Early years of string theory: Practically unknown facts

V.V. Nesterenko

(JINR, Dubna)

VIII Advanced Summer School
on
Modern Mathematical Physics
(Dubna, September 5 – 15, 2010)

String theory

- I highly recommend the source: Donald Marolf. *Resource Letter NSST-1 1: The Nature and Status of String* Theory. Am. J. Phys. 72 (2004) 730; arXiv:hep-th/0311044
- Who likes to get information in Internet: P. Schwarz, http://superstringtheory.com/basics/index.html

<u>Reviews:</u>

1. A.A. Tseytlin. Introductory Lectures on String Theory: arXiv:0808.0663
   Very simple, clear and condensed representation.

2. Gerard 't Hooft. INTRODUCTION TO STRING THEORY(with exercises) http://www.phys.uu.nl/~thooft/lectures/stringnotes.pdf

3. Rolf Schimmrigk, Applied String Theory: arXiv:0810.1743v1 Present-day level.

4. Matthew Headrick. A solution manual for Polchinski's "String Theory" arXiv:0812.4408v1 For fundamental studies

Books:

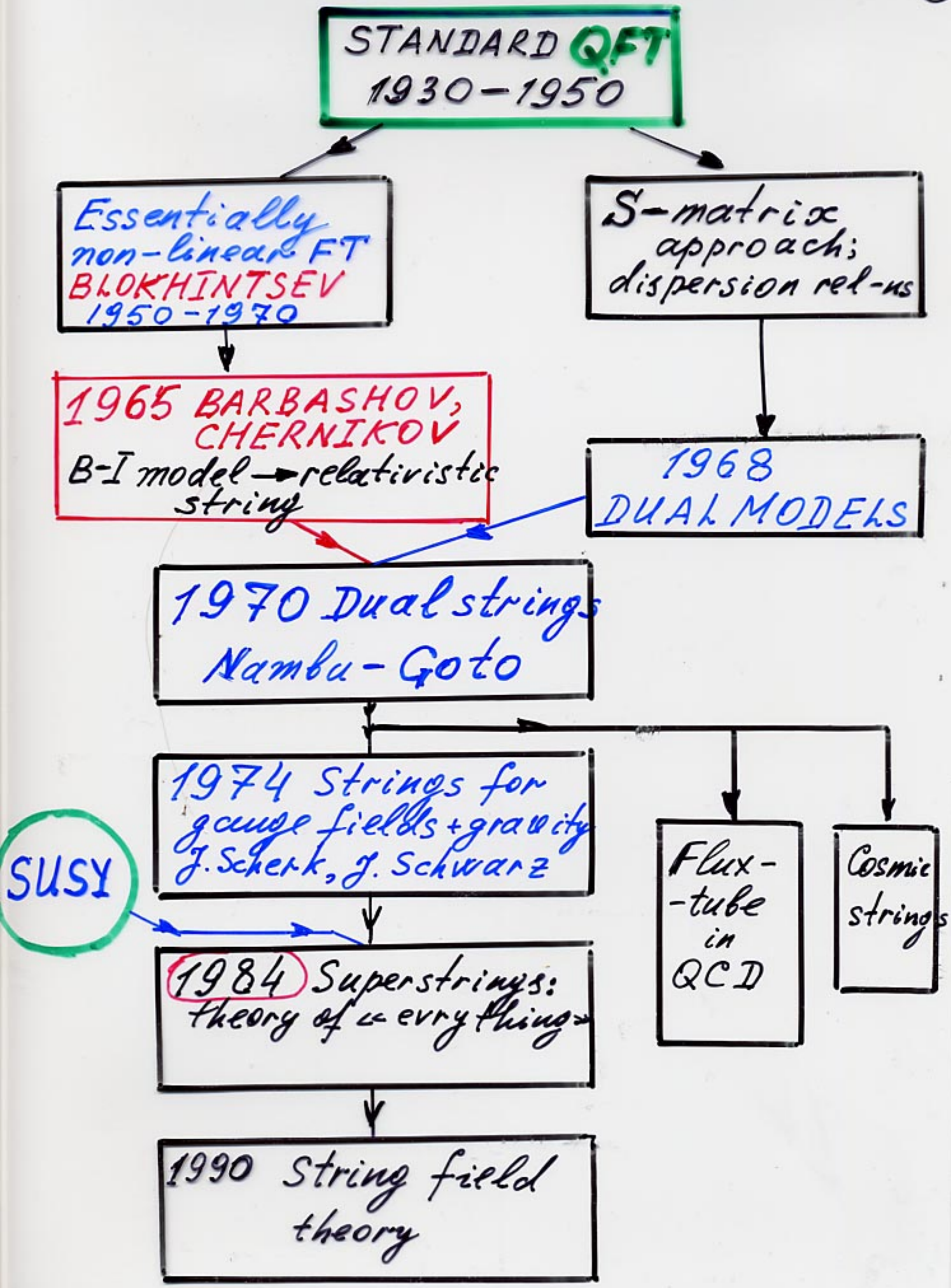
B.M. Barbashov and V.V. Nesterenko. Introduction to the Relativistic String Theory, World Scientific, Singapore, 1990, 250 p.

Critical look at string theory
Bert Schroer: arXiv: 0905.4006
0603.1112; 0611.132

The early years of string theory

1. John H. Schwarz. The Early Years of String Theory: A Personal Perspective. arXiv:0708.1917v3

2. Paolo Di Vecchia. The birth of string theory. Contribution to the volume "String theory and fundamental interactions", dedicated to Gabriele Veneziano on his 65th birthday. Lect. Notes Phys. 737:59-118,2008; arXiv:0704.0101v1.

