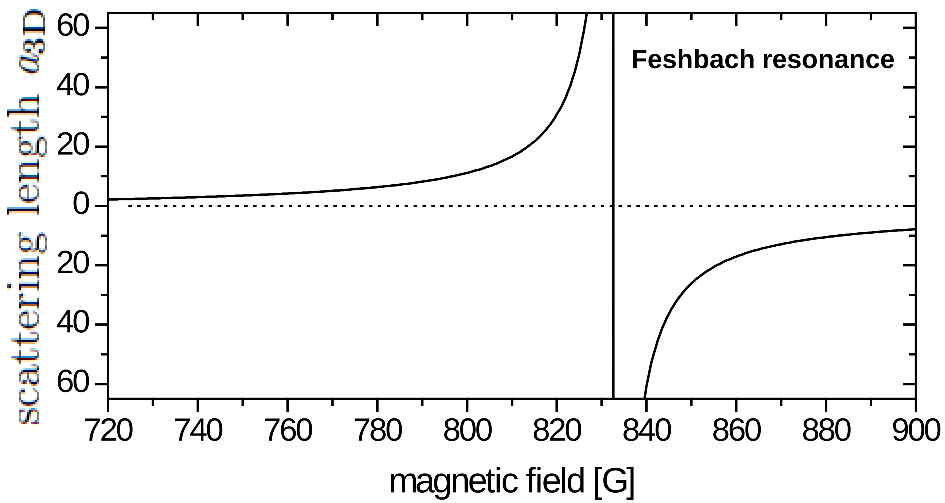
Tuning the interaction in 3D

3D



Tuning the interaction in 3D

3D

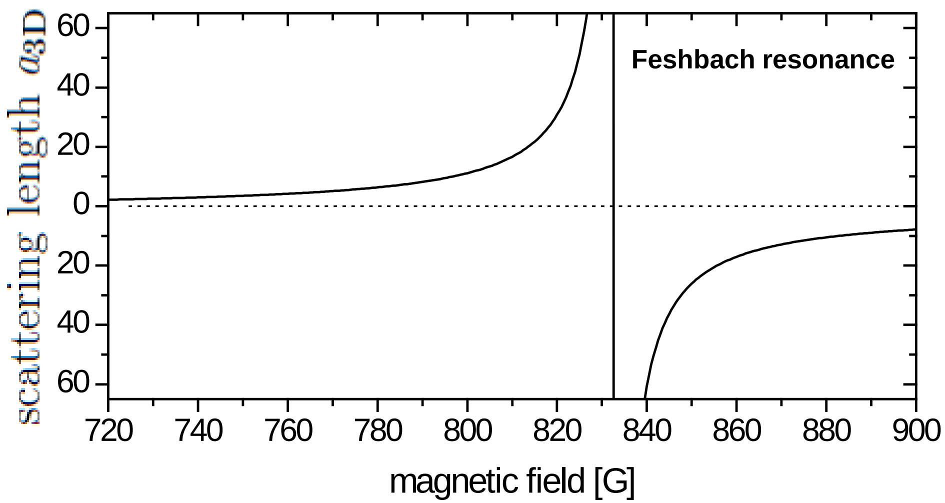


single-channel pseudopotential

$$\frac{2\pi\hbar^2 a_{3D}(\mathbf{B})}{\mu}\delta(\mathbf{r})$$

Tuning the interaction in 1D: B and ω

3D

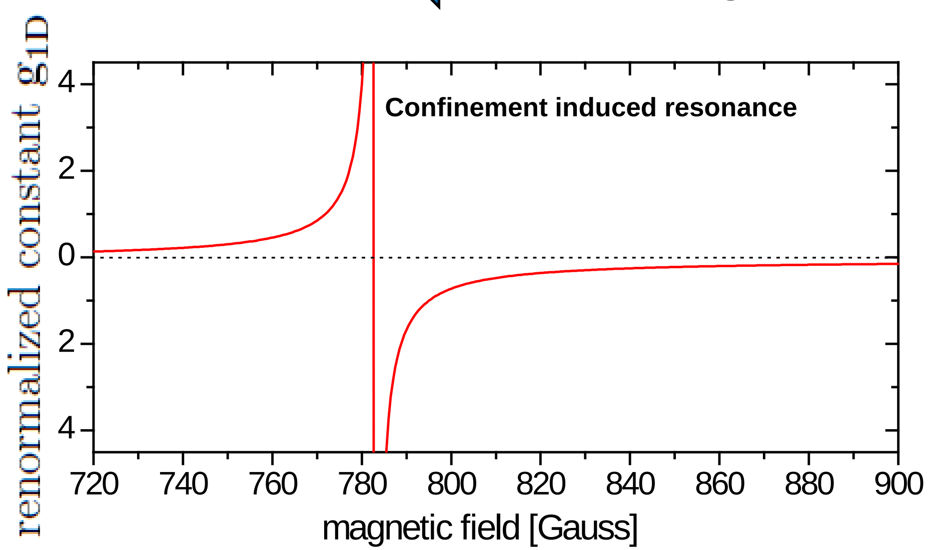


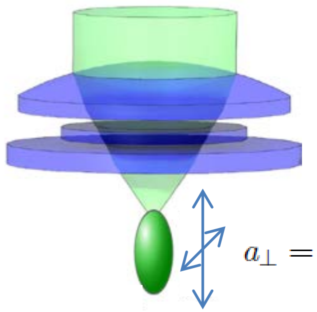
single-channel pseudopotential

$$\frac{2\pi\hbar^2 a_{3D}(B)}{\mu}\delta(\mathbf{r})$$

strong confinement

1D





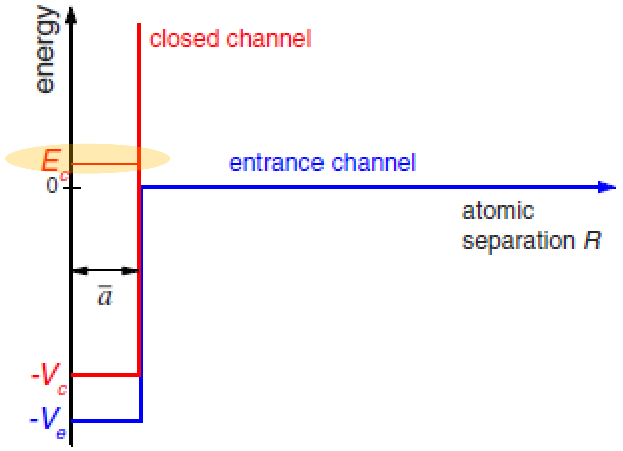
$$a_\perp = \sqrt{\hbar/(\mu\omega)}$$

single-channel pseudopotential with renormalized interaction constant

$$g_{1D} = \frac{2\hbar^2 a_{3D}(B)}{\mu a_\perp^2}\frac{1}{1 - C a_{3D}/a_\perp}$$

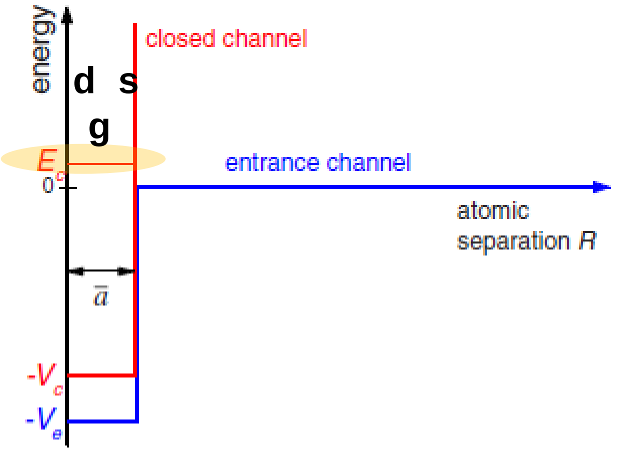
M. Olshanii, PRL 81, 938 (1998).

two-channel problem

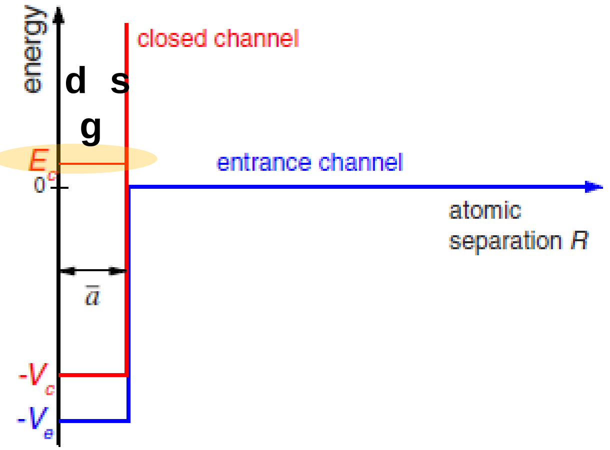


two-channel problem

tensorial sructure of molecular state



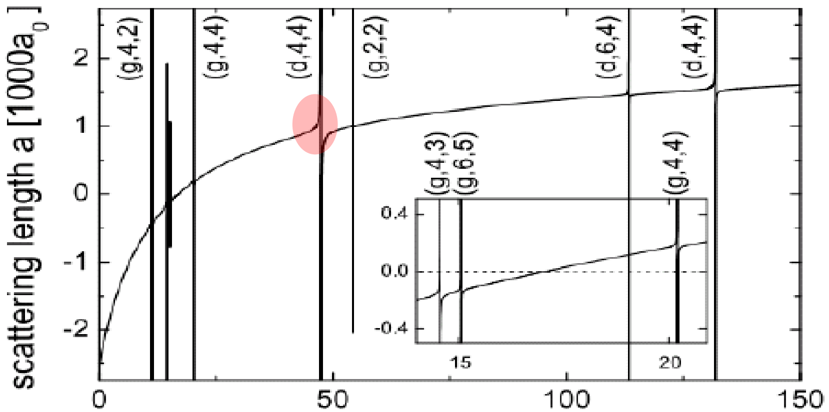
two-channel problem



tensorial sructure of molecular state

Innsbruck experiment with Cs atoms:

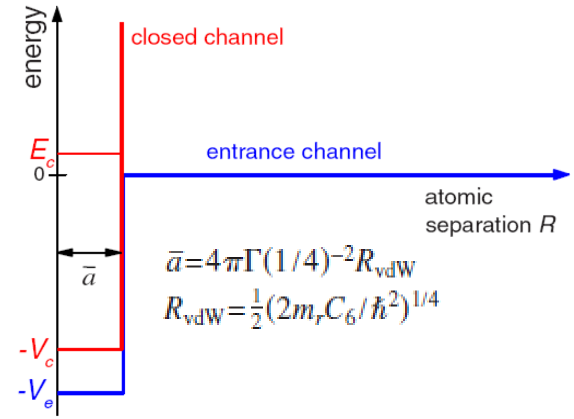
Feshbach Resonanzen



two-channel model of Lange et. al. Phys.Rev.79,013622(2009)

$$\hat{H}(r) = -\frac{\hbar^2}{2\mu}\hat{I}\frac{d^2}{d^2 r} + \hat{V}(r)$$

$$\hat{V} = \begin{bmatrix} -V_c & \hbar\Omega \\ \hbar\Omega & -V_e \end{bmatrix} \quad (\text{for } R < \bar{a}) = \begin{bmatrix} \infty & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{for } R > \bar{a})$$



3 fitting parameters:

$$\frac{1}{a-\bar{a}} = \frac{1}{a_{\rm bg}-\bar{a}} + \frac{\Gamma/2}{\bar{a}E_c}$$

$$E_c = \delta\mu(B - B_c)$$

$$\Gamma = \delta\mu\Delta$$

→ $V_e \; V_c \; \Omega$

TABLE I. Fitting parameters for the s-, d-, and g-wave Feshbach resonances, determining the scattering length in the magnetic-field range of interest; see Fig. 3. The background scattering length $a_{\rm bg}=1875a_0$, the mean scattering length of cesium, $\bar{a}=95.7a_B$, and the bare s-wave state magnetic moment $\delta\mu_1=2.50\mu_B$ [28] are set constant. Poles $B_{0,i}$ and zeros B_i^* of the scattering length are derived; see text. Uncertainties in the parentheses are statistical. The systematic uncertainty of the magnetic field is 10 mG.

