

History

- ▶ Vanyashin, Terentev (1965)
Massive vector field

$$\beta_0 = \left(\frac{11}{3} - \frac{1}{6} \right) C_A$$

$-1/6$ — the contribution of longitudinal vector bosons
(if the mass is generated by the Higgs mechanism, this
is the Higgs loop contribution)

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- ▶ Khriplovich (1969)
- ▶ 't Hooft (1972), unpublished
- ▶ Gross, Wilczek; Politzer (1973)

Coulomb gauge

$$\begin{aligned} \bullet \text{---} \bullet &= \frac{i}{\vec{q}^2} \\ \bullet \text{~~~~} \bullet &= -\frac{i}{q^2 + i0} \left(\delta^{ij} - \frac{q^i q^j}{\vec{q}^2} \right) \end{aligned}$$

No ghosts

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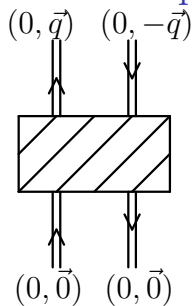
Infinitely heavy quark

$$\begin{array}{|c} \bullet \\ \hline \uparrow \\ \hline \bullet \end{array} = \frac{i}{p_0 + i0}$$

$$\begin{array}{|c} \bullet \\ \hline \uparrow \\ \hline \bullet \\ \hline \uparrow \\ \hline \bullet \end{array} \text{---} = ig_0 t^a$$

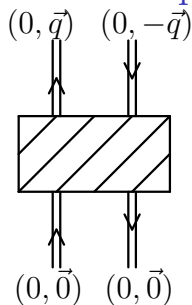
$$\begin{array}{|c} \bullet \\ \hline \uparrow \\ \hline \bullet \\ \hline \uparrow \\ \hline \bullet \end{array} \text{~~~~} = 0$$

Quark–antiquark potential



Quantum mechanics: $iU_{\vec{q}}$

Quark–antiquark potential



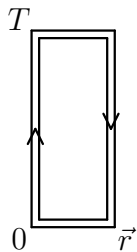
Quantum mechanics: $iU_{\vec{q}}$

$$= -iC_F g_0^2 D(0, \vec{q}) = iC_F \frac{g_0^2}{\vec{q}^2}$$

$$U_{\vec{q}} = C_F g_0^2 D(0, \vec{q}) = -C_F \frac{g_0^2}{\vec{q}^2}$$

$$U(r) = -C_F \frac{\alpha_s}{r}$$

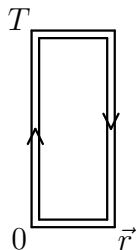
Wilson loop



$$T \gg r$$

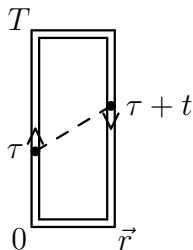
$$= e^{-iU(\vec{r})T} = 1 - iU(\vec{r})T$$

Wilson loop



$$= e^{-iU(\vec{r})T} = 1 - iU(\vec{r})T$$

$$T \gg r$$



$$= -i C_F g_0^2 T \int D(t, \vec{r}) dt$$

$$= -i C_F g_0^2 T \int \frac{d^{d-1}\vec{q}}{(2\pi)^{d-1}} D(0, \vec{q}) e^{i\vec{q}\vec{r}}$$

Self-energy and vertex corrections

$\sim \int \frac{d^{d-1}\vec{k}}{k^2} = 0$ Self-energy of a classical point charge

propagates along time, $\bullet - \bullet$ along space

$\sim D(t=0, \vec{r}=0) \sim \int \frac{d^{d-1}\vec{k}}{k^2} e^{i\vec{q}\cdot\vec{r}} \Big|_{\vec{r}=0} \sim U(\vec{r}=0) \Rightarrow 0$

Self-energy and vertex corrections

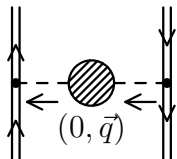
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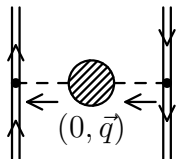
$= 0$