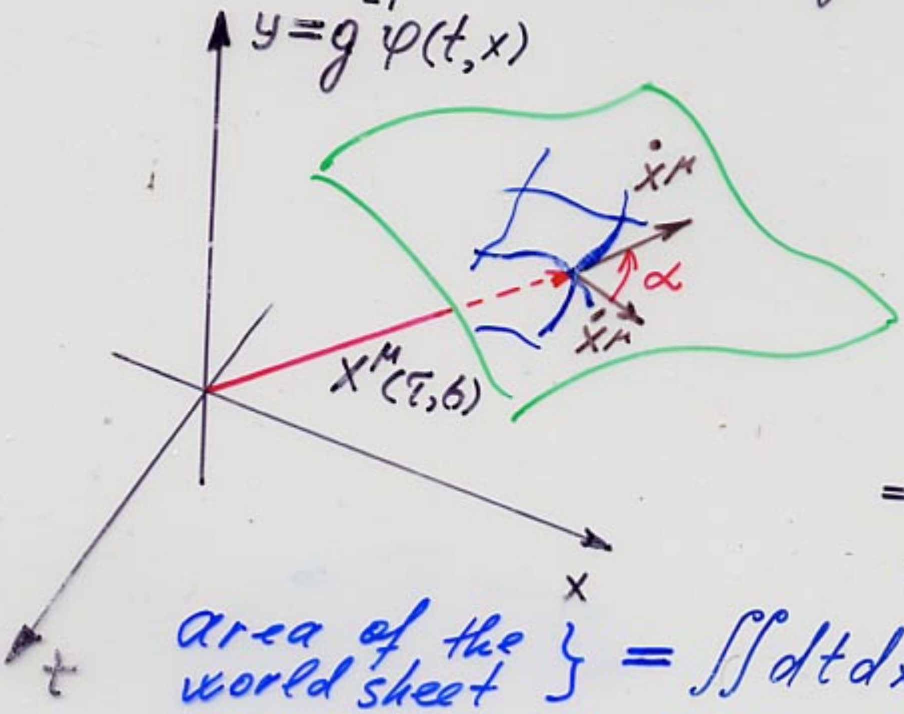

STANDARD QFT
1930 – 1950
Essentially non-linear FT
BLOKHINTSEV
1950 – 1970
S-matrix approach; dispersion rel-ns
1965 BARBASHOV, CHERNIKOV
B-I model → relativistic string
1968 DUAL MODELS
1970 Dual strings
Nambu – Goto
1974 Strings for gauge fields + gravity
J. Scherk, J. Schwarz
SUSY
Flux-tube in QCD
Cosmic strings
1984 Superstrings: theory of «evrything»
1990 String field theory

1965 B. M. Barbashov and N. A. Chernikov.
Solution and quantization of nonlinear two-dimensional Born–Infeld model. Sov. Phys.–JETP
Zh. Eksp. Theor. Fiz <u>50</u> (1966) 1296; <u>51</u> (1966) 658; Commun. Math. Phys. <u>5</u> (1966) 333.

$$\mathcal{L}_{B-I} = -g^2\sqrt{1-g^{-2}(\varphi_t^2-\varphi_x^2)}$$

$$\varphi=\varphi(t,x),\quad \varphi_t=\frac{\partial\varphi}{\partial t},\quad \varphi_x=\frac{\partial\varphi}{\partial x}$$

$$(g^2+\varphi_x^2)\varphi_{tt}-2\varphi_t\varphi_x\varphi_{tx}-(g^2-\varphi_t^2)\varphi_{xx}=0$$



$X^\mu(\tau,\sigma)$, $\mu=0,1,2$

$X^0(\tau,\sigma)=t(\tau,\sigma)$

$X^1(\tau,\sigma)=x(\tau,\sigma)$

$X^2(\tau,\sigma)=$
$=g^{-1}\varphi(t(\tau,\sigma),x(\tau,\sigma))$

$$\left.\begin{array}{l}\text{area of the}\\ \text{world sheet}\end{array}\right\} = \iint dt\,dx\,\mathcal{L}_{B-I}$$

$$d\Sigma = \dot{x}x' \sin\alpha \cdot d\tau d\sigma =$$

$$= \dot{x}x'\sqrt{1-\cos^2\alpha}\, d\tau d\sigma = \sqrt{\dot{x}^2 x'^2 - (\dot{x}x')^2}\, d\tau d\sigma$$

$$S = -\gamma \iint d\tau d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}$$

$$\delta S = 0$$

$$\downarrow$$

$$\begin{cases} \ddot{x}^\mu - x''^\mu = 0 \\ (\dot{x} \pm x')^2 = 0 \end{cases}$$

Quantization

$$x^\mu(\tau,\sigma) = x_+^\mu(\tau+\sigma) + x_-^\mu(\tau-\sigma)$$

$$x_\mu(\tau,\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\omega}} \left[a_\mu^+(k) e^{-i(k\sigma - \omega\tau)} + a_\mu^-(k) e^{i(k\sigma-\omega\tau)} \right]$$

$$[a_\mu^-(k), a_\nu^+(k')] = -g_{\mu\nu}\,\delta(k-k')$$

$$g_{\mu\nu} = \mathrm{diag}(+,-,-)$$

$$\Phi = \Pi\, a_\mu^+(k)\,|0\rangle$$

$$b(q) = \int e^{iqu^\pm} (\dot{x} \pm x')^2 du^\pm \quad , \quad u^\pm = \tau \pm \sigma$$

$$b(q) = \frac{1}{2} \int_{-\infty}^{+\infty} \alpha^\mu(q-p)\,\alpha_\mu(p)\,dp$$

