High Energy Scattering
and
Search for Extra Dimensions at the LHC

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Outline

I N T R O D U C T I O N

- **Main tasks for LHC**

  Higgs, Susy, extra-dimensions

- **Reasons to think about extra dimensions**
  - Kaluza-Klein,
  - Strings
  - D-branes
  - TeV-gravity scenario

- **Possible manifestations of Extra Dimensions**
  - **KK modes**
  - **Black Hole/Wormhole production**
  - **Signs of strong quantum gravity**

Possible manifestations of Extra Dimensions.

BH production in particles collisions. Hoop conjecture

1+1 → BH

Modified Thorn's hoop conjecture:

BH forms if the impact parameter $b < R_s(E,D)$,
E is the energy in c.m.

$$R_S(M_{BH}, D) = \gamma(D) \frac{1}{M_D} \left(\frac{M_{BH}}{M_D}\right)^{\alpha(D)}$$

Modified Thorn's hoop conjecture gives classical geometrical cross-section

$$\sigma(1+1 \to \mathrm{BH}) \sim \pi R_S^2$$

Modified Thorn's hoop conjecture:

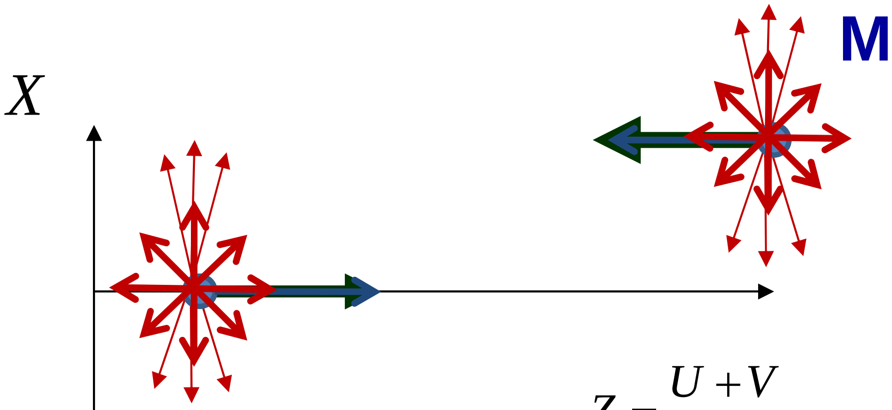
- It is not obvious that the hoop conjecture is applicable.

Trapped surface guarantees BH formation only for asymptotical free spacetimes

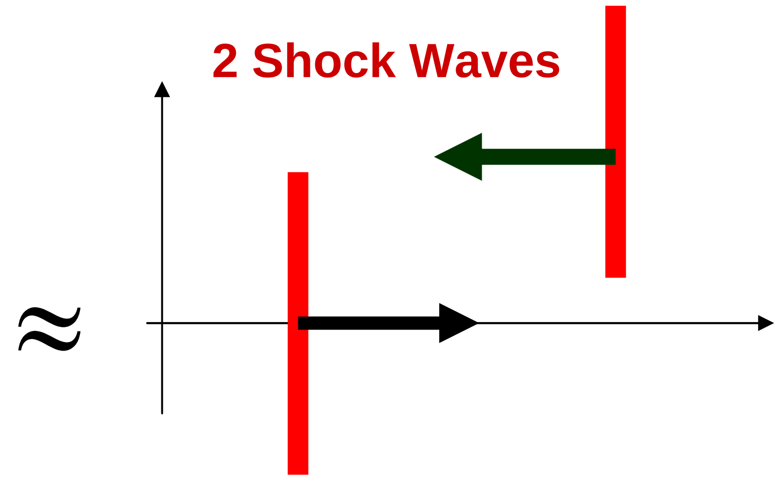
- It is not obvious that trapped surface arguments are applicable in the cases

