Mini-Black Holes at LHC

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Outlook

- ***Introduction:***
- **What is BH;**
- **Different types of BHs**

- ***Theoretical problems of BH formation:***
- **Semiclassical approach and geometrical cross section;**
- **Trapped surface arguments**

Outlook

BH formation in collision of matter

- 4-dim (astrophysics)

- TeV Gravity (micro-black holes) and LHC

  D-dim, D= 5, 6,…

- 5-dim AdS to 4-dim QCD (RHIC and LHC,

  Nica)

Outlook

- **Applications:**

- ***Black holes production in Tev Gravity:***
- **TeV Gravity**
- **BH production in M_D**

- ***Dual description of QGP(Quark Gluon Plasma)***
- **QGP in dual description;**
- **Trapped surface area and multiplicity;**
- **BH charge and chemical potential**

I. Introduction. Black Holes in GR

4-dimensional Schwarzschild Solution

$$R_{\mu\nu} = 0$$

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) dt^2 + \left(1 - \frac{R_S}{r}\right)^{-1} dr^2 + r^2 d\Omega_2{}^2$$

$$R_S = 2\, G_{Newton}\, M_{BH}$$

$$G_{Newton} = \frac{1}{M_{Pl}^2}$$

I. Introduction. Black Holes in GR

D-dimensional Schwarzschild Solution

$$R_{\mu\nu} = 0, \quad \mu,\nu = 0,1\ldots D-1$$

$$ds^2 = -\left(1-\left(\frac{R_S}{r}\right)^{D-3}\right)dt^2 + \left(1-\left(\frac{R_S}{r}\right)^{D-3}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}{}^2$$

$$R_S = \gamma_{BH}(D)\frac{1}{M_D}\left(\frac{M_{BH}}{M_D}\right)^{\alpha_{BH}}$$

$$\alpha_{BH} = \frac{1}{D-3}$$

$$\gamma_{BH}(D) = \frac{1}{\sqrt{\pi}}\left(\frac{8\Gamma(D-1/2)}{D-2}\right)^{1/(D-3)}$$

$$G_D = \frac{1}{M_D{}^{D-2}}$$

$$G_4 \equiv G_{Newton}$$

BLACK HOLES in GR. Historical Remarks

- The Schwarzschild solution has been found in 1916.

  The Schwarzschild solution(SS) is a solution of the vacuum Einstein equations, which is spherically symmetric and depends on a positive parameter M, the mass.

  In the coordinate system in which it was originally discovered, (t,r,theta,phi), had a singularity at r=2M
- In 1923 Birkoff proved a theorem that the Schwarzschild solution is the only spherically symmetric solution of the vacuum E.Eqs.
- In 1924 Eddington, made a coordinate change which transformed the Schwarzschild metric into a form which is not singular at r=2M
- In 1933 Lemaitre realized that the singularity at r=2M is not a true singularity
- In 1958 Finkelstein rediscovered Eddington's transformation and realized that the hypersurface r=2M is an event horizon, the boundary of the region of spacetime which is causally connected to infinity.

BLACK HOLES in GR. Historical Remarks

- In 1950 Synge constructed a systems of coordinates that covers the complete analytic extension of the SS
- In 1960 Kruskal and Szekeres (independently) discovered a single most convenient system that covers the complete analytic extension of SS
- In 1964 Penrose introduced the concept of null infinity, which made possible the precise general definition of a future event horizon as the boundary of the causal past of future null infinity.
- In 1965 Penrose introduced the concept of a closed trapped surface and proved the first singularity theorem (incompleteness theorem).
- Hawking-Penrose theorem: a spacetime with a complete future null infnity which contains a closed trapped surface must contain a future event horizon (H-E-books, 9-2-1)

$$R_{\mu\nu} = 0$$

$$ds^2 = -\left(1 - \frac{r_S}{r}\right)dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega_2{}^2$$

- **Asymptoticaly flat**
- **Birkhoff's theorem: Schwarzschild solution is the unique spherically symmetric vacuum solution**
- **Singularity**

$$R^{\mu\nu\rho\delta} R_{\mu\nu\rho\delta} = \frac{12 G^2 M^2}{r^6}$$

$$ds^2 = -\left(1-\frac{r_S}{r}\right)dt^2 + \left(1-\frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega_2{}^2$$