"Virasoro" algebra

$$[b(p), b(q)] = (p-q)\, b(p+q)$$

$$-\infty < p, q < +\infty$$

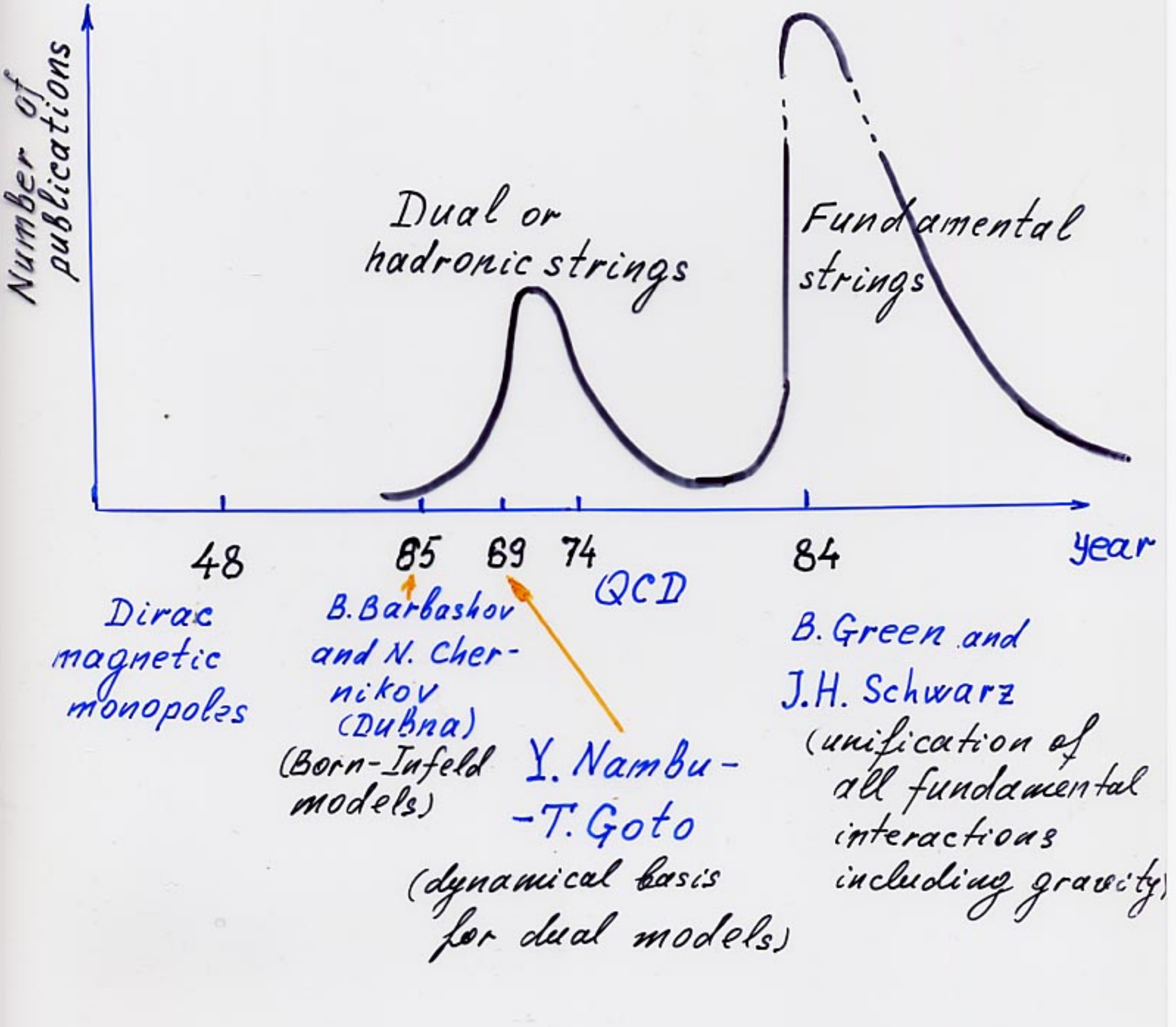
Anomaly term in the r.h.s.
is absent
(without central charge)

Introduction to the String Theory.

Basic Ideas and Applications.

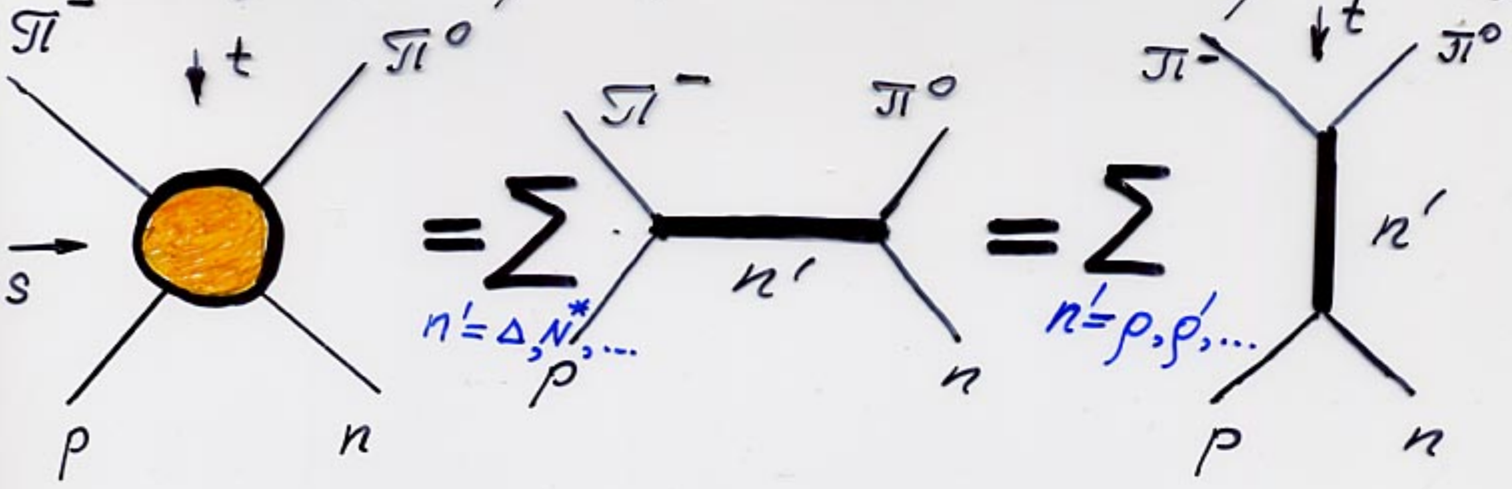
Lecture 1.

