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  Massive vector field

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$-1/6$ — the contribution of longitudinal vector bosons (if the mass is generated by the Higgs mechanism, this is the Higgs loop contribution)

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- Gross, Wilczek; Politzer (1973)

Coulomb gauge


$$= \frac{i}{\vec{q}^{\,2}}$$


$$= -\frac{i}{q^2 + i0}\left(\delta^{ij} - \frac{q^i q^j}{\vec{q}^{\,2}}\right)$$

No ghosts

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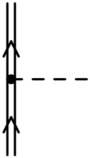
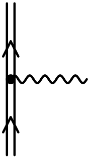
No ghosts

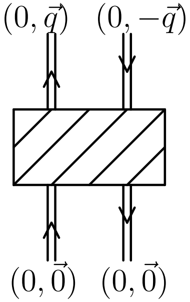
Infinitely heavy quark

 $= \dfrac{i}{p_0 + i0}$

 $= i g_0 t^a$

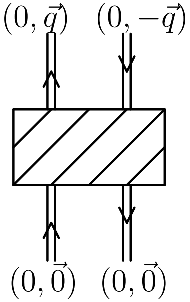
 $= 0$

Quark–antiquark potential



Quantum mechanics: $iU_{\vec{q}}$

Quark–antiquark potential



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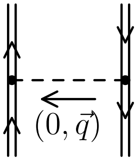
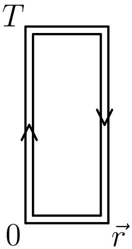


$$= -iC_F g_0^2 D(0,\vec{q}) = iC_F \frac{g_0^2}{\vec{q}^{\,2}}$$

$$U_{\vec{q}} = C_F g_0^2 D(0,\vec{q}) = -C_F \frac{g_0^2}{\vec{q}^{\,2}} \qquad U(r) = -C_F \frac{\alpha_s}{r}$$

Wilson loop

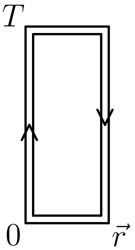


$$= e^{-i\,U(\vec{r})\,T} = 1 - i\,U(\vec{r})\,T$$

$$T \gg r$$

Wilson loop

$$T \gg r$$



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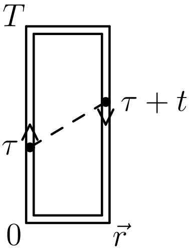
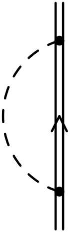


$$= -i\,C_F\,g_0^2\,T \int D(t,\vec{r})\,dt$$

$$= -i\,C_F\,g_0^2\,T \int \frac{d^{d-1}\vec{q}}{(2\pi)^{d-1}}\,D(0,\vec{q})\,e^{i\,\vec{q}\vec{r}}$$

Self-energy and vertex corrections



$$\sim \int \frac{d^{d-1}\vec{k}}{\vec{k}^{\,2}} = 0$$

Self-energy of a classical point charge

 propagates along time,  along space

$$\sim D(t=0, \vec{r}=0) \sim \int \frac{d^{d-1}\vec{k}}{\vec{k}^{\,2}} e^{i\vec{q}\cdot\vec{r}}\Big|_{\vec{r}=0} \sim U(\vec{r}=0) \Rightarrow 0$$

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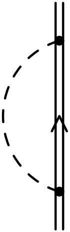

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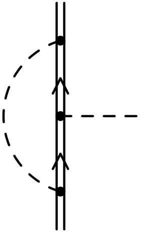
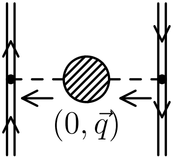


$$= 0$$

Vacuum polarization



$$U_{\vec{q}} = C_F g_0^2 D(0, \vec{q})$$

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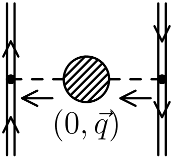

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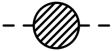
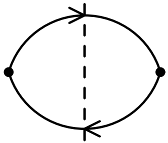


$$= -i\vec{q}^{\,2}\Pi(q)$$

$$D(q) = -\frac{1}{\vec{q}^{\,2}} \frac{1}{1 - \Pi(q)} = -\frac{1}{\vec{q}^{\,2}} \left(1 + \Pi(q)\right)$$

