Influence of nontrivial backgrounds and decoherence on vacuum decay

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Outline

- False Vacuum Decay
- Tunneling in Friedmann and Static Space-times
- Decoherence
- Localization of Vacuum Bubbles
- One-loop corrections

False Vacuum Decay

- scalar field theory with two minima

$$S_\phi = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

- transition amplitude given by Euclidean path integral

$$\langle \phi_t | e^{-HT} | \phi_f \rangle = N \int D\phi e^{-S_\phi} \approx A \exp(-S_E)$$

- Coleman's result for the imaginary part of the Euclidean action (assuming $O(4)$-symmetry)

$$\Im(S_E) = \frac{27\pi^2 S_1^4}{4\epsilon^3}, \quad S_1 = \int_{\phi_t}^{\phi_f} d\phi \sqrt{2V(\phi)}$$

- reduction of infinitely many field degrees of freedom to single quantum variable

Effective Action

- Bubble radius R as effective quantum variable ($O(3)$-symmetry)

$$\rightarrow S_R = \int dt \left(\int_{|\mathbf{x}| \leq R} d^3x \sqrt{-\eta}\epsilon - \int_{|\mathbf{x}|=R} d^2x \sqrt{-\gamma}\sigma \right)$$

- generalization to arbitrary space-time background

$$S_R = \int dt \left(\int_{|\mathbf{x}| \leq R} d^3x \sqrt{-g}\epsilon - \int_{|\xi|=R} d^2\xi \sqrt{-\gamma}\sigma \right)$$

- reduction of infinitely many degrees of freedom to *single* quantum mechanical degree of freedom as before

Friedmann Universe

- line element $ds^2 = a^2(y)(dy^2 - dx^2 - f^2(x)d\Omega^2)$

$$\Longrightarrow S_{x,FRW} = \int dy \Big(4\pi\epsilon a^4(y) \int_0^{x(y)} dx' f^2(x') \\ -4\pi\sigma a^3(y) f^2(x) \sqrt{1-\dot{x}^2(y)} \Big)$$

- ansatz $\sqrt{1-\dot{x}^2} = g(y)\dot{x}\,,\quad g$ arbitrary

$$\Longrightarrow \frac{\dot{a}}{a} - \frac{\dot{g}}{3g} - \frac{ga}{R_0} + \frac{2}{3}\frac{\partial_x f(x)}{f(x)\dot{x}} = 0$$

- from $x(y)$ (comoving radius) follows analytical expression for $a(y)$

Friedmann Universe

- scale factor for given shell trajectory

$$a(y) = \frac{\left(\frac{g(y)}{g(y_0)}\right)^{1/3} e^{-F(y)}}{C - \frac{1}{R_0}\int_{y_0}^{y} dy' \left(\frac{g(y')}{g(y_0)}\right)^{1/3} g(y') e^{-F(y')}}$$

  with

$$F(y) = \frac{2}{3}\int_{y_0}^{y} dy' \frac{\partial_x f(x)}{f(x)\dot{x}}$$

- shell trajectory determined by g

$$x(y) = \frac{R_{\text{phys}}}{a} = \int_{\tilde{y}_0}^{y} dy' \frac{1}{\sqrt{1+g^2(y')}}$$

Example I: De Sitter

- flat slicing

$$R_{\rm phys} = -\frac{1}{Hz}\sqrt{\alpha^2 + \left(z + \frac{\alpha}{R_0 H}\right)^2}$$

- open slicing

$$R_{\rm phys} = -\frac{1}{H\sinh(w)}\sqrt{\left(\frac{1+A^2}{1-A^2}\right)^2 \cosh^2(w+w_0) - 1}$$

with

$$A = \alpha\sqrt{1 + \frac{1}{R_0^2 H^2}}$$

Example I: De Sitter

- closed slicing

$$R_{\text{phys}} = \frac{1}{H\cos(y)}\sqrt{1 - \left(\frac{1-A^2}{1+A^2}\right)^2 \sin^2(y-y_0)}$$