Short history of the string models



Dual models and dual strings

Duality principle for hadronic amplitudes



Veneziano amplitude for meson-meson scattering obeying the duality principle

$$A(s,t,u) = F(s,t) + F(t,u) + F(u,s)$$

$$F(s,t) = g^2 \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \equiv g^2 B(-\alpha(s), -\alpha(t))$$

Γ and B are the Euler functions,

$$\alpha(s) = \alpha(0) + \alpha' s.$$

Beta function has the following expansions

$$B(-\alpha(s), -\alpha(t)) = \sum_{n=0}^{\infty} \frac{\Gamma(n+1+\alpha(t))}{n!\,\Gamma(1+\alpha(t))} \frac{1}{n-\alpha(s)} =$$

$$= \sum_{n=0}^{\infty} \frac{\Gamma(n+1+\alpha(s))}{n!\,\Gamma(1+\alpha(s))} \frac{1}{n-\alpha(t)}.$$

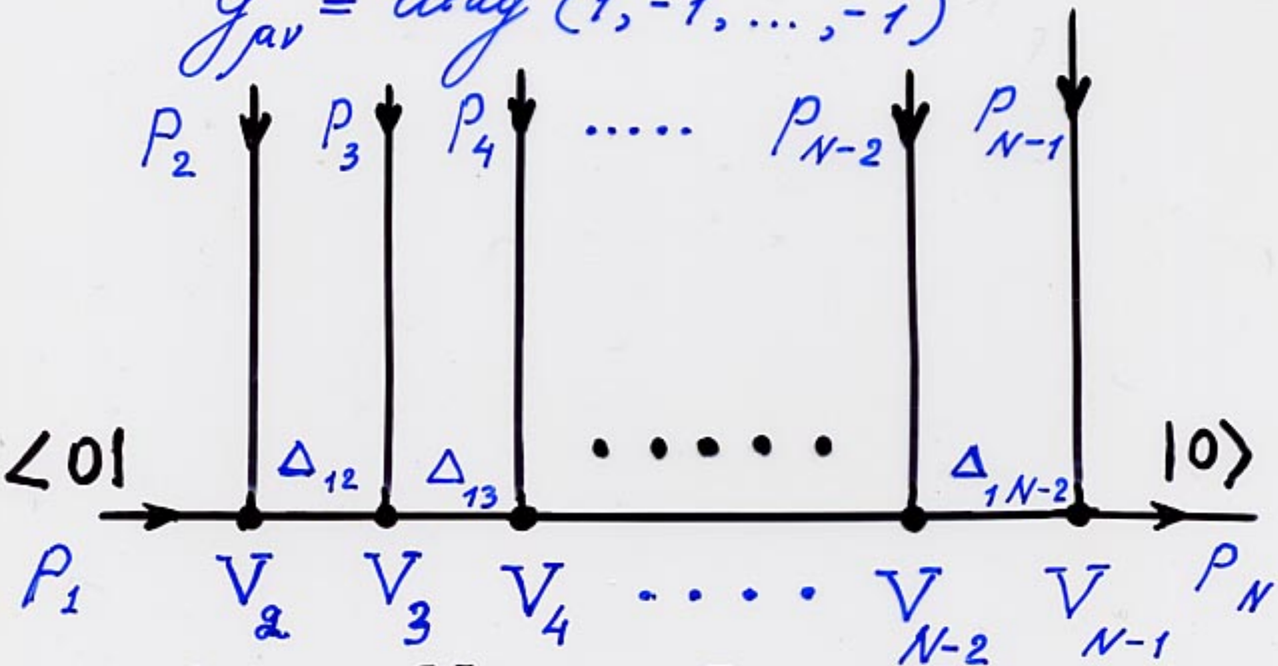
Operator formalism and diagram techniques for dual amplitudes

$$\alpha_n^\mu, \quad \mu = 0, 1, \ldots, D-1; \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\alpha_{n\mu} = \overset{*}{\alpha}_{-n\mu}$$

$$[\alpha_{m\mu}, \alpha_{n\nu}] = -m\, g_{\mu\nu}\, \delta_{n+m,0},$$

$$g_{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1)$$



$$B_N = \langle 0|V(p_2)\Delta_{12}V(p_3)\Delta_{13}\ldots V(p_{N-1})|0\rangle$$

Vertex operators are given by

$$V(p) = \exp\Big(i\sqrt{2\alpha'}\sum_{n=1}^{\infty} p^\mu \alpha^+_{n\mu}\Big)\cdot$$
$$\exp\Big(i\sqrt{2\alpha'}\sum_{n=1}^{\infty} p^\mu \alpha_{n\mu}\Big)$$