- Geodesics

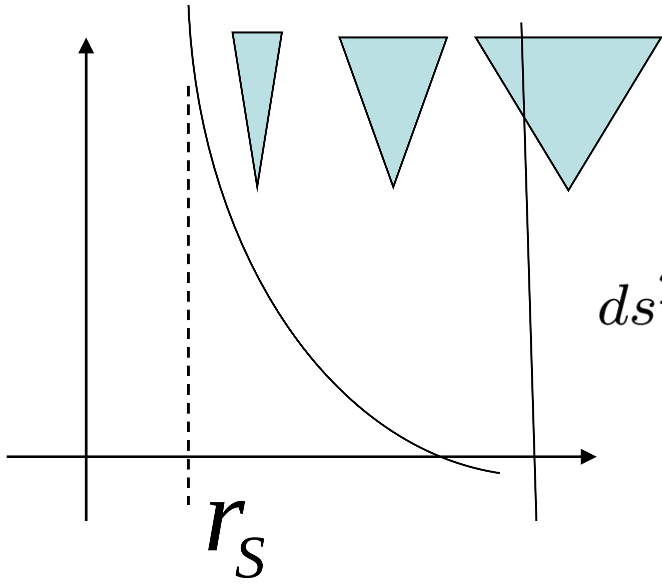
$$\frac{d^2x^\mu(\lambda)}{d\lambda^2} + \Gamma^\mu_{\rho\gamma}\frac{dx^\rho(\lambda)}{d\lambda}\frac{dx^\gamma(\lambda)}{d\lambda} = 0,$$

Null Geodesics

$$ds^2 = 0 = -\left(1-\frac{r_S}{r}\right)dt^2 + \left(1-\frac{r_S}{r}\right)^{-1} dr^2$$

Null Geodesics

$$\frac{dt}{dr} = \pm\left(1 - \frac{r_S}{r}\right)^{-1}$$



Regge-Wheeler coordinates

$$r^* = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$$

$$ds^2 = \left(1 - \frac{2GM}{r}\right)\left(-dt^2 + dr^{*2}\right) + r^2 d\Omega^2$$

Eddington-Finkelstein coordinates

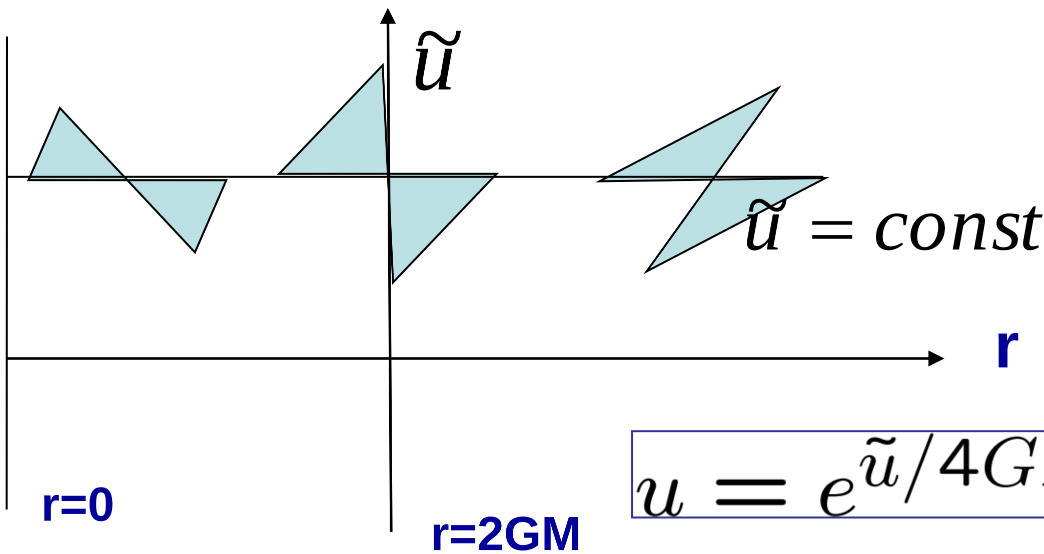
$$\tilde{u} = t + r^*$$
$$\tilde{v} = t - r^*$$

$$r^* = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) d\tilde{u}^2 + 2d\tilde{u}dr + r^2 d\Omega^2$$

