The Big Bang Cosmology: Lecture #4 Big Bang Nucleosynthesis

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Outline

Neutrino freeze-out

$T > m_e$ $e^+e^- \leftrightarrow \nu\bar{\nu},\ e\nu \leftrightarrow e\nu$

$$\sigma_\nu \sim G_F^2 E^2$$

neutrino interaction rate

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle} \sim \frac{1}{G_F^2 T^5}$$

$$\tau_\nu(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2}$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3\ \text{MeV}$$

Neutron decoupling $\quad p+e \longleftrightarrow n+\nu_e$

typical energy scales

$$T \gtrsim \Delta m = 1.3\,\mathrm{MeV}, \qquad T \gtrsim m_e = 0.5\,\mathrm{MeV}$$

neutron interaction rate

$$\tau_{n\leftrightarrow p} = \frac{1}{\Gamma_{n\leftrightarrow p}} = \frac{1}{C_n G_F^2 T^5}$$

neutron decoupling $\quad \Gamma_{n\leftrightarrow p}(T) \sim H(T) = T^2/M_{Pl}^*$

$$T_n = \frac{1}{\left(C_n M_{Pl}^* G_F^2\right)^{1/3}} \approx 1.4\,\mathrm{MeV}$$

$$g_* = 2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot 2\cdot N_\nu \qquad t = \frac{1}{2H(T_n)} = \frac{M_{Pl}^*}{2T_n^2} = 1.2\ \mathrm{s} \qquad T_n \approx 0.8\ \mathrm{MeV}$$

Neutron density after decoupling

$$n_n = g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_n - m_n}{T}} \qquad \mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\frac{n_n}{n_p} = \mathrm{e}^{-\frac{m_n - m_p}{T}} \mathrm{e}^{\frac{\mu_n - \mu_p}{T}}$$

for relativistic e^+ and e^-

$$n_{e^-} - n_{e^+} \sim \mu_e T^2 \longrightarrow \frac{\mu_e}{T} \sim \frac{n_{e^-} - n_{e^+}}{T^3} = \frac{n_p}{T^3} \sim \eta_B \sim 10^{-9}$$

$$\frac{n_n}{n_p} = \mathrm{e}^{-\frac{m_n - m_p}{T_n}} \equiv \mathrm{e}^{-\frac{\Delta m}{T_n}} \approx \frac{1}{5} \mathrm{e}^{-\frac{\mu_\nu}{T}}$$

$p+n \longrightarrow \mathrm{D}+\gamma$ Saha equation

$$n_n = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_n - m_n}{T}}\;, \qquad n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_p - m_p}{T}}\;,$$

Chemical equilibrium $\mu_A = \mu_p \cdot Z + \mu_n \cdot (A-Z)$

$$n_A = n_p^Z n_n^{A-Z} 2^{-A} g_A A^{3/2} \left(\frac{2\pi}{m_p T}\right)^{\frac{3}{2}(A-1)} \mathrm{e}^{\frac{\Delta_A}{T}}$$

$$X_A = \frac{A n_A}{n_{\mathrm{B}}} \qquad n_{\mathrm{B}} = \eta_{\mathrm{B}} \cdot n_\gamma = 0.24\eta_{\mathrm{B}} T^3$$

$$X_A = X_p^Z X_n^{A-Z} 2^{-A} g_A A^{5/2} \eta_{\mathrm{B}}^{A-1} \left(\frac{2.5T}{m_p}\right)^{\frac{3}{2}(A-1)} \mathrm{e}^{\frac{\Delta A}{T}}$$

Temperature of BBN T_{NS}: $X_{\rm D} \sim 1$

$$X_A = X_p^Z X_n^{A-Z} 2^{-A} g_A A^{5/2} \eta_{\rm B}^{A-1} \left(\frac{2.5T}{m_p}\right)^{\frac{3}{2}(A-1)} \mathrm{e}^{\frac{\Delta A}{T}}$$

$\Delta_{\rm D} = 2.23$ MeV

$$X_{\rm D}(T_{NS}) \sim \eta_{\rm B} \left(\frac{2.5T_{NS}}{m_p}\right)^{3/2} \mathrm{e}^{\frac{\Delta_{\rm D}}{T_{NS}}} \sim 1 \longrightarrow T_{NS} \approx 65 \text{ keV}$$

$$t_{NS} = \frac{1}{2H(T_{NS})} = \frac{M_{Pl}^*}{2T_{NS}^2} = 265 \text{ s} \approx 4.5 \text{ minutes} .$$

Helium density (chemical equilibrium)

$$X_A = X_p^Z X_n^{A-Z} 2^{-A} g_A A^{5/2} \eta_B^{A-1} \left(\frac{2.5T}{m_p}\right)^{\frac{3}{2}(A-1)} e^{\frac{\Delta A}{T}}$$