Propagator Δ_{ij} is

$$\Delta_{ij} = [s_{ij} + \alpha'\hat{M}^2 + \alpha(0)]^{-1},$$

$$s_{ij} = (p_i + p_{i+1} + \ldots + p_j)^2, \quad \hat{M}^2 = \sum_n \alpha_n^\mu \alpha_{n\mu}.$$

$$A_N = \sum_{\substack{\text{noncyclic}\\ \text{permutations}}} B_N(p_1, p_2, \dots, p_N),$$

$$B_N = \int_0^1 \cdots \int_0^1 \prod_{i=2}^{N-2} dx_i\, x_i^{-\alpha(s_{1i})-1} \prod_{2 \le i < j \le N-1} (1-x_{ij})^{-2\alpha' p_i p_j},$$

where

$$x_{ij} = x_i x_{i+1} \dots x_{j-1}, \quad \text{and} \quad \alpha(0) = 1$$

Virasoro conditions on the physical state vectors

$$L_n |\phi\rangle = 0, \quad n = 1, 2, \dots$$

$$(L_0 - \alpha(0))|\phi\rangle = 0, \qquad L_0 \equiv \hat{M}^2$$

$$L_n = -\frac{1}{2} \sum_{m=-\infty}^{+\infty} :\alpha_{n-m}\alpha_m:,$$

$$\alpha_{0\mu} = \sqrt{2\alpha'}\, p_\mu, \qquad \alpha_{-k} = \alpha_k^+ = \sqrt{k}\, a_k^+,$$

$$k = 1, 2, \dots$$

$$[a_k^\mu, a_\ell^{\nu +}] = -\delta_{k\ell}\, g^{\mu\nu}$$

$$|\Phi\rangle = a_{n_1}^{+\mu_1} a_{n_2}^{+\mu_2} \dots a_{n_m}^{+\mu_m} |0\rangle$$

When $\mu_j = 0$ we have the ghost states with negative norm.

For time-like components of a_n^μ we have

$$[a_k^0, a_l^{0+}] = -\delta_{kl}$$

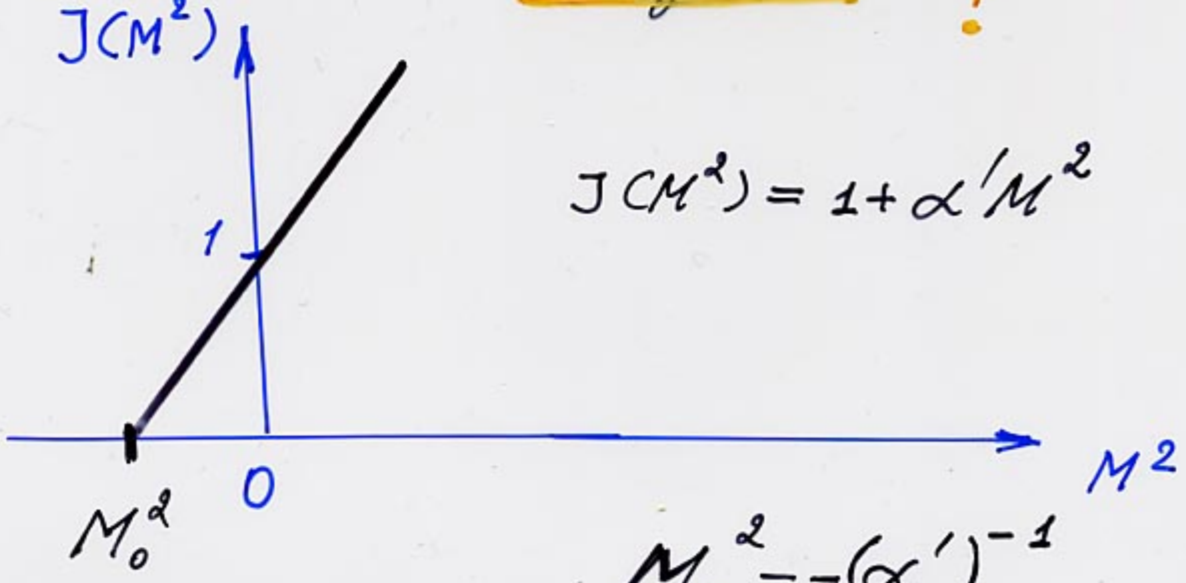
$$|g\rangle = a_k^{0+}|0\rangle\ ; \quad \langle g|g\rangle = \langle 0|a_k^0\, a_k^{0+}|0\rangle =$$

$$= -\langle 0|0\rangle = -1$$

In order to remove the ghost states one has to require

$$D = 26\,, \qquad \alpha(0) = 1$$

ground state is tachyonic !



$$J(M^2) = 1 + \alpha' M^2$$

$$M_0^2 = -(\alpha')^{-1}$$

To obtain the set of operators

$$\alpha_n^\mu\,, \quad n = 0, \pm 1, \pm 2, \ldots$$
$$\mu = 0, 1, \ldots, D-1$$

one has to quantize the ONE-DIMENSIONAL RELATIVISTIC OBJECT.