Res.	Γ_i/h (MHz)	$\delta\mu_i/\mu_B$	$B_{c,i}$ (G)	$B_{0,i}$ (G)	B_i^* (G)
s-wv.	11.6(3)	2.50	19.7(2)	−11.1(6)	18.1(6)
d-wv.	0.065(3)	1.15(2)	47.962(5)	47.78(1)	47.944(5)
g-wv.	0.0042(6)	1.5(1)	53.458(3)	53.449(3)	53.457(3)

extension of two-channel model of Lange et. al. to 1D geometry

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)

A four-channel square-well potential

$$\hat{V} = \begin{pmatrix} -V_{c,3} & 0 & 0 & \hbar\Omega_3 \\ 0 & -V_{c,2} & 0 & \hbar\Omega_2 \\ 0 & 0 & -V_{c,1} & \hbar\Omega_1 \\ \hbar\Omega_3 & \hbar\Omega_2 & \hbar\Omega_1 & -V_e \end{pmatrix}$$

$$|\psi\rangle = \sum_\alpha \psi_\alpha(\mathbf{r})|\alpha\rangle = \sum_\alpha \phi_\alpha(r) Y_{l_\alpha 0}(\hat{r})|\alpha\rangle$$

$\omega_\perp = 0$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l_\alpha(l_\alpha+1)}{2\mu r^2} + B_{\alpha\alpha}\right]\phi_\alpha(r) + \sum_\beta V_{\alpha\beta}(r)\phi_\beta(r) = E\phi_\alpha(r)$$

4-coupled radial equations

$$\psi_e(\mathbf{r}) \to \exp\{ikz\} + f(k,\theta)/r \,\exp\{ikr\}, \qquad \psi_{c,i}(\mathbf{r}) \to 0$$

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$$\left(\left[-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{1}{2}\mu\omega_\perp^2\rho^2\right]\hat{I} + \hat{B} + \hat{V}(r)\right)|\psi\rangle = E|\psi\rangle$$

4-coupled 2D equations in the plane $\{r, \theta\}$

$$\psi_e(\mathbf{r}) = [\cos(k_0 z) + f_e \exp\{ik_0|z|\}]\Phi_0(\rho), \quad \psi_{c,i}(\mathbf{r}) \to 0$$

$$T(B) = |1 + f_e(B)|^2$$

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scattering problem → boundary-value problem

V.Melezhik,C.Y.Hu,Phys.Rev.Lett.90(2003)083202
S.Saeidian,V.Melezhik,P.Schmelcher,Phys.Rev.A77(2008)042701

Shifts and widths of Feshbach resonances in atomic waveguides

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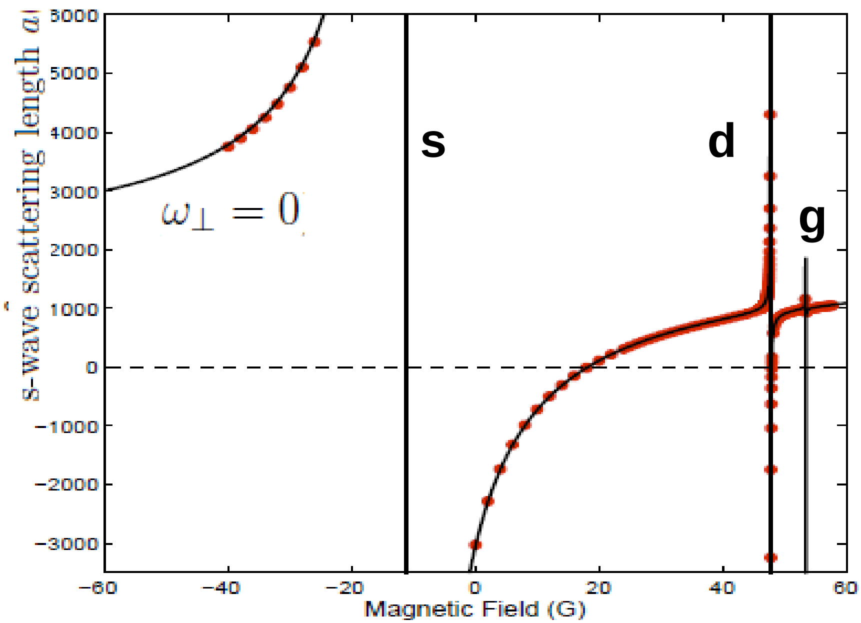

Feshbach resonances in the Cs ultracold gas in the 3D free space

$$H(r,\theta) = [-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{1}{2}\mu\omega_\perp^2\rho^2]\hat{I} + \hat{V}(r)$$

$$V = \begin{pmatrix} -V_{C_3} & 0 & 0 & \hbar\Omega_3 \\ 0 & -V_{C_2} & 0 & \hbar\Omega_2 \\ 0 & 0 & -V_{C_1} & \hbar\Omega_1 \\ \hbar\Omega_3 & \hbar\Omega_2 & \hbar\Omega_1 & -V_e \end{pmatrix}$$

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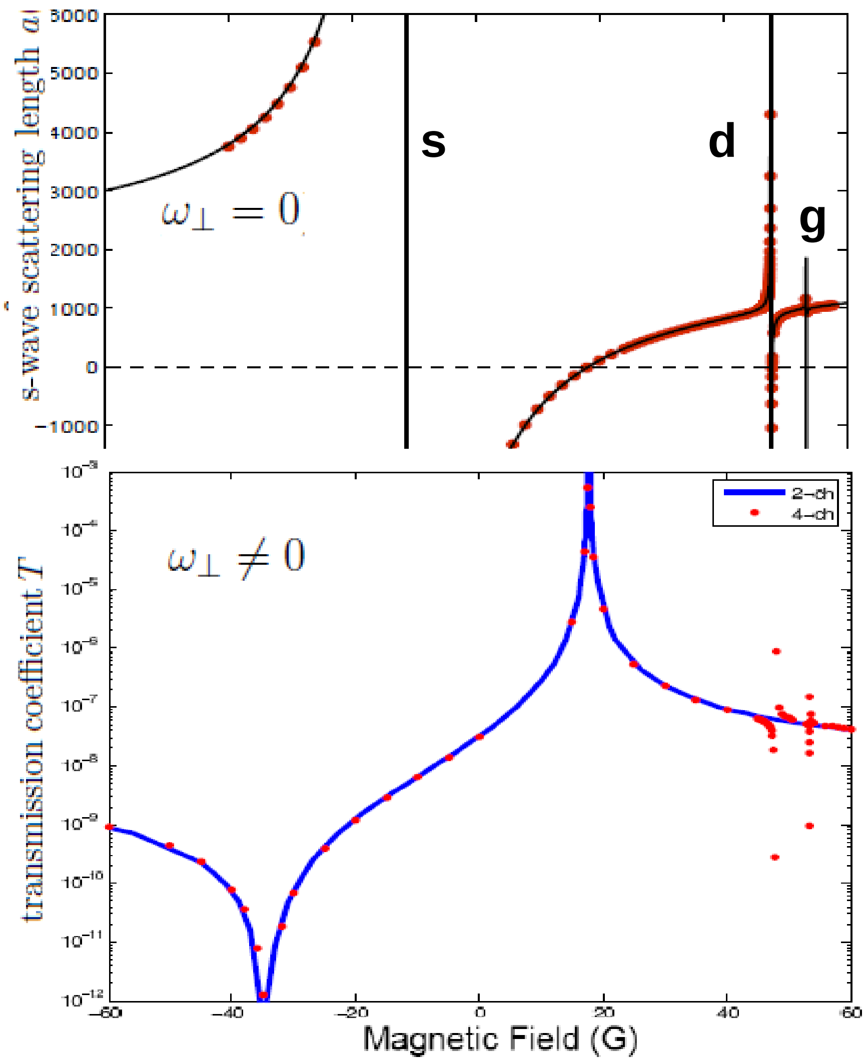

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in harmonic waveguides

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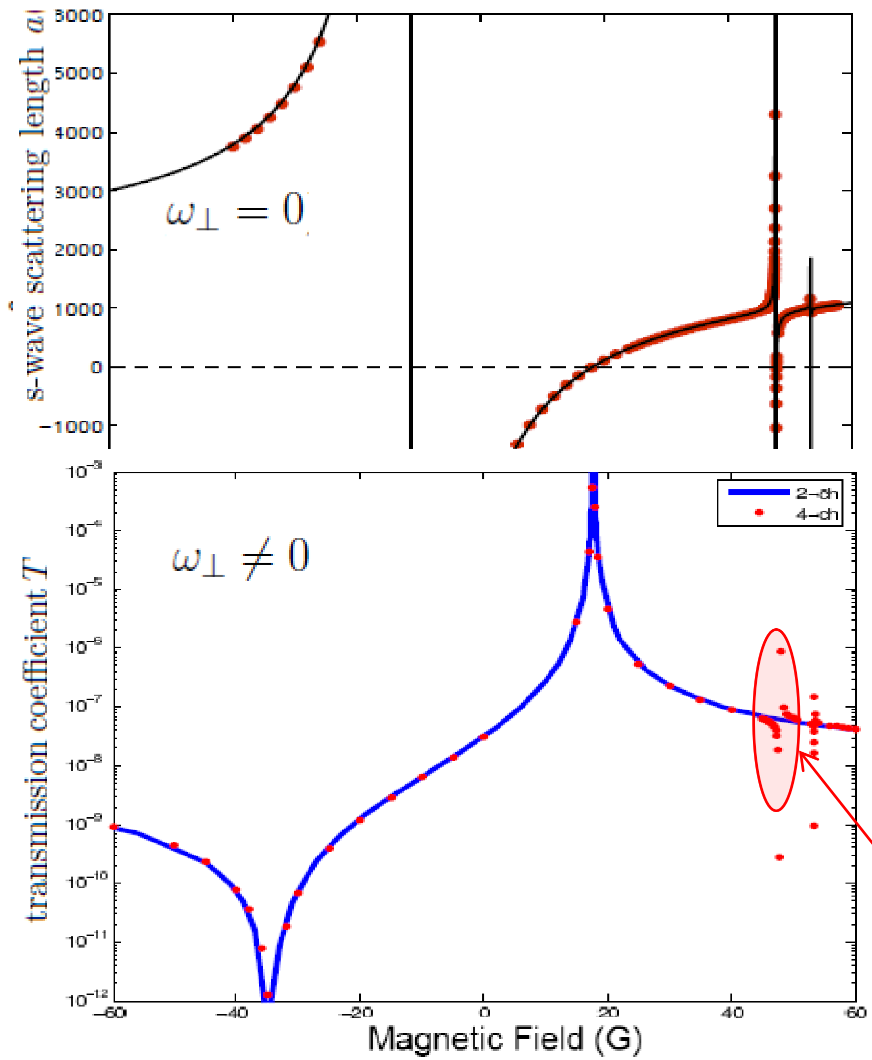

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in harmonic waveguides

region of Innsbruck experiment (d-wave Feshbach resonance)

Shifts and widths of Feshbach resonances in atomic waveguides

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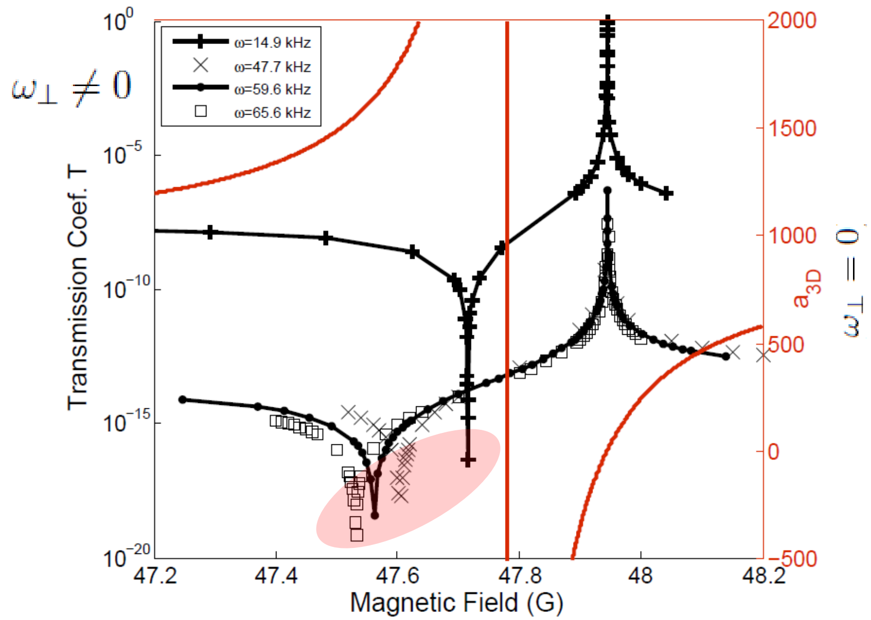


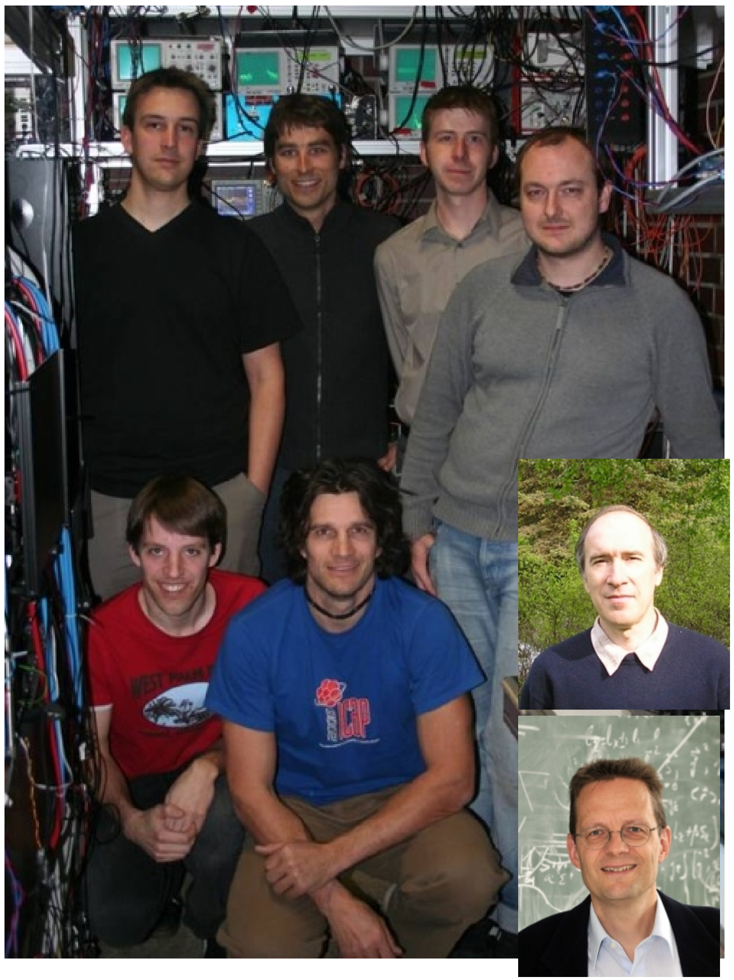
region of Innsbruck experiment
(d-wave Feshbach resonance)

PRL **104,** 153203 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

Confinement-Induced Resonances in Low-Dimensional Quantum Systems

Elmar Haller,[1] Manfred J. Mark,[1] Russell Hart,[1] Johann G. Danzl,[1] Lukas Reichsöllner,[1] Vladimir Melezhik,[2] Peter Schmelcher,[3] and Hanns-Christoph Nägerl[1]

