On black holes, cosmologies, superstrings, and dynamical systems describing them

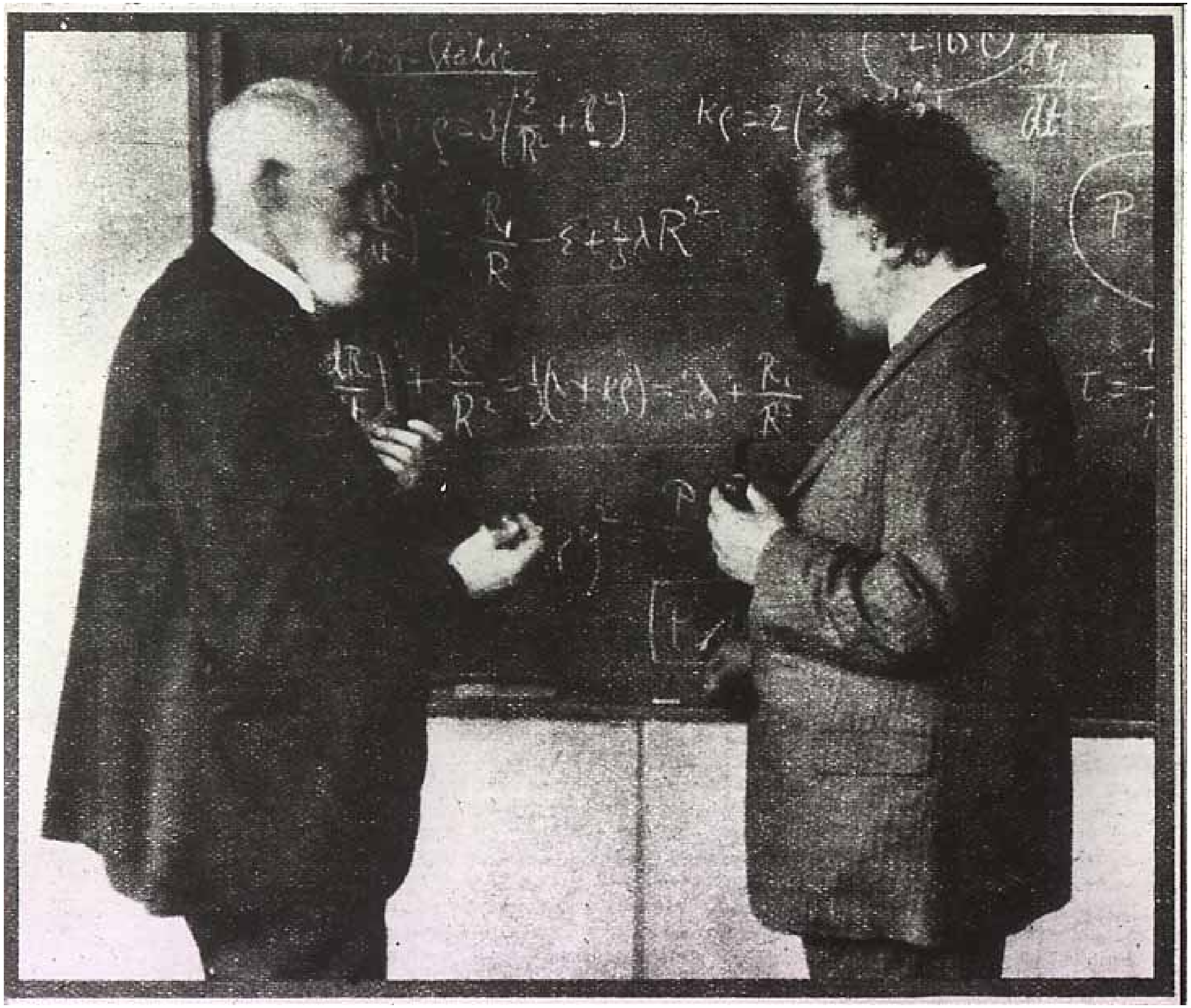
Remarks on history and brief summary of present status

Dark energy, dark matter, inflation, multiverse, black holes and some other mysterious and invisible things are touched up here. **We must be cautious while thinking about them!**

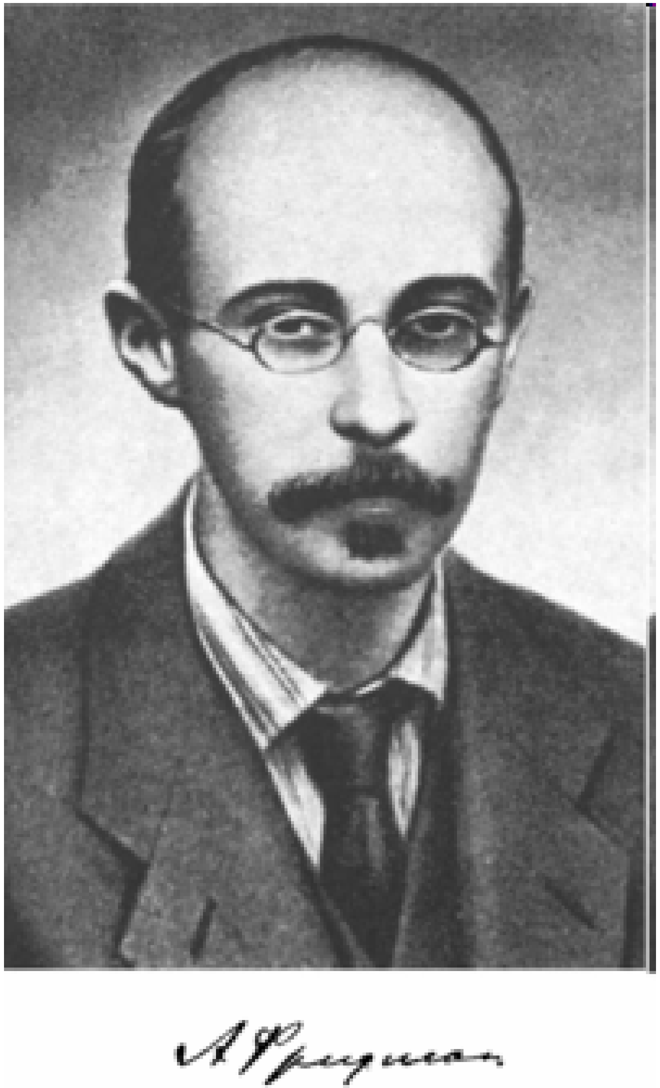
As I was going up the stair,
I met a man who wasn't there.
He wasn't there again today,
I wish, I wish he'd stay away.

Hughes Mearns "**Antigonish**" (1899)

Epigraph to: **S.Weinberg**, **The cosmological constant problem**, Rev. Mod. Phys. 61 (1988)



De Sitter and Einstein

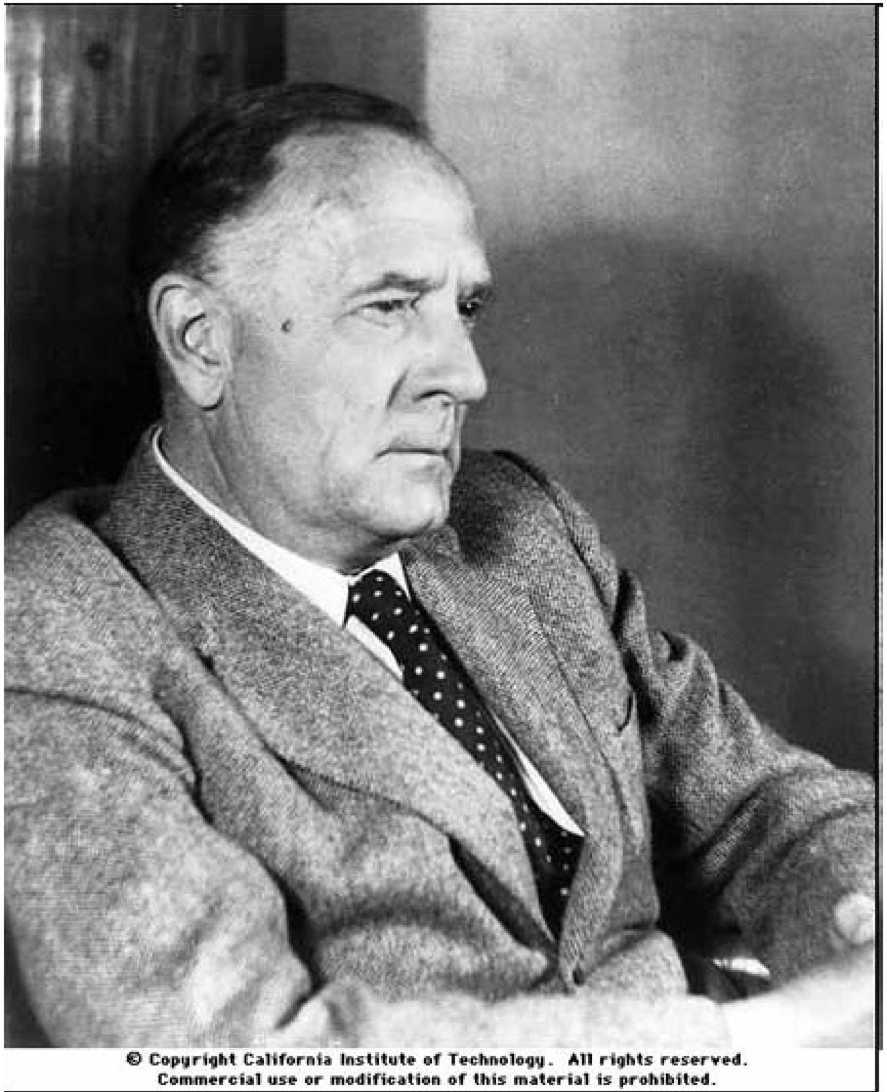


Alexander Friedmann
1888-1925

Georges Lemaitre, 1894-1966

Some history (up to Hubble)

	B.H.	Cosmol. (waves)	Generalizat. of gr.
			extra dimension
1916	Schwarzsch.		Nordstrøm ('14–..); P. Ehrenfest
1917	R.-N. B.H.	static (Λ) Einstein; de Sitter	
1918	.	.	H. Weyl ('18 – '23) (unif of Gr. and E.M.)
1919	.	.	A. Eddington ('19 – '23) (general Γ)
'21	.	.	Kaluza (extra d.)
'22	.	Friedman ('22–24)	.
'23	.	(fall of Λ)	Einstein (symm. Γ); Cartan (torsion etc.
'26	.		H. Mandel, V. Fock, O. Klein (extra dim., brane)
'27	....	G. Lemaître	
'29	.	Hubble	Einstein (N-S grav)



Edwin Hubble, 1889-1953

George Gamow, 1904-1968

Zur Herleitung der Feldgleichungen in der allgemeinen Relativitätstheorie.