Linear string model with the action S_{Lin} is not suitable

$$S_{Lin} \sim \frac{1}{2}\iint d\tau d\sigma \left(\dot{x}^2 - x'^2\right) \Longrightarrow \ddot{x}^\mu - x''^\mu = 0$$

$$x = x^\mu(\tau,\sigma), \quad \dot{x} = \frac{\partial x}{\partial \tau}, \quad x' = \frac{\partial x}{\partial \sigma},$$

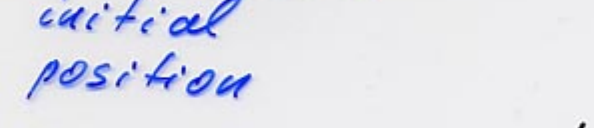
because here there are no Virasoro conditions. The required model should have a gauge symmetry to produce the Virasoro conditions in a consistent way

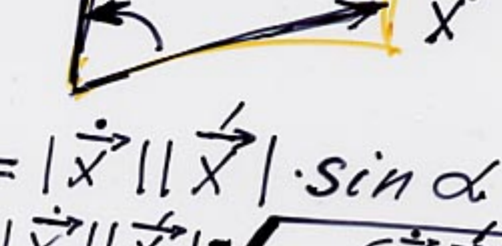
$$S_{N-G} = -\gamma \iint dS = -\gamma \int_{-\infty}^{+\infty} d\tau \int_{\sigma_0}^{\sigma_1} d\sigma \sqrt{-g}$$

$$g = \det g_{ij}, \quad g_{ij} = \partial_i x^\mu \partial_j x_\mu$$

$$-g = (\dot{x}x')^2 - \dot{x}^2 x'^2$$

$$\alpha' = (2\pi\gamma)^{-1}. \qquad \dot{x}^2 > 0, \; x'^2 < 0.$$





$$d\Sigma = |\vec{\dot{x}}||\vec{x}'| \cdot \sin\alpha \, d\tau d\sigma =$$

$$= |\vec{\dot{x}}||\vec{x}'|\sqrt{1-\cos^2\alpha} \, d\tau d\sigma = |\vec{\dot{x}}||\vec{x}'|\sqrt{1 - \frac{(\vec{\dot{x}}\vec{x}')^2}{\vec{\dot{x}}^2 \vec{x}'^2}} \, d\tau d\sigma$$

Y. Nambu. Lectures for the Copenhagen
Summer Symposium, August,
1970 (unpublished)

Strings, Lattice gauge theory,
High energy phenomenology.
Tata Institute of fundamental research,
Bombay, India.
Proceedings of the Winter School
PANCHGANI 25 January - 5 February
1986. Editors V. Singh and S.R. Wadia.
World Scientific 1987 Singapore.

pp. 573-596. Y. Namby «Duality and
hadrodynamics».
Notes prepared for the Copenhagen High
Energy Symposium, August, 1970
(unpublished and undelivered)

Lecture 3. Polyakov formulation of the string theory.

The basic disadventidge of the Nambu-Goto action is its nonlinearity in terms of $x^\mu(\tau,\sigma)$.

This problem is solved by using the following action

$$S = -\frac{T}{2}\iint du^0 du^1 \sqrt{|g|}\, g^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x_\mu ,$$

$$u=(u^0;u^1),\quad u^0=\tau,\ u^1=\sigma,\quad \mu=0,1,\dots,D-1;$$

$$\alpha,\beta=0,1,\qquad g_{\alpha\beta}g^{\beta\gamma}=\delta_\alpha^\gamma .$$

$g_{\alpha\beta}(u)$ is an auxiliary field (metric);
$x^\mu(u^0;u^1)$ is the string position vector.

Symmetry properties:

$$\delta u^\alpha = \xi^\alpha(u^0;u^1),\qquad \delta x^\mu = -\xi^\alpha\partial_\alpha x^\mu,$$

$$\delta g_{\alpha\beta} = \xi^\gamma\partial_\gamma g_{\alpha\beta} + g_{\gamma\beta}\partial_\alpha\xi^\gamma + g_{\alpha\gamma}\partial_\beta\xi^\gamma ;$$

$+$

Weyl transformations

$$g_{\alpha\beta}(u)\to \exp(\varphi(u))\cdot g_{\alpha\beta}(u)$$

global Poincaré transformations

$$\delta x^\mu = \omega^{\mu\nu}x_\nu + \varepsilon^\mu ;\quad \omega^{\mu\nu}+\omega^{\nu\mu}=0,\quad \delta g_{\alpha\beta}=0.$$

At the classical level, new action is completely equivalent to the Nambu-Goto action.

$$\frac{\delta S}{\delta x^\mu} = 0 \mapsto \Delta_g x^\mu = \frac{1}{\sqrt{|g|}}\left(\partial_\alpha g^{\alpha\beta}\sqrt{|g|}\,\partial_\beta x^\mu\right) = 0,$$

Laplace - Beltrami operator for metric field $g_{\alpha\beta}(u)$

$$\frac{\delta S}{\delta g^{\alpha\beta}} = 0 \mapsto \frac{1}{2}\sqrt{|g|}\,T_{\alpha\beta} =$$

$$= -\frac{T}{2}\sqrt{|g|}\left(\partial_\alpha x^\mu \partial_\beta x_\mu - \frac{1}{2} g_{\alpha\beta}\partial_\gamma x^\mu \partial_\delta x_\mu g^{\gamma\delta}\right) = 0 \quad (*)$$

$$dg = dg_{\alpha\beta}\cdot g^{\alpha\beta}\cdot g = -g_{\alpha\beta}\, dg^{\alpha\beta} g$$

$$\frac{\partial\sqrt{|g|}}{\partial g^{\alpha\beta}} = -\frac{1}{2}\sqrt{|g|}\, g_{\alpha\beta}$$

$T_{\alpha\beta}(u)$ is a "metric" symmetric energy-momentum tensor of string coordinates $x^\mu(u)$.