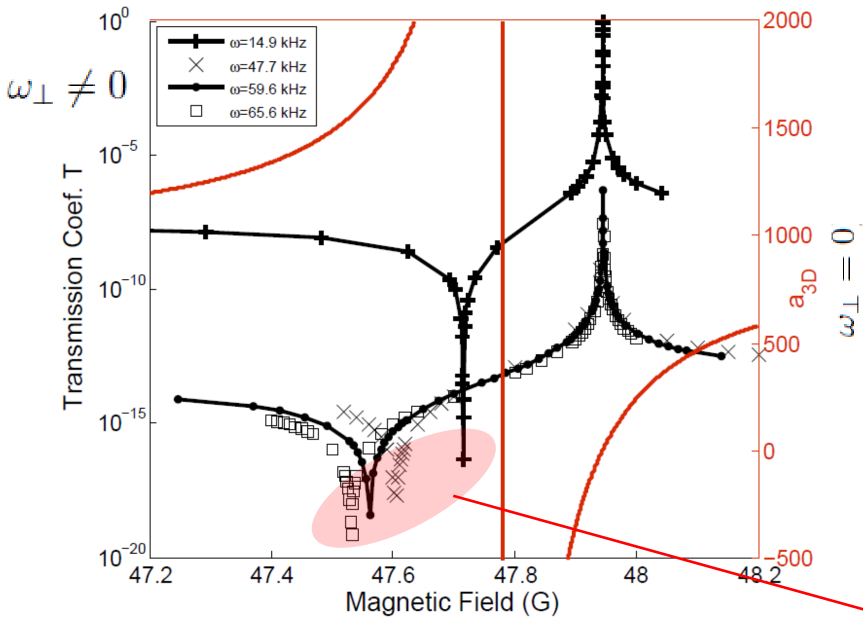
[1]*Institut für Experimentalphysik and Zentrum für Quantenphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*
[2]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980 Dubna, Russia*
[3]*Zentrum für Optische Quantentechnologien, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*
(Received 19 February 2010; published 14 April 2010)



In experiment performed in Innsbruck in collaboration with theoreticians from JINR and Hamburg, properties of ultracold Cs were studied by measuringthe atom loss in a 2D lattice formed by two retro-reflected laser beams

Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)



region of Innsbruck experiment
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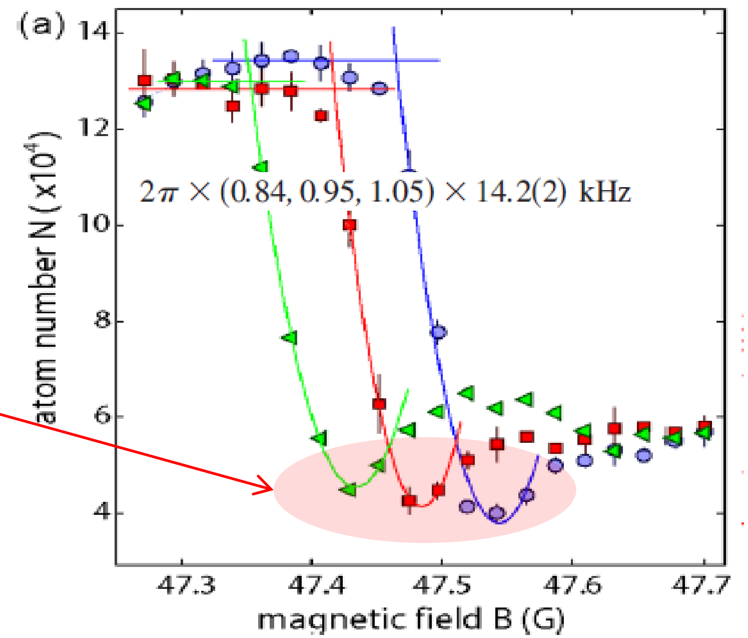
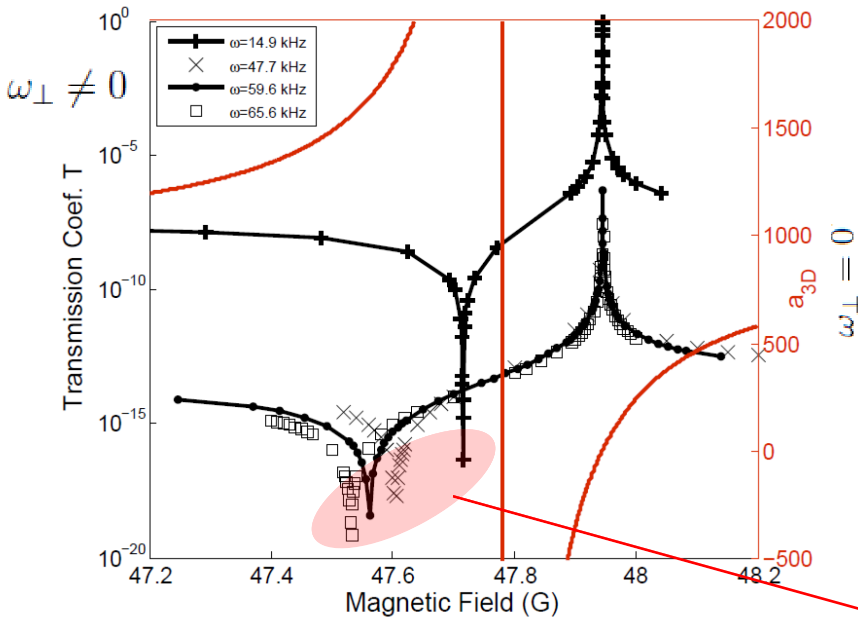


experiment:
E.Haller et.al. Phys.Rev.Lett.104, 153203 (2010)

Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)



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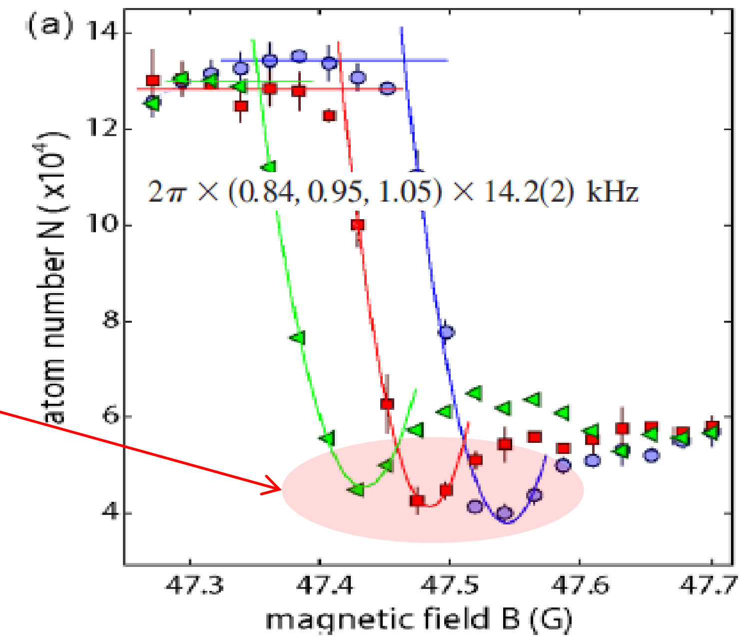

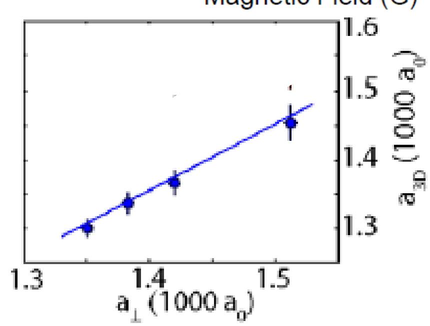

$$a_{3D} = a_{3D}(B)$$

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experiment:
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our multi-channel theory coincides with single-channel theory of M.Olshanii, Phys.Rev.Lett.81,938 (1998) : $a_{3D} = 0.64 a_\perp$

Shifts and widths of Feshbach resonances in atomic waveguides

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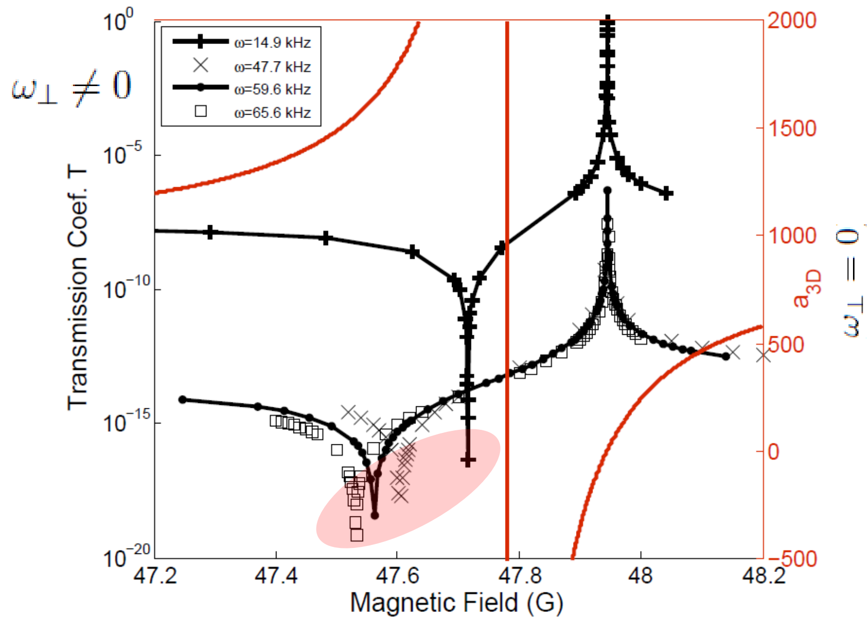

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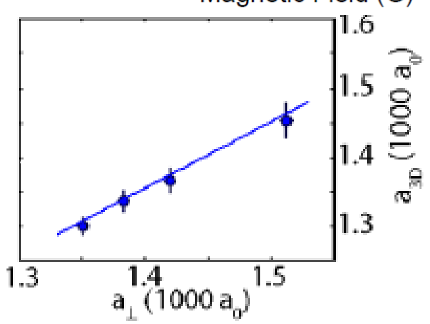

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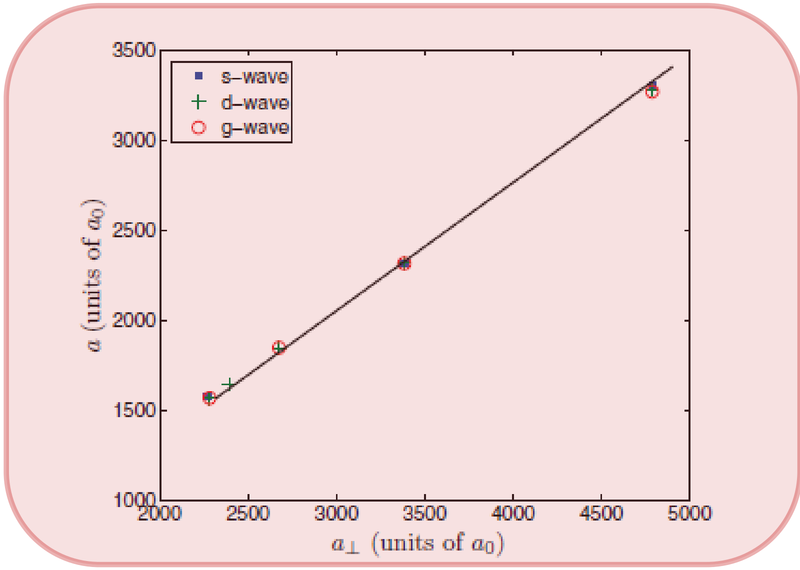

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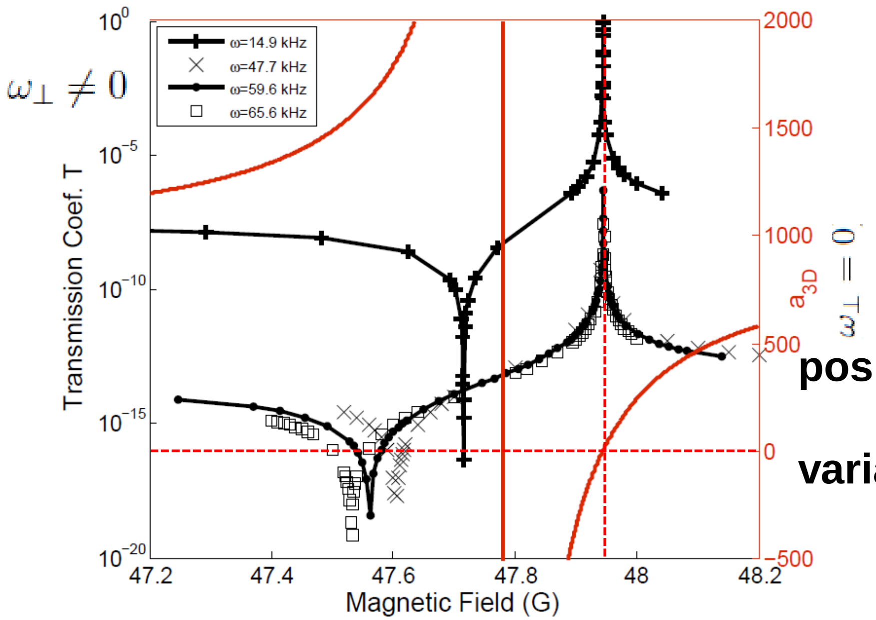

region of Innsbruck experiment
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position of T_{max} is stable with respect to variation of $\omega_\perp$ and coincides with $a_{3D} = 0$