The determinant of the metric is regular at r=2GM

$$g = -r^4 \sin^2 \theta$$

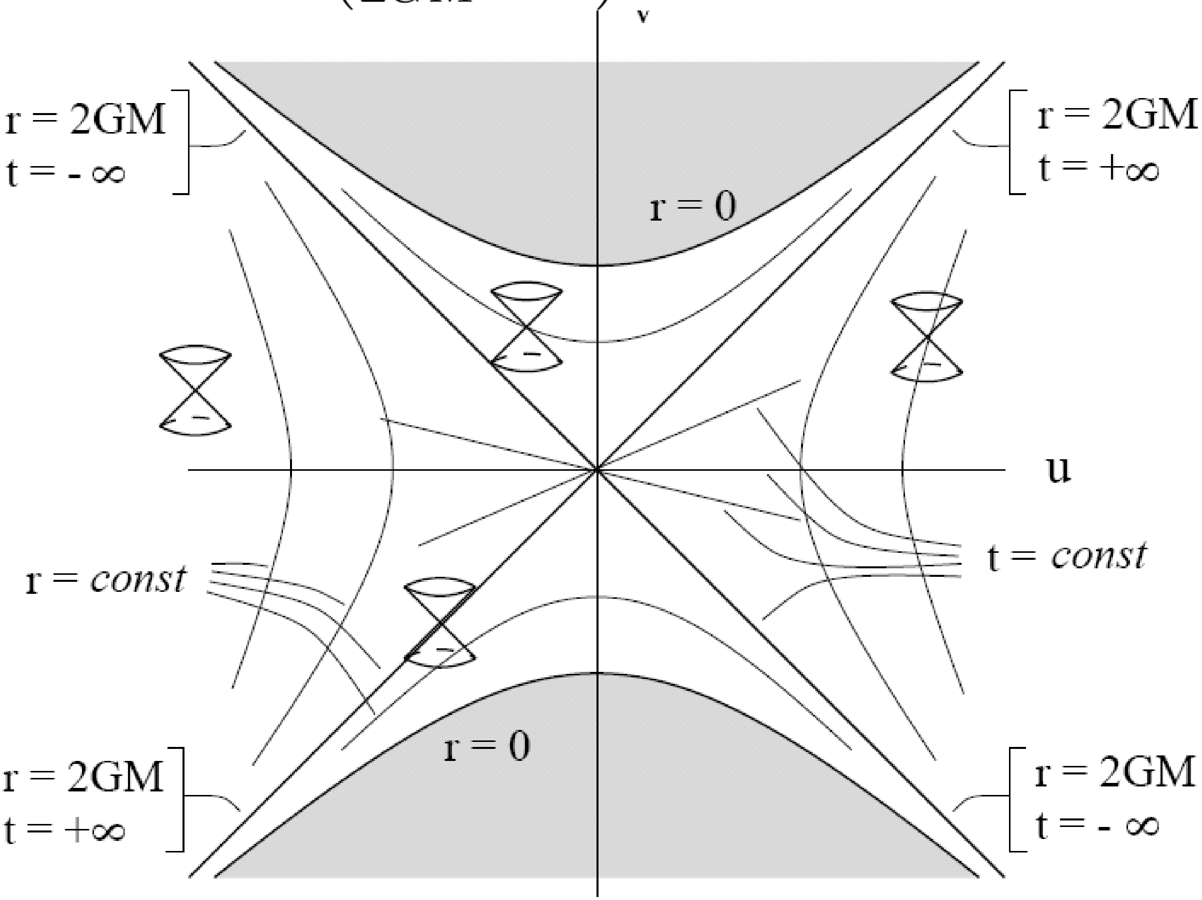


$$u = e^{\tilde{u}/4GM} \quad v = e^{-\tilde{v}/4GM}$$

Kruskal coordinates

$$ds^2 = \frac{32G^3M^3}{r}e^{-r/2GM}(-dv^2 + du^2) + r^2 d\Omega^2$$

$$(u^2 - v^2) = \left(\frac{r}{2GM} - 1\right) e^{r/2GM}$$



Kruskal diagram

II. BLACK HOLE FORMATION

- **Thorn's hoop conjecture:**

BH forms if the linear size of clumping matter *l* is

comparable to the Schwarzschild radius

R_S of a BH of mass *m*

$$l < R_S$$

$$R_S = 2Gm$$

II. BLACK HOLE FORMATION

Modified Thorn's hoop conjecture (for colliding particles):

BH forms if the impact parameter *b* is comparable to the Schwarzschild radius R_s of a BH of mass *E*.

