2-nd lecture

Outlook

- *Theoretical problems of BH formation:*
- Shock waves
- Semiclassical approach and geometrical cross section;
- Trapped surface arguments

Ultrarelativistic particle = shock wave

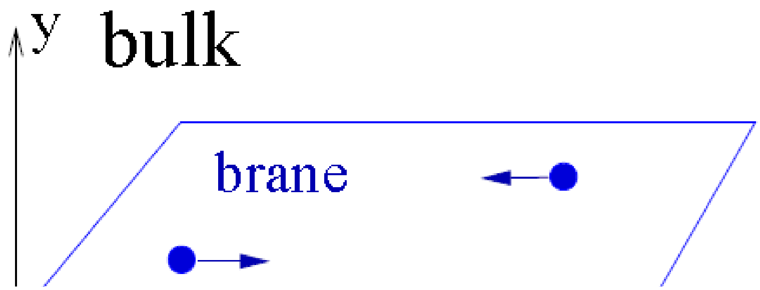
- Aichelburg-Sexl shock wave

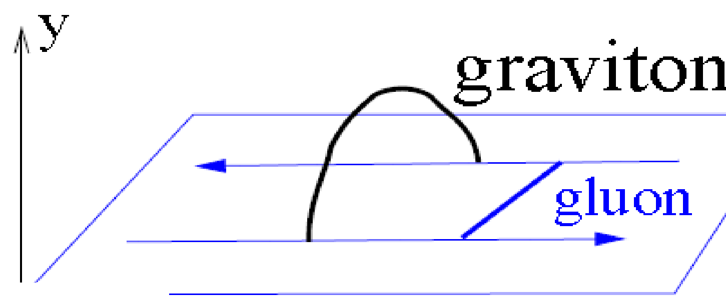
$$ds^2 = -dudv + dx^{i2} + F(x^i)\delta(u)\, du^2,$$

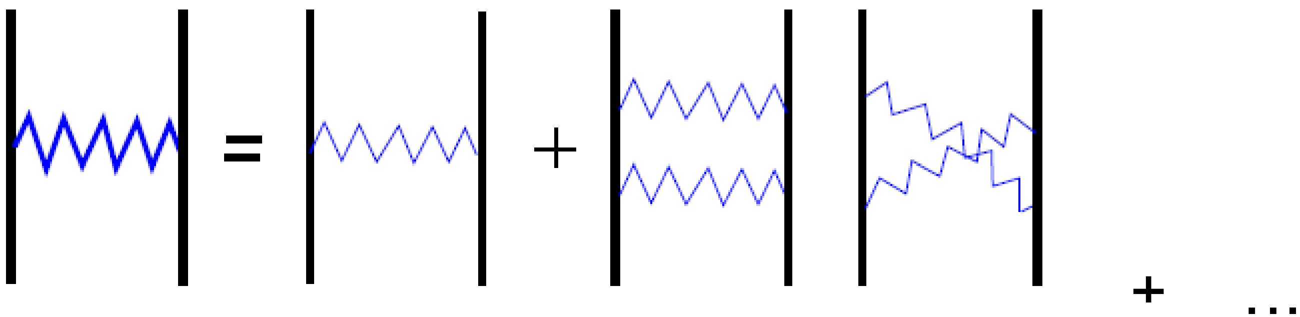
$$F(x^i) = \frac{c_k}{|\sum (x^i - x_{k0}^i)^2|^{(D-4)/2}}$$

Smooth coordinates: P.D'Eath coordinates, Dray and 't Hooft

Form of shock waves and eikonal approximation







$2 \to 2$ *small angle T-scattering amplitude*

Guidice, Rattazzi,Well, hep-ph/0112161,
Lodone, Rychkov, 0909.3519

Barbashov, Kuleshov, Matveev, Sissakian, TMP,1970

Kadyshevskii at al, TMP,1971

$$\mathcal{A}_{\rm eik}(\mathbf{q}) = \mathcal{A}_{\rm Born} + \mathcal{A}_{\text{1-loop}} + \ldots = -2is\int d^2\mathbf{b}\, e^{-i\mathbf{q}.\mathbf{b}}(e^{i\chi}-1)$$

$$\mathcal{A}_{Born}(\mathbf{q}) = \frac{-s^2}{M_D^{n+2}}\int \frac{d^n l}{q_\perp^2 + l^2}$$

$$\boxed{F(\mathbf{b}) = \chi(\mathbf{b})} = \frac{1}{2s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}.\mathbf{b}} \mathcal{A}_{\rm Born}(\mathbf{q})$$

$$ds^2 = -du\,dv + dx^{i2} + F(x^i)(x^i)\delta(u)\,du^2, \quad F(x^i) = \frac{c_k}{|\sum (x^i - x_{k0}^i)^2|^{(D-4)/2}}$$

Geodesics in the space-time with shock wave

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\,\delta(U)\,dU^2$$

$$\frac{d^2x^\mu(\lambda)}{d\lambda^2} + \Gamma^\mu_{\rho\gamma}\frac{dx^\rho(\lambda)}{d\lambda}\frac{dx^\gamma(\lambda)}{d\lambda} = 0, \qquad x = (U, V, X^i)$$

$$\boxed{\begin{aligned} \ddot{U} &= 0 \\ \ddot{V} - \frac{1}{2}F\delta'(U)\dot{U}^2 - F_{,i}\delta(U)\dot{U}\dot{X}^i &= 0 \\ \ddot{X}^i - \frac{1}{2}F_{,i}\delta(U)\dot{U}^2 &= 0 \end{aligned}}$$

$$U = \tau$$

$$\boxed{\begin{aligned} \ddot{V} - \frac{1}{2}F\delta'(U) - F_{,i}\delta(U)\dot{X}^i &= 0 \\ \ddot{X}^i - \frac{1}{2}F_{,i}\delta(U) &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} V &= V_0 + V_1 U + V_f\theta(U) + V_d\theta(U)U \\ X^i &= X_{i0} + X_{i1}U + X_{if}\theta(U) + X_{id}\theta(U)U \end{aligned}}$$

$$F = F(|X|^2) \text{ and } F_{,i} = 2F'X_i$$

$$\begin{aligned} V_f &= \frac{1}{2}F(|X_{i0}|^2) \\ X_{id} &= F'(|X_{i0}|^2)X_{i0} \\ V_d &= F'(|X_0^i|^2)X_{i0}(X_{i1} + \frac{1}{2}X_{id}) \end{aligned}$$