  of nonzero cosmological constant.
- Search for others approaches

Particles and Shock Waves



$$Z = \frac{U+V}{2}$$



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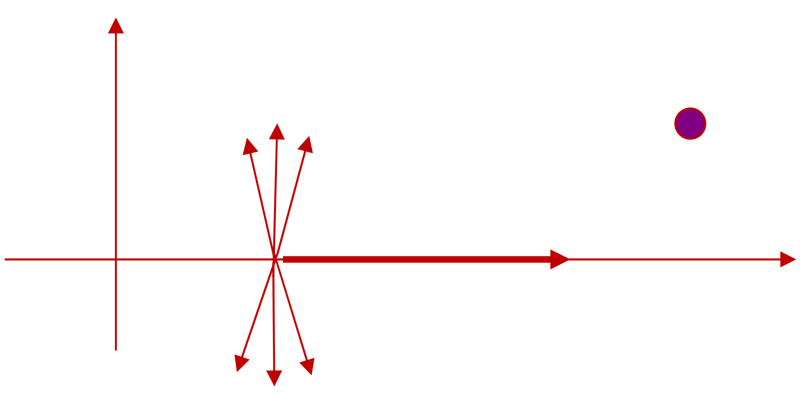
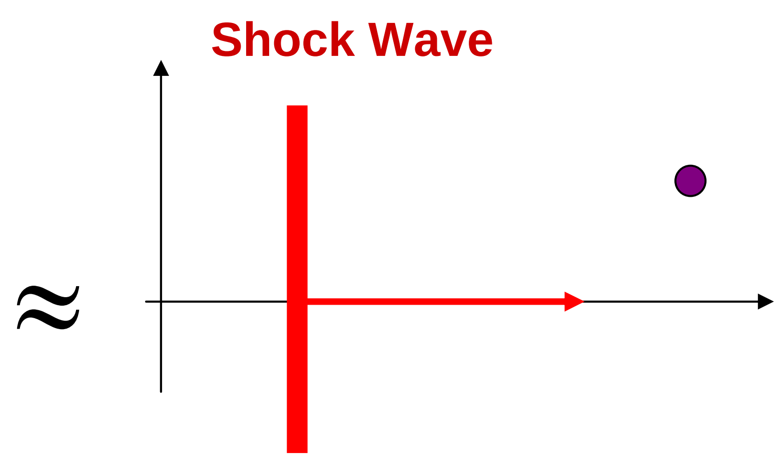




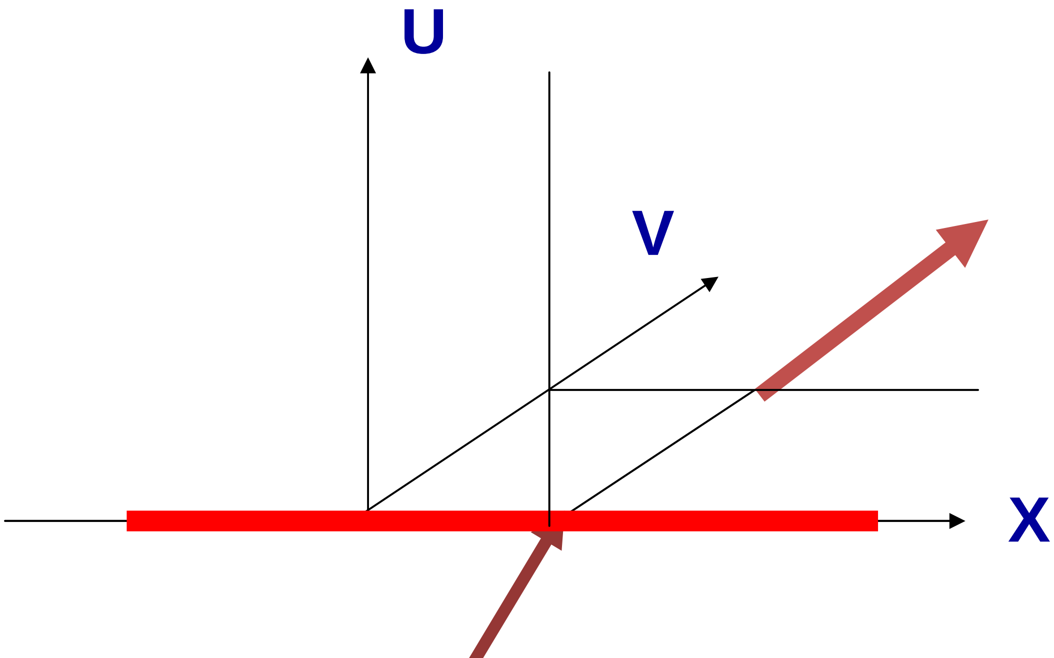
Geodesics in the space-time with shock wave