$X_{^4\mathrm{He}}$ @ $T = T_{NS}$? $\Delta_{^4\mathrm{He}} = 28.3\ \mathrm{MeV}$

$$X_{^4\mathrm{He}} = X_p^2 X_n^2 \cdot 8\eta_B^3 \left(\frac{2.5T}{m_p}\right)^{9/2} e^{\frac{\Delta_{^4\mathrm{He}}}{T}} \longrightarrow 10^{128}$$

Let $X_{^4\mathrm{He}} \sim 1$ $(2p, 2n)$, $n_n < n_p$ $X_p \sim 1$

$$X_n = X_{^4\mathrm{He}}^{1/2} \eta_B^{-3/2} \left(\frac{2.5T}{m_p}\right)^{-9/4} e^{-\frac{\Delta_{^4\mathrm{He}}}{2T}}$$

Light element densities (chemical equilibrium)

$$X_A = \left[\eta_B \cdot \left(\frac{2.5T}{m_p}\right)^{3/2}\right]^{\frac{3}{2}Z-\frac{1}{2}A-1} e^{\frac{\Delta_A - \Delta_{^4\mathrm{He}}(A-Z)/2}{T}} \simeq 10^{7.4(A+2-3Z)}\, e^{(A-Z)\frac{\Delta_A/(A-Z)-\Delta_{^4\mathrm{He}}/2}{T}}$$

Z	Nucleus	Δ_A	Δ_A/A	$\Delta_A/(A-Z)$	X_A
1	$^2H \equiv D$	2.23	1.11	2.23	10^{-79}
	$^3H \equiv T$	8.48	2.83	4.24	10^{-118}
2	^3He	7.72	2.57	7.72	10^{-51}
	$^4He \equiv \alpha$	28.30	7.75	14.15	1
3	^6Li	31.99	5.33	10.66	10^{-78}
	^7Li	39.24	5.61	9.81	10^{-116}
4	^7Be	37.60	5.37	12.53	10^{-55}
5	^8B	37.73	4.71	12.58	10^{-69}
6	^{12}C	92.2	7.68	15.37	10^{19}

Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

$$n_{^4\mathrm{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}) \,,$$

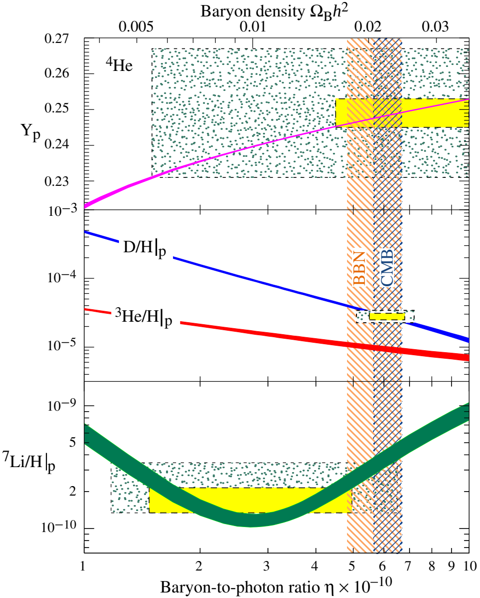
neutron-to-proton ratio $\tau_n \approx 886$ s

$$\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot \mathrm{e}^{-\frac{t_{NS}}{\tau_n}} \approx \frac{1}{7} \,,$$

$$Y_p \equiv X_{^4\mathrm{He}} = \frac{m_{^4\mathrm{He}} \cdot n_{^4\mathrm{He}}(T_{NS})}{m_p\left(n_p(T_{NS}) + n_n(T_{NS})\right)} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\% \,.$$

from observations of relic helium abundance:

$$\Delta N_{\nu,eff} \leq 1$$



Baryon density $\Omega_B h^2$
0.005
0.01
0.02
0.03
0.27
0.26
0.25
0.24
0.23
Y_p
^{4}He
10^{-3}
10^{-4}
10^{-5}
D/H|p
^{3}He/H|p
BBN
CMB
10^{-9}
5
2
10^{-10}
^{7}Li/H|p
1
2
3
4
5
6
7
8
9
10
Baryon-to-photon ratio $\eta \times 10^{-10}$

Main nuclear reactions

1. $p(n,\gamma)$D — deuterium production, BBN starts.
2. D$(p,\gamma)^3$He, D(D,n)^{3}He, D(D,p)T, ^{3}He(n,p)T — intermediate stage.
3. T(D,n)^{4}He, ^{3}He(D,p)^{4}He — production of ^{4}He.
4. T$(\alpha,\gamma)^7$Li, $^3\mathrm{He}(\alpha,\gamma)^7$Be, ^{7}Be$(n,p)^7$Li — production of the heaviest baryonic relics.
5. $^7\mathrm{Li}(p,\alpha)^4$He — ^{7}Li burning.