(Erste Mitteilung.)

Von **Heinrich Mandel** in Petersburg.

(Eingegangen am 12. Juli 1926.)

Ähnlich wie die nichteuclidische Maßbestimmung in einer zweidimensionalen Fläche durch ihre Beschaffenheit in einem höheren (2 + 1 dimensionalen) euclidischen Raum induziert wird, versuchen wir die nichteuclidische Maßbestimmung der vierdimensionalen Raum-Zeit-Welt, so weit sie aus der Erfahrung bekannt ist, dadurch zu erklären, daß man sich die Welt als eine vierdimensionale Hyperfläche in einem höheren ($4 + k$ dimensionalen) euclidischen Raume vorstellt. Der Materie-Energie-Tensor steht dann in enger Verbindung mit dem zweiten Fundamentaltensor dieser Hyperfläche. Die Weltlinien der Materie sind Krümmungslinien der Welt. Eine fünfdimensionale Betrachtungsweise scheint für das Verständnis der elektromagnetischen Eigenschaften der Materie wesentlich zu sein[1]).

[1]) Anmerkung bei der Korrektur: Dies ist schon von Th. Kalusa im Jahre 1921 bemerkt worden und in derselben Weise entwickelt (Th. Kalusa, Zum Unitätsproblem der Physik, Berl. Ber. 1921, S. 966), was mir erst aus einem Hinweis von O. Klein in seiner Arbeit (ZS. f. Phys. **37**, 895, 1926) bekannt wurde.

New age

1938 (BH collapse) (n-l. gr. waves) (RMXF-modern.)
39 • Oppenheimer, Snyder : Einstein, Rosen : Einst., Bergmann

(NS-gr.)
1948 • (Gamow) (B.B) Schrödinger ('48-'50)

'62-'63 Kerr Ehlers, Kundt (waves) {fiber bundles {gauge theories

'65 Newman (Penzias, Wilson) Pauli; YM, Utiyama.... ('50 →.)

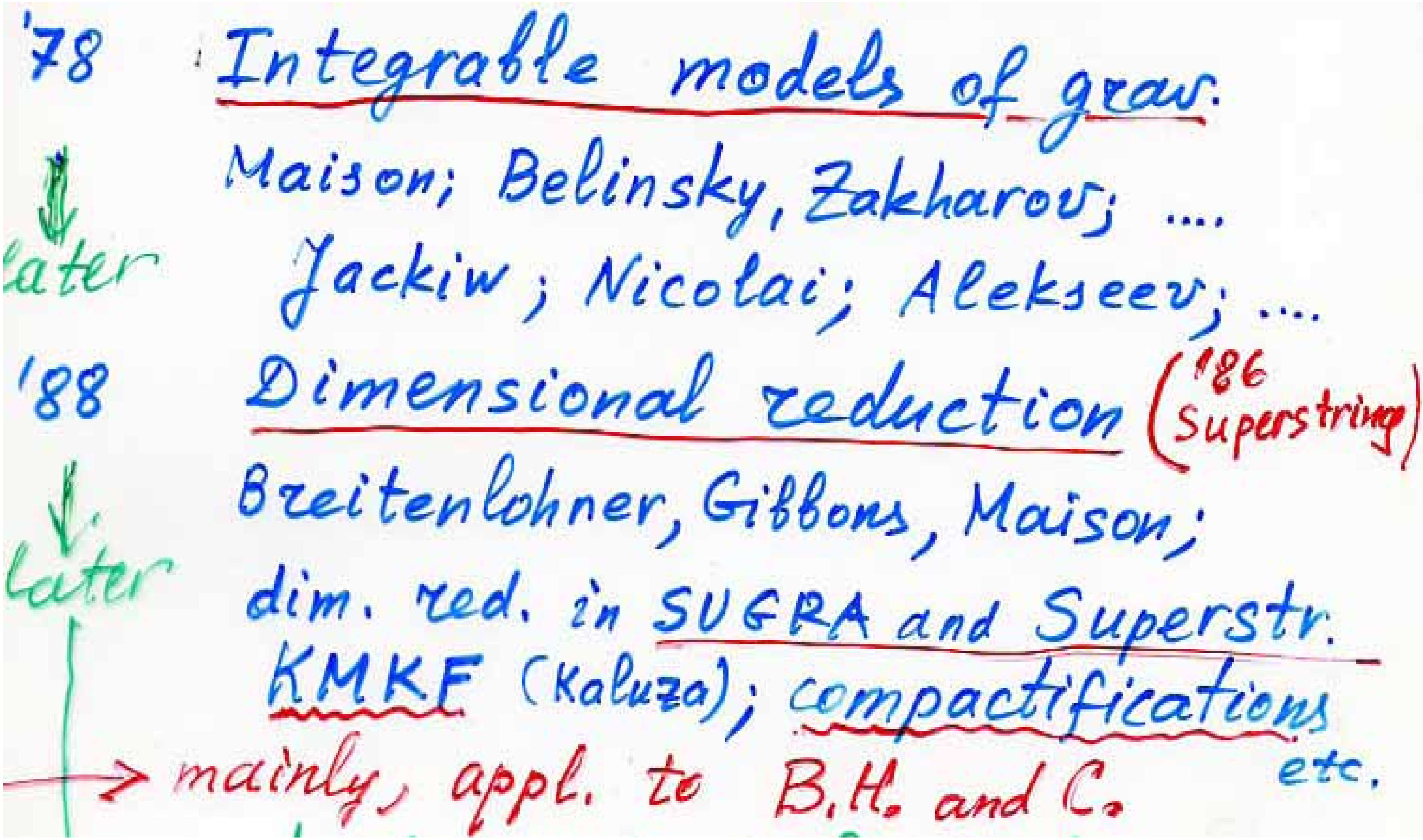
'67-'72 W-S, SUSY, strings
'74→ GUT, '76→ SUGRA, '78→ Integrable models

'85→ Superstring 'revolution' ..

'88→ Dimensional reduction ...

Great observations that completely changed the status of cosmology

Integrable models of black holes and of cosmologies


'78 Integrable models of grav.
Maison; Belinsky, Zakharov; ....
later Jackiw; Nicolai; Alekseev; ....
'88 Dimensional reduction ('86 Superstring)
Breitenlohner, Gibbons, Maison;
later dim. red. in SUGRA and Superstr.
KMKF (Kaluza); compactifications
→ mainly, appl. to B.H. and C. etc.

Group theoretical analysis of integrability and practical methods for solving integrable equations in gauge and other theories with symmetries (A. Leznov and M. Saveliev, ….)

Joel Scherk
1946 - 1980

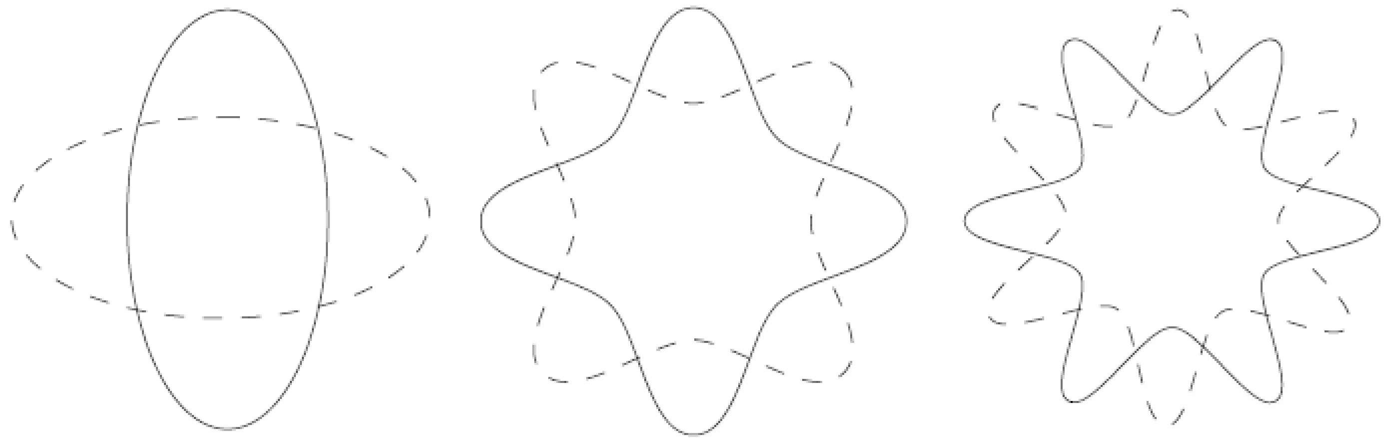
String investigation started around the end of sixties. Quantum strings – in the beginning of `70 (in relation to the dual resonance model of hadrons). Scherk and Schwartz boldly proposed fundamental strings in 1974

‘70 – ‘84 **String Theory** development : Veneziano, Nambu, Goto (an earlier work by Barbashov and Chernikov ‘66), Gervais, Neveu, Sakita, Virasoro, Mandelstam, Scherk, Schwartz, Green, Kaku, Kikkawa,GSO, Ramond – Neveu-Schwartz , Polyakov,…

SUSY and **SUGRA**: Yu. Golfand, D.Volkov, Wess, Zumino, Deser, Witten, Ferrara, Stelle, Howe, Cremmer, V.Ogievetsky, …

1984 - `revolution’ and intensive development of **Superstring Theory**: Green, Schwartz-Witten, Gross, Harvey, Martinec,Rohm, …

All particles are excited states of strings? A complete understanding of particle physics and of cosmology? A distant dream or mirage?