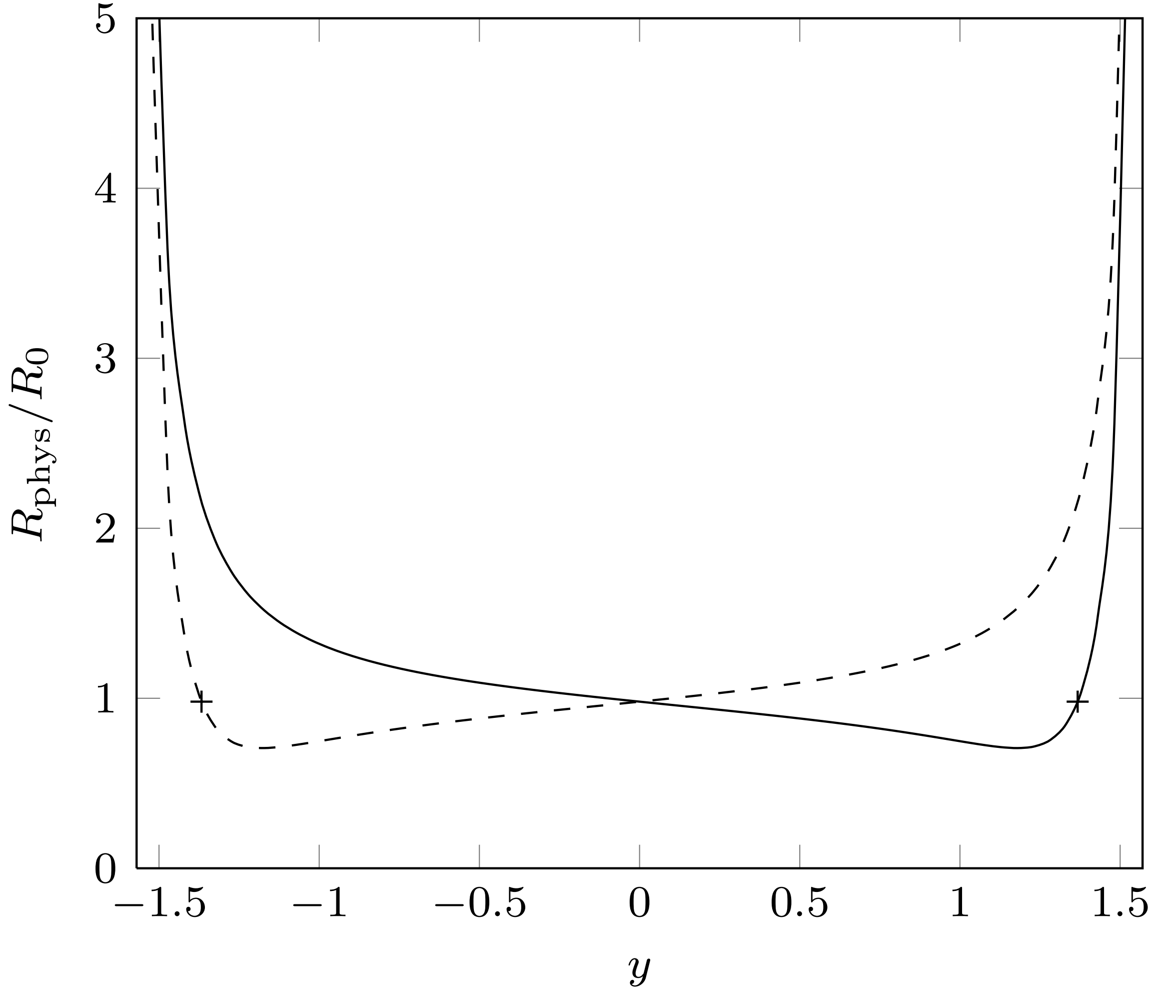
- decay rate does not depend on choice of coordinates (see coordinate invariant approach of Coleman)

- imaginary part of euclidean action (Parke 1983):

$$\Im(S) = \frac{\pi^2\epsilon}{3H^4}\frac{\left(1-\sqrt{1+R_0^2H^2}\right)^2}{\sqrt{1+R_0^2H^2}}$$

- $\epsilon \to 0 : \Im(S) = \pi^2\sigma/H^3$ and $H \to 0 : \Im(S) = 27\pi^2 S_1^4/(4\epsilon^3)$