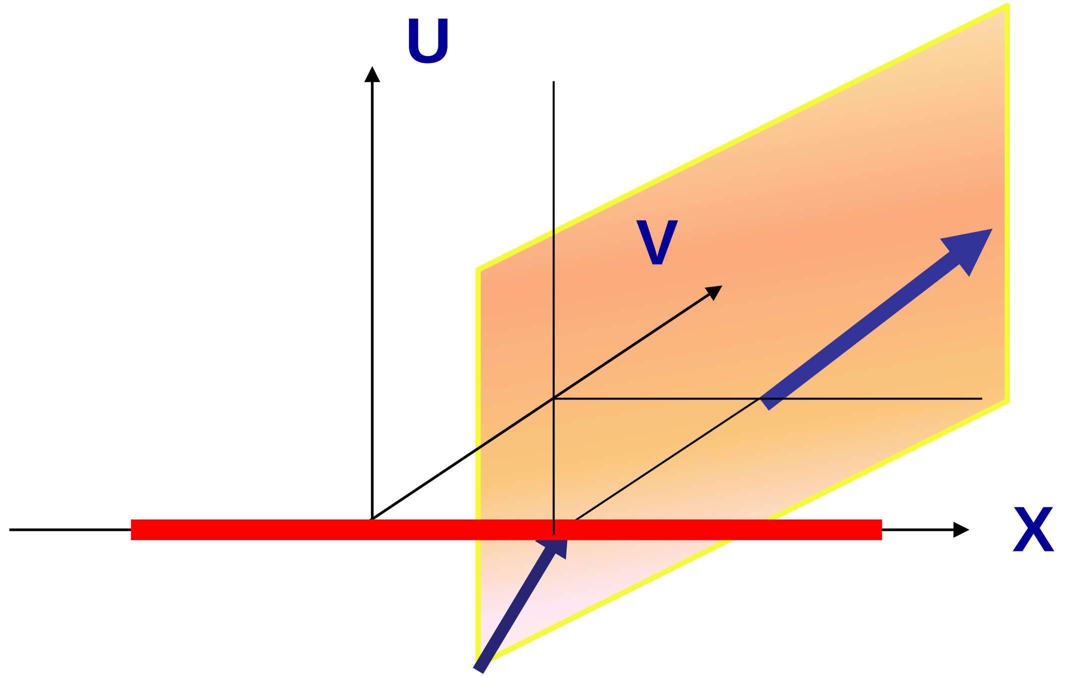
Geodesics in the space-time with shock wave