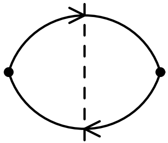
Quark loop



Lorentz-invariant $\rho_q(s) \geqslant 0$

$$\Pi_q(q^2) = \int \frac{\rho_q(s)\, ds}{q^2 - s + i0}$$

Quark loop



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$$= -C_F g_0^2 \left[\left(1 - \int \frac{\rho_q(s)\,ds}{s} \right) \frac{1}{\vec{q}^{\,2}} + \int \frac{\rho_q(s)\,ds}{\vec{q}^{\,2} + s} + \cdots \right]$$

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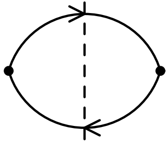

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$$U(r) = -C_F \frac{g_0^2}{4\pi r} \left[1 - \int \frac{\rho_q(s)\, ds}{s} + \int \rho_q(s)\, e^{-\sqrt{s}\, r}\, ds + \cdots \right]$$

Screening

$$\rho_q(s) = T_F n_f \frac{g_0^2 s^{-\varepsilon}}{(4\pi)^{d/2}} \left(\frac{4}{3} + \cdots \right)$$

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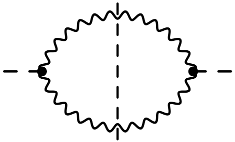
Transverse gluons



$$\Pi_t(q_0^2, \vec{q}^{\,2}) = \int \frac{\rho_t(s, \vec{q}^{\,2})\, ds}{q^2 - s + i0}$$

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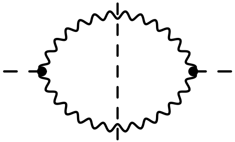

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$s \gg \vec{q}^{\,2}$:

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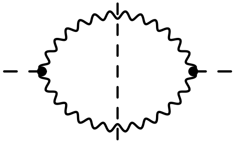

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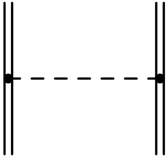
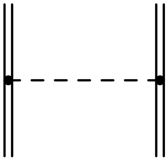
Coulomb gluon



+ transverse-gluon vacuum

$$E_0 = U(r)$$

Coulomb gluon



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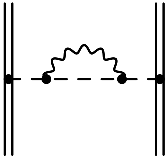
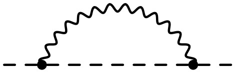
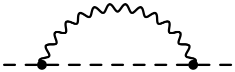


Second order of perturbation theory
Energy decreases — antiscreening



Depends on $\vec{q}^2$ but not on q^0

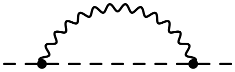
Is not given by a spectral representation



Depends on $\vec{q}^{\,2}$ but not on q^0

Is not given by a spectral representation

$$\Pi_c(\vec{q}^{\,2}) = \int \frac{d^d k}{(2\pi)^d} \frac{f(\vec{k}, \vec{q})}{k^2 + i0}$$



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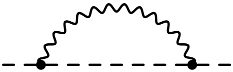

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$$\Pi_c(\vec{q}^{\,2}) = C_A \frac{g_0^2 \left(\vec{q}^{\,2}\right)^{-\varepsilon}}{(4\pi)^{d/2}} \left(\frac{4}{\varepsilon} + \cdots\right)$$

Results

$$U_{\vec{q}} = -\,C_F \frac{g_0^2}{\vec{q}^{\,2}} \left\{ 1 + \frac{g_0^2 (\vec{q}^{\,2})^{-\varepsilon}}{(4\pi)^{d/2}} \left[\left(\left(4 - \frac{1}{3} \right) C_A - \frac{4}{3} T_F n_f \right) \frac{1}{\varepsilon} \right. \right.$$

$$\left. \left. + \cdots \right] \right\}$$

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Renormalization

$$\frac{g_0^2}{(4\pi)^{d/2}} = \mu^{2\varepsilon} \frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma\varepsilon} \qquad Z_\alpha = 1 - \beta_0 \frac{\alpha_s}{4\pi\varepsilon}$$