Vacuum polarization



$$U_{\vec{q}} = C_F g_0^2 D(0, \vec{q})$$

Vacuum polarization

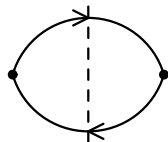


$$U_{\vec{q}} = C_F g_0^2 D(0, \vec{q})$$

$$-\text{[shaded circle]}- = -i\vec{q}^2 \Pi(q)$$

$$D(q) = -\frac{1}{\vec{q}^2} \frac{1}{1 - \Pi(q)} = -\frac{1}{\vec{q}^2} (1 + \Pi(q))$$

Quark loop

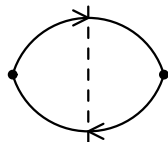


Lorentz-invariant

$$\rho_q(s) \geq 0$$

$$\Pi_q(q^2) = \int \frac{\rho_q(s) ds}{q^2 - s + i0}$$

Quark loop



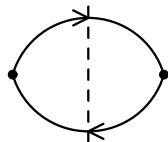
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$$\begin{aligned} U_{\vec{q}} &= -C_F \frac{g_0^2}{\vec{q}^2} \left[1 - \int \frac{\rho_q(s) ds}{\vec{q}^2 + s} + \dots \right] \\ &= -C_F g_0^2 \left[\left(1 - \int \frac{\rho_q(s) ds}{s} \right) \frac{1}{\vec{q}^2} + \int \frac{\rho_q(s) ds}{\vec{q}^2 + s} + \dots \right] \end{aligned}$$

Quark loop



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$$U(r) = -C_F \frac{g_0^2}{4\pi r} \left[1 - \int \frac{\rho_q(s) ds}{s} + \int \rho_q(s) e^{-\sqrt{s}r} ds + \dots \right]$$

Screening

$$\rho_q(s) = T_F n_f \frac{g_0^2 s^{-\varepsilon}}{(4\pi)^{d/2}} \left(\frac{4}{3} + \dots \right)$$

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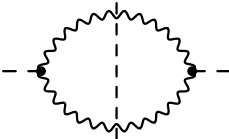
$$\int \frac{\rho_q(s) ds}{s + \vec{q}^2} \Big|_{\text{UV}} = \frac{4}{3} T_F n_f \frac{g_0^2}{(4\pi)^{d/2}} \int_{\sim \vec{q}^2}^{\infty} s^{-1-\varepsilon} ds = \frac{4}{3} T_F n_f \frac{\alpha_s}{4\pi\varepsilon}$$

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$$U_{\vec{q}} = -C_F \frac{g_0^2}{\vec{q}^2} \left[1 + \frac{4}{3} T_F n_f \frac{\alpha_s}{4\pi\varepsilon} + \dots \right]$$

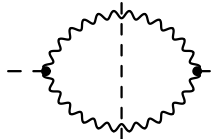
Transverse gluons



The diagram shows a gluon loop, represented by a wavy line forming a circle. A vertical dashed line passes through the center of the loop. Two external wavy lines enter and exit the loop at the left and right vertices, which are marked with black dots. The diagram is connected to the text on the right by a horizontal line.

$$\Pi_t(q_0^2, \vec{q}^2) = \int \frac{\rho_t(s, \vec{q}^2) ds}{q^2 - s + i0}$$

Transverse gluons



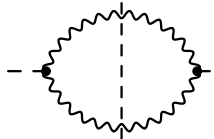
A Feynman diagram showing a gluon loop. The loop is represented by a wavy line forming a circle. A vertical dashed line passes through the center of the loop. Two external lines, represented by solid lines with dots at the vertices, enter and exit the loop from the left and right sides.

$$\Pi_t(q_0^2, \vec{q}^2) = \int \frac{\rho_t(s, \vec{q}^2) ds}{q^2 - s + i0}$$

$s \gg \vec{q}^2$:

$$\rho_t(s, \vec{q}^2) = C_A \frac{g_0^2 s^{-\epsilon}}{(4\pi)^{d/2}} \left(\frac{1}{3} + \dots \right)$$