Shifts and widths of Feshbach resonances in atomic waveguides

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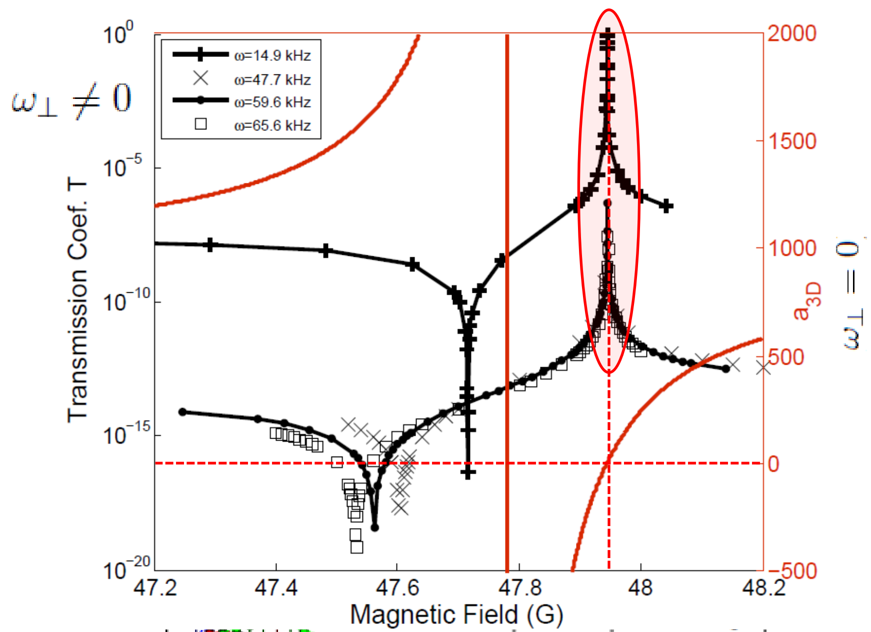

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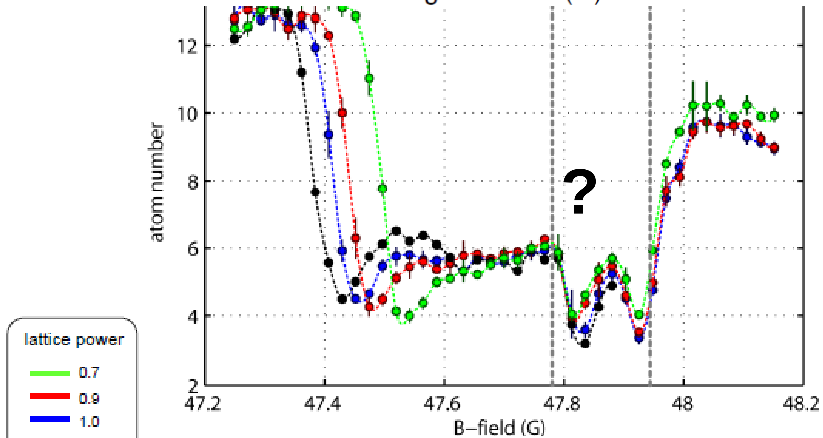

Innsbruck data, E.Haller (unpublished)

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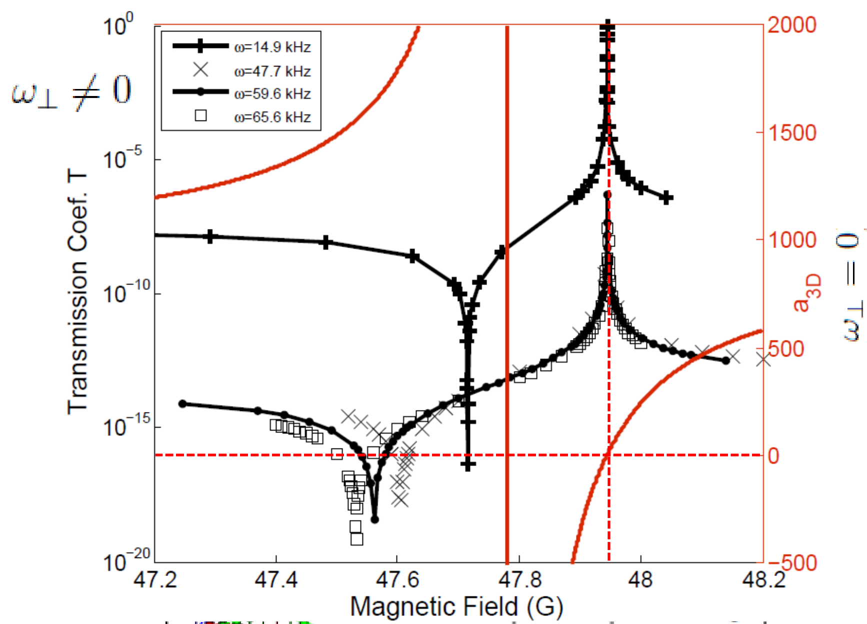

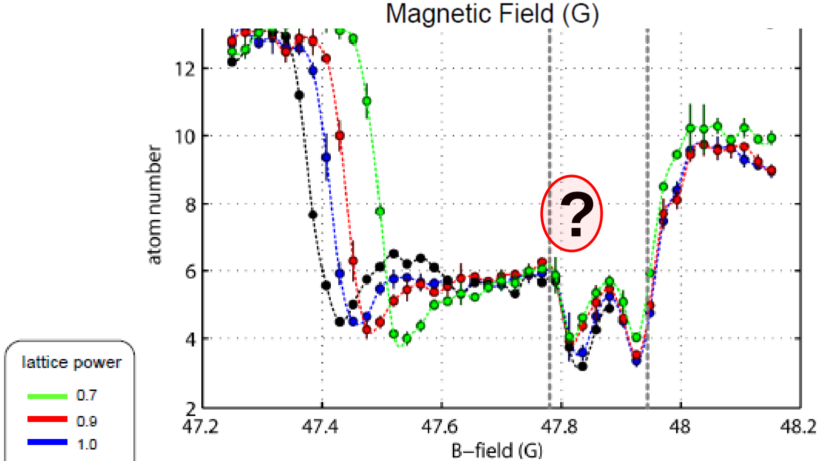

region of Innsbruck experiment

(d-wave Feshbach resonance)

?

1) **d-wave shape resonance**

2) **Efimov like resonance (3 body) :**

Shifts and widths of Feshbach resonances in atomic waveguides

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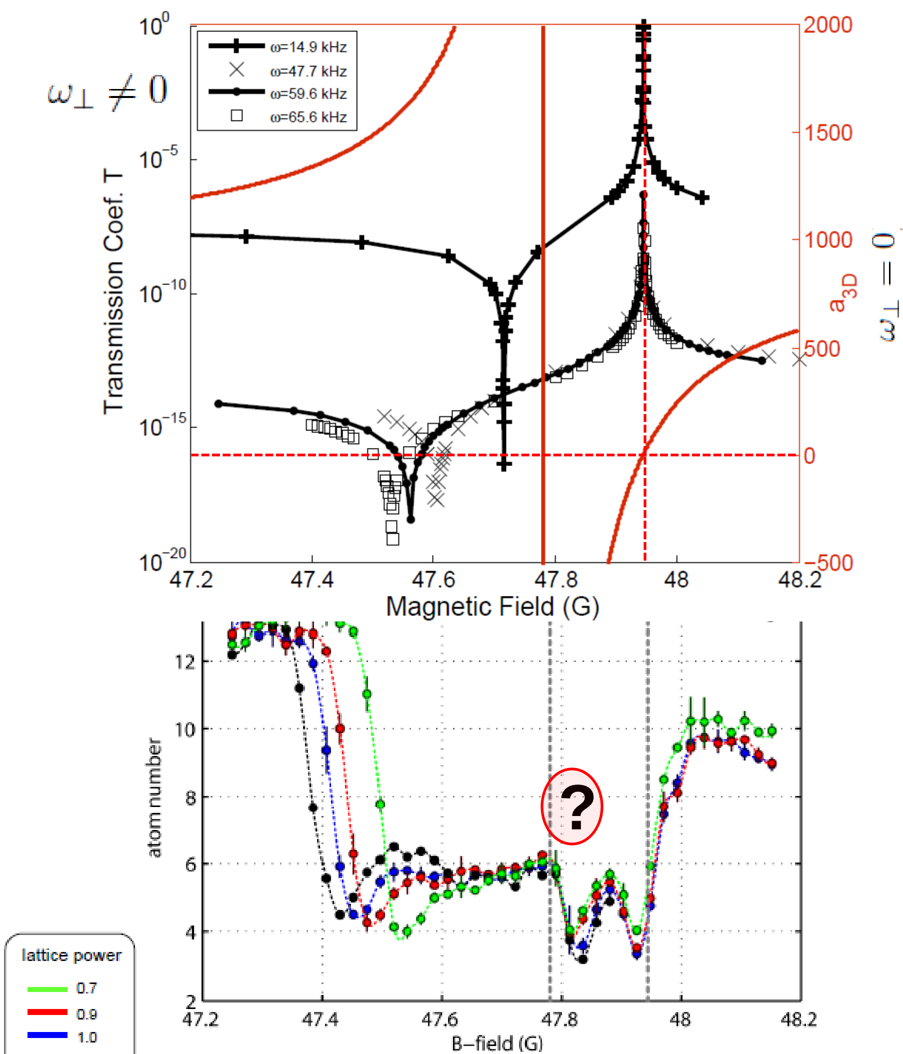

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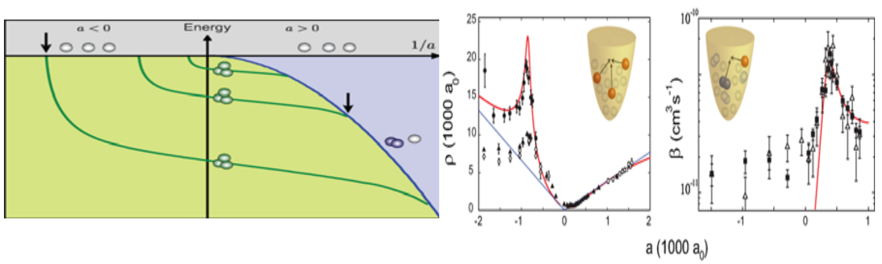
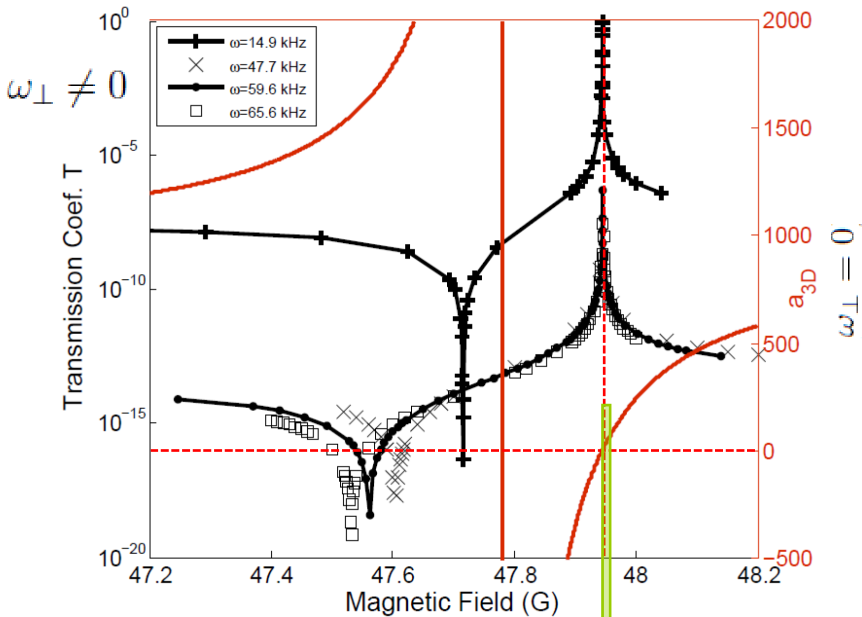


Shifts and widths of Feshbach resonances in atomic waveguides

Sh.Saeidian, V.S. Melezhik ,and P.Schmelcher, Phys.Rev. A86, 062713 (2012)



region of Innsbruck experiment
(d-wave Feshbach resonance)

$$\Gamma_i^* = \Delta_i \frac{\sqrt{2E_\parallel}}{\sqrt{\mu} a_{bg} \omega_\perp \gamma_i}$$

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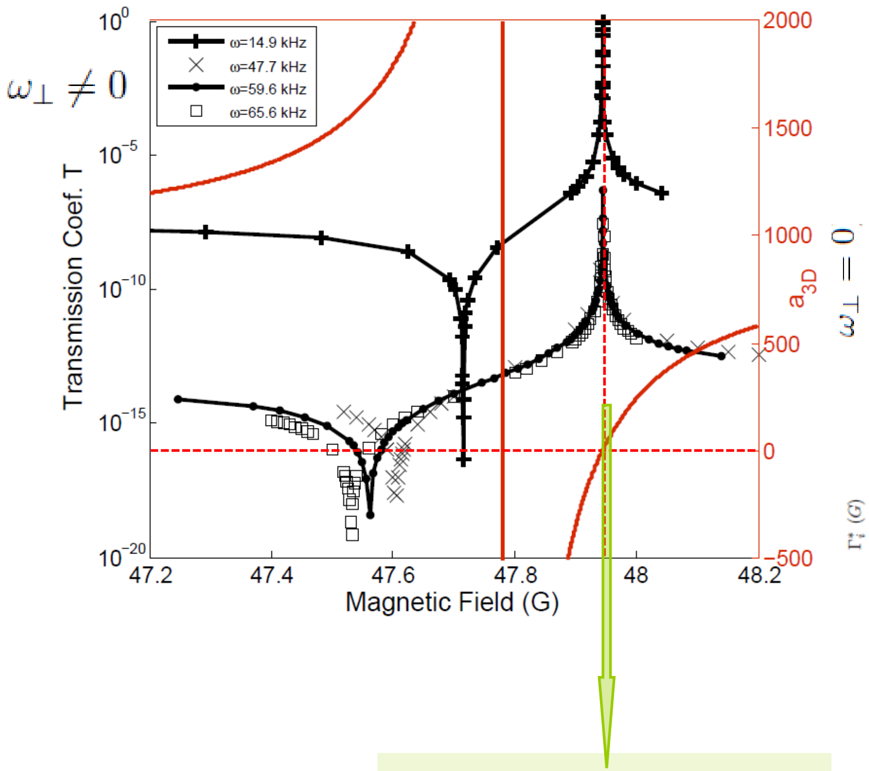