String theory arose in the late 1960s in an attempt to understand the strong nuclear force. This is the force that is responsible for holding protons and neutrons together inside the nucleus of an atom as well as quarks together inside the protons and neutrons. A theory based on fundamental one-dimensional extended objects, called strings, rather than point-like particles, can account qualitatively for various features of the strong nuclear force and the strongly interacting particles (or hadrons).

The basic idea in the string description of the strong interactions is that specific particles correspond to specific oscillation modes (or quantum states) of the string. This proposal gives a very satisfying unified picture in that it postulates a single fundamental object (namely, the string) to explain the myriad of different observed hadrons, as indicated in Fig. 1.1.

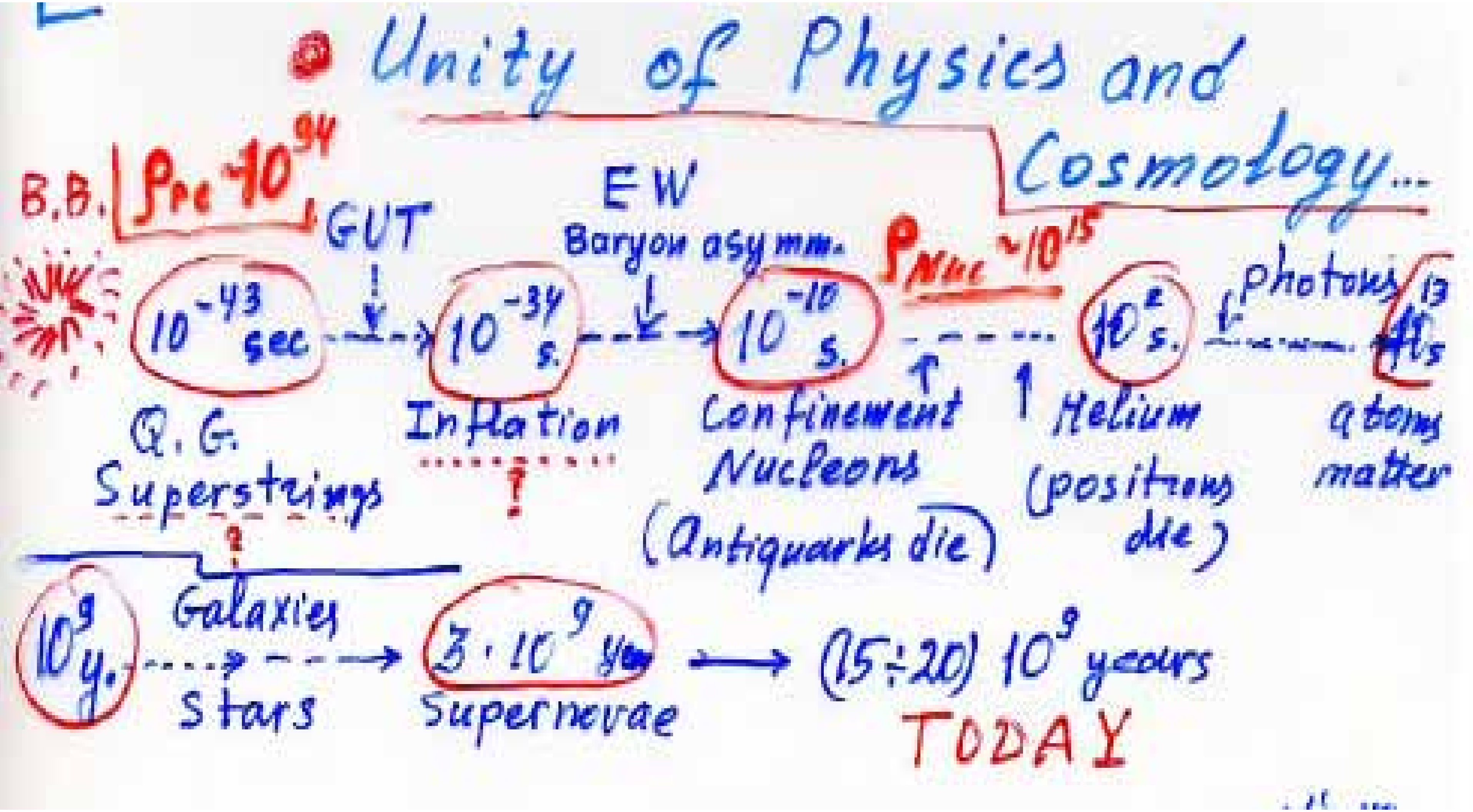
In the early 1970s another theory of the strong nuclear force – called quantum chromodynamics (or QCD) – was developed. As a result of this, as well as various technical problems in the string theory approach, string theory fell out of favor. The current viewpoint is that this program made good sense, and so it has again become an active area of research. The concrete string theory that describes the strong interaction is still not known, though one now has a much better understanding of how to approach the problem.

String theory turned out to be well suited for an even more ambitious purpose: the construction of a quantum theory that unifies the description of gravity and the other fundamental forces of nature. In principle, it has the potential to provide a complete understanding of particle physics and of cosmology. Even though this is still a distant dream, it is clear that in this fascinating theory surprises arise over and over.

STRING THEORY AND M-THEORY

KATRIN BECKER,
MELANIE BECKER,
JOHN H. SCHWARZ

Schematic illustration of



Is LHC the last super-high-energy accelerator in our lifetime?

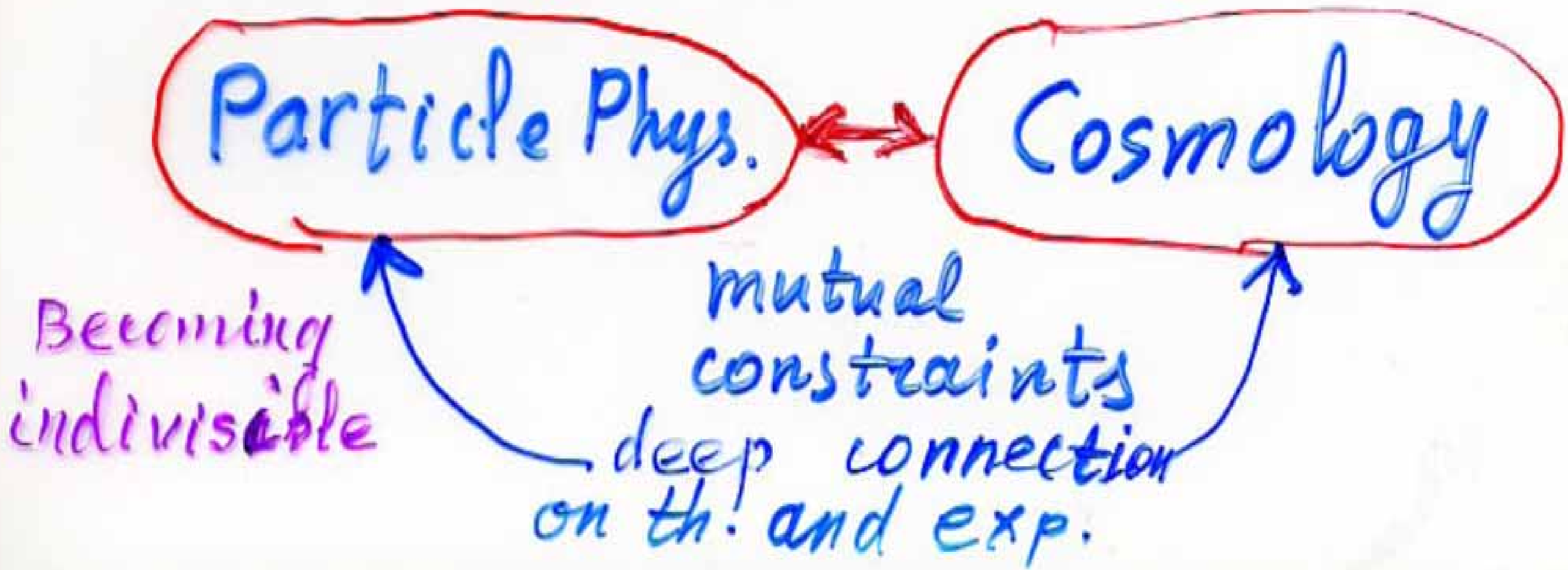
Grand DUALITY: Particle (String) Physics ←→ Cosmology