One has to compare reaction rates to the expansion rate

$$H(T_{NS} = 70 \text{ keV}) = 4 \cdot 10^{-3} \text{ s}^{-1}$$

to obtain nonequilibrium concentrations

Neutron burning $p + n \longrightarrow \mathrm{D} + \gamma$

@ $T = T_{NS} = 65$ keV

$$(\sigma v)_{p(n,\gamma)D} \approx 6 \cdot 10^{-20} \, \frac{\mathrm{cm}^3}{\mathrm{s}} \, .$$

for the rate (neutron disappearence when meets proton)

$$\Gamma_{p(n,\gamma)D} = n_p \cdot (\sigma v)_{p(n,\gamma)D} = \eta_B \cdot 2 \frac{\zeta(3)}{\pi^2} T^3 \cdot (\sigma v)_{p(n,\gamma)D} = 0.31 \; \mathrm{s}^{-1}$$

for $\eta_B = 6.15 \cdot 10^{-10}$ and $T = T_{NS}$

So, neutrons disappear very rapidly

$$\Gamma_{p(n,\gamma)D} \gg H(T_{NS}) = 4 \cdot 10^{-3} \; \mathrm{s}^{-1}$$

Deuterium burning $\mathrm{D}(\mathrm{D},n)^3\mathrm{He}, \quad D(D,\mathrm{p})T$

Coloumb barier: tunneling $T_9 \equiv T/(10^9\ \mathrm{K}) = T/(86\ \mathrm{keV})$

$$\langle\sigma v\rangle_{DD} = 3\cdot 10^{-15}\frac{\mathrm{cm}^3}{\mathrm{s}}\cdot T_9^{-2/3}\cdot \mathrm{e}^{-4.26\cdot T_9^{-1/3}}\,.$$

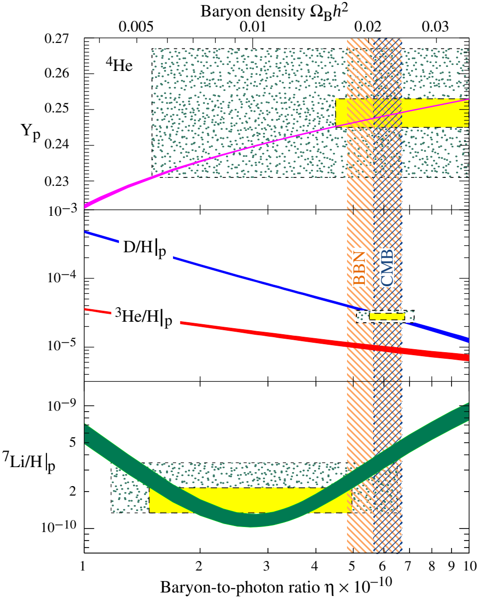
deuterium stops burning when $T = T_{NS}(T_9 = 0.75)$

$$\Gamma_{DD} = n_D(T)\cdot\langle\sigma v\rangle_{DD}(T) \sim H(T)\,.$$

Then relic deuterium abundance is estimated as

$$\frac{n_D}{n_p} = \frac{1}{0.75\eta_{\mathrm{B}}}\cdot\frac{n_D}{n_\gamma(T_{NS})} = 0.3\cdot 10^{-4}$$

for $\eta_{\mathrm{B}} = 6.15\cdot 10^{-10}$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Baryon-to-photon ratio $\eta \times 10^{-10}$

Production of ^{3}He & ^{3}H

$$\langle\sigma v\rangle_{^3\mathrm{He}(D,p)^4\mathrm{He}} = 10^{-15}\frac{\mathrm{cm}^3}{\mathrm{s}}\cdot T_9^{-1/2}\mathrm{e}^{-1.8T_9^{-1}}$$

