

Introduction to Cosmology:
Lecture #1
Overview of the relevant facts

Dmitry Gorbunov

Institute for Nuclear Research of RAS, Moscow, Russia

Outline

- 1 “Natural” units
- 2 The Universe today
 - General properties
 - Energy balance
- 3 The future Universe
- 4 The early Universe

"Natural" units in particle physics

$$\hbar = c = k_B = 1$$

measured in GeV: energy E , mass M , temperature T

$$m_p = 0.938 \text{ GeV}, \quad 1 \text{ K} = 8.6 \cdot 10^{-14} \text{ GeV}$$

measured in GeV^{-1} : time t , length L

$$1 \text{ s} = 1.5 \cdot 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ cm} = 5.1 \cdot 10^{13} \text{ GeV}^{-1}$$

$$\text{Gravity (General Relativity): } V(r) = -G \frac{m_1 m_2}{r} \quad [G] = M^{-2}$$

$$M_{\text{Pl}} = 1.2 \cdot 10^{19} \text{ GeV} = 22 \mu\text{g}$$

$$G \equiv \frac{1}{M_{\text{Pl}}^2}$$

"Natural" units in cosmology

$$1 \text{ Mpc} = 3.1 \cdot 10^{24} \text{ cm}$$

$$1 \text{ AU} = 1.5 \cdot 10^{13} \text{ cm}$$

$$1 \text{ ly} = 0.95 \cdot 10^{18} \text{ cm}$$

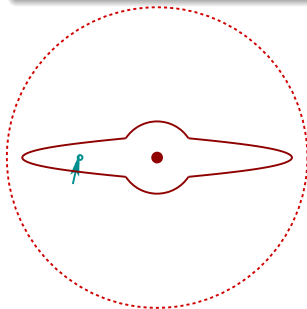
$$1 \text{ pc} = 3.3 \text{ ly} = 3.1 \cdot 10^{18} \text{ cm}$$

