

# High Energy Scattering and Search for Extra Dimensions at the LHC

**I.Ya.Aref'eva**

***Steklov Mathematical Institute, Moscow***

# Outline

I  
N  
T  
R  
O  
D  
U  
C  
T  
I  
O  
N

- **Main tasks for LHC**

**Higgs, Susy, extra-dimensions**

- **Reasons to think about extra dimensions**

- Kaluza-Klein,
- Strings
- D-branes
- TeV-gravity scenario

- **Possible manifestations of Extra Dimensions**

- **KK modes**



- **Black Hole/Wormhole production**

- **Signs of strong quantum gravity**

# Possible manifestations of Extra Dimensions.

Black Hole production in particles collisions. **BHs in Quantum Gravity**

**<2 particles | BH >**

$$\langle h'', \varphi'', \Sigma'' | h', \varphi', \Sigma' \rangle = \int_{\text{Sum over topologies}} \exp\left\{\frac{i}{\hbar} S[g, \Phi]\right\} dg d\Phi,$$

$$\Sigma'' : h_{ij}'', \varphi'' \quad ; \quad \Sigma' : h_{ij}', \varphi',$$

$$g|_{\Sigma''} = h'', \Phi|_{\Sigma''} = \varphi''; \quad g|_{\Sigma'} = h', \Phi|_{\Sigma'} = \varphi'$$

Transplanckian energy  $M_{\text{Pl}, D} < E$

**Classical approximation**

# Possible manifestations of Extra Dimensions.

BH production in particles collisions. **BH formation as classical process.**

- **Gravitational collapse**

**Oppenheimer, Snyder, Phys.Rev.,1939**

**Thorn's hoop conjecture:** BH forms if the linear size of clumping matter  $l$  is comparable to the Schwarzschild radius

**$R_S$  of a BH of mass  $M_{BH}$**

$$R_S \sim \frac{M_{BH}}{M_{Pl}^2}$$

# Possible manifestations of Extra Dimensions.

BH production in particles collisions. **Modified hoop conjecture**

**Modified Thorn's hoop conjecture:**

BH forms if the impact parameter  $b$  is comparable to the Schwarzschild radius  $R_s$  of a BH of mass  $E$ .

$$R_s \sim \frac{M_{BH}}{M_{Pl}^2} \quad \longrightarrow \quad R_s \sim \frac{E}{M_{Pl}^2}$$

Modified Thorn's hoop conjecture gives **classical geometrical cross-section**

$$\sigma(1 + 1 \rightarrow \text{BH}) \sim \pi R_s^2 \sim \pi E^2$$

# Possible manifestations of Extra Dimensions.

## BH production in particles collisions. **BH formation as classical process.**

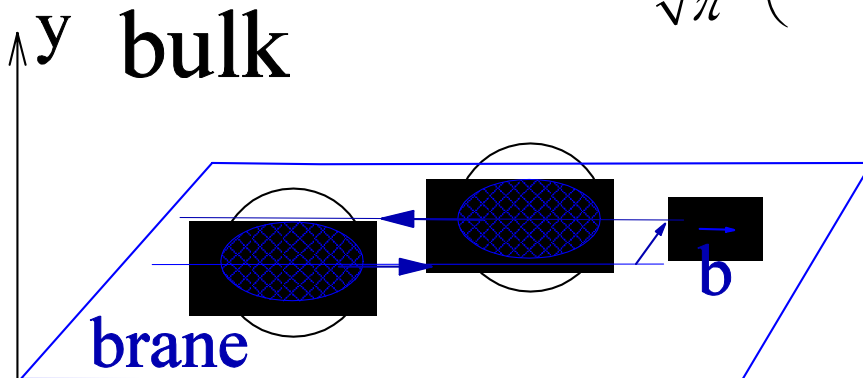
$$D > 4$$

$$R_S = \gamma(D) \frac{1}{M_D} \left( \frac{M_{BH}}{M_D} \right)^{\alpha(D)}$$

$$G_D = \frac{1}{M_D^{D-2}}$$

$$\gamma(D) = \frac{1}{\sqrt{\pi}} \left( \frac{8\Gamma(D-1/2)}{D-2} \right)^{1/(D-3)}$$

$$\alpha(D) = \frac{1}{D-3}$$



$$\sigma = \pi R_S^2(E) \square E^{2/D-3}$$

# Possible manifestations of Extra Dimensions.

BH production in particles collisions. **Modified hoop conjecture**

- Modified hoop conjecture has got support from trapped surface estimations

R.Penrose, unpublished, 1974  
D.M.Eardley and S.B. Giddings, 2002  
H.Yoshino and Y. Nambu, 2003  
S. B. Giddings and V. S. Rychkov, 2004

.....