Shell Trajectories



Example 2: Power-Law Scale Factors

- analytical solution only possible for degenerate vacua, i.e. $\epsilon = 0$
- choice of trajectory:

$$x(z) = \sqrt{\alpha^2 + \frac{2n-2}{2n+1} z^2}$$

- corresponding scale factor:

$$a(z) = \frac{1}{H} \left(-\frac{n-1}{n} z \right)^{-\frac{n}{n-1}} \left(1 + \frac{6(n-1)}{(2n+1)^2} \frac{z^2}{\alpha^2} \right)^{1/6} ,$$

- de Sitter limit for $n \to \infty$

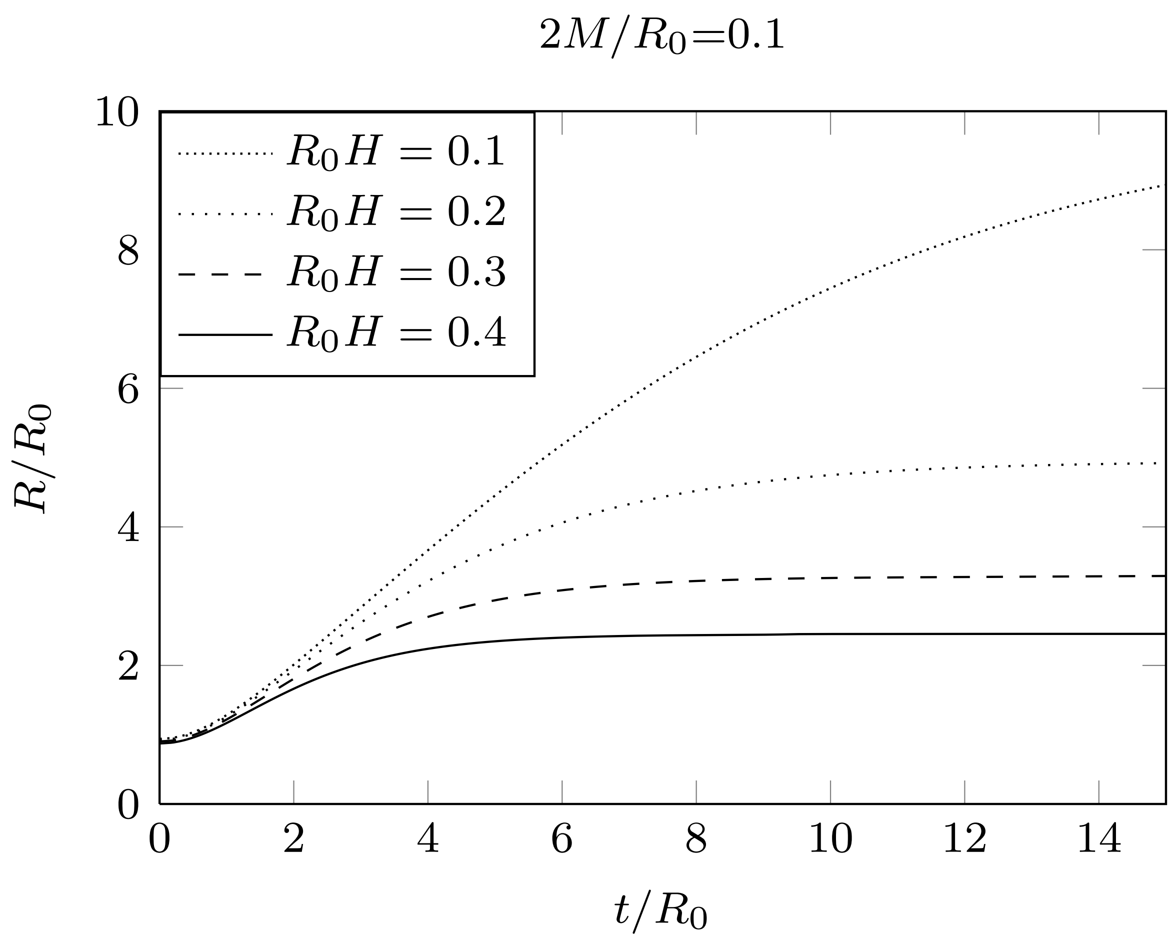
Static Space-times

- time-independent Lagrangian
  $\Longrightarrow$ equations of motion solved using conservation of energy
- $O(3)$-invariant line-element $ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega^2$
- reparametrization of square-root-Lagrangian leads to effective potential:

$$V(x) = \frac{2\pi}{3}\epsilon R_0^4 x^2 f^{-1}(f - x^2)$$

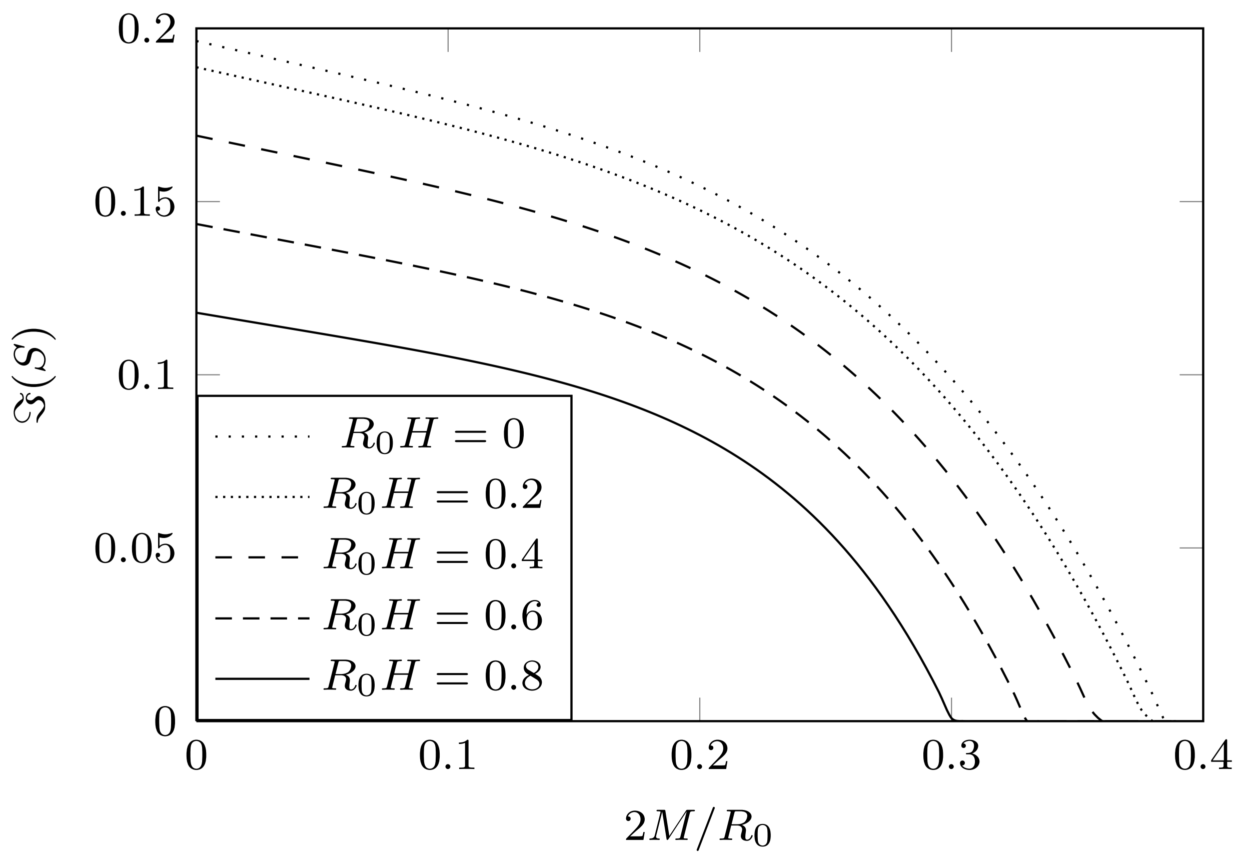
- examples: Schwarzschild-de Sitter (mass M and horizon H) and Reissner-Nordström (mass M and charge Q)