mean Earth-to-Sun distance

distance light travels in one year

$$1 \text{ yr} = 3.16 \cdot 10^7 \text{ s}$$

distance to object which has
a parallax angle of one arcsec



100 AU — Solar system size

1.3 pc — nearest-to-Sun stars

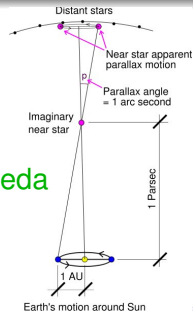
1 kpc — size of dwarf galaxies

50 kpc — distance to dwarves

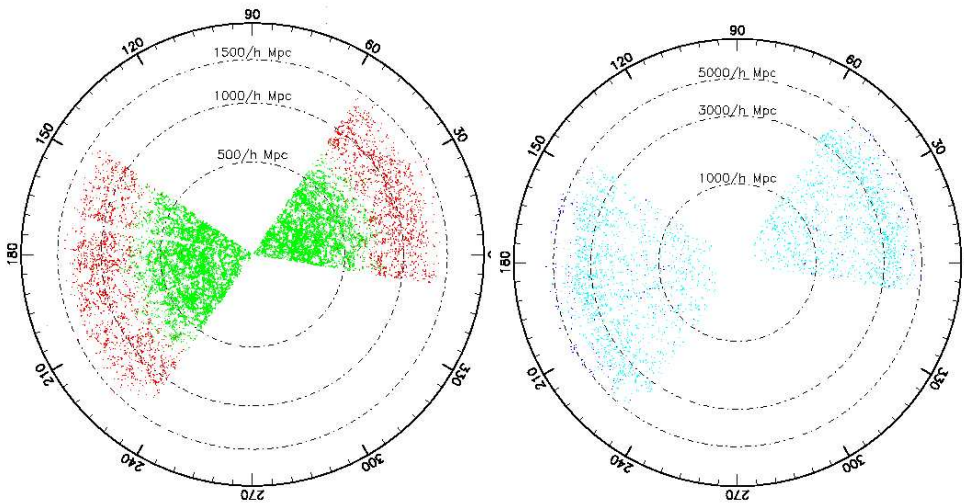
0.8 Mpc — distance to Andromeda

1-3 Mpc — size of clusters

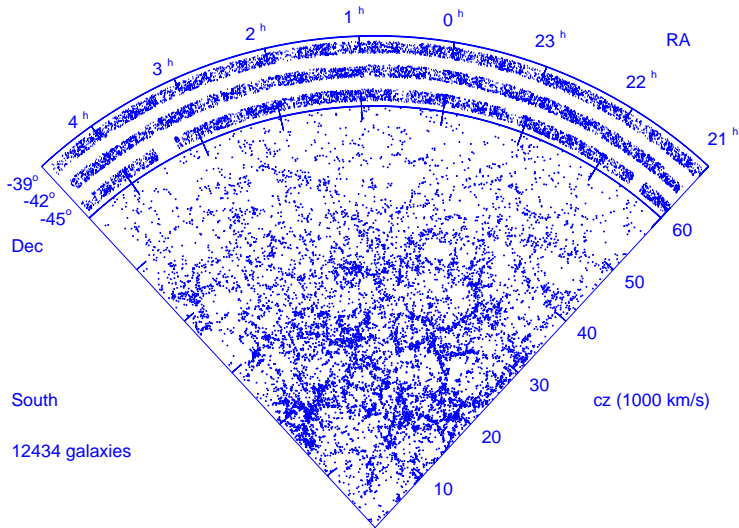
15 Mpc — distance to Virgo



Very large scales: homogeneity and isotropy



Large scales: structures

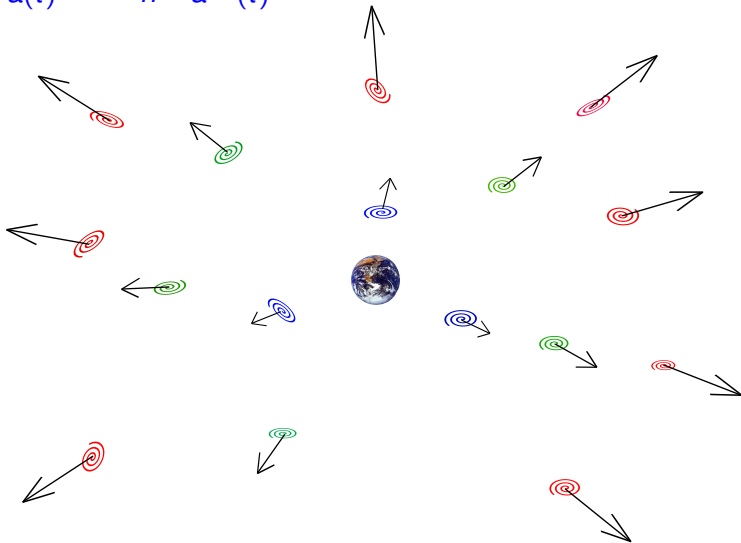


Expansion:

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

$$L \propto a(t)$$

$$n \propto a^{-3}(t)$$

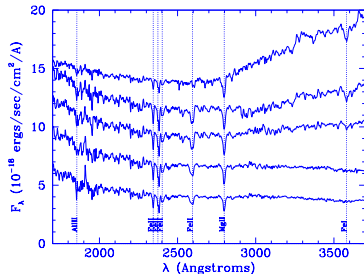
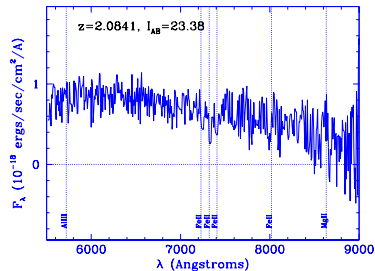


Expansion: redshift z

$$\lambda_{\text{abs.}} / \lambda_{\text{em.}} \equiv 1 + z$$

$$z \ll 1 \text{ Hubble law : } z = H_0 r$$

$$H_0 = h \cdot 100 \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \quad h = 0.705 \pm 0.015$$