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\,dU^2,$$

$$X(\tau) = b - \theta(\tau) v\tau$$

$$V(\tau) = \tau + \theta(\tau) v^2\tau - 2\theta(\tau) F(b)$$

$$v = F'(b)$$

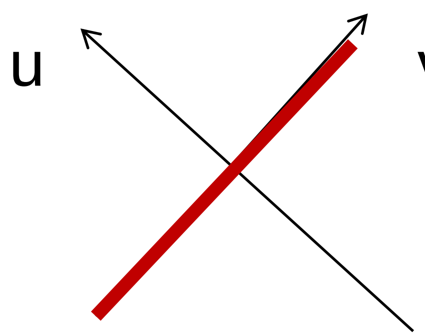


2 Ultrarelativistic particles = 2 shock waves

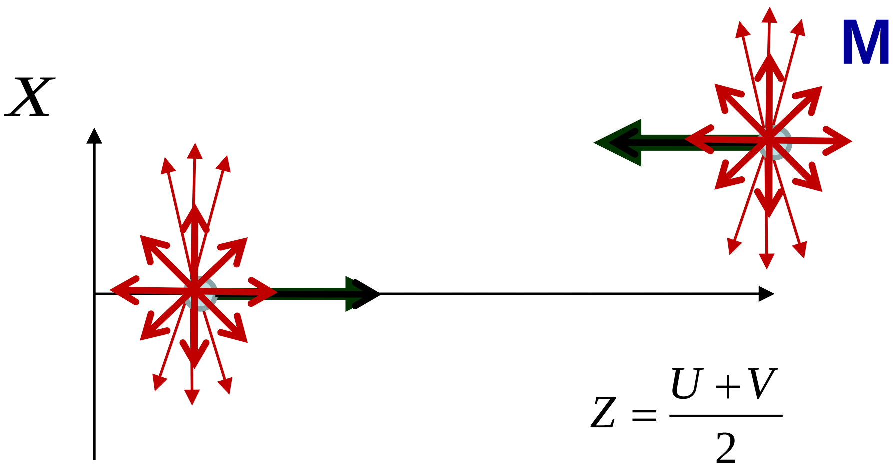
- 2 Aichelburg-Sexl shock waves

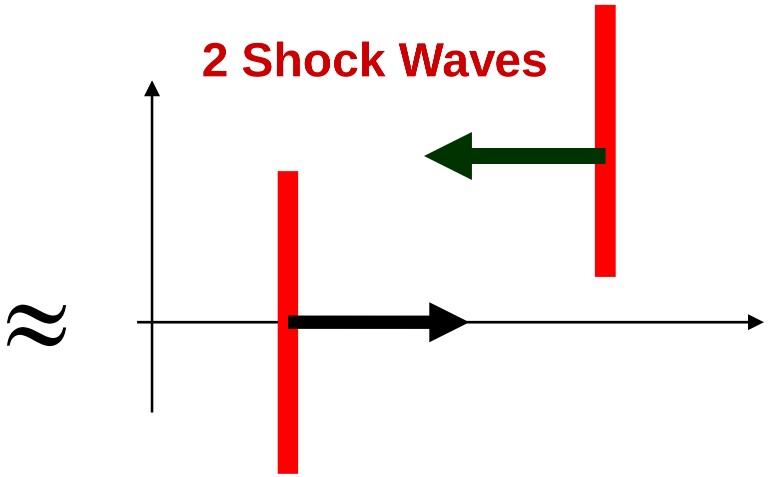
$$ds^2 = -dUdV + dX^{i2} + F_1(X^i)\delta(U)\, dU^2 + F_2(V^i)\delta(V)\, dV^2,$$

$$F_k(X^i) = \frac{c_k}{|\sum (X^i - X^i_{k0})^2|^{(D-4)/2}}$$

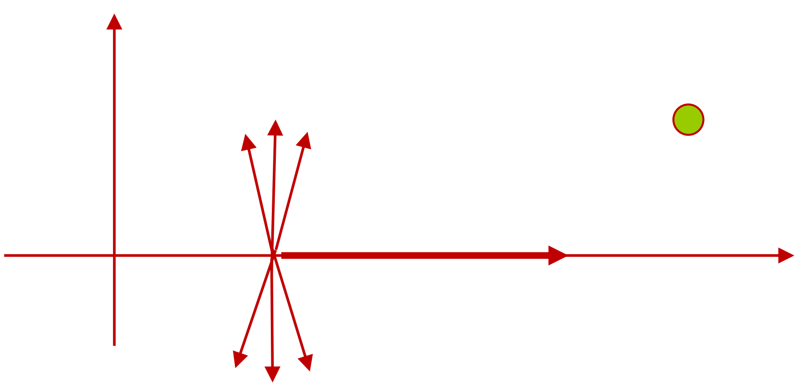


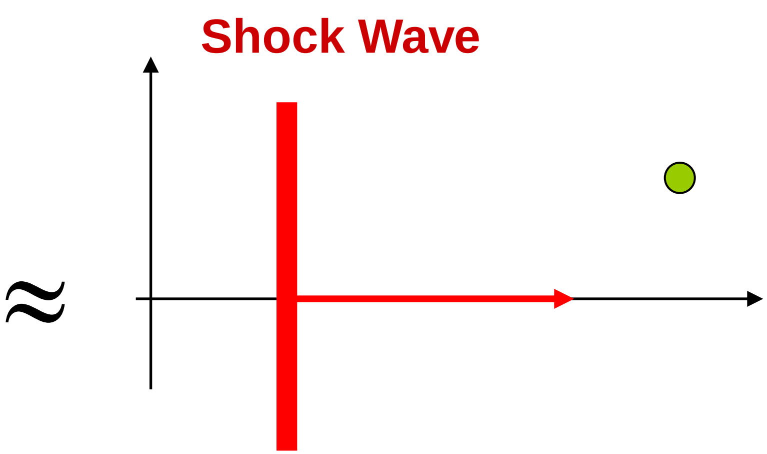
Particles and Shock Waves



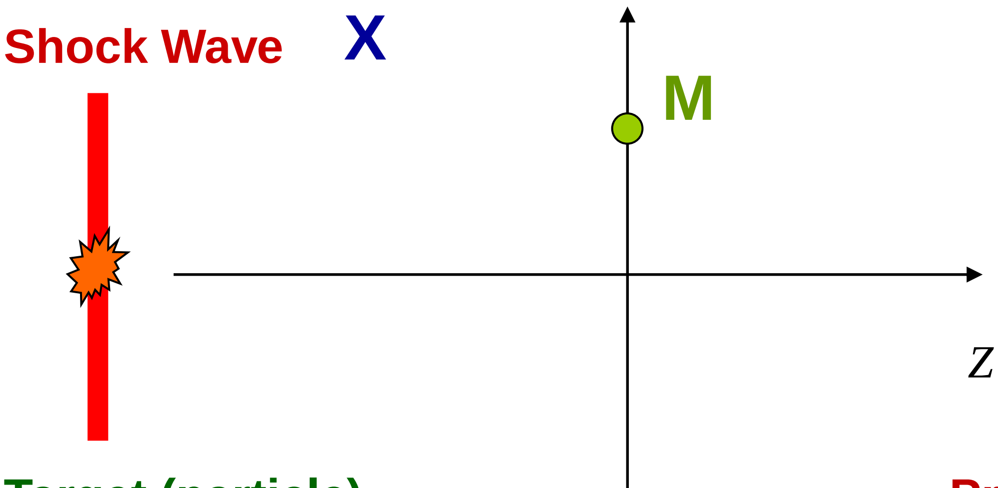


't Hooft,





Particle in Shock Wave



$$Z = \frac{U+V}{2}$$

Target (particle)

$$X = b - \theta(\tau) v \tau$$

$$Z = \theta(\tau)\frac{v^2 \tau}{2} - \theta(\tau) F(b)$$

$$t = \tau + \theta(\tau)\frac{v^2 \tau}{2} - \theta(\tau) F(b)$$

Projectile (shock wave)