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\,dU^2,$$

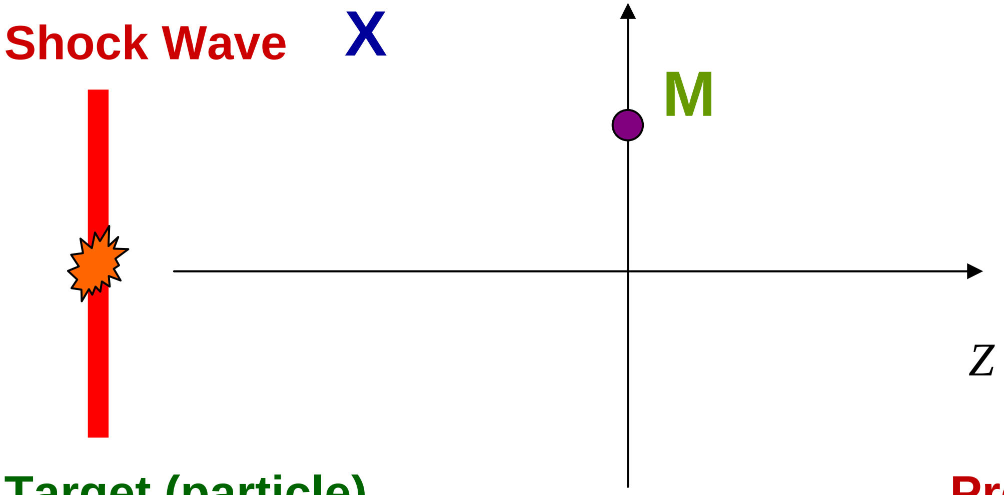
$$X(\tau) = b - \theta(\tau)v\tau$$

$$V(\tau) = \tau + \theta(\tau)v^2\tau - 2\theta(\tau)F(b)$$

$$v = F'(b)$$



Particle in Shock Wave



$$Z = \frac{U+V}{2}$$

Target (particle)

$$X = b - \theta(\tau) v\tau$$

$$Z = \theta(\tau)\frac{v^2\tau}{2} - \theta(\tau)F(b)$$

$$t = \tau + \theta(\tau)\frac{v^2\tau}{2} - \theta(\tau)F(b)$$

Projectile (shock wave)

$$X = 0$$

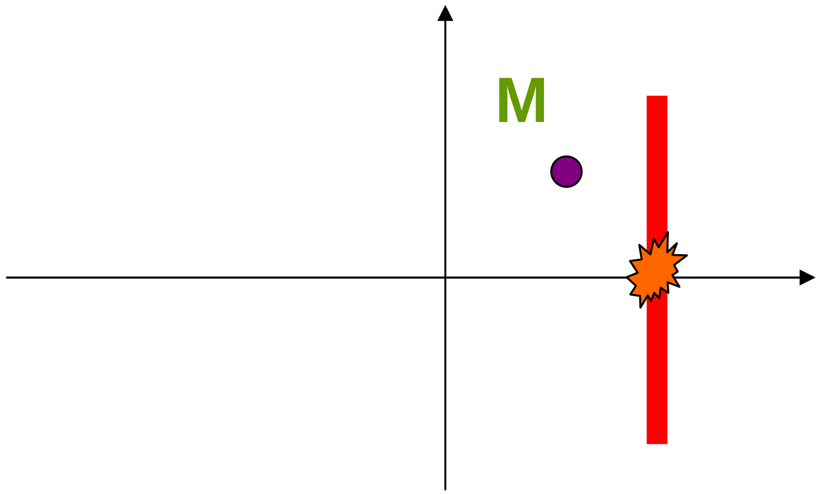
$$Z = t \rightarrow Z = \tau + \theta(\tau)\frac{v^2\tau}{2} - \theta(\tau)F(b)$$

M is chasing after the shock

$$v = F'(b)$$

$$v = \frac{p}{\pi M_{Pl}^2 b}$$

Capture or not capture



$$v = \frac{p}{\pi M_{Pl}^2 b}$$

$$R^2 = b^2 \left(1 - v \frac{\tau'}{b}\right)^2 + \tau'^2, \qquad \tau' = \frac{\tau}{1 + v^2/2}$$

$$R_{\min}^2 = \frac{b^2}{1+v^2}$$

Calculations show that just after collisions the relative velocity of m and M particles is small and we can use the nonrelativistic hoop conjecture

$$R_S(M) > R_{\min}(b) \qquad \boxed{b < b_*} \qquad b_*^2 = \frac{2Mp}{M_{Pl}^4} \quad \Rightarrow \quad \boxed{\sigma = \pi b_*^2}$$