@ $T = t_{NS}$ this rate exceeds the deuterium-burning rate

^{3}He stops burning when @ some $T = T_{^3\mathrm{He}} < T_{NS}$

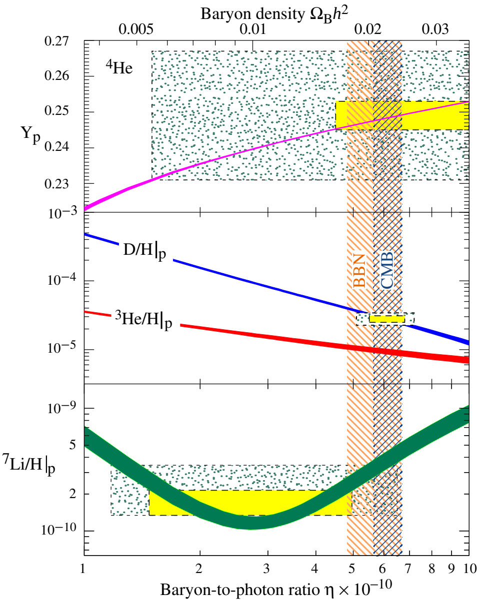
$$\langle\sigma v\rangle_{^3\mathrm{He}(D,p)^4\mathrm{He}}\cdot n_D \sim H\;, \quad T = T_{^3\mathrm{He}} \simeq 0.6 T_{NS}$$

^{3}He is produced via $\mathrm{D}+\mathrm{D}\rightarrow{}^3\mathrm{He}+n$ and for the Hubble time

$$n_{^3\mathrm{He}} \sim \langle\sigma v\rangle_{D(D,n)^3\mathrm{He}}\cdot n_D^2\cdot\frac{1}{H}\;, \quad T = T_{^3\mathrm{He}}\;.$$

$$\frac{n_{^3\mathrm{He}}}{n_D} \simeq \frac{\langle\sigma v\rangle_{D(D,n)^3\mathrm{He}}}{\langle\sigma v\rangle_{^3\mathrm{He}(D,p)^4\mathrm{He}}} \longrightarrow \frac{n_{^3\mathrm{He}}}{n_p} \simeq 0.9\cdot 10^{-5}$$

quite similar $\frac{n_T}{n_p} \simeq 2\cdot 10^{-7}$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BBN
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^{7}Li/H|p
2
10^{-10}
1 2 3 4 5 6 7 8 9 10
Baryon-to-photon ratio $\eta \times 10^{-10}$

Lithium in $T(\alpha,\gamma)^7$Li & ^{7}Li$(p,\alpha)^4$He

$$\langle\sigma v\rangle_{T(\alpha,\gamma)^7\mathrm{Li}} \sim 10^{-18}\,\frac{\mathrm{cm}^3}{\mathrm{s}}\cdot T_9^{-2/3}\mathrm{e}^{-8.0T_9^{-1/3}}\;.$$

tritium burning rate is smaller than H

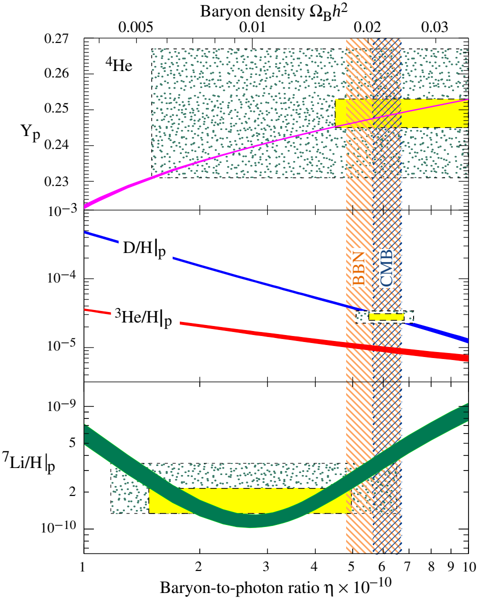
$$\langle\sigma v\rangle_{T(\alpha,\gamma)^7\mathrm{Li}}\cdot n_\alpha \simeq 1.5\cdot 10^{-4}\;\mathrm{s}^{-1}\,, \quad T_9 = 0.75$$

$$\langle\sigma v\rangle_{^7\mathrm{Li(p,\alpha)^4He}} \sim 10^{-15}\,\frac{\mathrm{cm}^3}{\mathrm{s}}\cdot T_9^{-2/3}\mathrm{e}^{-8.5T_9^{-1/3}}\;.$$

lithium burning rate exceeds H

$$\langle\sigma v\rangle_{^7\mathrm{Li(p,\alpha)^4He}}\cdot n_p \simeq 0.7\;\mathrm{s}^{-1}\,, \quad T_9 = 0.75\,, \;\; \eta_B = 6.15\cdot 10^{-10}$$

$$\frac{n_{^7\mathrm{Li}}}{n_T} \simeq \frac{\langle\sigma v\rangle_{T(\alpha,\gamma)^7\mathrm{Li}}}{\langle\sigma v\rangle_{^7\mathrm{Li(p,\alpha)^4He}}}\cdot\frac{n_\alpha}{n_p} \sim 2\cdot 10^{-5}$$



$^7Be(e^-, \nu_e)^7Li$
$^7Be(n,p)^7Li$