$$X = 0$$

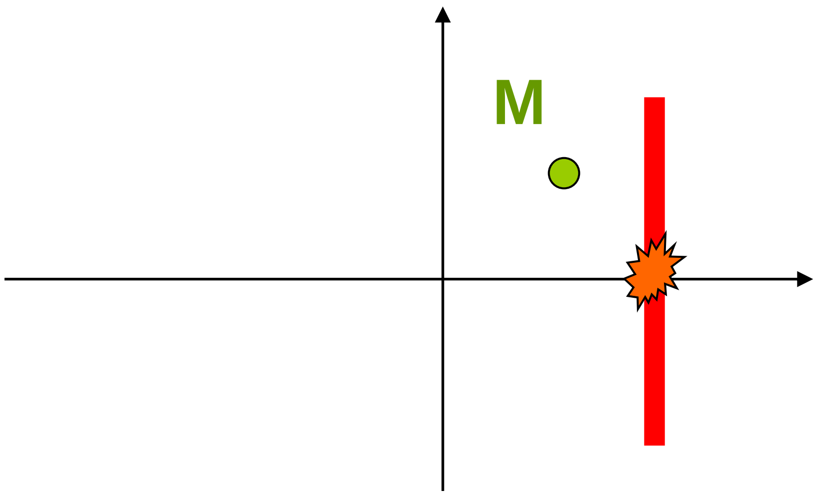
$$Z = t \rightarrow Z = \tau + \theta(\tau)\frac{v^2 \tau}{2} - \theta(\tau) F(b)$$

M is chasing after the shock

$$v = F'(b)$$

$$v = \frac{p}{\pi M_{Pl}^2 b}$$

Capture or not capture



$$v = \frac{p}{\pi M_{Pl}^2 b}$$

$$R^2 = b^2 \left(1 - v \frac{\tau'}{b}\right)^2 + \tau'^2, \qquad \tau' = \frac{\tau}{1 + v^2/2}$$

$$R_{\min}^2 = \frac{b^2}{1 + v^2}$$

Calculations show that just after collisions the relative velocity of m and M particles is small and we can use the nonrelativistic hoop conjecture

$$R_S(M) > R_{\min}(b) \qquad \boxed{b < b_*} \qquad b_*^2 = \frac{2Mp}{M_{Pl}^4} \qquad \boxed{\sigma = \pi b_*^2}$$

Modified Thorn's conjecture for charged particles

- Start from the Reissner-Nortstrom
- Boost
- Shock wave
- Apply the previous formula

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2 d\Omega_{D-2}^2$$

$$g(R) = 1 - \left(\frac{R_S}{R}\right)^{D-3} + \frac{Q^2}{R^{2(D-3)}} \qquad Q^2 = \frac{8\pi G_D q^2}{(D-2)(D-3)}$$

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\, dU^2,$$

$$F(|X|) = \frac{a_D G_D p}{|X|^{D-4}} + \frac{b_D G_D p_Q}{|X|^{2D-7}} \qquad \text{D>4}$$

Modified Thorn's conjecture for charged particles

- **Shock wave**
- **Apply the previous formula**

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\, dU^2,$$

$$F(|X|) = \frac{a_D G_D p}{|X|^{D-4}} + \frac{b_D G_D p_Q}{|X|^{2D-7}}$$

$$R_S(M) > R_{\min}(b)$$

$$b < b_*(p, p_Q)$$

$$\boxed{\begin{gathered} b_{*p_Q}(p, p_Q) = b_*(1 + f\ p_Q^2), \\ f < 0 \end{gathered}}$$

Cross section decreases

Modified Thorn's conjecture for charged dilaton

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\,dx \left(R - 2(\nabla\phi)^2 + e^{-2a\phi}F_2^{\,2} \right)$$

- ***Start from charged dilaton metric***
- **Boost**
- **Shock wave**
- **Apply the previous formula**

$$ds^2 = -A^2(R)dt^2 + A^{-2}(R)dR^2 + K^2 d\Omega_2^2$$

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\,dU^2,$$

$$F(|X|) = -8p\ln|X| + \frac{\alpha_{eff}}{|X|}$$

D=4

$$\alpha_{eff} = \frac{3-4a^2}{2(1-a^2)}$$

$$\boxed{\begin{aligned} b_{*\alpha_{eff}} &= b_*(1 + f\,\alpha_{eff}), \\ f &< 0 \end{aligned}}$$

Cross section increases for alpha<0

Technical details for (A)dS

- Shock waves;
- Geodesics in a spacetime with a shock wave

Technical details for (A)dS.

Shock waves from the Schwarzschild metric

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2 d\Omega_2^2$$

$$g(R) = 1 - \frac{R_S}{R} \mp \frac{a^2}{R^2}$$

- dS

+ AdS

Plane coordinates $\vec{Z} = \{Z^2, \dots, Z^{D-1}\}, \ -UV + \vec{Z}^2 \pm Z^{D^2} = \pm a^2$

$$U = Z^0 + Z^1, \ V = -Z^0 + Z^1$$

$$ds^2 = ds^2{}_{(A)dS} + ds^2{}_{pert}$$

$$ds^2{}_{pert} = m^2 K(|\vec{Z}|)(K_{00}(Z_0, Z_D)dZ_0^2 + K_{DD}(Z_0, Z_D)dZ_D^2 + K_{0D}(Z_0, Z_D)dZ_D dZ_0)$$

Boost

$$Z_0 = \gamma(Y_0 + vY_1),$$
$$Z_1 = \gamma(vY_0 + Y_1),$$

$$m = \frac{p}{\gamma}$$

$$\vec{Z} = \vec{Y}, Z_D = Y_D, \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$m \to 0, \gamma \to \infty, \gamma - fixed$$

Technical details for (A)dS .