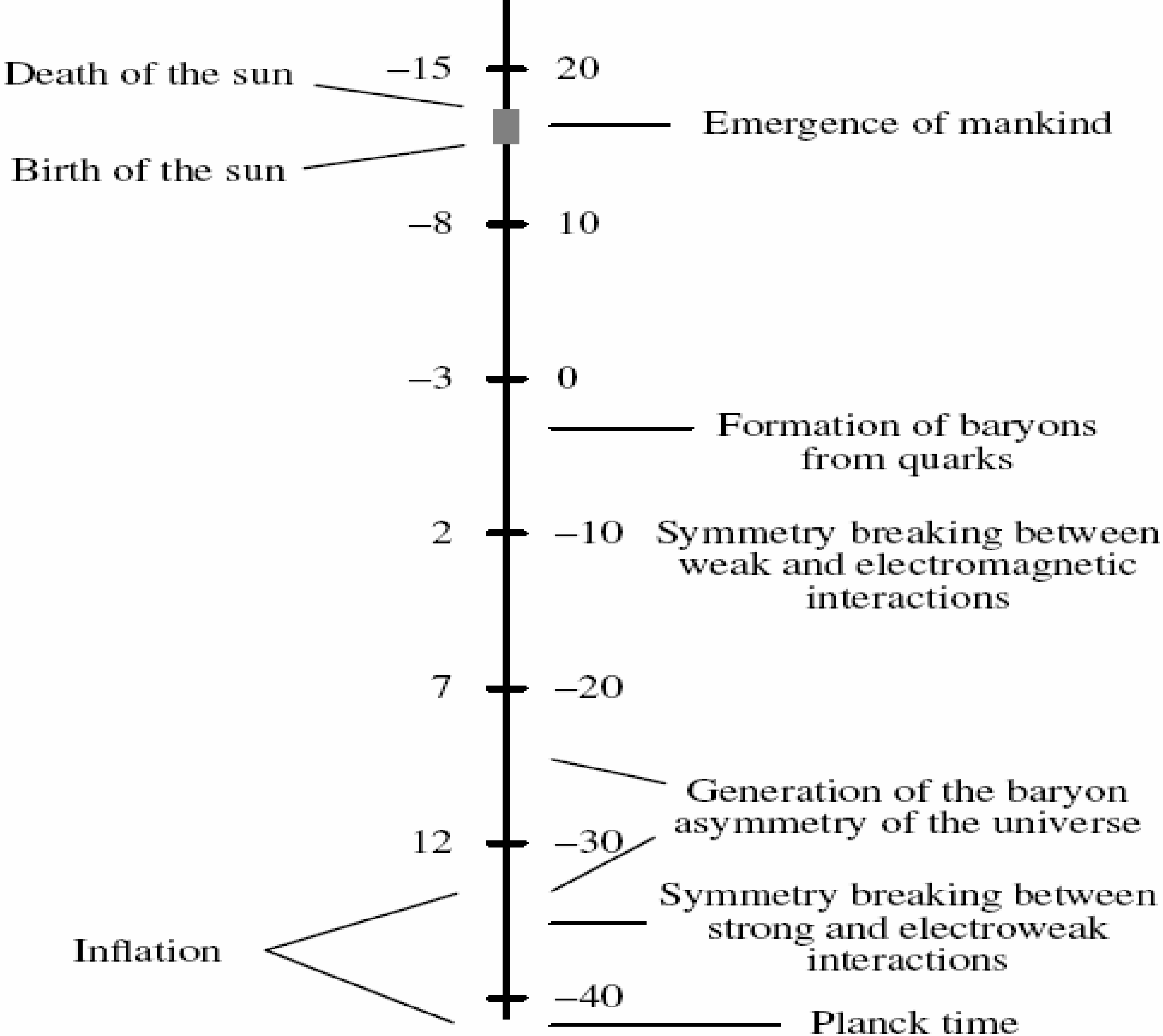
A chance for String theory to be tested?

- Available energies: $\lesssim 10^{7}$ GeV (accelerators)

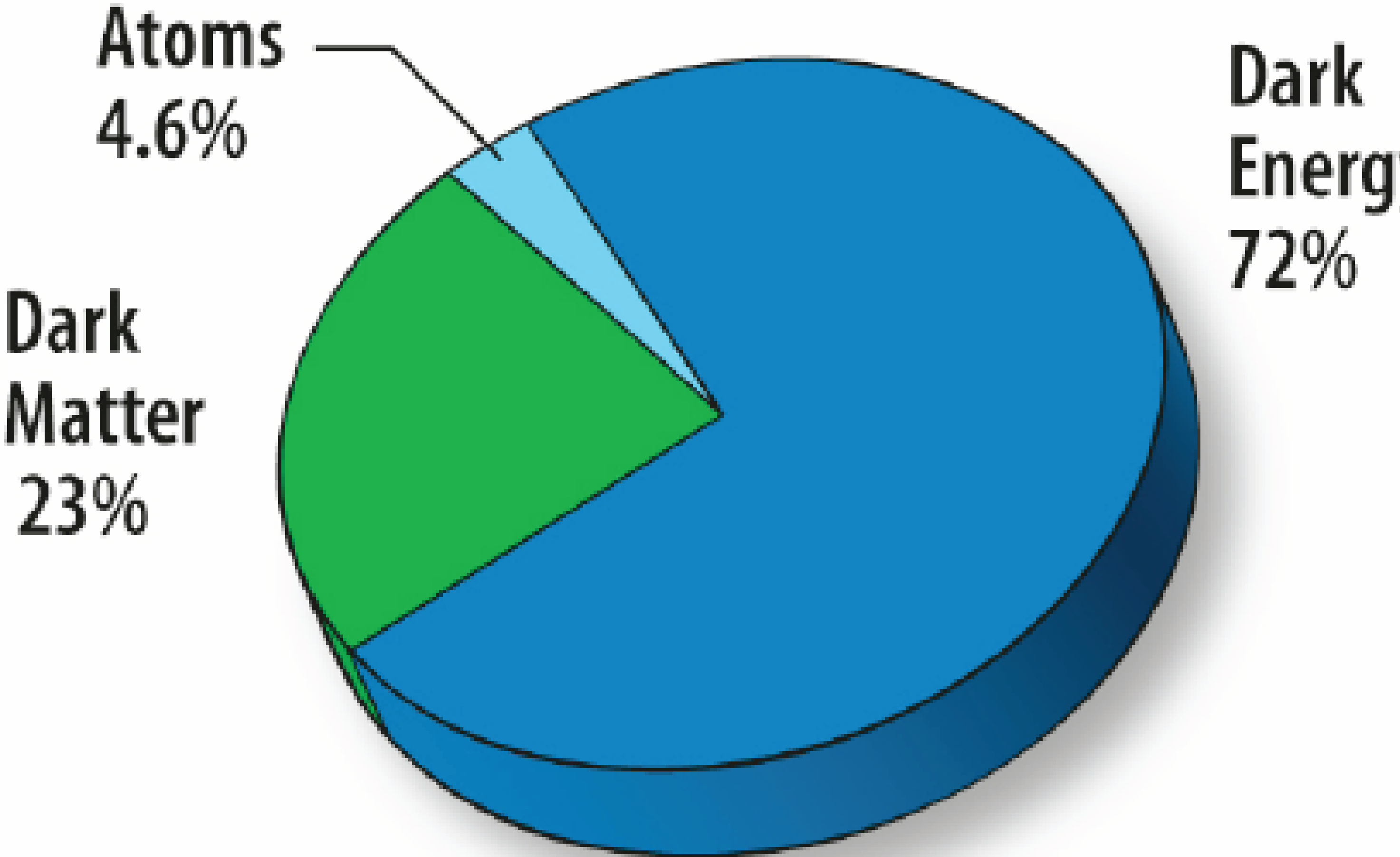
  $\sim 10^{7}$ GeV (cosmic rays)

  (to reach this energy in accel. the size should be $\gtrsim$ 5000 km !)