SDS Trajectories



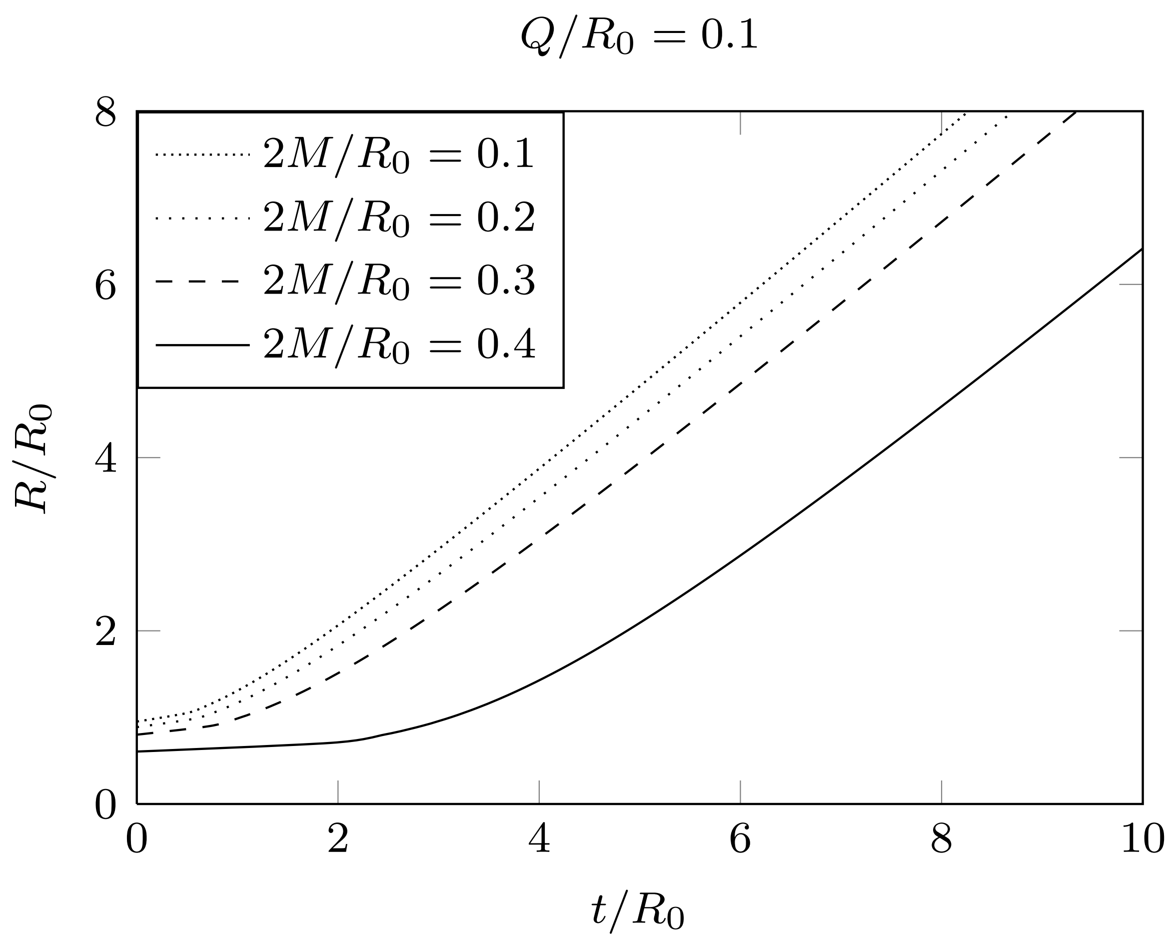
- tunneling always occurs between the horizons