Modified Thorn's conjecture for charged particles

- Start from the Reissner-Nortstrom
- Boost
- Shock wave
- Apply the previous formula

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2 d\Omega_{D-2}^2$$

$$g(R) = 1 - \left(\frac{R_S}{R}\right)^{D-3} + \frac{Q^2}{R^{2(D-3)}} \qquad Q^2 = \frac{8\pi G_D q^2}{(D-2)(D-3)}$$

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\, dU^2,$$

$$F(|X|) = \frac{a_D G_D p}{|X|^{D-4}} + \frac{b_D G_D p_Q}{|X|^{2D-7}}$$

D>4

Modified Thorn's conjecture for charged particles

- **Shock wave**
- **Apply the previous formula**

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\,dU^2,$$

$$F(|X|) = \frac{a_D G_D p}{|X|^{D-4}} + \frac{b_D G_D p_Q}{|X|^{2D-7}}$$

$$R_S(M) > R_{\min}(b)$$

$$b < b_*(p, p_Q)$$

$$b_{*p_Q}(p, p_Q) = b_*(1 + f\ p_Q^2),$$

$$f < 0$$

Cross section decreases

Modified Thorn's conjecture for charged dilaton

- **Start from charged dilaton metric**
- **Boost**
- **Shock wave**
- **Apply the previous formula**

$$ds^2 = -A^2(R)dt^2 + A^{-2}(R)dR^2 + K^2 d\Omega_2^2$$

$$ds^2 = -dUdV + dX^{i2} + F(X^i)\delta(U)\, dU^2,$$

D=4

$$F(|X|) = -8p\ln|X| + \frac{\alpha_{eff}}{|X|}$$

$$R - 2(\nabla\phi)^2 + e^{-2a\phi}F_2^2$$

$$\alpha_{eff} = \frac{3-4a^2}{2(1-a^2)}$$

$$b_{*\alpha_{eff}} = b_*(1 + f\,\alpha_{eff}),$$

$$f < 0$$

Cross section increases for alpha<0

Results

- Modified Thorn’s conjecture is confirmed for several examples;
- Results of trapped surface calculations are confirmed

Catalysts:

- A particular “dilaton” acts as a catalyst
- Lambda<0 acts as a catalyst

Technical details for (A)dS

- Shock waves;
- Geodesics in a spacetime with a shock wave

Technical details for (A)dS.

Shock waves from the Schwarzschild metric

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2 d\Omega_2^2$$

$$g(R) = 1 - \frac{R_S}{R} \mp \frac{a^2}{R^2}$$

- dS
+ AdS

Plane coordinates $\vec{Z} = \{Z^2, ..., Z^{D-1}\}, \; -UV + \vec{Z}^2 \pm Z^{D2} = \pm a^2,$

$$U = Z^0 + Z^1, \; V = -Z^0 + Z^1$$

$$ds^2 = ds^2{}_{(A)dS} + ds^2{}_{pert}$$

$$ds^2{}_{pert} = m^2 K(|\vec{Z}|)(K_{00}(Z_0, Z_D)dZ_0^2 + K_{DD}(Z_0, Z_D)dZ_D^2 + K_{0D}(Z_0, Z_D)dZ_D dZ_0)$$

Boost

$$Z_0 = \gamma(Y_0 + vY_1),$$
$$Z_1 = \gamma(vY_0 + Y_1),$$

$$m = \frac{p}{\gamma}$$

$$\vec{Z} = \vec{Y}, Z_D = Y_D, \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$m \to 0, \gamma \to \infty, \gamma - fixed$$

Technical details for (A)dS .

Shock waves from the Schwarzschild metric

$$ds^2 = ds^2{}_{(A)dS} + ds^2{}_{pert} \qquad m = \frac{p}{\gamma} \qquad m \to 0, \gamma \to \infty, \gamma - fixed$$

Lemma For an integrable function f takes place the identity

$$\lim_{v \to 1} \gamma f\left(\gamma^2 (Y_0 + vY_1)^2\right) = \delta(Y_0 + Y_1) \int f(x^2) dx$$

Proof

$$\gamma \int f\left(\gamma^2 (Y_0 + vY_1)^2\right) g(Y_0) dY_0$$

$$\gamma(Y_0 + vY_1) = x$$

$$\lim_{v \to 1} \int f\left(x^2\right) g(-vY_1 + \frac{1}{\gamma} x) dx = g(-Y_1) \int f\left(x^2\right) dx$$

Technical details for (A)dS .