Shock waves from the Schwarzschild metric

$$F(Z) = 2pa^2\ \mathcal{P} \int_{-\infty}^{\infty} \frac{(a^2(\pm Z^2 + x^2) + Z^2(x^2 \mp Z^2))}{(Z^2 \mp x^2)^2(\pm a^2 + x^2 \mp Z^2)^{\frac{D-1}{2}}} dx$$

$$F_{4,dS}(Z^4) = 4\ \mathrm{p}G_4 \left(-2 + \frac{Z^4}{a} \ln\left(\frac{1+\frac{Z^4}{a}}{1-\frac{Z^4}{a}}\right)\right)$$

$$F_{5,AdS}(Z^5) = \frac{3\pi\ \mathrm{p}G_5}{2a} \left(\frac{2\frac{Z^{5^2}}{a^2} - 1}{\sqrt{\frac{Z^{5^2}}{a^2} - 1}} - 2\frac{Z^5}{a}\right)$$

Shock wave in dS

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$

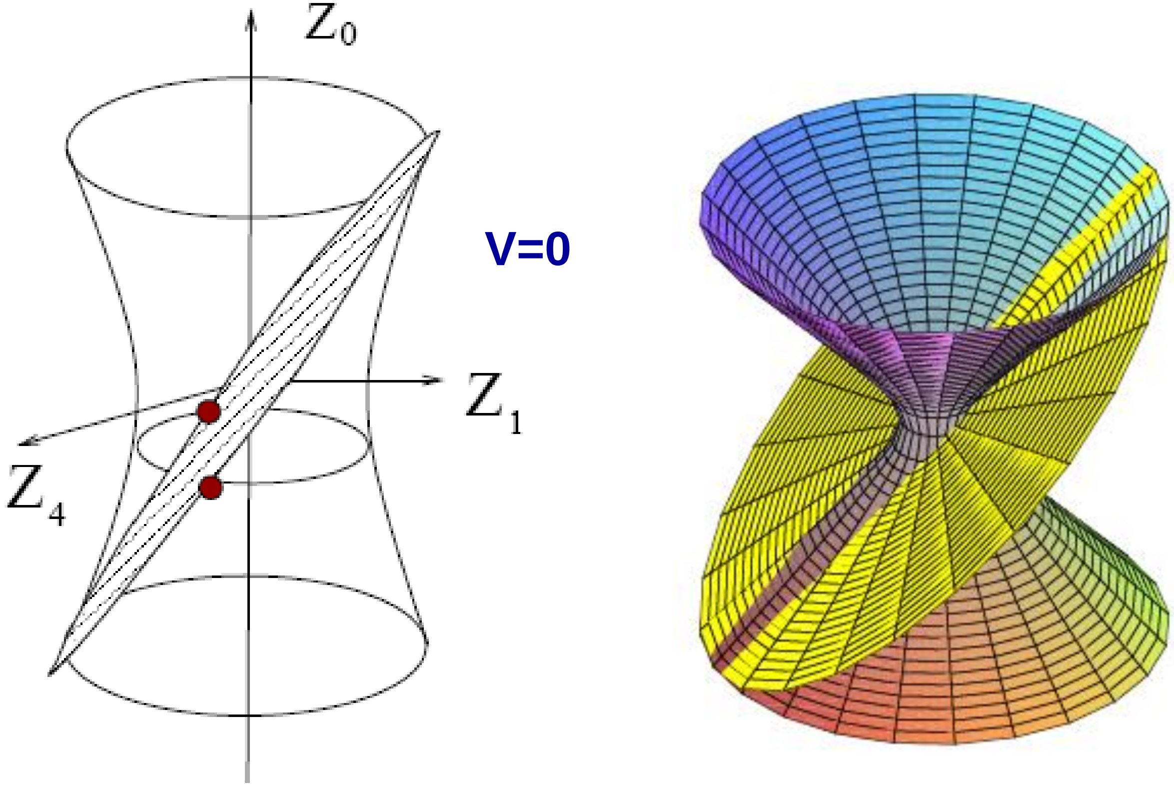
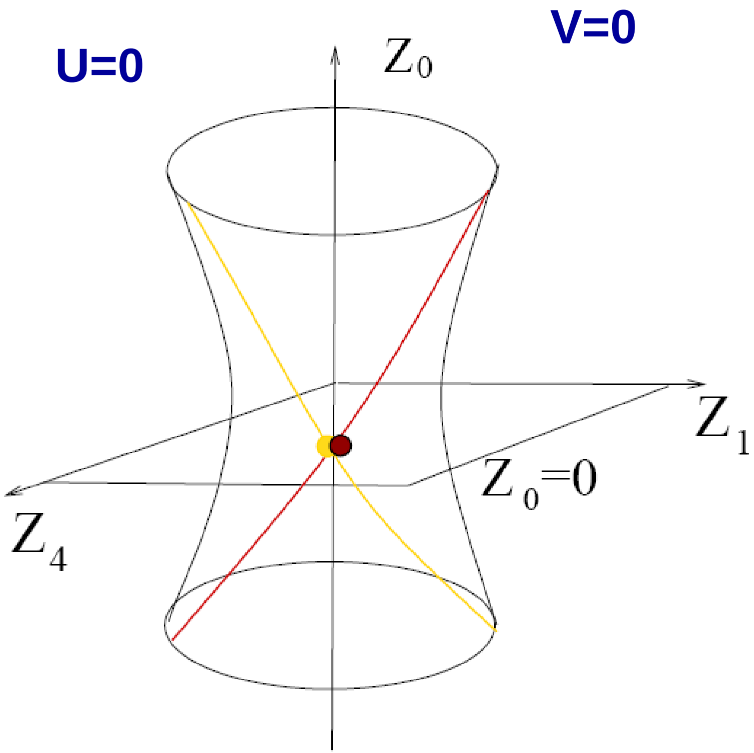


Figure 1: One shock wave in the de Sitter space presented as a hyperboloid is located on the intersection of the hyperboloid and the plane $Z_0 - Z_1 = 0$.Z_2 and Z_3 are suppressed At fixed "Z_0-time" this cross section consists of two points (small red ball in the left picture)