- **Classical geometrical cross-section**

$$\sigma(1+1 \to \mathrm{BH}) \sim \pi R_S^2 \sim G^2 \mathrm{E}^2$$

II. BLACK HOLE FORMATION

Historical remarks

4-dim

$$M_{Pl} \simeq 10^{19} Gev$$

D-dim, D= 5, 6,…

$$M_{Pl} \simeq 1\, Tev$$

II. BLACK HOLE FORMATION

Historical remarks. 4-dim $M_{Pl} \cong 10^{19} Gev$

- **In 1987 't Hooft and Amati, Ciafaloni and Veneziano conjectured that in string theory and in QG at energies much higher than the Planck mass BH emerges.**

  Aichelburg-Sexl shock waves to describe particles,

  Shock Waves ------ > BH

- **Colliding plane gravitation waves to describe particles**

  Plane Gravitational Waves ----- > BH

  I.A., Viswanathan, I.Volovich, Nucl.Phys., 1995

- **Boson stars (solitons) to describe particles**

  M.Choptuik and F.Pretorius, Phys.Rev.Lett, 2010

II. BLACK HOLE FORMATION

Historical remarks. D-dim, D= 5, 6,…

- **TeV Gravity (1998)**

  N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, I. Antoniadis, 1998

- **TeV Gravity to produce BH at Labs (1999)**

  Banks, Fischler, hep-th/9906038
  I.A., hep-th/9910269,
  Giuduce, Rattazzi, Wells, hep-ph/0112161
  Giddings, hep-ph/0106219
  Dimopolos, Landsberg, hep-ph/0106295,

……………………………

II. BLACK HOLE FORMATION

- Classical geometric cross-section has got support from **trapped surface** estimations

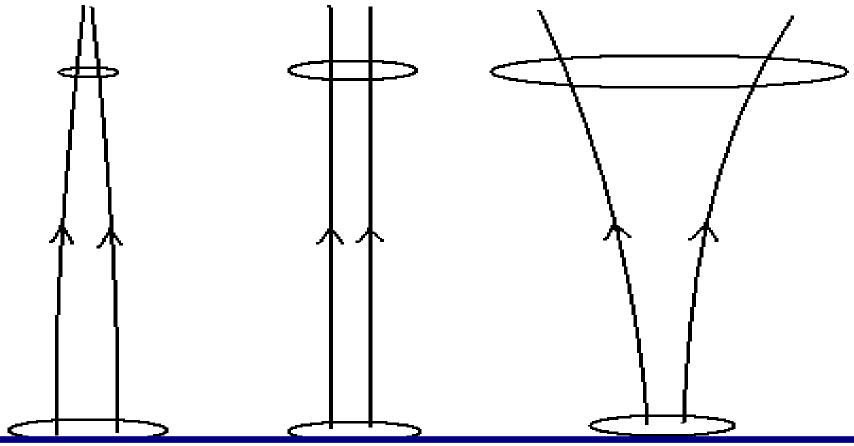
R.Penrose, unpublished, 1974
D.M.Eardley and S.B. Giddings, 2002
H.Yoshino and Y. Nambu, 2003
S. B. Giddings and V. S. Rychkov, 2004

……….

II. BLACK HOLE FORMATION

Trapped Surface

- A trapped surface is a two dimensional spacelike surface whose two null normals have negative expansion (=Neighbouring light rays, normal to the surface, must move towards one another)



D-dim gravitational model of relativistic particles

$$R > R_S, \quad ds^2 = (1-\frac{R_S}{R})dt^2 + (1-\frac{R_S}{R})^{-1}dR^2 + R^2 d\Omega_2^2$$

$R < R_S$

Tolman-Florides interior incompressible perfect fluid solution

or

Static spherical symmetric solitonic solution of gravity-matter E.O.M. (boson stars)

Tolman, 1933

$$R < R_S, \quad ds^2 = -\frac{1}{4}(3(1-\frac{a^2}{R_{st}^2})^{1/2} - (1-\frac{R^2}{R_{st}^2})^{1/2})^2 dt^2 + (1-\frac{R^2}{R_{st}^2})^{-1}dR^2 + R^2 d\Omega_2^2$$

