ELECTROMAGNETIC STRUCTURE OF CHARMED BARYONS

K. Utku CAN[1,2], G. ERKOL[2], B. Isildak[2], M. Oka[1], T. T. Takahashi[3]
[1] Department of Physics, H-27, Tokyo Institute of Technology, Meguro, Tokyo 152-8551 Japan
[2] Department of Natural and Mathematical Sciences, Faculty of Engineering, Ozyegin University, 34794 Cekmekoy, Istanbul Turkey
[3] Gunma National College of Technology, Maebashi, Gunma 371-8530, Japan

① Introduction

Electromagnetic form factors are good instruments to probe the internal structure of the hadrons and extract information about their sizes and shapes. In the non-perturbative regime, it is a challenge to determine the form factors from the quark-gluon degrees of freedom. Lattice QCD provides an *ab initio* scheme to perform such calculations. In the case of spectrum and form factors of the hadrons, previous years mostly saw the application of this method to the light-quark sector; which also has rich experimental results to compare with. However, one can question how the structure of the hadrons gets modified in the heavy-quark regime, like in the case of charmed hadrons. Unlike the light baryons, only the spectrum of the charmed baryons are accessible by the experiments for the time being. Future charm factories like BES-III and PANDA at GSI are expected to probe the charm sector. In this work, we revise our previous Ξ_{cc} calculation [1] and extend it to cover the Σ_c, Ω_c and Ω_{cc} baryons and calculate their electromagnetic form factors, as well as their electric/magnetic charge radii and magnetic moments in the framework of Lattice QCD.

② Baryon Form Factors and Lattice Formulation

Matrix element defining the vector current interaction of a spin-1/2 baryon can be parameterised as follows

$$\langle \mathcal{B}(p)|V_\mu|\mathcal{B}(p')\rangle = \bar{u}(p)[\gamma_\mu F_{1,\mathcal{B}}(q^2) + i(\sigma_{\mu\nu}q^\nu/2m_\mathcal{B})F_{2,\mathcal{B}}(q^2)]u(p),$$

where $q_\mu = p'_\mu - p_\mu$ is the transferred four-momentum. Here $u(p)$ denotes the Dirac spinor for the baryon with four-momentum p^μ and mass $m_\mathcal{B}$. The Dirac, $F_{1,\mathcal{B}}(q^2)$, and Pauli, $F_{2,\mathcal{B}}(q^2)$, form factors are related to the Sachs electric and magnetic form factors by the relations

$$G_{E,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + (q^2/4m_\mathcal{B}^2)F_{2,\mathcal{B}}(q^2), \quad G_{M,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + F_{2,\mathcal{B}}(q^2).$$

One can extract the Sachs form factors in the large Euclidean time limit by calculating the following ratio

$$R(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma};\mu) = \frac{\langle F^{\mathcal{B}\mathcal{V}_\mu\mathcal{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma})\rangle}{\langle F^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle F^{\mathcal{B}\mathcal{B}}(t_2-t_1;\mathbf{p};\Gamma_4)\rangle\langle F^{\mathcal{B}\mathcal{B}}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle F^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle F^{\mathcal{B}\mathcal{B}}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle\langle F^{\mathcal{B}\mathcal{B}}(t_1;\mathbf{p};\Gamma_4)\rangle\langle F^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}, \quad (1)$$

where $\langle F^{\mathcal{B}\mathcal{B}}(t;\mathbf{p};\Gamma_4)\rangle$ and $\langle F^{\mathcal{B}\mathcal{V}_\mu\mathcal{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma})\rangle$ are the baryonic two-point and three-point correlation functions with t_1 and t_2 being the current insertion and the sink particle's time slice respectively. Γ matrices are defined as $\Gamma_i = \gamma_i\gamma_5\Gamma_4$ and $\Gamma_4 \equiv (1+\gamma_4)/2$. When $t_2 - t_1$ and $t_1 \gg a$, the ratio in Eq. (1) reduces to the desired form $R(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) \xrightarrow[t_2-t_1\gg a]{t_1\gg a} \Pi(\mathbf{p}',\mathbf{p};\Gamma;\mu)$. We extract the form factors $G_{E,\mathcal{B}}(q^2)$ and $G_{M,\mathcal{B}}(q^2)$ by choosing appropriate combinations of Lorentz direction μ and projection matrices Γ:

$$\Pi(\mathbf{0},-\mathbf{q};\Gamma_4;\mu=4) = \left[\frac{(E_\mathcal{B}+m_\mathcal{B})}{2E_\mathcal{B}}\right]^{1/2}\mathbf{G_{E,\mathcal{B}}(q^2)}, \quad \Pi(\mathbf{0},-\mathbf{q};\Gamma_j;\mu=i) = \left[\frac{1}{2E_\mathcal{B}(E_\mathcal{B}+m_\mathcal{B})}\right]^{1/2}\epsilon_{ijk}\,q_k\,\mathbf{G_{M,\mathcal{B}}(q^2)}.$$

Charge radii of the baryons can be extracted from the slope of the form factors at zero momentum transfer,

$$\langle r^2_{E,M}\rangle = -\frac{6}{G_{\mathrm{E,M}}(0)}\frac{d}{dQ^2}G_{E,M}(Q^2)\Big|_{Q^2=0}.$$

We choose to model the form factors by a dipole form, $G_{E,M}(Q^2) = G_{E,M}(0)/(1+Q^2/\Lambda^2_{E,M})^2$, and thus evaluate the charge radii as, $\langle r^2_{E,M}\rangle = 12/\Lambda^2_{E,M}$. Magnetic moments are obtained from the magnetic form factor at zero momentum transfer by evaluating,

$$\mu_\mathcal{B} = G_M(0)\,(e/2m_\mathcal{B}) = G_M(0)\,(m_N/m_\mathcal{B})\,\mu_N.$$

③ Lattice Setup

- PCAS-CS generated $32^3\times 64$, $\beta = 1.9$, 2+1-flavor configurations [2],
- $a = 0.0907(13)$ fm, $a^{-1} = 2.176(31)$ GeV,
- Clover action valance quarks, $c_E = c_B = 1/(u_0)^3$,
- $\kappa_{u,d} = (0.13700, 0.13727, 0.13754, 0.13770)$
  $m_\pi \approx (700, 570, 410, 300)$
  $\kappa_s = 0.13640$, physical point,
- $\kappa_c = 0.1246$, tuned to 1S spin-averaged $M^{exp}_{\eta_c-J/\psi}$, M^{exp}_{D-D*}, $M^{exp}_{D_s-D_s^*}$,
- Shell source - Wall sink pairs, t=12a seperation.
- Number of configurations per $\kappa_{u,d}$

$\kappa_{u,d}$	0.13700	0.13727	0.13754	0.13770
(Σ_c, Ξ_{cc})	100	100	150	170
(Ω_c, Ω_{cc})	100	100	100	130

④ Results

Plateau, form factor and chiral fit plots are shown at the Plots section. Compiled physical point results are given below.

Table 1: Physical point results for Σ_c, Ω_c, Ξ_{cc}, Ω_{cc} baryons. Charge radii and magnetic moments are given in units of [fm^2] and nuclear magnetons, [μ_N], respectively. Chiral extrapolations of charge radii are better modelled by a quadratic, $a + b\,m_\pi^2 + c\,(m_\pi^2)^2$, form whereas magnetic moments fit to a linear, $a + b\,m_\pi^2$, form better.