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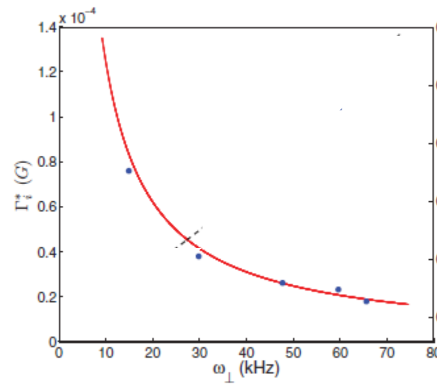

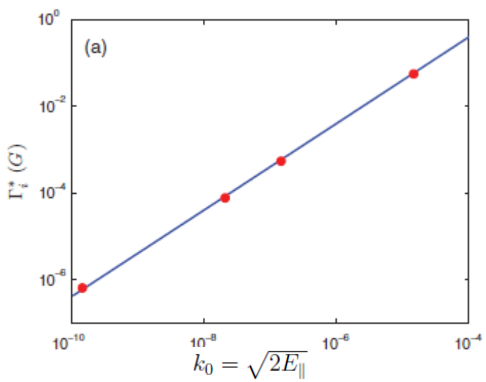

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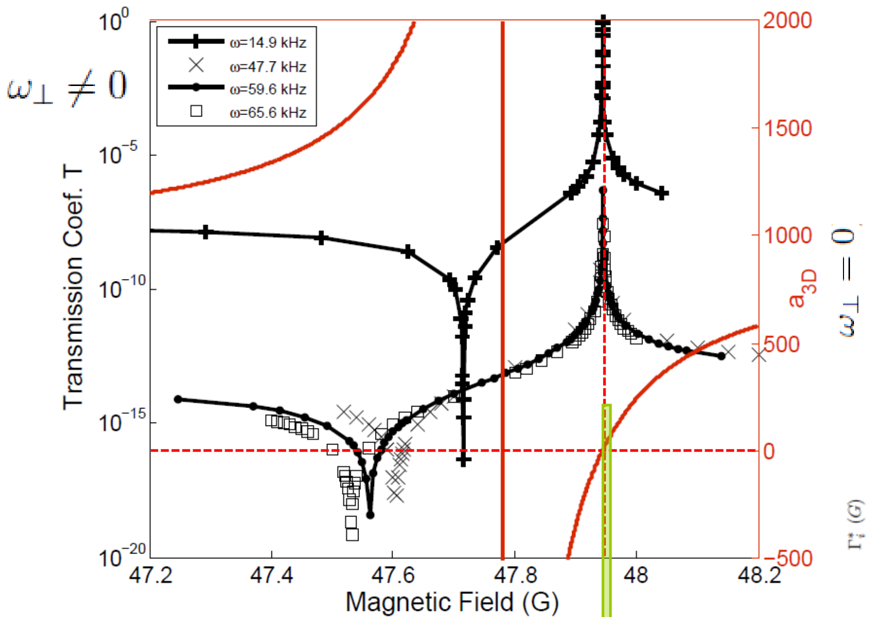

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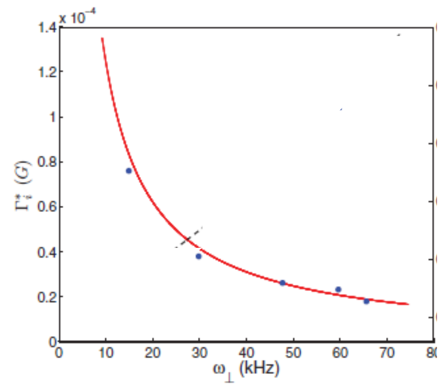
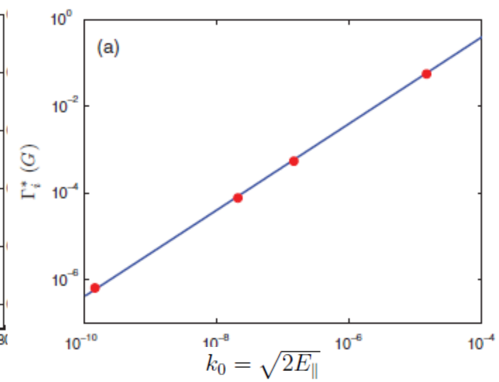




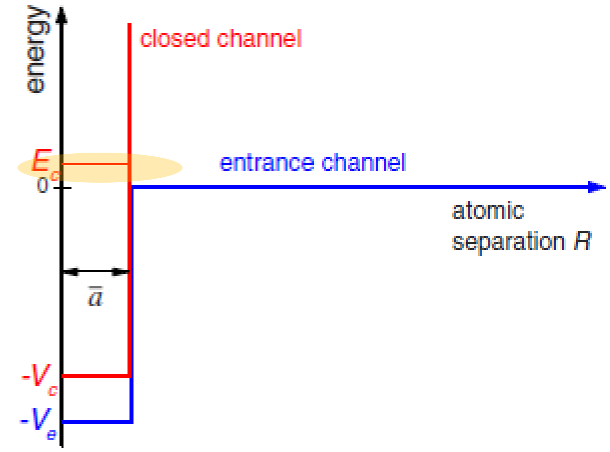
$$\Gamma_i^* = \Delta_i \frac{\sqrt{2E_\parallel}}{\sqrt{\mu} a_{bg} \omega_\perp \gamma_i}$$

narrowing width Γ^* with increasing $\omega_\perp$

$\Gamma^* \rightarrow$ longitudinal temperature $E_\parallel \sim k_B T$

Population of molecular states in free space and in the waveguide

$$P_i = 2\pi \int_0^{\infty} \int_0^{\pi} |\psi_{c,i}(r,\theta)|^2 r^2 dr \ \sin\theta \, d\theta$$



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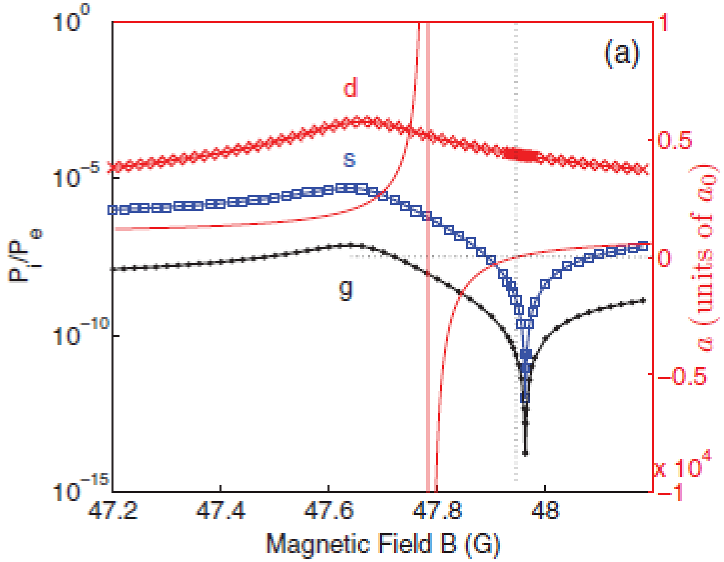

free space ($\omega_\perp = 0$)

d-wave FR at 47.8G

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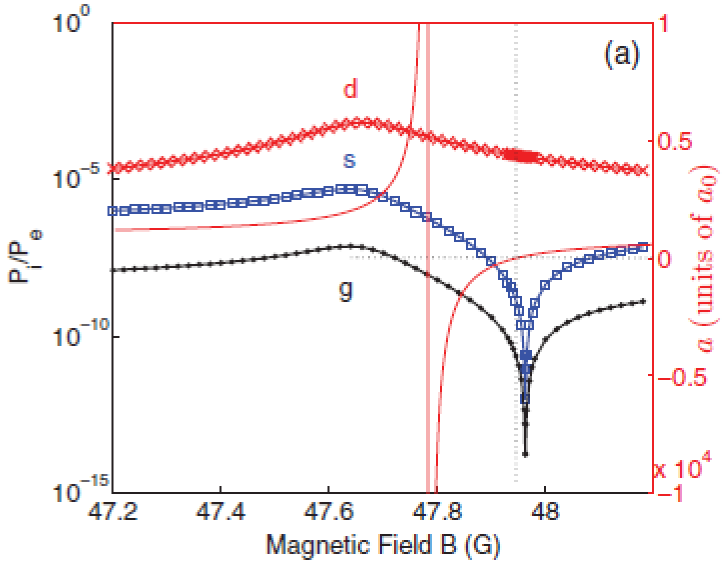

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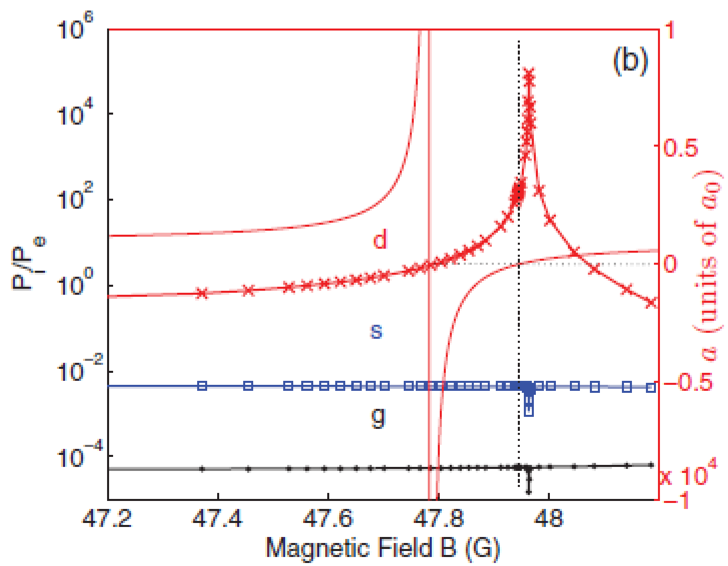

$\omega_\perp = 59.6$ kHz

Population of molecular states in free space and in the waveguide

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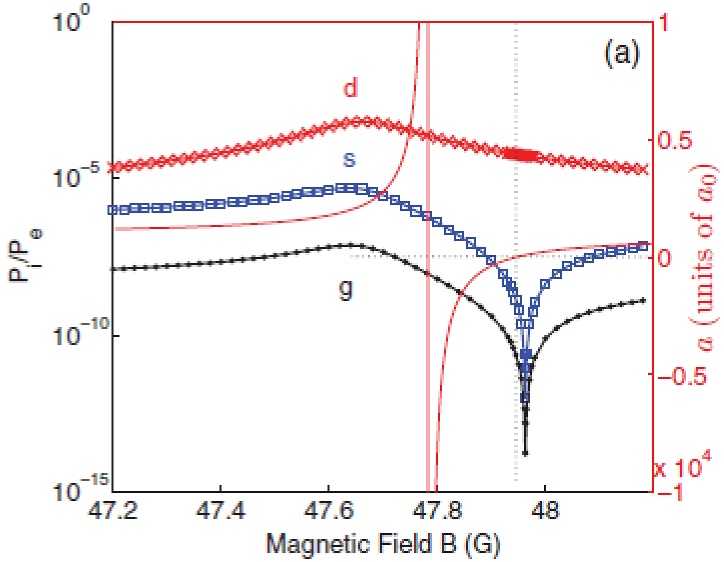

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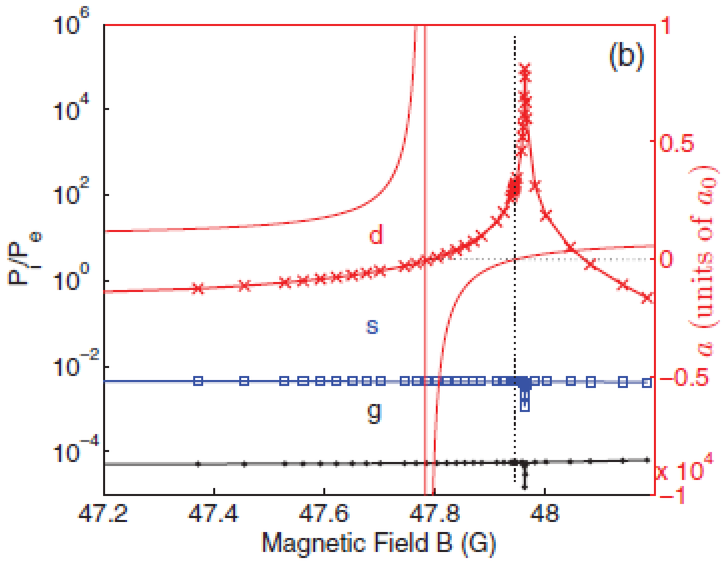

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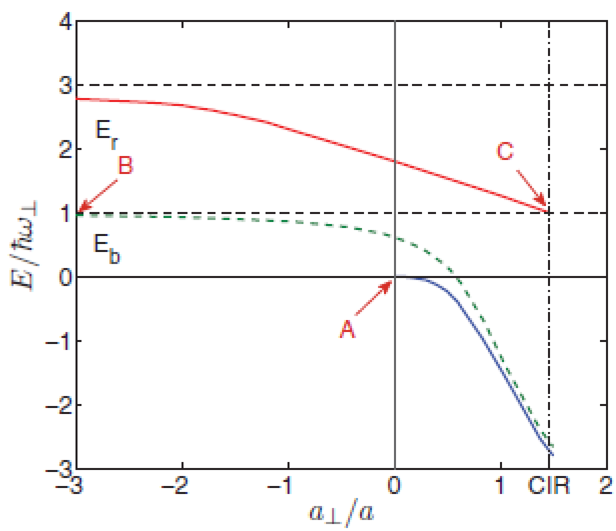