Shock waves from the Schwarzschild metric

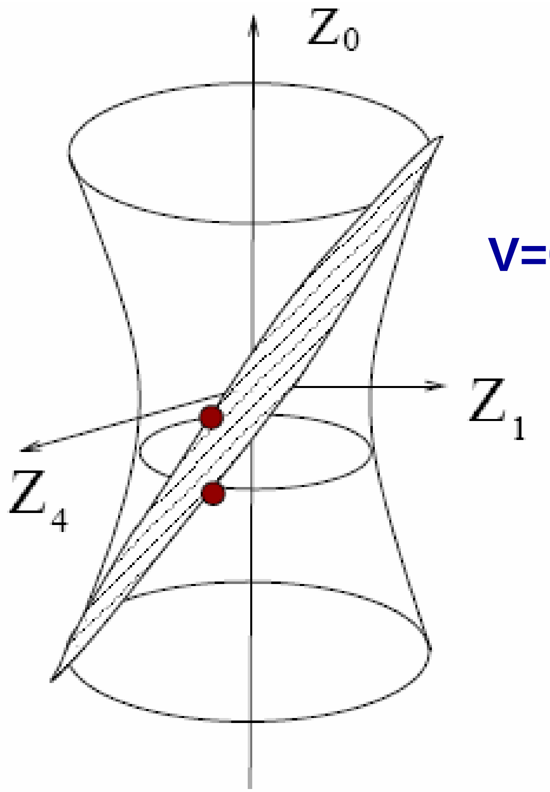
$$F(Z) = 2\,pa^2\,\boldsymbol{P} \int_{-\infty}^{\infty} \frac{(a^2(\pm Z^2 + x^2) + Z^2(x^2 \mp Z^2))}{(Z^2 \mp x^2)^2(\pm a^2 + x^2 \mp Z^2)^{\frac{D-1}{2}}}\,dx$$

$$F_{4,dS}(Z^4) = 4\ \mathrm{p}G_4 \left(-2 + \frac{Z^4}{a} \ln\left(\frac{1 + \frac{Z^4}{a}}{1 - \frac{Z^4}{a}}\right)\right)$$

$$F_{5,AdS}(Z^5) = \frac{3\pi\ \mathrm{p}G_5}{2a} \left(\frac{2\frac{Z^{5^2}}{a^2} - 1}{\sqrt{\frac{Z^{5^2}}{a^2} - 1}} - 2\frac{Z^5}{a}\right)$$

Shock wave in dS

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$



V=0

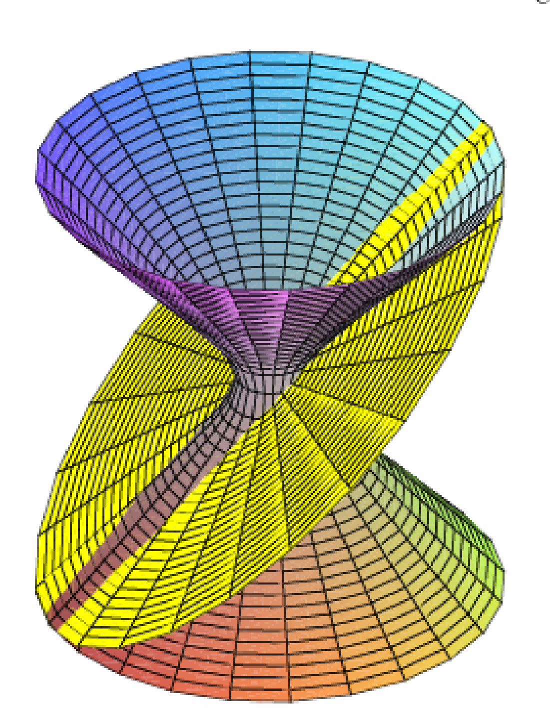


Figure 1: One shock wave in the de Sitter space presented as a hyperboloid is located on the intersection of the hyperboloid and the plane $Z_0 - Z_1 = 0. Z_2$ and Z_3 are suppressed At fixed "Z_0-time" this cross section consists of two points (small red ball in the left picture)