	Lin. Fit	Quad. Fit
$\langle r^2_{E,\Sigma_c^{++}}\rangle$	0.192(22)	0.234(37)
$\langle r^2_{E,\Xi_{cc}^{++}}\rangle$	0.136(8)	0.165(12)
$\langle r^2_{E,\Omega_{cc}^{+}}\rangle$	0.032(6)	0.043(11)
$\langle r^2_{M,\Sigma_c^{++}}\rangle$	0.410(81)	0.696(153)
$\langle r^2_{M,\Sigma_c^{0}}\rangle$	0.377(75)	0.650(126)
$\langle r^2_{M,\Omega_c^{0}}\rangle$	0.297(33)	0.354(54)
$\langle r^2_{M,\Xi_{cc}^{+}}\rangle$	0.135(10)	0.154(19)
$\langle r^2_{M,\Omega_{cc}^{+}}\rangle$	0.135(11)	0.148(21)
$\mu_{\Sigma_c^{++}}$	1.569(253)	2.220(505)
$\mu_{\Sigma_c^{0}}$	-0.852(133)	-1.073(269)
$\mu_{\Omega_c^{0}}$	-0.608(45)	-0.639(88)
$\mu_{\Xi_{cc}^{+}}$	0.411(15)	0.425(29)
$\mu_{\Omega_{cc}^{+}}$	0.405(13)	0.413(24)
$\langle r^2_{E,p}\rangle$	PDG Value:	0.770
$\langle r^2_{M,p}\rangle$	PDG Value:	0.604
$\langle r^2_{M,n}\rangle$	PDG Value:	0.862
μ_p	PDG Value:	2.793
μ_n	PDG Value:	-1.913

In Ref.[3] it is shown that the charm quark contributions are much smaller than the light quark's, which drive the hadrons to be more compact compared to the PDG values.

⑤ Conclusions

We have calculated the electromagnetic form factors of the Σ_c, Ξ_{cc}, Ω_c and Ω_{cc} baryons up to $\sim 1.6\,\mathrm{GeV}^2$ on 2 + 1-flavor lattices and as a by product extracted the electric and magnetic charge radii and the magnetic moments. Comparison with nucleon data shows that charmed hadrons' observables are smaller. This is believed to be due to the valance charm quarks. Simulations on *physical-point* configurations are ongoing.

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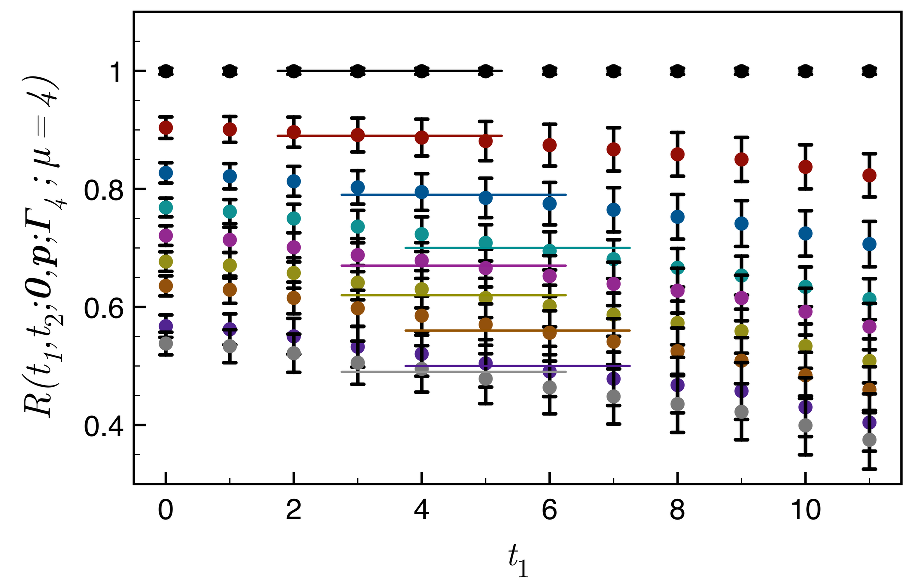
Plots