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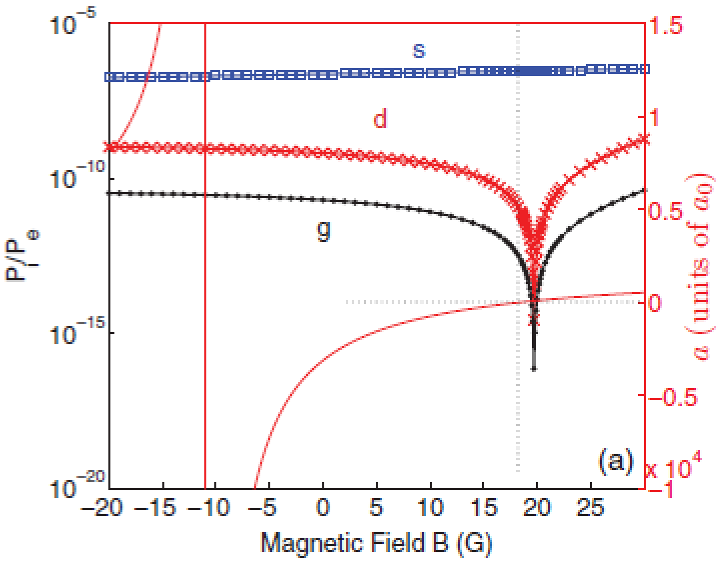
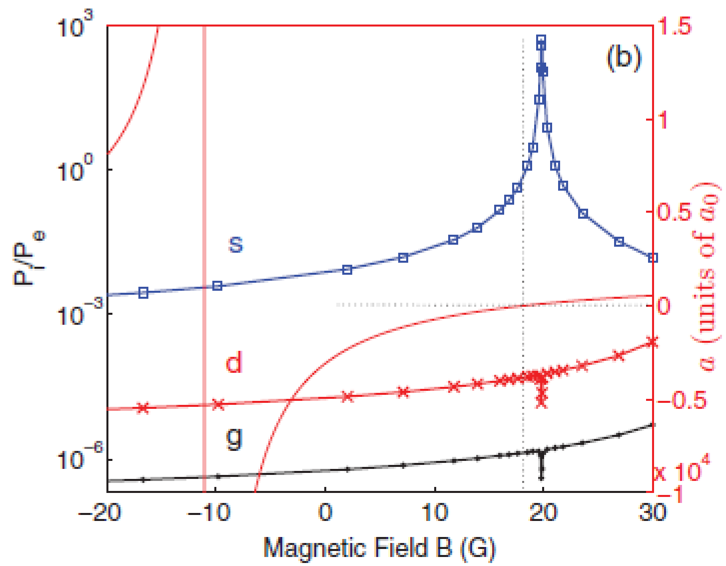
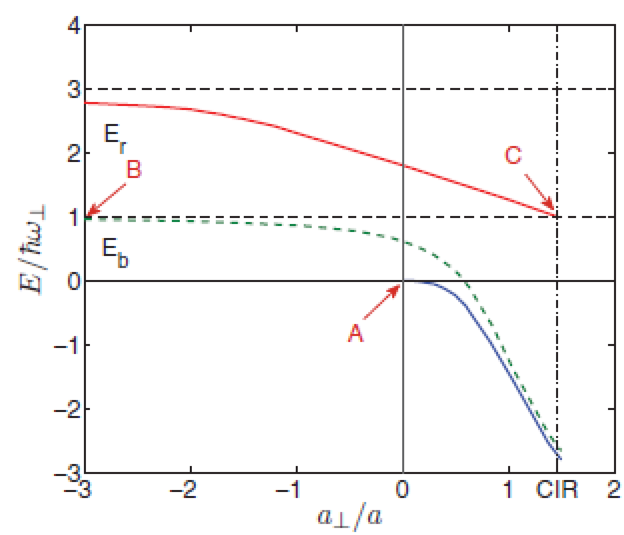


free space ($\omega_\perp = 0$)

broad s-wave FR at -12G



$\omega_\perp = 59.6$ kHz



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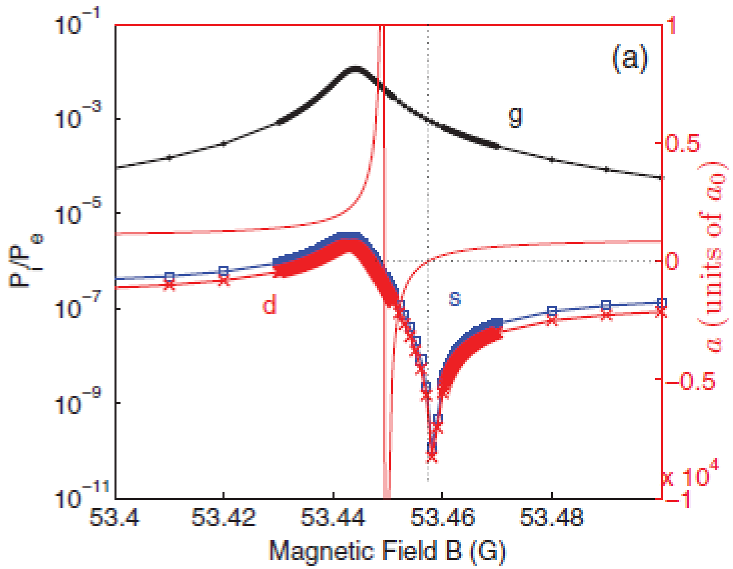
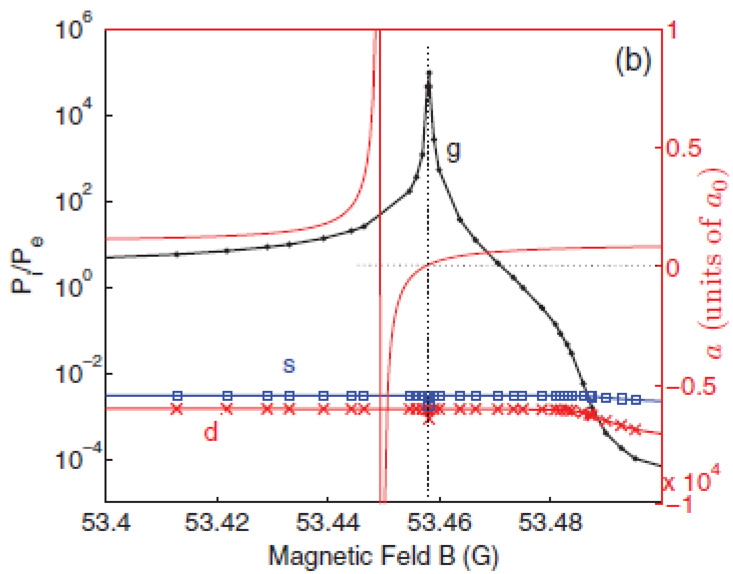
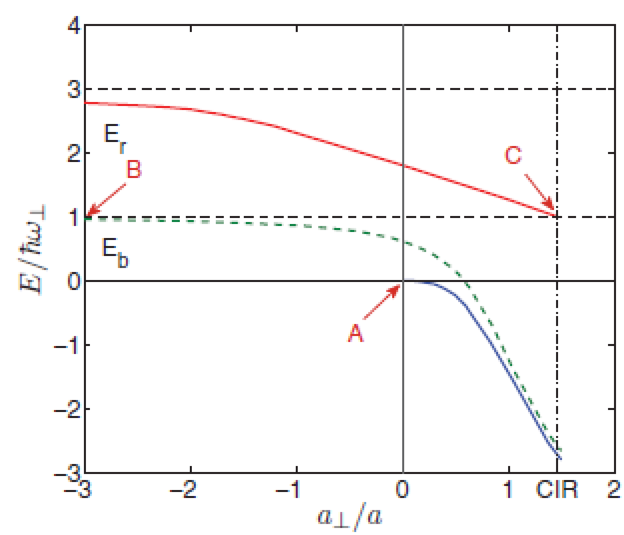


free space ($\omega_\perp = 0$)

g-wave FR at 53.45G



$\omega_\perp = 59.6$ kHz



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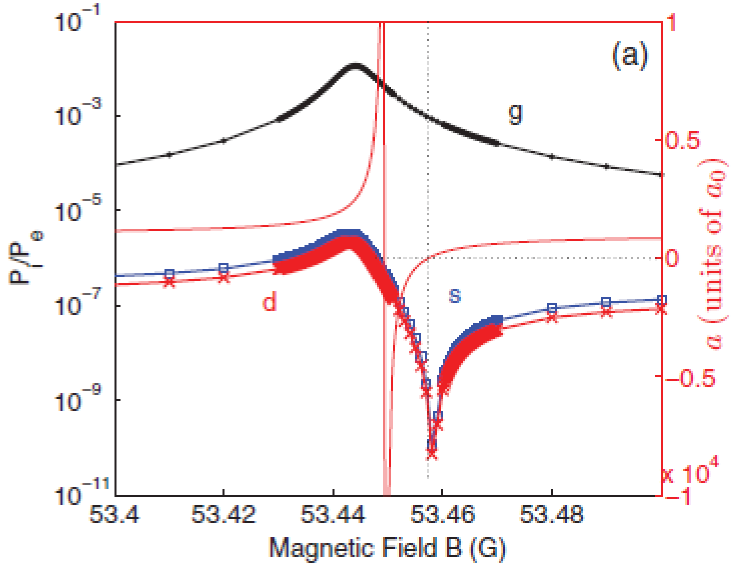

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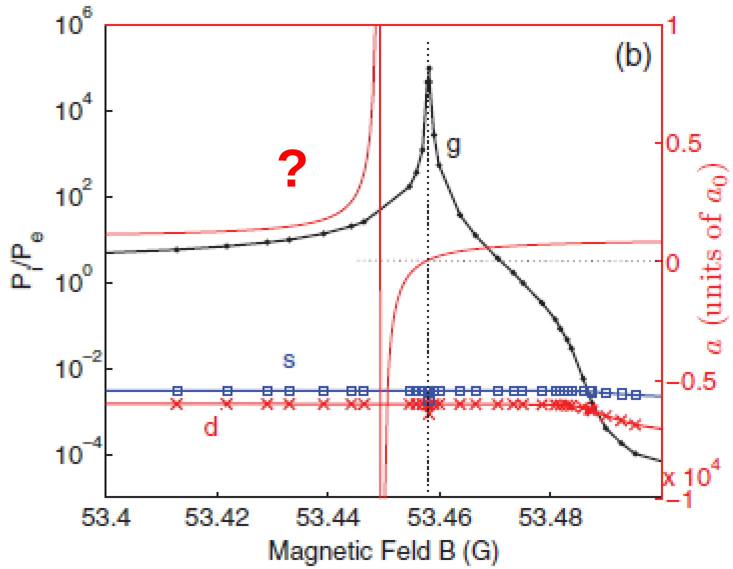
g-wave FR at 53.45G



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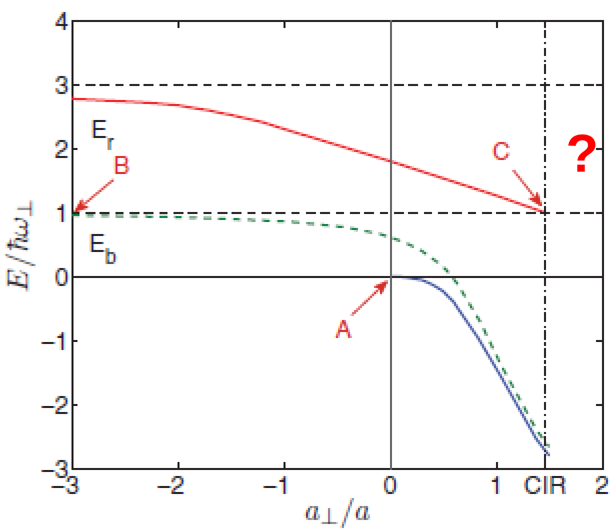
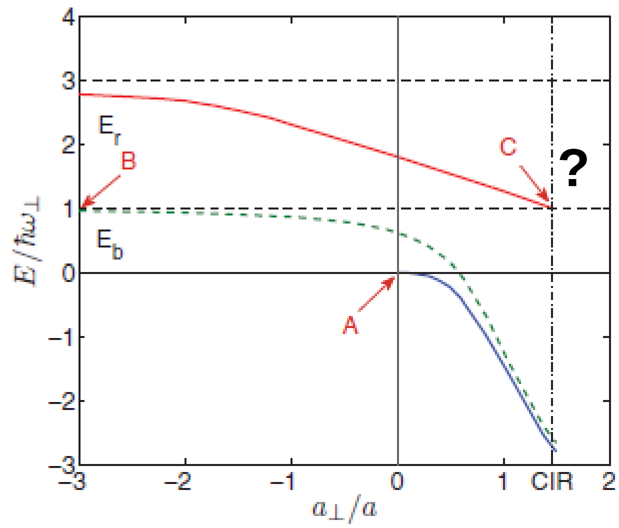
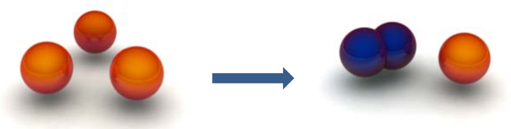
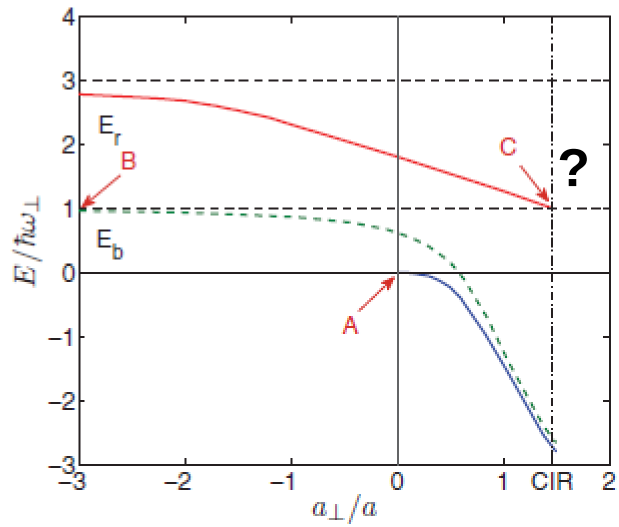


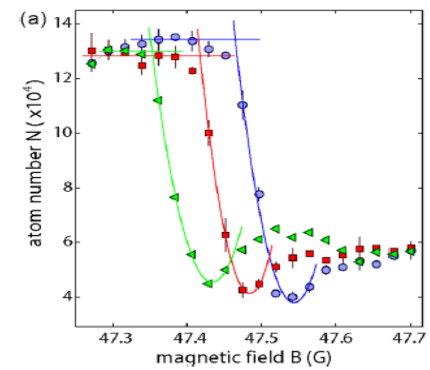


energy release ?