Two Shock waves in dS

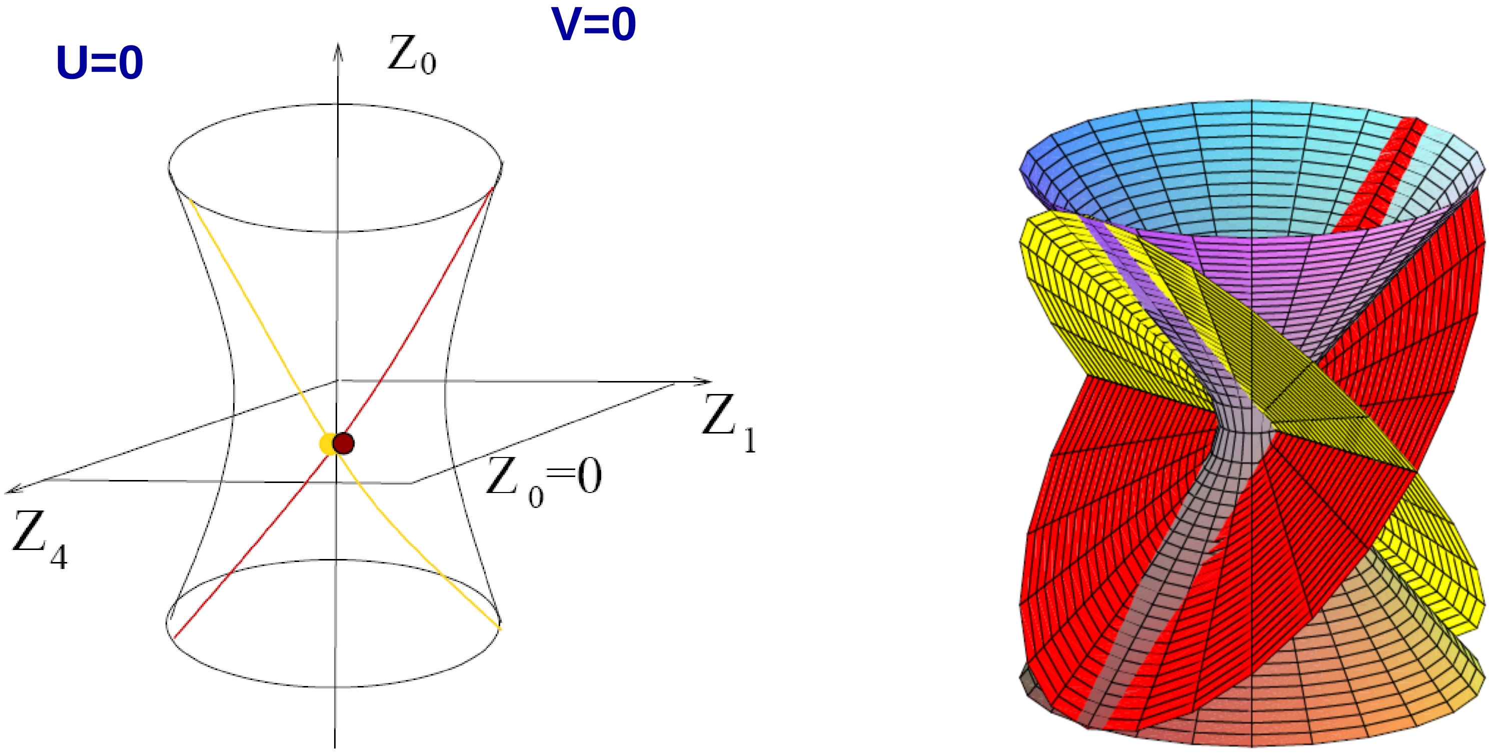


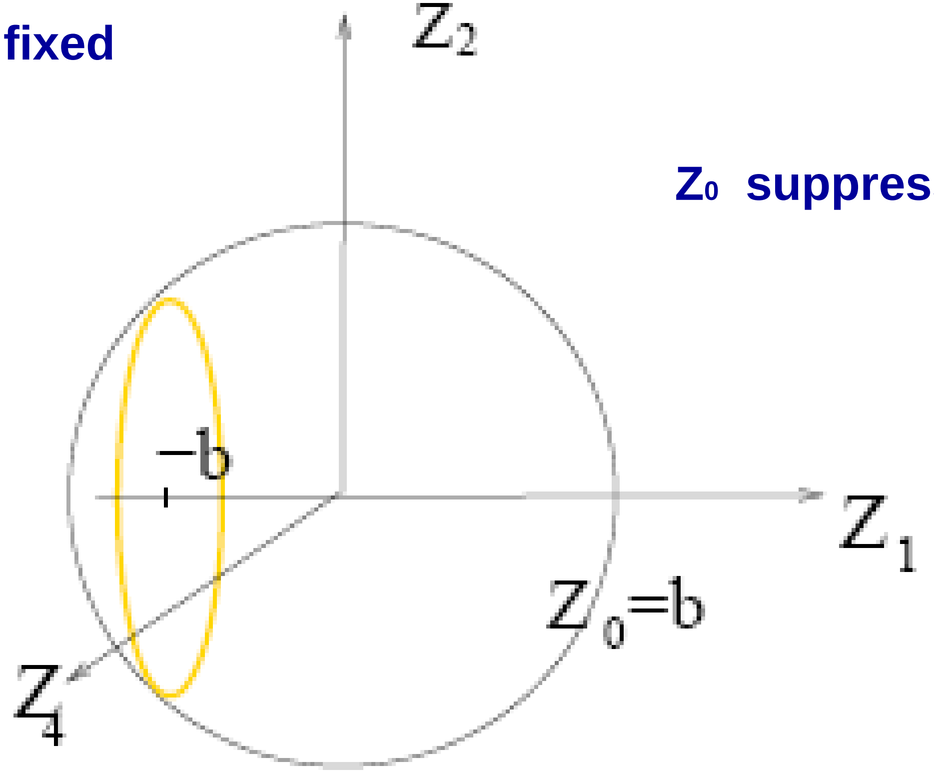
Figure 2: Two shock waves in the de Sitter space. A collision of two shock waves takes place at $Z_0 = 0$ and corresponds to