Expansion: redshift z

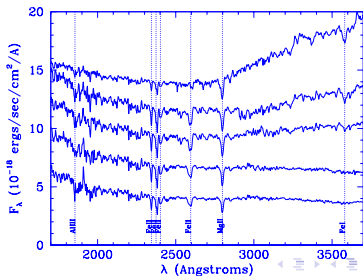
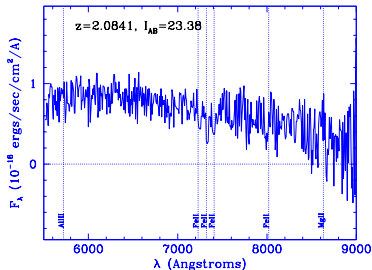
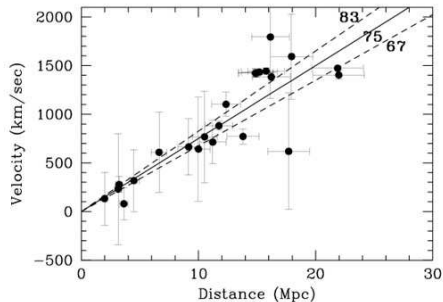
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standard candles

Hubble Diagram for Cepheids (flow-corrected)



The Universe: age & spatial geometry

$$[H_0] = L^{-1} = t^{-1}$$

time scale: $t_{H_0} = H_0^{-1} \approx 14 \cdot 10^9 \text{ yr}$

spatial scale: $l_{H_0} = H_0^{-1} \approx 4.3 \cdot 10^3 \text{ Mpc}$

age and size of the visible Universe

t_{H_0} is in agreement with various observations

homogeneity and isotropy in 3d:

flat, spherical or hyperbolic

Observations:

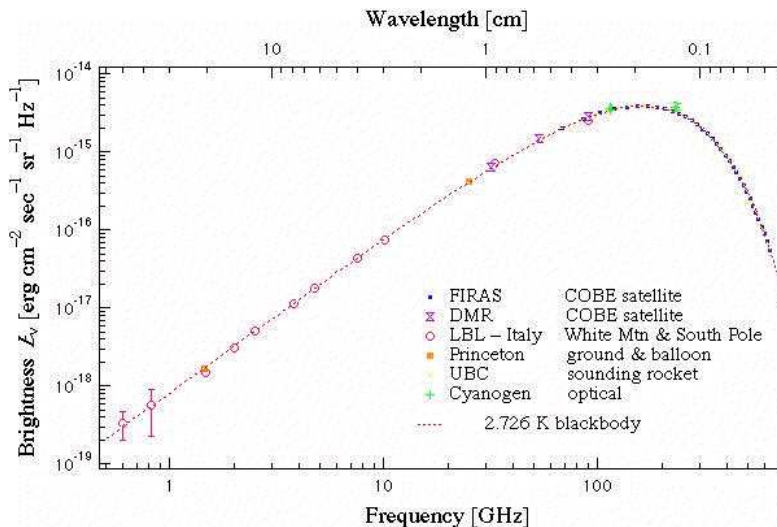
“very” flat

$$l_{H_0} / R_{\text{curv}} \lesssim 0.1$$

The Universe is hot:

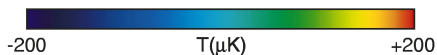
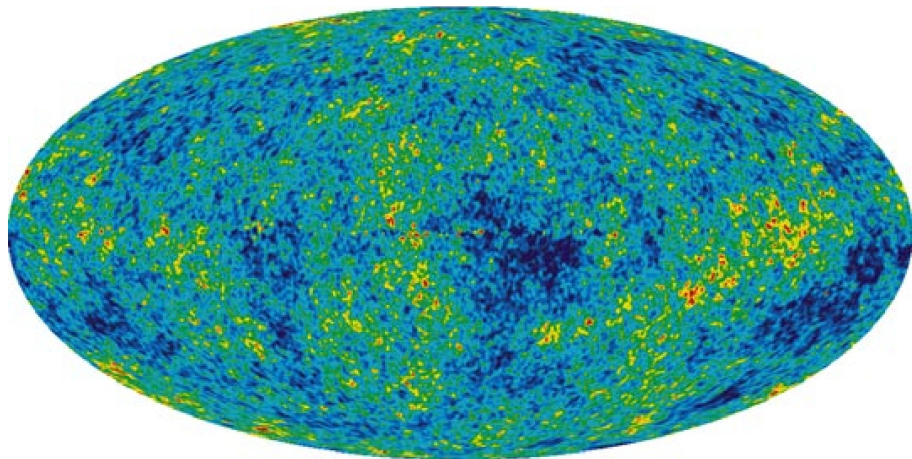
$$T_0 = 2.726 \pm 0.001 \text{ K}$$