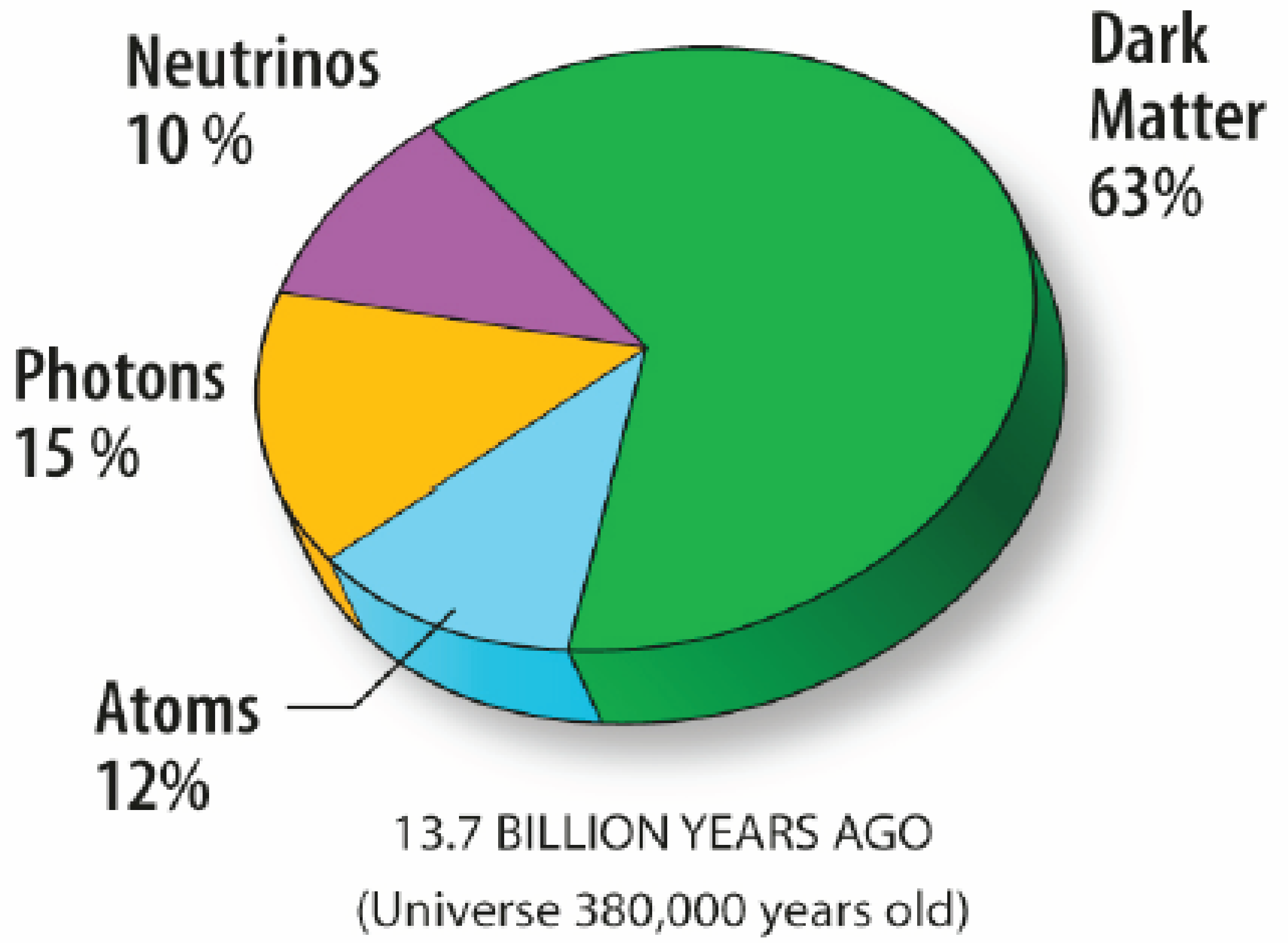




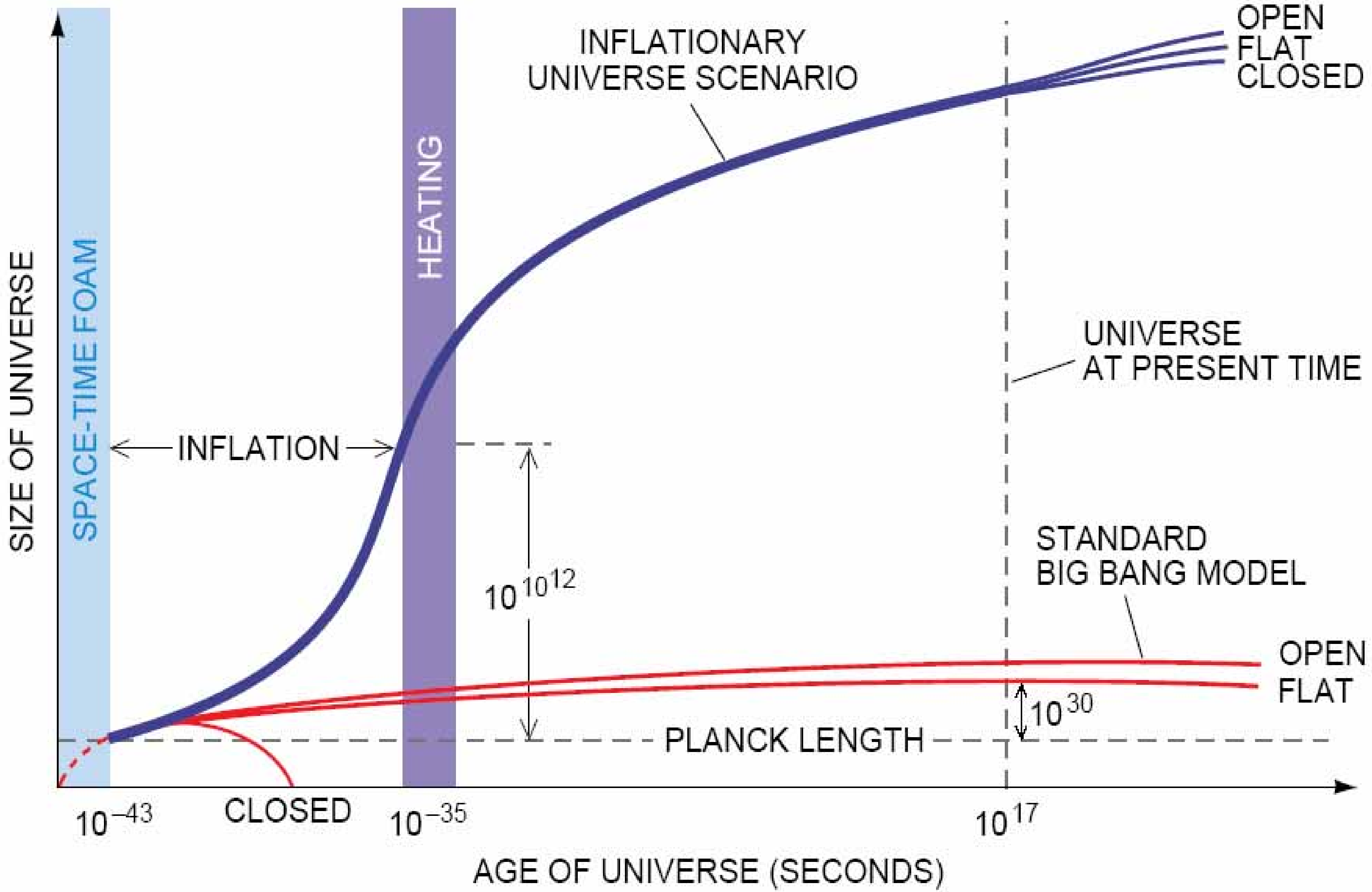
Death of the sun
Birth of the sun
−15
20
Emergence of mankind
−8
10
−3
0
Formation of baryons from quarks
2
−10
Symmetry breaking between weak and electromagnetic interactions
7
−20
Generation of the baryon asymmetry of the universe
12
−30
Symmetry breaking between strong and electroweak interactions
Inflation
−40
Planck time



TODAY


Neutrinos
10 %
Dark
Matter
63%
Photons
15 %
Atoms
12%
13.7 BILLION YEARS AGO
(Universe 380,000 years old)

t	$\rho^{1/4}$	Event
10^{-42} s	10^{18} GeV	Inflation begins?
$10^{-32\pm6}$ s	$10^{13\pm3}$ GeV	Inflation ends, Cold Big Bang begins?
$10^{-18\pm6}$ s	$10^{6\pm3}$ GeV	Hot Big Bang begins?
10^{-10} s	100 GeV	Electroweak phase transition?
10^{-4} s	100 MeV	Quark-hadron phase transition?
10^{-2} s	10 MeV	γ, ν, e, $\bar{e}$, n, and p in thermal equilibrium
1 s	1 MeV	ν decoupling, $e\bar{e}$ annihilation.
100 s	0.1 MeV	Nucleosynthesis (BBN)
10^4 yr	1 eV	Matter-radiation equality
10^5 yr	0.1 eV	Atom formation, photon decoupling (CMB)
$\sim 10^9$ yr	10^{-3} eV	First bound structures form
Now	10^{-4} eV (2.73 K)	The present.



INFLATIONARY
UNIVERSE SCENARIO
OPEN
FLAT
CLOSED
SIZE OF UNIVERSE
SPACE-TIME FOAM
HEATING
UNIVERSE
AT PRESENT TIME
INFLATION
$10^{10^{12}}$
STANDARD
BIG BANG MODEL
OPEN
FLAT
10^{30}
PLANCK LENGTH
10^{-43}
CLOSED
10^{-35}
10^{17}
AGE OF UNIVERSE (SECONDS)

We have Unified theory of Strong Electromagnetic and Weak interactions (**SM**).

It is supported by experiment. It should be somewhat extended (*neutrino masses and mixing*). The so called `Higgs particle' (*first introduced by Englert and Brout and soon, independently, by Higgs and by GHK*) is not yet discovered (waiting for **LHC** experiments).

SM has many parameters (***masses and coupling constants***) **not defined by the theory**. N**on-perturbative QCD** is also left to future.

Cosmological data (esp., ***DM, baryon asymmetry***) apparently **require further extending SM** (e.g., ***GUT, SUSY***, …).

We all hope to find some effects of these extensions at **LHC**.

Gravity remains separated in all these extensions.

A serious unification of **gravity** with **SM** might be expected in **Superstring theory**. In addition to **superparticles**, **SST predicts extra dimensions** of space.

If, somewhat miraculously, the extra dimensions are **large enough**, we could expect to find their effects in **LHC** experiments.

What is superstring theory? Who ordered it?

A **theoretical support for SST** recently came from a **new** interpretation of **BH thermodynamics and information paradox**.

SST theory does not solve directly **DE** and **inflation** problems but it certainly shed new light on these deep problems.

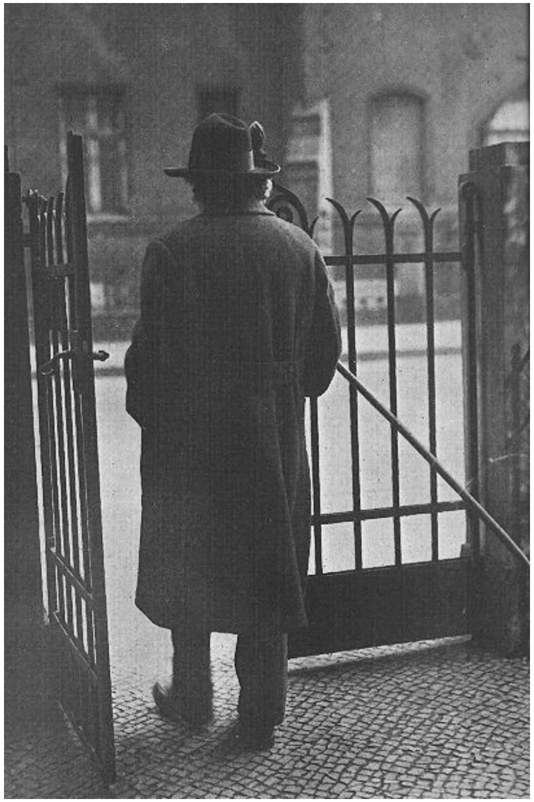
A bold idea to convert an apparent failure of the present **SST** into a potential success with the help of fantastic multiverse ideology became rather popular in literature and cinema but it is far from being formulated with a minimal scientific precision and probably will not be discussed at this school in detail.

I think it will be wise to concentrate on physics and mathematics of **BH** and **Cosmology** occasionally using **SST** as a dope for our imagination…(***yet, I confess to be a long time addict of the SST***…)

LECTURE 1

About new models based on ideas of Weyl, Eddington and Einstein 1919 -1923

H.Weyl



A.Eddington

Einstein in Berlin

AFFINE GENERALIZATIONS OF GRAVITY IN THE LIGHT OF MODERN COSMOLOGY

A.T. Filippov *

[+] *Joint Institute for Nuclear Research, Dubna, Moscow Region RU-141980*

arXiv:1008.2333 v1 (hep-th) and TMF;
arXiv:1003.0782v3 (hep-th) and TMF;
arXiv:0812.2616v2 (gr-qc) and TMF.

1 Summary

A **new interpretation** and a **higher-dimensional generalization** of the Weyl - Eddington - Einstein affine theory of gravity is proposed.

In addition to the standard GR it predicts:

dark energy (the cosmological constant, in the first approximation),

neutral massive vector field (a **dark matter** candidate),

massive scalar fields (**inflatons** and/or dark matter candidates).

The **mass terms are generated geometrically** as the new part of the **symmetric connection** γ^i_{jk} .