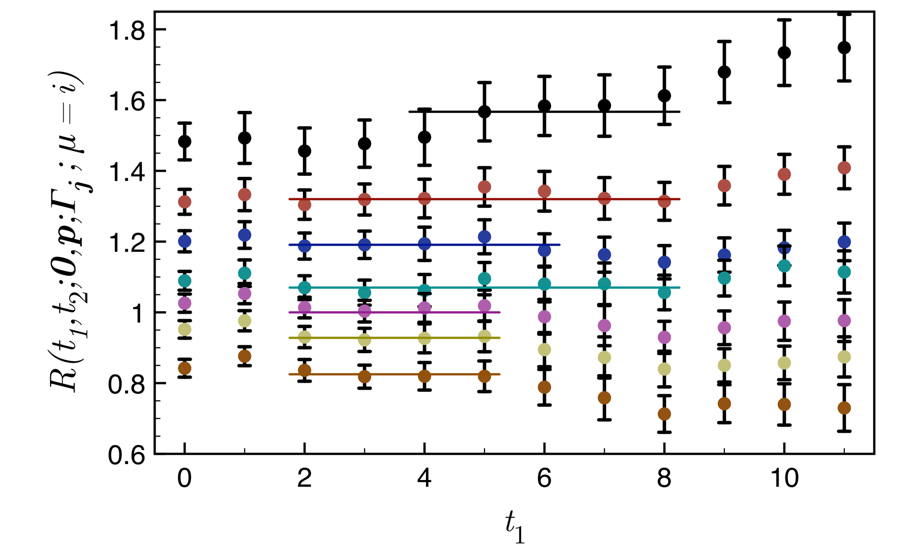
Upper two plots show the Euclidean time dependence of the ratio given in Eq.(1) for up to 9 (7) units of momentum insertions for electric (magnetic) data. Horizontal lines show our choice of fit regions determined by a p-value analysis. Plateaux are given for the $\kappa_{u,d} = 0.13770$ ($m_\pi \approx 300$ MeV) lattices.

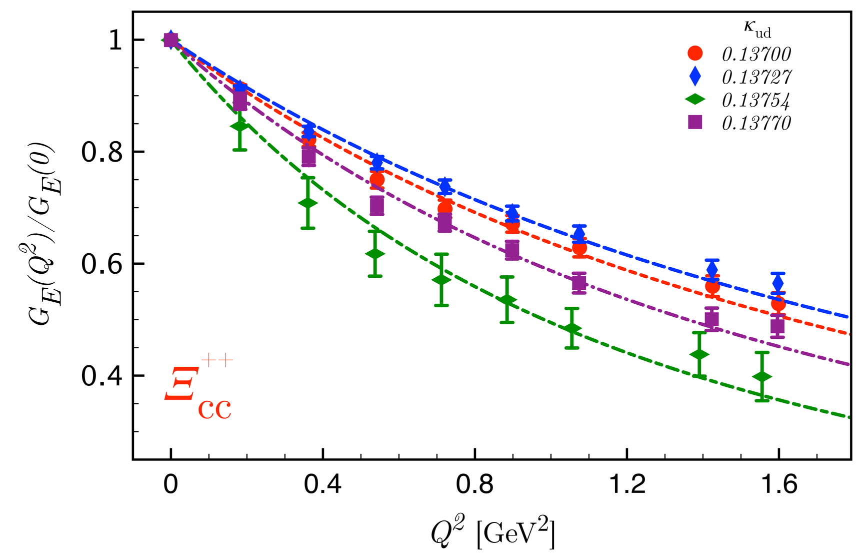
Middle plots show the form factors obtained on all sets of configurations.

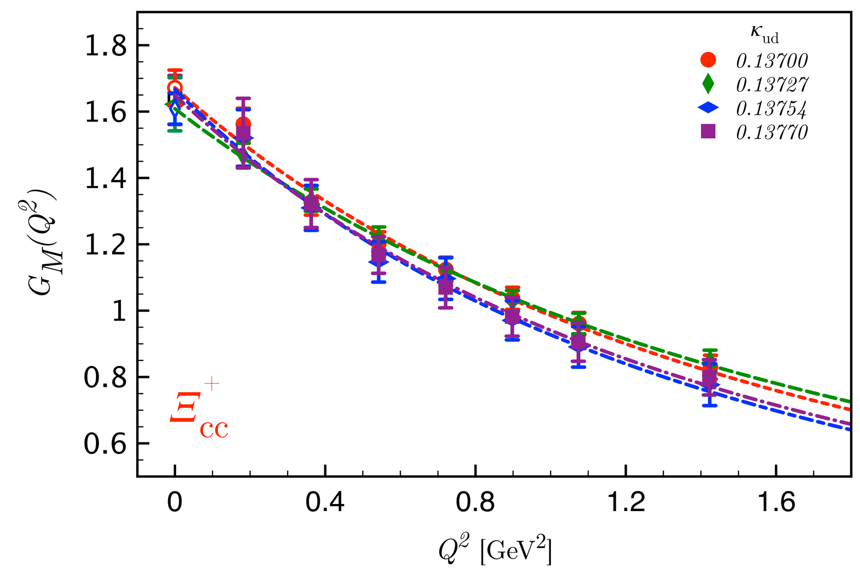
Extrapolations to the physical point for electric/magnetic charge radii and magnetic moments are given at the lower three plots.