Two Shock waves in dS



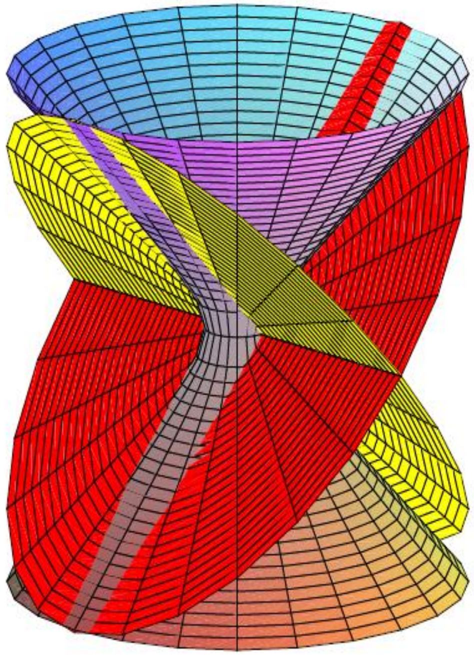


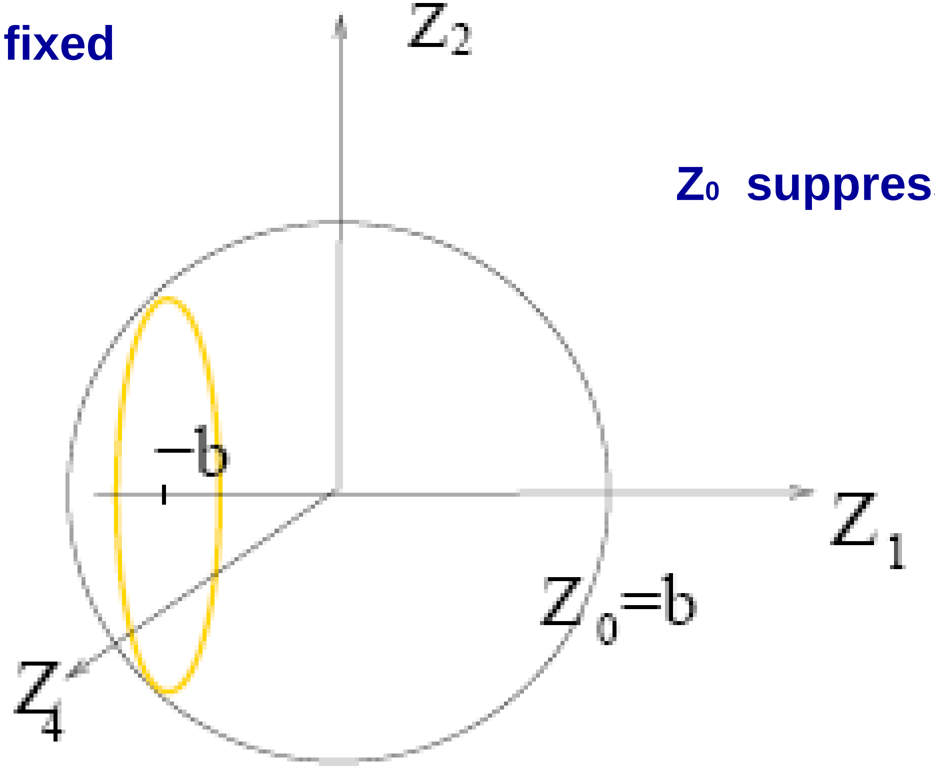
Figure 2: Two shock waves in the de Sitter space. A collision of two shock waves takes place at $Z_0 = 0$ and corresponds to a collision of red and yellow balls.

Shock wave in dS (nonexpanding shock waves).