$R_{st}^2 > a^2 > R,$

specified distributionby a and \rho,

a - radius of the sphere bounding the mass

$$\frac{1}{R_{st}^2} = \frac{8\pi}{3}\rho$$

$$m = \frac{4\pi}{3}\rho a^3$$

D-dim gravitational model of relativistic particles

Florides, 1974

$$ds^2 = -\left(1 - \frac{2m}{a}\right) \exp\left[\int_a^r \frac{2\mu(r)dr}{r^2(1-\frac{2\mu(r)}{r})}\right] dt^2 + \frac{1}{1-\frac{2\mu(r)}{r}} dr^2 + r^2 d\Omega_2^2$$

$$\mu(r) = 4\pi \int_0^r \rho(r) r^2 dr$$

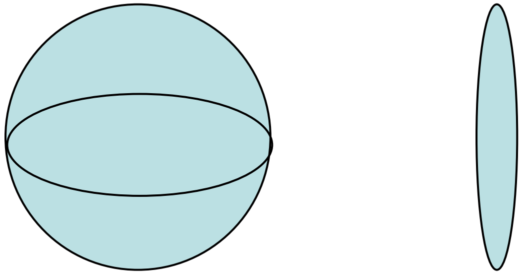
a is the radius of the sphere bounding the mass distribution and m=mu (a).

In the Florides interior solution the condition a>2m still holds, but there is nothing to prevent a from attaining values arbitrarily close to 2m

Shock waves from the Schwarzschild metric

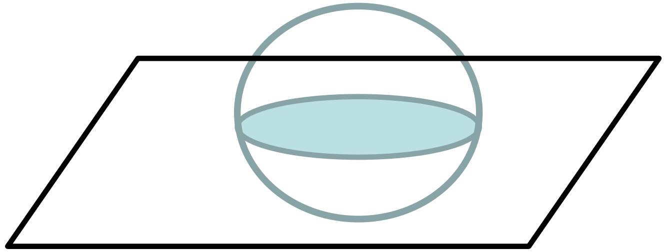
$$R > R_S, \quad ds^2 = (1-(\frac{R_S}{R})^{D-3})dt^2 + (1-(\frac{R_S}{R})^{D-3})^{-1}dR^2 + R^2 d\Omega^2_{D-2}$$

Boost → flattening of the Schwarzschild sphere



$$m = \frac{p}{\gamma}, \quad p - fixed$$

Perfect fluid on the brane? Does not matter? All information about interior is erased?



II. BLACK HOLE FORMATION

Metric of the space-time with shock wave

- M_D with a shock wave (Aichelburg-Sexl metric)

$$ds^2 = -du dv + dx^{i2} + F(x^i)\delta(u)\, du^2, \quad F(x^i) = \frac{c}{\rho^{D-4}}$$

Technical details. Schwarzschild metric ⇒ Shock waves

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2 d\Omega_2^2$$

$$g(R) = 1 - \frac{R_S}{R}$$

Plane coordinates

$$t, R, \theta, \varphi \Rightarrow u, v, z^2, z^3,$$

$$ds^2 = ds^2{}_{M^4} + ds^2{}_{(1,Sch)}$$

$$u = z^0 + z^1, V = -z^0 + z^1$$

$$ds^2_{(1),Sch} \quad = \quad \frac{2m}{R}\left(dt^2 + dR^2\right)$$

$$ds^2_{(1,Sch)} = \frac{2m}{R}\left(dt^2 + \frac{1}{R^2}\left(z_1 dz_1 + z_2 dz_2 + z_3 dz_3\right)^2\right)$$

Boost

$$t \;= \gamma(y_0 + vy_1),$$

$$z_1 = \gamma(vy_0 + y_1),$$

$$\vec{z} = \vec{y}, \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\gamma \rightarrow \infty$$

$$m \rightarrow 0, \gamma \rightarrow \infty, \gamma m = fixed \equiv p$$

Technical details.