$$n_\gamma = 411 \text{ cm}^{-3}$$



CMB anisotropy:

$$\delta T/T_0 \sim 10^{-5}$$



CMB anisotropy:

$$\delta T(\mathbf{n}) \equiv T(\mathbf{n}) - T_0 - \delta T_{\text{dipole}}$$

$$\delta T_{\text{dipole}} = 3.346 \pm 0.017 \text{ mK}$$

$$\delta T(\mathbf{n}) = \sum_{l,m} a_{l,m} Y_{lm}(\mathbf{n})$$

$$a_{l,m}^* = (-1)^m a_{l,-m}$$

observations

$\delta T(\mathbf{n})$ is Gaussian

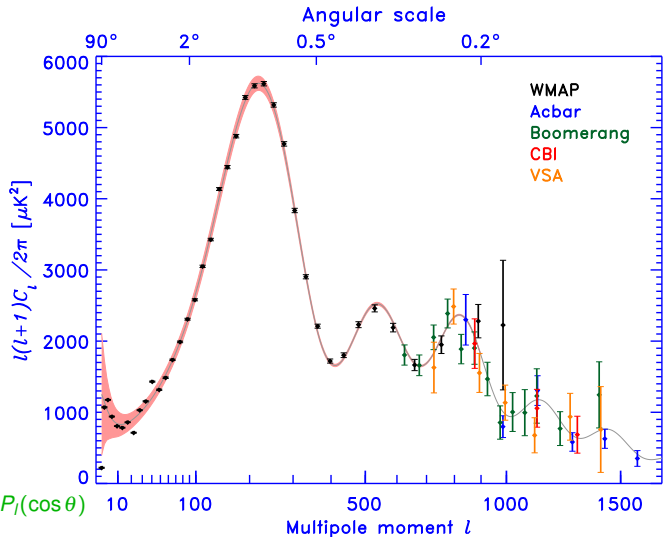
$$\langle a_{l,m} a_{l',m'}^* \rangle = C_{lm} \delta_{ll'} \delta_{mm'}$$

in isotropic Universe

$$C_{lm} = C_l$$

$$\langle \delta T(\mathbf{n}_1) \delta T(\mathbf{n}_2) \rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos \theta)$$

$$\langle \delta T^2 \rangle = \sum_l \frac{2l+1}{4\pi} C_l \approx \int \frac{l(l+1)}{2\pi} C_l d \ln l$$



Energy density

order-of-magnitude estimate:

$$G\rho_0 \sim H_0^2$$

Spatially flat Universe

$$\rho_c = \frac{3}{8\pi} H_0^2 M_{\text{Pl}}^2 \approx 0.53 \cdot 10^{-5} \frac{\text{GeV}}{\text{cm}^3}$$

CMB contribution:

$$\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \cdot 10^{-4}$$

Baryon contribution:

$$\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.046$$

Neutrino contribution:

$$\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$$

Dark Matter contribution:

$$\Omega_{\text{DM}} \equiv \frac{\rho_{\text{DM}}}{\rho_c} = 0.23$$

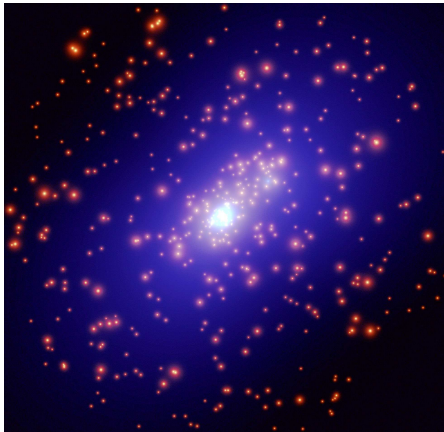
Dark Energy contribution:

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.73$$

Dark Matter in clusters

gravitational lensing

$$\rho_M \approx 0.25\rho_c$$



Dark Matter in clusters

X-rays from hot gas in clusters

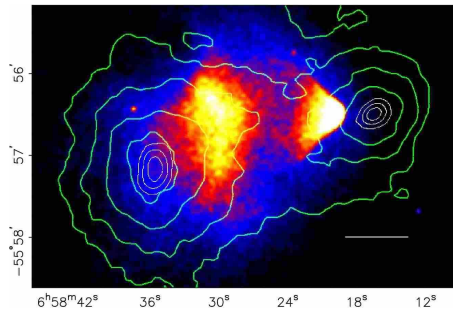
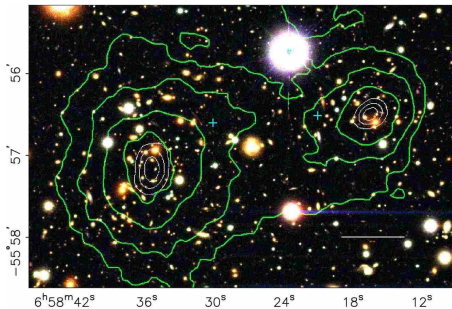
$$\frac{dP}{dR} = -\mu n_e(R) m_p \frac{GM(R)}{R^2}, \quad M(R) = 4\pi \int_0^R \rho(r) r^2 dr, \quad P(R) = n_e(R) T_e(R)$$

galaxies in clusters

$$3M \langle v_r^2 \rangle = G \frac{M^2}{R}$$

Milky Way: Virgo infall

Bullet clusters

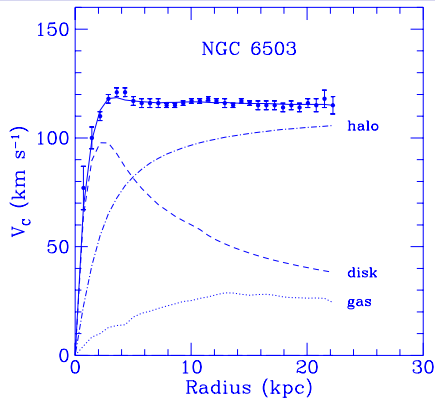


Dark Matter in galaxies:

flat rotation curves

$$v(R) = \sqrt{G \frac{M(R)}{R}}$$

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr$$



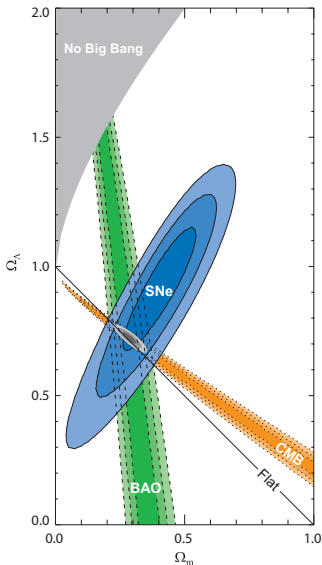
observations:

$$v(R) = \text{const}$$

visible matter:

filled regions $v(R) \propto \sqrt{R}$
 empty regions $v(R) \propto 1/\sqrt{R}$

Dark Energy: nonclumping matter?



- estimates of Matter contribution confined in galaxies and clusters
 $\rho_c - \rho_M \neq 0$ but the Universe is flat, so $\rho_{curv} \simeq 0$
- corrections to the Hubble law : red shift — brightness curves for standard candles — SN Ia
- The age of the Universe
- CMB anisotropy, large scale structures (BAO), etc

$$\rho_\Lambda = 0.73\rho_c$$

$$\rho_\Lambda \sim 10^{-5} \text{ GeV/cm}^3 \sim (10^{-11.5} \text{ GeV})^4$$

The future Universe

the same period with the same expansion rate