# D-dim gravitational model of relativistic particles

---

$$R > R_s, \quad ds^2 = \left(1 - \left(\frac{R_s}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{R_s}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega_{D-2}^2$$

$R < R_s$

**Tolman-Florides interior incompressible perfect fluid solution**

or

**Static spherical symmetric solitonic solution of gravity-matter**

**E.O.M. (boson stars)**

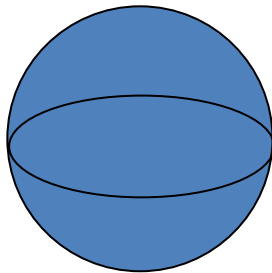


# Shock waves from the Schwarzschild metric

---

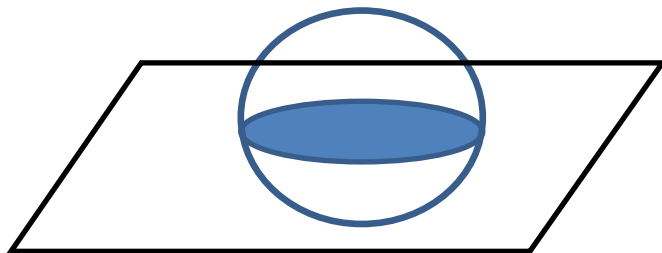
$$R > R_S, \quad ds^2 = \left(1 - \left(\frac{R_S}{R}\right)^{D-3}\right) dt^2 + \left(1 - \left(\frac{R_S}{R}\right)^{D-3}\right)^{-1} dR^2 + R^2 d\Omega_{D-2}^2$$

**Boost** → flattening of the Schwarzschild sphere



$$m = \frac{p}{\gamma}, \quad p - \text{fixed}$$

**Perfect fluid on the brane? Does not matter? All information about interior is erased?**



# Possible manifestations of Extra Dimensions.

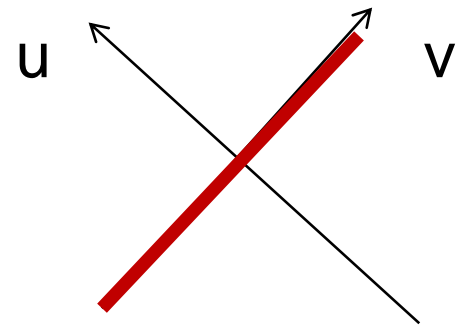
BH production in particles collisions. **Modified hoop conjecture**

---

Ultrarelativistic particle = **Aichelburg-Sexl** shock wave

$$ds^2 = -dudv + dx^{i2} + F(x^i)\delta(u) du^2,$$

$$F(x^i) = \frac{c}{\left| \sum x^{i2} \right|^{\frac{D-4}{2}}}$$



Smooth coordinates: P.D'Eath coordinates, Dray and 't Hooft

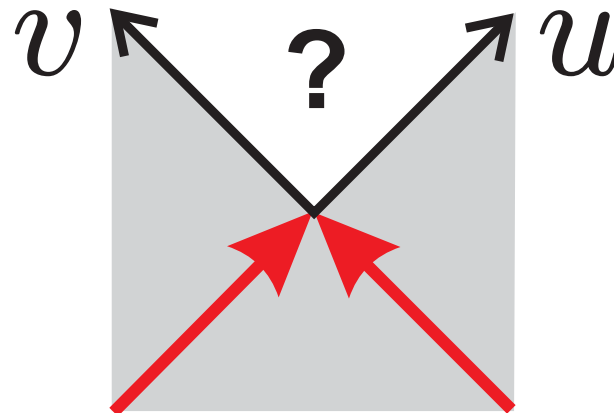
# Possible manifestations of Extra Dimensions.