SDS Action



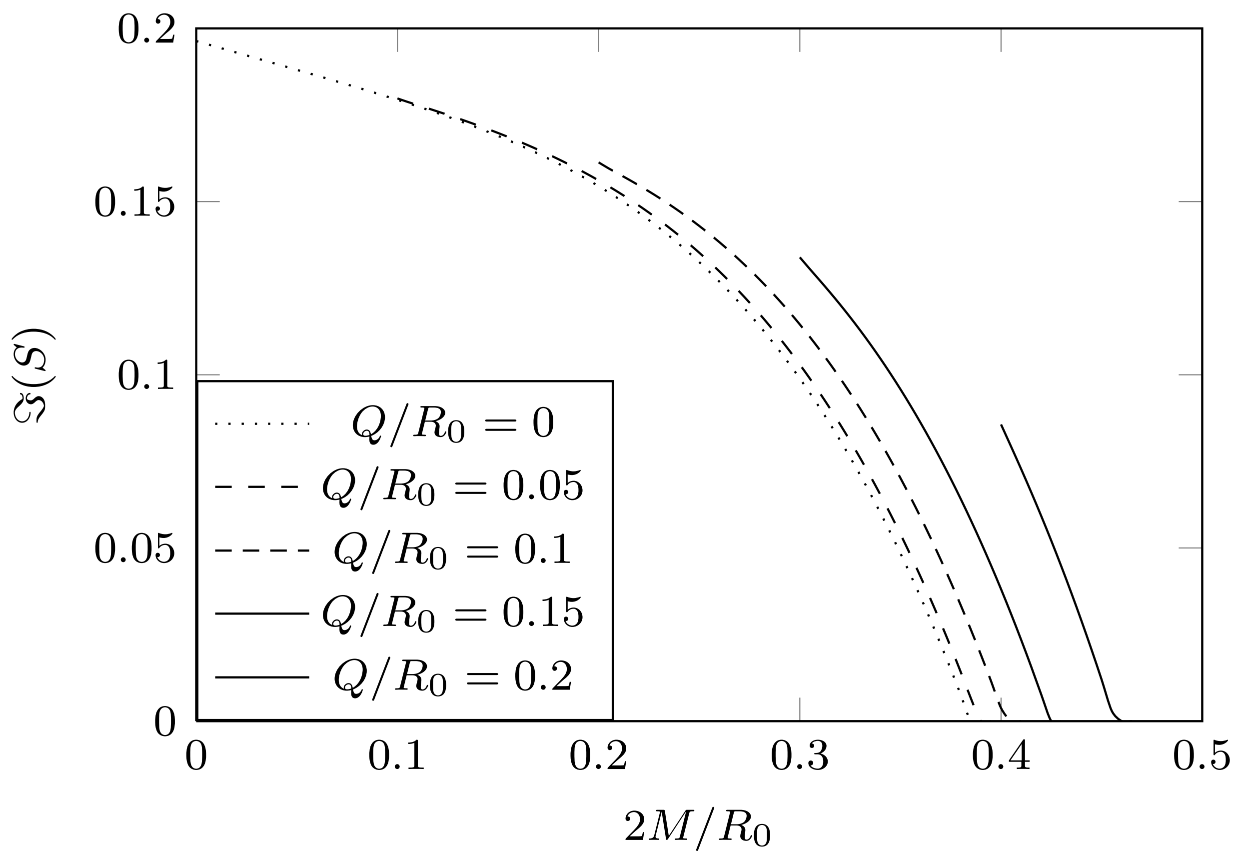
- barrier and action vanish for $M > R_0/\sqrt{27(1 + R_0^2 H^2)}$

RN Trajectories



- unlimited expansion of the bubble

RN Action



- tunneling observable for $Q^2 < M^2 < \frac{R_0^2}{54}\left(1 + 36\frac{Q^2}{R_0^2} + \left(1 - 12\frac{Q^2}{R_0^2}\right)^{3/2}\right)$

Decoherence

- $\hbar \to 0$ not sufficient for quantum-to-classical transition
- generation of entanglement between system and environment explains classical appearance
- phase relations delocalized in the environment
- coherences not locally observable, no interference effects
- $H_{\mathrm{int}} = \sum_n |n\rangle\langle n| \otimes A_n$

$$\Longrightarrow \sum_n c_n |n\rangle |\Phi_{\mathrm{in}}\rangle \to \sum_n c_n |n\rangle |\Phi_n\rangle$$

$$\Longrightarrow \rho_{\mathrm{sys}} = \sum_{nm} c_n c_m^* |n\rangle\langle m| \to \sum_{nm} c_n c_m^* \langle \Phi_n | \Phi_m \rangle |n\rangle\langle m|$$