There are many problems: the parameters are not defined by the theory, non-integrabitlity even of static and cosmological reductions, …

GEOMETRY OF SYMMETRIC CONNECTIONS

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + a^i_{jk}$$

$$\Gamma^i_{jk}[g] = \frac{1}{2} g^{il}(g_{lj,k} + g_{lk,j} - g_{jk,l})$$

$$r^i_{jkl} = -\gamma^i_{jk,l} + \gamma^i_{mk}\gamma^m_{jl} + \gamma^i_{jl,k} - \gamma^i_{ml}\gamma^m_{jk}$$

NONSYMMETRIC RICCI CURVATURE

$$r_{jk} = -\gamma^i_{jk,i} + \gamma^i_{mk}\gamma^m_{ji} + \gamma^i_{ji,k} - \gamma^i_{mi}\gamma^m_{jk}$$

Symmetric part of the Ricci curvature

$$s_{ij} \equiv \frac{1}{2}(r_{ij} + r_{j\,i})$$

Anti-symmetric part of the Ricci curvature

$$a_{ij} \equiv \frac{1}{2}(r_{ij} - r_{j\,i}) = \frac{1}{2}(\gamma^m_{jm,i} - \gamma^m_{im,j})$$

$$a_{ij,k} + a_{jk,i} + a_{ki,j} \equiv 0$$

VECTON: $\quad a_i \equiv a^m_{im}$

$$a_i \equiv \gamma^m_{mi} - \Gamma^m_{mi} \equiv \gamma_i - \partial_i \ln\sqrt{|g|}$$

$$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i}) \equiv -\frac{1}{2}(\gamma_{i,j} - \gamma_{j,i})$$

EDDINGTON'S SCALAR DENSITY

$$\mathcal{L} \equiv \sqrt{-\det(r_{ij})} \equiv \sqrt{-r}$$

$$s_{ij} = -\nabla^{\gamma}_{m}\gamma^{m}_{ij} + \frac{1}{2}(\nabla^{\gamma}_{i}\gamma_{j} + \nabla^{\gamma}_{j}\gamma_{i}) - \gamma^{m}_{ni}\gamma^{n}_{mj} + \gamma^{n}_{ij}\gamma_{n}$$

Expressing in terms of the `metric' and using notation $\nabla_i \equiv \nabla^g_i$

$$s_{ij} = R_{ij}[g] - \nabla_m a^m_{ij} + \frac{1}{2}(\nabla_i\, a_j + \nabla_j\, a_i) + a^m_{ni}a^n_{mj} - a^m_{ij}a_m$$

$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i})$ depends only on the vecton

‘GEODESICS’ (PATHS)

$$\ddot{x}^{\,i} + \gamma^i_{jk}\,\dot{x}^j\,\dot{x}^k = 0$$

TRANSFORMATIONS PRESERVING PATHS

$$\hat{\gamma}^i_{jk} = \gamma^i_{jk} + \delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j$$

GEO-RIEMANNIAN CONNECTIONS

$$\hat{\gamma}^i_{jk} = \Gamma^i_{jk}[g] + \delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j$$

$\alpha\beta$ - CONNECTION

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha(\delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j) - (\alpha - 2\beta)g_{jk}\,\hat{a}^i$$

Weyl: $\beta = 0$

geo-Riemannian: $\alpha = 2\beta$.

Einstein $\alpha = -\beta = \frac{1}{6}$

LINEAR TERMS in $s_{ij} - R_{ij}(g)$

$$(\alpha + \beta)(\nabla_i \hat{a}_j + \nabla_j \hat{a}_i) + (\alpha - 2\beta)\, g_{ij} \nabla_m \hat{a}^m$$

QUADRATIC TERMS in $s_{ij} - R_{ij}(g)$

$$\hat{a}_i \hat{a}_j \left[(\alpha - 2\beta)^2 - 3\alpha^2\right] + 2\, g_{ij} \hat{a}^2 (\alpha - 2\beta)(\alpha + \beta)$$

In addition to this dependence on the vecton, the generalized Einstein equations will depend on it through dynamics specified by the chosenLagrangian

FROM GEOMETRY TO DYNAMICS

REQUIREMENTS TO LAGRANGIAN DENSITIES

1. IT IS INDEPENDENT OF DIMENSIONAL CONSTANTS.

2. ITS INTEGRAL OVER SPACE-TIME IS DIMENSIONLESS.

3. IT CAN DEPEND ON TENSOR VARIABLES HAVING
a DIRECT GEOMETRIC MEANING and
a NATURAL PHYSICAL INTERPRETATION.

4. THE RESULTING GENERALIZED THEORY MUST AGREE WITH ALL ESTABLISHED EXPERIMENTAL CONSEQUENCES OF EINSTEIN'S THEORY.

r_{ij}, s_{ij}, a_{ij}, and $a_k \equiv a^i_{ik}$ satisfy requirement **3**.

Einstein's choice is $\mathcal{L} = \mathcal{L}(s_{ij}, a_{ij})$

A simple nontrivial choice of a geometric Lagrangian density generalizing the Eddington – Einstein Lagrangian ,

$$\mathcal{L} \equiv \sqrt{-\det(r_{ij})} \equiv \sqrt{-r} \ ,$$

is the following, depending on one dimensionless parameter:

$$\mathcal{L} = \mathcal{L}(s_{ij} + \nu a_{ij}) = \sqrt{-\det(s_{ij} + \nu a_{ij})}$$

$$\det(s_{ij}) < 0$$

When $\nu a_{ij} \to 0$ it will give Einstein's gravity with the cosmological constant.

Define the following densities of the weight two

$$d_0 \equiv 4!\,\det(s_{ij}) = \epsilon^{ijkl} s_{im} s_{jn} s_{kr} s_{ls} \epsilon^{mnrs} \equiv \epsilon \cdot s \cdot s \cdot s \cdot s \cdot \epsilon \,.$$

$$d_1 \equiv \epsilon \cdot s \cdot s \cdot s \cdot \bar{a} \cdot \epsilon \,, \qquad d_2 \equiv \epsilon \cdot s \cdot s \cdot a \cdot a \cdot \epsilon \,,$$

$$d_4 \equiv \epsilon \cdot a \cdot a \cdot a \cdot a \cdot \epsilon$$

where $\bar{a}$ denotes the matrix $a_i a_j$

$$\det(s_{ij} + \nu a_{ij}) = \frac{1}{4!}\left(d_0 + 6\nu^2 d_2 + \nu^4 d_4\right)$$

A more general Lagrangian

$$\mathcal{L} \equiv \alpha_0 \sqrt{|\, d_0 \; + \; \alpha_1 d_1 \; + \; \alpha_2 d_2 \; + \; \alpha_4 d_4 |}$$

Now we **define** (following Einstein) the metric and field densities by a Legendre-like transformation

$$\frac{\partial \mathbf{L}}{\partial s_{ij}} \equiv \mathbf{g}^{ij}\,, \qquad \frac{\partial \mathbf{L}}{\partial a_{ij}} \equiv \mathbf{f}^{ij} \qquad\qquad s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}\,, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$$

$$2\nabla_i^\gamma\, \mathbf{g}^{kl} \;=\; \delta_i^l\, \nabla_m^\gamma\, (\mathbf{g}^{km} + \mathbf{f}^{km}) + \delta_i^k\, \nabla_m^\gamma\, (\mathbf{g}^{lm} + \mathbf{f}^{lm})$$

$$\nabla_i^\gamma\, \mathbf{f}^{kl} \;=\; \partial_i\, \mathbf{f}^{kl} + \gamma_{im}^k\, \mathbf{f}^{ml} + \gamma_{im}^l\, \mathbf{f}^{km} - \gamma_{im}^m\, \mathbf{f}^{kl}$$

$$\nabla_i^\gamma\, \mathbf{f}^{ki} \;=\; \partial_i \mathbf{f}^{ki} \;\equiv\; \mathbf{a}^k. \qquad \nabla_i^\gamma\, \mathbf{g}^{ik} \;=\; -\frac{D+1}{D-1}\hat{\mathbf{a}}^k$$

$$\nabla_i^\gamma\, \mathbf{g}^{jk} \;=\; -\frac{1}{D-1}(\delta_i^j\,\hat{\mathbf{a}}^k + \delta_i^k\,\hat{\mathbf{a}}^j)$$

for **any** dimension **D**

Defining the Riemann metric tensor g_{ij} by the equations

$$g^{ij}\sqrt{-g} = \mathbf{g}^{ij}, \quad g_{ij}\, g^{jk} = \delta_i^k$$

$$\nabla_i\, g_{jk} = 0, \quad \nabla_i\, g^{jk} = 0 \qquad \hat{a}^k \equiv \hat{\mathbf{a}}^k / \sqrt{-g}$$

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha_D\, [\, \delta^i_j\, \hat{a}_k + \delta^i_k\, \hat{a}_j - (D-1)\, g_{jk}\, \hat{a}^i]$$

$$\alpha_D \equiv [(D-1)(D-2)]^{-1}, \qquad \beta_D \equiv -[2(D-1)]^{-1}$$

We thus have derived the connection using a rather general dynamics!

Using a simple dimensional reduction to the dimension 1+1 (similar to spherical or cylindrical reductions in the metric case) we can prove the relation between the **conjugate Lagrangians**:

$$\mathcal{L} = -\frac{1}{2}\sqrt{|\det(s + \lambda^{-1} a)|} = -2\Lambda\sqrt{|\det(\mathbf{g} + \lambda\mathbf{f})|} = \mathcal{L}^*$$

Λ having the dimension L^{-2}

Using the above definitions, we can then write the **generalized Einstein eqs.**

$$s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}\,, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$$

In **dimension D** we can similarly derive the relation

$$\mathcal{L}^* \equiv \sqrt{-\det(s_{ij} + \nu a_{ij})} \sim \sqrt{-g}\,[\det(\delta_i^j + \lambda f_i^j)]^{1/(D-2)}$$

With the simplest dimensional reduction to $D = 4$, the components of the vector field a_k with $k \geq 4$ become **real massive scalar fields**.

Thus we naturally derive a theory of **gravity with dark energy, vector dark matter**, and **massive scalar fields (inflatons?)**

The main parameters (mass, cosmological constant) cannot be predicted and **can be made arbitrary**.