## BH production in particles collisions. **Modified hoop conjecture**

**2 ultrarelativistic particles = 2 shock waves**

$$ds^2 = -dudv + dx^{i2} + F_1(x^i)(x^i)\delta(u) du^2 + F_2(x^i)\delta(v) dv^2,$$

$$F_k(x^i) = \frac{C_k}{\left| \sum (x^i - x_{k0}^i)^2 \right|^{\frac{D-4}{2}}}$$



# Possible manifestations of Extra Dimensions.

## BH production in particles collisions. Trapped surface formation

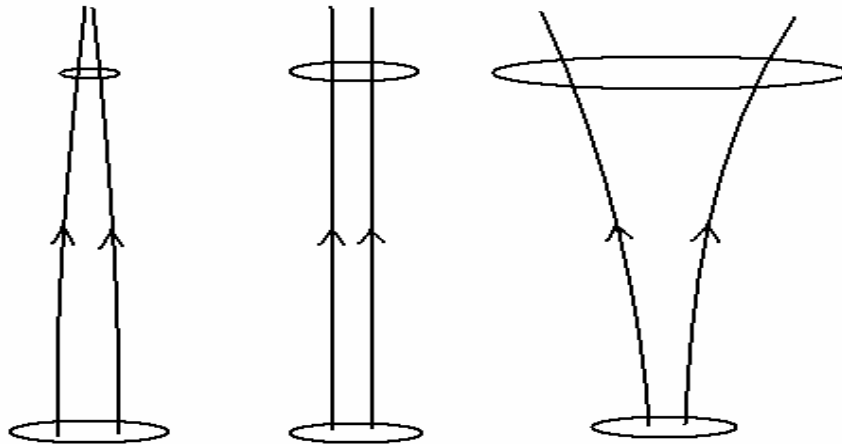
---

- **Black hole formation = Trapped surface formation**

**Theorem. (Hawking-Penrose)** A spacetime  $(M; g)$  with a complete future null infinity which contains a **closed trapped surface** must contain a future **event horizon**, the interior of which contains the trapped surface

# Trapped Surface

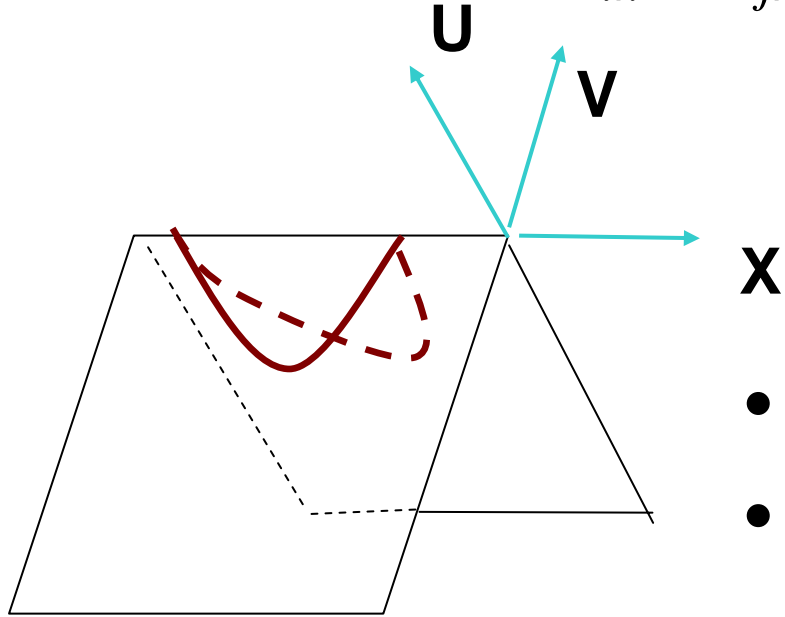
- A **trapped surface** is a two dimensional spacelike surface whose two null normals have **negative expansion** (=Neighbouring light rays, normal to the surface, **must** move towards one another)



# Trapped surface formation for 2 shock waves

Eardley, Giddings; Kang, Nastase,....

$$ds^2 = -dUdV + [H_{ik}^{(1)} H_{jk}^{(1)} + H_{ik}^{(2)} H_{jk}^{(2)} - \delta_{ij}] dX^i dX^j,$$



$$H^{(1)}_{ij} = \delta_{ij} + \frac{1}{2} \nabla_i \nabla_j F_1 u \theta(u)$$

- $\Psi_{1,2} > 0, X \in D, \Psi_{1,2} = 0, X \in \partial D$
- $\nabla^2 \Psi_{1,2} = \delta^{(D-2)}(X - X_{(1,2)}), X \in D,$