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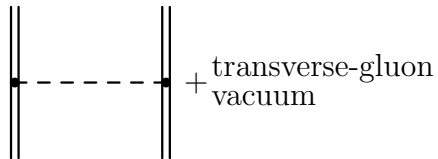

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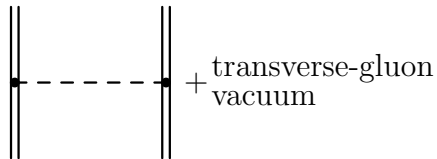
$$U_{\vec{q}} = -C_F \frac{g_0^2}{\vec{q}^2} \left[1 + \frac{1}{3} C_A \frac{\alpha_s}{4\pi\epsilon} + \dots \right]$$

Coulomb gluon

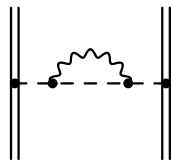


$$E_0 = U(r)$$

Coulomb gluon



$$E_0 = U(r)$$



Second order of perturbation theory
Energy decreases — **antiscreening**



Depends on \vec{q}^2 but not on q^0

Is not given by a spectral representation



Depends on \vec{q}^2 but not on q^0

Is not given by a spectral representation

$$\Pi_c(\vec{q}^2) = \int \frac{d^d k}{(2\pi)^d} \frac{f(\vec{k}, \vec{q})}{k^2 + i0}$$



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$$\Pi_c(\vec{q}^2) = C_A \frac{g_0^2 (\vec{q}^2)^{-\varepsilon}}{(4\pi)^{d/2}} \left(\frac{4}{\varepsilon} + \dots \right)$$

Results

$$U_{\vec{q}} = -C_F \frac{g_0^2}{\vec{q}^2} \left\{ 1 + \frac{g_0^2 (\vec{q}^2)^{-\varepsilon}}{(4\pi)^{d/2}} \left[\left(\left(4 - \frac{1}{3} \right) C_A - \frac{4}{3} T_F n_f \right) \frac{1}{\varepsilon} + \dots \right] \right\}$$

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Renormalization

$$\frac{g_0^2}{(4\pi)^{d/2}} = \mu^{2\varepsilon} \frac{\alpha_s(\mu)}{4\pi} Z_\alpha e^{\gamma\varepsilon} \quad Z_\alpha = 1 - \beta_0 \frac{\alpha_s}{4\pi\varepsilon}$$

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$$\beta_0 = \left(4 - \frac{1}{3} \right) C_A - \frac{4}{3} T_F n_f$$

Asymptotic freedom

Ward identity

External Coulomb line with incoming momentum $q = \omega v$
 (a dot means shifting the momentum by q)

$$\omega \text{---} \bullet \text{---} \bullet \text{---} \bullet = g_0 \text{---} \bullet \text{---} \bullet \text{---} \bullet \otimes \left[\text{---} \bullet \text{---} \bullet - \text{---} \bullet \text{---} \bullet \right]$$

$$\omega \text{---} \bullet \text{---} \bullet \text{---} \bullet = g_0 \text{---} \bullet \text{---} \bullet \text{---} \bullet \otimes \left[\text{---} \bullet \text{---} \bullet - \text{---} \bullet \text{---} \bullet \right]$$

$$\text{---} \bullet \text{---} \bullet \text{---} \bullet = \text{---} \bullet \text{---} \bullet \text{---} \bullet = 0$$

(colour structure) \otimes (Lorentz structure)

(In covariant gauges, the second identity contains extra ghost terms)

1 loop

$$\omega \text{ (diagram)} = g_0 \text{ (diagram)} \otimes \left[\text{diagram} - \text{diagram} \right]$$

The diagram on the left shows a fermion line with three vertices. The first vertex has an incoming fermion line labeled ω . The second vertex has an incoming dashed line with a downward-pointing triangle. The third vertex has an incoming fermion line with a dot. A wavy line connects the second and third vertices. The diagram on the right is the tree-level diagram g_0 with a wavy line between the second and third vertices. The two diagrams in the brackets are tree-level diagrams with a wavy line between the second and third vertices: the first has a dot on the fermion line between the second and third vertices, and the second has no dot.