a collision of red and yellow balls.

Shock wave in dS (nonexpanding shock waves).

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$

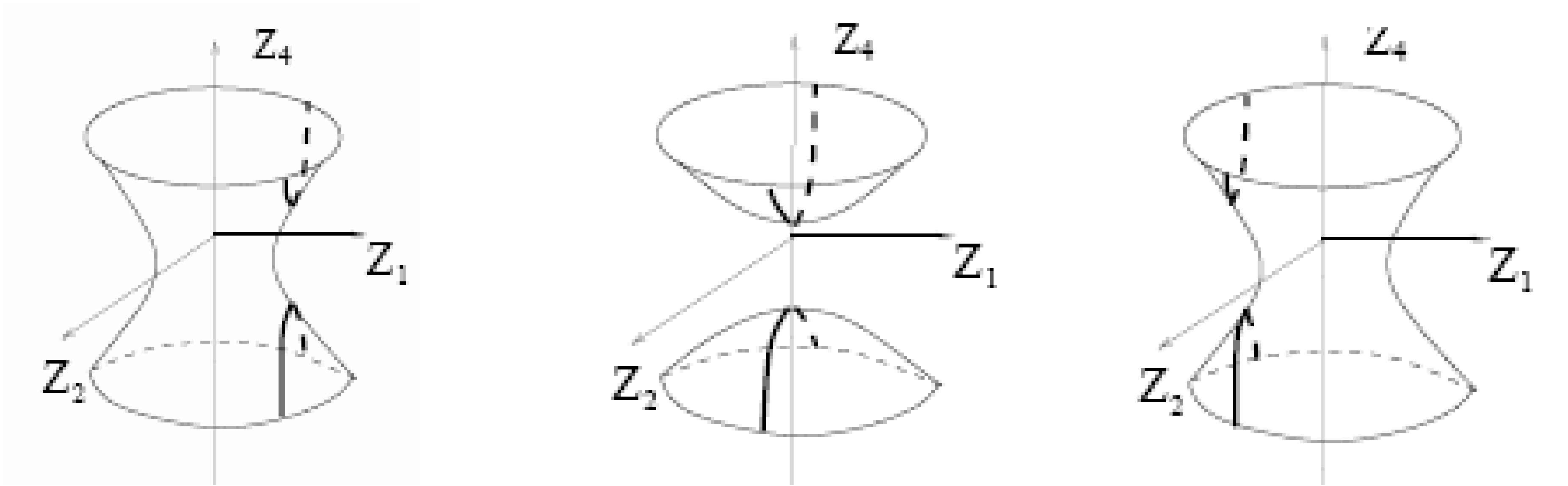
Shock wave $Z_1+Z_0=0$ → Z_1 fixed $\quad S^{D-1}$