- **triple collisions A + A + A → (AA) + A :**







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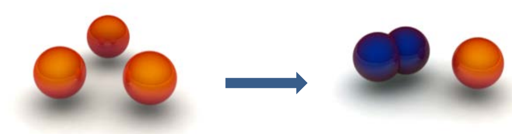
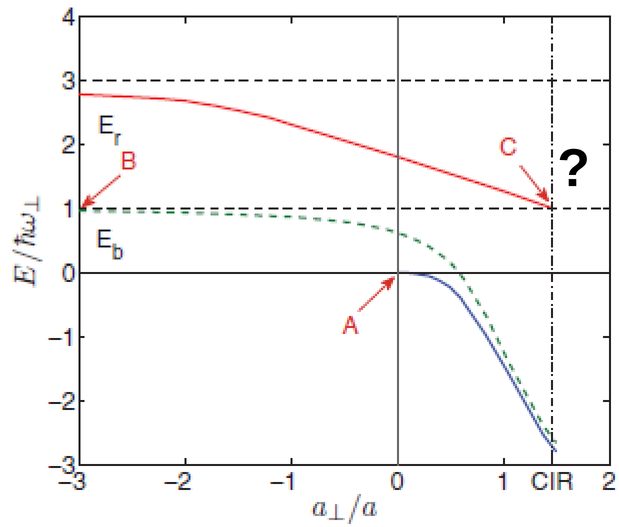
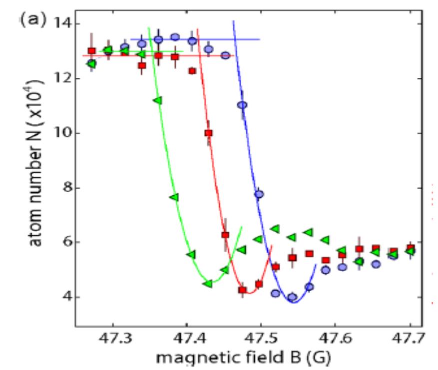


detection of the CIR by anincrease of three-body loss:

E.Haller, M.J. Mark, R. Hart, J.G. Danzl, L. Reichsoellner, V.Melezhik, P. Schmelcher and H.-C. Naegerl, Phys.Rev.Lett. 104 (2010)153203





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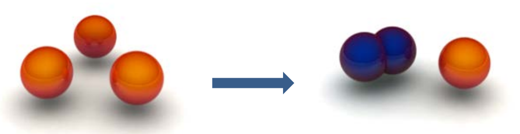

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- **pair collisions with CM excitation $A_{n1=0}+B_{n2=0} \rightarrow (AB)_{n=0,N=1}$**

Mechanism of molecule formation with transferring the energy release to CM excitation of forming molecule was considered in:

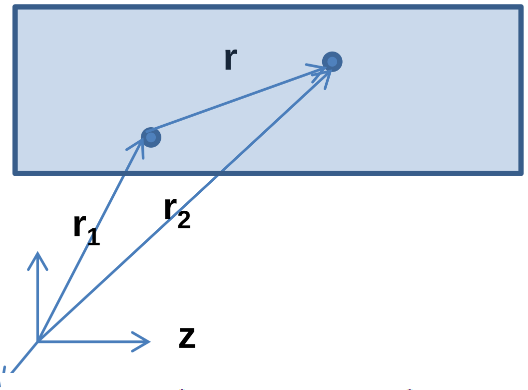
E.Bolda et.al. Phys.Rev. A71,033404 (2004) (in anharmonic lattices)

V.Melezhik &P.Schmelcher, New J.Phys.11,073031 (2009) (distinguishable atoms in harmonic waveguides)

non-separability of two-body problem in trap

(distinguishable atoms in harmonic trap or identical atoms in anharmonic trap)

V. Melezhik & P. Schmelcher, New J. of Phys. 11, 073031 (2009)



$$V(r) + \frac{1}{2}m_1\omega_1^2\rho_1 + \frac{1}{2}m_2\omega_2^2\rho_2$$

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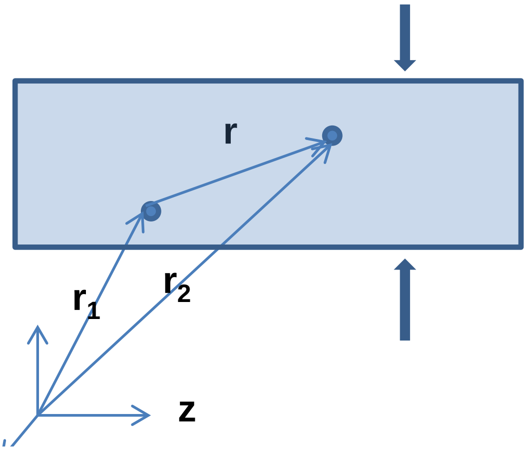

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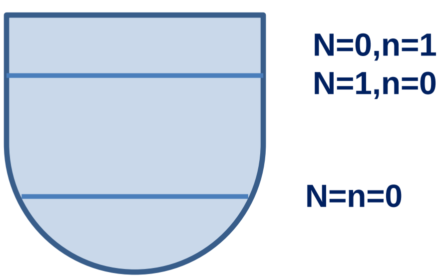

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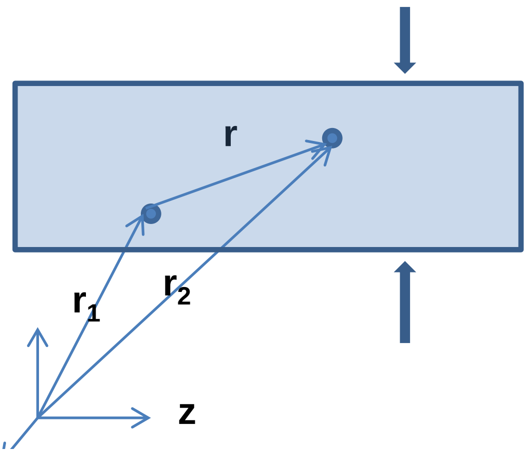

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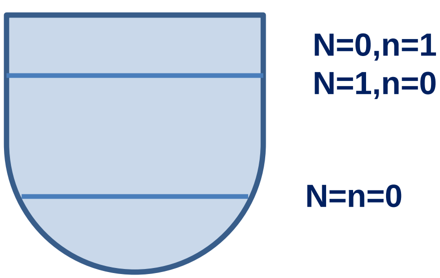


$$i\frac{\partial}{\partial t}\psi(\rho_R,\mathbf{r},t) = H(\rho_R,\mathbf{r})\psi(\rho_R,\mathbf{r},t)$$

$$H(\rho_R,\mathbf{r}) = H_{\mathrm{CM}}(\rho_R) + H_{\mathrm{rel}}(\mathbf{r}) + W(\rho_R,\mathbf{r})$$

$$H_{\mathrm{CM}} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{\rho_R^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{4\rho_R^2}\right) + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$H_{\mathrm{rel}} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + V(r)$$

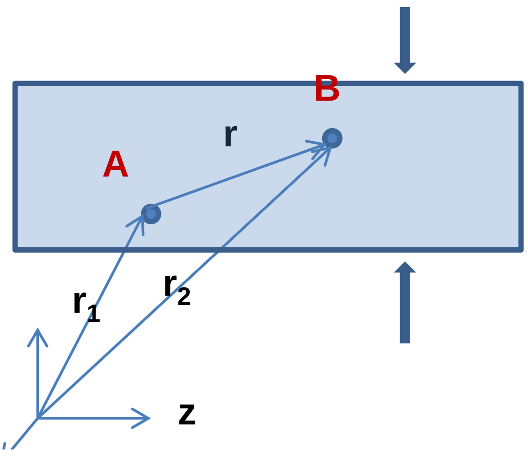
$$\frac{L^2(\theta,\phi)}{2\mu r^2} = -\frac{1}{2\mu r^2\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right)$$

$$W(\rho_R,\mathbf{r}) = \mu(\omega_1^2 - \omega_2^2)r\rho_R\sin\theta\cos\phi$$

→ **4D TDSE:** ρ_R, r, θ, ϕ

non-separability of two-body problem in trap
(distinguishable atoms in harmonic trap or identical atoms in anharmonic trap)

V. Melezhik & P. Schmelcher, New J. of Phys. 11, 073031 (2009)



$$V(r) + \tfrac{1}{2} m_1 \omega_1^2 \rho_1 + \tfrac{1}{2} m_2 \omega_2^2 \rho_2$$

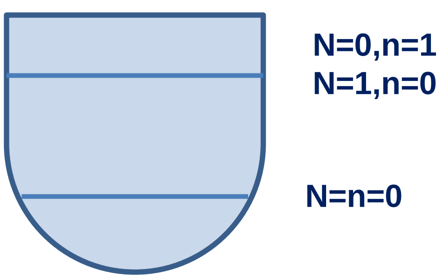
$$i \frac{\partial}{\partial t} \psi(\rho_R, \mathbf{r}, t) = H(\rho_R, \mathbf{r}) \psi(\rho_R, \mathbf{r}, t)$$

$$H(\rho_R, \mathbf{r}) = H_{\rm CM}(\rho_R) + H_{\rm rel}(\mathbf{r}) + W(\rho_R, \mathbf{r})$$

$$H_{\rm CM} = -\frac{1}{2M}\left(\frac{\partial^2}{\partial \rho_R^2} + \frac{1}{\rho_R^2}\frac{\partial^2}{\partial \phi^2} + \frac{1}{4\rho_R^2}\right) + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$H_{\rm rel} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + V(r)$$

$$\frac{L^2(\theta,\phi)}{2\mu r^2} = -\frac{1}{2\mu r^2 \sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right)$$



$$W(\rho_R, \mathbf{r}) = \mu(\omega_1^2 - \omega_2^2) r \rho_R \sin\theta \cos\phi$$ → **4D TDSE:** ρ_R, r, θ, ϕ

$A_{n1=0}$ + $B_{n2=0}$ → $(AB)_{n=0,N=1}$

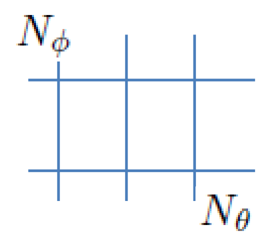
5D TDSE

- **Discretization of the angular subspace:**
  2D nondirect product discrete variable representation (npDVR)

$$\psi(\rho_R, r, \Omega, t) = \sum_{j=1}^{N} f_j(\Omega)\psi_j(\rho_R, r, t) \qquad \sum_{\nu=1}^{N} = \sum_{m=-(N_\phi-1)/2}^{(N_\phi-1)/2} \sum_{l=|m|}^{|m|+N_\theta-1}$$

$$f_j(\Omega) = \sum_{\nu=1}^{N} Y_\nu(\Omega)(Y^{-1})_{\nu j} \qquad \Omega_j = (\theta_{j_\theta}, \phi_{j_\phi})$$

$$Y_\nu(\Omega) = Y_{lm}(\Omega) = e^{im\phi} \sum_{l'} C_l^{l'} \times P_{l'}^{m}(\theta) \qquad Y_{j\nu} = Y_\nu(\Omega_j)$$