This is not bad in the context of **MULTIVERSE ideology**!

The generalized Einstein –Eddington Weyl model in dimension D

$$\mathcal{L}_{eff} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Restoring the dimensions and expanding the root term up to the second order in the vector and scalar fields

$$\mathcal{L}_{eff} \cong \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i + g^{ij}\partial_i\psi\,\partial_j\psi + m^2\psi^2\right)\right]$$

$$A_i \sim a_i,\; F_{ij} \sim f_{ij},\; \kappa \equiv G/c^4$$

NB: $\partial_i\psi$ Is proportional to F_{ij} for i < 4, j=4

LECTURE 2

About approximate cosmological models

Einstein – Weyl motivated model

$$\mathcal{L} = \sqrt{-g_4}\,[R_4 - V(\psi) - (\nabla\psi)^2 - \\ - F_{kl}F^{kl} - \mu^2 A_k A^k]$$

$$F_{kl} \equiv A_{k,l} - A_{k,l}$$

Pure Einstein – Weyl model:

$$\hat{\mathcal{L}} = \sqrt{-g}\,[R - 2\Lambda - F_{kl}F^{kl} - m^2 A_k A^k]$$

Spherical symmetry

$$ds_4^2 = e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}dr\,dt$$

$$e^{2\beta}[e^{-\alpha-\gamma}(\dot{A}_1 - A_0')^2 - e^{-\alpha+\gamma}(\psi'^2 + \mu^2 A_1^2) + e^{\alpha-\gamma}(\dot{\psi}^2 + \mu^2 A_0^2) - e^{\alpha+\gamma}(V + 2\Lambda)] + \mathcal{L}_{gr}$$

$$\mathcal{L}_{gr} \equiv e^{-\alpha+2\beta+\gamma}(2\beta'^2 + 4\beta'\gamma') - e^{\alpha+2\beta-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha}) + 2ke^{\alpha+\gamma}$$

Reduction to cosmological or static solutions

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' = \frac{1}{2}[\dot{\psi}\psi' + A_0 A_1]$$

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r),$$

$$\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0$$

Cosmological Lagrangian

$$6\bar{k}e^{\alpha+\gamma} - e^{2\beta}[e^{\alpha+\gamma}(V + 2\Lambda) - e^{\alpha-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha} - \dot{\psi}^2)]$$

Notation

$$\rho \equiv \frac{1}{3}(\alpha + 2\beta)\,, \quad \sigma \equiv \frac{1}{3}(\beta - \alpha)\,,$$

$$A_\pm = e^{-2\rho+4\sigma}(\dot{A}^2 \pm \mu^2 e^{2\gamma} A^2)\,, \quad \bar{V} \equiv V(\psi) + 2\Lambda$$

Lagrangian

$$e^{2\rho-\gamma}(\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2) + e^{3\rho-\gamma} A_- - e^{3\rho+\gamma}\,\bar{V}(\psi)$$

The energy constraint

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + A_- + e^{2\gamma}\,\bar{V} = 0$$

Equations of motion

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\mu^2 A = 0\,,$$

$$4\ddot{\rho} + 6\dot{\rho}^2 - 4\dot{\rho}\dot{\gamma} - 6\dot{\sigma}^2 + \frac{1}{3}A_- + \dot{\psi}^2 - e^{2\gamma}\,\bar{V} = 0\,,$$

$$\ddot{\sigma} + 3\dot{\sigma}\dot{\rho} - \dot{\sigma}\dot{\gamma} - \frac{1}{3}A_- = 0\,.$$

$$\ddot{\psi} + (3\dot{\rho} - \dot{\gamma})\dot{\psi} + \frac{1}{2}e^{2\gamma}\,\bar{V}_\psi = 0\,,$$

Anisotropic scalar cosmology

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + e^{2\gamma}\,\bar{V} = 0\,,$$

$$4\ddot{\rho} + 6\dot{\rho}^2 - 4\dot{\rho}\dot{\gamma} - 6\dot{\sigma}^2 + \dot{\psi}^2 - e^{2\gamma}\,\bar{V} = 0\,,$$

$$\ddot{\sigma} + 3\dot{\sigma}\dot{\rho} - \dot{\sigma}\dot{\gamma} = 0\,,$$

$$\ddot{\psi} + (3\dot{\rho} - \dot{\gamma})\dot{\psi} + \frac{1}{2}e^{2\gamma}\,\bar{V}_{\psi} = 0\,.$$

Standard gauge choice

$$\gamma = 0 : \quad \dot{\sigma} = C_0 e^{-3\rho} \, ,$$

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6C_0^2 e^{-6\rho} + \bar{V} = 0 \, ,$$

$$4\ddot{\rho} + 6\dot{\rho}^2 - 6C_0^2 e^{-6\rho} + \dot{\psi}^2 - \bar{V} = 0 \, ,$$

$$\ddot{\psi} + 3\dot{\rho}\dot{\psi} + \frac{1}{2}\bar{V}_\psi = 0 \, .$$

Unusual gauge choice
(good for constructing integrable models)

$$\gamma = 3\rho : \quad \ddot{\sigma} = 0 \,, \quad \dot{\sigma} = C_1 \,,$$

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6C_1^2 + e^{6\rho}\bar{V} = 0 \,,$$

$$4\ddot{\rho} - \frac{1}{2}e^{6\rho}\bar{V} = 0 \,, \qquad \ddot{\psi} + \frac{1}{2}e^{6\rho}\bar{V}_\psi = 0 \,.$$

Approximate (model) Lagrangian for the E-W cosmology

$$\mathcal{L}_c = -6\dot{\alpha}^2 e^{3\alpha-\gamma} - 2\Lambda e^{3\alpha+\gamma} + \dot{A}^2 e^{\alpha-\gamma} - \mu^2 A^2 e^{\alpha+\gamma}$$

ONLY MODEL!!

Not giving exact solution for D=4

Vecton dark matter can be produced in **strong** gravitational fields only. **Quantum gravity** is necessary!

Effects of nonlinear Lagrangians must be studied (like in `B-I cosmology')

Anyway, inflation and dark matter are crucial things to study and test the `Wedein' cosmological models

THE

END

WE DISCUSS NEW MODELS OF AN 'AFFINE' THEORY OF GRAVITY IN **MULTI-DIMENSIONAL SPACE-TIMES** WITH **SYMMETRIC CONNECTIONS**. WE USE AND DEVELOP IDEAS OF WEYL, EDDINGTON, AND EINSTEIN, IN PARTICULAR, EINSTEIN'S PROPOSAL TO SPECIFY THE SPACE - TIME GEOMETRY BY USE OF THE HAMILTON PRINCIPLE. MORE SPECIFICALLY, THE CONNECTION COEFFICIENTS ARE DETERMINED USING A 'GEOMETRIC' LAGRANGIAN THAT IS AN ARBITRARY FUNCTION OF THE GENERALIZED (NON-SYMMETRIC) RICCI CURVATURE TENSOR (AND, POSSIBLY, OF OTHER FUNDAMENTAL TENSORS) EXPRESSED IN TERMS OF THE CONNECTION COEFFICIENTS REGARDED AS INDEPENDENT VARIABLES.

SUCH A THEORY SUPPLEMENTS THE STANDARD EINSTEIN GRAVITY WITH **DARK ENERGY** (THE COSMOLOGICAL CONSTANT, IN THE FIRST APPROXIMATION), A **NEUTRAL MASSIVE (OR TACHYONIC) VECTOR FIELD**, AND **MASSIVE (OR TACHYONIC) SCALAR FIELDS**. THESE FIELDS COUPLE ONLY TO GRAVITY AND CAN GENERATE DARK MATTER AND/OR INFLATION. THE NEW FIELD MASSES (REAL OR IMAGINARY) HAVE A GEOMETRIC ORIGIN AND MUST APPEAR IN ANY CONCRETE MODEL.

THE CONCRETE CHOICE OF THE GEOMETRIC LAGRANGIAN DETERMINES FURTHER DETAILS OF THE THEORY, FOR EXAMPLE, THE NATURE OF THE VECTOR AND SCALAR FIELDS THAT CAN DESCRIBE MASSIVE PARTICLES, TACHYONS, OR EVEN 'PHANTOMS'. IN 'NATURAL' GEOMETRIC THEORIES, WHICH ARE DISCUSSED HERE, DARK ENERGY MUST ALSO ARISE. WE MAINLY FOCUS ON INTRICATE RELATIONS BETWEEN GEOMETRY AND DYNAMICS WHILE ONLY VERY BRIEFLY CONSIDERING APPROXIMATE COSMOLOGICAL MODELS INSPIRED BY THE GEOMETRIC APPROACH.

If the D-dimensional Lagrangian depends only on the **symmetric** and **antisymmetric** parts $(s_{ij}\,,\,a_{ij})$ of the **curvature tensor** r_{ij}, the connection is

$$\gamma_{kl}^{m} = \Gamma_{kl}^{m}(g) + \frac{1}{2}\Big[[(D-1)(D-2)]^{-1}[\,\delta_k^m\,a_l + \delta_l^m\,a_k - (D-1)g_{kl}\,a^m]\Big]$$

with an **arbitrary symmetric tensor** g_{mn} and a **vector** a_k

$$s_{ij} = R_{ij}(g) + [(D-1)(D-2)]^{-1}\,a_i a_j\;,$$

$$a_{ij} = [(D-1)(D-2)]^{-1}\,(a_{i,j} - a_{j,i})\;,$$

where R_{ij} is the standard Ricci curvature.

If we specify a concrete Lagrangian these relations give **equations of motion** (for $s_{ij}\,,\,a_{ij}$ written in terms $g_{mn}\,,\,a_k$).