Solution to eq. $(*)$ is

$$g_{\alpha\beta}(u) = f(u)\cdot\partial_\alpha x^\mu \partial_\beta x_\mu,$$ with $f(u)$ being arbitrary function

$$g_{\alpha\beta}\, g^{\beta\alpha} = 2.$$

$$\Delta_g x^\mu \Rightarrow \Delta_{ind.met.} x^\mu = 0$$

$x^\mu(u)$ should be a minimal surface.

$$S \to S_{N-G} = -T \iint d^2u \sqrt{|g|_{ind.}}$$

Orthonormal gauge

$$g_{\alpha\beta}(u) = e^{\varphi(u)} \eta_{\alpha\beta}, \quad \eta_{\alpha\beta} = \mathrm{diag}\,(1,-1)$$

$$T_{\alpha\beta} = 0 \to \begin{pmatrix} \dot{x}^2 - \frac{1}{2}(\dot{x}^2 - x'^2) & \dot{x}x' \\ \frac{1}{2}(\dot{x}^2 + x'^2) & \\ \dot{x}x' & \frac{1}{2}(\dot{x}^2 - x'^2) \end{pmatrix} = 0$$

$$\downarrow$$

$$\dot{x}^2 + x'^2 = 0, \quad x'\dot{x} = 0$$

$$(\dot{x} \pm x')^2 = 0$$

Quantization:

⊗ Conformal invariance is broken due to the quantum anomaly (or conformal anomaly).

Partition function

$$Z = \int Dg_{\alpha\beta}\, Dx^{\mu} \exp\left\{-\frac{T}{2}\iint d^2u \sqrt{g}\, g^{\alpha\beta}\partial_{\alpha}x^{\mu}\partial_{\beta}x^{\mu} - \mu_0^2 \iint d^2u \sqrt{g}\right\}$$

integration by parts

$$= \int Dg_{\alpha\beta}\, Dx^{\mu} \exp\left\{-\frac{T}{2}\iint d^2u \sqrt{g}\, x^{\mu}\Delta_g x^{\mu} - \mu_0^2 \iint d^2u \sqrt{g}\right\}$$

Let us put

$$e^{-F} = \int Dx^{\mu} \exp\left\{-\frac{T}{2}\iint d^2u \sqrt{g}\, x^{\nu}\Delta x^{\nu} - \mu_0^2 \iint d^2u \sqrt{g}\right\}$$

$$\frac{\delta}{\delta g^{\alpha\beta}} \qquad -\frac{2}{\sqrt{g}}\frac{\delta F}{\delta g^{\alpha\beta}} = \langle T_{\alpha\beta}\rangle$$

In the conformal flat metric

$$g_{\alpha\beta}(u) = \rho(u)\,\delta_{\alpha\beta}, \quad \rho(u) = e^{\varphi(u)}$$

$$\langle T_{\alpha}{}^{\alpha}\rangle = -2\frac{\delta F}{\delta\rho} = D\,Y(u,u';t)$$

$$\left.\begin{aligned} &\frac{d}{dt}Y(u,u';t) = -\Delta Y(u,u';t) \\ &Y(u,u';0) = \frac{1}{\sqrt{g}}\,\delta(u,u') \end{aligned}\right\}$$ "heat" equation

$$Y(u,u';t) = -\frac{i}{4\pi t} + \frac{R}{24\pi} + O(t)$$

$$R = -\frac{1}{\rho}\partial^2 \ln\rho$$

$$\langle T_\alpha{}^\alpha \rangle = \frac{D}{24\pi} R + \text{const}$$

$$-F = -\frac{D}{48\pi}\iint d^2u\left[\frac{1}{2}(\partial_i \ln\rho)^2 + \mu_0^2\rho\right]$$

Faddeev-Popov determinant

$$\Delta^{1/2}(\varphi) = \frac{1}{2}\ln\det(L) = \frac{26}{48\pi}\iint d^2u\left[\frac{1}{2}(\partial_i\varphi)^2 + \mu_0^2 e^{\varphi}\right],$$

where

$$L_{\alpha\beta} = \nabla_\alpha\nabla_\beta - \nabla_\beta\nabla_\alpha + g_{\gamma\delta}\nabla^\gamma\nabla^\delta\,\delta_{\alpha\beta}$$

Finally we obtain

$$Z = \int [D\varphi] \exp\left\{-\frac{26-D}{48\pi}\iint d^2u\left[\frac{1}{2}(\partial_i\varphi)^2 + \mu_0^2 e^{\varphi}\right]\right.$$

$$\partial_i^2\varphi + \mu_0^2 e^{\varphi} = 0$$ Liouville equation.

Total superstring action ($D=10$)

Dynamical variables:

$X^{\mu}(\tau,\sigma)$

$\left\{\begin{array}{l}\text{two anticommuting}\\ \text{space-time}\\ \text{spinors}\end{array}\right\} \to \Theta^{a}_{A}$ ← spinor index in 10-dimensional space-time

$A = 1, 2$

$a = 1, 2, \dots, 32 = 2^{[D/2]} = 2^{5}$

It is not a world sheet index !