*the outer null normals have zero convergence*

- $\nabla \Psi_1 \cdot \nabla \Psi_2 = 4, X \in \partial D$

*no  $\delta$  - function in convergence*

# Possible manifestations of Extra Dimensions. BH production in particles collisions. Trapped surface formation



## Conclusion:

Estimations of the trapped surface formation support  
**the modified hoop conjecture**

# Possible manifestations of Extra Dimensions.

## BH production in particles collisions. **Cross-section**

**Numbers ?**

### Classical geometric cross-section

$$b \leq r_S \approx \text{const } E^{1/D-3}$$

$$D = 6$$

$$\sigma \approx 1 \text{ nb}$$

$$t\bar{t} \approx 1 \text{ nb}$$

Rough estimation:  $\sigma \approx \frac{1}{(\text{TeV})^2} = 400 \text{ pb}$



# Possible manifestations of Extra Dimensions.

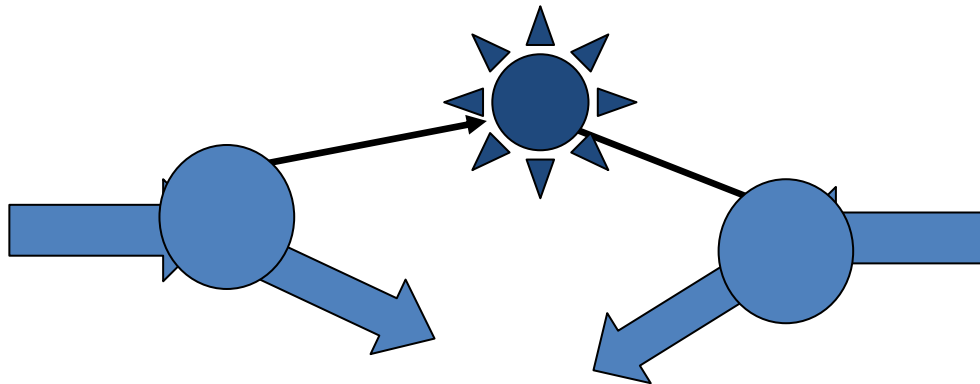
## BH production in particles collisions **and** Parton Structure

$$\sigma_{pp \rightarrow BH} \propto \sum_{ij} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} f_i(x) f_j(\tau/x) \sigma_{ij \rightarrow BH}(\tau S)$$

$S$  – the square of energy (in c.m.)

$x, \tau/x$  are the parton momentum fractions  
 $f_i$  the parton distribution functions

$\tau_m = M_{\min}^2 / S$ ,  $M_{\min}^2$  – *minimum mass*



**NUMBERS are small!**

# Possible manifestations of Extra Dimensions.

## Search for Catalysis of BH Production

---

- **Motivation:** small cross-section of micro BH production at the LHC
- **Question:** what we can do to increase cross-section
- **Answer:** modify the conditions of collisions

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = G_{\mu\nu}, \quad G_{\mu\nu} = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{(catalyst)})$$

# Possible manifestations of Extra Dimensions.

## Search for Catalysis of BH Production

---

- find effects related with nontrivial dynamics of 3-brane embedding in D-dim spacetime.
- Change the background (4-dim/D-dim), in particular, we can add the cosmological constant (AdS/dS)
- D-dim tail of 4-dim particles made from closed string excitations

# Possible manifestations of Extra Dimensions.

## Search for Catalysis of BH Production

---

### Framework of a search for a catalyst

- D-dim gravitational model of relativistic particles
- D-dim gravitational model of particles on brane with “tails” in the bulk.
- Shock-waves as an approx. for ultrarelativistic particles.
- “dilaton” shock-wave

# Possible manifestations of Extra Dimensions.

## Search for Catalysis of BH Production. Lambda Modifications

### Lambda

#### Modifications:

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{(\text{catalyst})})$$

$$T_{\mu\nu}^{(\text{catalyst})} \approx \Lambda g_{\mu\nu} \quad \Lambda > 0$$

$$\Lambda < 0$$

I.A., A.Bagrov,  
E.Guseva,  
0905.1087

Nastase;  
Gubser, Pufu,  
Yarom, 2008,2009;  
Alvarez-Gaume,  
Gomes,... 2008;  
Shuryak 2009,

### Colliding objects - shock waves with e,s,...

In flat space with charge: Yoshino, Mann 06;  
Spin - Yoshino, Zelnikov, V.Frolov 07.

With charge and Lambda: 0909.1292  
I.A., A.Bagrov, L.Joukowskaya