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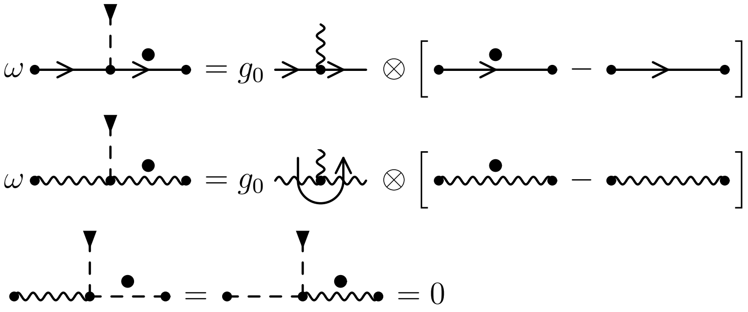
Asymptotic freedom

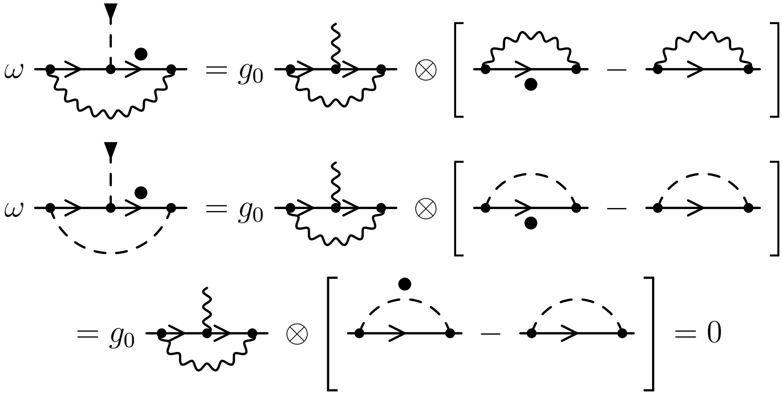
Ward identity

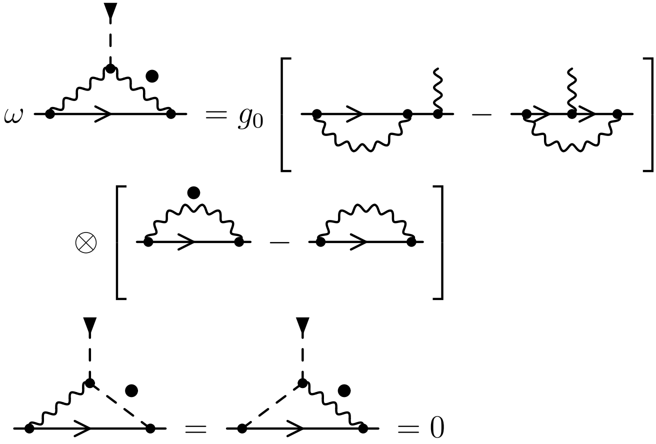
External Coulomb line with incoming momentum $q = \omega v$
(a dot means shifting the momentum by q)



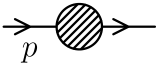
(colour structure) $\otimes$ (Lorentz structure)
(In covariant gauges, the second identity contains extra ghost terms)

1 loop

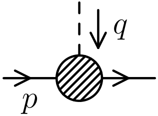



ω
= g0
−
⊗
−
=
= 0

Ward identity

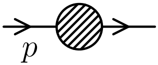


$$= -i\Sigma(p) \qquad S(p) = \frac{1}{\not{p} - m_0 - \Sigma(p)}$$



$$= i g_0 t^a \Gamma(p, q) \qquad \Gamma = \gamma_0 + \Lambda$$

Ward identity



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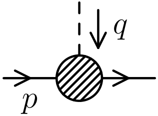

$= i g_0 t^a \Gamma(p, q)$

$$\Gamma = \gamma_0 + \Lambda$$

$$\omega\Lambda(p, \omega v) = \Sigma(p) - \Sigma(p + \omega v)$$
$$\omega\Gamma(p, \omega v) = S^{-1}(p + \omega v) - S^{-1}(p)$$

Coupling constant renormalization

$$g_0 = Z_\alpha^{1/2} g \qquad \Gamma = Z_\Gamma \Gamma_r \qquad S = Z_\psi S_r$$

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$$g_0 \Gamma Z_\psi Z_A^{1/2} = g \Gamma_r Z_\alpha^{1/2} Z_\Gamma Z_\psi Z_A^{1/2} = \text{finite}$$

$$Z_\alpha = \left(Z_\Gamma Z_\psi\right)^{-2} Z_A^{-1}$$

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Ward identity

$$Z_\Gamma Z_\psi = 1 \qquad Z_\alpha = Z_A^{-1}$$