Shock waves from the Schwarzschild metric

$$ds^2_{(1,Sch)} = \qquad \gamma \to \infty$$

$$= \frac{2p\gamma}{(\gamma^2(vy_0+y_1)^2+y_2^2+y_3^2)^{1/2}}((dy_0+vdy_1))^2$$

$$+ \frac{2p\gamma^3}{(\gamma^2(vy_0+y_1)^2+y_2^2+y_3^2)^{3/2}}(vy_0+y_1)^2(vdy_0+dy_1)^2$$

$$+ \frac{4p\gamma}{(\gamma^2(vy_0+y_1)^2+y_2^2+y_3^2)^{3/2}}(vy_0+y_1))(vdy_0+dy_1)(y_2dy_2+y_3dy_3)$$

$$+ \frac{1}{\gamma}\frac{4p}{\gamma^2(vy_0+y_1)^2+y_2^2+y_3^2)^{3/2}}(y_2dy_2+y_3dy_3)^2$$

$$m \to 0, \gamma \to \infty, \gamma m = \textit{fixed} \equiv p \qquad m = \frac{p}{\gamma}$$

Technical details.

Shock waves from the Schwarzschild metric

$$ds^2 = ds^2 + ds^2{}_{pert} \qquad m = \frac{p}{\gamma} \qquad m \to 0, \gamma \to \infty, p - fixed$$

Lemma For an integrable function f takes place the identity

$$\lim_{v \to 1} \gamma f\left(\gamma^2 (Y_0 + vY_1)^2\right) = \delta(Y_0 + Y_1) \int f(x^2) dx$$

Proof

$$\gamma \int f\left(\gamma^2 (Y_0 + vY_1)^2\right) g(Y_0) dY_0$$

$$\gamma (Y_0 + vY_1) = x$$

$$\lim_{v \to 1} \int f\left(x^2\right) g(-vY_1 + \frac{1}{\gamma} x) dx = g(-Y_1) \int f\left(x^2\right) dx$$

Technical details.

Shock waves from the Schwarzschild metric

Лемма 1 В $\mathcal{D}'(R^2)$

$$\frac{1}{\sqrt{w^2+\epsilon^2 z^2}} = \delta(w)\ln\frac{4}{C\epsilon^2} + \frac{1}{|w|} + \delta(w)\ln\frac{C}{z^2} + \mathcal{O}(\epsilon^2)$$

где $C > 0$ и

$$\left(\frac{1}{|w|}, g\right) = \int_{|w|<1} \frac{g(w)-g(0)}{|w|}dw + \int_{|w|>1}\frac{g(w)dw}{|w|}$$

$$I(\epsilon) = \int \frac{g(w)f(z)}{\sqrt{w^2+\epsilon^2 z^2}} = \int_{|w|<1} \frac{g(w)-g(0)}{\sqrt{w^2+\epsilon^2 z^2}} f(z)dwdz$$

$$+ \int_{|w|>1} \frac{g(w)}{\sqrt{w^2+\epsilon^2 z^2}} f(z)dwdz$$

$$+ g(0)\int_{|w|<1} \frac{1}{\sqrt{w^2+\epsilon^2 z^2}} f(z)dwdz$$

$$= \int_{|w|<1} \frac{g(w)-g(0)}{|w|} f(z)dwdz + \int_{|w|>1} \frac{g(w)f(z)dwdz}{|w|} + g(0)\int f(z)\ln\frac{4}{\epsilon^2 z^2} + \mathcal{O}(\epsilon^2)$$

Technical details.

Shock waves from the Schwarzschild metric

$$ds^2 = du\left(-dv + 4p\left(\frac{1}{|u|} + \delta(u)\ln\frac{4\gamma^2}{\hat{y}_2^2 + \hat{y}_3^2}\right)du\right) + d\hat{y}_2^2 + d\hat{y}_3^2 \cdot$$

$$\begin{aligned}
\bar{u} &= u \\
\bar{v} &= v + 4p\ln(\sqrt{u^2} - u) - 4p\ln 4\gamma^2\theta(u)
\end{aligned}$$

$$\begin{aligned}
d\bar{v} &= dv + 4p\frac{\frac{udu}{|u|} - du}{\sqrt{u^2} - u} + 4p\ln 4\gamma^2\delta(u)du \cdot \\
&= dv - 4p\frac{du}{|u|} - 4p\ln 4\gamma^2\delta(u)du
\end{aligned}$$

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$$ds^2 = -dudV + dx^{i2} + F(x^i)\delta(u)\,du^2, \quad F(x^i) = \frac{c}{|x^i|^{D-4}}$$

$$D = 4 \quad F(x^i) = p\ln|x^i|$$