$$\omega \text{ (diagram)} = g_0 \text{ (diagram)} \otimes \left[\text{diagram} - \text{diagram} \right]$$

The diagram on the left is identical to the one above. The diagram on the right is the tree-level diagram g_0 with a dashed line between the second and third vertices. The two diagrams in the brackets are tree-level diagrams with a dashed line between the second and third vertices: the first has a dot on the fermion line between the second and third vertices, and the second has no dot.

$$= g_0 \text{ (diagram)} \otimes \left[\text{diagram} - \text{diagram} \right] = 0$$

The diagram on the left is the tree-level diagram g_0 with a wavy line between the second and third vertices. The two diagrams in the brackets are tree-level diagrams with a dashed line between the second and third vertices: the first has a dot on the fermion line between the second and third vertices, and the second has no dot.

$$\omega \left[\text{Diagram 1} \right] = g_0 \left[\text{Diagram 2} - \text{Diagram 3} \right]$$

Diagram 1: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

Diagram 2: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

Diagram 3: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

$$\otimes \left[\text{Diagram 4} - \text{Diagram 5} \right]$$

Diagram 4: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

Diagram 5: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

$$\left[\text{Diagram 6} \right] = \left[\text{Diagram 7} \right] = 0$$

Diagram 6: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

Diagram 7: A triangle diagram with a wavy line on the left, a dashed line on the right, and a wavy line on top. A solid dot is on the top wavy line, and a solid dot is on the right wavy line. A downward-pointing triangle is above the top wavy line.

Ward identity


$$\text{Diagram: a circle with diagonal hatching, with an incoming arrow from the left labeled } p \text{ and an outgoing arrow to the right.} = -i\Sigma(p)$$

$$S(p) = \frac{1}{\not{p} - m_0 - \Sigma(p)}$$

$$\text{Diagram: a circle with diagonal hatching, with an incoming arrow from the left labeled } p \text{ and an outgoing arrow to the right. A dashed vertical line with a downward arrow labeled } q \text{ points to the top of the circle.} = ig_0 t^a \Gamma(p, q)$$

$$\Gamma = \gamma_0 + \Lambda$$

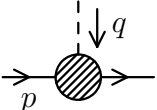
Ward identity



A Feynman diagram showing a fermion line with momentum p entering from the left and exiting to the right. A shaded circular loop is attached to the line. The equation is $= -i\Sigma(p)$.

$$\text{Diagram} = -i\Sigma(p)$$

$$S(p) = \frac{1}{\not{p} - m_0 - \Sigma(p)}$$



A Feynman diagram showing a fermion line with momentum p entering from the left and exiting to the right. A shaded circular loop is attached to the vertex. A dashed line with momentum q enters the loop from the top. The equation is $= ig_0 t^a \Gamma(p, q)$.

$$\text{Diagram} = ig_0 t^a \Gamma(p, q)$$

$$\Gamma = \gamma_0 + \Lambda$$

$$\omega \Lambda(p, \omega v) = \Sigma(p) - \Sigma(p + \omega v)$$

$$\omega \Gamma(p, \omega v) = S^{-1}(p + \omega v) - S^{-1}(p)$$

Coupling constant renormalization

$$g_0 = Z_\alpha^{1/2} g \quad \Gamma = Z_\Gamma \Gamma_r \quad S = Z_\psi S_r$$

Coupling constant renormalization

$$g_0 = Z_\alpha^{1/2} g \quad \Gamma = Z_\Gamma \Gamma_r \quad S = Z_\psi S_r$$

$$g_0 \Gamma Z_\psi Z_A^{1/2} = g \Gamma_r Z_\alpha^{1/2} Z_\Gamma Z_\psi Z_A^{1/2} = \text{finite}$$

$$Z_\alpha = (Z_\Gamma Z_\psi)^{-2} Z_A^{-1}$$

Coupling constant renormalization

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Ward identity

$$Z_\Gamma Z_\psi = 1 \quad Z_\alpha = Z_A^{-1}$$