V.Melezhik, Phys.Lett.A230(1997)203
V.Melezhik, AIP Conf.Proc.1479(2012)1200

- **Computational scheme: component-by-component split operator method**

$$i\frac{\partial}{\partial t}\psi_j(\rho_R, r, t) = \sum_{j'}^{N} H_{jj'}(\rho_R, r)\psi_{j'}(\rho_R, r, t) \qquad t_n \to t_{n+1} = t_n + \Delta t$$

interaction is diagonal in ndDVR $f_j(\Omega)$

kinetic energy operator is diagonal in $Y_\nu(\Omega) = Y_{lm}(\Omega)$

$$S_{j\nu} = \lambda_j^{1/2} Y_{j\nu}$$

V.Melezhik, Phys.Lett.A230(1997)203
V.Melezhik,J.I.Kim,P.Schmelcher, Phys.Rev.A76(2007)053611

economic computational scheme

V.Melezhik, Phys.Lett.A230(1997)203
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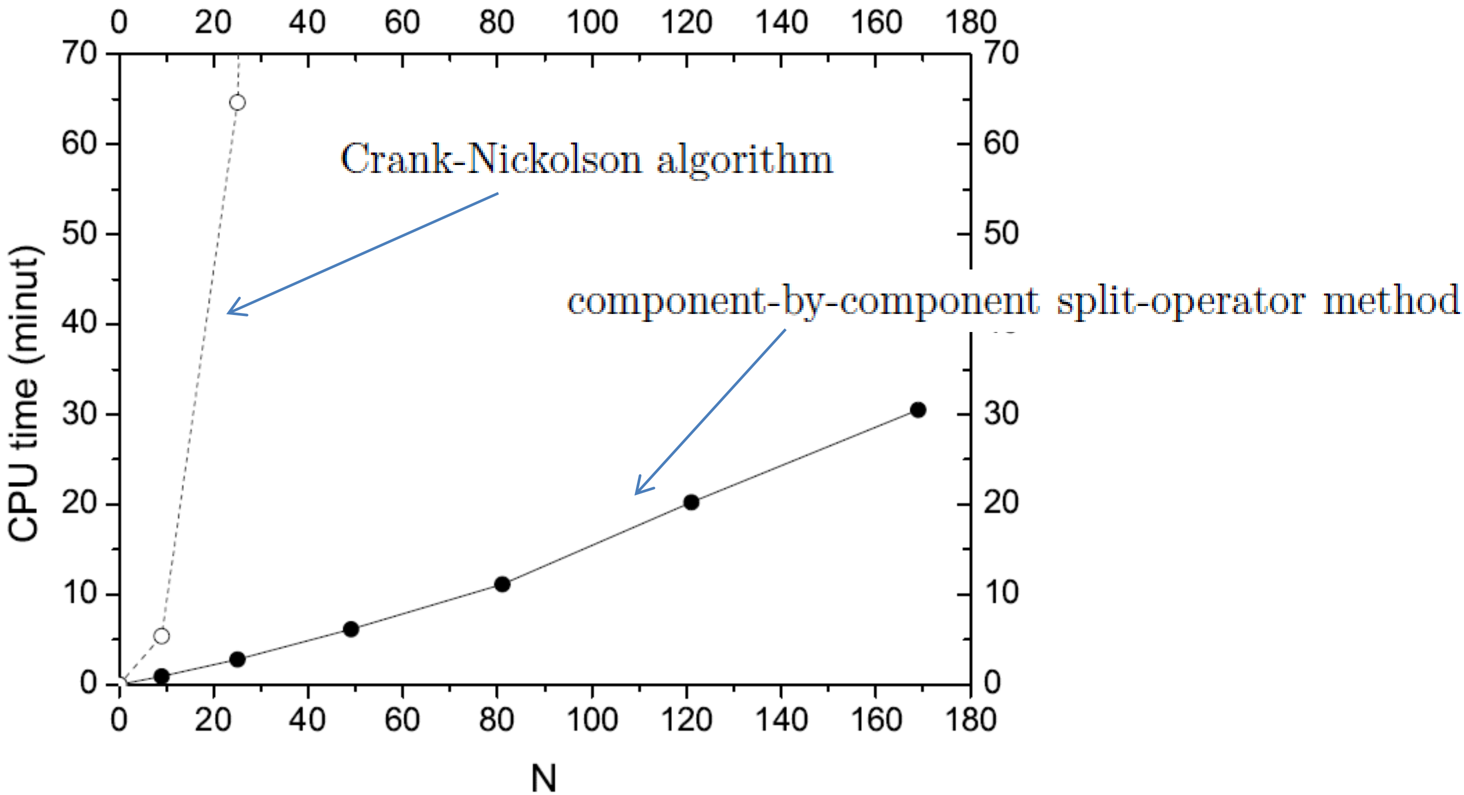


$$\psi(\rho_R, r, \Omega, t) = \sum_{j=1}^{N} f_j(\Omega)\psi_j(\rho_R, r, t) \qquad \sum_{\nu=1}^{N} = \sum_{m=-(N_\phi-1)/2}^{(N_\phi-1)/2} \sum_{l=|m|}^{|m|+N_\theta-1}$$

BLTP JINR two-core Intel processor Xenon 5160 with 3GHz frequency

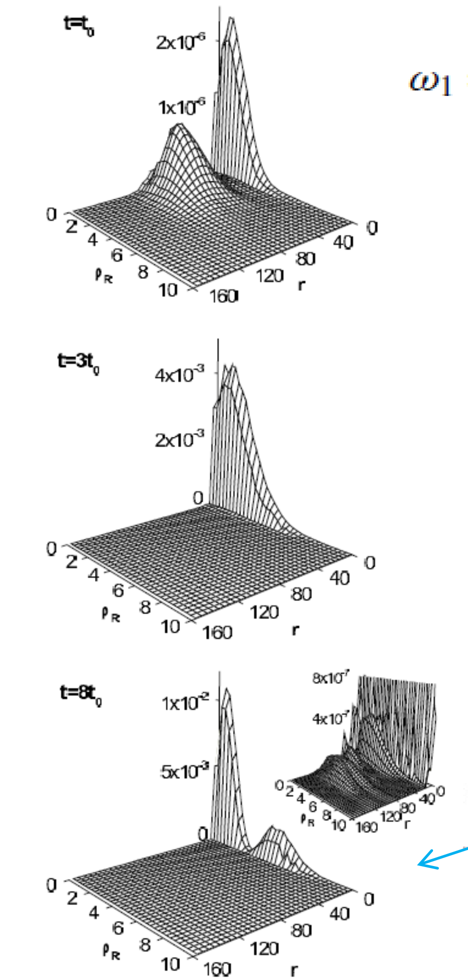
$A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}$

Time evolution of the probability density distribution during collision

$$W(\rho_R, r, t) = \int |\psi(\rho_R, r, \theta, \phi, t)|^2 (r^2 \rho_R)^{-1} \sin\theta \, d\theta \, d\phi$$

CM coupling with interatomic motion:

$\omega_1 \neq \omega_2$



molecule formation

$A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}$

Time evolution of the probability density distribution during collision

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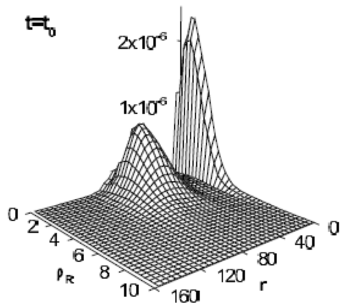
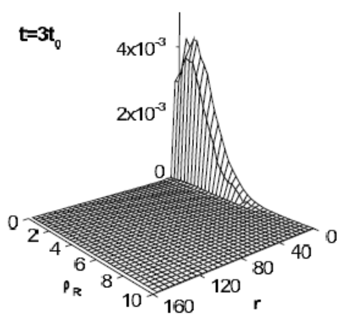
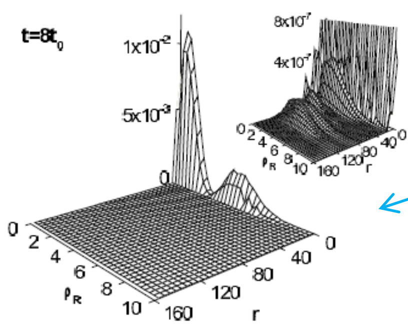
$\omega_1 \neq \omega_2$



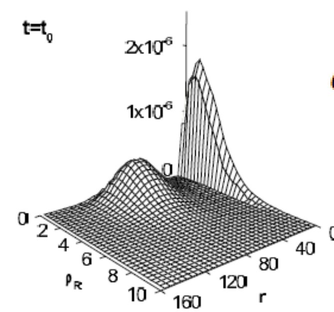


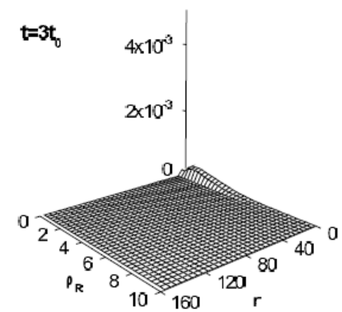


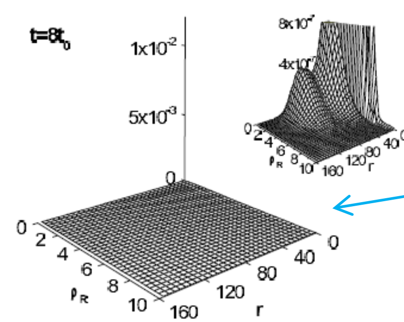
molecule formation

CM decouples from interatomic motion:

$\omega_1 = \omega_2$







no molecule

Resonant Formation of Ultracold Molecules in Waveguides

V. Melezhik & P. Schmelcher, New J. of Phys. 11, 073031 (2009)

coupling of the deatomic continuum with the CM of excited molecule at (N=1) in closed transverse channels:

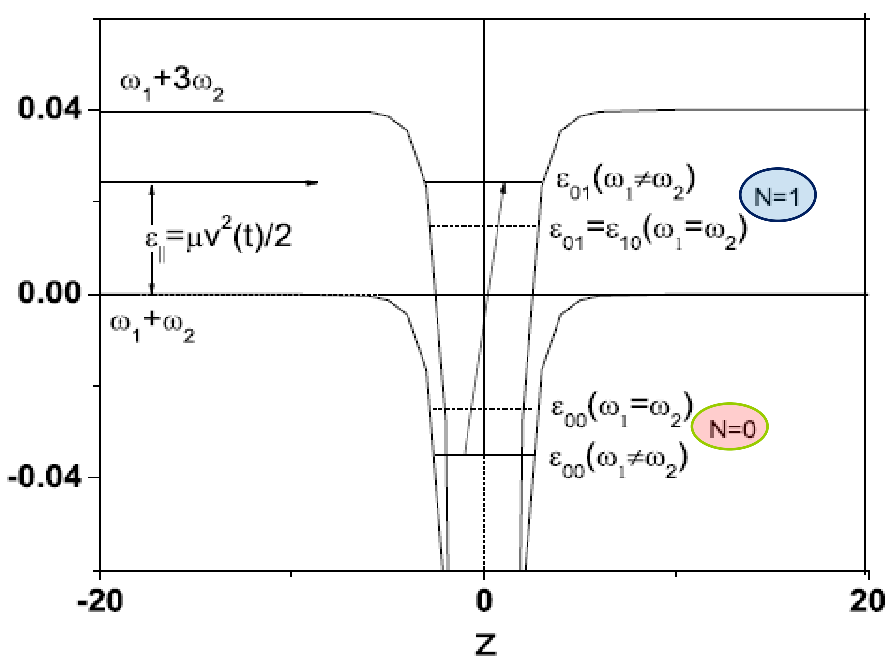
$$W(\rho_R, \boldsymbol{r}) = \mu\,(\omega_1^2 - \omega_2^2)\, r\, \rho_R \sin\theta \cos\phi$$

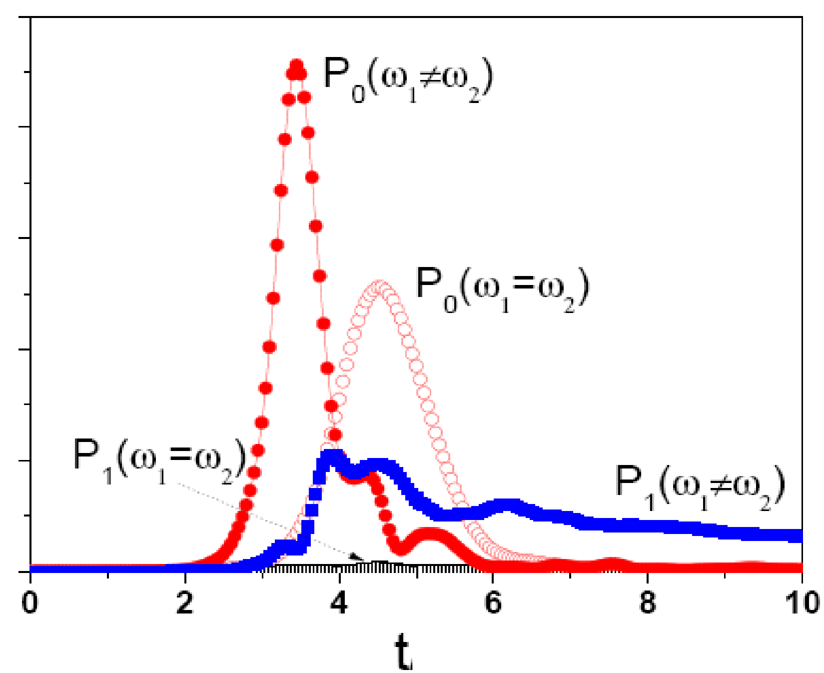
if the atoms in the colliding pair are identical, then coupling term goes to zero and the effect disappears.

TDSE: 4D

$\mathbf{A_{n1=0} + B_{n2=0} \rightarrow (AB)_{n=0,N=1}}$

Time evolution of the molecular states (N=0 and 1) population $P_N(t)$ during a pair collision:





in Heidelberg experiment , S.Sala et. al. Phys.Rev.Lett.110,203202 (2013),
the mechanism of molecule formation with transferring
energy release to CM molecule excitation was observed
in anharmonic waveguide

Quantum simulation with fully controlled few-body systems

control over: quantum states, particle number, interaction

- attractive interactions ⇨ BCS-like pairing in finite systems
- repulsive int.+splitting of trap ➡ entangled pairs of atoms
  (quantum information processing)
- + periodic potential ⟶ quantum many-body physics
  (systems with low entropy to explore
  such as quantum magnetism)
- ...

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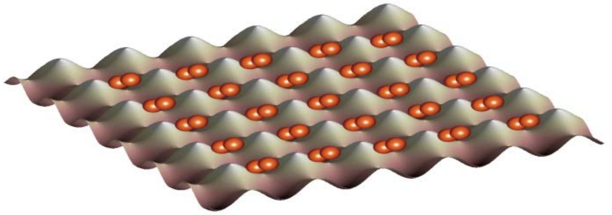
Bose-Hubbard Physics



Outlook

Experiments with:

Rb,Cs,K,Sr,Li …
Rb_2 , Cs_2 , RbK … **1D, 2D, 3D**

~ 80 experimental groups worldwide

new physics but time of simple models is over

Results were obtained in collaboration with

Peter Schmelcher (ZOQ,Hamburg)

Panagiotis Giannakeas (ZOQ,Hamburg)

Shahpoor Saeidian (IASBS,Zanjan,Iran)

Innsbruck experiment:

Elmar Haller

Hans-Chrisroph Nägerl