Z_0 fixed



Carton for different Z_0 time

Shock wave in AdS



Geodesics in dS with a shock wave

$$\int d\tau\, [\frac{dX^M(\tau)}{d\tau} G_{MN}(X(\tau)) \frac{dX^N(\tau)}{d\tau} - \lambda \left(X^M(\tau) g_{MN} X^N(\tau) - a^2 \right)]$$

Analogy with L.D.Faddeev, Dokl.Acad.Nauk 210(1973)807

$$G_{MN}[X] = g_{MN} + h_{MN}[X]$$

$$g_{MN} \;=\; -\delta^U_M \delta^V_N + \delta^N_M \delta^i_N, \quad h_{MN}[U,V,X] = \delta^U_M \delta^U_N F(X)\delta(U)$$

$$ds^2_{\rm dS} = -dX_0^2 + \sum_{M=1}^{D} dX_M^2 \qquad\qquad a^2 = -X_0^2 + \sum_{M=1}^{D} X_M^2$$

$$ds^2 = -ds^2_{\rm dS} + F(X_i)\delta(U)\; dU^2, \qquad U \;=\; \frac{X_0 + X_1}{\sqrt{2}}, \;\; X_i = X_2, \dots X_D$$

$$V \;=\; \frac{X_0 - X_1}{\sqrt{2}},$$

Geodesics

$$\mathcal{S} = \int d\tau \, [\frac{dX^M(\tau)}{d\tau} G_{MN}(X(\tau)) \frac{dX^N(\tau)}{d\tau} - \lambda \left(X^M(\tau) g_{MN} X^N(\tau) - a^2 \right)]$$

$$\begin{aligned} G_{MN} \frac{d^2 X^N(\tau)}{d\tau^2} + G_{MN} \Gamma^N_{KL} \frac{dX^K(\tau)}{d\tau} \frac{dX^L(\tau)}{d\tau} + \lambda g_{MN} X^N(\tau) &= 0 \\ X^M(\tau) g_{MN} X^N(\tau) - a^2 &= 0 \end{aligned}$$

Geodesics

$$\begin{aligned} \ddot{U} &= 0 \\ \ddot{V} - \frac{1}{2}F\delta'(U)\dot{U}^2 - F_{,i}\delta(U)\dot{U}\dot{X}^i &= -\frac{1}{2a^2}(-F + X^i F_{,i})V\delta(U)\dot{U}^2 \\ \ddot{X}^i - \frac{1}{2}F_{,i}\delta(U)\dot{U}^2 &= -\frac{1}{2a^2}(-F + X^j F_{,j})X^i\delta(U)\dot{U}^2 \end{aligned}$$

$$\tau = U$$

$$\begin{aligned} U\delta(U) &= 0, \\ \theta(U)\delta(U) &= \frac{1}{2}\delta(U) \end{aligned}$$

Equations to solve:

$$\begin{aligned} \ddot{V} - \frac{1}{2}F\delta'(U) - F_{,i}\delta(U)\dot{X}^i &= -\frac{1}{2a^2}(-F + X^i F_{,i})V\delta(U) \\ \ddot{X}^i - \frac{1}{2}F_{,i}\delta(U) &= -\frac{1}{2a^2}(-F + X^j F_{,j})X^i\delta(U) \end{aligned}$$

Solutions

$$
\begin{aligned}
V(U) &= V_0 + V_1 U + V_f \theta(U) + V_d \theta(U) U \\
X^i(U) &= X_{i0} + X_{i1} U + X_{id} \theta(U) U
\end{aligned}
$$

where

$$
\begin{aligned}
V_f &= \frac{1}{2} F \\
V_d &= \frac{1}{2} F_{,i} X_{i1} + \frac{1}{2a^2}(F - X_{i0} F_{,i}) V_0 + \frac{1}{8} F_{,i}^2 + \frac{1}{8a^2}(F^2 - (X_{i0} F_{,i})^2) \\
X_{id} &= \frac{1}{2} F_{,i} + \frac{1}{2a^2}\left(F - X_{j0} F_{,j}\right) X_{i0}
\end{aligned}
$$

F_i *is a given function of* X_{i0}

Results (for the 3-d lecture)

- Modified Thorn's conjecture is confirmed for several examples;
- Results of trapped surface calculations are confirmed

Catalysts:

- **A particular "dilaton" acts as a catalyst**
- **Lambda<0 acts as a catalyst**