Θ^{a}_{A} should be a Majorana-Weyl spinor in the index a

$$h^{ab}\Theta^{b}_{A} = 0, \quad \bar{\Theta}^{a}_{A} = \Theta^{b}_{A}(\gamma^{0})^{ba}, \qquad (*)$$

where h is the Weyl projection operator, $2h = 1 \pm \gamma^{11}$

This condition can be imposed only for $D = 2 \pmod 8$

$$D = 2, 10, 18, \dots$$

Conditions $(*)$ + boundary conditions + Dirac equation for Θ^{a}_{A} reduce the number of independent fermion degrees of freedom to $2^{3} = 8$

$$S = \frac{T}{2}\iint d\tau\, d\sigma\, (\mathcal{L}_1 + \mathcal{L}_2),$$

where

$$\mathcal{L}_1 = -\frac{1}{2\pi}\sqrt{-g}\, g^{\alpha\beta}\, \Pi^{\mu}_{\alpha}\, \Pi_{\mu\beta}, \quad \Pi^{\mu}_{\alpha} = \partial_\alpha x^\mu - i\bar{\theta}_A \gamma^\mu \partial_\alpha \theta_A;$$

$$\mathcal{L}_2 = -i\varepsilon^{\alpha\beta}\Big\{\partial_\alpha x^\mu\big[\bar{\theta}_1\gamma_\mu\partial_\beta\theta_1 - \bar{\theta}_2\gamma_\mu\partial_\beta\theta_2 +$$

$$+ \bar{\theta}_1\gamma^\mu\partial_\alpha\theta_1\,\bar{\theta}_2\gamma_\mu\partial_\beta\theta_2\big]\Big\}, \quad \alpha,\beta = 0,1;$$

$$\mu = 0,1,\dots,9.$$

Global $N=2$ supersymmetry

$$\delta\theta_A = \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\theta_A + \varepsilon_A, \quad \gamma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]_-,$$

$$\delta x^\mu = \omega^\mu_{\ \nu} x^\nu + a^\mu + i\bar{\varepsilon}_A\gamma^\mu\theta_A; \quad \delta g^{\alpha\beta} = 0$$

The boundary conditions reduce N to 1.

Light-cone gauge conditions

$$x^- = p^-\tau, \quad \gamma^-\theta_A = 0$$

θ_A become the Majorana world-sheet spinor.

String interaction (path integral approach)

(Mandelstam, Polyakov)

$$A \sim \int Dx_\mu \exp\{S_{str}\}$$

Vacuum amplitudes

String dual amplitudes

For these gauge groups

$SO(32)$

$E_{8\times 8}$

gauge anomaly are cancelled.

Field theory of interecting string

$$\Psi[x(\sigma)] = \{\varphi(x) + A^{(1)}_\mu(x)\alpha^\mu_{-1} + h_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} +$$

$$+ A^{(2)}_\mu(x)\alpha^\mu_{-2} + \dots\}|0\rangle$$

string field

Expansion in terms of the eigenvectors of the M^2 operator.

$\varphi(x)$ is a scalar field;

$A_\mu(x)$ is an electromagnetic field;

$h_{\mu\nu}(x)$ is ~~gr~~ tensor field with mass 1

and so on.

Gravitation field is described by closed strings.

Virasoro conditions

$$L_n \Psi[x(\sigma)] = \delta_{n,0}\Psi[x(\sigma)], \quad n = 0, 1, 2, \dots$$

$$L_0\varphi(x) = -\alpha' P^2\varphi(x) = \underbrace{\alpha'\partial^2\varphi(x) = \varphi(x)}_{\text{Klein-Gordon eq. with squared mass } = -(\alpha')^{-1}}$$

Klein-Gordon eq. with squared mass $= -(\alpha')^{-1}$

tachyonic field!

$$(\alpha'\partial^2 - 1 + n)A^{(n)}_\mu(x) = 0$$

$$\partial_\mu A^\mu = 0$$

$$(\alpha'\partial^2 + 1)h_{\mu\nu}(x) = 0 \quad \text{and so on}$$

Action for string fields

$$S \sim \Psi * \Psi * \Psi$$

$*$ string product

non commutativ geometry and so on

Complete field theory of interacting string is absent!

Relation with real physics through **compactification** of the space-time.

The solution to the equations of motion in the interacting string theory should have a form

$$V_4 \otimes K_6 ,$$

wher V_4 is the Riemannian space-time and K_6 is a compact manifold (with Plank dimensions).

K_6 maybe Kalabi-Yao manifold or something else (additional Z-boson)!

		Cl. Mech	QM	QFT
Point particle	Dynamical variable	$\vec{r}(t)$ trajectories	$\psi(\vec{r},t)$ waive funct.	functionals of $\psi(\vec{r},t)$
	Dynamical eqs.	$m\frac{d\dot{\vec{r}}}{dt}=\vec{F}$ ODE	$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi$ PDE	functional eqs (Schwinger--Dyson eqs.)
String	Dynamical variable	$r^\mu(\tau,\sigma)$ field	functionals of $r^\mu(\tau,\sigma)$	?
	Dynamical eqs.	$\ddot{r}^\mu - r''^\mu = 0$ $(\dot{r}^\mu \pm r'^\mu)^2 = 0$ PDE		functionals of functionals?

Basic variational problems in classical differential geometry and their applications in theoret. physics

Geometry	Physics
1. Minimal surfaces $A_1 = \alpha \iint dS$; $\delta A_1 = 0$ $\to$ $H = 0$	Relativistic strings, membranes, p-branes, hadronic strings, cosmic strings
2. Extremal surfaces enveloping fixed volume $A_2 = \alpha \oiint dS + \beta \iiint dV$ $\delta A_2 = 0, \to H = \frac{\beta}{\alpha} = const \neq 0$	W-geometry, W-strings the Laplace theory om membranes
3. Willmore surfaces $A_3 = \alpha \oiint H^2 dS$ $\delta A_3 = 0 \to$ Willmore eq. $\Delta H + 2H^3 + R \cdot H = 0$	Riged strings; biological (lipid or liquid) membranes, red blood cells

4. Willmore surfaces with fixed enclosed volume and area

Helfrich theory of liquid membranes and vesicles

$$A_4 = \alpha \oiint H^2 dS + \beta \oiint dS + \gamma \iiint dV$$