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$

Shock wave $Z_1 + Z_0 = 0$ → Z_1 fixed $\qquad S^{D-1}$

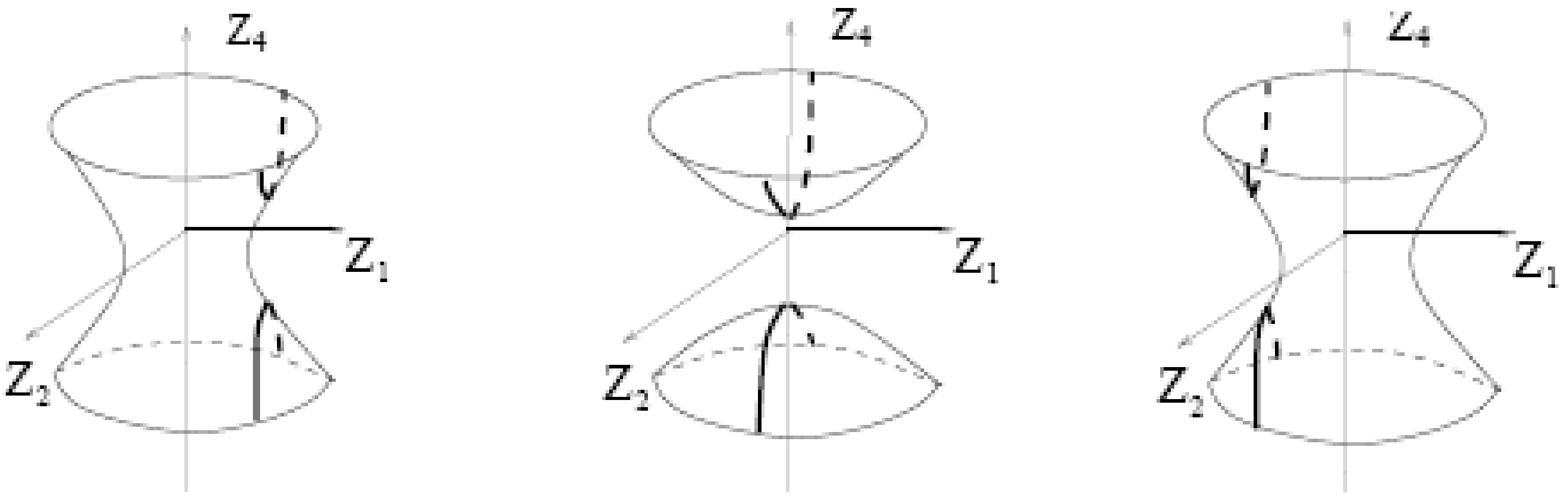
Z_0 fixed



Z_0 suppressed

Carton for different Z_0 time

Shock wave in AdS



Geodesics in dS with a shock wave

$$\int d\tau \, [\frac{dX^M(\tau)}{d\tau} G_{MN}(X(\tau)) \frac{dX^N(\tau)}{d\tau} - \lambda \left(X^M(\tau) g_{MN} X^N(\tau) - a^2\right)]$$

$$G_{MN}[X] = g_{MN} + h_{MN}[X]$$

Analogy with **L.D.Faddeev, Dokl.Acad.Nauk 210(1973)807**

$$g_{MN} \;\; = \;\; -\delta^U_M \delta^V_N + \delta^N_M \delta^i_N, \quad h_{MN}[U,V,X] = \delta^U_M \delta^U_N F(X) \delta(U)$$

$$ds^2_{\mathrm{dS}} = -dX_0^2 + \sum_{M=1}^{D} dX_M^2 \qquad\qquad a^2 = -X_0^2 + \sum_{M=1}^{D} X_M^2$$

$$ds^2 = -ds^2_{\mathrm{dS}} + F(X_i)\delta(U)\, dU^2, \qquad \begin{aligned} U &= \frac{X_0 + X_1}{\sqrt{2}}, \;\; X_i = X_2, \ldots X_D \\ V &= \frac{X_0 - X_1}{\sqrt{2}}, \end{aligned}$$

Geodesics

$$\mathcal{S} = \int d\tau \, [\frac{dX^M(\tau)}{d\tau} G_{MN}(X(\tau)) \frac{dX^N(\tau)}{d\tau} - \lambda \left(X^M(\tau) g_{MN} X^N(\tau) - a^2\right)]$$

$$\begin{aligned} G_{MN} \frac{d^2 X^N(\tau)}{d\tau^2} + G_{MN} \Gamma^N_{KL} \frac{dX^K(\tau)}{d\tau} \frac{dX^L(\tau)}{d\tau} + \lambda g_{MN} X^N(\tau) &= 0 \\ X^M(\tau) g_{MN} X^N(\tau) - a^2 &= 0 \end{aligned}$$

Geodesics

$$\begin{aligned}
\ddot{U} &= 0 \\
\ddot{V} - \frac{1}{2}F\delta'(U)\dot{U}^2 - F_{,i}\delta(U)\dot{U}\dot{X}^i &= -\frac{1}{2a^2}(-F + X^iF_{,i})V\delta(U)\dot{U}^2 \\
\ddot{X}^i - \frac{1}{2}F_{,i}\delta(U)\dot{U}^2 &= -\frac{1}{2a^2}(-F + X^jF_{,j})X^i\delta(U)\dot{U}^2
\end{aligned}$$

$$\tau = U$$

$$\boxed{\begin{aligned}
U\delta(U) &= 0, \\
\theta(U)\delta(U) &= \frac{1}{2}\delta(U)
\end{aligned}}$$

$$\boxed{\begin{aligned}
\ddot{V} - \frac{1}{2}F\delta'(U) - F_{,i}\delta(U)\dot{X}^i &= -\frac{1}{2a^2}(-F + X^iF_{,i})V\delta(U) \\
\ddot{X}^i - \frac{1}{2}F_{,i}\delta(U) &= -\frac{1}{2a^2}(-F + X^jF_{,j})X^i\delta(U)
\end{aligned}}$$

Solutions

$$
\begin{aligned}
V(U) &= V_0 + V_1 U + V_f \theta(U) + V_d \theta(U) U \\
X^i(U) &= X_{i0} + X_{i1} U + X_{id} \theta(U) U
\end{aligned}
$$

$$
\begin{aligned}
V_f &= \frac{1}{2} F \\
V_d &= \frac{1}{2} F_{,i} X_{i1} + \frac{1}{2a^2}(F - X_{i0} F_{,i}) V_0 + \frac{1}{8} F_{,i}^2 + \frac{1}{8a^2}(F^2 - (X_{i0} F_{,i})^2) \\
X_{id} &= \frac{1}{2} F_{,i} + \frac{1}{2a^2}\left(F - X_{j0} F_{,j}\right) X_{i0}
\end{aligned}
$$

F_i *is a given function of* X_{i0}

Shock waves. Smooth coordinates

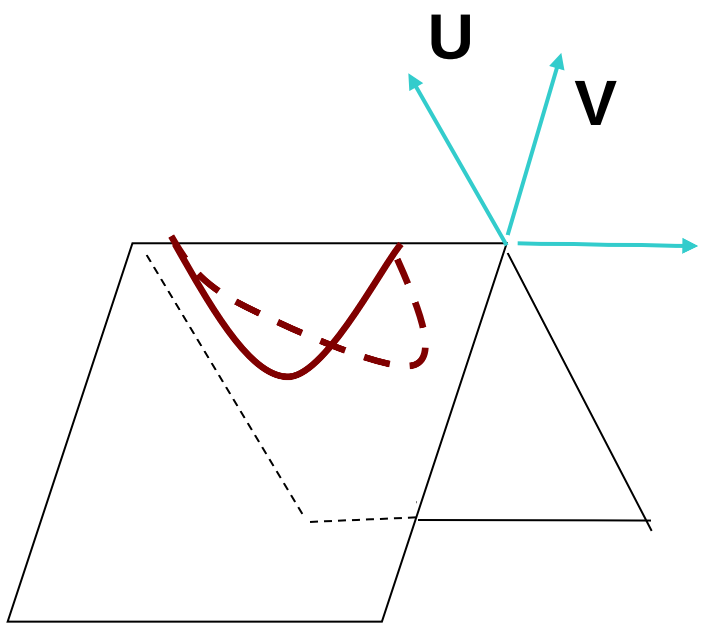
$$ds^2 = -dUdV + [H_{ik}H_{jk} - \delta_{ij}]dX^i dX^j,$$