Bubble-Environment Interaction

- environmental field ψ bi-linearly coupled to field ϕ

$$S_{\rm int} = g \int d^4x \phi\psi = g(\phi_t - \phi_f) \int dt \sum_{\mathbf{k}} \frac{4\pi}{k^3} \left(\sin(kR) - kR\cos(kR)\right) \psi_{\mathbf{k}}$$

- rough approximation: effective two-state system
- suppression of coherences for vanishing temperature:

$$\rho_{S,01}(t) \quad = \quad \rho_{S,01}(0) \exp\left[-\frac{4g^2}{9}(\phi_f - \phi_t)^2 R_0^6 \left(\frac{7}{4} + \ln\left(\frac{t}{2R_0}\right)\right)\right]$$

- compare with standard result from spin-boson model

$$\Gamma(t) = \ln[\rho_{S,01}(t)/\rho_{S,01}(0)] = -\lambda\frac{1}{2}\ln(1+\Omega^2 t^2) - \lambda \ln\left(\frac{\sinh(t\pi T)}{t\pi T}\right)$$

Growing Vacuum Bubble

- exact solution not possible $\to$ master equation
- factor ordering ambiguities due to the square-root term in the system Hamiltonian

$$H_{\mathrm{sys}} = \sqrt{16\pi^2 \hat{R}^4 \sigma^2 + \hat{P}_R^2} - \frac{4\pi \hat{R}^3 \epsilon}{3}$$

- assumption: momentum P_R dominates
- Heisenberg operators:

$$\begin{aligned} \hat{R}^H(t) &= \hat{R}_0 \pm |t| \\ \hat{P}_R^H(t) &= \hat{P}_R(0) + \frac{4\pi\epsilon}{3}\left((\hat{R}_0 \pm |t|)^3 - \hat{R}_0^3\right) . \end{aligned}$$

Reduced Density Matrix

- solution of the master equation for $t \gg R, R'$

$$\rho(R,R',t) \approx \rho(R,R',0)\exp\left[-\frac{g^2}{90}(\phi_t-\phi_f)^2 t^4 (R-R')^2\right] .$$

- result sensitive to vacuum bubble size
- t^4-dependence represents increasing space-time volume of the expanding bubble
- quantum-to-classical transition immediately after nucleation
- analogous effect: decay of alpha particles

Modified Tunneling Rate

- dissipation leads to friction term in equations of motion

$$\frac{1}{R^2}\frac{d}{dT}\left(\frac{4\pi\sigma R^2\dot{R}}{\sqrt{1+\dot{R}^2}}\right) = -4\pi\epsilon + \frac{8\pi\sigma}{R}\sqrt{1+\dot{R}^2}$$

$$+\frac{4g^2}{3}(\phi_t-\phi_f)^2\int_{-\infty}^{\infty}dT'\,\frac{R^3-{R'}^3}{(T-T')^2}$$

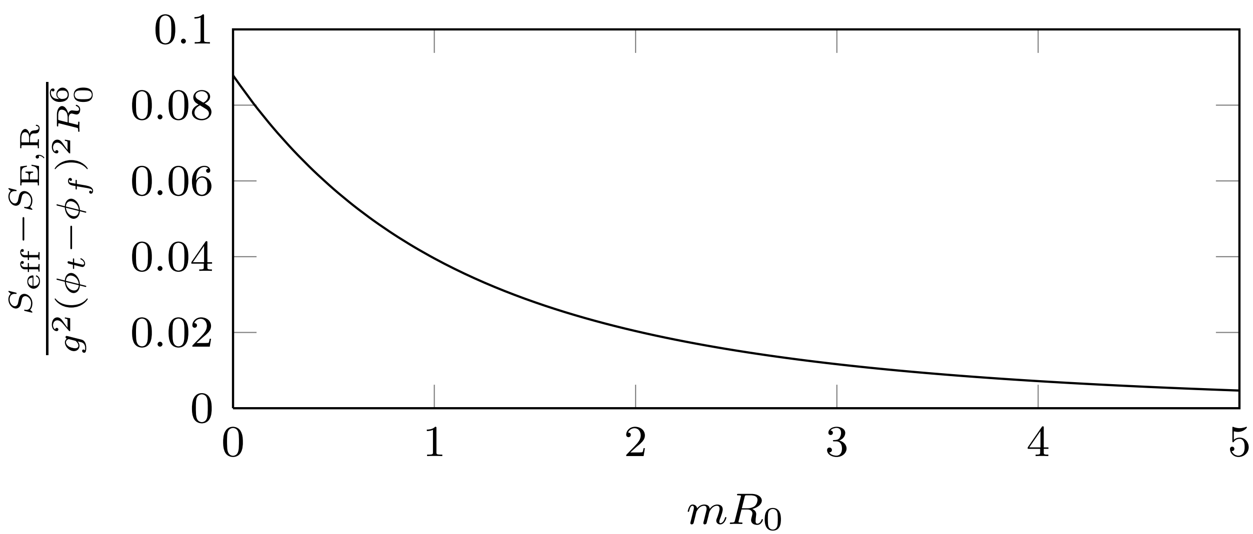
- effective action has positive definite correction term