The simplest 'geometrical' Lagrangian is **Eddington's scalar density** (our generalization is the sum of three independent scalar densities):

$$\mathbf{L}(\gamma^i_{jk}) = \sqrt{-\det[r_{mn}(\gamma^i_{jk})]}\,.$$

In this case, the **effective Lagrangian** contains the Einstein - Hilbert term, the **vecton mass term** and the **nonlinear term** ($f_{ij} \sim a_{i,j} - a_{j,i}$, $f^i_{\ j} \equiv g^{ik} f_{kj}$)

$$\Delta\mathbf{L}_{eff} = \sqrt{-g}\,[\det(\delta^i_j + f^i_{\ j})]^{1/D-2}\,.$$

The **physical interpretation** is simplified by expanding the **effective Lagrangian** in powers of f_{ij} up to f^2-terms).

The approximate D-dimensional Lagrangian describes the standard gravity with the **cosmological term** plus a **real massive vector field**.

Geometry and dynamics of the affine theory

$$\gamma_{kl}^{m} = \frac{1}{2}[g^{mn}(g_{nk,l} + g_{ln,k} - g_{kl,n}) + \alpha(\delta_k^m\, a_l + \delta_l^m\, a_k) - (\alpha - 2\beta)g_{kl}\, a^m]$$

$$g_{ij}g^{jk} = \delta_j^i$$

$$r^i_{klm} = -\gamma^i_{kl,m} + \gamma^i_{nl}\gamma^n_{km} + \gamma^i_{km,l} - \gamma^i_{nm}\gamma^n_{kl}$$

$$r_{kl} = -\gamma^m_{kl,m} + \gamma^m_{nl}\gamma^n_{km} + \gamma^m_{km,l} - \gamma^m_{nm}\gamma^n_{kl}$$

The starting point for Einstein $\mathbf{L} = \mathbf{L}(s_{ij}, a_{ij}).$

$$s_{ij} \equiv \frac{1}{2}(r_{ij} + r_{ji}) \qquad a_{ij} \equiv \frac{1}{2}(r_{ij} - r_{ji})$$

$$\gamma_{ij}^{k} = \Gamma_{ij}^{k} + \frac{1}{6}(\delta_i^k a_j + \delta_j^k a_i) - \frac{1}{2} g_{ij} a^k$$

$$s_{ij} = R_{ij} + \frac{1}{6} a_i a_j \, , \qquad a_{ij} = \frac{1}{6}(a_{i,j} - a_{j,i})$$

1/6 to 1/(D-1)(D-2); 1/2 to 1/(d-2)

$$\mathbf{L} \equiv \sqrt{-\det(r_{ij})} = 4\sqrt{-\det(\mathbf{g}^{ij} + \mathbf{f}^{ij})} \equiv 4\sqrt{-\det(g_{ij} + f_{ij})}$$

$$\mathbf{L}_{eff} = -2\Lambda\sqrt{-\det(g_{ij} + f_{ij})} + \sqrt{-\det(g_{ij})}\left[R(g) - \frac{1}{6} g^{ij} a_i a_j\right]$$

$$\det(r_{ij}) \equiv \frac{1}{4!}\epsilon^{ijkl}\epsilon^{mnrs} r_{im} r_{jn} r_{kr} r_{ls} \, ,$$

$$\det{}'(r_{ij}) \equiv \frac{1}{4!}\epsilon^{ijkl}\epsilon^{mnrs} r_{im} r_{jn} r^T_{kr} r^T_{ls} \, ,$$

$$\det{}''(r_{ij}) \equiv \frac{1}{4!}\epsilon^{ijkl}\epsilon^{mnrs} r_{im} r_{jn} r_{kr} r^T_{ls} \, .$$

$$\mathbf{L} \equiv \alpha\sqrt{-\det(r_{ij})} + \alpha' \sqrt{-\det{}'(r_{ij})} + \alpha'' \sqrt{-\det{}''(r_{ij})} \, .$$

The above Lagrangian define the vecton mass proportional to the cosmological constant. Let Us try to deform it by introducing num. constant:

$$\sqrt{|\det(g_{ij}+\lambda f_{ij})|} = \sqrt{-g}\sqrt{\det(\delta^i_j+\lambda f^i_j)} = \sqrt{-g}\left(1+\frac{1}{4}\lambda^2 f_{ij}f^{ij}+...\right)$$

$$R_{ij}-\Lambda\, g_{ij} = -\lambda^2\Lambda\left[f_{ik}f^k_j - \frac{1}{4}g_{ij}f_{kl}f^{kl}\right]+\frac{1}{6}a_i a_j$$

$$\Lambda\, f_{ij} = \frac{1}{6}(\partial_i a_j - \partial_j a_i)$$

$$\kappa^{-1} \equiv 36\,\lambda^{-2}\Lambda\, c_A^2$$

$$e_i = a_{0i}\,, \quad \tilde{e}_i \equiv e_i/\sqrt{s_0 s_i}\,,$$

$$h_i \equiv \epsilon_{ijk}\, a_{jk}\,, \quad \tilde{h}_i \equiv h_i/\sqrt{s_j s_k}$$

$$\det(r_{ij}) = [1 - \tilde{e}^2 + \tilde{h}^2 + (\tilde{e}\cdot\tilde{h})^2] \prod_i s_i$$

$$\det{}'(r_{ij}) = [1 + \frac{1}{3}\tilde{e}^2 - \frac{1}{3}\tilde{h}^2 + (\tilde{e}\cdot\tilde{h})^2] \prod_i s_i$$

$$\det{}''(r_{ij}) = [1 - (\tilde{e}\cdot\tilde{h})^2] \prod_i s_i$$

INFLATION hypothesis

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H^{-1}}{a}\right) < 0 \quad \Rightarrow \quad \frac{\mathrm{d}^2 a}{\mathrm{d}t^2} > 0 \quad \Rightarrow$$

$$\rho + 3P < 0$$

First realized by
Starobinsky;
Guth; Linde;

The simplest scalar
model of inflation in
Friedmann universe

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0\,.$$

Scalar inflation

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$$

$$H^2 = \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

Simple example: $V = \frac{1}{2}m^2\varphi^2$

$$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$$

$$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi}{\dot{\varphi}}$$

Inflationary solution $\qquad \dot{\varphi}_{\mathrm{atr}} \approx -\frac{m}{\sqrt{12\pi}}$

Many, many questions:

What was before the Big Bang?

We do not know yet

Why is our universe so **homogeneous**?
Why is it **not exactly** homogeneous?
Why is it **isotropic** (same in all directions)?
Why all of its parts started expanding simultaneously?
Why is it **flat** **($\Omega = 1$)**?
Why is it so **large**?
Where are monopoles and other unwanted relics?

Answered by inflation

Why vacuum (dark) energy is so small but not zero?
Why there is 5 times more dark matter than normal matter?
Why there is about 4 times more dark energy than dark matter?
Why **$w = -1$**?

Possible answers are given by a combination of particle physics, string theory and eternal inflation

Why multiverse ?

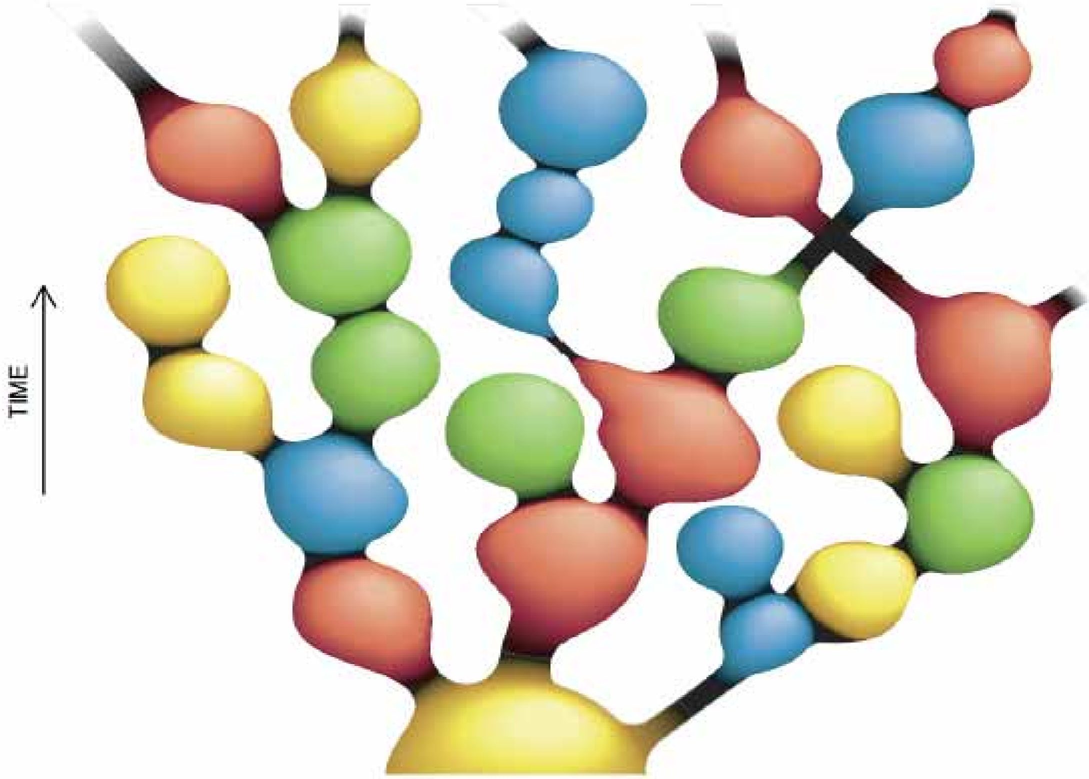
Uniformity of our world is explained by **inflation**:
Exponential stretching of the new-born universe makes it almost exactly uniform.

However, inflationary fluctuations eternally produce new parts of the universe with different properties.

Inflationary **universe** becomes a **multiverse**

Inflationary Multiverse

Inflationary universe may consist of many parts with different properties depending on the local values of the scalar fields, compactification, fluxes, etc.



How many different universes are in the multiverse

There are perhaps $\sim 10^{500}$ vacua in string theory landscape

If these vacua appear as a result of bubble formation which is not followed by slow roll inflation, then each of these vacua is equally unimportant, because nobody can live there. Thus one could think that the number of possibilities is much smaller than 10^{500}. However, the number of different universes which may emerge as a result of eternal slow roll inflation is much greater than 10^{500}.