Smooth coordinates: P.D'Eath coordinates, Dray and 't Hooft

$$H_{ij} = \delta_{ij} + \frac{1}{2}\nabla_i\nabla_j F \;\; u\theta(u)$$

$$ds^2 = -dUdV + [H^{(1)}_{ik}H^{(1)}_{jk} + H^{(2)}_{ik}H^{(2)}_{jk} - \delta_{ij}]dX^i dX^j,$$

Trapped Surface for two shock waves



X
- $\Psi_{1,2} > 0,\ X \in D,\ \ \Psi_{1,2} = 0, \quad X \in \partial D$
- $\nabla^2 \Psi_{1,2} = \delta^{(D-2)}(X - X_{(1,2)}), \quad X \in D,$

  the outer null normals have zero convergence
- $\nabla \Psi_1 \cdot \nabla \Psi_2 = 4, \ \ X \in \partial D$

  no $\delta-$ function in convergence

TS comprises two halves, which are matched along a "curve"

Eardley, Giddings; Kang, Nastase,….

Outlook

- *Introduction:*
- What is BH;
- Different types of BHs

- *Theoretical problems of BH formation:*
- Semiclassical approach and geometrical cross section;
- Trapped surface arguments

Outlook

- Applications:

- *Black holes production in Tev Gravity:*

- *Dual description of QGP(Quark Gluon Plasma)*
- QGP in dual description;
- Trapped surface area and multiplicity;
- BH charge and chemical potential

III. Black holes production in TeV Gravity

INTRODUCTION. Reasons to think about extra dimensions

- Kaluza-Klein
- Strings
- D-branes
- TeV-gravity scenario (an alternative to SUSY in addressing the hierarchy problem)

III. Black holes production in TeV Gravity

Reasons to think about extra dimensions. TeV-gravity scenario

N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, I. Antoniadis, 1998

$$M_{SM} \approx 1\, TeV$$

$$\boxed{M_D \approx 1\, TeV}$$

$$M_{Pl} \approx 1.2 \cdot 10^{16}\, TeV$$

$$\boxed{S = M_D^{D-2} \int d^D x \sqrt{-g}\, R(g)}$$

$$\boxed{S = M_{Pl}^2 \int d^4 x \sqrt{-g}\, R(g)}$$

$$\boxed{G_D = \frac{1}{M_D{}^{D-2}}}$$

$$M_{Pl}^2 = M_D^2 \left(\frac{M_D}{M_c}\right)^n$$

$$If \quad \frac{M_D}{M_c} >> 1 \quad \Rightarrow M_D^2 << M_{Pl}^2$$

INTRODUCTION. Reasons to think about extra dimensions.

TeV-gravity scenario. Scale of Extra Dimensions

$$M_{Pl}^2 = M_D^2 \left(\frac{M_D}{M_c}\right)^n$$

$$L_c = \frac{1}{M_c}$$

$$\boxed{L_c = M_D^{-1} \left(\frac{M_{Pl}}{M_D}\right)^{2/n}}$$

$$M_{Pl} \approx 10^{16} TeV$$

$$M_D \approx TeV$$

$$n = 2, \quad L_c \approx 10^{-1} cm$$

$$n = 4, \quad L_c \approx 10^{-9} cm$$

$$n = 6, \quad L_c \approx 10^{-12} cm$$

$$L_{Pl} \approx 10^{-33} cm, \quad L_{SM} \approx 2 \cdot 10^{-17} cm,$$

INTRODUCTION. Reasons to think about extra dimensions.

TeV-gravity scenario. Modification of the Newton law

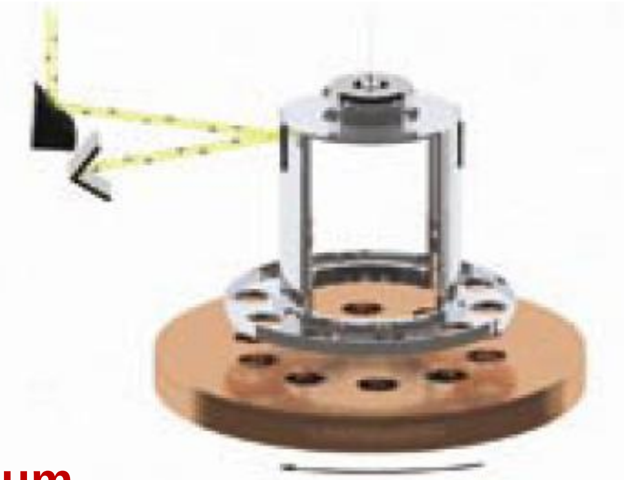
$$F = \frac{G_{Newton}}{r^2} m_1 m_2 \quad \Rightarrow \quad F = \frac{G_{Newton}}{r^2} m_1 m_2 \qquad for\ r \geq L_c$$

$$F = \frac{V_n}{r^n} \frac{G_{Newton}}{r^2} m_1 m_2 \qquad for\ r \leq L_c$$

Tabletop experiments:

Schematic drawing of the Eöt-Wash group's search for deviations from Newton's Law of Gravitation.

Image: Eöt-Wash group, University of Washington



Cavendish-type experiments using torsion pendulum