$$S_{\rm eff} = S_{\rm E,R} + \int_{-\infty}^{\infty}dT'\int_0^{T_0}dT\sum_{\mathbf{k}}\frac{e^{-\sqrt{k^2+m^2}|T-T'|}}{8\mathcal{V}\sqrt{k^2+m^2}}(f_k(R)-f_k(R'))^2$$

- $\Longrightarrow$ reduction of tunneling rate (Caldeira-Leggett model)

Modified Tunneling Rate

- spectral density is determined through Lorenz-invariance of the field theory
- integral is UV- and IR convergent
- mass-dependence of correction term (neglecting back-reaction):



One-loop corrections

- correction of order $\hbar$ are determined by ratio of functional determinants:

$$\Gamma = \frac{S_{\text{eff}}^2}{4\pi^2} \left| \frac{\operatorname{Det}(\delta^2 \tilde{S}_{\text{eff}}/\delta\phi^2)}{\operatorname{Det}'(\delta^2 S_{\text{eff}}/\delta\phi^2)} \right|^{1/2} \exp(-S_{\text{eff}})$$

- for $g = 0$: determinant has one negative eigenvalue (Coleman 1980)
- summation over instantons and anti-instantons:

$$E_{1/2} = \mathcal{V}T(\rho_0 \pm \Gamma)$$

- $\Longrightarrow$ small imaginary correction to eigenenergies
- vacuum becomes perturbatively unstable
- is this also true for $g \neq 0$?

One-loop corrections

- effective action for field ϕ

$$S_{\rm eff} = S_{{\rm E},\phi} + \frac{g^2}{(4\pi)^2} \int dT \int dT' \int dx^3 \int dy^3 \times$$
$$\times \left(\frac{\phi(|\mathbf{x}|,T)\phi(|\mathbf{y}|,T)}{|\mathbf{x}-\mathbf{y}|^2 + |T-T'|^2} - \frac{\phi(|\mathbf{x}|,T')\phi(|\mathbf{y}|,T)}{|\mathbf{x}+\mathbf{y}|^2 + |T-T'|^2} \right)$$

- choose embedding: $\phi_\lambda = \bar\phi(|\mathbf{x}|/\lambda, T/\lambda)$ and vary action with respect to λ

- effective action is positive and at least one negative eigenvalue

$$S_{\rm eff}(\bar\phi) = \frac{1}{6} \int d^4x (\partial_\mu \bar\phi)^2 > 0\,,$$
$$\left.\frac{d^2 S_{\rm eff}(\phi_\lambda)}{d\lambda^2}\right|_{\lambda=1} = -2 \int d^4x (\partial_\mu \bar\phi)^2 < 0$$

Conclusions

- we determined tunneling rates and tunneling trajectories of vacuum bubbles for different space-time backgrounds
- nucleation rate increases with H and M
- no gravity back-reaction was taken into account (inclusion of Ricci scalar)
- decoherence explains quantum-to-classical transition of growing bubble
- reduction of tunneling rate due to dissipation