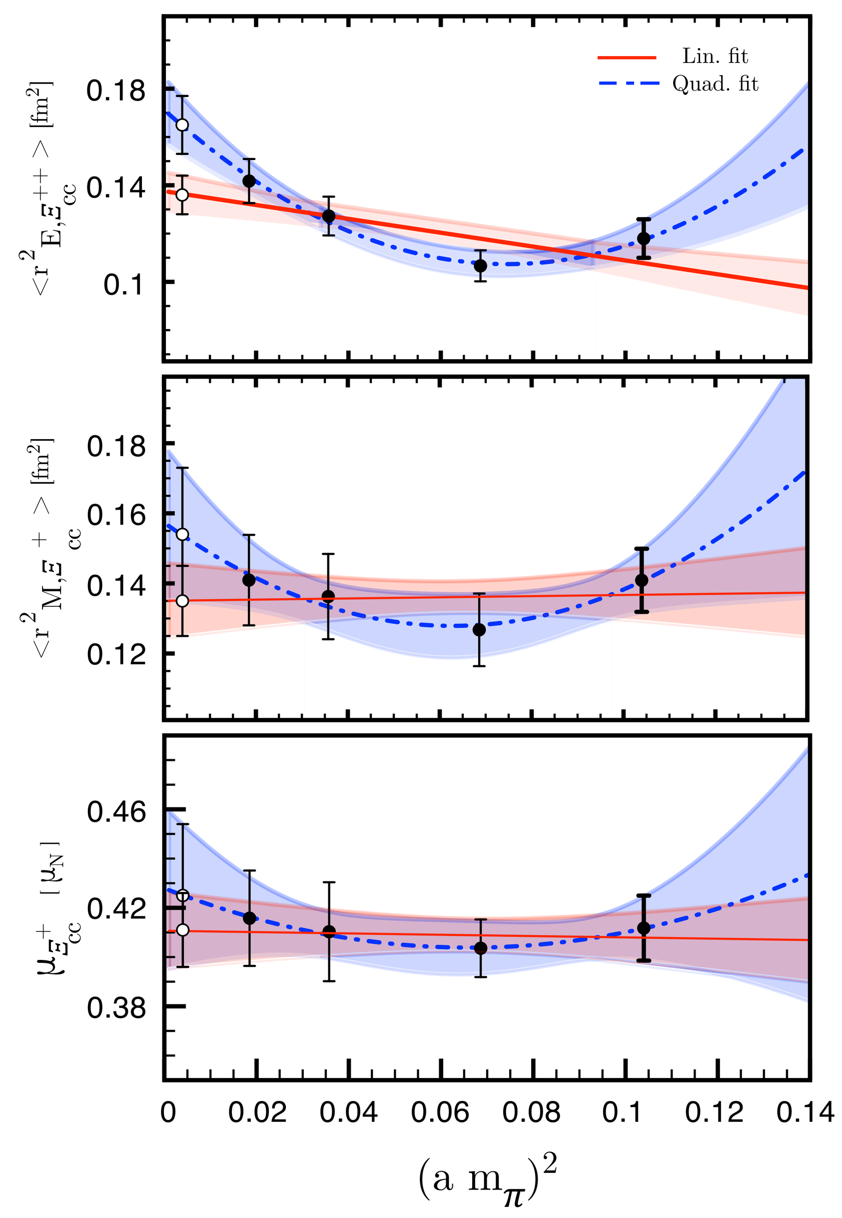
Only the Ξ_{cc} figures are given for illustrative purposes. Σ_c, Ω_c, Ω_{cc} plots can be found in Ref.[3].











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